

376 | Juni 1979

## SCHRIFTENREIHE SCHIFFBAU

J.V. Wehausen

### I. Georg-Weinblum-Gedächtnis- Vorlesung

**TUHH**

*Technische Universität Hamburg-Harburg*

## I. Georg-Weinblum-Gedächtnis-Vorlesung

J.V. Wehausen, 1. Auflage, Hamburg, Technische Universität Hamburg-Harburg, 1979

© Technische Universität Hamburg-Harburg

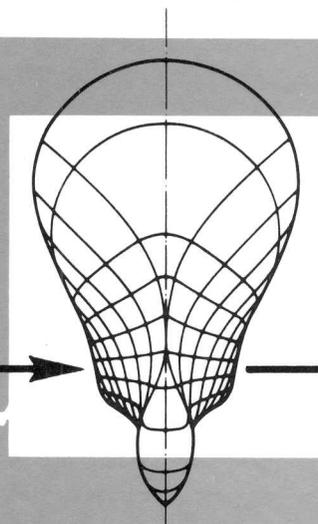
Schriftenreihe Schiffbau

Schwarzenbergstraße 95c

D-21073 Hamburg

<http://www.tuhh.de/vss>

INSTITUT FÜR SCHIFFBAU  
DER UNIVERSITÄT HAMBURG



## I. Georg-Weinblum-Gedächtnis-Vorlesung

gehalten von  
J.V. Wehausen, Berkeley

Juni 1979

Bericht Nr. 376

INSTITUT FÜR SCHIFFBAU DER UNIVERSITÄT HAMBURG

Bericht Nr. 376

I. Georg-Weinblum-Gedächtnis-Vorlesung

Ship Theory, Ship Design and Georg Weinblum

gehalten von

J. V. Wehausen, Berkeley

am 22. November 1978 in Berlin

und

am 29. März 1979 in Washington, D.C.

Hamburg, Juni 1979

Ship Theory, Ship Design and Georg Weinblum

by

J. V. Wehausen

If one had to categorize Georg Weinblum among naval architects, it would be fair, I believe, to call him a ship theorist. He himself liked the term "ship theory", his own research was primarily devoted to this, he supported it among others, and one of his chief goals as a teacher was to try to provide prospective naval architects with a good understanding of the fundamentals of their profession.

I have been rereading many of Georg Weinblum's papers during recent months and one aspect that has struck me is his constant concern for the application of ship theory to ship design. In almost every paper I have looked at, some part of it is devoted to its implications for design. Some are almost completely oriented in this direction.

This isn't surprising, I suppose. Georg Weinblum was serious about his profession, in fact, more than that, he was enthusiastic about it and he was convinced that the study of ship theory could and would result in the improvement of ships. It is just this last point that I should like to examine. Does ship theory really play an important role in ship design, and if not, could it? This is a question that any ship theorist must ask himself from time to time.

Note that I am not asking if ship design needs ship theory. It seems clear, fairly clear anyway, that ship designers could get along by the method of trial and error, of cut and try. In a field like naval architecture, with a long history behind it, one has the option of starting any more-or-less conventional design problem from a safe position. The designer may indulge his spirit of creativity by making small modifications of a known acceptable design. Whether well-founded or not, they do not usually result in disaster, and if the consequences are observed and recorded, these small changes contribute to the advancement of the art of design. I suppose that much of the progress in technologies of all sorts has been a result of such an empirical process. Of course, there are examples where a "small" change does result in a disaster, but these are also learned and avoided.

If one doesn't actually need ship theory, where then does it fit in? One obvious place is in those design situations where there is no tradition to start from, and another is where something more than a "small" modification is required to achieve a real advance in design practice. Let us look at some examples.

Our field is particularly rich these days in examples of the first kind, those situations where one cannot rely upon a well tested body of experience, codified in rules, but only upon one's intuition,

model tests and the laws of mechanics. I am thinking, of course, of the design of the various off-shore platforms, some floating, some fixed after they are towed into position. Their diversity is enormous, as one can easily see by thumbing through some issues of Ocean Industry, and they attest to the inventiveness of their designers. But this ingenuity in conception needs to be supported either by a tradition of experience or by calculations based upon the laws of mechanics. Since the former is lacking, reliance must be placed upon the latter.

To be faced with such problems is, of course, a frightening prospect. One is suddenly painfully aware of the limitations in the various theoretical developments. For example, in calculating hydrodynamic forces, may one really neglect viscosity or is it just that we don't know what else to do, are linearized approximations adequate, what sort of spectrum should one assume for the incident waves, what does a 100-year wave really signify, etc? A particular problem can no longer be set aside because it is "messy" or not a good academic research problem. It must be accepted as it has been presented by circumstances, although it is indeed prudent to confirm that the "right" problem has been presented if it has been formulated by someone else.

Still, decisions have to be made and calculations carried through, and it is just the academic research based upon clear-cut problems that must provide the background and basis for dealing with the more complex problems. Indeed, unless one can examine simple problems first, one cannot usually analyze the complicated problems presented to an engineer. A direct attack upon a too complicated problem may be in danger of not uncovering the underlying principles.

In recent years I suppose that more than half of our students have ended up doing ocean engineering rather than naval architecture in its strict sense. However, it is essentially ship theory that they apply, for if they have understood, for example, the uses of potential theory in calculating the motion of a ship in a seaway, they also know how to deal with floating bodies of different configurations. Indeed, such computations have become close to commonplace. And even more esoteric ones, such as the second-order drift forces acting on floating bodies moored in waves, occur as part of the design process. The number of such examples can be increased considerably and they cover most of ship hydrodynamics. Moreover, parallel and more pressing ones exist in ship-structure theory.

Does this mean that every naval architect or ocean engineer must be an expert in the mathematics of the equations of fluid dynamics and structures? I think not. Just as one can make effective use of the telephone without understanding, or even being very interested in how it works, one can make effective use of ship theory without digging into the details. Georg Weinblum's papers on ship motions illustrate the possibilities. In an early paper [Z. VDI 78 (1934), 1373-79] concerning the motion of a ship in a seaway he has emphasized that in using the linearized equations of motion, it is important to take into account the added-mass terms derived from the hydrodynamic force, as well as the damping terms. The actual calculation of either of these terms is, of course, difficult and methods are still being developed for

doing it, especially in three dimensions. However, once one knows that such terms must be taken into account, one may try to find their values by other means than mathematical calculation, by model tests for example. Ship theory has still played an essential role by identifying quantities easily overlooked in a more elementary analysis. Indeed they were overlooked by Krylov in his classic papers on ship motions.

In my opinion, applications of ship theory of this sort, i.e. ones that clarify the nature of the underlying physics and point out the appropriate equations to express it, are at least as important as the specific calculations they may lead to. It is a consequence of this point of view that the education of naval architects should include enough ship theory so that these fundamental aspects are always within their grasp. Even though detailed calculations may be left to specialists, the designer needs to recognize when certain physical phenomena are important, how they show up in the calculations, and to know that they can indeed be calculated by specialists. These were essentially Georg Weinblum's pedagogical principles. He introduced them at the Institut für Schiffbau in Hamburg and also at the University of California in Berkeley. Since then they have spread world wide. The aim is not to make every naval architect a ship theorist, but rather to instruct him to recognize when and where ship theory is useful and how to make use of it when it is.

I should like to think that wave-resistance theory has played a role in practical design similar to ship-motion theory, especially since Georg Weinblum devoted so much effort to it, but I believe that I would be overstating the case if I claimed this. The insights that wave-resistance theory, in the form of Michell's integral, could have offered to ship design had already been discovered empirically through model-series testing. Furthermore, it has not proved useful as a computational procedure for predicting wave resistance; it is simply not accurate enough for ships of normal dimensions. For ship motions linearized theory seems to work well enough, for wave resistance it does not. Perhaps one must be content to call it bad luck. Nature is not always kind.

This brings us to the second category of situations where ship theory can make a contribution to ship design, those where small modifications of existing designs will not disclose a possible significant advance. Here again we have a splendid example, and one in which Georg Weinblum played a part, the bulbous bow. It doesn't seem reasonable to assume that the large bulbs used on contemporary ships could have been developed by a step-by-step process from the small bulbs used earlier. They are too far from the norm of earlier days and would have offended the aesthetic sense and probably the common sense of almost any practicing naval architect. Something more fundamental was needed that would allow one to get beyond this barrier of tradition. And indeed it was the analytical investigations of Inui, Takahei and Kumano followed by model tests, that showed the substantial improvements that could be obtained with large bulbs. However, in this case the analytical investigation had to come first and to suggest that a radical revision of the conventional ideas concerning bulbs was necessary. Of course, once Inui and his colleagues had completed these pioneering investigations, others could begin to

improve bulb design by "small steps". I think I should also add that Inui's analytical investigations were based upon an application of a modification of the linearized wave-resistance theory that I have just discredited. The fact that theory does not yield useful quantitative information does not mean that it cannot give important qualitative information that can then be improved and refined by empirical or other means.

I have mentioned that Georg Weinblum played some part in the development of bulbs. Because of his interest in design, he had concerned himself for most of his professional life with ships of minimum wave resistance. In 1934 both he and Wigley, more or less simultaneously and independently, published papers whose purpose was to explain the working of the bulbous bow. In each paper part of the analysis was devoted to the optimum size and position of the bulb. In my opinion either one of these two could have discovered the efficacy of large bulbs at this time. Better computing machines would have helped, of course, but a more serious handicap, I believe, was the technical climate. A serious suggestion at that time that bulbs of a type now commonplace should be used would not likely have fallen upon sympathetic ears. Indeed, I imagine that 35 years later it required some courage on the part of Inui to make this suggestion.

The story of the bulbous bow cannot be terminated, of course, without mentioning that remarkable bit of serendipity, the even greater and unanticipated success of the bulbous bow for ships in ballast at low speeds. As everyone now knows, this was finally explained by the ingenious experimental investigations of S. D. Sharma, described in a paper by Eckert and Sharma, [Jbuch STG, 64(1970), 129-171, esp. pp. 140-158]. However, can one really attribute this astonishing advance in ship design to an application of ship theory? In the narrowest sense, certainly not. The theoretical computations of Inui and his colleagues certainly did not include the possibility of eliminating wave breaking at low speeds by ships in ballast, nor would they have supported the use of a large protruding bow at low speeds. On the other hand, without these calculations and the consequent introduction of large bulbs, it is unlikely that their effectiveness in such situations could have been discovered. But perhaps in this case we should be content with our good fortune and not inquire too deeply into its origins.

