383 | Juli 1979

SCHRIFTENREIHE SCHIFFBAU

K. Wieghardt

Preliminary Flow Tests with Sand, Sand/Water Mixtures and Mush Ice

TUHH
Technische Universität Hamburg-Harburg

Preliminary Flow Tests with Sand, Sand/Water Mixtures and Mush Ice

K. Wieghardt

Hamburg, Technische Universität Hamburg-Harburg, 1979

Schriftenreihe Schiffbau
Schwarzenbergstraße 95c
D-21073 Hamburg

http://www.tuhh.de/vss

INSTITUT FÜR SCHIFFBAU
DER UNIVERSITÄT HAMBURG

Preliminary flow tests with sand, sand/water mixtures and mush ice

K. Wieghardt

Juli 1979

Bericht Nr. 383

INSTITUT FÜR SCHIFFBAU DER UNIVERSITÄT HAMBURG

Bericht Nr. 383

Preliminary flow tests with sand, sand/water mixtures and mush ice.

K. Wieghardt

Dedicated to Prof.Dr. D.h.c. Henry Görtler on the occasion of his 70th birthday in thankful memory of our 4o-year-old friendship

Juli 1979

Preliminary flow tests with sand, sand/water mixtures and mush ice.

Friction in a few particulate systems has been measured in a kind of Couette viscosimeter with free surface flow.

1) Test apparatus

A revolving vessel (diameter 1.4o m) is filled, e.g. with sand up to a height of o.5 m. Into this sand bed a piece of a large tube with outer radius R = o.352 m and wall thickness s = 5 mm is immersed cocentrically to a depth of h = o.o5 to o.2 m. On this tube the holding moment M is measured when the vessel containing the sand is set into rotation.

When this is done slowly by hand, a high starting moment M_{st} is measured instantaneously. After that, when the material is sliding slowly past the tube (say with a few cm/s), the moment drops down to a much lower value M_s.

In other test series the revolving vessel is driven by a motor and the angular velocity ω_o can be kept constant. The relative velocity at the tube wall then is $v = R \cdot \omega_o$ = o.2 to 1.6 m/s; such tests give M(h,v).

2) Results for dry sand

The sand used here is made of polystyrol which has a specific weight of γ_m = 1o6o kp/m^3, i.e. only 6% higher than for water. Apparently, this sand has been produced by cutting a wire into pieces; tests with different screens give a mean grain diameter of 1.43 mm. The sand was given us by Bundesanstalt für Wasserbau, Außenstelle Küste, Hamburg, where also shear tests have been made. The packing is characterized by κ = grain volume/total volume; e.g. for homogeneous spheres in densest packing $\kappa = \pi/3\sqrt{2}$ = o.74o and in cubical packing

$\kappa = \pi/6$ = o.524. For the polystyrolsand the angle ϕ of inner friction can be approximated by

$$\mu = \tan\phi \approx 3.3\kappa - 1 \text{ for } o.53 < \kappa < o.60, \qquad (1)$$

whether the sand is dry or wet. The normal pressure in the shear box was between o.5 and 2 kp/cm^2 and the shear velocity o.9 mm/min. The bulk weight $\gamma = \gamma_M \cdot \kappa$ varied here from 562 to 636 kp/m^3.

Fig.1 gives the starting moment as measured with the same polystyrolsand, yet on different days. Unfortunately, there is a large difference for the absolute values of M_{st}. Perhaps, the original packing was different and also the work to be done to dilate the sand in the neighbourhood of the fixed tube into a looser packing to enable sand flow. However, in both series M_{st} increases with the square of immersion depth

$$M_{st} \sim h^2. \qquad (2)$$

Hence, we assume the pressure p to be proportional to depth y and to bulk weight γ as in a fluid $p = \gamma y$. With Coulomb's friction coefficient μ the wall shear stress becomes

$$\tau = \mu p = \mu \gamma y \qquad (3)$$

and the moment

$$M = \int_o^h \left[2\pi R^2 + 2\pi (R-s)^2\right] \tau \, dy + 2\,(R-s/2)^2 s \quad (h) \qquad (4)$$

$$= \mu\gamma(o.767\ h^2 + o.oo4\ h) \text{ for } R = o.352 \text{ m, h in m.}$$

Since $h \leqslant o.2$ m we may neglect the second term.

In Fig.2 speed tests at $v \geqslant o.2$ m/s are shown: M/h^2 vs v. Obviously, the test points do not quite form a single street. It might be mentioned that they do so - at least for $v < o.5$ m/s or so - when M/h^2 is plotted vs $v \cdot h$, as if there existed in this dry sand a viscosity ν with the dimension m^2/s and a Reynolds number vh/ν .

In any case, Fig.2 shows that the moment depends strongly on speed at low velocity. Hence, the measurements of the sliding moment M_s at small, yet finite speed are not significant since this small velocity could not be measured; so these results are shown in brackets only. In the shear box tests at $v \approx 0$ it has been found after equ.(1):

	$\varkappa$	μ	γ	$\mu\gamma$
loose packing	o.53	o.749	562 kp/m^3	421 kp/m^3
	o.55	o.815	583 "	475 "
dense packing	o.58	o.914	615 "	562 "
	o.60	o.980	636 "	623 " .

The right hand scale in Fig.2 for $\mu\gamma$ shows that these values for $v \approx 0$ might well fit as limiting values for $v \rightarrow 0$ if one assumes a dense packing in the rotating sand due to unavoidable vibrationes, e.g. $\varkappa$ = o.58, γ = 615 and $\mu\gamma$= 562 kp/m^3.

For greater velocities ($v >$ o.5 m/s) the decrease of the moment is not only due to a decrease of friction with speed, but also because the sand surface becomes parabolical and the "wetted" surface of the fixed tube is also changed. On the rotating sand surface $y_s(r)$ the inner normal must have the same direction as the resultant of centrifugal and gravitational force on a sand grain there with mass Δm:

$$\frac{d\, y_s}{dr} = \frac{\Delta m\; r\, \omega^2}{\Delta m\; g} \; . \tag{5}$$

If ω = const = ω_o this gives

$$y_s = \frac{v^2}{2g}\,\frac{r^2}{R^2} + \text{const.} \tag{6}$$

where $v = R\,\omega_o$ is a constant characterizing each test.

Yet, in contrast to water, there is apparently no flow underneath the open edge of the fixed tube so that there is no exchange of sand inside and outside the tube. When each single sand volume inside the tube ($0 < r < R$) and outside ($R < r < R_V$= radius of vessel) remains constant, and if the angular velocity were the same everywhere, there were two parabolas

$$y_s - y_o = \frac{v^2}{2g} \left(\frac{r^2}{R^2} - \frac{1}{2} \right) \quad \text{for } 0 < r < R, \text{ and} \tag{7}$$

$$y_s - y_o = \frac{v^2}{2g} \left(\frac{r^2}{R^2} - \frac{R_V^2 + R^2}{2R^2} \right) \quad \text{for } R < r < R_V, \tag{8}$$

with y_o = original surface height at $v = 0$. With $R_V = 2R$ the sand surface would increase at the inner wall by

$$\Delta h_i = \frac{1}{2} v^2/2g \tag{9}$$

and decrease at the outer wall by

$$\Delta h_o = -\frac{3}{2} v^2/2g. \tag{10}$$

Centrifugal forces yield indeed an increase of sand height Δh_i inside and a decrease Δh_o outside the tube as shown in Fig.3. But there are also two boundary layers where the angular velocity $\omega(r)$ varies from a low value at the inner and outer tube wall to the constant value ω_o outside the boundary layer. Usually, there was some slip so that $0 < \omega(r=R) < \omega_o$. Because of this dependence of ω on radius r the surface in the two boundary layers decreased steeply towards the wall. Outside the tube the angle of repose increased with speed up to 45^o and more; the boundary layer thickness was a few cms. Since the boundary layer flow has not been measured in detail, it does not seem profitable to calculate this surface deformation starting from equ.(5). Exactly in this region dilatancy should also be taken into account.

