

Seakeeping simulation of surfaced submarines and statistical validation based on model tests in open waters as part of the development of the new stability regulation DMS 1030-2 for the German navy

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ABSTRACT

This paper explains the need for the new surface stability regulation recently developed for the submarines of the German Navy and gives an overview of its general setup and main principles. Further, the test setup and main findings of open water model tests using two submarine hull forms on Eckernförde Bay are presented, which contributed towards the development of the new standard. Building on this introduction of the model tests, the results of an exemplary test run at specific combination of speed, stability, sea state and encounter angle are compared via statistical analysis with seakeeping simulation results using E4-ROLLS, a potential flow theory-based computation method. The comparison aims to check the suitability of E4-ROLLS for analyzing stability-dependent large-amplitude roll motions of surfaced submarines in natural seaways. The quality of results is discussed, whereby limitations of the simulation method are pointed out and the relevance of the presented research becomes apparent.

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Introduction

Although submarines are mainly designed for submerged operation, there are situations during which submarines are required to operate at the sea surface, aside from harbour maneuvers. Where military submarines are concerned, diving is not always permitted, as this is strictly regulated and strongly depends on the sovereignty of the waters, the objective of the operation (e.g. different rules apply to transfer voyages than to exercise assignments) and last but not least, on the Naval administration in charge. Furthermore, the possibility to submerge may also be restricted by the water depth at the current position, since a minimum water depth is required in order to dive deep enough to escape the main impact of the waves, which is still present underneath the surface and decreases with increasing diving depth. It is a well-known fact, that the widely cylindrical hull forms of modern submarines are far from ideal in regard to seakeeping and that surfaced submarines thus tend to roll heavily in waves, as stated by Gabler (2000). In recent years, the topic seakeeping of submarines at the surface is reappearing with increased interest within the research community. Nonetheless, the publicly available information in this area is still poor, due to the confidential nature of most submarine research performed. This paper aims to give some insights into recent developments and findings of research that

was jointly conducted by Hamburg University of Technology, the German Naval Arsenal and thyssenkrupp Marine Systems concerning stability requirements and seakeeping of surfaced submarines. The presented research builds upon the current state of the art, which is summarized in the following section.

State of the art

The state of the art concerning the regulation of surface stability is usually limited to deterministic requirements, which address either a minimum metacentric height or certain characteristics of the GZ-curves, as can be found, for example, in the stability regulations for military submarines of either the Royal Australian Navy (Australian Government Department of Defence 2009), Bureau Veritas (2016) or Det Norske Veritas Aksam (2022). A detailed overview and discussion of the most relevant existing stability regulations, including also regulations for civil underwater vehicles as well as the previous regulation for submarines of the German Navy, is given by Konovalov (2015). An extensive, more recent account can be found in (2024) by Büsken. One main issue of the existing regulatory frameworks for submarines, that becomes evident, is that the origin of the applied limiting values is largely unknown and that the reasoning behind specific deterministic criteria is not

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stated. Hence, it is not clear, whether the application of these non-transparent requirements, especially of historical threshold values, to modern submarine designs is reasonable. Aside from the Naval Submarine Code (North Atlantic Treaty Organization 2022), which follows a goal-based approach similar to the Naval Ship Code (North Atlantic Treaty Organization 2023), however does not provide its own criteria for verifying compliance with the therein defined objectives, all of the existing stability regulations apply a solely deterministic approach. Therein lies another point of criticism of the existing standards. Deterministic criteria can only cover the topics dynamic stability and especially seakeeping to a limited extent. To successfully include seakeeping in a stability regulation, a probabilistic standard is required. Thus, a modern stability standard should ideally combine the advantages of the deterministic and the probabilistic approach, as has been realized for surface vessels e.g. within the stability requirements for military ships of the German Navy, see (Bundesamt für Ausrüstung, Informationstechnik und Nutzung der Bundeswehr 2022). A similar approach for submarines, which includes seakeeping at the surface probabilistically, was yet to be developed, before the research presented here was conducted.

Aside from the state of the art in submarine surface stability regulations, the topic of the numerical simulation of the seakeeping performance of submarines at the surface still leaves plenty of room for improvement. An issue that hinders the further development of suitable simulation methods in this area is that there is usually no data available to validate the methods. Hardly any model test data on the seakeeping of surfaced submarines is publicly available, as also stated recently by Courdier (2023). One exception, are the public model test results given in Hermanski and Kim (2010) by Hermanski and Kim, which include data obtained via seakeeping model tests of a Victoria Class submarine within a model testing facility. Courdier also presents results of model tests performed within a basin in his dissertation, see Courdier (2023). However, these tests focus on the heave and pitch motion and neglect the roll motion, which in turn is the main topic of interest when it comes to seakeeping from a ship safety point of view. Concerning the stability-dependency of the seakeeping of surfaced submarines, the existing research, especially involving model tests, is limited. For surfaced cylinders, Vugts shows as early as 1968 that the coupling of roll and sway are affected by the position of the vertical center of gravity based on model tests (Vugts 1968). More recent research on the impact of the stability on the roll motion of a submarine has been performed for a surfaced Collins Class submarine: Hedberg develops a CFD-simulation method and compares the results with model test data of a simplified mock-up of this

submarine class in Hedberg (2006). Thereby, both metacentric height and sea states are varied. This research focusses on the roll motion and in particular the dependency of the roll damping on appendages such as bilge keels. Further, Corden-McKinley performs model tests using a simplified Collins Class submarine model, in order to assess the influence of the stability on the roll motion in beam seas, see (Corden-McKinley 2012). Generally, any model test data that is available for surfaced submarines, is restricted to model tests performed indoors, i.e. in an enclosed environment. In order to assess seakeeping, much longer test runs are however essential than can be realized within a basin. A sufficiently long test run duration is necessary, in order to capture the stochastic character of the seaway and thus of the vessel motions. To this end, model tests in natural waters are required, such as the ones performed for ships in the 60s and 70s under Prof. Wendel in Germany, respectively under Prof. Paulling in the U.S.A., see the works of Roden (1962a), Roden (1962a, 1962b), Kastner (1964), Kastner (1973), Haddara et al. (1972) and Paulling et al. (1972). These historical model tests also provide a test setup for the assessment of the influence of the stability on the seakeeping, that had not been utilized in the same way for submarines until recently, as described by Büsken and Krüger (2023). Such model tests performed on a natural body of water are referred to as *open water tests* in this paper, based on the description by Kastner (2007). In addition to the literature mentioned above, a significant part of the research on surfaced submarines deals with the aspect of roll damping and the influence of the free-flooding water between outer hull and pressure hull, as summarized in Büsken (2024).

Based on the above overview of the state of the art, the following section illustrates the problem statement of the presented research, by explaining the necessity of a new surface stability standard for German Naval submarines in detail.

Necessity of a new surface stability regulation for submarines of the German navy

In this paper, the recently developed new stability regulation is referred to as DMS 1030-2 as an abbreviation for its long German title *Deutscher Marinestandard für Wasserfahrzeuge der Bundeswehr Teil 1030-2 Hydrostatik von Ubooten*. The DMS 1030-2 is applicable for all manned submarines of the German Navy and addresses the hydrostatics of submarines in submerged, as well as in surfaced condition and further includes requirements for the stability during the transition phase. However, in this paper, only the parts concerning the stability of an intact submarine in surfaced condition are referred to. Due to

reasons of confidentiality, only main principles can be given here and any quantitative information concerning the DMS 1030-2 is omitted. This section describes, why the development of the new stability regulation was necessary.

Before the development of the DMS 1030-2 the surface stability of the submarines of the German Navy was solely regulated by given threshold values requiring a minimum metacentric height in surfaced condition in dependency of the submarine's displacement, as can be found in the previous stability regulation BV 1033-2, see (Bundesamt für Wehrtechnik und Beschaffung 2023). This old stability standard dates back to the 1960s and it is therefore not clear, whether it is still suitable for modern submarine hull forms and sizes. Most importantly, the origin of the limiting values for the minimum metacentric height in surfaced condition in the BV 1033-2 is not known. It is reasonably suspected, that these threshold values were simply adopted from the regulations for ships in force at the time and directly transferred to submarines. Bearing in mind the calculation accuracies at that time, it is further assumed that the limits in the BV 1033-2 mainly aimed at ensuring that the surface stability of a submarine remains positive.

