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Stabilization Columns for Embankment Support – Investigation, Verification and Further Development of Analytical Analyses

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Abstract. As a technical and economical alternative to foundations on piles, but also to shallow foundations on improved soil, in recent decades a high number of soil improvement methods have been developed and established. Many of these methods use non-reinforced, cylindrical load bearing elements. A very common application of stabilizing columns is the improvement of a few meters thick soft soils below dams and embankments. But especially for this application, many failure cases are documented worldwide. In the contribution the substantial content and results are presented for investigation, testing and further development of methods for evaluating the slope stability. After a description of the problem and consequential tasks the contribution contains main results of the investigations of international sources with the stepwise development of analytical solutions. Next to the in practice well-known approaches for gravel columns, less common approaches from Scandinavia are explained. The contribution is completed with a presentation and discussion of an illustrative example, taking into account a number of different failure modes of the columns and the surrounding soil. The example was compared and validated with a 3D Model using the Finite Element Method.

1. Introduction

Within the German Geotechnical Society (*DGGT*) the working group 2.8 is working on the revision of recommendations for manufacturing, design and quality control of non-reinforced, cylindrical load bearing elements, further referred as stabilization columns. In the working group the column types are distinguished due to their manufacturing processes and the nature of the contribution of the column materials. A very common yet highly sensitive application of stabilizing columns is the improvement of several meters thick, low load-bearing soils below dams and traffic embankments. Analytical design programs (based on the the state of equilibrium) are currently not able to consider the stability columns in the safety analyses in terms of a slope stability failure. Therefore, the effect of the columns must be incorporated indirect into the programs, so that the effect can be taken into account in the slope stability evaluation.

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Figure 1. Failure of an embankment, supported by stabilization columns [17]

Motivation for the authors was to investigate, compare and further develop approaches for evaluating the slope stability using software based on the state of equilibrium. This may lead for arbitrary constellations to safe and economical dimensioning and arrangement of the columns. In this paper, main content and results are presented for the development of the analytical concepts, discussed and tested on a concrete example using a 3D numerical model.

2. Research and Analysis

General information on the design, analysis and design for applications in geotechnical engineering can be found currently in DIN EN 1997-1: 2009-09 (Eurocode 7). The guideline is part of a series of standards for influence and design, which only makes sense in the package. For national application also individual parameters are defined in Annexes (in Germany - NAD: DIN EN 1997-1 / NA: 2010-12). Additional regulations also can be found in DIN 1054: 2010-12. In addition to the European standards for specifications to ground and ground failure calculations reference is made to DIN 4084: 2009-01.

For a consideration of stability columns in an analytical calculation, using an equilibrium analysis, two basic approaches are very common:

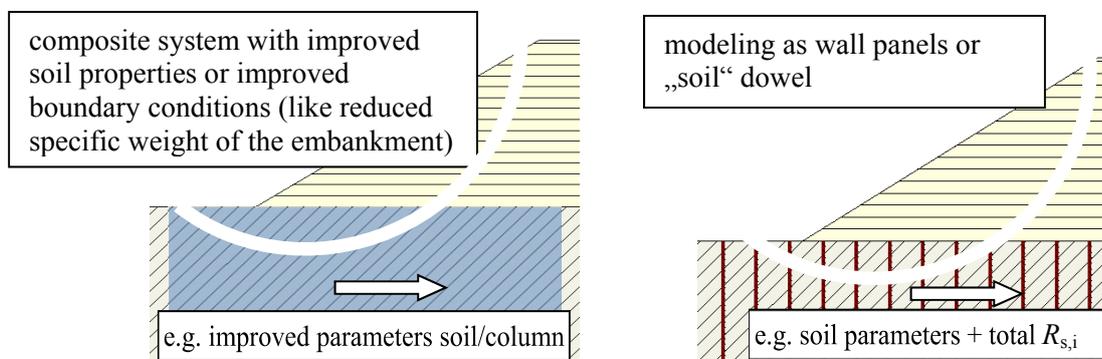


Figure 2. Basic approaches for consideration of stability columns in a calculation: “global“ by improved soil properties of a composite system or by improved boundary conditions (left figure); by modelling the columns as wall panels or “soil” dowel (right figure)

For estimating of improved soil properties of the composite system many approaches exist which take into account the columns according to a load-component and an area-ratio e.g. according to [7], [12] and [13].

