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Surface Waves for the Ray Point of View

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Seventh Georg Weinblum Memorial Lecture

SURFACE WAVES FROM THE RAY POINT OF VIEW

by

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The Seventh Georg Weinblum Memorial Lecture:

SURFACE WAVES FROM THE RAY POINT OF VIEW

Marshall P. Tulin¹
Presidential Professor

INTRODUCTION

In commenting on a paper of Bessho (1966), Georg Weinblum noted:

"Science is subject to fashion as much as other human activities. Recently the thin ship and surrogates have completely dominated the field, but in the twenties (and earlier) the pressure system has been considered as being an equally important hydrodynamic model (at least in principle) as the Michell ship, especially suitable for picturing fast shallow-draft and planing vessels. By Dr. Bessho's paper a sound equilibrium has been established. The present speaker had emphasized the similarity of the Hogner and the Michell integral (Zamm, 1930) and thus inspired Sir Thomas Havelock to derive the simple relation between source-sink distributions σ and pressure systems p

$$4\pi\rho g\sigma = c \frac{\partial p}{\partial x} \quad "$$

These days the fashion in ship waves has been very much with so-called low-speed theories, which can be implemented through digital computation. The question has arisen, Keller (1979), as to the nature of true asymptotic low speed theory. In that paper he proposed a ray theory. In the present paper, which I have prepared especially for this lecture, I have chosen to explore the ray theory and to begin by combining it with a very old fashioned subject, and one which early attracted the attention of Weinblum himself, the waves made by a moving pressure patch. In this case, assuming light loading, linearizing assumptions are valid and the theory takes a simple form. It is therefore very useful for sharpening our tools and insight.

After that start we tackle the ship problem, as Keller already has. We made no assumptions concerning the thickness of the ship. We repeat some of his findings. We also find some waves issuing from a limited portion aft and from the ends. We have formulae for these waves. At the ends the situation is, however, ambiguous because of insufficient knowledge of the displacement flow giving rise to the waves.

A few words concerning ray theory. Its antecedents are found in geometrical optics. In dispersive systems it arises with group velocity as a product of asymptotic integration and is inherent in Kelvin (W. Thomson) (1887) and Havelock (1908), and later works, and then much later in Stoker (1957) who considered the wave pattern created by a ship moving in a curved trajectory. All of these assumed no displacement flow in the water. But in problems of optics and acoustics, the inhomogeneity of the medium had long been considered. For dispersive systems, at least for ship waves, this was first discussed and the basic relations given by Ursell (1960), and independently by Whitham (1961). At about the same time, the fundamentals of the interaction between waves and currents were laid out in a series of important papers: Longuet-Higgins and Stewart (1960,1961) and Whitham (1960,1962); see also the discussion in Phillips (1966). The basic assumption of the ray theory is that the waves are short in comparison to the scale over which the flow changes (it is this assumption which is at question near the ends of the ship), so that the waves may be assumed to have locally the same dispersive relation as in undisturbed water (it is this assumption which Eggers (1981) questions at the bow of a ship).

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A detailed study of the implications of ray theory for ship waves was begun and later extended by Keller (1974, 1979), while the actual application was begun by Inui and Kajitani (1977) who assumed the waves generated as in linear theory and utilized ray theory to calculate the bending of the rays. Yim (1981) has later made extensive ray tracing studies in the same spirit.

Here we are motivated to understand how ships generate waves, in this case in the true low speed limit; as many questions are eventually raised as are answered.

1. WAVES GENERATED BY A STEADY MOVING PRESSURE PATCH IN THREE DIMENSIONS

Introduction

The problem of the waves generated by a moving real ship contains serious non-linear features. On the other hand, the waves generated by a pressure patch of finite size moving on top of the water surface can perhaps be treated using linearizing assumptions, provided that the magnitude of the pressures are suitably small. And the solution to this problem can yield valuable understanding of wavemaking, just as Kelvin's treatment in 1887 of a concentrated pressure patch. This is certainly the reason why Georg Weinblum (1930) undertook to study this problem. Much later it became of some importance in connection with the performance of air cushion vehicles like the Hovercraft, and was studied by several workers, notably Nick Newman (1962).

Here we study this problem using ray (asymptotic) techniques and obtain some important results; then we go on to the case of a real ship, which is considerably more complicated; but non-linear effects can be treated with interesting results. These ray techniques which we use, first in the linear pressure patch case and then in the non-linear ship case, are generally applicable in the limit of small Froude number.

The usual linearizing assumptions which we employ in the pressure patch problem are: i) the deep water waves propagate locally as progressive waves of small amplitude, for which the dispersion relation between the wave frequency, ω , and wave number, k , is: $\omega^2 = gk$; ii) waves are generated at each point on the free surface under the pressure patch, and the amplitude of the generated waves is proportional to the excess pressure, p_0 , associated with the patch; iii) the waves generated at each point add linearly; iv) the waves, once generated, propagate over the water surface as if it were at rest. The last assumption is decisive and is equivalent to assuming that the moving pressure patch does not induce any significant motion in the water, aside from the waves themselves. This is certainly not true in the case of displacement ships, where the water is forced to go around the ship as well as to issue waves; we shall correct for this vital difference later.

We imagine a patch of constant pressure moving over the water at a speed $U(t)$, from right to left along the x -axis, Figure 1.1. Our technique will be to observe the waves, $\eta(t)$, arriving at an observer point, $P(x,z) = \zeta$, at a distance of many wavelengths from the moving pressure patch. We assume to begin with that these waves can arrive along any ray passing from the observer point to the pressure patch (later we show that only distinct rays contribute at any time, t). We designate the rays by their angle of inclination, β , to the horizontal axis. In our present approximation, since no motion occurs in the water to bend the rays, they are straight lines.

The Far Field Plane Wave Spectrum

An elemental radial wave is generated as if by a concentrated imposition of pressure p_0 (a delta function) at each point of the water over which the pressure patch passes. Those waves generated along a fixed ray bundle (origin at ζ) of mean ray angle β and width, $(\zeta - \zeta')d\beta$, and observed at the fixed point, $\zeta(x,z)$, can be represented as an integration of the generator points, ζ' , along the mean ray, and over time, $t' < t$:

$$\frac{d\eta}{d\beta}(\zeta, t, \beta) = R \cdot \int_{\zeta_l}^{\zeta_u} dt' \int \frac{p_0}{\rho} [A](\zeta - \zeta') e^{i\{k(\zeta - \zeta') - \omega(t - t')\}} d\zeta' ; \quad C_g(k) = \frac{\zeta - \zeta'}{t - t'} \quad [1.1]$$

where ζ_l, ζ_u are the intersections of the ray with the upper and lower boundaries of the patch, and $C_g = d\omega/dk$. The function $[A]$ is found from the asymptotic (ray) theory solution for the wave due to a concentrated imposition of pressure. It is, see Havelock (1908) or Lamb (1932):

$$[A] = g(t - t')^{3/2} \pi (\zeta - \zeta')^4 = g/2^{7/2} \pi (\zeta - \zeta')^3 c_g^3 \quad [1.2]$$

The result of this collection of waves generated within the ray bundle is a plane wave of wave number k and amplitude, $\text{Amp}(\beta)$:

$$\frac{d\eta}{d\beta} = R \cdot [\{\text{Amp}(\beta)\} e^{i[k\xi - \omega t]}] ; \text{ far field} \quad [1.3]$$

It was originally pointed out by Thomas Havelock (1934), that the far field wave may be represented by a distribution (spectrum, $\frac{d\eta}{d\beta}$) of such waves:

$$\eta(\xi, t) \cong \int_0^\pi \frac{d\eta}{d\beta}(\xi, t, \beta) d\beta \quad [1.4]$$

and the wave resistance (in the case of uniform motion) given simply as:

$$\text{Res.} = \pi \rho U_0^2 \int_{\pi/2}^\pi [R \cdot \{\text{Amp}^2(\beta)\} + I \cdot \{\text{Amp}^2(\beta)\}] \cos^3 \beta d\beta \quad [1.5]$$

a result we shall refer to later. In consequence, the problem of determining wave resistance is equivalent to the determination of $\frac{d\eta}{d\beta}$, [1.1], in the far field ($\xi \rightarrow \infty$).

The Boundary Sources

The integral, [1.1], has no stationary phase points so its asymptotic form can be determined through repeated integration by parts. The result can be represented as a sum of terms with coefficients k^{-1} , k^{-2} , etc. In the present case, where the observer is many wavelengths from the generator point, the first term in this series is dominant and:

$$\frac{d\eta}{d\beta}(\xi, t, \beta) \cong R \cdot \int_0^t \frac{p_0}{\rho k} \{ [A(\xi - \xi_u^*; t - t')] (\xi - \xi_u^*) \cdot e^{if_u} - [A(\xi - \xi_\ell^*; t - t')] (\xi - \xi_\ell^*) \cdot e^{if_\ell} \} dt' \quad [1.6]$$

where: $f_{u,\ell} = k(\xi - \xi_{u,\ell}^*) - \omega(t - t')$, and $\xi_{u,\ell}^*$ refer to the intersection of the ray with the patch boundary at time t' .

We see, [1.6], that waves seem to originate only on the boundary of the pressure patch.

Radiation from the Boundary: Ray Theory

The integration of these wave generators (i.e. in t') represented by [1.6] is facilitated by the application of Kelvin's method of stationary phase, see Stoker (1957); this method assumes asymptotic conditions; i.e. that the wave length is much shorter than the range of integration and that $[A]$ varies slowly enough. We recall for reference, if:

$$b(\xi) = R \cdot \int_{t_1}^{t_2} a(\xi, t') e^{i\psi(\xi, t')} dt'$$

then,

$$b(\xi) \cong R \cdot \sum_p e^{i\{\psi(\xi, t_s) + \pi/4 \text{sgn } \psi_s''\}} a(\xi, t_s) \left[\frac{2\pi}{|\psi_s''|} \right]^{1/2} + R \cdot \sum_p e^{i\psi(\xi, t_p)} \cdot a(\xi, t) \frac{\Gamma(1/3)}{\sqrt{3}} \left(\frac{6}{|\psi_s''|} \right)^{1/3} \quad [1.7]$$

where $\psi'_s = \frac{\partial \psi}{\partial t'}(t) = 0$, defines the points $t'=t$ where the dominant contribution to the integral arises when $\psi''_s \neq 0$; and where $\psi'_s = \psi''_s = 0$ defines the points $t = t_p$ where the dominant contribution arises when $\psi''_s = 0$; i.e. if $\psi''_s \neq 0$ sum only over s , and when $\psi''_s = 0$, sum only over p . In this latter case, rays focus by bunching and the resultant wave is more observable. This condition is called a caustic.