The above may be a reasonable approach for submarines, seeing that submarines usually do not suffer insufficient stability in surfaced condition, as long as they meet the submerged stability requirements. Here, a short repetition of submarine hydrostatics is helpful: In submerged condition the submarine is weight-stable due to its vertical center of gravity being below the center of buoyancy. Its metacenter is a real center and located at the same point as the center of buoyancy. In case of circular frames, this point coincides with the center of the circle. In surfaced condition, the submarine's metacenter is higher than in submerged condition, due to the existence of a waterline area. Furthermore, the center of buoyancy is located at a lower point than before, since the submarine is less submerged without the water in the diving cells. If the vessel is heeled in surfaced condition, the center of buoyancy can move off-center, resulting in movements of the metacenter and in restoring forces as depending on the submarine's hull form. Nonetheless, this form-resultant part of the submarine's surface stability is relatively small compared to the part resulting from the low center of gravity. Based on this combination of weight- and form-stability, much smaller initial metacentric heights ensure *sufficient* surface stability when it comes to submarines than is the case for ships. Usually the range of metacentric height, which can be used as a descriptive value for the stability, that has been found to be practicable for surfaced operation of submarines is approximately ten times smaller than for naval surface ships.

Even in view of the above considerations, the impact of the surface stability on the behaviour of a submarine should be investigated thoroughly, before a new stability regulation is defined for modern hull forms. In case of a limiting threshold value, this value should be verified using submarines of various shapes and sizes and it must be reasonable and transparent. To this end, a threshold could be found by balancing heeling and righting levers in surfaced condition for a representative selection of submarine classes. Hereby, it should also be checked, whether the dependency of the minimum stability on the submarine's displacement is expedient. In defining a minimum threshold value for the initial surface stability, it is further essential to avoid too high requirements, since these would lead to a stiff behaviour of the submarine and thus result in dangerously high dynamic lateral accelerations whenever the submarine rolls, e.g. in severe sea states. Finally, it is not clear, whether a higher stability of the surfaced submarine necessarily ensures a better seakeeping performance, as such a limiting threshold might suggest. It is well known that submarines are prone to exhibit large-amplitude roll motions in harsh sea conditions, due to the fact that most modern submarine hull forms are optimized for submerged condition and exhibit low roll-damping characteristics. Some phenomena are known to lead to large amplitude roll motions of submarines, such as waves crashing against the sail and direct excitation in beam seas. Another known phenomenon is that sudden high roll angles can occur, if the submarine is significantly washed over by waves in following seas. However, it is assumed that additional phenomena may occur and it is further unclear, how the stability of the surfaced submarine may affect these and whether they can be avoided by an increase in stability. In this regard, a better knowledge of the seakeeping of surfaced submarines is required, especially including an investigation of the stability's influence on the seakeeping performance, if this aspect is to be included in a preferably more holistic stability regulation.

Based on the above considerations, dedicated research on the seakeeping of surfaced submarines as dependent on their stability was performed within a joint industry project between the German Naval Arsenal (*Marinearsenal*), the submarine-building company thyssenkrupp Marine Systems, as well as the Institute of Ship Design and Ship Safety of Hamburg University of Technology from spring 2021 to autumn 2023. The main objective of this research was the development of a new stability regulation, which addresses the surface stability of submarines professionally, thereby includes seakeeping and is applicable for a large range of submarine sizes and hull forms. The DMS 1030-2 is the result of this research, was signed in November 2023 and is now

mandatory for all manned submarines of the German Navy, also for already-built submarines, if not agreed otherwise, see Bundesamt für Ausrüstung, Informationstechnik und Nutzung der Bundeswehr (2023).

Overview of the three-level approach for proof of sufficient surface stability in the DMS 1030-2

This section gives a short overview of the main principles of the part of the DMS 1030-2 that regulates the surface stability of an intact submarine. Similar to the corresponding stability regulation for naval ships (DMS 1030-1), the new submarine stability standard also consists of a three-level approach. The three levels are treated as equals as shown in Figure 1, even though more or less calculation effort is required for the proof of sufficient surface stability depending on the chosen level.

The first level requires the smallest computational effort and is comparable to the previous stability regulation BV 1033-2. It utilizes a limiting value for the minimum initial metacentric height \overline{GM}_0 in surfaced condition, which is denoted as $\overline{GM}_{min,1}$ here. This threshold value has been checked for a total of twelve different submarines, including both historical classes from the 1960s as well as modern hull form designs and has been found reasonable for submarines based on balancing heeling and righting levers in still water and waves. The calculations performed as part of the development of the DMS 1030-2 show no clear dependency of required stability on displacement based on the twelve investigated submarines. Thereby, the twelve submarines cover a surface displacement range from about 500 tons to approximately 4000 tons. Therefore, such a dependency of limiting metacentric height on displacement is no longer included in the regulation. In order to avoid excessive lateral accelerations, a recommendation for a maximum metacentric height is given in the DMS 1030-2,

which is based on the results of seakeeping simulations for the twelve submarines. This limiting value \overline{GM}_{max} is complimentary for the first level and chosen in a way that does not conflict with the requirements for the submerged operation of the submarine.

In case the submarine cannot meet the requirement stated by the first level, i.e. if the initial metacentric height is less than the first threshold value ($\overline{GM}_{min,1}$), it is possible to provide proof of sufficient surface stability by fulfilling the second level. This is only possible, as long as the initial stability \overline{GM}_0 is within a certain range, i.e. still greater than or equal to a second, slightly reduced threshold value $\overline{GM}_{min,2}$ for which the following holds true:

$$\overline{GM}_{max} \gg \overline{GM}_{min,1} > \overline{GM}_{min,2} > 0 \quad (1)$$

As long as $\overline{GM}_0 \geq \overline{GM}_{min,2}$ holds, sufficient surface stability can be proven by balancing the heeling levers against the righting levers of the submarine, as defined in the stability regulation DMS 1030-2. This balance distinguishes between still water condition and waves and utilizes respective GZ-curves for the submarine in still water or in specific positions in a seaway to this end. On the heeling lever side, the balance according to level two includes submarine-specific heeling levers, which were identified during the open water tests performed on Eckernförde Bay. In order to account for energy being shifted into the system by periodic stability alterations, areas under the righting curves in the seaway conditions are further included as part of the balance. The balance includes threshold values for permissible heeling angles as well as minimum requirements for the residual righting lever at a specific reference angle. These limiting values are different to those defined for ships and require a specified modelling of the submarine, as given in the stability regulation. Summing up the second level, with the new stability regulation it is possible to design a submarine with a slightly lower initial stability than according to level one, as long as the submarine is still able to withstand the expected heeling moments in surfaced condition, both in still water and waves.