Especially for the use of stiff column materials with binder the advices of the authors [4], [9], [10] and [11] have to be taken into account:

- The load transfer behavior is decisively influenced by the column stiffness.
- Columns with very low stiffness transfer loads in interaction with the surrounding soil better than columns with high stiffness. The failure modes can be approximated to an improved/composite system.
- Columns with high stiffness behave towards the surrounding soil with low bearing capacity as a pile-like element. A column failure is assessed as more critical due to the brittle material behavior (progressive failure or "zipper" principle possible).

It is therefore necessary to distinguish between two basic modes of operation:

- behavior as a block or as a composite system for granular columns without binder and for "soft" columns with binder
- behavior as a pile-like element for "rigid" columns with binder

3. Approaches und Separation

3.1. Granular Columns (gravel columns)

For granular columns, the approach of [12], supplemented by [13] is recommended. In this approach improved soil properties of a composite system are calculated considering the load-component and the area-proportion of column and surrounding soil. Compared with a just area proportional determination of the improved soil properties, this may easily lead to an overestimation of the improvement in slope stability considerations, especially for deep slip circles. In [13] approaches are explained which deal with this problem, e.g. via calculating improved soil properties with consideration of the ratio between the respective load above the improved layer (Q_{top}) and the total load (Q_{total}).

3.2. "Soft" Columns with Binder (deep mixing method)

For "soft" stabilization columns the specifications according to [7] are recommended. Basis of this approach is the determination of improved soil properties according to the area ratio of column and the surrounding soil (see (1) and (2)) and the limitation of shear strengths of the columns of 150 kN/m² or max. 200 kN/m² depending on the arrangement of the columns below an embankment or a dam. A distinction is made between the location of the column between "passive zone", "shear zone" and "active zone". Furthermore, the approach provides a separate consideration of undrained and drained conditions (less favorable is relevant). Other failure mechanisms of the columns are not considered. Therefore an effective interconnection between column and surrounding soil as well as a shear failure along a slip surface through soil and columns is assumed, analogous to the approach for granular columns. This is considered to be realistic when strength or stiffness between columns and soil does not vary too much or if the ratio of the improved area is greater than 50%. As a result of numerical benchmark calculations, [11] indicates 15 psi (100 kN/m) for a column compression strength as an upper limit suitability for a calculation with improved soil properties.

$$\varphi_{avg} = A_S/A \cdot \tan\varphi_S + (1 - A_S/A) \cdot \tan\varphi_B \quad (1)$$

$$c_{avg} = A_S/A \cdot c_S + (1 - A_S/A) \cdot c_B \quad (2)$$

Especially in northern Europe groups of stabilization columns are used for settlement reduction and stabilization of dams. In this case, the area ratio of the pillars from the surrounding soil is usually well below 30%. In contrast to this in the Japanese region, also with frequent applications of similar improvement methods, the area ratio is from 30% to 50% and higher [11]. For this column arrangements with high area ratio separate recommendations have been published [15].

3.3. „Rigid“ Columns with Binder

The approach for “soft“ columns with binder with a determination of improved soil properties and evidence of equilibrium conditions along a sliding body assumes an uniform deformation behavior of columns and surrounding soil (similar strengths of columns and soil). This approach leads to an overestimation of the improvement for the use of groups of “rigid“ columns with binder. [9] and [4] evaluated analyses of experimental studies on the failure behavior in centrifuge tests and numerical analyses. They pointed out that for “rigid“ columns with binder in addition to shear failure along a sliding surface other failure modes of the columns and the surrounding soil will occur. Particularly in areas below or next to embankments an arrangement of the columns in overlapping form is much safer. Groups of individual columns should not be used in next to embankments [15] and [10].

4. Application Example for “Rigid” Columns with Binder

In this chapter an application example for the approach for groups of individual “rigid“ columns with binder according to [9] and [4] is illustrated. In modified approach, six major failure modes of the columns were evaluated (see also [1]). For the traffic dam, the overall stability has to be proofed in the limit state GEO-3 for the design situation BS-P. The applied load on the dam crest is composed of $q_{Q,k} = 60.0$ kN/m with a width of 2×2.6 m and $q_{G,k} = 12.5$ kN/m with a width of 8.5 m (see Figure 3). The water table is 20 m below the dam crest.

