Global Masters in Parallel Condensation of Eigenvalue Problems

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Abstract

In the dynamic analysis of structures condensation methods are often used to reduce the number of degrees of freedom to manageable size. Substructuring and choosing the master variables as the degrees of freedom on the interfaces of the substructures yields data structures which are well suited to be implemented on parallel computers. This paper discusses a parallel condensation method in the presence of generalized global masters which are obtained in reanalysis or from prolongation of coarse grid approximations, e.g.

Keywords: generalized eigenvalue problem, condensation, parallel method, global masters

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1 Introduction

In the analysis of the dynamic response of structures using finite element methods very often prohibitively many degrees of freedom are needed to model the behaviour of the system sufficiently accurate. Static condensation is frequently employed to economize the computation of a selected group of eigenvalues and eigenvectors. These methods choose from the degrees of freedom a small number of master variables which appear to be representative for the dynamic behaviour of the structure.

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Neglecting inertia terms the remaining variables (termed slaves) are eliminated leaving a much smaller problem for the master variables only.

The condensation process requires the factorization of the restriction of the stiffness matrix to the slave degrees of freedom. This can be done efficiently by choosing the master variables appropriately. Partitioning the structure under consideration into substructures and choosing the degrees of freedom on the interfaces of the substructures as masters and the interior knots of the substructures as slaves the matrix which has to be factorized becomes block diagonal. At the same time one obtains data structures and formulae which are well suited to be implemented on parallel computers. Taking advantage of these properties Rothe and the third author obtained a fully parallel condensation method for generalized eigenvalue problems (cf. [8]).

It has frequently been noted in the literature that the quality of the eigenvalue and eigenvector approximations produced by static condensation is satisfactory only for a very small part of the lower end of the spectrum. Several attempts have been made to improve the approximation properties, most of them being very time consuming since every wanted eigenvalue has to be corrected individually by an iterative process (cf. [3], [4], [5],[10], e.g.). In [7] Mackens and the third author introduced general masters which allow to implement a priori information of the eigenmodes (such as eigenmodes of similar structures considered in reanalysis or prolongations of eigenvector approximations obtained on a coarser grid) into the condensation process, thus enhancing the approximation properties without an iterative process.

In [6] it was shown that the parallelization concept of [8] can be generalized to the case of general masters with local support (i.e. the support of each general master is contained in a single substructure). Approximate eigenvectors in general will not have this property but its support usually will be the entire structure. This type of masters is called global. One way of applying the parallelization concept from [6] in the presence of global masters is to split it onto the substructures under consideration as was done in [6]. The disadvantage of this approach is that for each approximate eigenvector the number of additional masters is increased by the number of substructures. In this paper we present a parallel condensation method without splitting generalized masters which have a global support.

Our paper is organized as follows. In Section 2 we briefly sketch nodal condensation and substructuring, Section 3 introduces condensation with general masters, and Section 4 contains the derivation of the parallelization concept of condensation in the presence of global masters. In Section 5 we summarize our parallelization approach, and Section 6 demonstrates the favourable properties of the method by a numerical example. We consider the finite element model of a container ship with 35262 degrees of freedom where we take into account hydrodynamic masses. As general masters we use a few dry eigenmodes of the container ship, i.e. eigenmodes of the FE model without hydrodynamic masses.

2 Nodal Condensation and Substructuring

We consider the general eigenvalue problem

$$Kx = \lambda Mx \tag{1}$$

where $K \in \mathbb{R}^{(n,n)}$ and $M \in \mathbb{R}^{(n,n)}$ are symmetric and positive definite matrices which are usually the stiffness and mass matrix of a finite element model of a structure, respectively. The dimension n is supposed to be very large.

To solve this problem it is necessary to reduce the system to manageable size. In this section we consider the reduction of the dimension by nodal condensation. To this end we assume that (1) is decomposed and ordered into the block form

$$\begin{pmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{pmatrix} \begin{pmatrix} x_m \\ x_s \end{pmatrix} = \lambda \begin{pmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{pmatrix} \begin{pmatrix} x_m \\ x_s \end{pmatrix}, \tag{2}$$

where $x_m \in \mathbb{R}^m$ and $x_s \in \mathbb{R}^s$ indicate the master part and the slave part, respectively, with $m \ll s < n$.