No explanation can be given why in Fig.3 at original depths h = 7.5 and 10 cm (at v = 0) the change of sand height on the inner tube wall is negative whereas usually it is positive. At the other immersion depths the decrease of wetted surface on the outer wall is roughly almost compensated by an increase on the inner tube wall.

3) Results for a dense sand/water mixture

In the present tests with a large free surface the behaviour of a sand/water mixture is quite different from that of dry sand, although the same angle of inner friction - as measured in the shear box - has been found for dry or wet sand.

The bulk weight of the mixture depends on the specific weight of the sand material $\gamma_M = 1060$ kp/m^3, that of water $\gamma_W = 1000$ kp/m^3, and of the fractional solid content κ of the packing

$$\gamma = \gamma_M \kappa + \gamma_W (1 - \kappa) = 1000 + 60 \kappa \ [\mathrm{kp/m^3}] \qquad (11)$$

If the sand packing is the same as in dry sand: $\kappa \approx 0.58$ and $\gamma \approx 1035$ kp/m^3.

An earth pressure can now originate only from the difference

$$\Delta\gamma = \gamma - \gamma_W = (\gamma_M - \gamma_W)\kappa = 60 \kappa \ [\mathrm{kp/m^3}] \qquad (12)$$

Hence, one would expect the starting moment M_{st} to be much smaller here than in dry sand, viz. by the ratio of $\Delta\gamma$ to γ of the dry sand: $60\kappa / 1060\kappa = 0.0566$. Yet, after Fig.4 the actual moment is much higher: 105/(455 or 835) = 0.23 or 0.13 instead of 0.0566. Here, besides the tube with R = 0.352 m a smaller one was also used with R' = 0.207 m. After equ.(4) the ratio of the moments should be

$$(2\pi R^2 + 2\pi (R-s)^2)/(2\pi R'^2 + 2\pi (R'-s)^2) = 2.92.$$

This is confirmed when one assumes again $M \sim h^2$, as $105\ h^2 / 36\ h^2 = 2.92$. The moment at h = 0.05 m was sometimes too small to be measured by this test equipment (two points in brackets).

The starting moment depends on frictional tangential shear at rest, or rather its limiting value before motion sets in, and on the work to dilate sand into a looser packing. Obviously, these phenomena cannot be converted here simply by replacing the bulk weight γ of dry sand by $\Delta\gamma$ for the sand/water mixture. Yet, for the passive normal earth pressure on a body dragged through sand or sand/water at vanishing speed this procedure works very well as is shown in the appendix.

At non-zero speed v the behaviour of the mixture is even qualitatively quite different from that of dry sand. Fig.5 shows how intricate the dependence on M(v) is; the moment depends here on depth h rather linearly than with h^2.

The measurements in Fig.5 were made when there was a water surplus of about 4 cm above the sand at rest. Without such a surplus, i.e. when at rest the water just covers the sand, the moment is considerably greater. The difference ΔM of the moments without and with water surplus is after Fig.6 roughly independent of speed and immersion depth. This is again an effect of sand dilatancy. In the inner and outer boundary layer of the tube with relative motion of sand grains the packing is loosened and more water is needed there. Hence, the free water/air surface is locally dragged into the narrow interstices of the sand arousing additional capillary forces. This effect was also observed in resistance tests where it could be much reduced by a detergent reducing surface tension.

The rotating sand surface (with or without water surplus above it) is a single paraboloid now and the sand height at the fixed tube is indeed reduced by $v^2/2g$ at the inner and outer wall. This follows also from equ.(6) with $\omega = \omega_0$ i.e. when the boundary layers are neglected.

Hence, replotting the values M(h,v) from Fig.5 vs wetted depth $(h-v^2/2g)$ gives Fig.7. For all depths h = o.1, o.15 and o.2 m at higher velocities between o.5 and 1.2 m/s we now get

$$\begin{aligned} &\text{with water surplus} \quad M = 4.2\cdot(h - v^2/2g) \text{ in mkp, and} \\ &\text{without " \quad "} \quad M = 0.29 + 4.2\cdot(h - v^2/2g). \end{aligned} \tag{13}$$

Apparently, at $v > 0.5$ m/s the flow in the boundary layers has become turbulent, the sand grains are whirled up and their small overweight is no longer important. The tubulent shear stress is practically independent of depth and velocity in the test range; its mean value follows from

$$M(h,v) = 2\cdot 2\pi(R-s/2)^2(h - v^2/2g)\overline{\tau} \qquad (14)$$

$$\overline{\tau} = 2.7 \text{ kp/m}^2.$$

Unfortunately, no solution seems available for this three-dimensional Couette flow (with an open end) to compare with a Newtonian fluid. After J.Happel and H.Brenner, "Low Reynolds number Hydrodynamics" p.463, Nordhoff, Leyden, one would expect the viscosity of this dense sand/water mixture to be 1o or 2o times that of water.

4) Results for mush ice

For similar tests Dr.J.Schwarz, HSVA, supplied us with broken ice from his ice tank. Much water was added to produce a somewhat homogeneous ice/water mixture. On the first testing day (2nd May 1979) it contained some large floes of up to about 1o cm diameter. For the second tests on 4th May these large pieces were taken out or crushed. To describe the consistence of this mush ice: one could write with a finger in it and the traces held on for a few hours. Probably it reached down to the bottom of the vessel without a sheet of water there; when the vessel was put into rotation the whole mixture was taken along at once as with a rigid body.

After Fig.8 the starting and sliding moments on the tube with R = o.352 m are of the same order of magnitude as in Fig.4 for the sand/water mixture. But, here the moment M_{st} depends only linearly on depth h, or possibly like $h^{3/2}$. When we evaluate the second series only because this mixture was more homogeneous we must assume $M \sim h$, which again means that the shear stress on the inner and outer wall of the tube was independent of depth and weight

$$M = 2\cdot 2\pi(R - s/2)^2 h\,\tau = 1.53\ h\,\tau\ . \qquad (15)$$

For depths h = o.1, o.15 and o.2 m we get

starting moment	M/h = 25 kp	or τ = 16 kp/m^2
sliding "	17 "	11 "
v = o.2 m/s	6.3"	4 " .

This finite shear in a slip plane (without a visible boundary layer flow) which is independent of normal pressure reminds one of cohesion. This is an essential material property - e.g. in clay in contrast to cohesionless sand - but also here in mush ice. At small gliding speed it decreases rapidly at first and increases slowly for $v > 0.2$ m/s.

At constant speed $v = 0.2$ or 0.4 m/s on part of the outer wall of the fixed tube a crevice was observed, perhaps caused by a slight excentricity of the tube. It was filled up by tapping (vertically) the rotating ice next to the tube; without this precaution the moment was about half as small as in Fig.8.