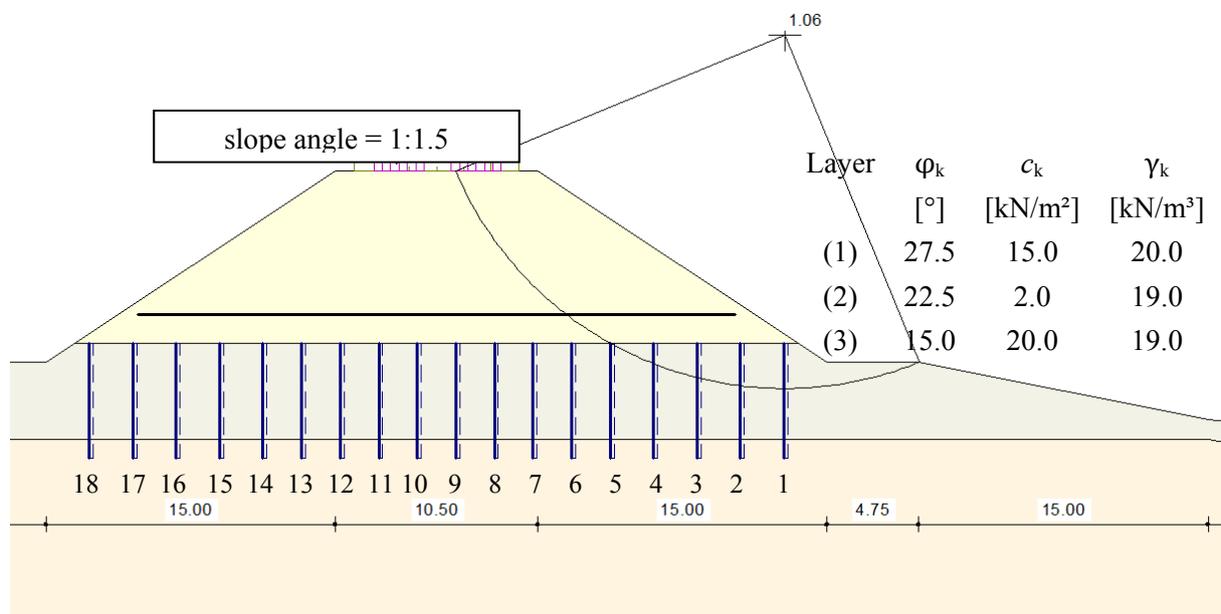


Figure 3. Geometry and basic details of the embankment, decisive slip circle and utilization factor μ_{\max} without consideration of stability columns (only illustration of the proposed column arrangement)

For the initial situation (without columns) a utilization factor $\mu_{\max} < 1.0$ was determined for the limit state GEO-3 in the design situation BS-A. Using the partial safety factors obtained for a design situation BS-P leads to an utilization factor $\mu_{\max} = 1.06$ (an increase of the overall stability is required, s. Figure 3).

In Table 1 the required geometric data which is necessary to calculate the applied load Q_E according to (3) is collected:

Table 1. Determination of the vertical load per unit cell (dead load dam).

number of column	1	2	3	4	5	6	7	8
height of embankment h_i (m)	0.5	2.0	3.5	5	6.5	7.8	9.0	9.0
length of column above the slip surface H_1 (m)	2.4	2.2	1.8	1.1	0.0	0.0	0.0	0.0
length of column below the slip surface H_2 (m)	3.6	3.8	4.2	4.9	6.0	6.0	6.0	6.0
load per unit cell Q_E (kN)	48.1	192.4	336.7	480.9	559.6	595.2	384.7	384.7

$$Q_E = \gamma \cdot h_i \cdot A_E \quad (3)$$

Decisive for the moment capacity of the stability columns M_u is the vertical stress in the columns (see also Table 2). Depending on the type and design of the load distribution layer an increased load component of the columns m' has to be estimated using an appropriate method for example according to [6], [16] oder [8]. For this example it is assumed that an increased load component of the columns m is 0.8 (80%) for the entire column length.

$$Q_S = Q_E \cdot m' \quad (4)$$

$$\sigma_{v,S} = Q_S / A_S \quad (5)$$

$$M_u = f_{c,d} \cdot A_{pl} \cdot e_{pl} \quad (6)$$

$$A_{pl} = \sigma_{v,S} / f_{c,d} \cdot A_S \quad (7)$$

$$e_{pl} = \frac{d_S}{2} \cdot \left(1,65 \left(\frac{A_{pl}}{A_S} \right)^4 - 4,05 \left(\frac{A_{pl}}{A_S} \right)^3 + 3,49 \left(\frac{A_{pl}}{A_S} \right)^2 - 2,08 \left(\frac{A_{pl}}{A_S} \right) + 1 \right) \quad (8)$$