Are there other examples where a theoretical insight has opened up new design possibilities not likely to have been discovered by taking only small steps? I don't think that one should anticipate finding many of these, but at least one more comes to mind, supercavitating propellers. Without Marshall Tulin's initial development of a linearized theory for cavitating hydrofoils and of the consequent foil shapes of least drag/lift ratio, it is unlikely that the idea of designing supercavitating propellers would have presented itself. Propellers behaving in this fashion would certainly have occurred, but Tulin's discoveries allowed the presence of a cavity to be included in the design and the performance to be optimized under this condition.

In the above remarks, I have restricted myself to accomplishments of ship theory that have taken place in relatively recent times, but ship-theoretical calculations that have become

commonplace are still ship theory. Even the hydrostatic calculations were once revolutionary. The story of Archimedes' excitement upon discovering a hydrostatic law is too well known to need repeating. Those developments in ship theory that prove useful in design will certainly, in the future, become as much a part of a naval architect's tool kit as hydrostatic calculations or Froude's Hypothesis are today. And this is, of course, what Georg Weinblum foresaw and planned for when he proposed a modernization of the curriculum of study for naval architects that would allow them to grasp the significance of the most recent developments in ship theory. In the years to come this may prove to be his greatest contribution to Naval Architecture.

A Phenomenon Observed in Transient Testing

by

O. J. Sibul, W. C. Webster and J. V. Wehausen

## Introduction

The phenomenon that will be discussed in this paper was observed in the course of some experiments with a quite different aim. The original purpose was to investigate the effect of bottom irregularities upon a ship moving in water shallow enough so that the bottom could be "felt" by the ship. In the initial experiments the ship, a model of Series 60, block 60, was held fixed and the "bottom irregularity" consisted of a rectangular box lying on the floor of the towing tank and spanning it. The forces acting on the struts holding the model were then recorded for several different model speeds. What had been anticipated was a force response localized in time to which we hoped to apply Fourier analysis and thence to predict the response to bottom obstructions of other shapes. Instead of the localized response, the record showed something that might be interpreted as this followed by a slowly decaying almost periodic response that we decided to call "ringing". The word "almost" should be noted, for, as we shall see below, careful inspection of the records often showed more than one period. Figure 1 shows a typical record taken for water depth  $h = 1.191$  ft, model velocity  $v = 2.67$  ft/sec and model length  $L = 5$  ft.

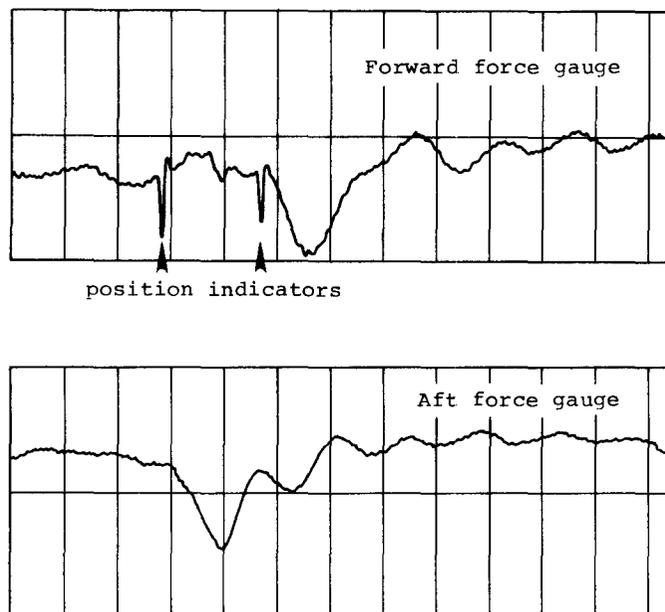


Figure 1.

Varying the height or length of the obstruction had no appreciable effect upon the measured period of the ringing. It also appeared to be independent of the clearance of the hull over the obstruction and of the hull shape. On the other hand, there was a very definite dependence upon speed and upon water depth.

One other phenomenon was observed in carrying out these experiments. Unless considerable care was taken to have the towing carriage accelerate slowly up to its test speed, the record showed a periodic response also before the model passed over the obstruction, with a period the same as that observed in the ringing.

It was evident that the configuration of the obstruction or of the hull played only a very small role in the observed period of ringing. The remaining variables that seemed relevant were the ship length  $L$  and the tank width  $b$ . If  $T$  is the period,  $h$  the water depth and  $v$  the speed, it appeared that

$$T = f(v, h, L, b).$$

If we include also  $g$ , but assume a negligible influence of viscosity, this may be written nondimensionally as follows:

$$\frac{gT}{v} = F\left(\frac{v}{\sqrt{gh}}, \frac{L}{h}, \frac{b}{h}\right).$$

### Why the Ringing?

The circumstances under which ringing occurs give a hint as to the physical cause. An abrupt start, a so-called "hard start", piles up the water in front of the ship model. This water must then dissipate itself in some fashion. Similarly, when the model passes over an obstruction, the water between the obstruction and the ship is squeezed upwards and again must dissipate itself. The precise form of these "humps" of water and of the associated velocities is, of course, difficult to determine. However, as we shall see, some quantitative conclusions can be drawn without its being necessary to have detailed information.

The situation that prevails just after the hard start or just as the model passes over the obstruction is similar to that in a classical problem in the theory of water waves, usually called the Cauchy-Poisson problem. In this problem one starts with some initial configuration of the free surface and/or initial velocity distribution and asks for the future motion of the fluid. Because the solution of this problem is important to us, let us review it.

### The Cauchy-Poisson Problem

Because the two-dimensional version is easier we begin with it. We shall assume irrotational motion and use the linearized boundary conditions on the free surface. Let the fluid at rest occupy the strip  $-h < y < 0$ ,  $-\infty < x < \infty$ , with  $y = 0$  being the surface of the undisturbed fluid. Let  $y = Y(x, t)$  be the equation for the free surface at time  $t$ . We suppose that  $Y(x, 0)$  is given and that the fluid is at rest at  $t = 0$ . Define

$$E(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(x,0) e^{ikx} dx. \quad (1)$$

Then it can be shown that  $Y(x,t)$  is given by

$$\begin{aligned} Y(x,t) &= \int_{-\infty}^{\infty} e^{-ikx} E(k) \cos[\sigma(k)t] dk \\ &= \frac{1}{2} \int_{-\infty}^{\infty} E(k) [e^{-i(kx - \sigma t)} + e^{-i(kx + \sigma t)}] dk \\ &= Y_R(x,t) + Y_L(x,t) . \end{aligned} \quad (2)$$

Here  $\sigma(k)$  is the "dispersion equation". For gravity waves in water of depth  $h$  it is given by

$$\sigma^2 = gk \tanh kh . \quad (3)$$

It is evident that  $Y(x,t)$  is a superposition of infinitely many periodic waves, of which one set moves to the right and the other to the left. Here  $k$  is the wave number,  $2\pi/k = \lambda$  the wave length,  $\sigma$  the frequency,  $2\pi/\sigma = T$  the period, and  $c = \sigma(k)/k = \lambda/T(\lambda)$  the velocity for a given periodic component.

A classical result in this problem is an asymptotic expansion for a large  $x$  or large  $t$ . Let  $k$  be determined by the equation

$$\sigma'(k) = x/t \quad (4)$$

for given  $x$  and  $t$ . It is then possible to establish that

$$Y_R(x,t) \approx \begin{cases} \operatorname{Re} \frac{1}{2} E(k) \left[ \frac{-2\pi\sigma'(k)}{x\sigma''(k)} \right]^{\frac{1}{2}} e^{-i(kx - \sigma t + \pi/4)} , & x < t\sqrt{gh} \\ 0 , & x > t\sqrt{gh} . \end{cases} \quad (5)$$

The asymptotic form is the same for either large  $x$  or large  $t$ , with (4) being used to modify (5) if a form for large  $t$  is desired. There is an expression of similar form for an initial impulse on the surface.

One may interpret this formula in several ways. Suppose that an observer is moving so that  $x/t = \text{const}$ . Then from (4)  $k$  is constant, i.e., the observer is just keeping pace with waves of wave length  $\lambda = 2\pi/k$ . On the other hand, the crests of these waves are moving with velocity  $\sigma(k)/k > \sigma'(k)$ , so that they are passing the observer by, that is, the water surface seen by the observer is undulating periodically. Figure 2 shows both  $c(k)$ , the "phase velocity", and  $c_g(k) = \sigma'(k)$ , the "group velocity", plotted against  $kh$ .

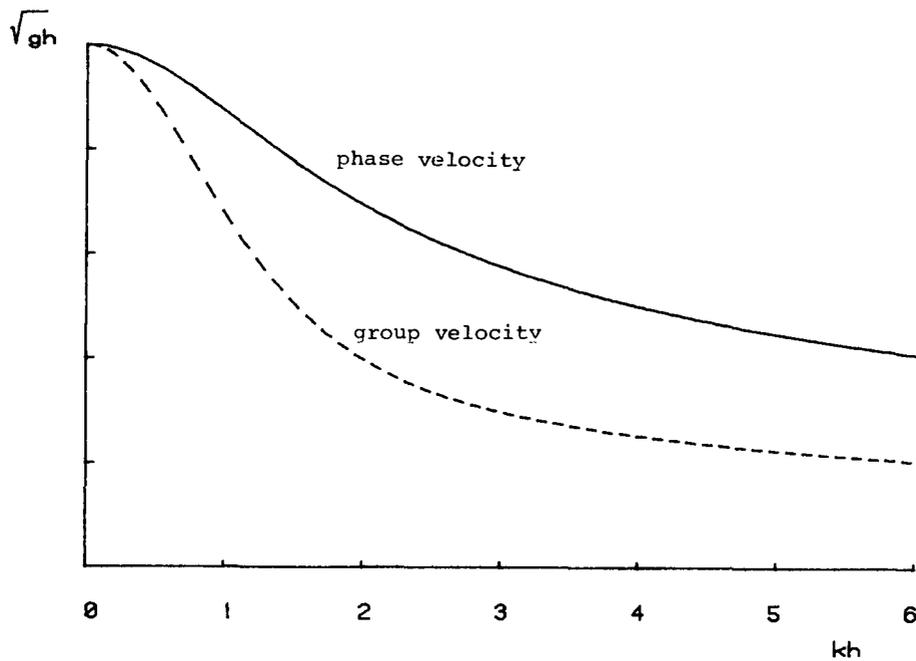


Figure 2.