Neglecting inertia terms in the second equation, solving for x_s, and substituting x_s into the first equation one obtains the statically condensed problem

$$K_0 x_m = \lambda M_0 x_m \tag{3}$$

where

$$\begin{array}{rcl} K_0 & := & K_{mm} - K_{ms}K_{ss}^{-1}K_{sm}, \\ M_0 & := & M_{mm} - K_{ms}K_{ss}^{-1}M_{sm} - M_{ms}K_{ss}^{-1}K_{sm} + K_{ms}K_{ss}^{-1}M_{ss}K_{ss}^{-1}K_{sm} \end{array} \tag{4}$$

which was introduced by Guyan [1] and Irons [2].

The condensation can be performed completely in parallel if the slave variables can be chosen such that the matrices K_{ss} and M_{ss} are block diagonal (cf. [8]). Suppose that r substructures are considered and that they connect to each other through the master variables on the interfaces only. If the slave variables are numbered appropriately, then the stiffness matrix is given by

$$K = \begin{pmatrix} K_{mm} & K_{ms1} & K_{ms2} & \dots & K_{msr} \\ K_{sm1} & K_{ss1} & 0 & \dots & 0 \\ K_{sm2} & 0 & K_{ss2} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ K_{smr} & 0 & 0 & \dots & K_{ssr} \end{pmatrix},$$

and the mass matrix M has the same block form.

It is easily seen that in this case

$$K_0 = K_{mm} - \sum_{j=1}^{r} K_{mmj} := K_{mm} - \sum_{j=1}^{r} K_{msj}K_{ssj}^{-1}K_{smj}$$

and

$$M_0 = M_{mm} - \sum_{j=1}^{r} M_{mmj},$$

where

$$M_{mmj} := K_{msj}K_{ssj}^{-1}M_{smj} + M_{msj}K_{ssj}^{-1}K_{smj} - K_{msj}K_{ssj}^{-1}M_{ssj}K_{ssj}^{-1}K_{smj}.$$

Hence, taking advantage of the blockstructure of K and M the reduced matrices K_0 and M_0 can be calculated substructurewise, and therefore, completely in parallel.

3 Condensation with General Masters

The nodal condensation has the disadvantage that it produces accurate results only for a small part of the lower end of the spectrum. In [7] general masters were considered in order to introduce preinformation about the modes of the structure into the condensation process.

In the previous section we split a vector x into a master part x_m and a slave part x_s by the equation $x = \binom{x_m}{x_s}$. This idea can be generalized by splitting x into a master and a slave part belonging to orthogonal subspaces.

Let a basis $z_1, \ldots, z_m$ of the space of master vectors be given, and complement it by $n-m$ linearly independent vectors $y_{m+1}, \ldots, y_n$ which are orthogonal to $z_1, \ldots, z_m$. We define $Z := (z_1, \ldots, z_m) \in \mathbb{R}^{(n,m)}$ and $Y := (y_{m+1}, \ldots, y_n) \in \mathbb{R}^{(n,n-m)}$. Then $x \in \mathbb{R}^n$ has the unique representation

$$x = Zx_m + Yx_s.$$

If we insert this into the original problem (1) and premultiply it by $(Z, Y)^T$ we obtain the following eigenvalue problem

$$\begin{pmatrix} K_{zz} & K_{zy} \\ K_{yz} & K_{yy} \end{pmatrix} \begin{pmatrix} x_m \\ x_s \end{pmatrix} = \lambda \begin{pmatrix} M_{zz} & M_{zy} \\ M_{yz} & M_{yy} \end{pmatrix} \begin{pmatrix} x_m \\ x_s \end{pmatrix} \tag{5}$$

where

$$L_{zz} := Z^T L Z,\ L_{zy} := Z^T L Y =: L_{yz}^T,\ L_{yy} := Y^T L Y,\ L \in \{K, M\}. \tag{6}$$

Equation (5) is of the same structure as equation (2). Hence, in principle it could be employed to reduce the eigenvalue problem (1) using $\{z_1, \ldots, z_m\}$ as master degrees of freedom in a similar way as in nodal condensation yielding the reduced problem (3), (4). However, since in practice only the small set of masters is available, but the large set of slave vectors $\{y_{m+1}, \ldots, y_n\}$ is definitely not the matrices $K_{zy}, K_{yy}, M_{zy}, M_{yy}$ are usually not at hand (Note that in our numerical example in

Section 6 we consider a finite element model of a container ship with $n = 35262$ degrees of freedom. This model is reduced to $m = 2097$ master unknowns. In this case we would have to determine $s = 33165$ slave vectors in $\mathbb{R}^{35262}$ by a Gram-Schmidt process if we wanted to apply Guyan reduction to problem (5) directly).