Table 2. Determination of the moment capacity M_u per column according to [9].

number of column	1	2	3	4	5	6	7	8
Q_S (kN)	38.48	153.90	269.33	384.75	447.65	476.17	307.74	307.74
$\sigma_{v,S}$ (kN/m ²)	136.08	544.31	952.54	1360.77	1583.23	1684.11	1088.40	1088.40
A_{pl} (m ²)	0.0030	0.0121	0.0212	0.0302	0.0352	0.0374	0.0242	0.0242
e_{pl} (m)	0.2934	0.2751	0.2587	0.2438	0.2364	0.2331	0.2536	0.2536
M_u (kNm)	11.29	42.34	69.67	93.81	105.81	111.00	78.03	78.03

In addition to the failure modes shown in Figure 3, columns with less strength can fail by shearing, compressing or buckling. For columns with medium strength or as in this example, high strength, these failure modes are unlikely and only expected in areas in the centre of a dam ("active zone"). Other failure modes according to [9] for groups of individual columns next to the slope ("passive zone") are not considered.

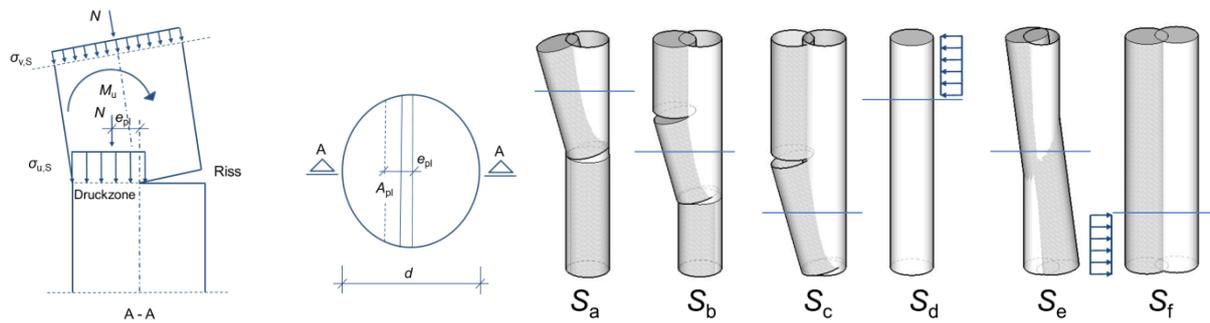


Figure 4. Review of the moment capacity M_U of the columns (left) and identification of the column forces $R_{S,i}$ depending on different failure modes (pictured right)

According to [9], the column resistance S (Figure 4) is defined as the resistance acting parallel to the slip surface and is made up of units of normal force and column strength. The column force T corresponds to the horizontal resistance of the column and is hereinafter referred as $R_{S,i}$. With (9) to (14) the resistance values are determined for the failure modes a to f.

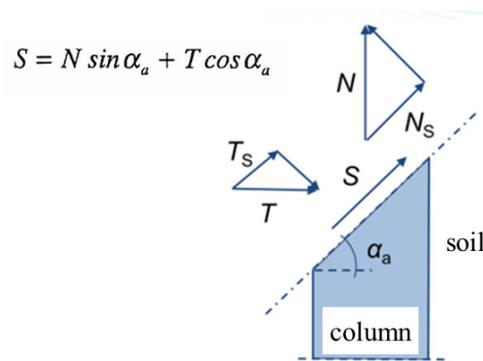


Figure 5. Approach to calculate the column resistance as a function of normal force and column strength according to [9]

$$R_{S,a} = kc_u d_s^2 \left(\sqrt{\frac{4}{9} \left(\frac{H_1}{d} \right)^2 + \frac{4}{3} \frac{M_u}{kc_u d_s^3} - \frac{1}{3} \left(\frac{H_1}{d_s} \right)} \right) \quad (9)$$

$$R_{S,b} = \sqrt{2M_u kc_u d_s} \quad (10)$$

$$R_{S,c} = kc_u d_s^2 \left(\sqrt{\frac{4}{9} \left(\frac{L - H_1}{d} \right)^2 + \frac{4}{3} \frac{M_u}{kc_u d_s^3} - \frac{1}{3} \left(\frac{L - H_1}{d_s} \right)} \right) \quad (11)$$