Let us next suppose that the position  $x$  is fixed and that  $t$  increases. Then, since  $x/t$  is decreasing, it follows from (4) and Figure 2 that  $k$  is increasing, or  $\lambda$  decreasing, with increasing  $t$ . Figure 3, reproduced from paper by J. E. Prins (1958) shows a set of records for a rectangular initial hump.

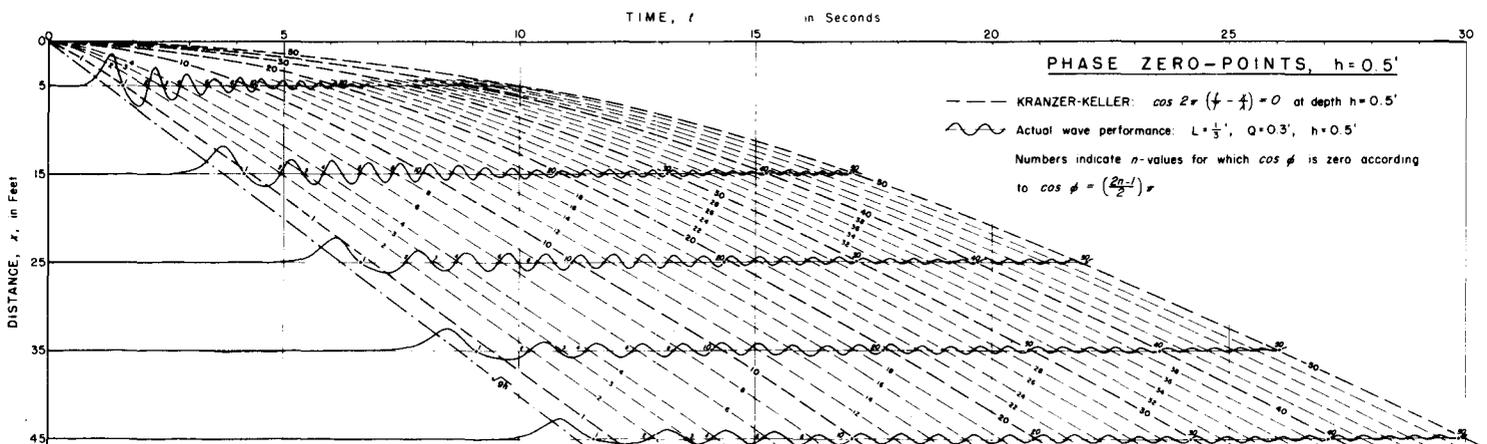


Figure 3.

According to the linearized theory, these curves should start with a discontinuity at  $t = x/\sqrt{gh}$ . In fact, only a steep rise is observed. The gradually decreasing wave lengths are clearly visible. Fixing  $t$  and letting  $x$  increase is equivalent to taking a snapshot of the water surface at time  $t$ . It is seen

that wave lengths will get longer as  $x$  increases and that the profile should end with a discontinuity at  $x = t\sqrt{gh}$ . A direct measurement of this profile would be difficult, but Figure 4 shows a qualitative sketch. As stated above, if there were a sequence of such curves for increasing values of  $t$ , the region associated with a given wave number  $k$  would move to the right with group velocity  $\sigma'(k)$  whereas a crest, for example, in the same region would be moving to the right with phase velocity  $\sigma(k)/k$ .

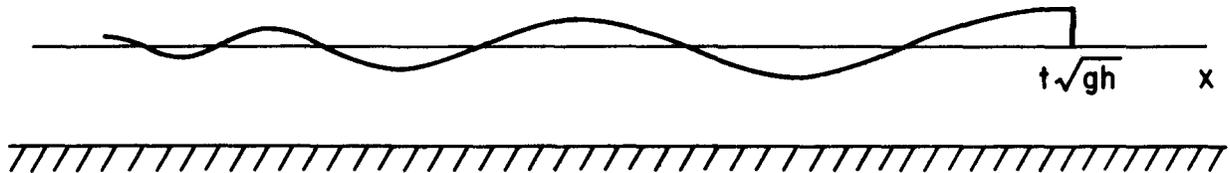


Figure 4.

The analogous problem in three dimensions has been worked out for the radially symmetric case by Kranzer and Keller (1959) the results are shown below. Let

$$\bar{E}(k) = \int_0^{\infty} Y(r,0) J_0(kr) r dr . \quad (6)$$

Then

$$Y(r,t) = \int_0^{\infty} \bar{E}(k) \cos[\sigma(h)t] J_0(kr) k dk . \quad (7)$$

The asymptotic form, written for large  $r$ , is given by

$$Y(r,t) \approx \begin{cases} \frac{\bar{E}(k)}{r} \left[ \frac{k\sigma'}{-\sigma''} \right]^{\frac{1}{2}} \cos(kr - \sigma t), & r < t\sqrt{gh} , \\ 0, & r > t\sqrt{gh} , \end{cases} \quad (8)$$

where  $k$  is determined by

$$\sigma'(k) = r/t . \quad (9)$$

The most noticeable difference between (5) and (9) is the nature of the dependence upon  $x$  and  $r$ , respectively. Interpretations of the three-dimensional results are almost identical with those already given for two dimensions and need not be repeated. A more complete three-dimensional problem in which axial symmetry is not assumed could be easily worked out, but would not add in an essential way to the information needed here.

It was mentioned above in connection with the two-dimensional problem that a similar asymptotic expansion can be obtained for an initial impulse. An expression analogous to (8) has been worked out by Keller and Kranzer (1959) for this case. Since we wish to make use of this result later on, the asymptotic expression is also given here. Let  $I(r)$  be the initial impulse on the free surface and define

$$\bar{I}(k) = \int_0^\infty I(r) J_0(kr) r dr .$$

Then

$$y(r,t) = \begin{cases} \frac{-\bar{I}(k)}{\rho h r \sqrt{gh}} \left[ \frac{\sigma' h \tanh(kh)}{-\sigma''} \right]^{\frac{1}{2}} \sin(kr - \sigma t), & r \leq t\sqrt{gh} \\ 0, & r \geq t\sqrt{gh} \end{cases} \quad (10)$$

The most important difference between (8) and (10) is the absence of the discontinuity at the leading edge of the disturbance in (10), i.e. at  $r = t\sqrt{gh}$ .

Finally some remarks concerning the region of applicability of the asymptotic expansions seem necessary, for they are derived for "large" values of  $t$ , or of  $x$  and  $r$ . Unfortunately, we are not aware of any systematic study of this point and we have not attempted it ourselves. We shall assume, however, that the asymptotic expansion gives a useful approximation from almost the beginning, say for  $r/h > 3$  and  $t\sqrt{gh} > 10$ .

#### The Predicted Ringing Frequency

We shall now exploit the model that we have proposed earlier. We suppose that at time  $t=0$  a hump of water has been formed near the bow of the ship and that the ship is moving forward with velocity  $U$ . At time  $t > 0$  the configuration of ship plus wave (see Figure 4) might appear as in Figure 5.

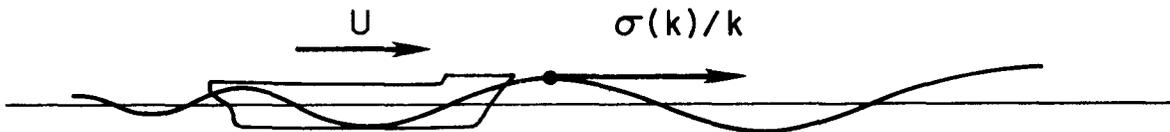


Figure 5.

Since the ship is moving with constant velocity  $U$ , its bow is just keeping pace with waves of length  $\lambda = 2\pi/k$  determined by

$$U = \sigma'(k) \quad . \quad (11)$$

These waves, however, are moving forward with phase velocity  $\sigma(k)/k > \sigma'(k)$ , i.e., the ship is moving in a following sea of its own creation. The frequency with which these waves are passing the bow is determined by the equation

$$\sigma_e = (c - U) \frac{2\pi}{\lambda} = \left[ \frac{\sigma(k)}{k} - \sigma'(k) \right] k = \sigma(k) - k\sigma'(k) \quad . \quad (12)$$

With  $\sigma$  given by (3), this becomes

$$\sigma_e(k) = k\sigma'(k) \left[ \frac{\sinh(2kh) - 2kh}{\sinh(2kh) + 2kh} \right] < k\sigma' \quad . \quad (13)$$

This can be made dimensionless as follows:

$$\begin{aligned} \frac{U\sigma_e}{g} &= 2\pi \left( \frac{gT_e}{U} \right)^{-1} = \frac{U^2}{gh} kh \left[ \frac{\sinh(2kh) - 2kh}{\sinh(2kh) + 2kh} \right] \\ &= \frac{1}{4} \tanh(kh) \left[ 1 - \left( \frac{2kh}{\sinh(2kh)} \right)^2 \right] \quad . \end{aligned} \quad (14)$$

Since

$$\frac{U}{\sqrt{gh}} = \frac{\sigma'}{\sqrt{gh}} = \frac{1}{2} \left[ \frac{\tanh(kh)}{kh} \right]^{\frac{1}{2}} \left[ 1 + \frac{2kh}{\sinh(2kh)} \right] \quad , \quad (15)$$

one may eliminate  $kh$  and plot  $U\sigma_e/g$  or  $gT_e/U$  directly against  $U/\sqrt{gh}$ . This is shown in Figure 6.