The third level is available for those cases, which meet all requirements of the lever balance according to level two at an even lower initial metacentric height than the reduced threshold value ($\overline{GM}_0 < \overline{GM}_{min,2}$). Level three is only fulfilled, if both the level two-criteria concerning the balance are fulfilled, as well as if a sufficiently safe seakeeping performance can be shown using direct calculations. This latter requirement is defined in detail within the DMS 1030-2 and comprises extensive time-domain seakeeping simulations in all six degrees of freedom, in which at least the roll motion is treated fully non-linear on the one hand, and a probabilistic assessment of the calculated

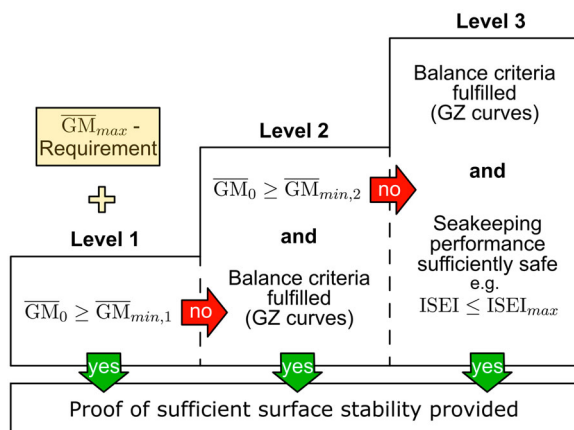


Figure 1. Three level approach for proving sufficient surface stability in DMS 1030-2.

motions and dynamic lateral accelerations at a specified point of interest on the other hand. The results of the probabilistic assessment are indices as a measure of the safety, which quantify the probability that the submarine encounters a dangerous situation under the analyzed conditions. The approach in level three, which utilizes a probabilistic assessment of the computed seakeeping performance, has been successfully adopted for ships by way of the *Insufficient Stability Event Index* (short: ISEI) developed by Kluwe (2009). This approach is now similarly included in the DMS 1030-2 for submarines, however with different failure criteria to define a dangerous situation than for ships. Furthermore, for submarines, different threshold values for the limiting $ISEI_{max}$ are proposed based on extensive seakeeping calculations for the twelve different submarines, see, Büsken (2024). In the currently adopted version of the DMS 1030-2 the third level requirements are only included within the appendix using preliminary threshold values. If the attained ISEI is less than or equal to the maximum permissible value ($ISEI_{max}$) the seakeeping of the submarine is regarded as sufficiently safe. The calculations for the twelve submarines have shown, that realistic modelling of the roll damping characteristics of the submarine is essential in order to obtain viable results. Therefore, this aspect should be given special attention in the seakeeping simulation preparations.

As shown above, the three-level approach increases in complexity from the first to the third level due to the rising number of requirements in the second and especially the third level. Hereby, the third level also involves the largest computational effort. At the same time, it is ensured that conservatism decreases with increasing complexity. This becomes apparent e.g. in the descending limiting values for the minimum required metacentric height.

With the above procedure, the new stability regulation allows for a more flexible submarine design by providing three equal levels for the proof of sufficient surface stability, now including reviewed threshold values and a probabilistic approach to account for the seakeeping performance. All three levels are physically justified and based on the following interim studies:

- Open water model tests using two modern submarine hull forms in Eckernförde Bay
- Balances of heeling and righting levers for 12 submarines at large \overline{GM}_0 -range
- Time-domain seakeeping simulations using E4-ROLLS for 12 submarines at large \overline{GM}_0 -range and various combinations of sea state, forward speed and encounter angle
- Probabilistic assessment (ISEI) of calculated seakeeping performance of the 12 submarines in

regard to both dynamic lateral accelerations as well as critical maximum absolute roll angles

The following section gives an overview of the open water tests on Eckernförde Bay which strongly contributed to the development of the DMS 1030-2. The model test results not only show the main heeling moments that attack a submarine in surfaced condition. They also enable the identification of relevant phenomena, that lead to large amplitude roll motions in a seaway and further serve as reference data for the validation of the utilized seakeeping computation method E4-ROLLS, as discussed later on.

Open water model tests on eckernförde bay using two submarine hull forms

Between autumn 2021 and spring 2022 model tests in open waters were performed on Eckernförde Bay, a natural bay area in Northern Germany, using two submarine hull forms. These two models are between 3.5 and 4.5 metres long and their hull forms are each based on a modern submarine class developed by thyssenkrupp Marine Systems. The test setup of these open water model tests can be summarized as follows: Both models are free-running and equipped with an autopilot. They can be controlled remotely by WLAN-connection. The models each contain an inertial measurement unit (IMU) of the type *Ellipse-D Dual Antenna RTK INS*, which measures the GPS position, the rotation rates and the accelerations and combines the measured data to determine the heading of the model as well as the motions in all six degrees of freedom. During the model tests the waves in the test area are measured using a wave buoy of the type Spooindrif Spotter by SPOONDRIFT moored at a fixed location in the bay, see Figure 2.

The wind speed and direction are measured using wind sensors of the type WeatherStation® WX200 by AIRMAR. A special feature of the experimental setup of the open water tests on Eckernförde Bay is, that the submarine models have masts, which are located fore and aft of the sail as visible in Figure 3, which shows the first model. Along these masts weight plates can be shifted and secured so that the vertical center of gravity is adjustable. This way, different settings of stability can be realized so that it is possible to analyze the stability's influence on the seakeeping of the surfaced submarine.

The model test runs are aimed at delivering statistically representative results. Bearing in mind the stochastic character of natural seaways and thus also seakeeping, long test runs are essential to this end. Therefore, the test runs on Eckernförde Bay are performed with durations of about 15–20 min in model scale each, which amounts to approximately one hour in full scale per set combination of stability, speed, encounter angle and seaway. Throughout this



Figure 2. Map of testing area and mooring locations of wave buoys within Eckernförde Bay

paper the term ‘full scale’ refers to scaled up values according to the model scale factor, whereas actually measured values from the open water model tests are referred to as ‘model scale’. A more detailed description of the model tests along with an account of their main advantages and disadvantages is given by Büsken and Krüger (2023).

The open water model tests with the two hull forms show, firstly, which heeling moments attack a submarine in surfaced conditions aside from the waves and

secondly, which qualitative phenomena occur, that lead to large amplitude roll motions.

The external heeling moments observed during the seakeeping tests on Eckernförde Bay are mainly due to wind on the one hand, as well as due to the propeller at high forward speeds. The heeling effect of the wind is to be expected, especially in beam wind conditions. Hereby, the influence of the wind is heightened in the model tests due to the increased lateral area containing masts and weight plates compared to the full-

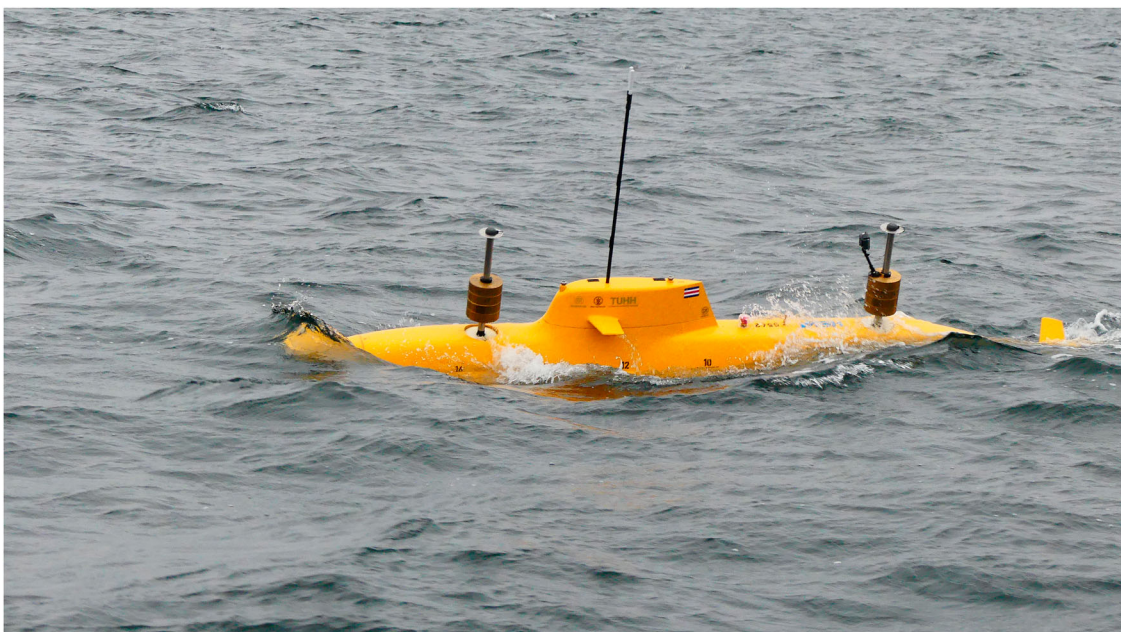


Figure 3. First model on the bay on the 17th of November 2021 at the lowest stability setting.

scale submarine, and also because the wind speeds in the model tests are higher at an observed model sea state than in a realistic seaway. Less expected was the model test observation, that the sail of the submarine acts as an airfoil in bow-quartering head wind conditions. This is observed especially at high forward speeds in combination with slightly oblique inflow or at high wind speeds attacking with slight angular offsets from the front. The heeling effect of the propeller at large forward speeds is also unexpected. This occurs at high propeller revolutions. Hereby, the propeller causes a pressure distribution at the stern of the submarine that leads to a notable stationary inclination, here towards port for both models. Depending on the surface stability of the vessel as well as on the forward speed the stationary heel angle is larger for lower stability conditions and larger forward speeds. Both the airfoil effect of the submarine sail as well as the heeling effect caused by the propeller are isolated and checked during the open water model tests.