Hence, the straightforward transfer of Guyan's method to perform the reduction in the presence of general masters does not apply. In [7] two methods were given to generate the condensed problem corresponding to the decomposition (5) with the basis $z_1, \ldots, z_m$ only, but not the complementary vectors $y_{m+1}, \ldots, y_n$.

Theorem 1 *Let $Z = (z_1, \ldots, z_m) \in I\!R^{(n,m)}$ have maximal rank. Then the condensed eigenproblem with general masters $z_1, \ldots, z_m$ is given by*

$$P^T K P x_m = \lambda P^T M P x_m \tag{7}$$

with the projection matrix

$$P = K^{-1} Z \left(Z^T K^{-1} Z \right)^{-1} Z^T Z. \tag{8}$$

Since $\left(Z^T K^{-1} Z \right)^{-1} Z^T Z \in \mathbb{R}^{(m,m)}$ is a nonsingular matrix the condensed problem is equivalent to the projection of problem (1) to the space spanned by the columns of $K^{-1}Z$. Hence, equation (8) demonstrates that condensation is nothing else but one step of simultaneous inverse iteration with initial space $X = M^{-1}Z \in \mathbb{R}^{(n,m)}$. In particular, nodal condensation corresponds to inverse iteration with initial guess $Z = I_m$. We can expect improved approximation properties of condensation if we include general masters $z_j := Mx_j$ where x_j are approximate eigenvectors of problem (1) corresponding to the desired eigenvalues.

The reduced problem (7), (8) is equivalent to the projected eigenproblem with projection matrix $\tilde{P} := K^{-1}Z$, and it seems that the latter one is less costly to obtain. However, it can be shown that if we choose the interface degrees of freedom of the substructures as nodal masters and if we add a general master z_j which has its support in a single substructure then the corresponding column p_j of P has its support in the same substructure. This was the basis of the parallelization concept in [6] for condensation in the presence general local masters. To generalize this concept to condensation with global masters (without splitting them to the substructures) we make use of the following characterization of the projection matrix P and the reduced stiffness Matrix $P^T K P$ contained in Theorem 2 which was also proved and discussed in [7].

Theorem 2 *Let $Z^T Z = I_m$ for $Z \in I\!R^{(m,n)}$. Then the projection matrix P of the condensed problem with generalized masters $z_1, \ldots, z_m$ can be calculated from*

$$\begin{pmatrix} K & -Z \\ -Z^T & O_{m,m} \end{pmatrix} \begin{pmatrix} P \\ S \end{pmatrix} = \begin{pmatrix} O_{n,m} \\ -I_m \end{pmatrix}. \tag{9}$$

Moreover, the condensed stiffness matrix is given by

$$P^T K P = S. \tag{10}$$

4 General Masters and Substructuring

In this section we combine the technique of general masters and its good approximation quality with the method of substructuring and its time efficient implementation.

We consider free vibrations of a structure divided into r substructures. The master vectors are split into two groups: All degrees of freedom on the interfaces of the substructures are chosen as nodal masters to guarantee that the substructures have no direct connection. Furthermore we allow general masters to exploit a priori information about eigenmodes. We do not assume that each of the general masters has its support in exactly one substructure since for this case a parallelization concept has already been developed in [6] and has been applied in [11] and [9]. We consider the more general case of general masters having global support.

We assume that the $m+g$ masters are split into m interface masters and g general masters. Furthermore, we assume that all the masters are orthonormal. Then, the matrix of masters has the form

$$Z = \begin{pmatrix} I_m & O_{m,g} \\ O_{n-m,m} & \tilde{Z} \end{pmatrix} = \begin{pmatrix} I_m & O_{m,g} \\ O & Z_1 \\ \vdots & \vdots \\ O & Z_r \end{pmatrix}, \text{ with } Z^T Z = I_g,$$

where the right column of Z denotes the matrix of general masters and Z_j denotes that part of the general masters corresponding to the j-th substructure.