Upon applying Kelvin's method to the integration of the boundary waves, [1.6], we find the main contribution arising from points where (stationary phase point):

$$\frac{df_{u,\ell}}{dt} = \frac{d}{dt} \{k(\xi - \xi_{u,\ell}^*) - \omega(t - t')\} = 0$$

or,

$$\frac{d\xi_{u,\ell}^*}{dt} = \omega/k = C_p(k) \quad [1.8]$$

where C_p is the phase velocity of the wave; this condition defines the wave number of the wave arising at each point of the path boundary; i.e. the phase velocity of the outgoing boundary wave is equal to the velocity of the boundary along the ray.

We have so far considered an arbitrary velocity of the pressure patch. When the patch moves with constant horizontal speed, U_0 , then all of the waves in the resulting pattern must be stationary in body coordinates (the frame of reference moving with the patch) and it follows that the phase speed, C_p , of the wave traveling along a ray at angle β (Figure 1), is:

$$C_p(k) = -U_0 \cos \beta \quad [1.9]$$

This follows from the fact that the frequency of wave encounter to an observer traveling with velocity \vec{V}_{obs} is: $k|\vec{V}_{obs} \cdot e^{i\beta - C_p}|$.

Notice that for waves moving outward toward the observer point above the patch, $C_p > 0$, so that $\pi/2 < \beta < \pi$. These waves may be classified in the usual way: those traveling on rays closest to the vertical (divergent waves) are short, while those on rays closest to the horizontal (transverse waves) are longest.

This condition of stationarity, [1.9], when combined with the stationary phase condition, [1.8], leads to a relationship between the local ray angle, $\beta(k)$, and the local patch angle measured from the horizontal, α^* . First it may be shown taking into account the cutting angle, β , of the ray as it traces out $\xi_{u,\ell}(t')$ that:

$$\frac{d\xi_{u,\ell}^*}{dt} = \frac{U_0 \tan \alpha^*}{\cos \beta (\tan \beta - \tan \alpha^*)} = \frac{U_0 \sin \alpha^*}{\sin(\beta - \alpha^*)} \quad [1.10]$$

and then combining [1.8-1.10] we finally find that the wave number vector is normal to the patch boundary!:

$$(\beta - \alpha^*) = \pi/2 \quad [1.11]$$

This is a result we might have expected. It is well known that in the case of non-dispersive wave systems (optics or acoustics), that the signal is primarily due to excitation from the point on the body which lies closest to the observer and therefore arrives on the ray normal to the body.

We note: each and every point of a smooth patch boundary produces (at a fixed observer point above the patch) a single wave (single k , corresponding to β); however each wave (k) receives a contribution from all points sharing the same patch boundary angle, α^* ; the transverse waves arise particularly from the blunt ends and the divergent waves from the moderately sloped sides. The waves traveling to an observer above the patch will originate from the upper forward and rear aft sectors of the patch boundary, see Figure 1.2. For smooth shapes, then, each wave will be excited at two generator points, one in each of the contributing sectors, provided that a normal to the ray exists (for boundaries with non-blunt ends, the range of β will be limited). Finally a simple relation exists for the phase velocity at any point: $C_p = U_0 \sin \alpha$.

The wave energy moves away from the boundary generation points, $\xi_{u,\ell}^*$, along the rays, $\beta(\alpha^*)$, at the group velocity, $\dot{\xi}_{u,\ell}^g(k)$. The waves from each generator point will thus be seen by an observer moving with the pressure patch along an angle, γ , to the horizontal, given by the argument of the vector $\dot{\xi}_{u,\ell}^g(k) - \dot{U}_0$. The ray β is thus transformed to body coordinates with the result (we also use [1.11]):

$$\tan \gamma = \frac{-\sin \beta \cos \beta}{1 + \sin^2 \beta} = \tan \alpha^* \cdot \left[\frac{\cos^2 \alpha^*}{1 + \cos^2 \alpha^*} \right] \quad [1.12]$$

We note that the body rays pass through the pressure patch over the forward part of the hull ($\gamma < \alpha^*$). Of course this is permitted here.

Caustics

Waves created from different points on the boundary may cross in the water; this is permitted. If they approach tangency while merging (bunching), then a caustic is created; this corresponds

to the zero of $\left(\frac{d^2 f_{u,\ell}}{dt'^2} \right)$ at a stationary phase point, then the waves correspond to the p-waves of

[1.7]. We differentiate [1.8] again and finally find (we suppress the asterisk on $\xi_{u,\ell}^*$ for simplicity):

$$\frac{d^2 f_{u,\ell}}{dt'^2} = -k \left\{ \frac{2C_g^2}{(\xi - \xi_{u,\ell})} + \xi_{u,\ell}'' \right\} = -k \left\{ \frac{(\xi'_{u,\ell})^2}{2(\xi - \xi_{u,\ell})} + \xi_{u,\ell}'' \right\} \quad [1.13]$$

so that a caustic ($f''_{u,\ell} = 0$) will form out in the water along every ray originating at a point where $d^2 \xi_{u,\ell} / dt'^2 < 0$; that is everywhere the patch is concave from within. These caustics represent the merging of the rays originating at different points along the patch-boundary. They merge at a finite position along the ray and therefore disappear in the far field; their location is: $(\xi_c - \xi_{u,\ell}) = -(\xi'_{u,\ell})^2 / 2\xi_{u,\ell}''$, where:

$$\xi_{u,\ell}'' = +U_0^2 \frac{d\alpha}{dx} \sin \beta = \frac{-U_0^2}{R^*} \quad [1.14]$$

where R^* is the radius of curvature of the patch, positive when concave from within.

In the far field ($\xi \rightarrow \infty$), then new caustics will appear upon the vanishing of $\xi_{u,\ell}''$, corresponding to points of flatness ($R^* \rightarrow 0$).

The Resultant Amplitude Spectrum for the Boundary Wave

We allow the patch boundary to have both convex and concave regions with a local point of flatness separating them. Then the far field wave is obtained by combining various relations, [1.1-1.4], using only the waves in [1.7]. The result for $\text{Amp}(\beta)$, see the definition according to [1.3], is:

$$\frac{1}{R^*} > 0,$$

$$\text{Amp}(\beta) = \frac{1}{\sqrt{\pi}} [F_L][P_0] \left\{ |R^*|^{1/2} \cdot e^{-i[k(y_{u,\ell} \cos \beta + x'_{u,\ell} \cos \beta) \pm \pi/4]} \right\} \begin{matrix} \swarrow \text{concave} \\ \searrow \text{convex} \end{matrix} \Big|_l^u \quad [1.15]$$

$$\frac{1}{R^*} = 0,$$

$$\text{Amp}(\beta) = \frac{\Gamma(1/3)}{\pi^{1/6}} [F_L^{2/3}] [\bar{p}_0] \left\{ (\sin\beta) d^2 \alpha^* / dx'^2 \right\}^{-1/3} \cdot e^{-i[k(y_{u,\ell} \csc\beta + x'_{u,\ell} \cos\beta)]} \Bigg\}_\ell^u \quad [1.16]$$

(locally)

where: $x'_{u,\ell}$ is measured aft from the bow; $\bar{p}_0 = p_0 / \rho U_0^2$; $R^* = R^* / L$; F_L (Froude number) = $U_0 / (gL)^{1/2}$; and L is the length of the patch.

In the neighborhood of a point of flatness, the correct result, [1.16], must be used and the other result, [1.15] (valid elsewhere), appropriately merged with it.

We note: all of the waves are driven by the patch excess pressure, \bar{p}_0 ; interference effects between forward and aft generator points occur and are described by the exponential term in each relation; the strongest wave at low Froude numbers arises from the caustic at the inflection point (local flatness) in the boundary shape, $O(F_L^{1/3})$; for a curved boundary most of the waves are proportional to the (local radius of curvature)² and $O(F_L)$.

The End Waves

In addition to the foregoing boundary waves, which have arisen from the stationary phase contribution to the above integral, we must consider the possibility of end waves arising from integration by parts of the integral over the boundary represented by y_2 [1.3]. In this case the result can be represented as a sum of terms with coefficients, k^1 , k^2 , etc. Again in the far field the first term is dominant. It is:

$$\frac{d\eta}{d\beta} \cong R \cdot \left\{ \frac{p_0 [A] \zeta}{\rho k^2} \left[\frac{1}{c_p - \zeta'_u} - \frac{1}{c_p - \zeta'_\ell} \right] \cdot i e^{i[k\zeta - w(t - t_{b,s})]} \right\}_{t_b}^{t_s} \quad [1.17]$$

where C_p is given by [1.9], ζ'_ℓ by [1.10], and t_s is the time required for the pressure patch to cross the ray (we take $t_b = 0$). The far field amplitude function for the end waves is thus:

$$\text{Amp}(\beta) \cong \frac{i \bar{p}_0}{\sqrt{2\pi}} (F_L)^2 [G_s(\beta) \cdot e^{ikL^* \cos\beta} - G_b(\beta)] \quad [1.18]$$

where $G_{b,s} = \frac{\bar{\zeta}'_u - \bar{\zeta}'_\ell}{(1 - \bar{\zeta}'_u)(1 - \bar{\zeta}'_\ell)}$; $\bar{\zeta}'_{u,\ell} = \zeta'_{u,\ell} / c_p$; and L^* is the horizontal distance between the initial

forward (bow) and final aft (stern) intersection of a ray with the hull ($L^* = L$ for rays sufficiently near the vertical, but may be less than L for rays inclined near the horizontal, whose initial and/or final intersections may be tangent to the boundary at a point inboard from the bow and/or stern).

We note that the end waves are $O(F_L^2)$ and therefore weaker at low Froude number than the boundary waves. For $\bar{\zeta}'_{u,\ell} = 1$, the stationary phase point is realized and the correct value of $\text{Amp}(\beta)$ is that given by [1.15] or [1.16]. For prolonged flatness at the ends (wedge shapes), special considerations must be made, which we will not undertake here.

The results given here provide the solution of Weinblum's problem for sufficiently small Froude number, allowing the prediction of both wave patterns around the pressure patch and of the wave resistance. Some of the important results are: i) waves are generated on the boundary of the patch and at the ends; ii) for the strongest waves, the boundary wave number vector is normal to the patch boundary, and transverse waves are generated at the blunt ends and divergent waves on the near horizontal sides; iii) boundary waves observable above the patch originate on the forward-upper and near-lower sides of the patch; iv) for smooth boundaries each boundary wave will be excited at two generator points, one in each of the contributing sectors; v) the boundary waves can form caustics in the near field, which can be predicted; vi) the waves from the upper and lower boundary sectors interfere with each other; vii) the amplitude function of the boundary waves is weighted locally by the (radius of curvature)² of the patch boundary and is therefore

larger where the surface is flat than where it is highly curved; viii) points of local flatness (inflection) cause stronger waves (caustics) locally than other points; ix) the boundary wave amplitude grows like (F_L) except for the inflection point wave which grows like $(F_L^{2/3})$; x) the "end" waves originate at points of initial or final intersection (or tangency) of a ray with the pressure patch boundary and grow as (F_L^2) .

