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SIMPLIFIED FLUTTER PREDICTION FOR BRIDGES WITH BLUFF CROSS-SECTION

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Abstract—Wind-induced flutter of bridges occurs as coupled torsional and vertical oscillation, or uncoupled torsional oscillation. Bridges with bluff, non-streamlined sections are prone to torsional flutter. Prediction of torsional flutter on the basis of measured aerodynamic derivatives essentially corresponds to the prediction of coupled flutter, although, compared to the latter, it can be largely simplified. The theoretical background is discussed and the given formulae are checked against empirical data. It is found that simplification is recommendable only up to a certain degree.

NOTATION

b	half chord of bridge deck
$c_{\alpha\alpha}$	complex aerodynamic derivative function
$c'_{\alpha\alpha}$, $c''_{\alpha\alpha}$	real and imaginary parts of $c_{\alpha\alpha}$
g_α	damping coefficient for rotational (or torsional) oscillation
I	mass moment of inertia per unit length
i	imaginary unit
k	reduced frequency
k_α	rotational stiffness
m	mass per unit length
$\mathbb{R}$	set of real numbers
r	reduced mass radius of gyration
t	time
v	(critical) wind velocity
α	rotational displacement
$\tilde{\alpha}$	amplitude of rotational oscillation
μ	relative density
ξ_α	damping ratio-to-critical for rotational (or torsional) oscillation
ρ	density of air
ω	circular frequency (of flutter)
ω_α	natural circular frequency of rotational (or torsional) oscillation

INTRODUCTION

Wind-induced flutter of bridges with streamlined cross-section is a coupled motion of torsion and vertical bending. Its analytical prediction by means of a two-dimensional analytical model with two degrees of freedom requires simultaneous solution of the generalized equations of rotational and vertical motion (a complex second-order eigenvalue problem). In these equations, aerodynamic forces are described by six or eight real aerodynamic derivative functions which are analytically or experimentally determined.[1,2] If the vacuum-vibration mode shapes of bending and torsion are strongly non-affine (i.e. different), utilization of a two-dimensional model may be inadmissible. This case requires a more refined, spatial modeling, e.g. by means of finite elements,[3,4] which leads to an eigenvalue problem of higher order.

For bridges with bluff cross-section, e.g. non-streamlined box or channel-shaped section, the aerodynamic forces may be such that torsional and vertical motions remain nearly uncoupled, and flutter in a mainly torsional mode occurs.[5,6] Theoretical evaluation of the aerodynamic derivative functions for such sections is not yet possible. Determination must be made through wind-tunnel measurements on sectional models. Due to missing or extremely weak coupling, however, analytical prediction based on experimentally found derivatives can be enormously simplified. Firstly, the number of derivatives can be drastically reduced; the procedure described in Ref. [2], e.g., requires measurement of only one real derivative function. Secondly, mode shapes and, by this, system spatiality, are of little relevancy. Generalization by means of a two-dimensional analytical model is largely relieved of uncertainty in regard to spatiality. Solution of merely one equation of motion is required.

In the following section, the theoretical fundamentals of simplified prediction methods with application to torsional flutter are elucidated. Discussion is limited to the two-dimensional model. For verification of the given formulae, example calculations and comparisons are made using measurements provided by other authors. It is found that simplification is expedient only up to a certain degree. The recommended analytical procedure requires measurement of two real aerodynamic derivative functions.

SIMPLIFIED FLUTTER PREDICTION

The uncoupled equation of rotational motion when employing the exponential solution function $\alpha = \tilde{\alpha}e^{i\omega t}$, leads to:

$$[(1 + ig_\alpha)\,k_\alpha - \omega^2(I + \pi\rho b^4 c_{\alpha\alpha})]\,\tilde{\alpha} = 0. \quad (1)$$

The complex (i.e. bivalent) aerodynamic derivative:

$$c_{\alpha\alpha} = c'_{\alpha\alpha} + ic''_{\alpha\alpha} \quad (2)$$

is a function of the reduced frequency:

$$k = \frac{\omega b}{v};\ k \in \boldsymbol{R}. \quad (3)$$

It corresponds to the nonstationary aerodynamic coefficients according to Scanlan[7] ($c'_{\alpha\alpha} \hat{=} {}^{16}\!/\!_{\pi} A_3^*$, $c''_{\alpha\alpha} \hat{=} {}^{16}\!/\!_{\pi} A_2^*$; cf. Ref. [3]). The structural damping force is introduced as being a fraction (damping coefficient g_α) of the elasticity-induced restoring force and in phase with the structural velocity.[8,9] The damping coefficient g_α defined in this way corresponds to $2\xi_\alpha$, i.e. twice the damping ratio-to-critical.

