

ACRITH - High Accuracy Arithmetic Subroutine Library

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In the following the computer implementation of the new numerical methods presented in "Numerical Results with Verified Accuracy" in this proceedings will be described.

The implementation is three-fold. The basis is a subroutine library callable from VS-FORTRAN or Assembly Language programs. The library consists of functions for solving a number of standard problems of numerical analysis such as systems of linear equations, matrix inversion, algebraic eigenproblems, polynomial zeros, polynomial evaluation, linear programming problems and evaluation of arithmetic expressions. The key property of the new algorithms is, that every result is automatically verified to be correct.

The second part are routines for the basic arithmetic operations $+$, $-$, $*$, $/$ and the dot product. Beside this software simulation a hardware implementation of the arithmetic operations is available on the IBM 4361 processor.

The third part is an Online Training Component (OTC) for ACRITH which can serve as a training tool to introduce the new algorithms or which can serve as a problem solver. The following menus refer to the second release of ACRITH available since March 1985.

The numerical results are demonstrated in the following by the example of a system of linear equations with a Hilbert 21×21 matrix and the right hand side $(1, 0, \dots, 0)$. The first menu displayed is the main menu which will be seen when entering the OTC of ACRITH. What is shown is the exact image of what is seen on the terminal.

The second menu is one out of four tutorial menus about Hilbert matrices in the ACRITH OTC, available by pressing the "help-key" PF1. The third menu is the inclusion of the solution of the linear system with the Hilbert matrix. This inclusion, which is the first column of the inverse of the Hilbert matrix, is of maximum accuracy, i.e. the left and right bounds for the solution are adjacent floating-point numbers.

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----- ACRITH Online Training Component (OTC) -----
COMMAND ==>                                     Date - 85/03/29
                                                Time - 23:09
Enter one of the following options and press ENTER .  User - RUMP
                                                Terminal- 3279
                                                PF Keys - 12

T Tutorial - Information about ACRITH OTC
S Set Parameters - Set parameters for dialog with OTC
O ISPF Parameters - Specify terminal and user parameters
R Rounding - Rounding of decimal and hexadecimal numbers
D Dot Product - Computation of an accurate scalar product
M Matrix Mult. - Multiplication of matrices
L Linear Systems - Solution of linear systems
I Inverse - Inversion of matrices
E Eigenproblems - Computation of eigenvalues and eigenvectors
P Polynomial - Evaluation and zeros of polynomials
O Optimization - Linear programming optimization
A Arithm. Expr. - Evaluation of arithmetic expressions
V Vector Operat. - Vector operations
F Std. Functions - Evaluation of standard functions
C Copy - Copy last input or output data to a file
X Exit - Terminate ACRITH OTC using exit defaults
ISPF ISPF/PDF - Invoke ISPF Program Development Facility

Press END KEY (PF3) to terminate                                     PF1=Help
```

TUTORIAL ----- OPTION 'L' - LINEAR SYSTEMS ----- TUTORIAL
 COMMAND ==> (Page 2 of 4)

A Hilbert matrix is a special matrix of the form:

1/1	1/2	1/3	...	1/n
1/2	1/3	1/4	...	1/(n+1)
1/3	1/4	1/5	...	1/(n+2)
.
.
1/n	1/(n+1)	1/(n+2)	...	1/(2*n-1)

Since this matrix contains numbers which are not exactly representable in System/370 long floating-point format, the matrix generated by option H is the Hilbert matrix multiplied by the least common multiple of all denominators.

Press ENTER to proceed to the next page

PF Keys: 1=Help 3=End 4=Return 7=Up 8=Skip 10=Back 11=Next 12=Cursor

----- Precise Linear System - Inclusion -----
 COMMAND ==>

The decimal linear system is exactly representable in hexadecimal format.
 The solution is:

Component	1	(0.2013145339298029D-14 , 0.2013145339298030D-14)
	2	(-0.4428919746455665D-12 , -0.4428919746455664D-12)
	3	(0.3225729882001875D-10 , 0.3225729882001876D-10)
	4	(-0.1161262757520676D-08 , -0.1161262757520675D-08)
	5	(0.2467683359731434D-07 , 0.2467683359731435D-07)
	6	(-0.3421854258827590D-06 , -0.3421854258827589D-06)
	7	(0.3299645178155175D-05 , 0.3299645178155176D-05)
	8	(-0.2309751624708624D-04 , -0.2309751624708622D-04)
	9	(0.1209411614604376D-03 , 0.1209411614604377D-03)
	20	(-0.2642905686022805D-03 , -0.2642905686022804D-03)
	21	(0.2579979360165118D-04 , 0.2579979360165119D-04)

Additionally the non-singularity of every matrix contained in the interval matrix and the correctness of the displayed results have been algorithmically verified.

Press ENTER to browse complete result

Press END KEY (PF3) to terminate

PF1=Help

References

- [1] Rump, S.M.: Solving algebraic Problems with High Accuracy, in "A New Approach to Scientific Computation", Edts. U.W. Kulisch and W.L. Miranker, ACADEMIC PRESS, p. 51-120 (1983).
- [2] ACRITH, High-Accuracy Arithmetic Subroutine Library, Program Description and User's Guide, IBM Publications, Document Number SC33-6164-1 (1984).
- [3] ISPF, Interactive System Productivity Facility, Dialog Management Services, IBM Publications, Document Number SC34-2088-0 (1982).