The Michell Ship as an Extended Pressure Patch

Finally we should point out that the Michell ship may readily be treated as a pressure patch extending to infinity with varying patch pressure corresponding to (p_M/ρ) , where p_M is the zero Froude number (double model) pressure in the field about the Michell hull. The far field wave, $d\eta/d\beta$, will be found by integration of the pressure sources along a straight ray characteristic, β . When the ray intersect the hull line on the x-axis, then a discontinuity in the pressure gradient, $\nabla_\beta(p_M/\rho)$ occurs; otherwise the pressure p_M is smooth.

Upon integration by parts along the ray, waves will originate at the hull boundary, driven by the pressure gradient there: $\nabla_\beta p = \cos\beta \partial p/\partial x + \sin\beta \partial p/\partial z$. Except at the ends of the hull, $\partial p/\partial x$ is continuous across the hull, so that the jump arises from the pressure gradient normal to the hull, and it is this gradient which drives the hull waves, see Figure 1.3. Immediately at the ends, the gradient in x becomes discontinuous at a stagnation point, and must be accounted for.

We note that upon integrating these boundary generated waves in t' (along the hull), no stationary phase contributions arise, since the hull is taken on the x-axis (to permit the hull to be distinct from the x axis and otherwise ignore the displacement flow would be inconsistent). Therefore all of the waves made by a smooth Michell hull will arise at the ends as a result of integration by parts.

2. THE WAVES GENERATED BY A SHIP

Introduction

The flow about a ship differs from that under a moving pressure patch in a number of important ways:

- a) there exists no externally imposed pressure to drive the waves,
- b) the waves which are created must be prevented from crossing the hull, which is of course impermeable,
- c) there exists a substantial flow about the ship, which bends the rays, and,
- d) the waves as they travel along the rays are effected by the displacement flow and alter their characteristics.

Our Method, Fundamentals.

These differences are formidable and render the problem non-linear. However, most of these differences can be conceptually dealt with, as we show below. (The small letters, a.) etc., refer to the letters above.)

a) We consider that the flow about the ship has been calculated by the method of the "naive" Froude number expansion (the potential is represented as a series of terms whose coefficients are integer powers of F). The first term, in which the water surface is flat, represents the zero Froude number flow about the ship. It produces an elevation, η_0 , of the water surface which is $O(F^2)$. This in turn produces a displacement flow of $O(F^2)$. We shall assume that it is this related elevation of the water surface which relaxes and in the process produces a wave pattern. Note that the elevation is simply, $\eta_0 = p_D/\rho g$, where p_D is the pressure in the displacement flow. The latter is the double model flow to zero order in F , but differs from it in $O(F^2)$. We discuss at the end of the paper whether the displacement flow calculated in this way is adequate at the points of the bow and stern.

b) Waves of a given far-field wave number direction, $\tilde{\beta}$, which arrive at a distant observer point, ζ , above the hull, originate at time t' along some ray, $S_R(\beta, t')$, which intersects the hull surface in the time interval $(t'_b - t'_s)$, between contact first with the bow and then with the stern, Figure 2.1.

What conditions must be imposed on the wave vector, \vec{k} , at the hull? The hull is impermeable; i.e. there can be no energy flux through it. This condition is automatically satisfied when waves originate on the hull whose vector \vec{k} is parallel to the hull, or if their group velocity, c_g , is zero. The latter corresponds, we later show, to wave vectors normal to the hull.

Does this mean that no other wave vectors are allowable at the hull? The answer is no. The pair of an outgoing wave vector, $\vec{k} = ke^{i\beta_o}$, and an incoming vector, $ke^{i\beta_i}$, whose sum is normal to the hull is also allowable. In other words, the outgoing vector is the reflection (optical) from the wall of the incoming vector. Therefore the locus, S_o , along which energy originates for the observer, must include an incoming portion, too, for which the preceding conditions are satisfied; quite apparently this incoming energy is reflected at the wall toward the observer, see Figure 2.1

As a result of the reflection of the locus at the wall, a discontinuity in the gradient of the pressure occurs on the locus at the wall, and we shall see that this discontinuity drives the waves and they appear as if originating at the hull. This is the same result as emerged in the conceptually simpler case of the Michell ship, discussed earlier.

The question of conditions leading to allowable rays is later discussed in the forthcoming section: The Rays at the Hull.

c) There is a free surface velocity $\vec{q}_o = q_o e^{i\alpha_o}$ associated with the displacement flow, where α_o is the angle of the flow on the free surface relative to the x-axis; we assume this angle is not significantly different from the corresponding angle for the projection of the free surface velocity on the horizontal plane. This velocity is measured relative to ship (moving) coordinates and is therefore stationary. At a fixed point in water (fixed) coordinates, there corresponds to it a velocity, $\vec{v}_o = v_o e^{i\delta_o} = q_o e^{i\alpha_o} - U_o$, which is not stationary, see Figure 2.2.

The wave energy propagates in the water with the sum of the group velocity, $\vec{C}_g(k)$, and the convective velocity, \vec{v}_o . If, $\vec{C}_g(k) + \vec{v}_o = u_\mu e^{i\mu}$, and $\vec{C}_g(k) + \vec{q}_o = u_\gamma e^{i\gamma}$ then the ray angles in fixed coordinates, μ , and in ship coordinates, γ , everywhere in the flow are:

$$\tan\mu = \frac{[\sin\alpha_o(1+\cos^2\beta) - \cos\alpha_o\sin\beta\cos\beta]}{[\cos\alpha_o(1+\sin^2\beta) - \sin\alpha_o\sin\beta\cos\beta - 2U_o/q_o]} \quad [2.1]$$

$$\tan\gamma = \frac{-\{\sin\beta\cos\beta\} + \tan\alpha_o\{1+\cos^2\beta\}}{\{1+\sin^2\beta\} - \tan\alpha_o\{\sin\beta\cos\beta\}}$$

see Figure 2.3, a most important diagram. To derive [2.1] we have used a result, $C_g = (-q_o/2)\cos(\beta-\alpha_o)$, derived from stationarity (see d) below).

d) The waves at any point in the flow must be stationary when observed in ship coordinates. That is:

$$C_p + v_o \cos(\beta-\delta_o) = -U_o \cos\beta \quad [2.2]$$

This can be shown to be equivalent to:

$$C_p = (g/k)^{1/2} = -q_o \cos(\beta-\alpha_o) \quad [2.3]$$

In addition, since $\vec{k} = \text{grad}(\text{phase})$, the condition of irrotationality, $\nabla \times \vec{k}$, must be satisfied in the wave field. In our notation this can be written:

$$\frac{\partial(k \cos\beta)}{\partial y} - \frac{\partial(k \sin\beta)}{\partial x} = 0 \quad [2.4]$$

When this is expanded and the derivatives $\partial k/\partial x$, $\partial k/\partial y$ are substituted with values determined by differentiating [2.3], then it is found that:

$$\frac{\partial \beta}{\partial y} \{-q_0 \cos \beta \sin(\beta - \alpha_0) - \frac{C_p}{2} \sin \beta\} + \frac{\partial \beta}{\partial x} \{+q_0 \sin \beta \sin(\beta - \alpha_0) - \frac{C_p}{2} \cos \beta\} = F$$

$$F = \{-\cos \beta \cos(\beta - \alpha_0) \frac{\partial q_0}{\partial y} + \sin \beta \cos(\beta - \alpha_0) \frac{\partial q_0}{\partial x} - q_0 \cos \beta \sin(\beta - \alpha_0) \frac{\partial \alpha_0}{\partial y} + q_0 \sin \beta \sin(\beta - \alpha_0) \frac{\partial \alpha_0}{\partial x}\} \quad [2.5]$$

Then it can be shown that the characteristic form of this equation is:

$$\frac{d\beta}{d\ell} = F/[C_g \cos(\beta - \alpha_0) + q_0 \cos(\gamma - \alpha_0)] \quad [2.6]$$

where ℓ is the distance along the ray, γ . This latter equation together with [2.1] and [2.3] allow the tracing of a ray in the ship's flow field. Extensive calculations of this kind have been carried out by Yim (1981).

The local wave frequency on a wave, β , in water coordinates is ($\omega_0 = (kg)^{\frac{1}{2}}$):

$$\omega = \omega_0 + kv_0 \cos(\beta - \delta_0) \quad [2.7]$$

Finally, the wave along a ray in water coordinates can be described in general as:

$$\eta \sim a(s, t) e^{i\psi(s, t)} \sim a(s, t) e^{i\left\{\int k_\mu ds - \int \omega dt\right\}}; \quad \frac{d\omega}{dk_\mu} = \frac{ds}{dt} \quad [2.8]$$

where k_μ , ω , q_0 , and δ_0 , determined in water coordinates vary along s and with time too; k_μ is the apparent wave number along s , $k_\mu = k \cos(\beta - \mu)$.

The Rays at the Hull

Under what circumstances can an outgoing ray exist at the hull, which will be seen by an observer above the hull? We should require: i) that the ray in ship coordinates, γ , does not lie inside the hull, and ii) that the energy flow outward along the ray, toward the observer; i.e. that $C_p > 0$. The first of these requirements is equivalent to: on the upper side of the hull, that $\gamma^* > \alpha^*$, and that on the lower side that $\gamma^* < \alpha^*$. The second of these requirements is equivalent to: $\pi/2 \leq (\beta^* - \alpha^*) \leq \pi$, see [2.3]. The asterisks refer to conditions on the hull.

We observe the ray diagrams and note that the second requirement above is met between the two hull lines ($\gamma^* = \alpha^*$), Figure 2.4, and then that the first of these is met only on the upper surface of the hull, Figure 2.3. Therefore, only on the upper surface of the hull can rays originate which will be seen by an observer above the hull. We notice that this is a different result than in the pressure patch case, where we had only the second requirement; but in this case it was allowed for the rays to cross inside the patch boundary, and in the ship case this kind of behavior is forbidden.

Does this mean that any wave angle, β , satisfying the above requirements can exist at each point on the upper hull? The answer is yes. Of course the range of γ possible at a given point will be restricted to a minimum value, $\gamma^* = \alpha^*$, and a maximum value corresponding to the peak of the curve γ vs. β , α^* being held fixed (this peak corresponds to the caustic curve in Figure 2.4). This conclusion applies as well to the ends as elsewhere, except that at the corner made by a finite entrance or leaving angle, many flow angles may exist, as we discuss in the next section.

Finally we notice that according to the ray diagram, Figure 2.3, there exists no dependence of the local flow speed on the relation between the local angles, γ , α_0 , and β . Therefore, within the assumptions of ray theory, aspects of the flow geometry determined solely by these variables (not including local wave lengths, which are speed dependent) will not vary with speed except insofar as α_0 is itself speed dependent. Of course the displacement flow is speed dependent; for example, in the high speed limit, the flow would seem to approach the hull with little lateral deviation, reducing α_0 in comparison to the low speed limit.