Appendix

For reproducible future measurements in mush ice it could be useful to characterize the respective ice by the result of a simple test. Hence, the drag of a vertical rod was also measured when the vessel was set into rotation. Fig.9 gives the starting and sliding moment D_{st} and D_s for two rods with diameter $d = 4.5$ resp. 3 cm at depths $h = 0.1$, 0.15 and 0.2 m.

Admittedly, the scatter of the results for the thin rod is perhaps too large for any serious evaluation, yet we can not resist trying. The drawn lines correspond to

$$D_{st} = 21\,\gamma\, h\, d^2 \quad \text{and} \quad D_s = 17\,\gamma\, h\, d^2, \text{ with } \gamma = 1000 \text{ kp/m}^3.$$

On the other hand, one would expect that here not the spec. weight γ is important but rather the cohesion τ. Yet, a formula like $D \sim \tau \cdot h \cdot d$ could fit only to one diameter d, e.g. with $\tau = 16$ kp/m^2 (from p.7) for the thick rod $D_{st} = 59\,\tau\, h\, d = 42.5\, h$. Yet, for the thin rod this would give $D_{st} = 28\, h$, which is too great compared with test results. More tests are needed here; possibly the ratio rod diameter to seize of average ice particle is important too.

On the other hand, interpretation of such drag tests in sand and sand/water mixtures is possible even including the

the dependence of drag on velocity. Many tests with rods of various cross sections have proved that here $D \sim h^{5/2} b^{1/2}$ with b = greatest breadth normal to flow. The pressure p on the front part of the rod has also been measured as $p \sim \gamma h \sqrt{h/b}$. Accordingly, a distorted Froude number is useful in Fig.1o and 11:

$$v_* = \frac{v}{\sqrt{gh}} \cdot \left(\frac{b}{h}\right)^{1/4} \sim \sqrt{\varrho v^2/p}, \quad (\text{with } \varrho = \gamma/g)$$

where either the bulk weight γ or the difference $\Delta\gamma$ is introduced. At vanishing speed in a mixture $\Delta\gamma$ yields a passive earth resistance just as in dry sand γ (where the air weight is negligible). At $v \neq 0$ the momentum of displaced bulk mass of the mixture gives an additional drag of $8 \varrho hbv^2$ independent of gravity. In dry sand this mass is much smaller and the friction decreases with speed, so that total drag decreases somewhat at first with speed.

(The sand/water mixture at high speed becomes diluted at the test station R = o.5 m by centrifugal force because the sand grains are denser than water and so the drag is reduced.)

All measurements were performed by Herr F.Meyer whose expertise is duly acknowledged.