Equation (1) represents an eigenvalue problem for a system with one degree of freedom. Nontrivial solution is possible only when the determinant—here the term in brackets—vanishes. Through application of this condition to real and imaginary parts and limitation to $\omega\varepsilon\boldsymbol{R}$ (at the critical point simple harmonic motion occurs), the real equations:

$$\omega^2 \doteq \frac{k_\alpha}{I + \pi\rho b^4 c'_{\alpha\alpha}} = \frac{\omega_\alpha^2}{1 + \dfrac{c'_{\alpha\alpha}}{\mu r^2}}, \quad (4)$$

$$c''_{\alpha\alpha} \doteq \frac{g_\alpha k_\alpha}{\omega^2 \pi\rho b^4} = \left(\frac{\omega_\alpha}{\omega}\right)^2 g_\alpha \mu r^2 \quad (5)$$

are obtained. Eliminating ω by substitution leads to:

$$c''_{\alpha\alpha} \doteq g_\alpha(\mu r^2 + c'_{\alpha\alpha}). \quad (6)$$

The dependent parameters appearing here are defined as follows:

$$\left.\begin{array}{lll} \omega_\alpha^2 = \dfrac{k_\alpha}{I}; & \omega_\alpha \rightarrow & \text{natural circular frequency of rotational motion} \\ \mu = \dfrac{m}{\pi\rho b^2} & \rightarrow & \text{relative density} \\ r = \dfrac{1}{b}\sqrt{\dfrac{I}{m}} & \rightarrow & \text{reduced mass radius of gyration} \\ \Rightarrow \mu r^2 = \dfrac{I}{\pi\rho b^4} & \rightarrow & \text{relative pitching inertia.} \end{array}\right\} \quad (7)$$

It has been proposed to assume the flutter frequency of torsional flutter ω as being identical to the natural frequency of rotational (i.e. torsional) motion ω_α [10]. As can be seen in Eq. (4), this corresponds to neglecting derivative $c'_{\alpha\alpha}$. Hence, conditional Eq. (6) reduces to:

$$c''_{\alpha\alpha} \doteq g_\alpha \mu r^2. \quad (8)$$

This equation is solved for k. The critical wind speed v can then be determined by means of the fundamental relation (3). If the simplified assumption referring to ωw is not used, k is obtained by solving the more intricate Eq. (6). With k, ω follows from Eq. (4) and v can ultimately be calculated from Eq. (3).

In both cases, determination of k is affected by only two structural system parameters: the damping coefficient g_α and the relative pitching inertia μr^2. The expression:

$$g_\alpha \mu r^2 \simeq \frac{2\xi_\alpha I}{\pi\rho b^4}, \quad (9)$$

corresponds to the mass-damping parameter defined elsewhere (cf. Ref. [11]), for which the denomination "Scruton number" has recently been proposed.

In the case of vanishing structural damping ($g_\alpha = 0$), both conditional equations reduce to the simple requirement:

$$c''_{\alpha\alpha} \doteq 0. \quad (10)$$

Consequently, k is obtained solely as a function of cross-section geometry and becomes independent from structural system parameters.

Recent experimental studies,[12,13] which have been confined to the measurement of $c''_{\alpha\alpha}$ (there: A^*_2) and structural damping, tacitly assume validity of Eq. (8). The critical wind speed calculated in this way deviates from the measured value by only 10% for torsional flutter of a ⊔-section. In such cases, a particularly simple expression for critical wind speed may be given:

$$v = \frac{\omega_\alpha b}{k(c''_{\alpha\alpha} = g_\alpha \mu r^2)}. \quad (11)$$

The respective influences of system parameters become clear in this formula. In particular, it can be inferred that flutter stability improves with increasing natural frequency in torsion and growing deck width.