$$R_{S,d} = kc_u d_s H_1 \quad (12)$$

$$\text{only if } M_{max} = kc_u d_s \frac{H_1^2}{2} < M_u$$

$$R_{S,e} = \frac{kc_u d_s}{2} \left(\sqrt{3L^2 - 4LH_1 + 4H_1^2} - L \right) \quad (13)$$

$$\text{only if } H_1 \approx H_2$$

$$R_{S,f} = kc_u d_s (L - H_1) \quad (14)$$

$$\text{only if } M_{max} = kc_u d_s \frac{(L - H_1)^2}{2} < M_u$$

The load capacity factor k is for large deformations above or below a sliding surface at up to 9 [9]. Particularly for large differences in strength between column and the surrounding soil, a failure of the columns is assumed, just before the soil can mobilize its full shear strength. For small deformations in the soil k should be set to 2.0 according to [4]. This approach takes into account the very limited ductility of the columns and is used in the further calculation example. In recognition of $k > 2$, the deformation compatibility of columns and soil should be considered and proofed separately.

For determining the column resistance in accordance to the cutting angle α_a proportions for normal force and column strength have to be considered (Figure 4). The approach of the normal force leads, in particular for large cutting angles α_a to computationally higher column resistances. Weaknesses by imperfections and/or by tilting of the columns are not considered. Subsequently, the column resistance $R_{s,i}$ was simplistically equated with the column strength T . This corresponds to columns in which the angle of cutting angle α_a is small as in this example. Subsequently, the column forces $R_{s,i}$ are determined as depending on the failure modes a to f. The minimum value ($\min R_{s,i}$) is decisive.

Table 3. Determination of the relevant column forces $R_{s,i}$.

number of column	1	2	3	4	5	6	7	8
$R_{s,a}$ (kN)	20,24	29,77	37,15	44,58	52,85	54,13	45,39	45,39
$R_{s,b}$ (kN)	21,14	40,95	52,52	60,95	64,73	66,30	55,59	55,59
$R_{s,c}$ (kN)	26,80	35,20	42,37	49,27	55,62	56,33	51,68	51,68
$R_{s,d}$ (kN)	47,52	43,56	35,64	21,78	0,00	0,00	0,00	0,00
$R_{s,e}$ (kN)	25,44	26,08	27,90	32,64	43,48	43,48	43,48	43,48
$R_{s,f}$ (kN)	71,28	75,24	83,16	97,02	118,80	118,80	118,80	118,80
$\min R_{s,i}$ (kN)	20,24	29,77	35,64	21,78	0,00	0,00	0,00	0,00

light grey: condition $M_{\max} < M_u$ (failure mode d and f) or/and $H_1 = H_2$ (failure mode e) not satisfied

The sum of the forces in the relevant slip circle and the exceeding of the stresses (ΔE) may be determined as followed (see Figure 3):

$$\begin{aligned}
 \sum \min R_{s,i} &= 107.43 \text{ kN} \\
 \Delta E &= \Delta M / \text{radius } (r) \quad \text{with } \Delta M = E \cdot M - r \cdot M \\
 &= (19,601.5 \text{ kNm} - 18,413.6 \text{ kNm}) / 20.66 \text{ m} \\
 &= 57.5 \text{ kN}
 \end{aligned}$$

For this example calculation the review of the column forces $\sum \min R_{s,i}$ to ΔE leads to sufficient capacity of the improvement ($107.4 \text{ kN} > 57.5 \text{ kN}$). In the analytical model, soil resistances and the corresponding column strength $R_{s,i}$ can be added analogous to Figure 3 for further iteration. For the evidence of the overall stability it has to be proofed whether a reassessment is required on a different relevant slip circle. In this example, the column resistance for column 1 to 5 and a limited soil resistance were used in the analytical model in limit state GEO-3 and the design situation BS-P. The result of the calculation no significantly changed the position of the slip circle.

5. Comparative Calculation with Finite Element Method (FEM)

The results obtained with the approach of "rigid" binder columns according to [9] and [4] has to be recalculated and verified with a numerical model. For this purpose, the FEM is used with the algorithm of the software Plaxis3D [3].