Before comparing measured periods with the curve of Figure 6, we mention a possible difficulty suggested by Figure 5. We have assumed a hump of water centered near the bow at  $t=0$ , and the computed frequency  $\sigma_e$  of (12) is measured at that location. Near the stern of the ship, however, the wave length will be smaller, and hence the phase velocity also. After sufficient time, the difference will be negligible, for the region where the wave length is between  $\lambda$  and  $\lambda + \Delta\lambda$  continually increases with time. In the beginning, however, this region may not be large and one can even imagine a situation where the stern is overtaking the local waves while just the opposite is happening at the bow (although it is probably pressing too far the applicability of the asymptotic expansion to use it for so small a  $t$ ). The frequency at any point along the hull is calculable without great difficulty. However, unlike the frequency at the point near the bow where the initial disturbance was centered, it will depend upon the time  $t$  or the distanced travelled  $p = Ut$ . We shall postpone the discussion of this calculation until later when it will be included in a more general one in which the observer measuring the frequency is not only behind the center of the disturbance but also to one side of it.

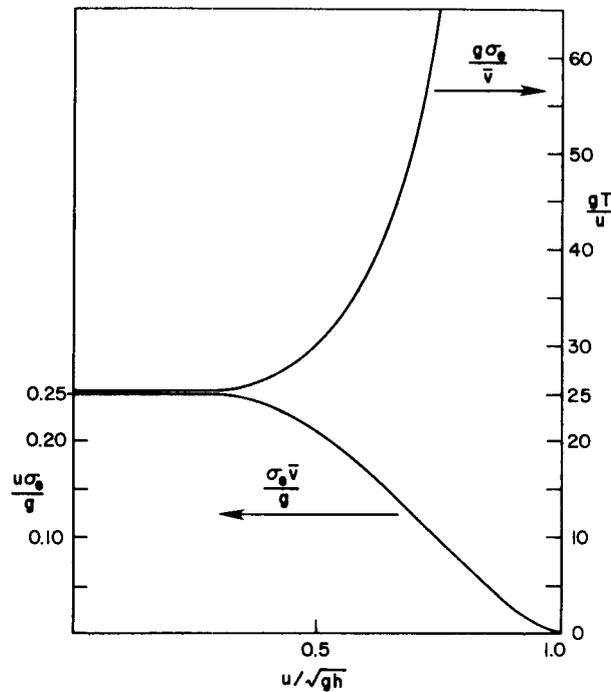


Figure 6.

#### Comparison with Measured Values

As was stated in the Introduction, ringing could be observed either from passing over an obstruction or from a hard start. But this observed ringing was in the force measurements in the forward and after struts holding the model. If the explanation proposed in the last section is correct, one should be able to observe it equally well in the records of a wave gauge attached to the model. Indeed, if our goal has now become to explain the ringing, the wave-gauge record is better than the force records, which in a sense are averages of wave records over the hull. We shall show some data with both.

Experiments were made in the Ship Towing Tank at the Richmond Field Station of the University of California at Berkeley. The tank is 200 ft long and 8 ft wide. Water depth was changed during the tests. Models were generally 5 ft long and the hull forms were Series 60,  $C_B = 0.60, 0.70$  or  $0.80$ . In addition, some tests were made with models 3.75 ft and 7.5 ft long, these being symmetric fore and aft with both entrance and run being that of the entrance of Series 60,  $C_B = 0.80$ . These two models were constructed because at one point in the course of the investigation it was believed that a phenomenon to be described below was associated with the model length. This was proved to be incorrect.

Figure 7 shows a record taken with the 7.5 ft model following a hard start. For this test  $h = 5.4$  ft and  $v = 2.182$  ft/sec. The top two records show the forces in the forward and after struts respectively. These were located 2 ft forward and 2 ft aft of the centerplane section. The next two show wave records, the first gauge being 2 in ahead of the bow and  $7\frac{1}{2}$  in to the starboard,

the second at the stern and 10 in to the starboard of the center-plane. The wave-gauge data are clearly easier to analyze than the force data, which have already been passed through a 5 Hz filter before amplification.

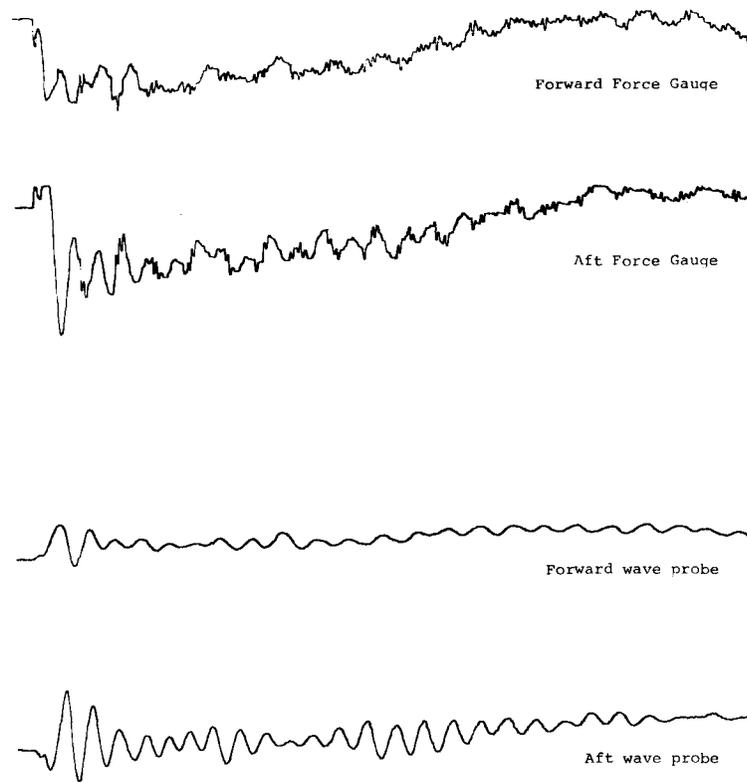


Figure 7.

We present now the results of the analysis of such data. We begin with some results derived from tests made with  $h = 1.191$  ft and a rectangular obstruction (1.25 ft long, 0.729 ft high) on the bottom. Figure 8 shows  $gT_e/U$  plotted against  $F_h = U/\sqrt{gh}$  for the measured values of  $T_e$ . The results are striking. For  $F_h > 0.55$  the measured values  $T_e$  lie very close to the predicted theoretical curve, but for  $F_h < 0.55$  they seem to obey some quite different law. The matter was further confused by a closer analysis of our data. For the region near  $F_h = 0.55$  it was possible to read the longer period on the forward force gauge at the same time that one read the shorter one on the after force gauge. It was even possible to read the shorter period near the beginning of a record and then find a somewhat abrupt change to the longer one near the end. Two values obtained from a wave gauge 6 inches ahead of the bow conformed very well with the theoretical curve.

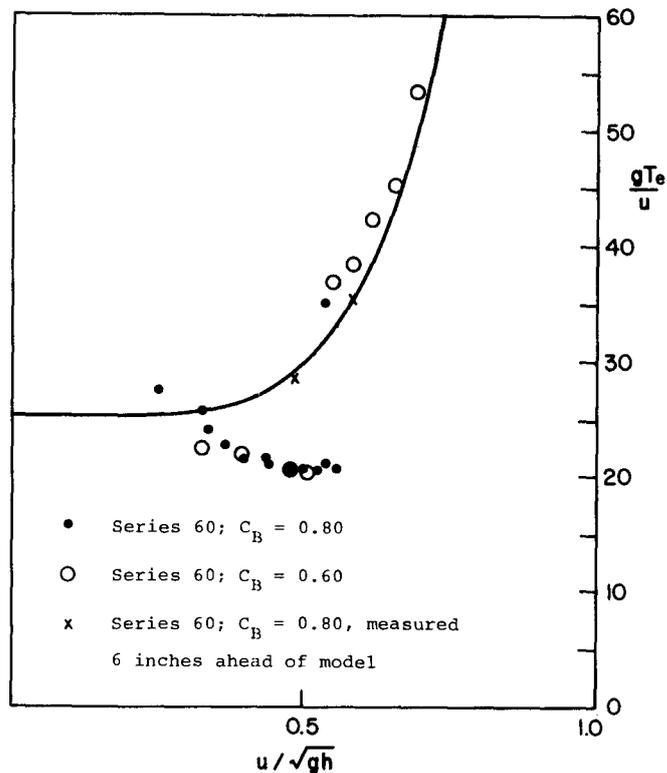


Figure 8.

The results became even more puzzling after the nature of the obstacle and the hull form had been eliminated as possible explanations. It did not seem likely that it could be an effect of water depth, for the measured data conformed well with the theoretical curve for larger values of  $F_h$ , the region where one might have expected discrepancies if depth effects were the cause. In any case, this cause was eliminated by some tests in "deep" water, in this case in water with  $h = 5.5$  ft. Figure 9 shows some values of  $gT_e/U$  taken from the records of a wave gauge 5 in ahead of the bow and 17 in to the starboard on several different models as described in the legend. Figure 10 shows values of  $gT_e/U$  for both force gauges and for two wave gauges, one mounted 5 in ahead of the bow (and 17 in to the starboard) and another mounted at the stern and 7 in to the starboard of the centerplane. Many more data of this sort are available but they do not really add any essentially different information.

It is clear from Figures 9 and 10 that depth does not play a role in determining the apparently discontinuous behavior of  $gT_e/U$ . Furthermore, the indications from the three model lengths are that length is also not a determining factor. This leaves only tank breadth  $b$  as a likely parameter. Varying tank breadth is somewhat difficult and would entail constructing a movable false wall. Moreover, even if this were done and tests confirmed that  $b$  were a controlling factor in the position of the discontinuity, it would not throw any light upon the physical mechanism for this behavior. Since it seems important to clarify the reasons for the observed behavior, let us consider the effect of the tank walls.