EMPIRICAL CHECK

In Ref. [14], wind-tunnel tests which investigate the flutter behaviour of various bridge deck sectional models are described, and the aerodynamic derivative functions are measured and completely documented. Expressions (6), (8) and (10) are reviewed by means of this empirical data. In order to facilitate numerical evaluation, Eq. (6) is used in rearranged form:

$$\frac{1}{g_\alpha} c''_{\alpha\alpha} - c'_{\alpha\alpha} \doteq \mu r^2. \qquad (6a)$$

Derivatives $c'_{\alpha\alpha}$ and $c''_{\alpha\alpha}$ for various k, and the left side of Eq. (6a) are numerically presented in Table 1. In order to determine k (and if necessary, $c'_{\alpha\alpha}$), linear interpolation between the specified data is effected. System parameters and results of computation are compiled in Table 2. Deviations of k and v are given in percent—on grounds of better comparability in reference to the results of accurate

Table 1. Input data for simplified flutter prediction (according to Ref. [14])

	Section A				Section C				
			$^1/_{g_\alpha} c''_{\alpha\alpha} - c'_{\alpha\alpha}$				$^1/_{g_\alpha} c''_{\alpha\alpha} - c'_{\alpha\alpha}$		
k	$c'_{\alpha\alpha}$	$c''_{\alpha\alpha}$	Case 1	Case 2	$c'_{\alpha\alpha}$	$c''_{\alpha\alpha}$	Case 1	Case 2	Case 3
0.075					14.1	4.05	32.8	38.6	25.9
0.100		5.23			9.15	1.45	7.63	9.71	5.16
0.150	−32.7	3.44	96.1			−0.21			
0.200	−18.5	2.60	66.4	69.9					
0.250	−11.7	2.07		52.6					

Table 2. Simplified flutter prediction for sectional models

System parameters								
Model	Parameters	μ	r	g_α	$\omega_\alpha[^1/_s]$	b[m]	μr^2	$g_\alpha \mu r^2$
A	Case 1	258.2	0.5076	0.0543	50.2	0.100	66.5	3.61
	Case 2	258.2	0.5076	0.0506	45.5	0.100	66.5	3.37
C	Case 1	105.9	0.359	0.0864	25.2	0.102	13.6	1.18
	Case 2	105.9	0.359	0.0769	25.7	0.102	13.6	1.05
	Case 3	105.8	0.359	0.1013	26.3	0.102	13.6	1.38
Evaluation according to Eqs (6) respectively (6a), and Eq. (4)								
Model	Parameters	k	$c'_{\alpha\alpha}$	$\omega[^1/_s]$	$v[^m/_s]$	Dev. k	Dev. v	
A	Case 1	0.200	−18.5	59.1	29.6	−4%	5%	
	Case 2	0.210	−17.1	52.8	25.1	−4%	5%	
C	Case 1	0.094	10.3	19.0	20.6	−20%	16%	
	Case 2	0.097	9.74	19.6	20.6	−24%	22%	
	Case 3	0.090	11.1	19.5	22.1	−18%	13%	
Evaluation according to Eq. (8), i.e. with $\omega = \omega_\alpha$								
Model	Parameters	k	$c'_{\alpha\alpha}$	$\omega[^1/_s]$	$v[^m/_s]$	Dev. k	Dev. v	
A	Case 1	0.145	0	50.2	34.6	−30%	23%	
	Case 2	0.154	0	45.5	29.5	−30%	23%	
C	Case 1	0.108	0	25.2	23.8	−8%	34%	
	Case 2	0.112	0	25.7	23.4	−13%	39%	
	Case 3	0.102	0	26.3	26.3	−8%	35%	
Evaluation according to Eq. (10), i.e. for $g_\alpha = 0$; with $\omega = \omega_\alpha$								
Model	Parameters	k	$c'_{\alpha\alpha}$	$\omega[^1/_s]$	$v[^m/_s]$	Dev. k	Dev. v	
A	Case 1	no	0	50.2	—	—	—	
	Case 2	solution	0	45.5	—	—	—	
C	Case 1	0.144	0	25.2	17.9	23%	1%	
	Case 2	0.144	0	25.7	18.2	12%	8%	
	Case 3	0.144	0	26.3	18.6	30%	−5%	

evaluation [3], rather than in reference to the respective flutter tests.*

Equation (6) yields fairly good results for section A (H-section), but provides only moderate accuracy for section C (truss-stiffened section). This finding corresponds to the respective degree of dominance of the rotational motion component, which is reasonably distinct for section A only.[3] For both sections, evaluation by means of Eq. (8) results in greater inaccuracy for values of v. Equation (10) is not applicable to section A, for $c''_{\alpha\alpha}$ does not vanish in the investigated (and plausible) range of k. In this case, a lower limit of the critical velocity range cannot be established. The good agreement for section C with respect to v should therefore be regarded as accidental. This view is supported by strong deviations and scattering relative to k.

