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On Problems of Wave Resistance Research

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ON PROBLEMS OF WAVE RESISTANCE RESEARCH

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Synopsis

The impact of theory on shipbuilding practice is modest, even model research does not make proper use of theoretical solutions. This state of affairs is largely due to erroneous **judgment**, but to inspire actual design, efforts must be made to develop basic theory as well as its application and evaluation and experimental research. The synopsis is restricted to uniform motion in calm water. - Some problems of ship geometry and generation of bodies by singularities ~~are~~ discussed. Results of linearized theory are enumerated dealing with wave pattern (shortly) and with resistance. The problem of optimisation is reviewed. Second order and non linear effects are briefly touched upon. Experimental methods are appraised and results of an investigation on resistance of simple ships are communicated.

Introduction

During the process of planning the program of the symposium I asked my colleague Wehausen to contribute an introductory synopsis on our subject, eventually as a joint enterprise. My humble request has been declined essentially by pointing out that such an attempt is superfluous at a meeting consisting of specialists. I agree to a wide extent with this reasoning - a summary of the discussions produced at the conference will be much more productive. However, since the participants come from different camps and our family is being rejuvenated, some general if by no means systematic remarks on our subject may be nonetheless justified. The purpose is to point out some weak spots in the earlier approach and to suggest increased activity in the field of basic theory, of its application and evaluation and of crucial experimental work. Occasionally the survey of literature may lead to a communication of less known information.

When we consider as starting point the surveys given by Wigley [1] in the early thirties the principal trends in the development of our subject could be followed till recently by studying Havelock's work [2] and such highlights as the wave resistance conference in Moscow 1937 [3], the publication of the already famous II. volume of the Japanese Ship Theory [4] and the Symposium at Wageningen 1960 [5]. We wish that the present seminar may play a similar role.

Several years ago a prominent member of the Moscow conference remarked to me that there is no need to toil about the wave resistance theory since ships can remain afloat without any hydrodynamic theory; the latter has a more _

or less decorative value only - an opinion shared by many practical people. This questionable statement is justified in so far as up^tnow the impact of theory on actual shipbuilding practice is modest only. There are some obvious indications of this sad state of affairs; a less serious one stems from the fact that the numerous ITTC Congresses have not acknowledged officially the existence of our theory, (i.e. it has not been a topic of its meetings notwithstanding attempts to make it presentable at this court). As a more serious shortcoming we consider the fact that the rather popular systematic model series are being planned without making use of wave resistance theory.

It is the definite purpose of our meeting to promote the development of theory as well as its application to model work^r and ship design. Obviously, our difficult subject could and can be treated theoretically by introducing drastic simplifications and abstractions only; but endeavours should be made in due course to relax restrictions and to enlarge the scope of problems presented by practice which frequently are unpleasant and difficult to handle. Further, when practical application is aimed at the influence of other design criteria on wave resistance research must be studied. Such considerations have been made but in a rather cursory fashion.

Contrary to the pessimistic or ignorant opinions quoted it is our contention that theory should represent already now an indispensable tool in model research work and thus should influence at least indirectly design.

There are from the viewpoint of practice two main problems which theory must solve - the determination of resistance for a given hull form and the development of shapes of least (low) resistance, although in principle, an adequate theory is able to handle both problems. Since, however, only approximate methods^s were available these practical aims have produced different trends in research.

I welcome the fact that our kind hosts have included the discussion of an experimental determination of wave resistance in our program as far as these experiments are connected with scientific ideas. For a long time there has not been any progress at all in pertinent experimental methods; a more satisfactory approach has been developed actually not earlier than the application of theoretical means. Looking back, my teacher Föttinger was right in asserting (1924) that his proposal concerning double models was almost the only new basic idea since W. Froude.