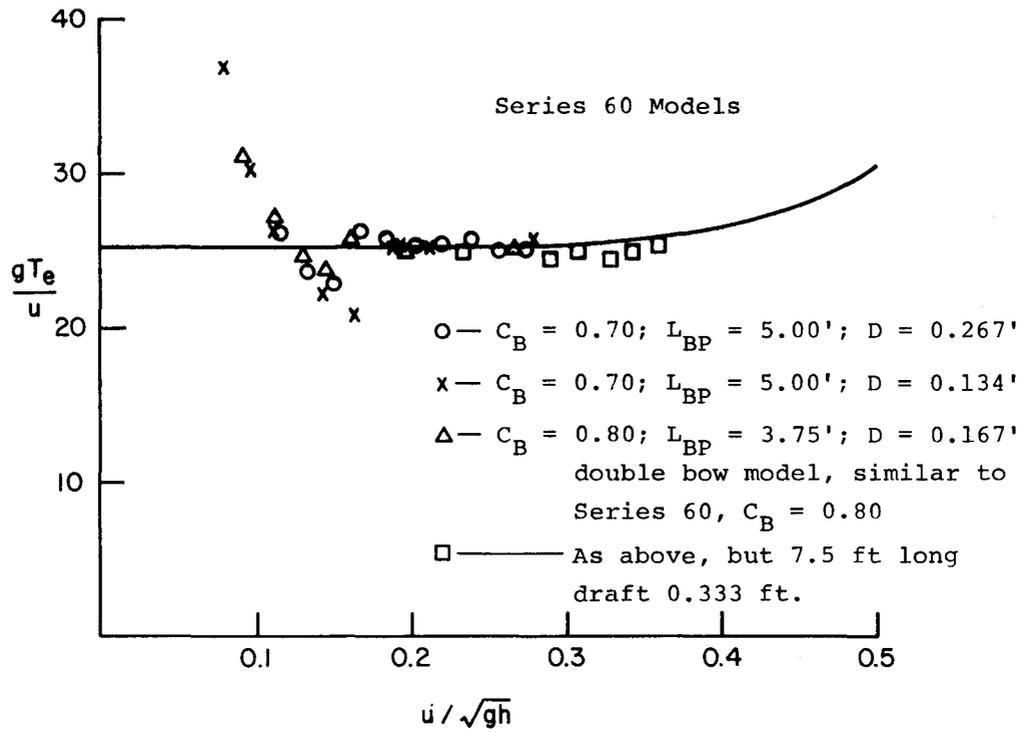


Figure 9.

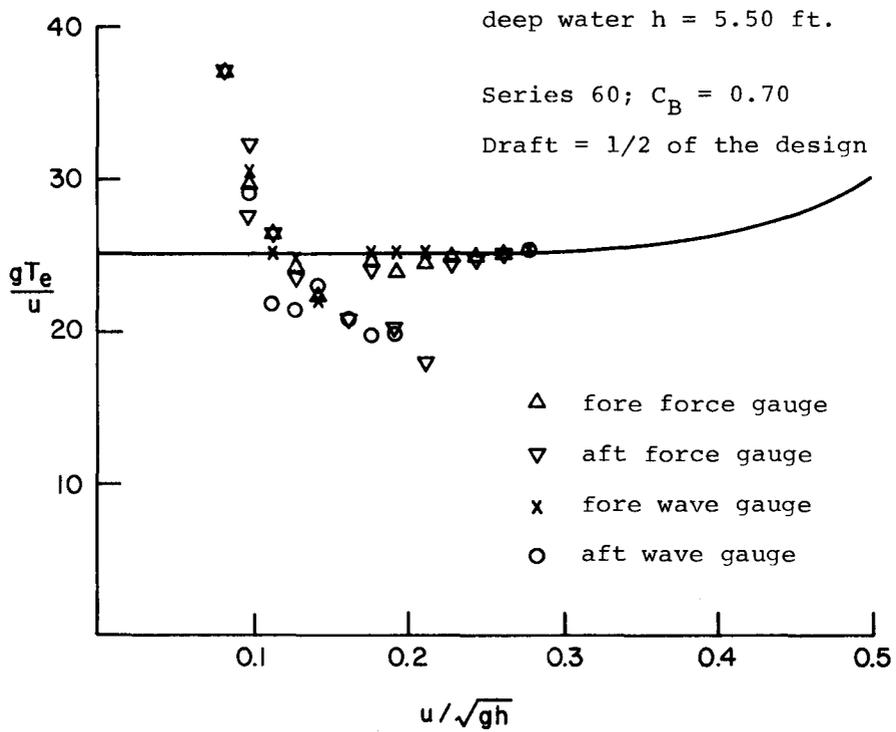


Figure 10.

## Effect of Tank Walls

It seems evident that the only way in which the tank walls can play a role is through reflection of the primary wave and subsequent interference with it. There can, of course, be multiple reflections, and one can imagine that a rather complicated wave record could be produced. This possibility had occurred to us early on in the experiments but had then been discarded because observation showed almost no change of water level at the walls. This was evidently an erroneous conclusion, and the possible role of reflections was again examined. This is most easily done conceptually by imagining a whole row of ships spaced  $b$  apart and starting simultaneously. We calculate the effect of the "phantom" ships as though each were in unbounded fluid. As a result, the composite of all of the corresponding solutions of the initial-value problem satisfies the appropriate boundary condition on the walls. It would not, however, be strictly correct to say that a "blockage correction" is being made, since in the subsequent development the ship itself is not included. For example, the diffraction of the Cauchy-Poisson waves upon the ship is not taken into account. Nevertheless, a true blockage correction might be necessary for a model in a very narrow tank or for a model travelling near  $F_h = 1$ . We assume it is not necessary here.

We now turn to the problem discussed in the paragraph following Figure 6. Let the observer be at a distance  $s$  behind the center of disturbance and a distance  $d$  to the left of it (see Figure 11). The disturbance source may be thought of as one

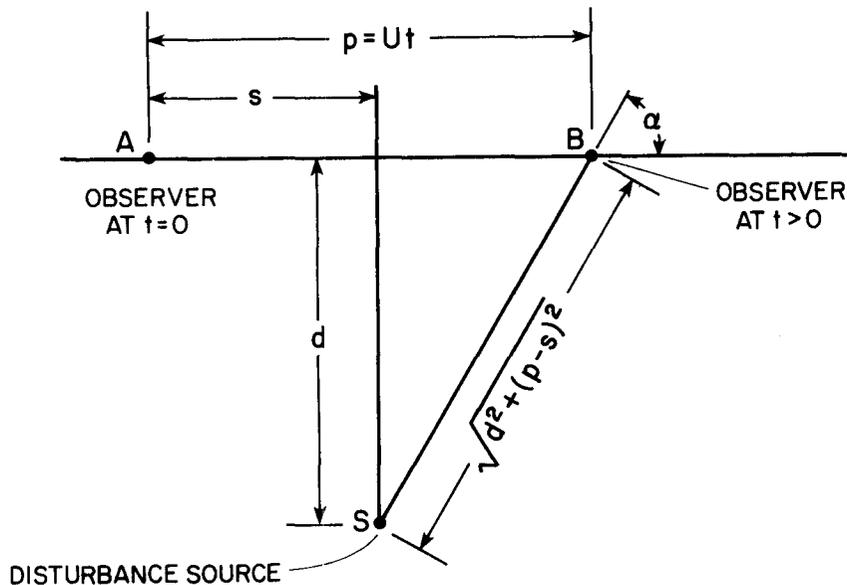


Figure 11.

of the images in a tank wall of the original disturbance source near the bow of the ship. Waves of wave number  $k$  arriving at position B of the observer will have travelled a distance

$$[d^2 + (p - s)^2]^{\frac{1}{2}}$$

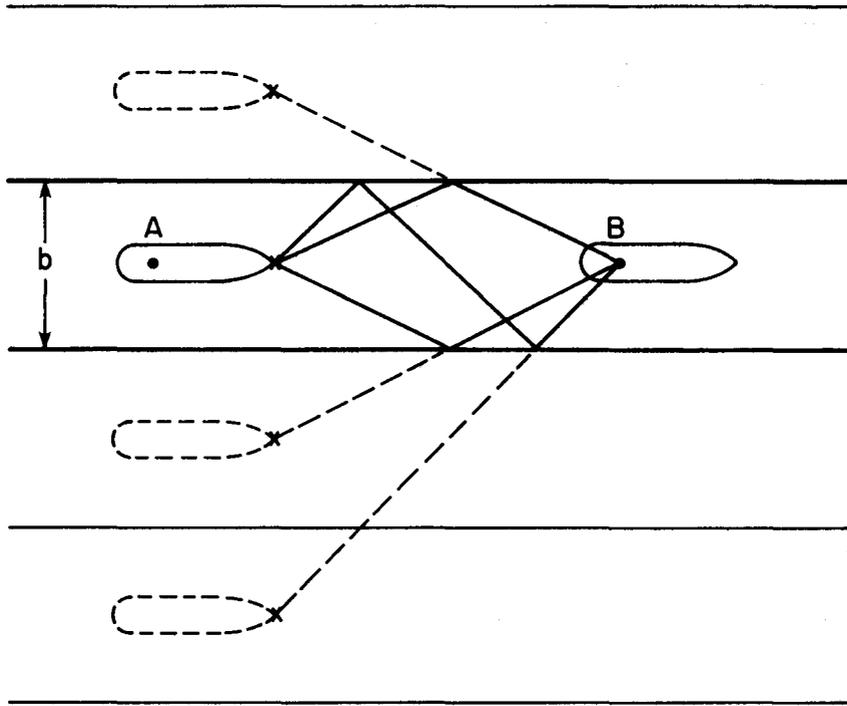


Figure 12.

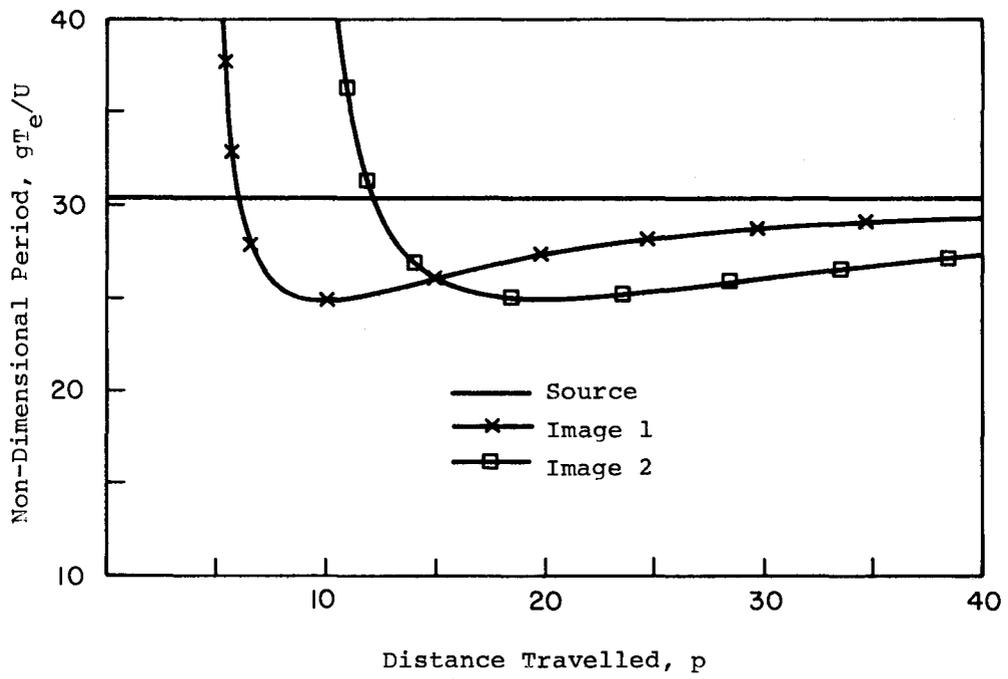


Figure 13.