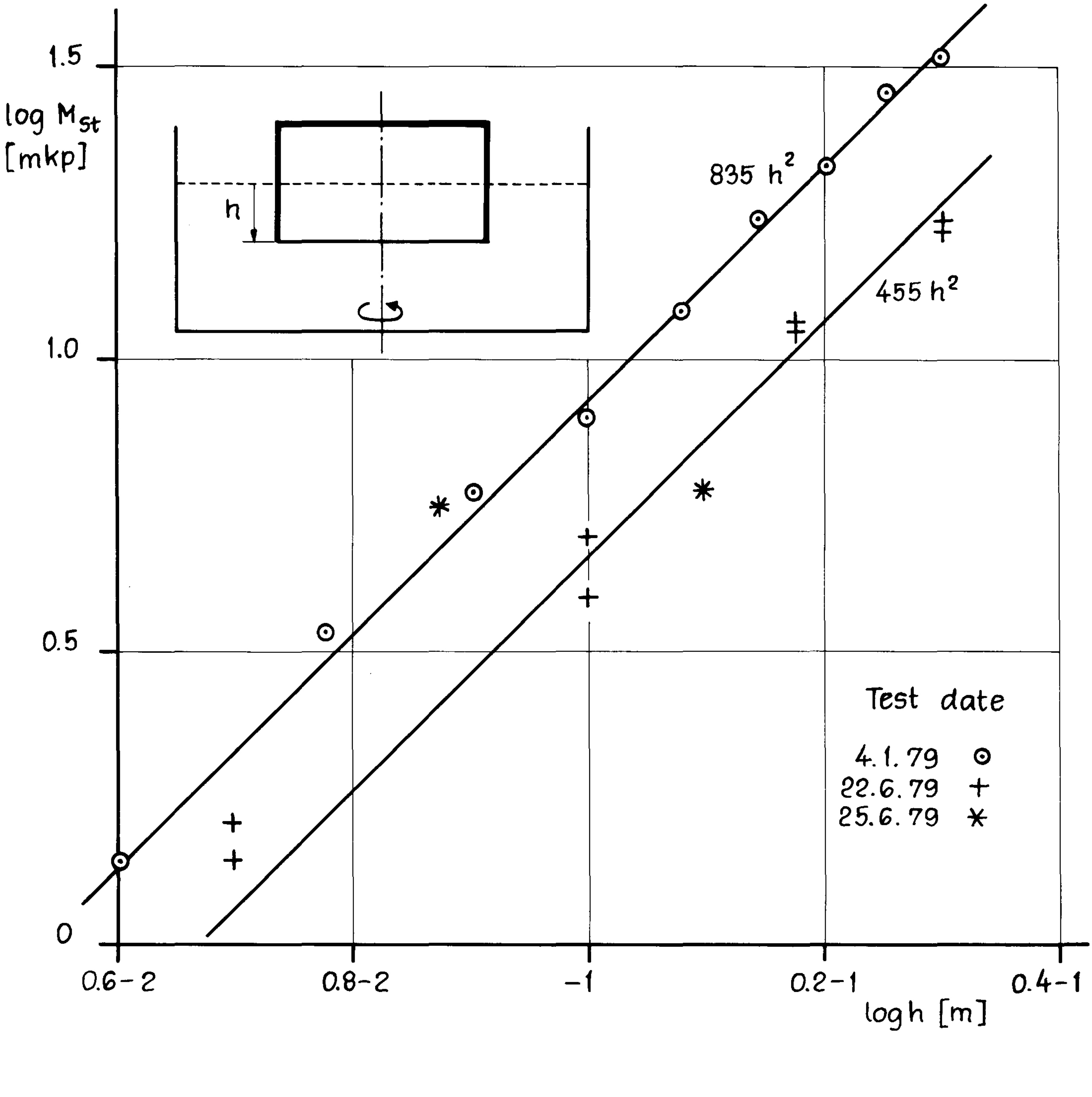

Fig. 1

Dry polystyrolsand : starting moment M_{St} vs depth h

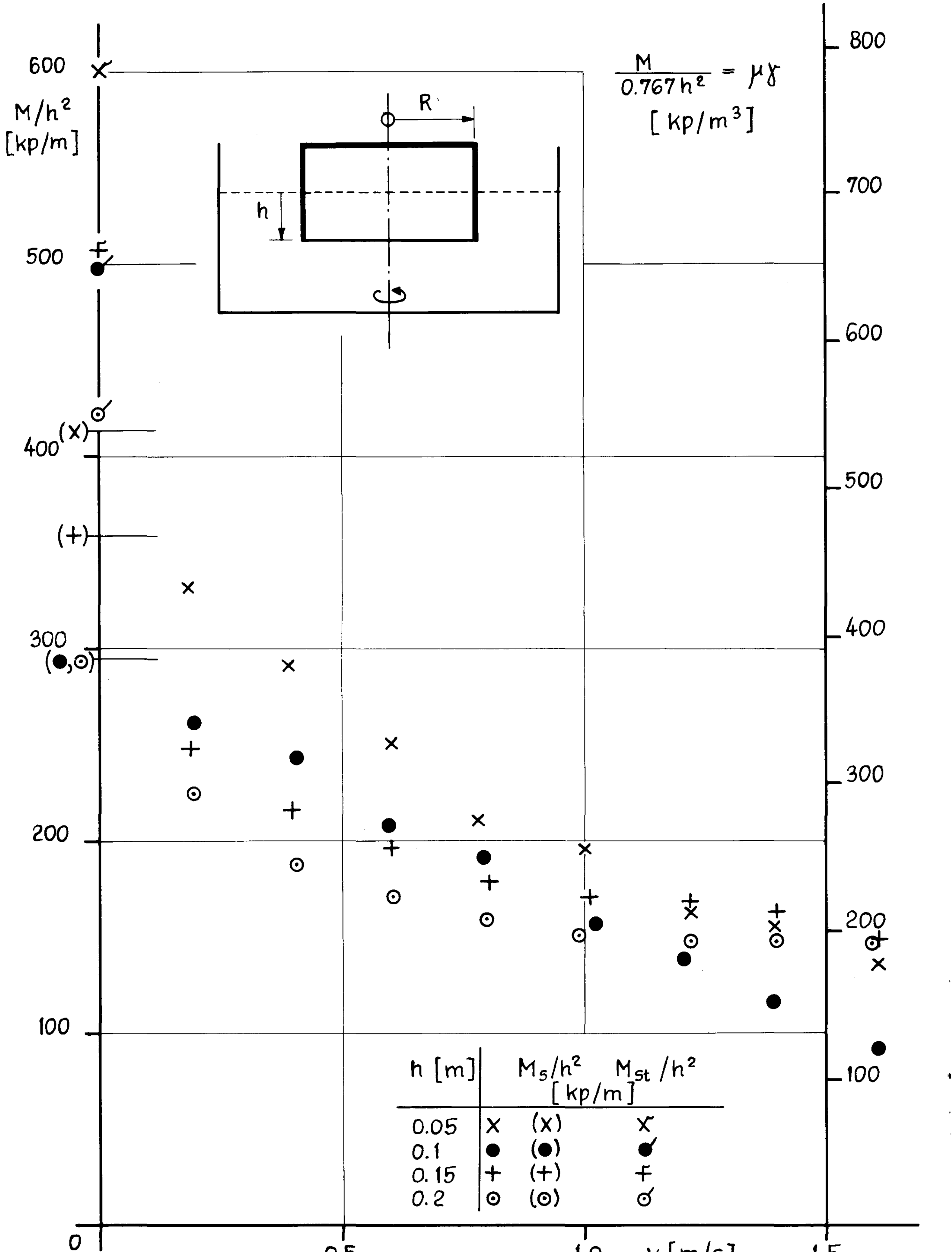

Fig. 2

Dry polystyrolsand : moment at various depth vs speed $v = R\omega$

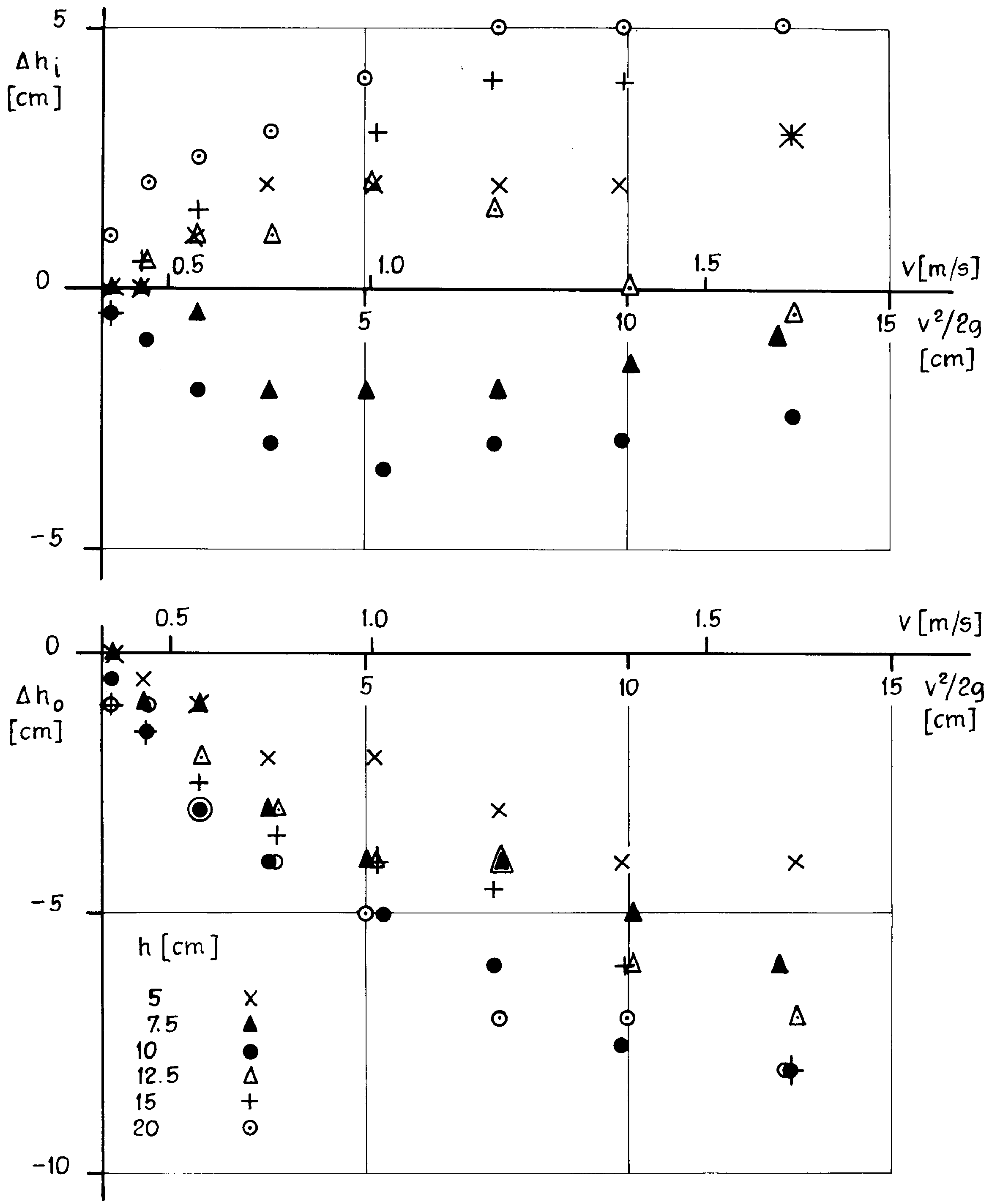

Fig. 3

Dry polystyrolsand : change of immersion depth h at the inner wall Δh_i or outer wall Δh_o of the fixed tube due to change of the rotating sand surface.