Besides the identified heeling moments, specific qualitative phenomena are observed that lead to large amplitude roll motions during the open water tests on Eckernförde Bay. The following three main phenomena are observed: firstly, waves crashing against the submarine sail or upper casing, secondly, sudden loss of stability due to significant overwashing of the submarine in waves and thirdly, sudden large roll angles in a wave trough after dynamic emergence. The first phenomenon is well-known for surfaced submarines and typically occurs in beam and oblique beam seas. The phenomenon cannot be avoided by an increase in stability, at least not within the range that is practicably possible in conventional submarine designs. Past investigations have however already

shown, that a minimum stability can be found, so that the resulting submarine motions are kept within bounds. A beam sea situation, in which the second model is subjected to high waves crashing against the upper casing and also the sail is displayed in [Figure 4](#). Hereby, the submarine is moving with a speed over ground of approximately 7.9 knots at an initial metacentric height of 0.24 metres in waves of significant height of around 9.44 metres attacking from starboard (all values in full scale).

The second phenomenon is also already known to endanger surfaced submarines especially in following seas, as described e.g. by Gabler (2000). Whenever the upper casing of the submarine is significantly washed over by waves, sudden high roll amplitudes may occur, due to a temporary loss of transverse stability in the overwashed condition. Such significant overwashing is displayed in [Figure 5](#), which depicts the second model in following waves of about 8,40 metres significant height at a speed over ground of around 13.8 knots at an initial metacentric height of 0.163 metres (full scale). To fully avoid this phenomenon, the initial metacentric height would have to be in the range of metres, which is not practicable for submarines. Therefore, it is instead advisable to use numerical methods that are able to predict this behaviour and thus find combinations of speed and encounter angle, which do not lead to such significant repeated overwashing of the hull at a given seaway and surface stability.

The third phenomenon occurs whenever the model is subjected to high restoring forces or moments as a result of large heave respectively pitch motions. Large amplitude motions in these two degrees of freedom are observed especially in head seas and bow-quartering seas, and consequently this phenomenon

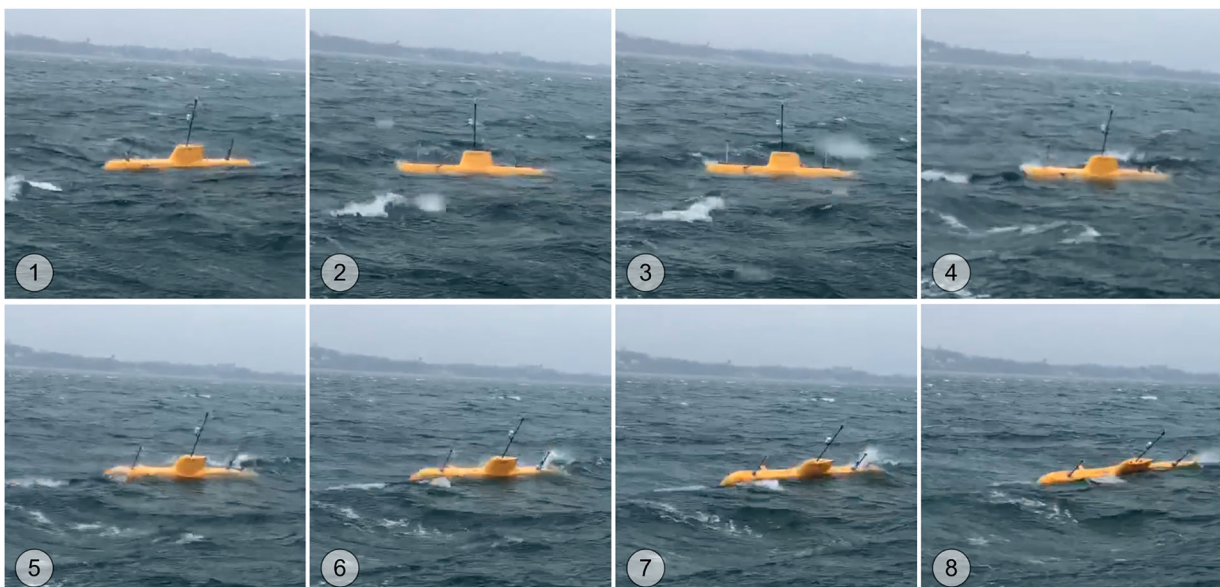


Figure 4. Waves crashing against upper casing and sail of the second model in high waves in beam seas.

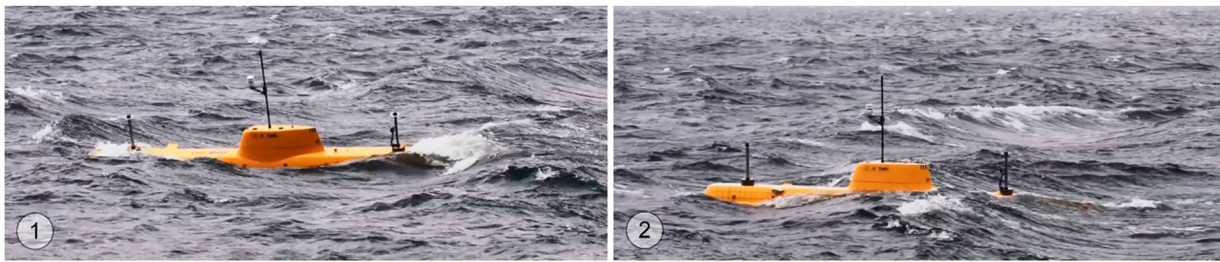


Figure 5. Significant overwashing of the second model in following seas.

mostly occurs within these encounter angle sectors. Due to the high restoring forces the upper deck respectively bow part of the submarine resurfaces by emerging from the water in a large dynamic motion. If this dynamic emergence coincides with the submarine's location in a wave trough situation, it tends to suddenly fall over, which becomes visible in sudden strikingly large roll angles. A critical situation is shown in [Figure 6](#). In this figure, the second model is moving in bow-quartering seas, i.e. at an encounter

angle of 150° , in waves of approximately 6.72 metres significant height at a speed over ground of 8.6 knots. The stability setting is the same as in the previous figure ([Figure 5](#)). As visible in [Figure 6](#), the model shows significant heave and pitch motions at this combination of parameters and is intermittently almost submerged completely ([Figure 6](#), Subfigure 4). Subsequently, the model emerges extensively and dynamically and falls over to starboard in Subfigures 8–11.