Next, we partition the projection matrix P and the matrix S of equation (9) in the following way:

$$P =: \begin{pmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \\ \vdots & \vdots \\ P_{r0} & P_{r1} \end{pmatrix}, \quad S =: \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix},$$

with $P_{00} \in \mathbb{R}^{(m,m)}, P_{01} \in \mathbb{R}^{(m,g)}, P_{j0} \in \mathbb{R}^{(s_j,m)}, P_{j1} \in \mathbb{R}^{(s_j,g)}, S_{00} \in \mathbb{R}^{(m,m)}, S_{01} \in \mathbb{R}^{(m,g)}, S_{10} \in \mathbb{R}^{(g,m)}, S_{11} \in \mathbb{R}^{(g,g)}$, $j = 1, \ldots, r$, where s_j denotes the number of nodal coordinates of the j-th substructure.

Then system (9) obtains the following form:

$$\begin{pmatrix} K_{mm} & K_{ms1} & \ldots & K_{msr} & -I_m & O \\ K_{sm1} & K_{ss1} & \ldots & O & O & -Z_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ K_{smr} & O & \ldots & K_{ssr} & O & -Z_r \\ -I_m & O & \ldots & O & O & O \\ O & -Z_1^T & \ldots & -Z_r^T & O & O \end{pmatrix} \begin{pmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \\ \vdots & \vdots \\ P_{r0} & P_{r1} \\ S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} = \begin{pmatrix} O & O \\ O & O \\ \vdots & \vdots \\ O & O \\ -I_m & O \\ O & -I_g \end{pmatrix}. \quad (11)$$

From the penultimate equation one gets $P_{00} = I_m$ and $P_{01} = O_{m,g}$. Furthermore S_{00} and S_{01} depend on the first row only. Therefore, system (11) could be solved by skipping the penultimate row and column of (11) to solve the subsystem

$$\begin{pmatrix} K_{ss1} & \dots & O & -Z_1 \\ \vdots & \ddots & \vdots & \vdots \\ O & \dots & K_{ssr} & -Z_r \\ -Z_1^T & \dots & -Z_r^T & O \end{pmatrix} \begin{pmatrix} P_{10} & P_{11} \\ \vdots & \vdots \\ P_{r0} & P_{r1} \\ S_{10} & S_{11} \end{pmatrix} = \begin{pmatrix} -K_{sm1} & O \\ \vdots & \vdots \\ -K_{smr} & O \\ O_{g,m} & -I_g \end{pmatrix} \tag{12}$$

and afterwards solving the first equation of (11)

$$S_{00} = K_{mm} + \sum_{i=1}^{r} K_{msi} P_{i0}. \tag{13}$$

The corresponding equation for S_{01} has not to be solved since S is symmetric, and therefore $S_{01} = S_{10}^T$ is already available after S_{10} has been derived from equation (12).

Observe that only those columns of K_{smj} are different from the null vector which correspond to the master degrees of freedom on the boundary of substructure j. Therefore, only those columns have to be considered in the computation.

We first restrict our attention to the first column of system (12) only. It can be written in the form

$$K_{ssj} P_{j0} - Z_j S_{10} = -K_{smj},\ j = 1, \dots, r \tag{14}$$

$$-\sum_{j=1}^{r} Z_j^T P_{j0} = O_{g,m}. \tag{15}$$

From equation (14) we get

$$P_{j0} = K_{ssj}^{-1} Z_j S_{10} - K_{ssj}^{-1} K_{smj}. \tag{16}$$

Inserting this into (15) yields

$$\sum_{j=1}^{r} Z_j^T K_{ssj}^{-1} Z_j S_{10} = \sum_{j=1}^{r} Z_j^T K_{ssj}^{-1} K_{smj}. \tag{17}$$

Notice, that the matrices $T_j := Z_j^T K_{ssj}^{-1} Z_j$ and $W_j := Z_j^T K_{ssj}^{-1} K_{smj}$ can be calculated in parallel. Thus, only the summation needs communication between the processes corresponding to the substructures.