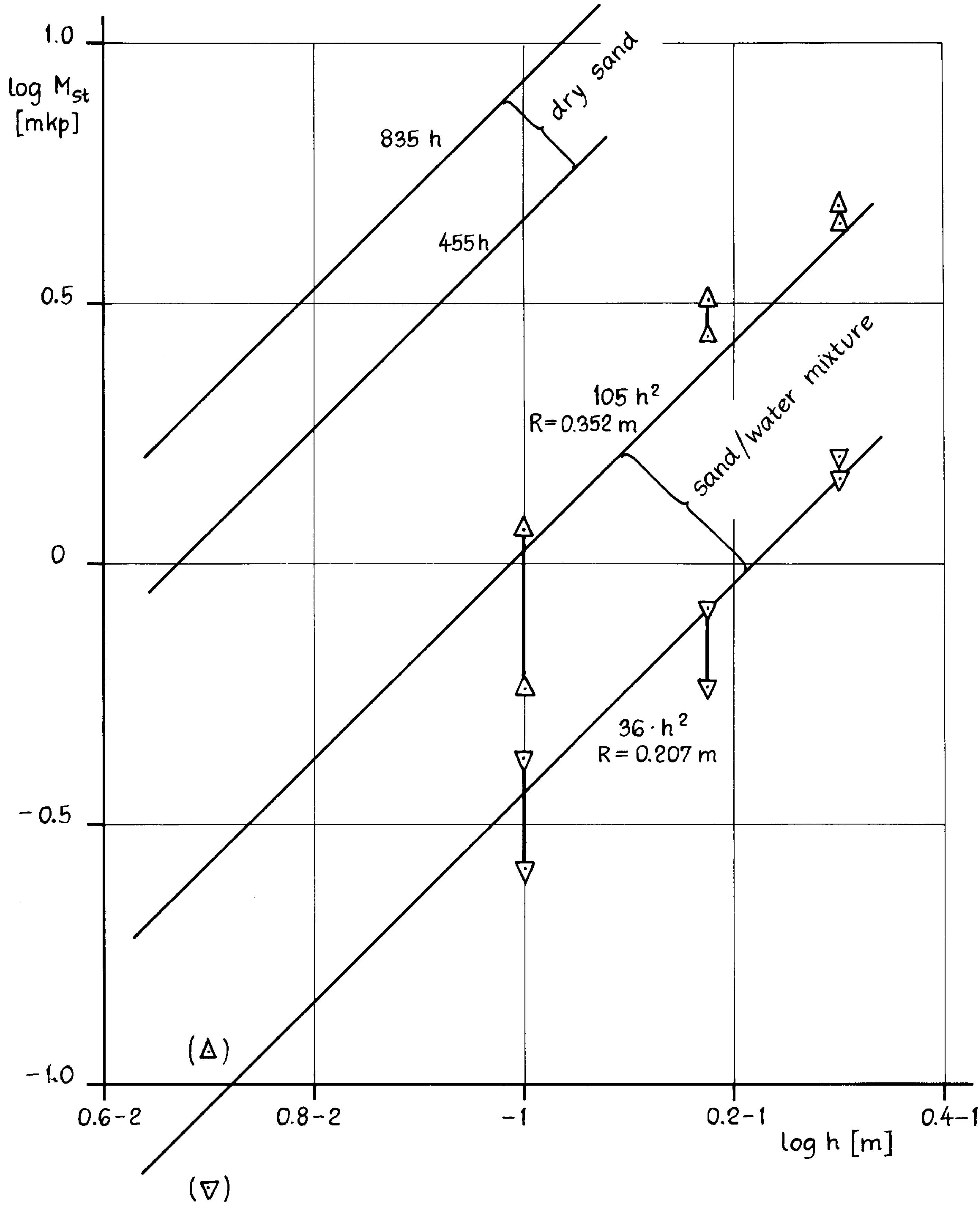

Fig. 4

Sand / water – mixture : starting moment M_{st} vs depth h for two fixed tubes (R = 0.352 m or 0.207 m)