The Rays at the Ends

We imagine a bow with a finite entrance angle, α_e . Directly at the bow then the flow direction changes discontinuously; i.e. the flow direction, α , is zero on the streamline approaching the stem, while it has the angle, $\pm\alpha_e$, on the hull streamline itself. In the corner between these streamlines the angle of the flow takes on various angles, $0 < \alpha < \alpha_e$, which depend on how the corner is approached; i.e. the flow angle, α , at the corner is a function of γ , the ray angle at the corner, see Figure 2.5. What do we know about $\alpha(\gamma)$? On the hull itself, $\gamma = \alpha_e$. On the approaching streamline, $\alpha = 0$ and $\gamma = \pi$. In between we must solve for the flow. As far as the zero Froude number flow is concerned, it seems necessary for the flow at the wedge bow to approach the planar (2 dimensional) wedge flow in the limit of small (thickness/draft). In this case, which we give as an example, a linear relation exists between α and γ :

$$\begin{array}{l} \text{Wedge bow,} \\ \text{Small (thickness/draft).} \end{array} \quad \gamma = \left[\frac{\alpha_e - \pi}{\alpha_e} \right] \alpha_0 + \pi \quad [2.9]$$

This relation can be superimposed on the ray diagram; we give an example in Figure 2.5. The solution extends between the hull boundary loci. On the upper, $\beta = \alpha + \pi/2$, and $C_g = 0$. On the lower, $\beta = \alpha + \pi$ and $C_g = -\alpha_0/2$. In between there are two distinct values of β for each value of γ less than γ_c , which is a limiting value and corresponds to a caustic; so all the waves are contained within, $\alpha_c < \gamma < \delta_c$. In this respect the wave pattern is qualitatively similar to the Kelvin wave. Notice that on the hull ray, $\gamma = \alpha_e$, the wave crests are either normal or parallel to the hull. In the latter case $C_g \equiv 0$ and the wave does not therefore penetrate the hull. In the former case the wave vector is parallel to the hull and does not penetrate it either.

This problem of determining the kinematical wave pattern at the bow of a ship emphasizes the importance of having a better quantitative understanding of the singular flow at ship bows; but at least the example here illustrates how knowledge of $\gamma(\alpha_0)$ can be used with the ray diagram to determine the initial wave pattern, $\beta(\gamma)$. In addition, since reduction in draft would reduce the disturbance away from the hull, it seems likely that the planar case is more than an example, but represents an upper limit, i.e. $\alpha_0(\gamma)|_{3D} < \alpha_0(\gamma)|_{2D}$ and in consequence $\gamma(\alpha_0)|_{3D} < \gamma(\alpha_0)|_{2D}$.

The limiting ray angle, γ_c , is readily determined by combining $\gamma(\alpha_0)$, for example [2.9], with the locus of the caustics in the ray diagram, Figure 2.4. The latter (we are dealing with the locus which intercepts the Kelvin far field curve) is:

$$\gamma = \gamma_K + \alpha_0 \quad [2.10]$$

where γ_K is the Kelvin angle, 19.5° . The solution of [2.9] and [2.10] is:

$$\gamma_c = \alpha_e (1 - \gamma_K/\pi) + \gamma_K \quad [2.11]$$

We note that $\gamma_c > \gamma_K$. This relation, [2.11] is given as Figure 2.6. We would expect for ships of finite draft that the limiting ray, γ_c , would lie between the two curves labeled hull and bow (theory).

The same considerations apply at the stern if we ignore the viscous wake. If we assume the planar wedge solution there, too, we find:

$$\begin{array}{l} \text{Wedge Stern} \\ \text{Small thickness/draft} \end{array} \quad \gamma = - \left[\frac{\alpha_e - \pi}{\alpha_e} \right] \alpha_0 \quad [2.12]$$

and the intersection of this relation and the caustic locus gives for the caustics at the stern:

$$\gamma_c = \left[\frac{\alpha_e - \pi}{2\alpha_e - \pi} \right] \cdot \gamma_K \quad [2.13]$$

which is also plotted as Figure 2.6. We note that in this case, $\gamma_c < \gamma_K$.

Finally we note that our bow wave results do not at all coincide with the observations of the "non-dispersive" bow waves of the Tokyo school, see Inui (1980) or Miyata (1980). In their experiments on wedge bows, carried out at speeds between 0.5-1.5 m/sec. and drafts between 1-15 cm., the observed "shock" angle increased with draft and with speed. At the lowest speed the observed

shock angle was far in excess of the predictions here; furthermore as the entrance angle decreased to zero the shock angle did not approach the Kelvin angle but an angle of about 30° ; we observe that for these tests (speed = 50 cm/sec.) the local speeds near the bow may have been too close to 23 cm/sec. the minimum phase speed for capillary gravity waves. At the highest speed (Froude number based on draft between 1.5-6) the observed shock angles were much reduced, reducing further with decrease in draft, and did not approach the Kelvin angle as α_c approached zero. This behavior suggests that the "shock" wave observed by Miyata does not correspond to the limiting gravity wave (caustic) described here. Additional data are required at speeds safely above the capillary regime and at Froude numbers based on draft $O(10^{-1})$.

We shall return later at the end of the paper to discuss the vexing problems associated with the ends of the ship. Meanwhile we explore the implications of the ray theory which we have just defined, and which have been largely based on the assumptions of Ursell (1960) and Whitham (1961).

The Far Field

We may now proceed to examine in further detail the question of how a displacement ship makes waves. Our technique is identical to that observed already in the case of the moving pressure patch, although important differences arise in the results.

From each point in the flow outside the ship an elemental radial wave is generated. Of course this radial wave is distorted in time as the rays from the generation point travel out and are bent; this effect is taken into account by ray tracing according to [2.1], [2.3], and [2.5].

Again, we take a bundle of rays (bent) corresponding to a far field wave number angle, $\tilde{\beta}$, originating at an observer point, s , and width, $(s-s')d\beta$, and we represent the resulting plane wave, $d\eta/d\beta$, at s , as an integration of the elemental waves along the generator points, s' , and over time, t' , see Figure 2.1. The ship is moving from right to left:

$$\frac{d\eta}{d\beta}(s, t, \tilde{\beta}) = R \cdot \int_0^t dt' \int_{s'}^s \frac{p_D}{\rho} [A](s-s') i e^{i\psi(s-s'; t-t')} ds'; \quad [2.15]$$

where ψ is defined in [2.8] and: $dw/dk_\mu = d(s-s')/d(t-t')$.

We recognize that the locus, $S_0(\tilde{\beta})$, on which waves destined for the fixed observer appear, will vary with time, t' , approaching the fixed far field observer point, s , as a ray with a fixed angle, β . This movement of the locus results from the effect of the displacement flow about the ship, which is non-stationary in the observers frame.

The function $[A]$ in [2.15] refers, as before, to the spreading function for the wave due to a concentrated imposition of pressure, p_D/ρ . In the absence of a displacement flow this has the form, [1.2], which may also be written: $[A] = (E)^2(s-s')$, where E is proportional to the energy density at the wave number being considered. The displacement flow allows a mechanism for exchange of energy between the wave and the displacement flow, resulting in changes of E during the travel of the wave group from the hull to the far field. The wave resistance is in reality therefore manifested both in radiation of wave energy to the far field and in changes in the displacement flow. Here we take the point of view that the energy exchange in the near field does not significantly influence the pressure field on the hull and that the wave resistance can be calculated from the far field (pseudo) waves, themselves predicted as if energy exchange does not occur. Of course the pseudo wave amplitude spectrum will not agree with measured spectra, unless the energy exchange happens to be insignificant. In keeping with this point of view, we take $[A]$ identical with [1.2] putting $C_3 = C_3(\text{hull})$; this form conserves the initial energy. Finally, we add, the possibility for predicting the energy exchange in the near field utilizing ray theory does exist and could be implemented, using the conservation equation of Whitham (1962).

This integral, [2.14], can be represented asymptotically (short waves relative to the scale of the flow field) as a wave arising from the hull and perhaps from waves arising in the water at stationary phase points of $\psi(s, t)$. We neglect the latter in the present work; it is difficult to see how they might influence the wave resistance.

In keeping with the earlier discussion, we take the locus, $S_0(\tilde{\beta})$, to include its reflection at the hull. We then find, upon integrating by parts over s' along the entire locus, that the dominant term arises from the pressure gradient at the hull:

$$\frac{d\eta}{d\beta}(s, t, \tilde{\beta}) \cong R \cdot \int \frac{1}{\rho(k_\mu^*)^2} J^*[\nabla_\mu p_D][A](s-s^*) i e^{ih} dt' \quad [2.16]$$

where $h = \int_{s^*}^s k_\mu ds'' - \int_{t'}^t \omega dt''$; s^* represents the intersection of the ray and hull at time t' ;

and J^* represents the jump in the gradient of p_D , i.e. $(\nabla_{\mu_i} p_D)^* - (\nabla_{\mu_i} p_D)^*$. The direction of

the outgoing ray is μ_0 and the incoming, μ_i ; the latter is determined from the reflection condition on \bar{k} , see Figure 2.1. Finally,

$$J^*[\nabla_{\mu} p_D] = \frac{\partial p}{\partial \sigma} [\cos(\mu_0^* - \alpha_0^*) - \cos(\mu_i^* - \alpha_0^*)] + \frac{\partial p}{\partial n} [\sin(\mu_0^* - \alpha_0^*) + \sin(\mu_i^* - \alpha_0^*)] \quad [2.17]$$

where σ is direction along the hull and n , normal to it.

The values of $\mu_{0,i}$ can be determined using [2.1] and the following relation, which follows from the reflection property:

$$(\beta_i^* - \alpha_0^*) = 2\pi - (\beta_0^* - \alpha_0^*) \quad [2.18]$$

The "Dominant" Waves

This integral, [2.14], may be integrated by applying Kelvin's method of stationary phase, where the main contribution arises from points where $d^2h/dt'^2 = 0$, and also through integration by parts. The former produces waves which are stronger than the latter at low Froude numbers.

Applying first Kelvin's method, we can show that

$$dh/dt' = \omega - (k_\mu) \cdot ds^*/dt' \quad [2.19]$$

so that the stationary phase condition becomes:

$$ds^*/dt' = [w/k_\mu] = [w/k \cos(\beta^* - \mu)] \quad [2.20]$$

Stationarity requires that: $w/k = -U_0 \cos\beta^*$, so that the stationary phase condition is:

$$ds^*/dt' = -U_0 \cos\beta^* / \cos(\beta^* - \mu) \quad [2.21]$$

We have already given a relationship for $d\xi_{u,\theta}/dt'$, [1.7], and it applies to the hull case too.

$$ds^*/dt' = U_0 \sin\alpha^* / \sin(\mu - \alpha^*) \quad [2.22]$$

These equations taken together, [2.21-2.22], require:

$$\text{Either, } \beta = \alpha_0^* + \pi/2 \quad [2.23]$$

$$\text{or, } \mu = 0, \pi \quad [2.24]$$

In the former case, [2.23], the wave number is normal to the hull boundary exactly as in the case of the pressure patch, the group velocity is therefore zero, see [2.3], and these waves are allowable. Therefore in ship coordinates the ray is initially tangent at each point to the hull ($\gamma = \alpha_0^*$). This condition is represented in the ray diagram, Figure 2.3, by the upper boundary.