Once we computed S_{10} from (17) we can compute P_{j0} by equation (16).

In the same way we derive similar formulae for the second column of (12) and get S_{11} and P_{j1}, $j = 1, \dots, r$, from

$$\sum_{j=1}^{r} T_j S_{11} = I_g,\ P_{j1} = K_{ssj}^{-1} Z_j S_{11}.$$

Therefore, we completely solved system (12) and S_{00} results from equation (13).

After having calculated the matrices P and S one can compute the reduced mass matrix $M_0 := P^T M P$ by

$$\begin{aligned} M_0 &= \begin{pmatrix} I_m & P_{10}^T & \dots & P_{r0}^T \\ O & P_{11}^T & \dots & P_{r1}^T \end{pmatrix} \begin{pmatrix} M_{mm} & M_{ms1} & \dots & M_{msr} \\ M_{sm1} & M_{ss1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ M_{smr} & 0 & \dots & M_{ssr} \end{pmatrix} \begin{pmatrix} I_m & O \\ P_{10} & P_{11} \\ \vdots & \vdots \\ P_{r0} & P_{r1} \end{pmatrix} \\ &= \begin{pmatrix} M_0^{(1,1)} & \sum_{j=1}^{r} \left(M_{msj} + P_{j0}^T M_{ssj} \right) P_{j1} \\ \sum_{j=1}^{r} P_{j1}^T \left(M_{smj} + M_{ssj} P_{j0} \right) & \sum_{j=1}^{r} P_{j1}^T M_{ssj} P_{j1} \end{pmatrix} \end{aligned} \tag{18}$$

with

$$M_0^{(1,1)} := M_{mm} + \sum_{j=1}^{r} \left(P_{j0}^T M_{smj} + P_{j0}^T M_{ssj} P_{j0} + M_{msj} P_{j0} \right).$$

On account of Theorem 2 the condensed stiffness matrix simply is

$$K_0 = \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix}. \tag{19}$$

5 Parallel Condensation

In this section we collect the results from Section 4 and present the parallel condensation method in the presence of global masters.

To each substructure we assign a (slave-) process named 'S_j', and we attach to the master information one further process called 'Ma'.

We assume that the matrices K_{mm}, M_{mm} and the substructuring are known to the master process and that for every $j = 1, \dots, r$ the j-th slave process owns the matrices K_{ssj}, M_{ssj}, K_{smj}, M_{smj}, and Z_j.

1. For every $j = 1, \dots, r$ process 'S_j' does:
   factorize $K_{ssj} =: L_j L_j^T$
   solve $L_j Y_j = Z_j$ for Y_j
   solve $L_j U_j = K_{smj}$ for U_j
   $T_j := Y_j^T Y_j$
   $W_j := Y_j^T U_j$
   isend T_j and W_j to 'Ma' (nonblocking communication)
   solve $L_j^T Q_j = U_j$ for Q_j
   solve $L_j^T R_j = Y_j$ for R_j

2. Masterprocess 'Ma' does:
   compute $T := \sum_{j=1}^{r} T_j$ and $W := \sum_{j=1}^{r} W_j$
   solve $TS_{10} = W$ for S_{10}
   solve $TS_{11} = I$ for S_{11}
   broadcast S_{10} and S_{11} to all 'S_j', $j = 1, \ldots, r$

3. For every $j = 1, \ldots, r$ process 'S_j' does:
   $P_{j0} := R_j S_{10} - Q_j$
   $P_{j1} = R_j S_{11}$
   $A_j := P_{j0}^T M_{ssj}$
   $B_j := (M_{msj} + A_j) P_{j1}$
   $C_j := A_j P_{j0}$
   $D_j := M_{msj} P_{j0}$
   $E_j := P_{j1}^T M_{ssj} P_{j1}$
   $F_j := K_{msj} P_{j0}$
   $G_j := C_j + D_j + D_j^T$