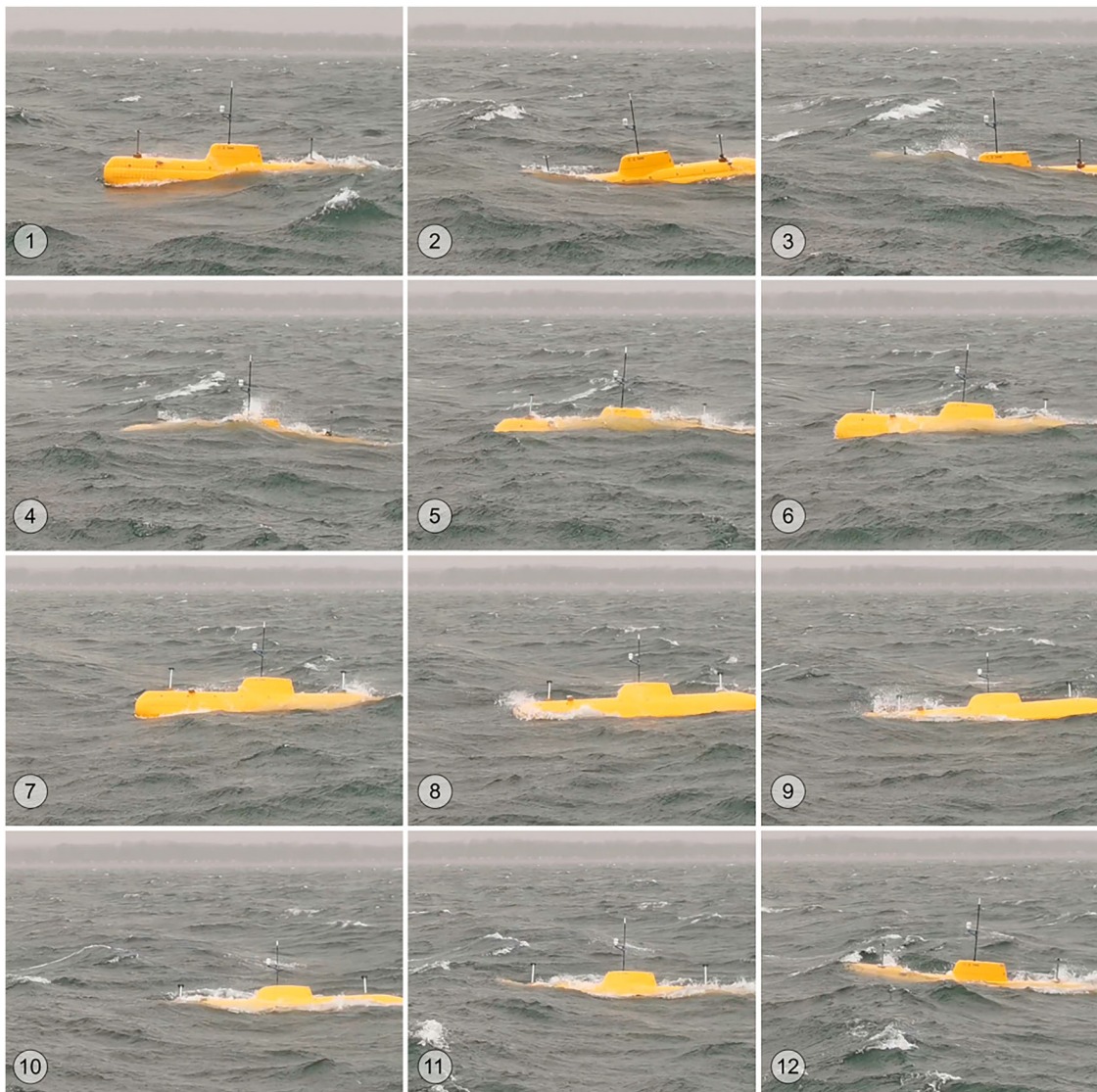


Figure 6. Second model falls over after dynamic emergence in bow-quartering seas with high pitch and heave motions.

Also for the prediction of this phenomenon, adequate seakeeping simulation methods are required in order to avoid this critical behaviour, seeing that it cannot be avoided by providing a feasible minimum surface stability either.

The model tests on Eckernförde Bay further clearly show, that the seakeeping of surfaced submarines is a complex problem, which involves all six degrees of freedom. For example, pitch-roll coupling and roll-yaw coupling are frequently observed during the open water tests on Eckernförde Bay. Furthermore, situations are observed, wherein the models tend to build up to higher and higher roll motions, indicating resonant behaviour. Hereby, simply increasing the stability of the submarine does not necessarily ensure a safe situation, i.e. avoid large amplitude roll motions. Instead, resonant situations can occur depending on the respective combination of parameters, i.e. speed, encounter angle, seaway and stability. These resonant areas are shifted along with the stability. Nonetheless, a certain minimum stability is reasonable, in order to limit the stationary heeling angles caused by the acting heeling moments.

A more detailed account of the main observations of the model tests, including the qualitative phenomena, is given by Büsken, Russell and Frühling in Büsken et al. (2022) (in German) and by Büsken and Krüger (2023) (in English).

The observations of the open water model tests on Eckernförde Bay presented above show, that numerical methods for the simulation of a surfaced submarine's seakeeping performance in natural seaways as dependent on its stability are required, in order to ensure a safe operation of the submarine, if large roll amplitudes are to be avoided. Such numerical methods must be validated, e.g. by comparison with model test data. The validation of the utilized computation method E4-ROLLS is therefore an important part of the research conducted towards the development of the new stability standard DMS 1030-2. The base code of the utilized computation method, called 'rolls', is summarized by Söding et al. (2013) and its potential and extensions examined by el Moctar et al. (2021). An overview of the underlying theory of the implementation, that is integrated within the design environment 'E4' and used here (referred to as E4-ROLLS) is further given by Römhild et al. (2022) and therefore omitted here. The publication referred to further shows that this numerical method is able to predict stability-dependent seakeeping and even capsizing of model tests of the utilized setup, at least when it comes to ships. This comparison was performed in the initial decision phase of the project. Based on the results of the comparison, both the test setup as well as E4-ROLLS were found to be useful for seakeeping analysis with focus on stability and roll motions and their application transferred to

submarines. Based on this initial assessment, further validation of the computation method is required specifically for submarines. To this end, the open water model tests performed on Eckernförde Bay supply the required reference data. The following section contains an exemplary validation case, which shows, that E4-ROLLS is also suited to predict the seakeeping of surfaced submarines, especially when focusing on the roll motion.

Comparison via statistical analysis of the motions of surfaced submarines

In order to assess, whether the potential flow theory-based computation method E4-ROLLS is able to predict the seakeeping of a surfaced submarine with sufficient accuracy, comparative calculations are performed using full scale numerical models of the respective submarines. Thereby, great attention is placed on the aspects weight distribution and inertia, stability and floating condition, roll damping characteristics as well as wind lateral areas when setting up the numerical models, so that these correspond well with their respective counterpart, i.e. physical submarine model. For example, in order to capture the roll damping characteristics of the submarine, dedicated roll decay tests are performed at zero speed and at different forward speeds with the physical submarine models. This way, reliable linear and quadratic roll damping coefficients are obtained to account for viscous effects that affect a submarine's roll damping in a simplified way. Since it is based on potential flow theory, E4-ROLLS does not directly include the viscous components which contribute to the roll damping of the submarine, e.g. due to eddy formation around sharp edges, sonar flank arrays (similar effect as a bilge keel) or caused by skin friction on the hull. Instead, the resulting damping forces and moments are modelled using the obtained damping coefficients. Comparisons of simulated roll decay tests with E4-ROLLS with the measured decay tests show that the speed-dependency of the roll damping is successfully captured by the numerical model. It should be noted, that in case of submarines the free-flooding water in between the pressure hull and the outer hull may have an additional damping influence on the surfaced submarine depending on the number, size, location and geometry of the flooding slots. However, this is not relevant for the validation presented here, since the models used here are intentionally constructed with a fully-buoyant outer hull.

In this paper, the results of exemplary comparative calculations are presented, which are performed for the first model using the environmental input data and thus combination of parameters of a selected test run of the open water model tests. The simulation results are contrasted with the full scale motions

derived from the analyzed test run. To this end, both the simulated roll, pitch and heave motions are analyzed statistically as well as the corresponding motions in the model tests (full scale). As part of the analysis, histograms of these three degrees of freedom are generated. The height of the bars in the histogram give the relative frequency of the motions that fall into the corresponding bin. Hereby, the relative frequency gives the ratio between the number of local extrema that fall within the respective range (bin) and the total number of all measured local extrema of this degree of freedom, as also described by Büsken and Krüger (2023) during the analysis of selected open water model tests. Besides the histograms, the results are also compared via characteristic values of the motions. Hereby, the significant value denotes the threshold value, above which the highest third of all absolute local extreme values lies, see Büsken and Krüger (2023). The significant value of the respective degree of freedom thus allows for a preferably descriptive designation of the submarine motions via a single characteristic value. Aside from this, the overall extrema, i.e. the maximum and minimum value are given, as well as the arithmetic mean over time with the corresponding standard deviation. Further, the analysis includes the arithmetic mean of the absolute local extrema and the standard deviation from this value. However, from a ship safety point of view, the significant value remains most representative to describe the submarine motions.