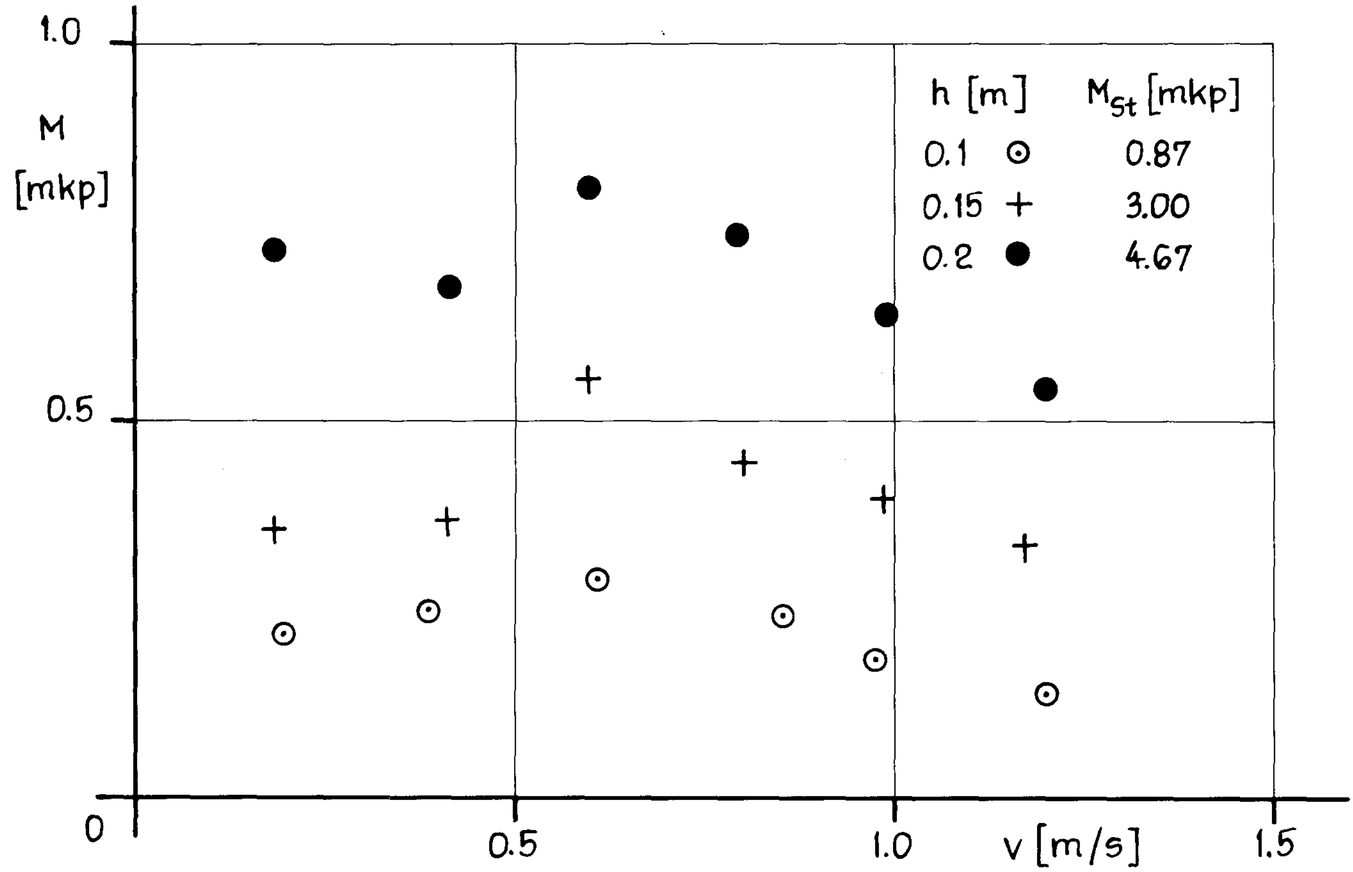

Fig. 5

Sand / water mixture with water surplus:
moment M(h,v) vs velocity

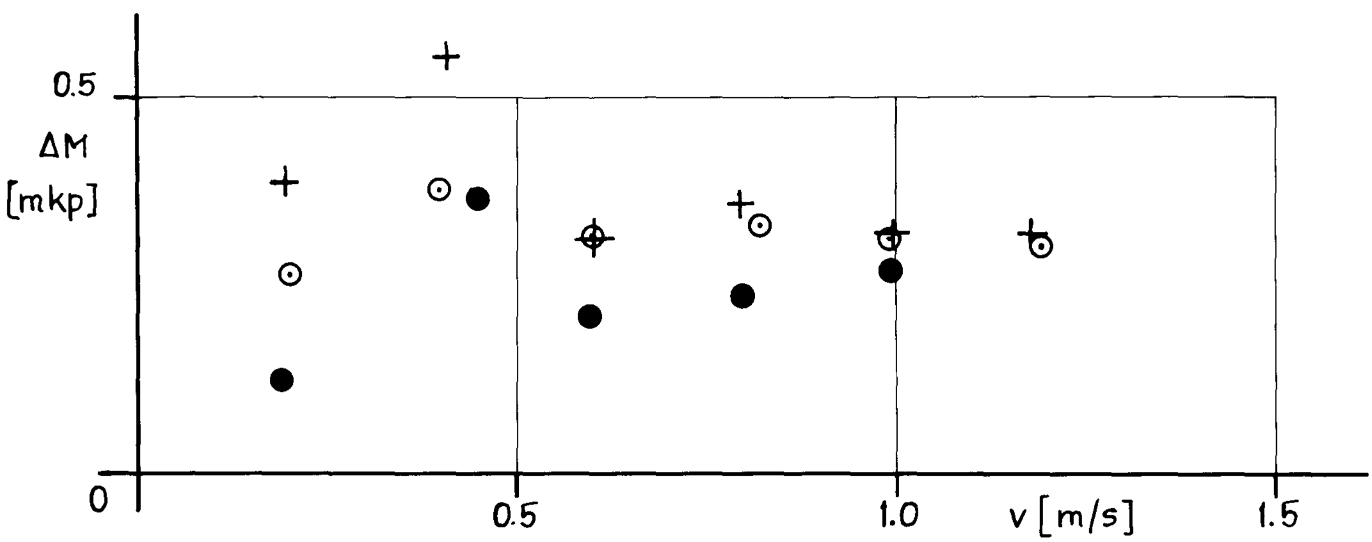

Fig. 6

Difference of moments without and with water surplus

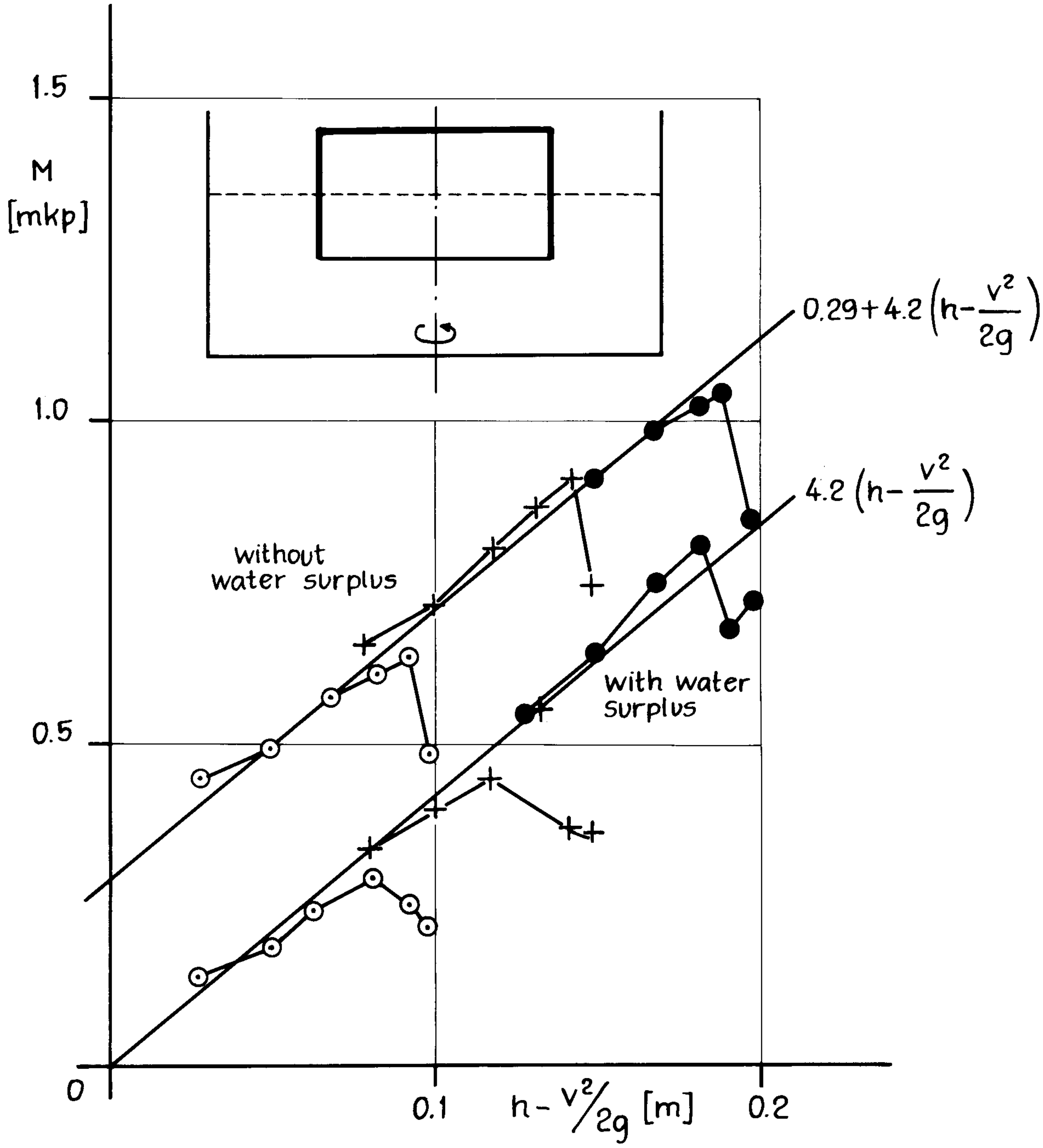

Fig. 7

Polystyrolsand / water mixture : moment vs wetted depth. For $v > 0.5$ m/s wall shear is independent of depth and velocity; with water surplus