4. Determine
   $M_0^{(1,1)} := M_{mm} + \sum_{j=1}^{r} G_j$
   $M_0^{(1,2)} := \sum_{j=1}^{r} B_j$
   $M_0^{(2,2)} := \sum_{j=1}^{r} E_j$
   $S_{00} := K_{mm} + \sum_{j=1}^{r} F_j$
   by a fan in process such that at the end $'Ma'$ owns the matrices

$$M_0 := \begin{pmatrix} M_0^{(1,1)} & M_0^{(1,2)} \\ (M_0^{(1,2)})^T & M_0^{(2,2)} \end{pmatrix} \quad \text{and} \quad K_0 := \begin{pmatrix} S_{00} & S_{01} \\ S_{01}^T & S_{11} \end{pmatrix}$$

5. Determine desired eigenvalues $\lambda_1, \ldots, \lambda_k$ and corresponding eigenvectors $u_1, \ldots, u_k$ of the reduced eigenvalue problem

$$K_0 u = \lambda M_0 u,$$

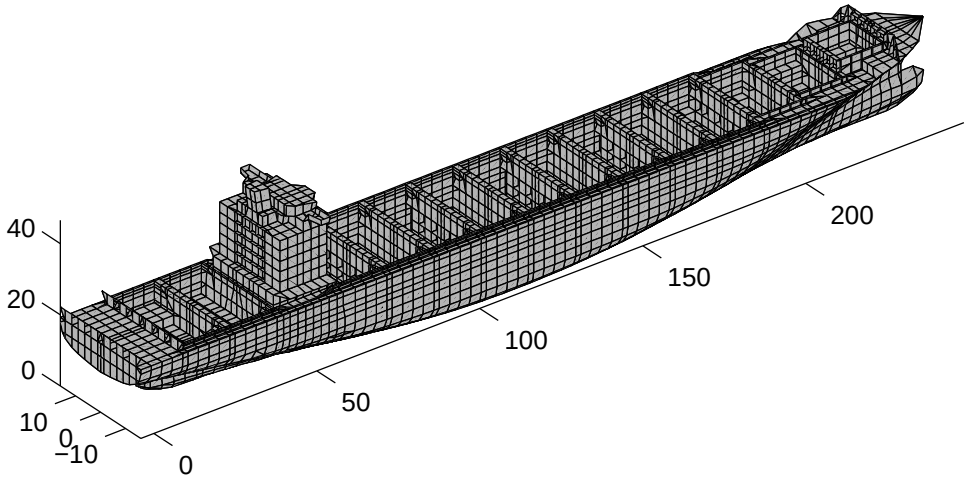
   and broadcast $\lambda_1, \ldots, \lambda_k$ and $u_1, \ldots, u_k$ to all 'S_j'

6. For every $j = 1, \ldots, r$ process 'S_j' does:
   For $i = 1, \ldots, k$ restrict u_i to the vector $\tilde{u}_i$ of interface degrees of freedom
   solve $(K_{ssj} - \lambda_i M_{ssj})\tilde{u}_{ji} = (K_{smj} - \mu_i M_{smj})\tilde{u}_i$ for $\tilde{u}_{ji}$

At the end $\lambda_1, \ldots, \lambda_k$ are eigenvalue approximations of problem (1) and $(\tilde{u}_i^T, \tilde{u}_{1i}^T, \ldots, \tilde{u}_{ri}^T)^T$ are approximations of the corresponding eigenvectors.

The reduced eigenproblem in step 5 can be solved sequentially or in parallel depending on its dimension and on the machine in use. Notice that in the first step the communication is overlapped by the computation of Q_j and R_j.

Figure 1: Container ship



6 A Numerical Example

In this section we discuss details of our implementation of the algorithm in the preceding section and of one numerical example.

We implemented the algorithm in FORTRAN 90 using LAPACK 3 and BLAS routines for the linear algebra and MPI 1.05 for message passing. Depending on the dimension of the reduced problem in step 5 we solved it with the LAPACK routine dspgvx or the SCALAPACK routine pdsygvx. In both routines the generalized eigenproblem is transformed to a standard eigenvalue problem, the matrix is reduced to tridiagonal form, and the desired eigenvalues are determined by bisection.