The model test run analyzed and used for the comparison with the simulations is the open water test with measurement number (MNr.) 13 of the first model. In this test run, the first model is moving in stern-quartering waves at the mast setting ‘Mast510’, which results in a stability condition with an initial metacentric height of approximately 0.179 metres (all values in full scale). The speed of the model over ground corresponds to about 11.98 knots. The duration of the test run is almost 1.5 h in full scale. The analysis of the wind sensor data reveals that the true wind speed is approximately 6.275 beaufort on average. Similarly, the direction of the true wind is recorded. In combination with the course of the model, the wind direction results in a mean encounter angle of approximately 2.1° between submarine and wind, i.e. the wind is attacking almost directly from behind during the test run with slight component from starboard. The encounter angle between submarine and the main propagation direction of the waves differs from the wind: The waves are attacking with component from port at a stern-quartering encounter angle of about 30° . The data given by the wave buoy shows, that the submarine is subjected to a seaway, whose energy distribution can be described using a JONSWAP-spectrum with a peak period of 5.28 s, i.e. a characteristic period of 4.404 s, and a significant wave height of 1.648 metres in full scale, whereby a

peak enhancement factor of 3.3 is applied. Based on this analysis of both the environmental conditions and the other parameters (stability and lateral area at mast setting as well as speed) present during the test run, three comparative simulations are performed using E4-ROLLS. Short crested irregular waves are realized in the simulations by scattering wave components of different encounter angles and frequencies. Hereby, a specific composition of the seaway is realized for each comparative simulation, whereby the frequencies, phase shifts and encounter angles are scattered randomly within given intervals in order to generate a preferably realistic natural seaway, as described in detail e.g. by Kluwe (2009). Each unique composition of the seaway has the same energy distribution and is referred to by using pseudo-random numbers, denoted by the letter ‘Z’ here, see Table 1. Depending on the respective seaway composition, the resulting motions in the simulations differ, due to the stochastic character of the seaway. Therefore, it is recommended to perform more than one comparative simulation in order to obtain representative results. According to the measurement duration transferred to full scale of the model test run in question, each comparative simulation has a simulated time of 5320 s. Hereby, a time step of 0.5 s is chosen, resulting in a total of 10,640 steps each. Time-dependent wind is included in the comparative simulations, which varies in both wind speed and direction around the input average wind speed, respectively direction, as described by Römhild et al. (2022). The heeling influence of the propeller is also included in the simulations by applying an external speed-dependent stationary heeling moment to port, which is based on the results of the model tests. Even though a nonlinear treatment of the surge motion is possible in E4-ROLLS, this degree of freedom is treated linearly here, in order to reduce the computation effort. In advance of the time domain simulations using E4-ROLLS, motion response amplitude operators (RAOs) are calculated with a strip method in the frequency domain for

Table 1. Simulation settings used for the comparative simulations for test run MNr. 13 of the first model.

Simulation parameter	Symbol/ Abbrev.	Value	Unit
Loading condition (mast setting)	‘Mast510’	X	[-]
Forward speed (RAOs)	v	11	kn
Significant wave height	$H_{1/3}$	1.648	m
Characteristic wave period	T_1	4.404	s
Encounter angle to waves	$\mu_{\lambda,ROLLS}$	-30.0	deg
Wind speed	v_{Wind}	6.275	BFT
Encounter angle to wind	$\mu_{Wind,ROLLS}$	2.1	deg
Simulated time	T_{Sim}	5320	s
Time step	δt	0.5	s
Number of time steps	n_{Steps}	10,640	[-]
Pseudo-random numbers	Z	0.23456; 0.65421; 0.79136	[-]

the given loading condition (Mast510) and forward speed (11 knots). The resulting RAOs are used to calculate the surge, sway, heave, pitch and yaw motions in a simplified linear way during the simulations, while the roll motion is treated fully nonlinear. A summary of the most important simulation input data, which is used for the comparative calculations, is given in Table 1 (full scale).

Based on the simulation settings above, each time domain simulation is performed in less than two seconds. This extremely short computation time is only possible by utilizing the concept of Grim's equivalent wave, see Grim (1961), as well as the simplifications above. The fast computation presents one of the main advantages of E4-ROLLS and allows to perform large numbers i.e. many variations of fast time domain seakeeping simulations to assess the seakeeping performance of a vessel, if the focus is placed on the roll motion.

For a first comparison of the simulated roll, pitch and heave motions with the data from the open water test an extract of the motions over time is given in Figure 7 in full scale. Hereby, the extract showing the motions calculated using E4-ROLLS and displayed on the right-hand side is taken from the simulation with $Z = 0.23456$ as an example. This exemplary comparison of time-series is included here for illustrative purposes only, in order to give a general impression of the simulated motions compared to the measured motions. Thereby, the extracts are selected based mainly on the magnitude and qualitative course of the roll motions, each extract showing an extreme event. The depicted results show that the submarine builds up to higher roll amplitudes, both in the open water test as well as in the simulation. The largest absolute roll angle amounts to just below 25° in both cases. Furthermore, a slight difference in the mean roll angle can be perceived. Thereby, the stationary inclination towards port (negative roll angle) appears to be marginally underestimated in the simulation. There is a more notable difference between the open water test and the simulation results

concerning the mean pitch angle: This is negative in the open water test, which suggests a stationary forward trim, yet in the simulation the mean pitch angle is greater than zero. According to the simulated results, the submarine would be trimmed towards the stern. This difference is discussed in more detail later based on the results of the statistical analysis including the histograms. The heave motion is hard to compare visually at this scale, but seems to be in good agreement at first glance.

Bearing in mind the relatively low significant wave height, it is interesting to find out why the submarine shows this kind of behaviour with quite high roll amplitudes. A simplified check based on a comparison of the eigenfrequencies and the encounter frequency shows, that in case of MNr. 13 the encounter frequency of the waves is close to the natural roll frequency of the submarine. In other words, the high roll amplitudes are due to the fact that the submarine, which is excited mainly parametrically in stern-quartering seas based on the nearly periodic stability alterations in waves, experiences a critical near-resonant situation close to 1:1-resonance here. It can further be observed, that the submarine starts to build up to higher roll amplitudes whenever the periods of the heave and pitch motion are close to each other and almost in phase.

A more suitable procedure to compare the results of open water test and simulations than the above comparison of extracts taken from the time-plots of the motions, is the statistical analysis of the data and their visualization using histograms. The direct comparison of the motions over time is not a satisfactory representation of the results for validation purposes here, especially since the sequence of waves at the submarine is not necessarily the same in the comparative simulations as in the model tests, seeing that the exact wave elevation at the model was not measured in this test setup. Therefore, only the results of the statistical analysis are used for validation purposes. Such a more representative comparison can be taken from the