Can these rays, initially tangent at their formation, leave the hull? The answer lies with the first order ray equation, [2.5] and [2.6]. It is easy to verify that the solution for β along the ray, with the initial condition, $\beta^* = \alpha_0^* + \pi/2$, is everywhere just $\partial\beta/\partial\ell = \partial\alpha_0^*/\partial\ell$, and this means that the ray is not only tangent to the hull at its formation, but coincident with it; therefore none of these waves leave the hull boundary. Keller (1979) had earlier identified these waves ($C_g \equiv 0$) and also concluded that they could not leave the hull.

In the other stationary phase case, [2.19], the ray angle in water coordinates, μ , is required to leave the hull parallel to the direction of motion. This leads to two possibilities. First, at a stagnation point, then $\tan \mu^* = 0$, see [2.1], while the ray angle, γ^* , can take on a range of values, $\alpha_0^* < \gamma^* < \gamma_c$, as we have discussed in a previous section: The Rays at the Ends. Second, in the absence of a stagnation point, the selected rays will emerge from the hull in the horizontal direction; i.e. $\gamma = 0$. This is not possible on the forepart of the hull as these rays will pierce the hull, but it is possible over a part of the stern.

The wave number direction, β^* , for these aft waves is given by, see [2.1]:

$$\tan \alpha_0^* = \frac{\sin \beta^* \cos \beta^*}{[1 + \cos^2 \beta^*]} \quad [2.25]$$

which is represented in the ray diagram, Figure 2.3, by the intersection of the curves $\alpha_0 = \text{const.}$ and the horizontal axis ($\gamma = 0$). Notice that all values of β from $\pi/2$ to π are represented, two for each value of α_0 smaller than zero and larger than -20° , which represents a limiting value and is a caustic ($d\gamma/d\beta = 0$). This is the spectrum created on the upper aft part of the ship.

As these rays move aft in the water behind the ship, their angle changes according to the ray diagram. We would expect them to travel upward (α_0 increasing toward zero) from the horizontal axis ($\gamma = 0$) along a trajectory which must be determined from ray tracing, terminating in the far field on the Kelvin boundary ($\alpha_0 = 0$). Of course the final value of β on the ray, i.e. β , will generally be different from the initial value, β^* , and must be determined by ray tracing. The same remark applies, too, in the case of the end waves.

The Aft Waves: Their Strength

Upon applying Kelvin's formula, [1.7], to the far field wave integral, [2.16], the strength of the aft waves may be obtained. The far field wave takes the form:

$$\frac{d\eta}{d\beta}(s, t, \beta) \cong R \cdot \left\{ \frac{C_p J^{*i}}{\rho g \sqrt{\pi} \cos^2 \beta h''} \cdot e^{i[h' + \pi/4 \operatorname{sgn} h'']} \right\}_{x^*} \quad [2.26]$$

where the entire term is to be evaluated at the hull at a particular location, say x^* , where x^* is the distance aft of the bow, and is reached by the ray in time $t'(x^*) = x^*/U_0$, where we put $t'(0) = 0$. Of course the value $\alpha_0^*(x^*)$ must be allowable, i.e. $-20^\circ < \alpha_0^* < 0$. We can calculate $g''(\mu = 0)$ as follows:

$$h' = \int_{s^*(x^*)}^s k_\mu ds'' - \int_{x^*/U_0}^t w dt'' \quad [2.27]$$

$$h' = 0 = -k[(s^*)' \cos(\beta - \mu) + U_0 \cos \beta]$$

therefore, $(s^*)'_{\mu=0} = -U_0$, and

$$h''(\mu=0) = -k[(s^*)'' \cos \beta - U_0(\mu') \sin \beta] \quad [2.28]$$

$$\text{but, } (s^*)''_{\mu=0} = -U_0 \frac{\cos \alpha_0}{\sin \alpha_0} \cdot \mu' \quad [2.29]$$

and, finally, using [2.24], and $d\mu/dt' = -U_0 d\mu/dx$:

$$h''(\mu=0) = -\frac{2kU_0^2}{\sin \beta} d\mu/dx \quad [2.30]$$

In the far field,

$$g \cong [ks - \omega t] + \tilde{\phi}(x^*) \quad [2.31]$$

where $\tilde{\phi}$ is a phase function which must be determined by ray tracing, after combining [2.31] and [2.26]. Using this result, [2.31], the amplitude function corresponding to [2.25] becomes (we have non-dimensionalized everything):

$$\text{Amp}(\beta) \cong \left\{ \frac{(F_L)^3 (\bar{q}_0 \cos(\beta - \alpha_0))^2 (\sin \beta)^{\frac{1}{2}}}{\sqrt{2\pi} \cos^2 \beta \partial \mu / \partial x} [\bar{J}] e^{i[\tilde{\phi} - \pi/4 \text{sgn } \partial \mu / \partial x]} \right\}_{x^*} \quad [2.32]$$

where $\partial \mu / \partial x$ can be calculated by differentiating [2.1]. A lengthy calculation arises involving the necessity to determine $\partial \alpha_0 / \partial x$ and $\partial \beta / \partial x$. The former is known from the displacement field and the latter is determined from [2.6], the ray formula. We have to recognize that a caustic may exist ($\partial \mu / \partial x = 0$) in which case the p waves must be substituted for the s waves, see [1.7], and the wave becomes locally stronger. Obviously these aft waves need further detailed (and numerical) study. Finally we note that these waves are $O(F_L^3)$.

The Point of the Bow

We mean by this phrase, the hull-water intersection in the mid-plane of the ship. Our ray theory predicts two possible sources of waves immediately at the bow or stern ($x^* = 0$): i) in the case where $q_0(0) = 0$, then a fan of waves corresponding to $\mu = 0$ will arise (provided a stagnation point exists there), whose strength is given by [2.27] appropriately evaluated at the bow; these waves are $O(F_L^3)$; ii) waves arising from integration by parts over the hull, giving rise to waves of $O(F_L^4)$ whose strength also depends on $J^*(x^*)$, i.e. on the pressure gradients in the displacement flow at the point of the bow; thus this wave does not depend on the existence of a stagnation point there. These waves can readily be calculated in a similar manner as in the case of the pressure patch, but we do not carry out the calculation here.

The asymptotic theory of bow waves thus predicts that the energy release depends entirely on flow quantities (including their gradients) evaluated immediately at the point of the bow. This is the result of Keller (1979). Now we can even propose formulae for the wave amplitude. But in what situation does this place us?

We have assumed so far that normal wave theory applies: that wave energy propagates with the group velocity advected with the displacement flow, that the group velocity is 1/2 the phase velocity and that waves are conserved. Under what conditions does such theory apply? In connection with the singular region near the bow, the conclusions of K. Eggers (1981) are important. He claims a region near the bow of the double model flow where waves cannot exist. It corresponds to the region $(q_0)^2 < 1/3$. Other than his investigation, the question of the validity of normal assumptions in regions of small local flow scales does not seem to have been systematically studied.

Here we take the point of view that for such theory to apply, the wave lengths in the field must be smaller than the local scale of the velocity field. We scale the former with k^{-1} and latter with $(\frac{1}{q_0} \partial q_0 / \partial r)^{-1}$. Remember that $k \sim g C_g^{-2} \sim g q_0^{-2}$, see [2.3], so that:

$$\frac{\text{Wave Length}}{\text{Local Flow Length}} \sim (F_L)^2 \partial \bar{p}_0 / \partial \bar{r} \sim \partial \bar{\eta}_0 / \partial \bar{r} \quad [2.14]$$

where $\bar{p} = p / \rho U_0^2$, $\bar{r} = r / L$, $\bar{\eta}_0 = \eta_0 / L$. Therefore, we would conclude that this asymptotic theory applies provided that the slopes of the elevation, η_0 , which drives the waves are sufficiently small.

However, in ray theory the entire energy release at the bow depends on conditions immediately at the point of the bow. What is the speed and wave slope in the displacement flow at this point? In particular, is the wave slope small? Are we even able to predict it with existing theories?

In the case of high Froude number based on draft, the situation near a blunt bow was authoritatively discussed by Fernandez (1981) incorporating an inner flow comprising a jet, first proposed by Dagan and Tulin (1972). In the case of low Froude number we have available only the suggestion of the naive Froude number expansion, and even in this case we do not have, as far as I know, actual numerical solutions for practical bow shapes such as wedges alone and wedges incor-

porating bulbs, which we know have a profound effect on the flow at the bow; see, for example, Sharma (1966).

It has in fact been generally proposed to base "low speed" theories including ray theory on a displacement flow calculated according to the naive Froude number expansion. In this case we are forced to have a stagnation point at zero Froude number and $q_0 \sim r^n$, where $n = 1$ if the bow is blunt ($\alpha_0 = \pi/2$) and $n < 1$ for other wedges (for planar flows $n = \alpha_0 / \pi - \alpha_0$). We thus face the vexing situation that, based on the double model flow, either $\nabla \eta_0$ (second order) $\sim \nabla p_0 \sim \nabla q_0 \rightarrow 0$ at the point of sufficiently blunt bows [no wave energy release to $O(F_L^3)$] or that $\nabla \eta_0$ is singular, in which case the ray theory does not apply. We are therefore forced to the conclusion that ray theory based on η_0 to second order is not useful. Whether the stagnation point and the singular behavior ($q_0 \sim r^n$) is removed in the second order displacement flow remains to be seen; this would only cause a change in η_0 at $O(F_L^4)$.

Is the naive Froude number expansion even applicable (uniformly convergent) in the neighborhood of the point of the bow? There exists a good chance that it is not. I say that because in nature it is normal on wedge models, see Standing (1974), to find the highest point on the free surface at some distance aft of the point of the bow (as Michell's theory predicts!); is it possible that this behavior is reflected at all Froude numbers on a scale near the bow which decreases with speed, perhaps as U_0^2 , creating an inner flow at the point of the bow for which the naive Froude number expansion is an outer flow?

Finally we seem to face two possibilities. Either: a) the slopes are sufficiently small at the point of the bow that ray theory is applicable and can be used, provided that the displacement flow there is known, or b) the usual ray theory, for one reason or another, is not applicable there. In either case we still have before us to understand, the question how a ship hull generates waves in the asymptotic limit of small Froude number.