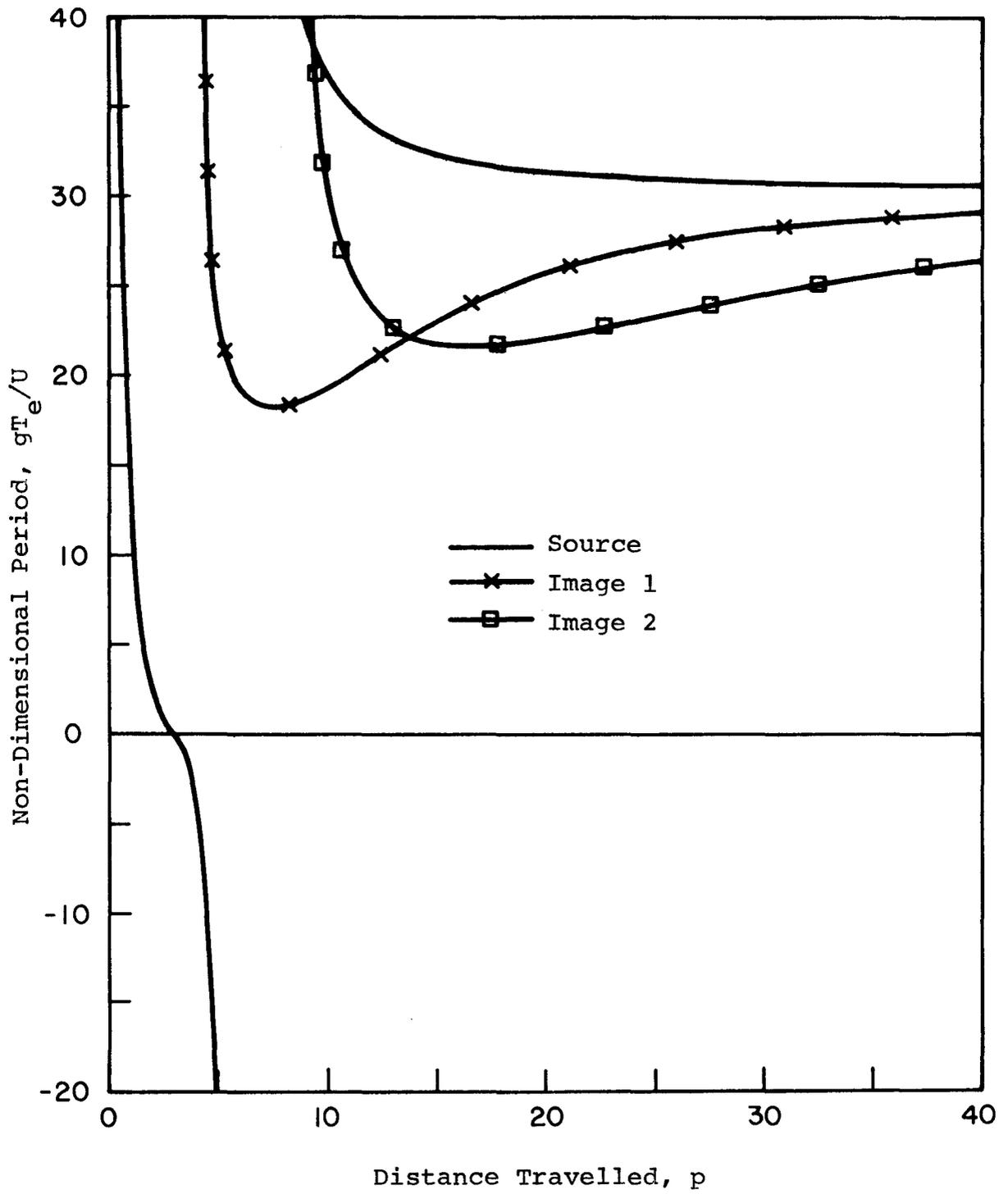


Figure 14.

with a group velocity  $c_g(k)$ . Consequently  $k$  is determined by the equation

$$\frac{\sqrt{d^2 + (p-s)^2}}{t} = c_g(k) \quad (16)$$

or, expressed dimensionlessly, by

$$G(p) \equiv \frac{\sqrt{d^2 + (p-s)^2}}{p} = \frac{c_g}{U}, \quad p = Ut. \quad (17)$$

The encounter frequency at B is given by

$$\sigma_e = \sigma - kU \cos\alpha$$

or, again expressed dimensionlessly, by

$$\frac{\sigma_e U}{g} = \frac{\sigma U}{g} - \frac{kU^2}{g} \cos\alpha. \quad (18)$$

From a relatively straightforward calculation one finds the following expression:

$$\frac{\sigma_e U}{g} = khF_h \left[ \frac{c}{\sqrt{gh}} - \frac{F_h}{G(p)} \frac{d^2 + \sqrt{(G^2 - 1)d^2 + G^2 s^2}}{d^2 + s^2} \right], \quad (19)$$

a function of  $kh$ ,  $F_h$  and  $p$ . From

$$F_h = \frac{c_g(k)}{\sqrt{gh}} \cdot \frac{1}{G(p)} \quad (20)$$

one can solve for  $kh$  as a function of  $F_h$  and  $p$ . This allows one to express  $\sigma_e U/g$  as a function of the two variables  $F_h$  and  $p$ . As in (14)

$$\frac{gT_e}{U} = 2\pi \left( \frac{\sigma_e U}{g} \right)^{-1}. \quad (21)$$

In order to apply (19) to the situation at hand, one may consult Figure 12. The distance  $d$  will be taken as  $b$ ,  $2b$ ,  $3b$ , etc. Since reflections take place from each wall and the model is in the center, each one must be doubled. This has, of course, no influence on the period  $T_e$ . Figures 13 and 14 show, for  $F_h = 0.5$ ,  $gT_e/U$  plotted against  $p$  in feet for  $s = 0$  and  $s = 3$  ft. The tank width was taken as 8 ft. For  $s = 0$ ,  $gT_e/U$  is just 30.50 for the waves associated with the primary disturbance source (see Figure 6). Both the first and second reflections, however, vary with distance travelled, each approaching 30.50 asymptotically from below. For  $s = 3$  ft the period associated with the primary source now also varies with  $p$ . This is explained in the paragraph following Figure 6. The vertical asymptote corresponds to the instant when  $c = 8$  (see (18)).

The information obtained from Figures 13 and 14 (and many other similar ones) shows that other periods can be expected than those

derived from the first analysis that led up to Figure 6. However, these various periods need to be superposed in some fashion. The superposition of the primary and various reflected waves will produce a composite wave, and it was presumably from such a wave that we were taking our measurements. In order to be reasonably sure that reflections were the cause of the unexpected observed behavior, it seemed necessary to assume a model for the disturbance and to calculate the superposition of the primary and reflected waves. The analysis for such a calculation for finite  $h$  is in the paper by Kranzer and Keller (1959) cited earlier, and the necessary results have been reproduced in (8) and (10). Although either one of these could have been used, there was some slight convenience in using (10), for no discontinuity was involved (and had not been observed).

In applying (10) the following initial impulse distribution was assumed:

$$I(r,0) = I_0 [1 + 2(r/R)^2]^{-3/2}, \quad \bar{I} = \int_0^{\infty} I(r,0) J_0(kr) dr \quad (22)$$

where  $R = 0.15L$  and  $I_0 = 1 \text{ lb sec/ft}^2$ . (Since the problem is linear, the value of  $I_0$  does not affect the periods of waves at any later time.) The observer was stationed on the ship at a distance  $s = 0.8L$  behind the center of the disturbance in its initial location (see Figure 12). The asymptotic expression (10) was "placed" at each image point of the primary disturbance and a finite number of these summed. Before writing down an expression for this it will be convenient to rewrite (10) in a dimensionless form. We introduce the following dimensionless variables:

$$\hat{k} = kh, \quad \hat{L} = L/h, \quad \hat{\sigma} = \sigma L/U, \quad \hat{t} = tU/L, \quad \hat{r} = r/L, \\ \hat{I}(\hat{k}) = \bar{I}/\rho L^2 h \sqrt{gh}, \quad \hat{Y}(\hat{r}, \hat{t}) = Y/L. \quad (23)$$

The asymptotic expression (10) may then be written as follows:

$$\hat{Y}(\hat{r}, \hat{t}) = \begin{cases} \frac{\hat{I}(\hat{k})}{\hat{r}} \left[ \frac{\hat{\sigma}'(\hat{k}) \tanh \hat{k}}{-\hat{\sigma}''} \right]^{\frac{1}{2}} \sin(\hat{k}\hat{r}\hat{L} - \hat{\sigma}\hat{t}), & \hat{r} \leq \hat{t}/F_h, \\ 0, & \hat{r} \geq \hat{t}/F_h \end{cases} \quad (24)$$

where  $\hat{k}$  is determined by

$$\hat{\sigma}'(\hat{k}) = \hat{L} \hat{r}/\hat{t}$$

and

$$\frac{\hat{\sigma}' \tanh \hat{k}}{-\hat{\sigma}''} = \frac{2\hat{k} [2\hat{k} + \sinh(2\hat{k})] \sinh(2\hat{k}) \tanh(\hat{k})}{\sinh^2(2\hat{k}) + 8\hat{k}^2 \cosh(2\hat{k}) - 4\hat{k} \sinh(2\hat{k}) - 4\hat{k}^2}$$