5.1. Numerical Model

For the numerical analysis, a three-dimensional model of the dam is used. In addition to the formation of slip surfaces and the determination of the stability, the interaction between the columns is considered. Taking advantage of the symmetries, a limited section of the dam is modeled for a spacing of one column. The geometry used, groundwater conditions and the material properties are illustrated in Figure 3. At the bottom and at the lateral edges of the model the displacement degrees of freedom are kept in horizontal and vertical directions. Figure 6 shows the discretization of the FE model. The mesh consists of about a hundred thousand 10-node tetrahedral elements.

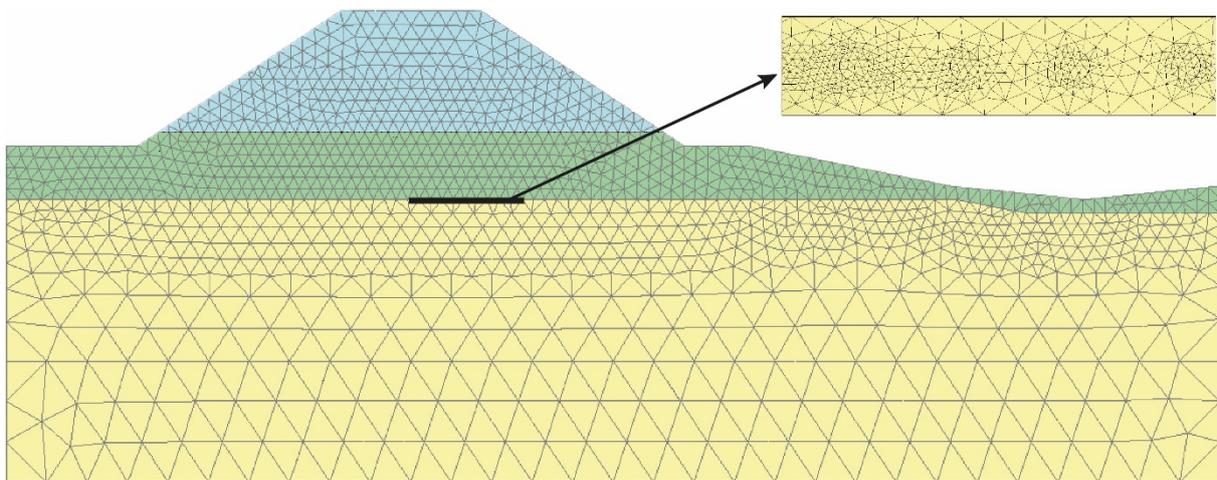


Figure 6. FE model with the used discretization

5.1.1. Load History

At the beginning of the analysis the initial stress state is calculated before modeling the dam in two phases. Another plastic step is to stabilize the equilibrium state in a so-called "zero phase" without further design changes. Now, the stability of the dam is determined without use of stabilizing columns under the previously applied loads. Alternatively the columns are considered wished-in-place, in the next step the loads are applied and the stability of this condition is calculated.

5.1.2. Modeling of the Soil

The global stability of the slope is determined by an ϕ -c reduction in Plaxis3D. According to the recommendations of the Working Group "Numerical Analysis in Geotechnical Engineering" [5] the safety of numerical stability analyzes is less dependent on the choice of material model. Thus, for the FE analysis also as in the above calculation, the linear elastic, perfectly plastic material model with boundary condition according to Mohr-Coulomb (MCM) and the material parameters illustrated in figure 3 were used.

5.1.3. Modeling of the Columns

The columns are modeled as volume elements with MCM using fitted parameters for the material (C30/37), to take into account the resistance reduction of the column. To map the yield surface of the concrete, the material parameters listed in Table 4 are used. This allows an approximate consideration of the plastic behavior of the column and the automatic ϕ -c-reduction contained in Plaxis3D also for the columns.

Table 4. Material parameters for mapping the stabilization column (C30/37) to the linear elastic, perfectly plastic material model with the boundary condition according to Mohr-Coulomb.

Parameter	Unit	C30/37	Note
γ/γ_r	kN/m ³	24/24	
E	MN/m ²	33,000	according to EC2-1-1, section 3.1.3
ν	-	0.2	
φ'	°	50.7	$\tan \varphi' = (f_{cm} - 5/3 f_{ct}) / (f_{cm} + 5/3 f_{ct})$ *
c'	kN/m ²	6,776	$c' = (0,5 f_{cm} (1 - \sin \varphi')) / \sqrt{1 - (\sin \varphi')^2}$ *
$\sigma_{\text{Tension cut off}}$	kN/m ²	2,900	
* according to [14]			