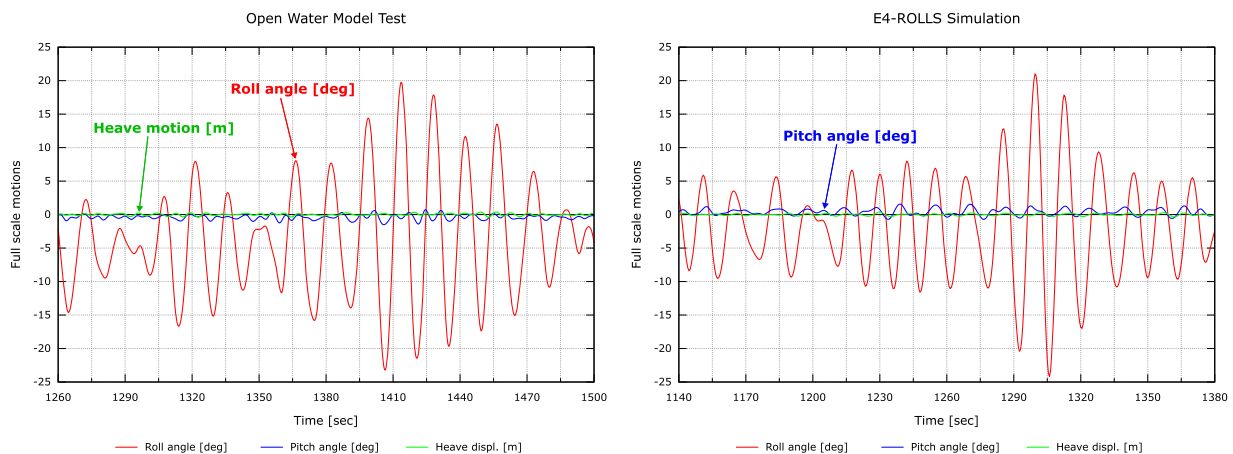


Figure 7. Extract of measured (left) and simulated (right) roll, pitch and heave motions over time (full scale).

results in Table 2 and from the histograms of relative frequency of the roll angle (Figure 8), the pitch angle (Figure 9) and the heave motion (Figure 10). These figures consist of three subfigures each, depending on the respective seaway composition denoted by the Z-number.

The histograms of the roll angle clearly show that the simulated roll motions for this measurement number are very close to the model test results. Only the stationary heel angle to port is slightly too low in the simulations, whereby the arithmetic mean value may also be distorted by outliers of the roll angle to port. The significant value of the roll motion, which is around 16.2° in the open water test, is much more descriptive. In particular, the results of the comparative simulations with the seaway compositions using random numbers $Z = 0.23456$ and $Z = 0.79136$ are particularly close to the open water test data. These are visualized in the colour navy in the left and right subfigure of Figure 8. In these simulations, the significant roll angle amounts to approximately 16.1° to 16.9° . In summary, the simulated roll motions are in very good agreement with the model test results, even with regard to the extreme values of the rolling motion. In the comparative simulations, these range from -24.4° (port) to 22.8° (starboard). In the open water test, roll angles of a minimum of -24.1° (port) and a maximum of 19.8° (starboard) occur, see Table 2.

The low wave height is noticeable in a good accuracy of the simulated heave and pitch motion, even if this is not obvious in regard to the pitch motion at first glance. The histograms of the pitch angle do not show a good agreement between simulation and open water test results. In fact, the histogram of the simulated pitch motion appears to be shifted to the right by about one degree compared to the histogram

of the open water test data. This deviation is also apparent in the difference of the respective arithmetic mean over time as well as in the extreme values. Nonetheless, the magnitude of the simulated pitch motion is close to the model tests. This is visible in the significant value of the pitch angle, which amounts to around 1.0° in the open water test and 1.1° in the simulations. The reason for the visible difference in the mean value of the pitch motion that was already visible in the comparison of the time-plots in Figure 7 can be found in the submarine's dynamic forward trim, which occurs in the open water test of MNr. 13 but is not included in the simulations in E4-ROLLS. At high forward speeds, submarines typically generate a bow wave that runs onto the upper casing at the submarine's bow. This run-up of the bow wave is clearly visible in the video recording of the corresponding test run. Since neither bow wave nor resulting dynamic trim are modelled in the current implementation of E4-ROLLS, the stationary trim in the comparative simulations is towards the stern, corresponding with the initial floating condition, in which the submarine is trimmed aft by approximately 0.5 degrees.

The good agreement between the simulation and open water test results is more apparent for the heave motion, as the histograms are well aligned for this degree of freedom, see Figure 10. Thereby, the heave motion is slightly underestimated in the comparative simulations. This is visible in the slightly larger width of the histograms of the open water test (green), i.e. the values of the largest motions in terms of magnitude. However, this mainly concerns the extreme values of the movements. The area in which most of the heave motions occur, is well covered in the simulations, as becomes apparent in the overlapping areas. The high quality of the simulation results at the given low wave height is also quantifiable

Table 2. Results of the statistical analysis of open water test and the comparative simulations for MNr. 13 of the first model.

tab	Parameter from statistical analysis	Simulation Z = 0.23456	Simulation Z = 0.65421	Simulation Z = 0.79136	Test data MNr. 13
Roll angle [deg]	Minimum	-24.195	-20.313	-24.397	-24.076
	Maximum	21.129	17.267	22.793	19.750
	Arithmetic mean over time	-1.957	-1.984	-1.961	-4.094
	Standard deviation over time	7.806	7.009	8.413	7.288
	Mean local extrema	10.109	9.216	11.006	9.610
	Standard deviation loc. extr.	4.917	4.246	5.075	5.562
	Signific. value (1/3 loc. extr.)	16.057	14.265	16.873	16.154
Pitch angle [deg]	Minimum	-1.026	-1.002	-0.970	-1.960
	Maximum	1.766	2.033	1.741	1.627
	Arithmetic mean over time	0.402	0.402	0.402	-0.414
	Standard deviation over time	0.399	0.416	0.401	0.360
	Mean local extrema	0.556	0.575	0.556	0.528
	Standard deviation loc. extr.	0.404	0.411	0.400	0.386
	Signific. value (1/3 loc. extr.)	1.077	1.086	1.072	1.006
Heave motion [m]	Minimum	-0.387	-0.461	-0.374	-0.464
	Maximum	0.438	0.460	0.397	0.544
	Arithmetic mean over time	-0.000	0.000	-0.000	-0.000
	Standard deviation over time	0.125	0.129	0.118	0.132
	Mean local extrema	0.154	0.162	0.147	0.160
	Standard deviation loc. extr.	0.080	0.082	0.079	0.100
	Signific. value (1/3 loc. extr.)	0.255	0.261	0.247	0.295

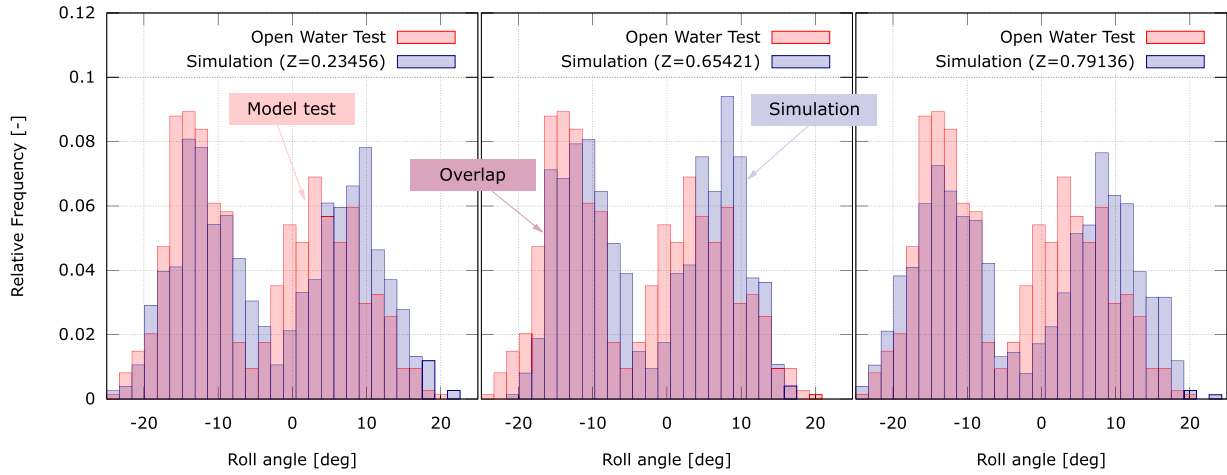


Figure 8. Histograms of the roll angle in the open water test (red) and the comparative simulations (navy).