Concluding Remarks

Using the same procedure for the calculation of the far field spectrum as in the case of the pressure patch, we have shown again that in asymptotic theory the boundary of the ship generates waves; for an observer above the hull only the upper side of the hull can generate waves. One set of these waves must have their wave vectors normal to the ship's hull, just as in the case of the pressure patch. However, because of the condition that the displacement flow follows the ship hull these waves have zero group velocity. They therefore propagate on a ray in ship coordinates tangent to the ship's hull and cannot leave the hull. These results are the same as those of Keller (1979). Another set of waves leave the aft portion of the hull on rays initially parallel to the ship's path, provided that the inclination of the hull is not steeper than -20° . We provide formulae for calculating the strength of these waves which are in general both transverse and divergent. In principal this set of waves includes a fan at the bow (or stern) provided that a stagnation point exists at the point of the bow; the strength of these waves depends entirely on the gradient of the pressure (elevation) in the displacement at that point, see Figure 2.7. We conclude that our present knowledge is inadequate either to know whether the conventional ray theory is valid near the ends of the hull or if it is, to use it effectively.

We have also examined the geometry of the wave flow which could be expected in the vicinity of a wedge bow or stern, utilizing ray theory. A limiting ray angle is found which correspond to the Kelvin angle (19.5°) for vanishing entrance angle, and increases linearly at the bow with increasing entrance angles; it is not dependent on the flow speed, except through changes in the displacement flow. At the stern we find a limiting ray angle which increases very slowly from the Kelvin angle (and not linearly) with the stern angle.

We hope these results will help in future efforts to provide an adequate understanding of the difficult question: how do ships generate waves?

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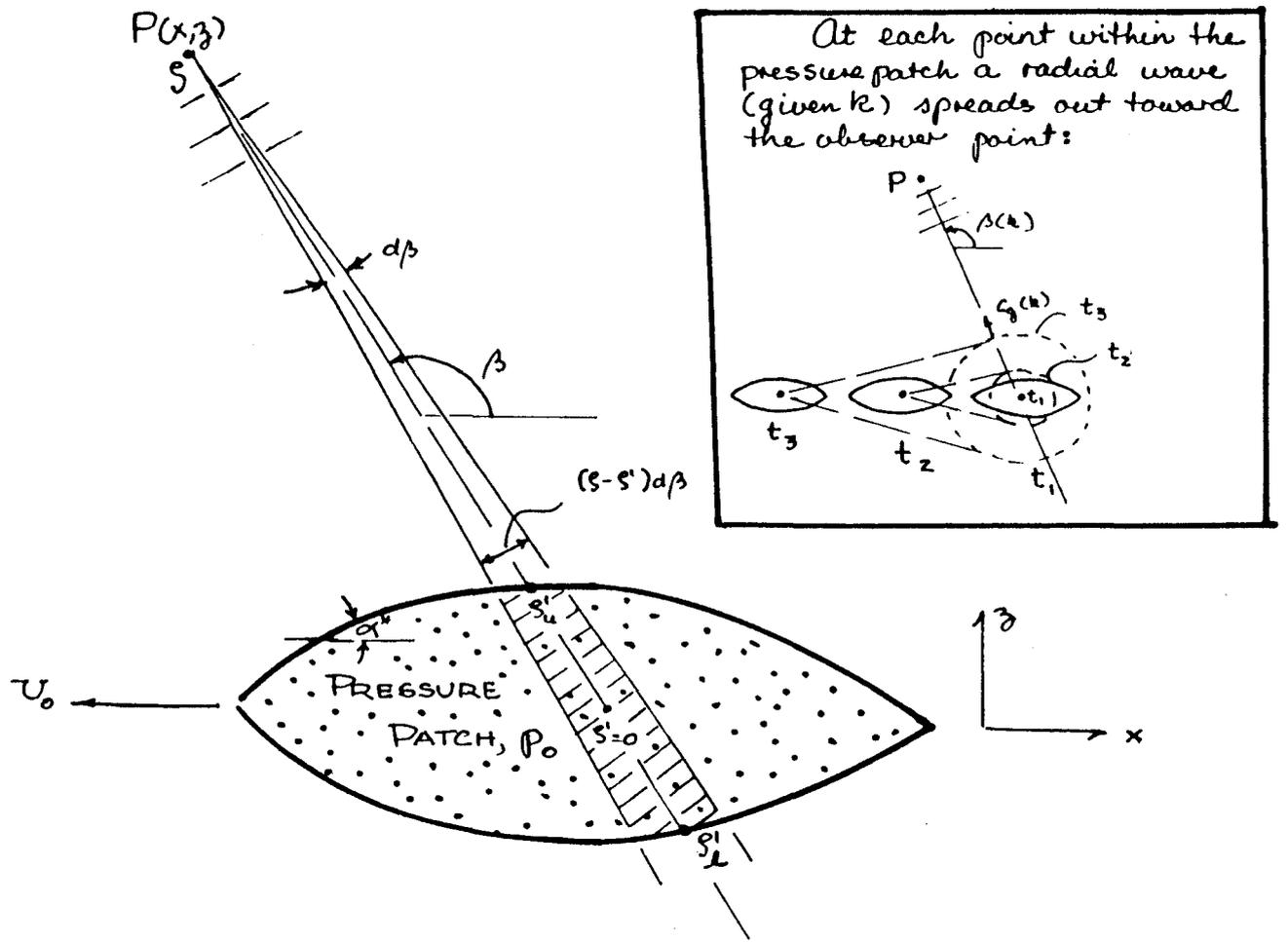


FIGURE 1.1 WAVEMAKING BY A PRESSURE PATCH

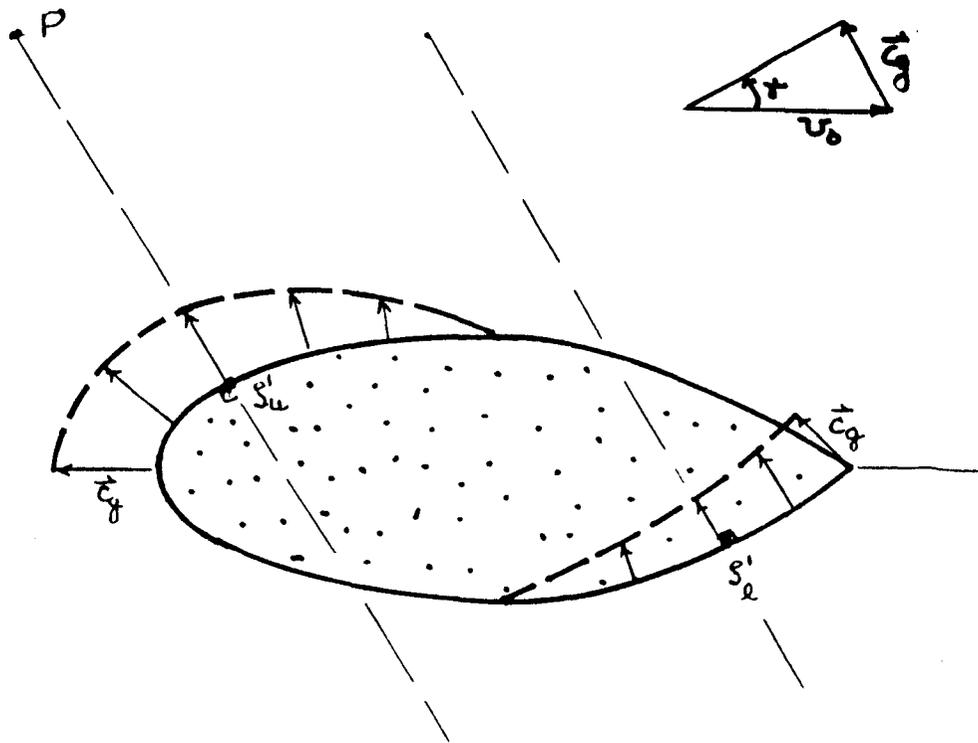


FIGURE 1.2 BOUNDARY WAVE GENERATION

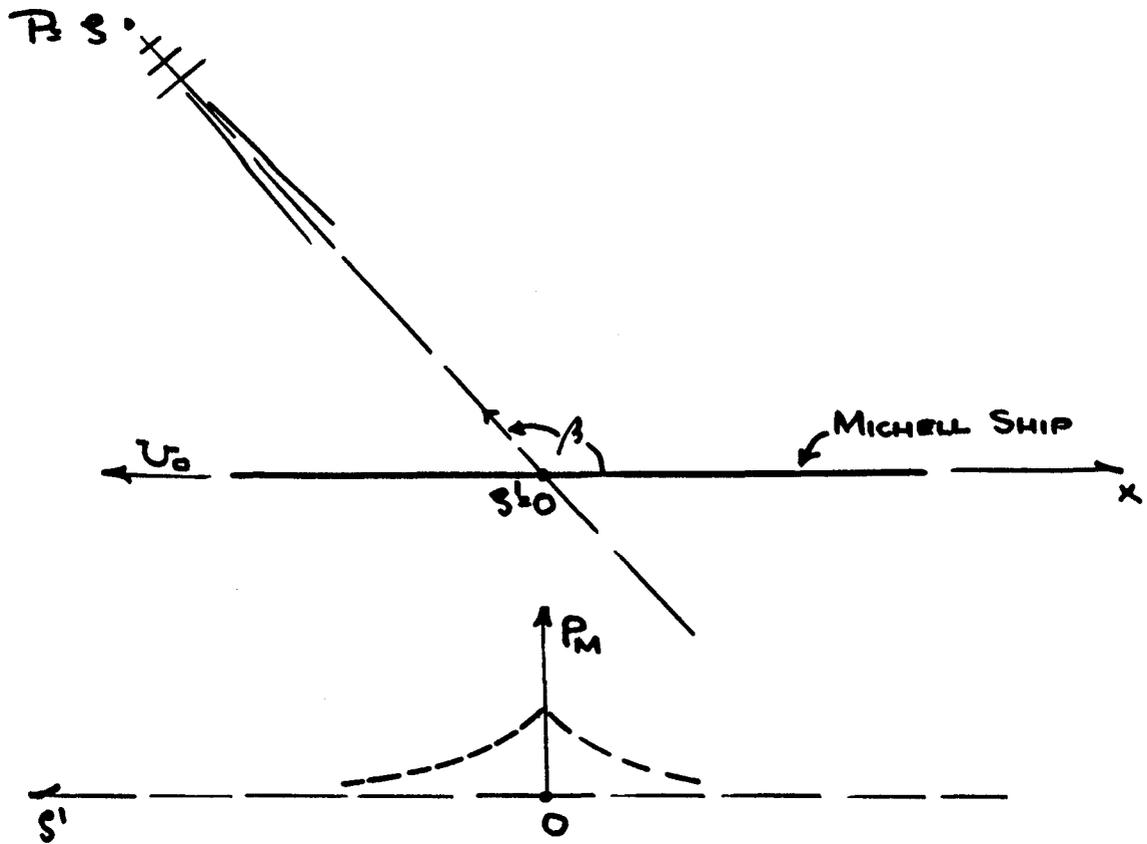
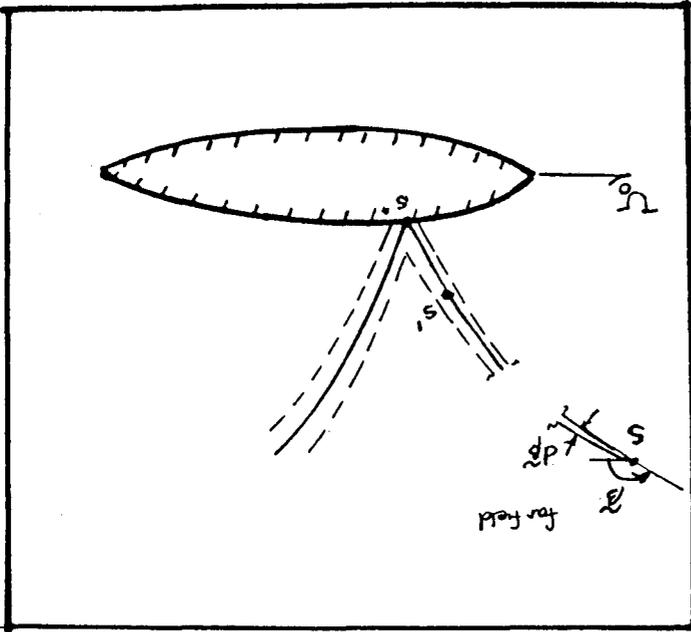