A direct force transmission between soil and columns is assumed. To determine the internal forces and moments, as well as the deformation of the columns, a beam element is added in the axis of the column. The column stiffness of the volume elements and of the beams has to be equal to the overall stiffness. In this case, the stiffness of the beam is taken as one thousandth of the overall stiffness, whereas the stiffness of the volume elements corresponds to the overall stiffness. The resulting difference of 0.1% for the overall stiffness is tolerable. The ratio of the cutting forces corresponds to (16) and (17).

$$u_{\text{column}} = u_{\text{beam}} \quad (15)$$

$$M_{\text{column}} = \frac{(EI)_{\text{column}} + (EI)_{\text{beam}}}{(EI)_{\text{beam}}} M_{\text{beam}} = 1001 M_{\text{beam}} \quad (16)$$

$$N_{\text{column}} = \frac{(EA)_{\text{column}} + (EA)_{\text{beam}}}{(EA)_{\text{beam}}} N_{\text{beam}} = 1001 N_{\text{beam}} \quad (17)$$

5.2. Results

5.2.1. Increase of Stability

The essential for a proof of the slope is the stability of the observed slope. In the FEM φ - c -reduction the utilization factor can be increased with considering the stability columns by approximately 15%. This result is comparable to the 12% increase in the analytical calculation.

5.2.2. Failure Mechanism

In order to assess the influence of the columns on the failure mechanism for the depth of the dam, a section in the plane of the column axis (A) and between the columns (B) is evaluated.

Figure 7 (top) shows the shear plane of the non-stabilized embankment by illustrating the incremental shear strain $\Delta\gamma$. As expected, due to the symmetrical loading of the embankment, an identical failure arises on both slopes. The failure surface is constant over the depth (in the direction of dam axis) and is almost equivalent to the failure image of the calculations with the equilibrium software (see Section 4, Figure 3), a single monolithic failure body. The radius and the center of the slip circle differ slightly from each other.

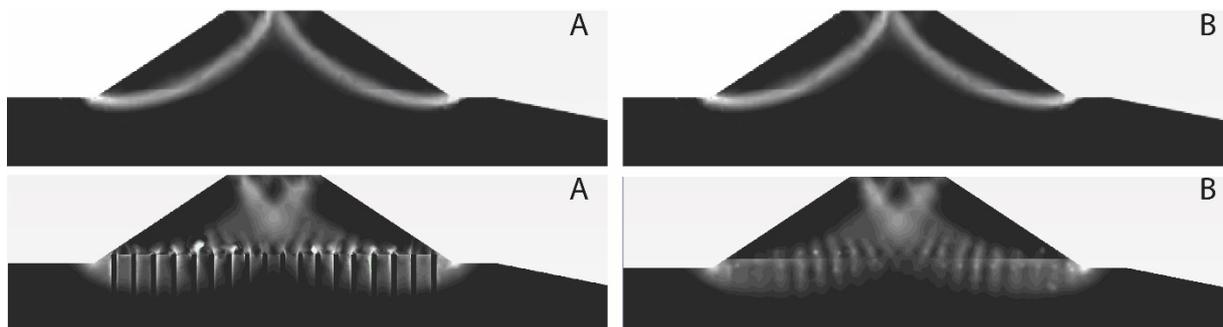


Figure 7. Failure mode of the dam after strength reduction in section A and section B without stabilizing columns (top) and with stabilization column (below)

In Figure 7 (below) the failure body in the ultimate limit state for the dam is illustrated with stabilizing columns. The left picture shows the positions of the columns are clearly visible due to the very low shear strains. The columns lead to a shifting of the shear plane and a strong dispersion of the shear plane towards a shear band. The shear strains are also significantly reduced. Section B also illustrates that the failure body is almost constant in the depth. Thus, the selected distance is sufficient to ensure interaction between the columns and to prevent a flow of soil between the columns. In this case, a two-dimensional calculation of the system is justified as in the approach of "rigid" binder columns according to [9] and [4].

5.2.3. Comparison with the Approach of "Rigid" Columns with Binder

For the verification of the method for "rigid" binder columns according to [9] and [4] in particular the attachable lateral forces at the level of the slip surface are considered. The lateral force curves calculated in the FEM are shown for the right slope in Figure 8.