$$\tau = \frac{M / (h - v^2/2g)}{4\pi\,(R - s/2)^2} = 2.7 \text{ kp/m}^2$$

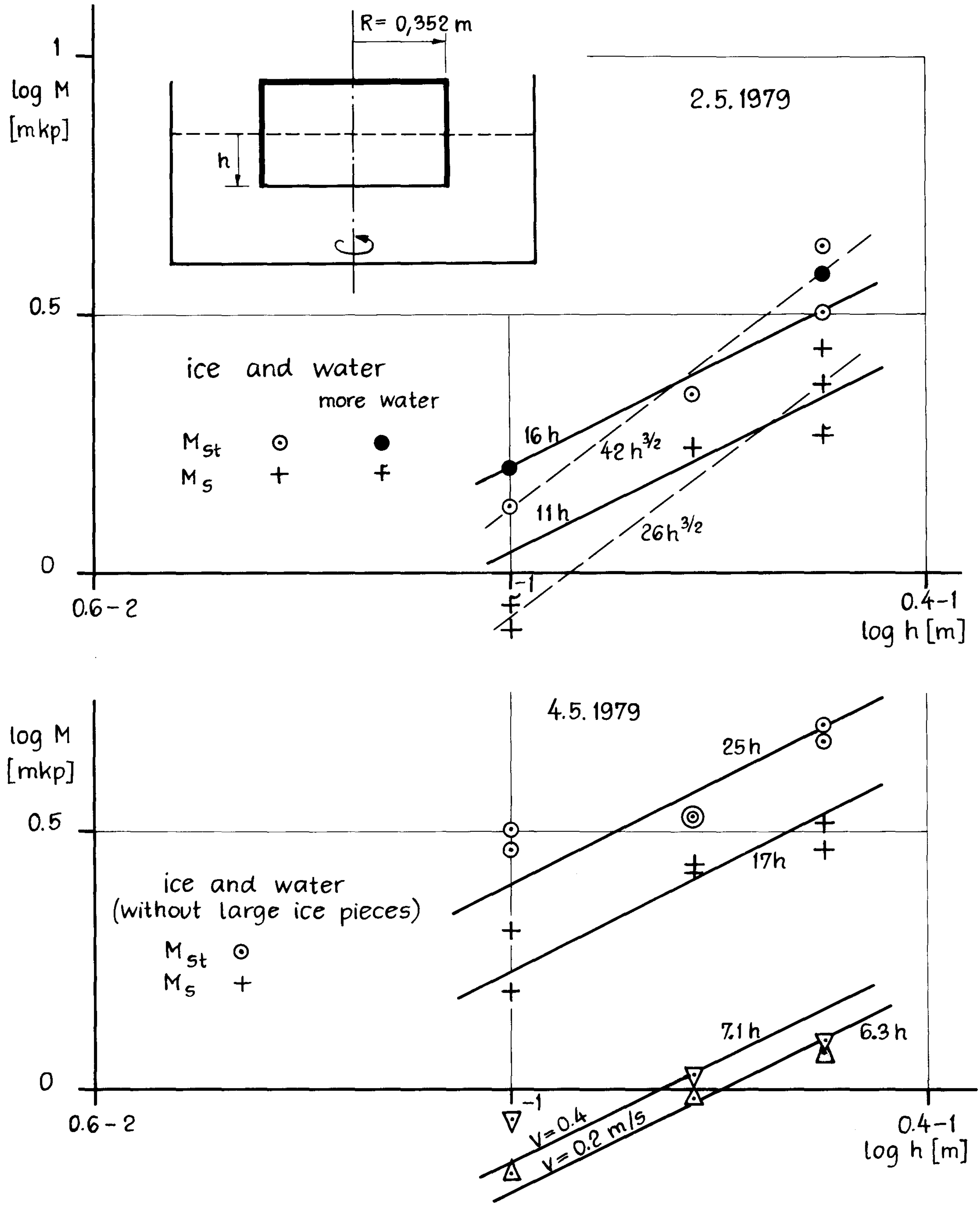

Fig. 8

Mush ice : starting and sliding moment vs depth h and M (h, v) for v = 0.2 or 0.4 m/s

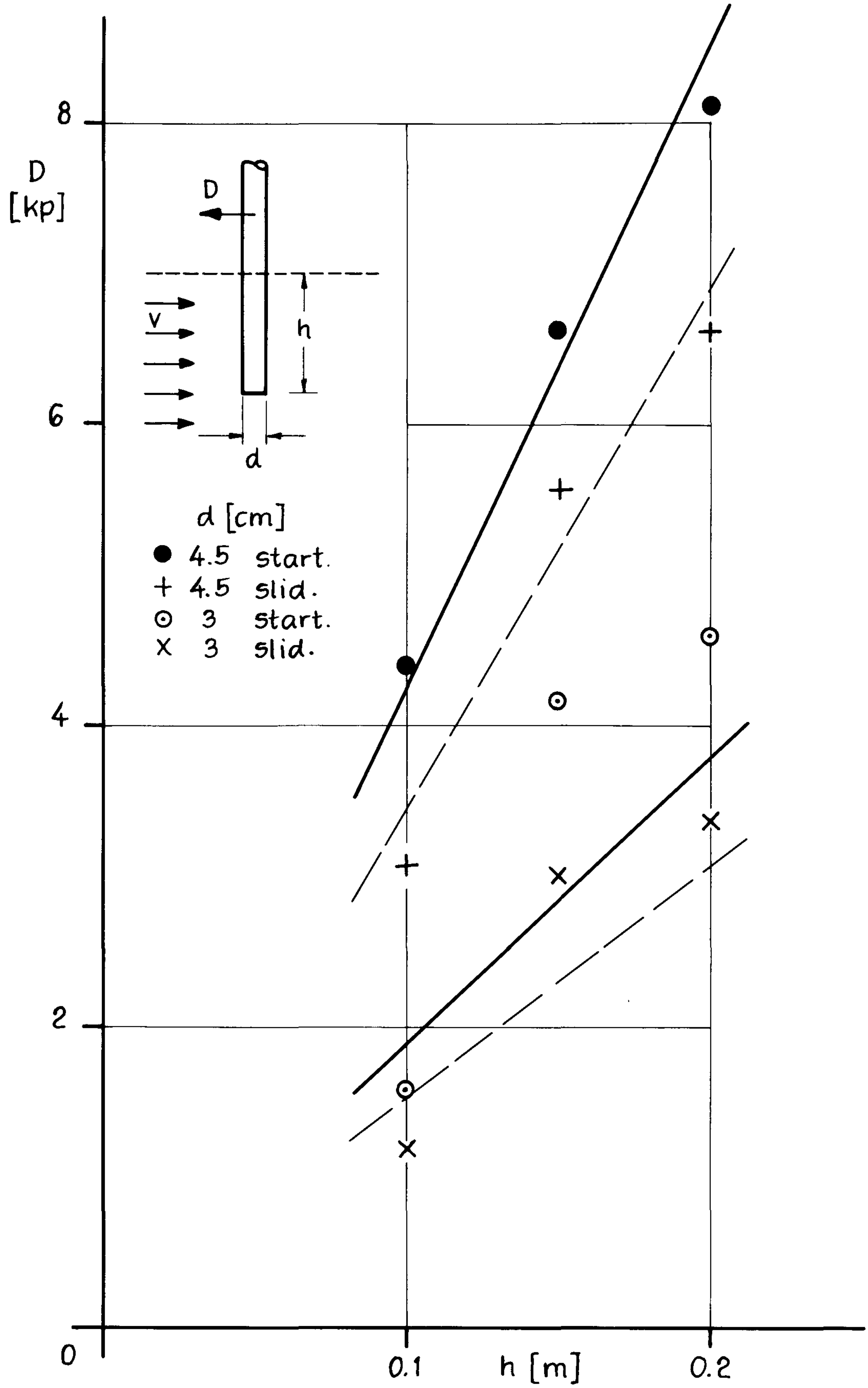

Fig. 9

Drag of a vertical rod in mush ice; diameter d, depth h.