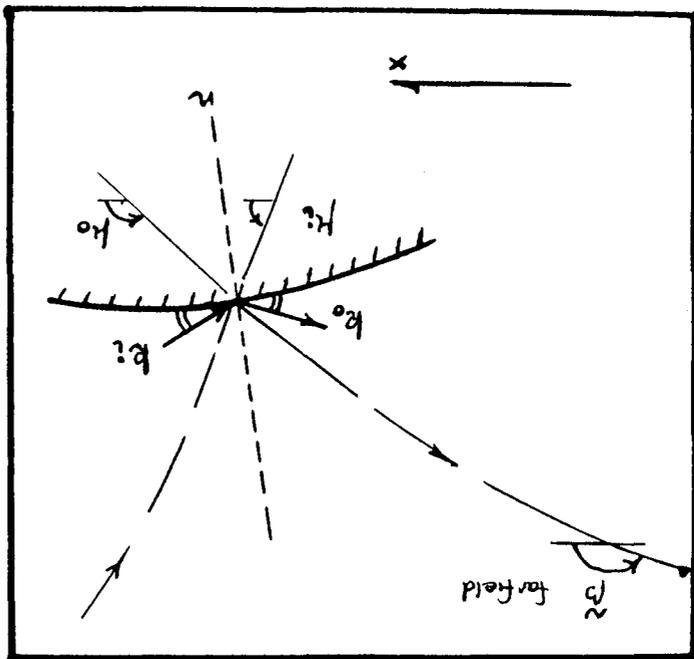
FIGURE 1.3 THE MICHELL SHIP (SCHEMATIC)