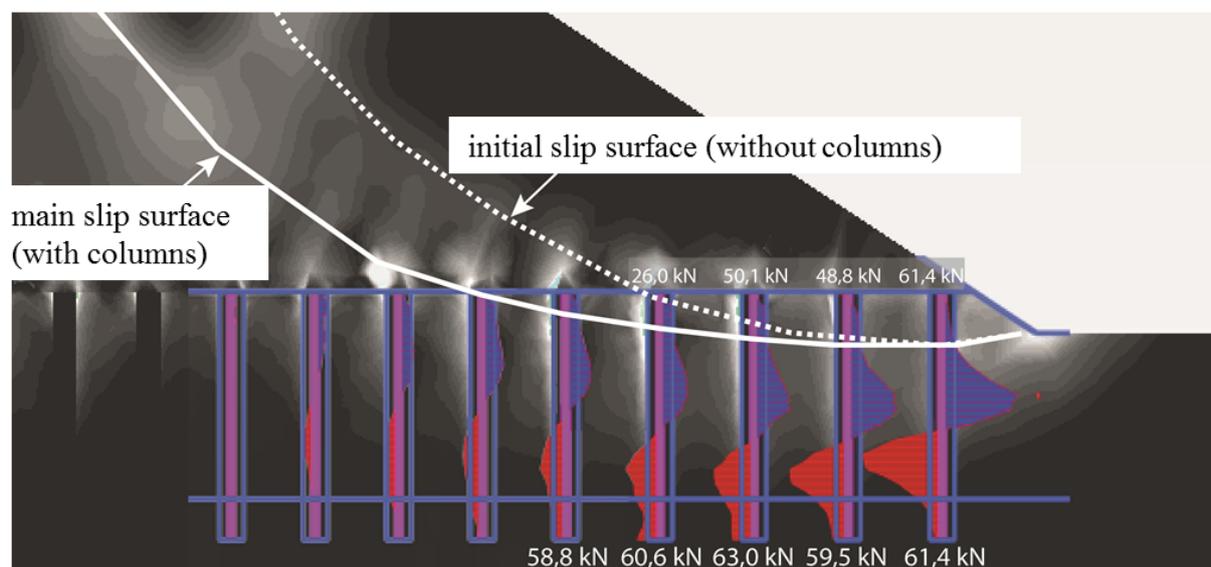


Figure 8. Lateral force curves as well as cutting forces at the level of the slip surface in the ultimate limit state

The shear surface is influenced by all columns, but only the outermost five stability columns are in the initial slip surface and therefore taken into account for the analytical calculation. The procedure for "rigid" binder columns is based on the initial slip surface without columns. This leads to attachable transverse forces R_s in the order of 20 to 40 kN. The transverse forces calculated with the FEM are based on the slip surface of the embankment stabilized with columns and are between 59 to 63 kN. By shifting the slip surface more columns are considered. Tab. 5 shows a comparison of the calculated forces.

Table 5. Summary of attachable lateral forces of the approach for "rigid" binder columns (BMS) and the stability analysis with FEM.

number of column	6-8	5	4	3	2	1
min $R_{s,i}$ ("rigid" columns) [kN]	0	0	21.8	35.6	39.8	20.2
Q_s (FEM) [kN]	0	58.8	60.6	63.0	59.5	61.4
Q_s (FEM; initial slip surface) [kN]	0	0	26.0	50.1	48.8	61.4

In addition, the shear forces at the level of the original slip surface are indicated in the table. Due to the more accurate replica of the approach, this leads to a significantly better agreement with the forces min $R_{s,i}$ ("rigid" columns). However, these are also smaller. By shifting the slip surface in the range of stability columns always occurs an increase in the attachable columns or shear forces compared to the initial slip surface.

6. Summary

As a result of the conducted research and analysis of analytical approaches for estimating the slope stability with consideration of stabilizing columns, essential and appropriate approaches were identified. The current types of non-reinforced cylindrical load bearing elements have to be separated between granular columns and "soft" binder columns with a consideration via improved soil properties of a composite system of columns and surrounding soil. For "rigid" binder columns, an approach was presented and discussed with consideration of different failure modes of the individual columns in a concrete example using the analytical software GGU Stability. In addition, a comparative analysis with the FEM has been illustrated and discussed on this example. The analytical approach is in principle suitable and plausible in the form shown and it has reserves in the safety level compared to the numerical calculation. The comparative calculation with FEM using "rigid" binder columns also illustrates a significant widening / scattering of the failure zone.

Thus, with the presented approaches to computational consideration and assessment of stability columns, the current types of stability columns can be suitable considered and measured in a slope stability analyses in limit state GEO-3.

7. References

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