The primary wave plus  $2n$  reflected waves (in pairs because of the two tank walls) may be expressed as the following sum:

$$\hat{Y}_T(\hat{t}) = \hat{Y}(\hat{r}_0, \hat{t}) + 2\hat{Y}(\hat{r}_1, \hat{t}) + \dots + 2\hat{Y}(\hat{r}_n, \hat{t}), \quad (25)$$

where

$$\hat{r}_m = \left[ \left( \hat{t} - \frac{s}{L} \right)^2 + \left( m \frac{b}{L} \right)^2 \right]^{\frac{1}{2}} \quad (26)$$

and where  $\hat{k}$  in (24) must be determined separately for each term of (25) from (17) and (20). In computing the sum (25) for a given  $\hat{t}$  one need take into account only those  $\hat{Y}(\hat{r}_m, \hat{t})$  for which  $\hat{r}_m < \hat{t}/F_h$ . Hence the number of terms is always finite, but increases as  $\hat{t}$  increases and as  $F_h$  decreases. In the dimensionless variables the equation for  $\hat{k}_m$  is

$$\hat{\sigma}'(\hat{k}_m) = \hat{L}G(\hat{t}) = \hat{L} \frac{[(\hat{t} - s/L)^2 + (mb/L)^2]^{\frac{1}{2}}}{\hat{t}}, \quad (29)$$

so that a transcendental equation must be solved. It appears as if  $F_h$  has dropped out of the calculations, which obviously should not happen. If one introduces the equation (3) for  $\sigma$  and then computes  $\hat{\sigma}'(\hat{k})$ , one finds that (29) takes the following form:

$$\frac{1}{2} F_h^{-1} \left[ \frac{\tanh \hat{k}_m}{\hat{k}_m} \right]^{\frac{1}{2}} \left[ 1 + \frac{2\hat{k}_m}{\sinh 2\hat{k}_m} \right] = \frac{[(\hat{t} - s/L)^2 + (mb/L)^2]^{\frac{1}{2}}}{\hat{t}}. \quad (30)$$

Hence (25) becomes a function of  $\hat{t}$  with parameters  $s/L$ ,  $L/h$ ,  $b/L$  and  $F_h$ .

A program was prepared for carrying through these calculations. For the results to be shown here  $s/L = 0.8$ ,  $L/h = 4.1$ ,  $b/L = 1.6$  and  $R/L = 0.15$  (see equation (22)). For  $L = 5$  ft, these correspond to  $h = 1.22$  ft,  $b = 8$  ft and  $s = 4$  ft, all values corresponding closely to the measured values of Figure 8. The dimensionless time (or distance)  $\hat{t}$  was computed at intervals of 0.04 up to the point where 12 ship lengths had been travelled. Computations were made for  $F_h = 0.40, 0.45, 0.50, 0.55, 0.55, 0.60$  and  $0.65$ . For  $F_h = 0.40$  a total of 18 reflections was computed, for  $F_h = 0.45$ , 15 reflections, and for the others 12 reflections. Figures 15, 16 and 17 show graphically the results for  $F_h = 0.40, 0.50$  and  $0.60$ . Each figure shows the computed value of  $\hat{Y}_T(\hat{t})$  and also separately the contributions from the first and the second term of (25), labelled "source" and "first reflection", respectively. In addition, for  $F_h = 0.50$  the total for 3 reflections is shown. The total number of reflections included depended upon the number that would have an effect in the range  $0 \leq \hat{t} \leq 12$ . For  $F_h = 0.50$  the totals of 3 and of 12 reflections are the same for  $\hat{t} < 3$ . Thereafter the divergence between the two increases. That the first reflection dominates the primary wave is a consequence of its being doubled by reflections from both walls.

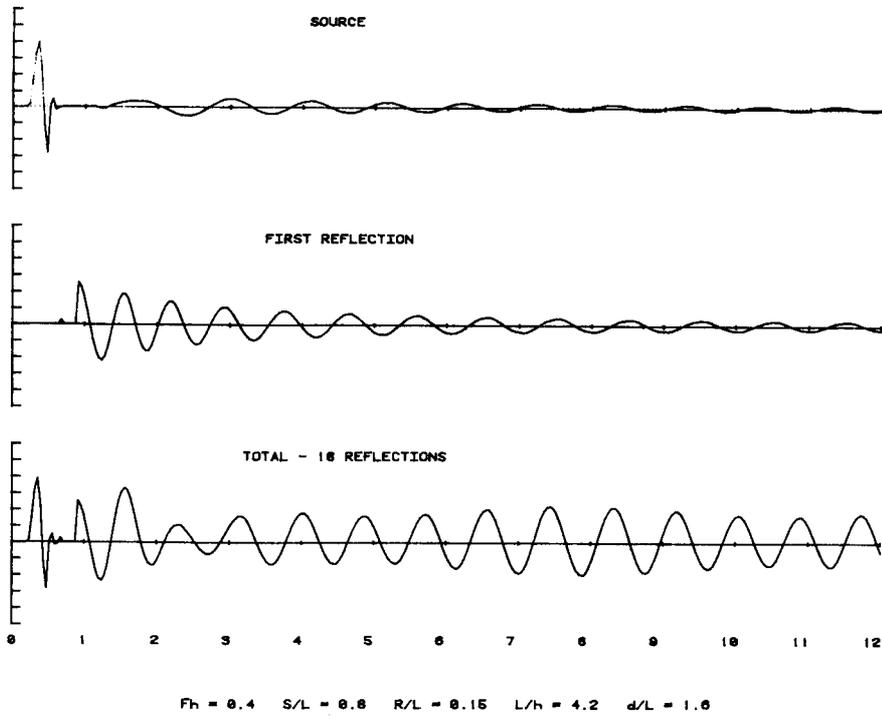


Figure 15.

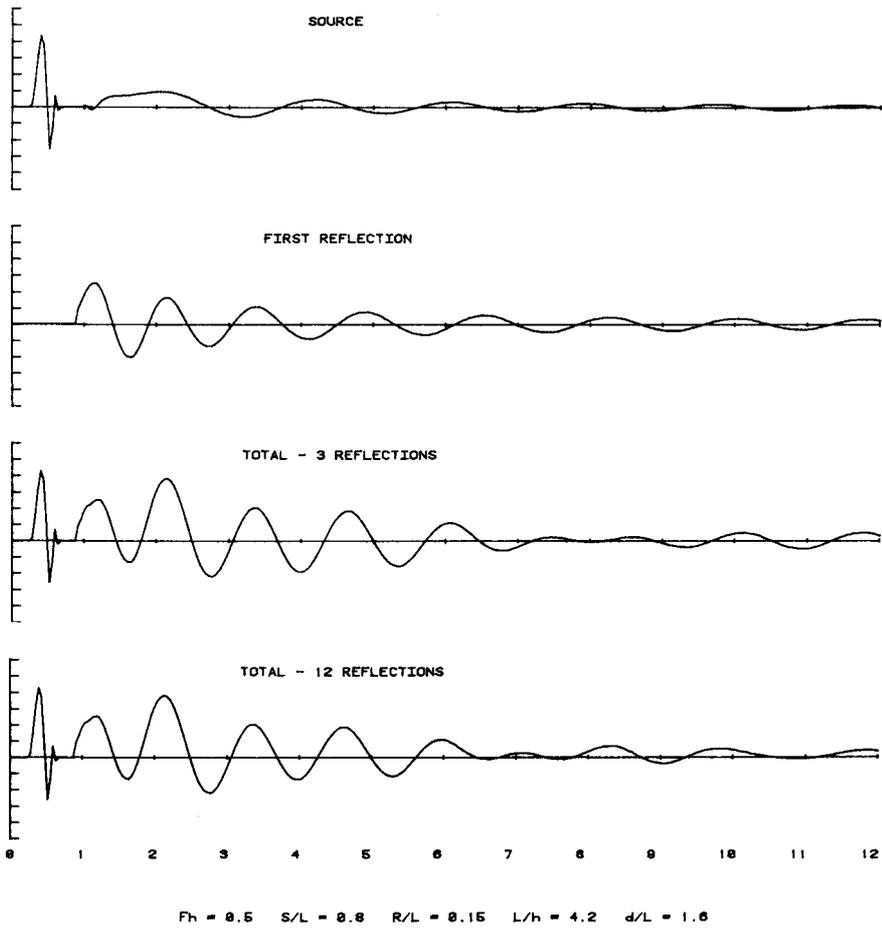


Figure 16.

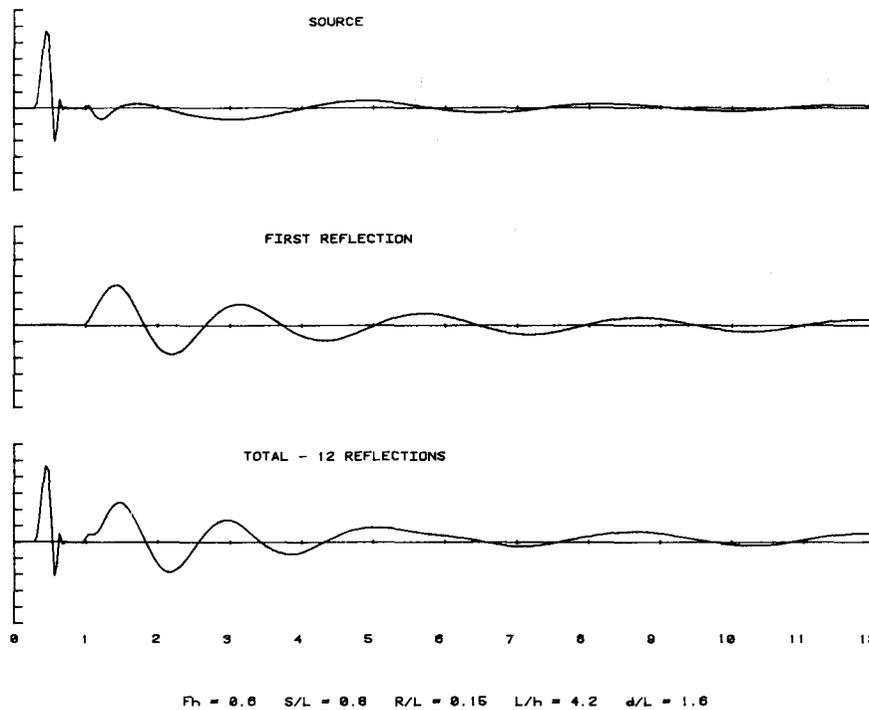


Figure 17.