FIGURE 2.1 RAY BUNDLE GEOMETRY

NEAR THE SHIP



AT THE HULL



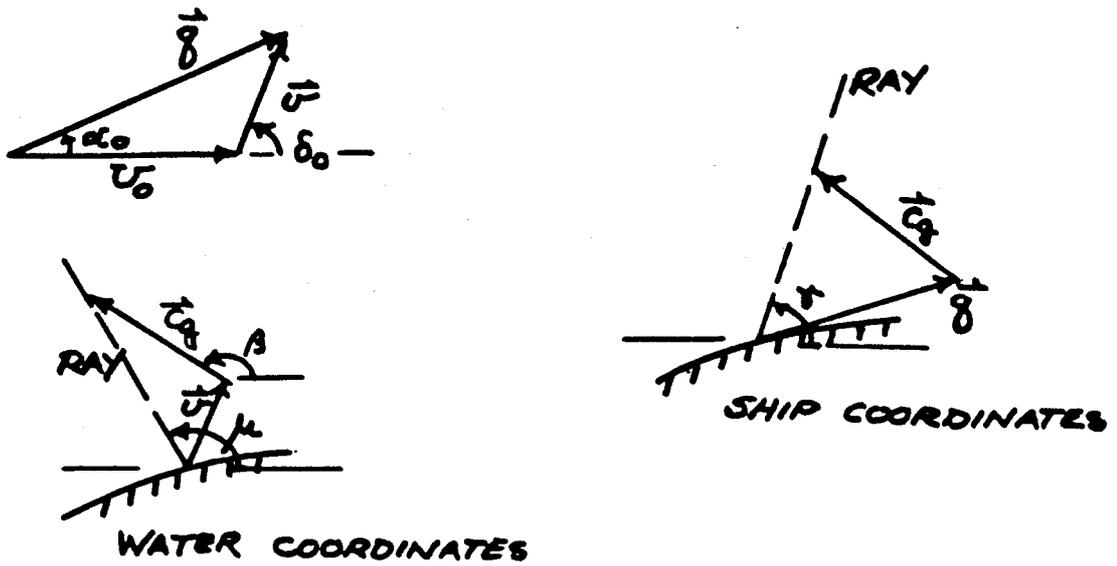


FIGURE 2.2 RAY AND OTHER DEFINITIONS

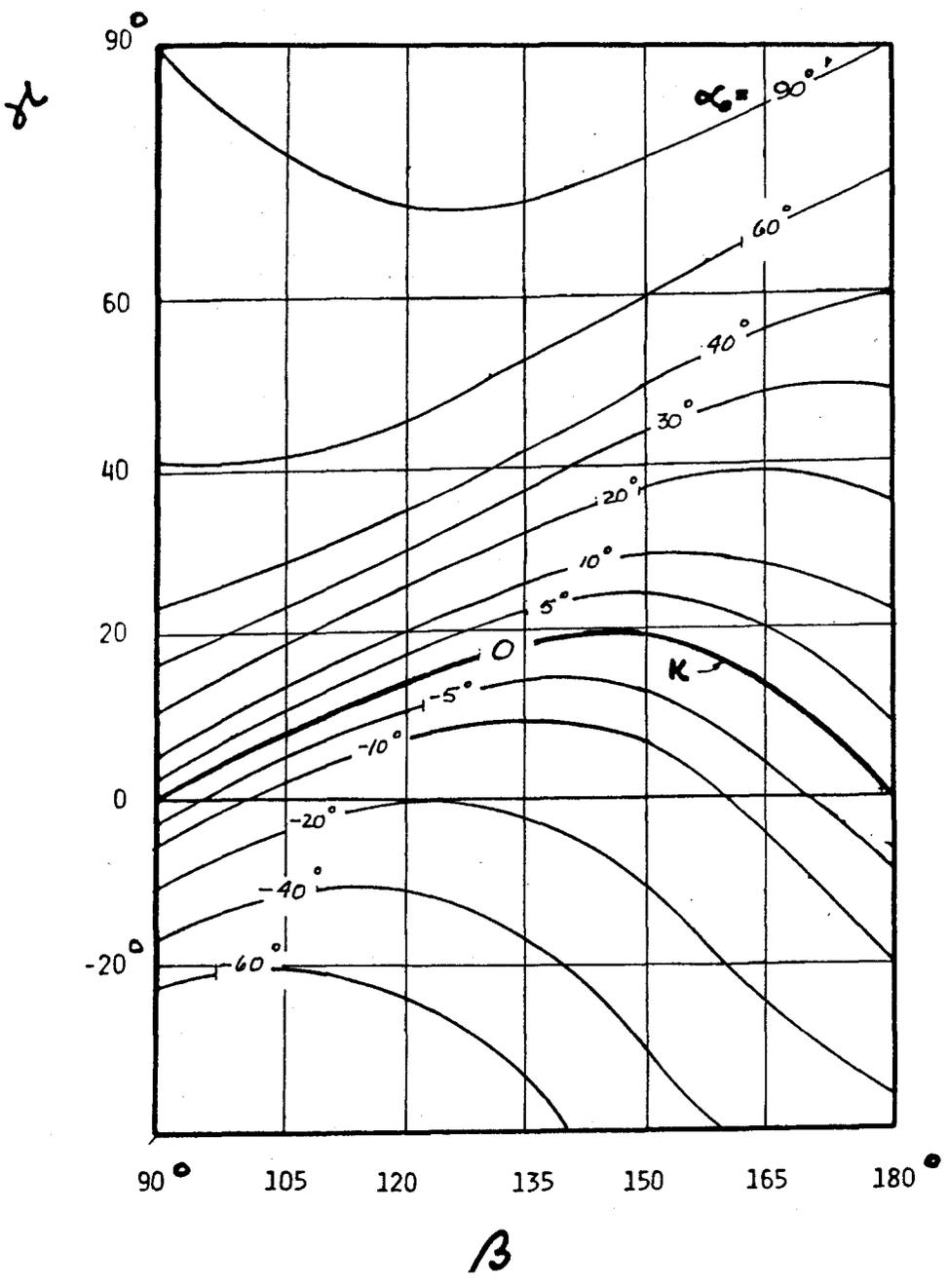


FIGURE 2.3 THE RAY DIAGRAM

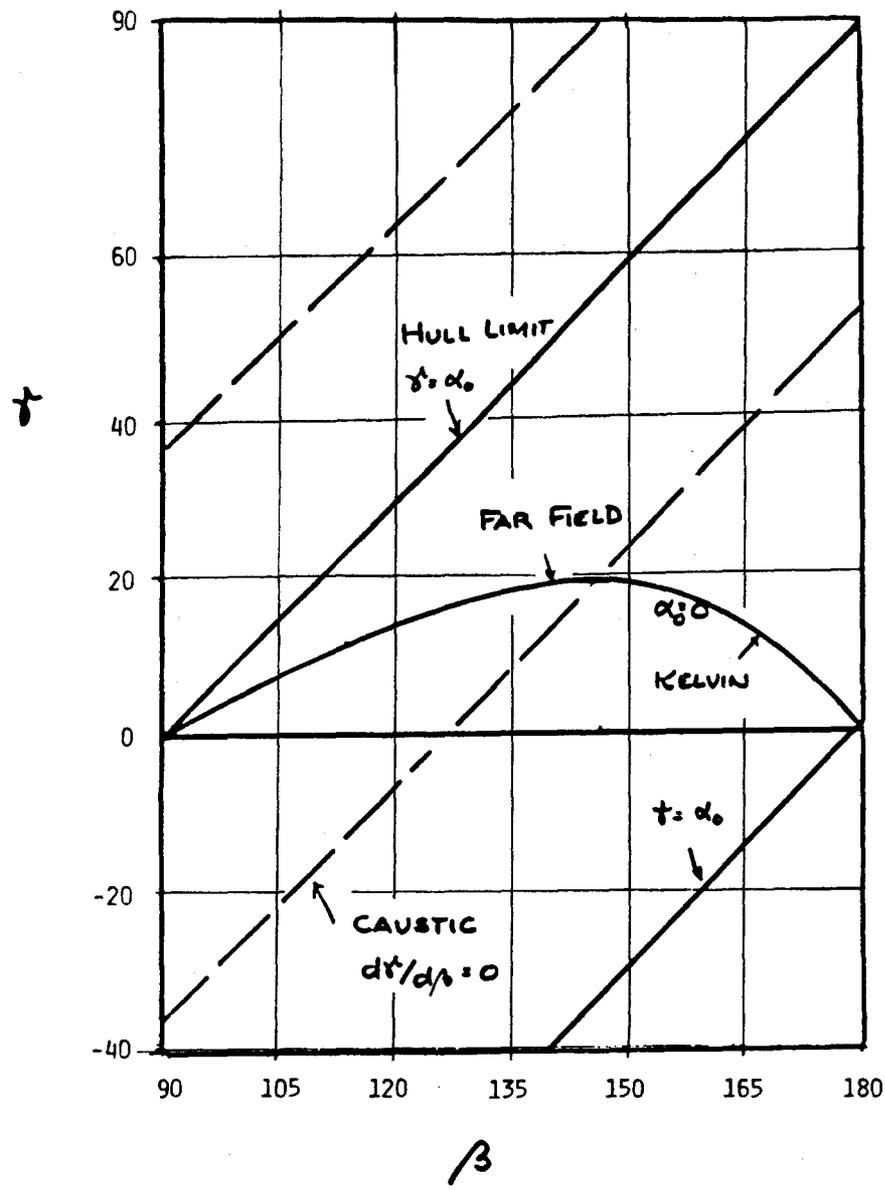


FIGURE 2.4 HULL BOUNDARY, CAUSTIC LIMITS

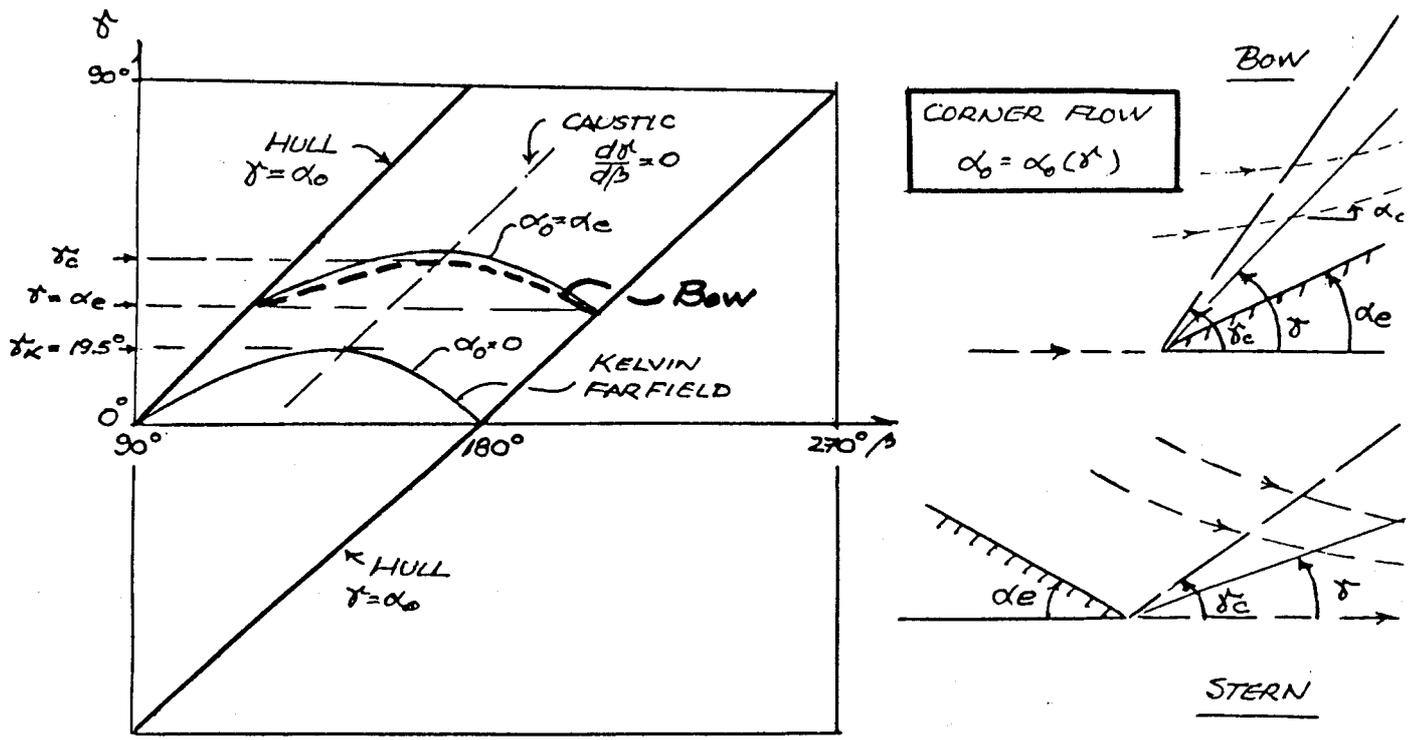


FIGURE 2.5 THE END FLOW RAY DIAGRAM (SCHEMATIC)

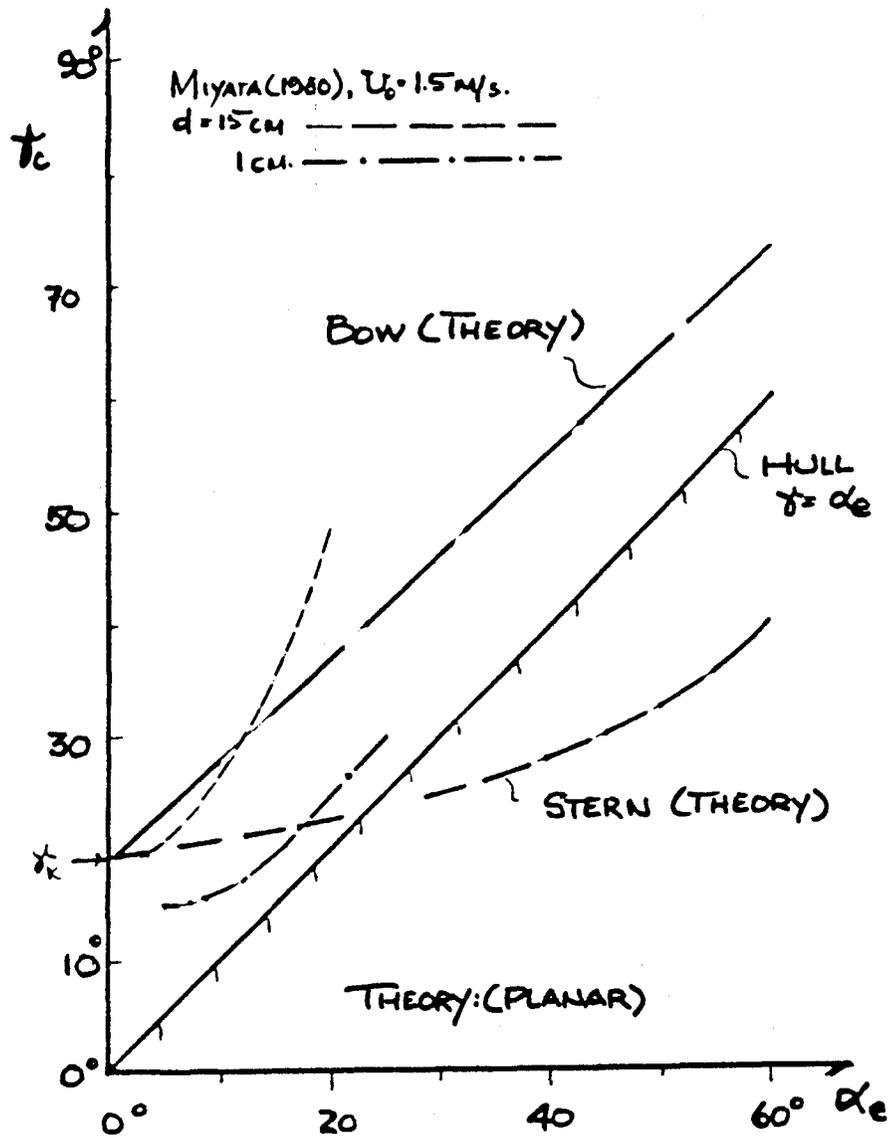


FIGURE 2.6 LIMITING RAY ANGLE, γ_c

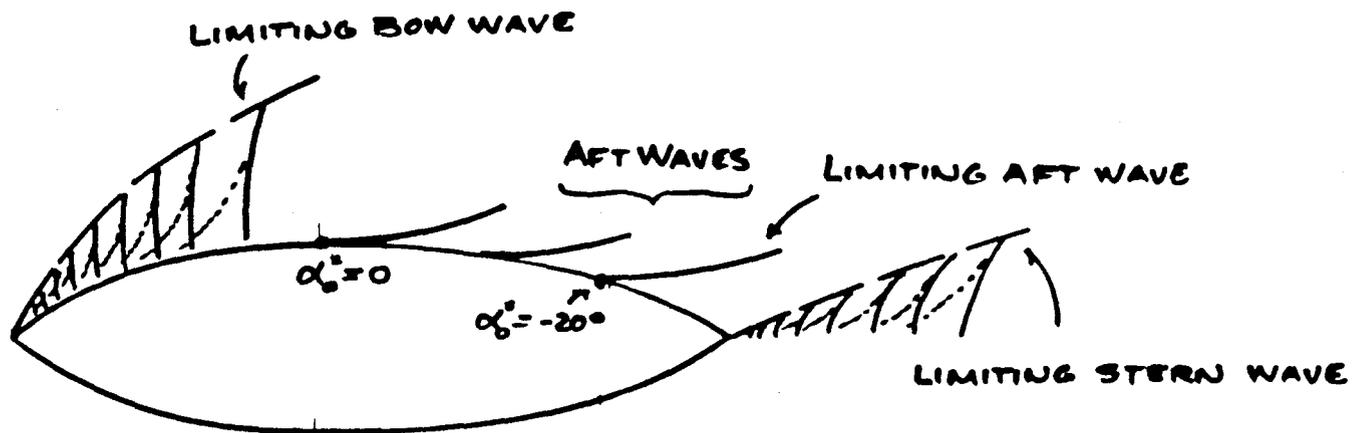


FIGURE 2.7 ASYMPTOTIC SHIP WAVE SYSTEM (SCHEMATIC)