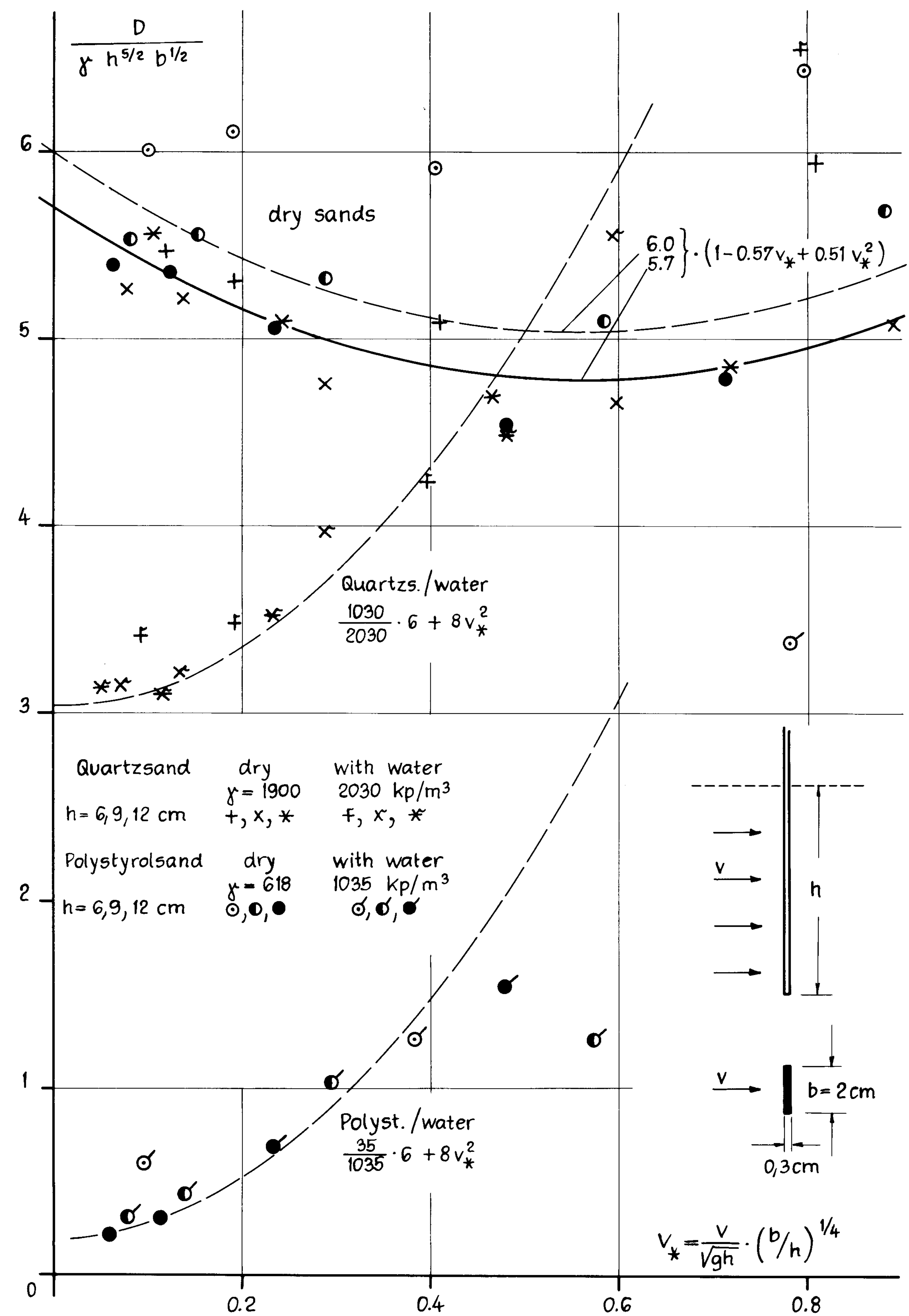

Fig. 10 Drag of a flat bar in various media vs speed

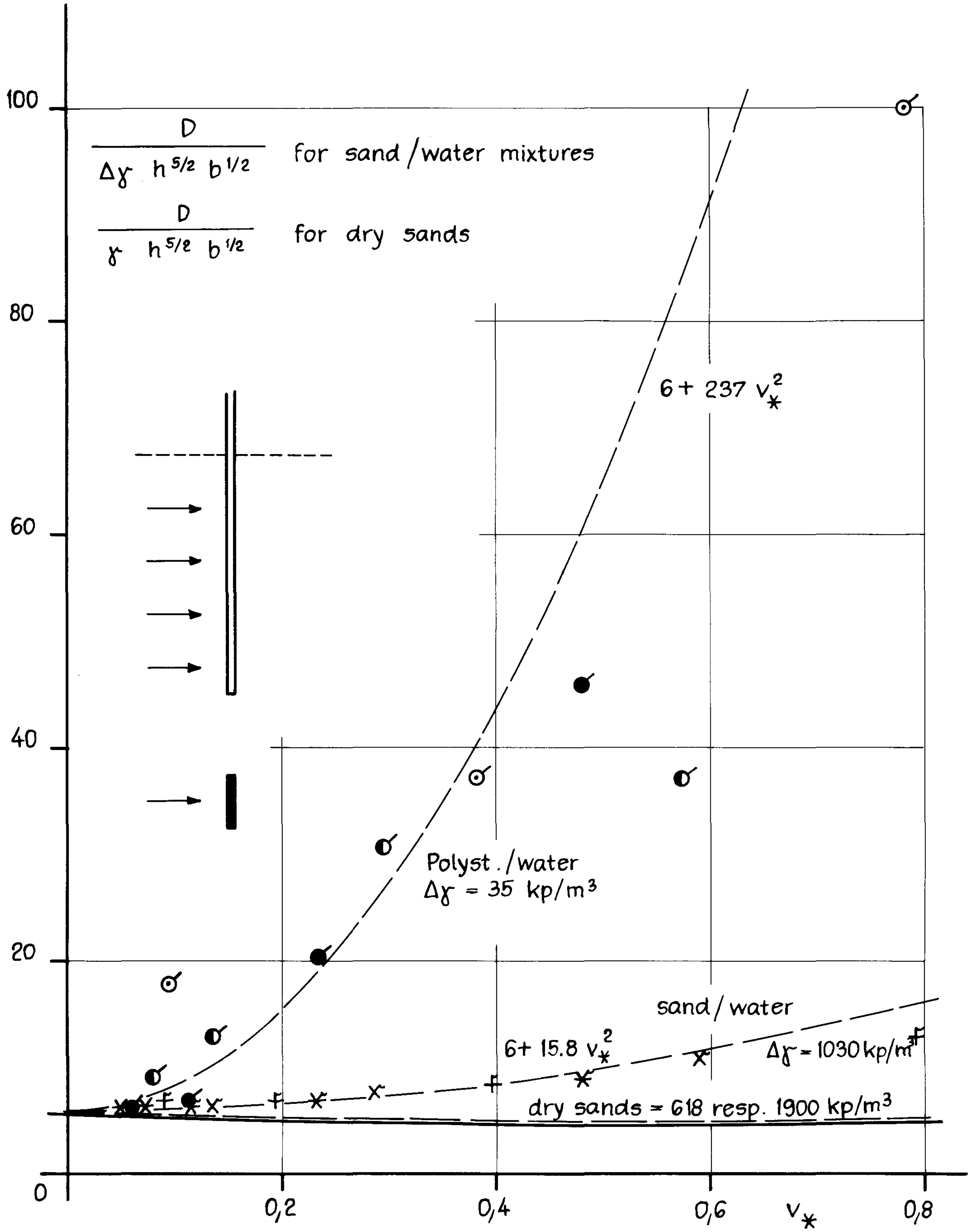

Fig. 11

Drag D of a flat bar in various media vs speed