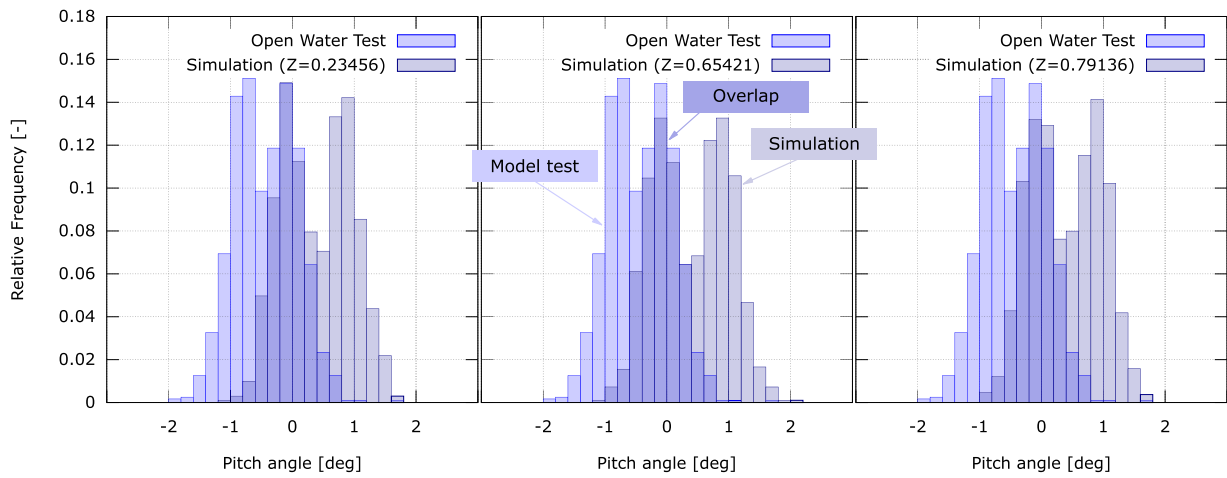


Figure 9. Histograms of the pitch angle in the open water test (royal blue) and the comparative simulations (navy).

by comparison of the significant value of the heave motion. This amounts to 0.295 metres in the open water test and ranges from 0.247 metres to 0.261 metres in the comparative simulations.

The above discussion of the results shows that E4-ROLLS is well suited to simulate the seakeeping

performance of a surfaced submarine in natural seaways. For this low wave height, a good quality of results is even achieved for the linearized degrees of freedom, as is visible in the agreement of the simulated pitch and heave motions with the open water test data.

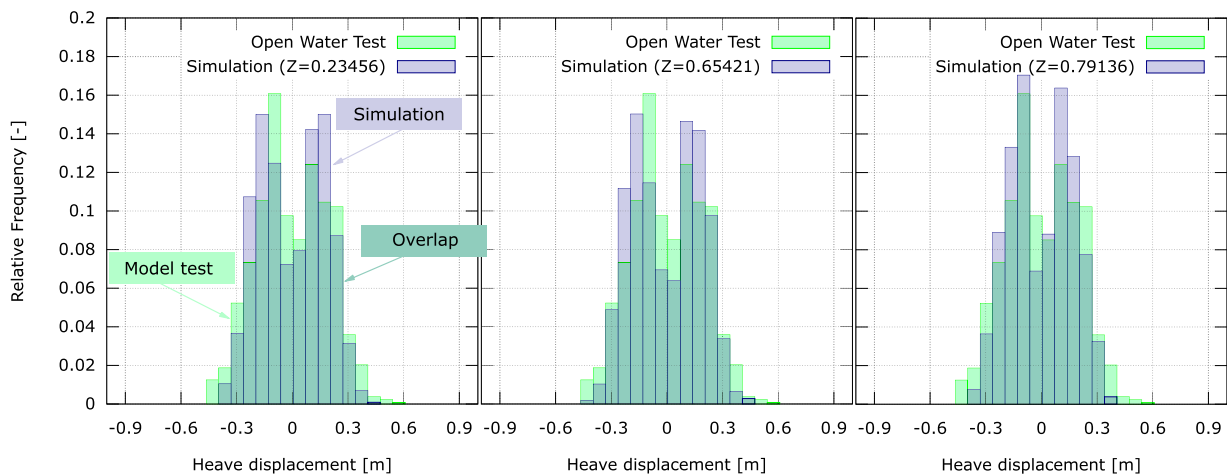


Figure 10. Histograms of the heave motion in the open water tests (green) and the comparative simulations (navy).

Similar comparisons by way of statistical analysis have been performed for other combinations of parameters (speed, seaway, etc.) based on additional selected open water test runs. These further validation cases specifically cover different encounter angle sectors and mast settings and can be found for both submarine models in Büsken (2024). These additional validation cases show, that E4-ROLLS is especially suited to simulate the roll motion of surfaced submarines depending on its stability. However, in higher waves the linearized treatment of the pitch and the heave motion becomes problematic and is no longer reasonable, so that the simplifications utilized by E4-ROLLS lead to unrealistic results in these degrees of freedom, as is to be expected. Nonetheless, the simulated roll motions are still in relatively good agreement with the open water test data, even in these cases. Consequently, E4-ROLLS is a suitable method for seakeeping simulation of surfaced submarines, if the main focus is on stability-dependent roll motions.

Limitations of the numerical method

When performing seakeeping simulations for surfaced submarines with E4-ROLLS, the limitations of the numerical method should be kept in mind. These can be summarized as follows for the current implementation:

- Waves crashing against the sail or upper casing are not included.
- The airfoil-effect of the sail is not yet included.
- In beam seas the roll motions are often overestimated in the simulations.
- Heave and pitch motions are linearized and thus strongly simplified.

Depending on the respective encounter angle and on the wave height, these limitations may affect the quality of results more or less. It should be noted, that for the purpose of the presented research conducted towards the development of the new stability standard DMS 1030-2 the utilized computation method is chosen deliberately, as this method is able to capture stability-related nonlinear roll motions within exceptionally low computation times. The project partners are well aware that the use of E4-ROLLS is a compromise, especially concerning the pitch and heave motions, which should not be treated as linear when it comes to submarines.

Conclusion and relevance of results

The presented validation via statistical analysis shows that E4-ROLLS is also able to simulate the seakeeping of surfaced submarines, especially the roll motion. If the wave height is relatively low, also the pitch and

heave motions are calculated with good accuracy, despite the strong simplification of these degrees of freedom. Besides decreasing accuracy of the latter motions with increasing wave height, some of the presented qualitative phenomena or heeling moments are not yet captured by the method. However, these limitations are tolerated given the main objectives of this study.

The relevance of the results becomes apparent in the inclusion of such seakeeping simulations as part of the third level of the new stability standard DMS 1030-2. Only using a method such as E4-ROLLS it is possible to assess the seakeeping performance of a surfaced submarine by probabilistic approach based on direct seakeeping simulations. This is because such a method must simulate the submarine's motions in all six degrees of freedom in short crested irregular waves in the time domain, whilst treating the roll motion fully nonlinear and stability-dependent, and all this within a very short computation time. A low computation time, e.g. less than one second, is essential here, because of the high number of parameters, which must be varied in a seakeeping analysis, in order for the analysis to be in accordance with the third level-requirements of the DMS 1030-2. Such a seakeeping analysis includes a detailed variation of both the encounter angle as well as the forward speed per investigated seaway and stability condition. At each combination of parameters (i.e. stability, characteristic wave period, encounter angle, and forward speed) a critical significant wave height is determined iteratively, at which a limiting failure criterion is reached (e.g. a critical roll angle). This iteration is only possible, if the computation time is sufficiently short, seeing that each time domain simulation must be performed for a statistically representative time span (e.g. 10,000 s at each combination of parameters). Neither the number of parameters to be varied nor the minimum simulated time span required for a third level seakeeping analysis can be reduced without sacrificing either the informative value of the analysis or the statistically representative nature of the simulated results. So far, no numerical method has been developed, that is able to not only treat the roll motion fully non-linear but also heave and pitch whilst still fulfilling the requirements above. Therefore, up to date, the poorer quality of results in heave and pitch motions must be tolerated in this kind of analysis and only other computation methods, which are also based on Söding's *rolls* and utilize the concept of Grim's equivalent wave to account for the impact of stability alterations in waves on the roll motion, can be used similarly.

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