Figure 2: Substructuring of container ship

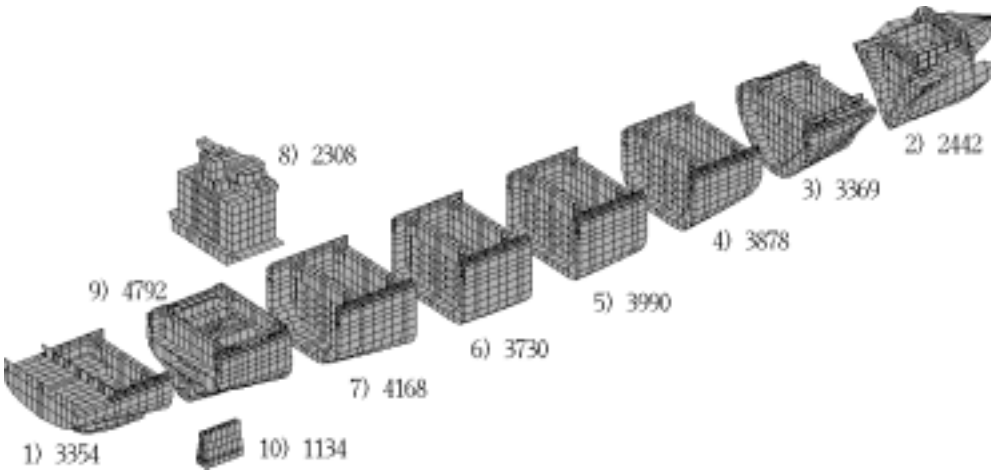


Table 1: Relative errors

#	rel. dist.	nodal cond.	gen. masters
1	3.06e-01	4.83e-05	1.38e-07
2	1.51e-01	6.01e-05	8.78e-07
3	2.71e-01	2.35e-05	1.59e-06
4	7.48e-01	1.15e-04	2.72e-07
5	1.04e+00	3.93e-04	2.32e-07
6	8.88e-01	4.55e-04	1.40e-07
7	6.00e-01	6.43e-03	5.65e-07
8	6.66e-01	1.89e-02	9.99e-06
9	3.39e-01	2.08e-02	1.86e-05
10	4.42e-01	8.92e-02	1.66e-04
11	4.68e-01	1.01e-01	4.24e-03
12	7.53e-01	1.09e-01	1.24e-02

Our numerical example was a finite element model of a container ship with 35262 degrees of freedom given in Figure 1. The bandwidth of the stiffness matrix is 1072. We subdivided the problem into 10 substructures which yielded a reduced problem of dimension $m = 2097$ and slave subproblems of dimensions s_j between 1134 and 4792 (cf. Figure 2). This model which does not consider hydrodynamic forces is called the dry model of the ship, and the eigenmodes are called dry modes. In order to take into account forces of the surrounding water we added hydrodynamic masses which yielded the so called wet model of the ship.

Although the eigenfrequencies of these two models differ quite a bit (column 2 of Table 1 displays the relative differences of the smallest 12 eigenvalues of the dry and the wet ship) the approximation properties of the condensation method are enhanced considerably if we add a small number of dry modes as general masters (which we assume to be known from previous calculations) to the interface masters when solving the wet model. Columns 3 and 4 of Table 1 contain the relative errors of the approximations to the 12 smallest eigenvalues obtained with interface masters only and considering 12 additional dry modes as general masters, respectively.

We tested the program on a heterogeneous workstation cluster consisting of one HP C3000, one HP J2240, and five HP 9000, 712/100 connected by fast ethernet. In this situation communication is much more expensive than algebraic operations. We therefore chose a very coarse grained parallelization concept the communication of which consists only of one fan in, two broadcasts and one nonblocking communication, which is overlapped by useful computations.

The usual performance measurements in parallel computing such as speed up or load balancing do not make sense in this situation. The main reason to solve large models on workstation clusters is that usually computing resources are available on the

various workstations of a group and the individual workstations do not have enough storage to solve the entire problem. Moreover, the substructuring (and therefore the dimensions of the slave subproblems) is dictated by the whole structure. Modifying the substructuring to enhance the load balancing usually will increase the number of master unknowns and thus the computing time to solve the reduced problem in step 5.

Moreover we tested our algorithm on an HP N-class parallel computer with 8 HP-PA 8500/440 Mhz processors organized as a tightly coupled shared memory node. Again on this machine performance measurements as mentioned above are of doubtful use since the computer is run in a multi-user environment, and the user is not able to distribute the processes to the processors of the cluster but the local scheduling is organized by the operating system.

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