CONCLUSIONS

The following conclusions can be drawn:

- For certain bluff cross-sections, simplified evaluation according to Eqs (6) and (8) provides rough approximation of the critical wind speed.
- Good approximation can be expected only for sections which are prone to nearly uncoupled torsional flutter, but only if, in addition to $c''_{\alpha\alpha}$, derivative $c'_{\alpha\alpha}$ is included [Eq. (6)].
- In contrast to the coupled flutter situation, structural damping has a substantial effect on uncoupled torsional flutter and should be considered for flutter prediction.

The given formulae can be applied directly to spatial bridge systems. The implied generalization is exact when structural and aerodynamical system parameters are uniform along span. Variability occurring in practice is generally of little influence.

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* Accurate evaluation takes into consideration all eight aerodynamic derivative functions and is based upon complete solution of the coupled equations of rotational and vertical motion. The numerical results found in this way agree well with the respective flutter tests using sectional models.[3,14]

REFERENCES

1. K. Klöppel and F. Thiele, "Modellversuche im Windkanal zur Bemessung von Brücken gegen die Gefahr winderregter Schwingungen," *Der Stahlbau* **36**, 353–365 (1967).
2. E. Simiu and R. H. Scanlan, *Wind Effects on Structures*, 2nd edn, Wiley, New York. 1986.
3. U. Starossek, *Brückendynamik—Winderregte Schwingungen von Seilbrücken (Bridge Dynamics—Wind-Induced Vibration of Cable-Supported Bridges)*, Doctoral Thesis, Universität Stuttgart, 1991, by Friedr. Vieweg and Sohn, Braunschweig, 1992.
4. U. Starossek, "Flatternachweis von Brücken mittels Finiter Balkenelemente (flutter prediction for bridges by means of finite beam elements)," *Stahlbau* **61**, 203–208 (1992).
5. R. H. Scanlan and J. J. Tomko, "Airfoil and bridge deck flutter derivatives," *ASCE, Journal of the Engineering Mechanics Division*, **97**, 1717–1737 (1971).
6. R. L. Wardlaw, "The wind resistant design of cable-stayed bridges," *Proceedings, ASCE National Convention, Session Cable-Stayed Bridges*, Nashville, Tennessee, May 9, pp. 46–61 (1988).
7. R. H. Scanlan, "An examination of aerodynamic response theories and model testing relative to suspension bridges," *Proceedings of the Third International Conference on Wind Effects on Buildings and Structures*, Tokyo, Part IV, pp. 941–951, 1971.
8. Fr. Bleich, "Dynamic instability of truss-stiffened suspension bridges under wind action," *ASCE, Transactions* **114**, 1177–1232 (1949).
9. H. W. Försching, *Grundlagen der Aeroelastik*. Springer, Berlin, 1974.
10. E. H. Dowell, H. C. Curtiss Jr and R. H. Scanlan, *A Modern Course in Aeroelasticity*. Sijthoff and Noordhoff, Alphen aan de Rijn, 1978.
11. H. Ruscheweyh, *Dynamische Windwirkung an Bauwerken*. Vols 1, 2. Bauverlag, Wiesbaden, 1982.
12. B. Bienkiewicz, "Wind-tunnel study of effects of geometry modification on aerodynamics of a cable-stayed bridge deck," *Journal of Wind Engineering and Industrial Aerodynamics* **26**, 325–339 (1987).
13. B. Bienkiewicz, J. E. Cermak and J. A. Peterka, "Wind-tunnel study of aerodynamic stability and response of a cable-stayed bridge deck," *Journal of Wind Engineering and Industrial Aerodynamics* **26**, 341–352 (1987).
14. N. Ukeguchi, H. Sakata and H. Nishitani, "An investigation of aeroelastic instability of suspension bridges," *Symposium on Suspension Bridges*, Paper No. 11, Lisbon, 1966.
15. M. Herzog, "Vereinfachte Beurteilung der aerodynamischen Stabilität von Hängebrücken," *Bauingenieur* **57**, 393–399 (1982).