The graphs for these computed wave records were now treated in the same fashion as the experiment records discussed earlier. Periods were measured near the right-hand ends of the curves and also near the left-hand ends. The latter usually show a gradually increasing period, so that a spread of values was read. Finally, as a kind of check upon the accuracy of the procedure, the periods near the right-hand ends of the "source" curves were also measured to confirm that they conformed with the values obtained from (14) and (15). The results were then plotted on Figure 6; this is shown in Figure 18. It is evident that the same sort of behavior is observed here as was observed earlier in analyzing the experiment records for  $h = 5.5$  ft and  $h = 1.191$  ft. Indeed, if the points on Figure 18 are plotted on Figure 8, the two sets agree well with each other.

### Conclusion

We believe that the originally puzzling behavior has been adequately explained. It is the result of the interference of multiply reflected waves with each other and with the primary wave. A reasonable objection to the calculation procedure is that the assumed disturbance near the bow of the ship is probably not very close to the actual one. This is apparently not a serious objection in the present situation, for what we are really examining are consequences of the dispersion relation and of reflection. We are not trying to reproduce exactly the wave records.

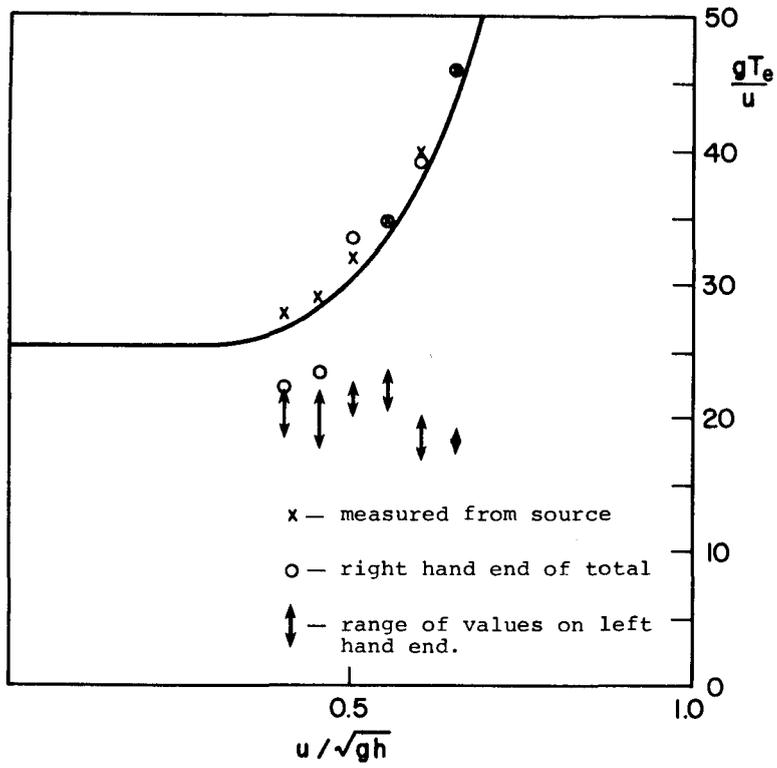


Figure 18.

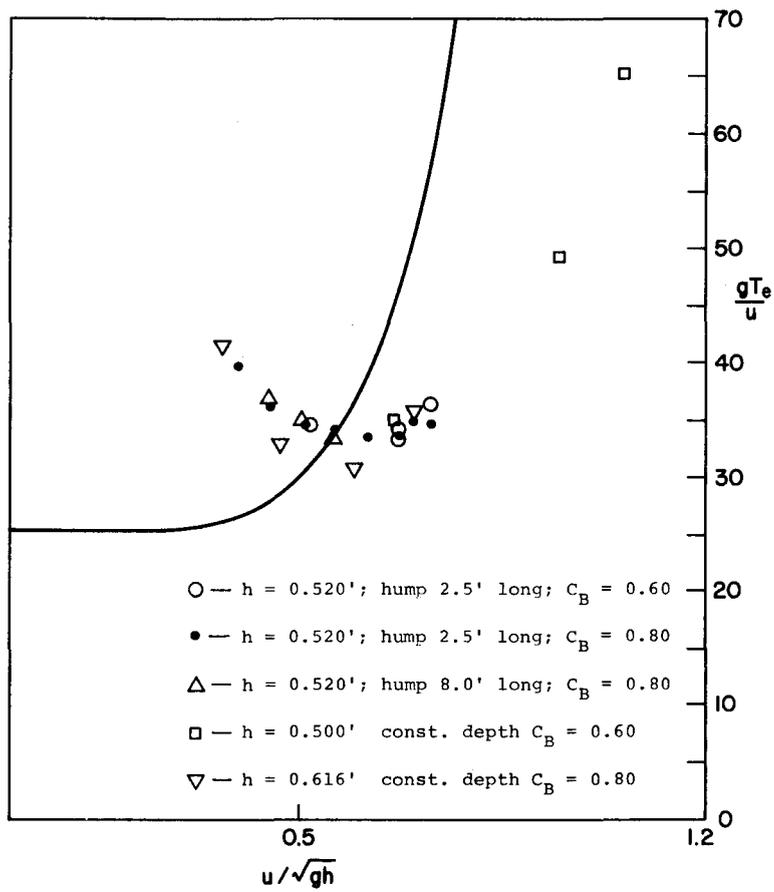


Figure 19.

There still remain some unexplainable phenomena. Up to now we have not shown the results of experiments carried out at very shallow water depths. Figure 19 shows some results for various depths near  $h = 0.5$  ft. They do not conform at all with the theoretical curve of Figure 6. Although the divergence for the smaller values of  $F_h$  may be explainable by reflection, that for the larger values certainly cannot. In particular, the occurrence of the values for  $F_h > 1$  is contrary to the predictions of the linearized theory. It is evident that one needs a solution to a Cauchy-Poisson problem that takes into account nonlinear terms in the boundary conditions. This has not yet been examined.

Finally there remains another question. Is the observed phenomenon of "ringing" of any practical significance for ships? Probably not under ordinary circumstances. The height of the waves generated by either a hard start or by passing over an obstruction was small, not more than 0.2 in. There is, however, one circumstance where ringing might have a noticeable effect. If there is a sequence of obstructions so spaced that the ringing is in resonance with the frequency of passing the obstructions, the effect will be constantly reinforced. It is easy to compute the circumstances under which this should occur. If  $d$  is the spacing between obstructions and  $T_e$  the period of ringing after passing one, then resonance occurs when  $T_e = d/U$  or, in terms of dimensionless variables, when

$$\frac{d}{h} = F_h \frac{2gT_e}{U} \quad (31)$$

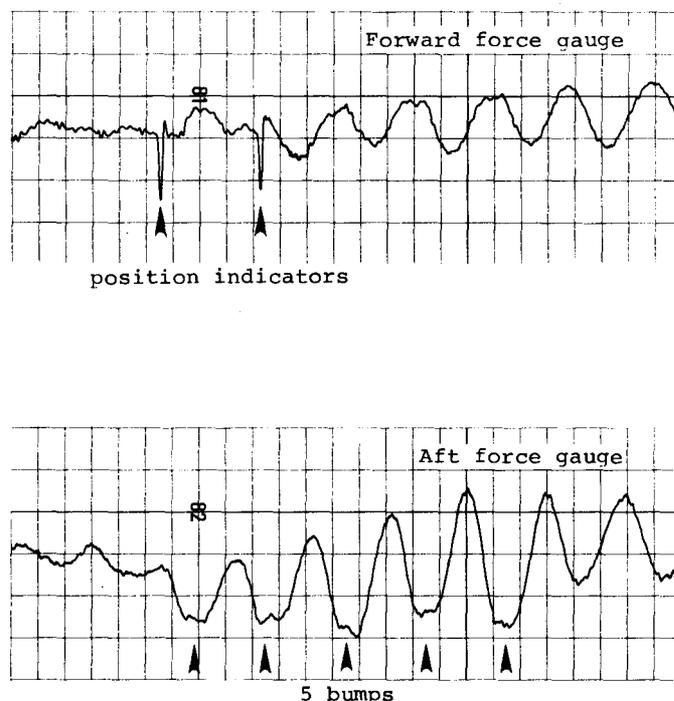


Figure 20.

Figure 20 shows the measured force in the fore and aft struts when the model passed over five rectangular obstructions spaced 7.75 ft apart at their centers. The model velocity was 2.541 ft/sec, the water depth  $h = 0.50$  ft and the measured  $T_e = 2.93$  sec. Although not completely in resonance ( $d/U = 3.05$  sec), the model is responding with increasing force in the struts until the last obstruction is passed by. On a larger scale, obstructions spaced 335 ft apart in water of depth 45 ft might expect to cause this resonance in a ship moving at a speed of 11.25 knots with the resonance period being  $T_e = 17.75$  sec. Or, in water of the same depth a ship moving at 15.8 knots over obstructions spaced 1150 ft apart could expect to experience a resonance period of 30 sec. It would be interesting to know if such resonances have been observed.

Although we have mentioned the possible occurrence of ringing in practical ship operations it seems evident that it is primarily a towing-tank phenomenon that must be confronted in certain situations. It occurs not only in passing over an obstruction on the bottom in shallow water and in hard starts in water of any depth, but also in passing closely by obstacles or from sudden changes in the configuration of a wall which a model is moving parallel to. In fact, it can occur in any situation where the geometry is such that the water may suddenly pile up in a localized region. It is not a phenomenon that can be avoided by taking extra precautions, for it occurs naturally in certain circumstances, as in the motion over an obstruction, and its presence cannot be properly "subtracted out" from experimental measurements.

#### Acknowledgment

The experimental part of the work reported on here has been supported by the National Science Foundation with grant number ENG77-15896-Whsn-2-04/80.

#### References

- Kranzer, H. C.; Keller, J. B. Water waves produced by explosions. J. Appl. Phys. vol. 30 (1959), pp. 398-407.
- Prins, J. E. Characteristics of waves generated by a local disturbance. Trans. Amer. Geophys. Un. vol. 39 (1958), pp. 865-874.