Although wave resistance is the 'economically' most important 'free surface effect' phenomenon as far as ships are concerned other generalized forces can be determined almost as byproducts in our field. It appears that they will be studied especially in the case of the motion in a seaway. Although there is a certain danger that the scope of our seminar may become too ample, it may be still more dangerous to restrict ourselves artificially to resistance only.

Obviously, mathematicians and naval architects sometimes have different opinions as to what is important in studies of wave resistance. Some painstaking developments are a bogey to engineers; in their opinion methods are needed which lead to explicit results; the difficult work should not be burdened by mathematical niceties. However, it is now generally understood that investigations, e.g. on second order and non linear effects represent an indispensable prerequisite of the practical as well as of the scientific progress.

Even when we restrict ourselves to the resistance only, the scope of our studies is ample: Steady and unsteady motion, rectilinear and curved path, unrestricted and restricted water, smooth and corrugated water surface (the latter regular and irregular), displacement vessels of various types and hydrodynamic craft - in fact an impressive list!

In what follows I shall confine myself to remarks on the simplest problems - the resistance experienced by ships moving uniformly rectilinearly at or under a smooth water surface. We shall briefly discuss the classes of ships but eliminate completely hydrofoil craft since this topic has been the subject of other recent meetings.

Ship theory is widely indebted to aerodynamics. Aerodynamicists have ridiculized our inefficiency in handling problems of ship resistance (although they themselves have frequently failed when they condescended to deal with the (water!) wave resistance). We

admit that ship resistance has been investigated frequently more industriously rather than intelligently; this applies to experimental work as well as to theory and especially its application. Possibly the productivity of thinking has suffered in our field by the need for tedious auxiliary work caused by the complicated hull form. In the experimental field it has been disastrous that ^{tanks} depended rather on 'running models' than on investigating resistance problems.

It had been almost a dogma amongst naval architects that it was practically impossible to calculate the wave resistance of ships. Thus Havelock's and Wigley's work opened a new ~~dera~~ era in ship theory. Naturally, after the long stagnation romantic feelings arose as to possibilities furnished by the existing theory. The present writer was especially responsible for pressing it hard from the point of view of application to practice. He feels grateful for the opportunity given here to express a more moderate appraisal of earlier results.

On the other hand erroneous deprecating statements have been made occasionally by prominent theoreticians on the practical value of Michell's theory based on a superficial comparison of calculated and measured results which actually refer to non-identical ship forms! [] Obviously, the practical merits of theory can no more be supported to-day by demonstrating such a common place as the correct dependence of the resistance upon $\phi = c_p$ in a well known range of the Froude number. In order to be helpful theory must be able to disclose finer form effects. To make a modest contribution in this direction (with success) some we have repeated

earlier experiments which had served as a proof of the important fact that small changes in ship form may correspond to large changes in wave resistance.

The recent development of computers has changed basically the aspect of our scientific policy. Occasionally, it had been considered easier to develop/exhaustively the existing integrals. Resistance formulas were preferred which lend themselves to an easy numerical treatment; more general and farther reaching results were neglected as will be pointed^{out} later. At present interest centers around the physical content of 'theories'; complexity of the relations involved is no more an objection. We expect that in the wake of this seminar the present tendency will be strengthened to evaluate thoroughly all available advanced resistance formulas. This will be a further step in establishing theory as a guide for practice in determining resistance and in developing 1. new ship forms and 2. devices to reduce wave resistance by local action, 3. in appraising properly wellknown fundamental hull features like the cruiser and the transom stern. One can expect that this work will stimulate the discovery of new solutions.

We are planning to prepare a list of references; perhaps this can be settled during the conference. Apart from recent publications my present exposé is based essentially on books mentioned in the bibliography 2 - 8 amongst them primarily on the excellent work by Kocmłoko which is by far the best synopsis of our subject.

I. Ship Geometry

Expressions for wave resistance of basic geometrical bodies like circular and elliptic cylinder, sphere, spheroid and /new formulae than to evaluate

ellipsoid are well known. For the vertical plate moving normally to its plane experimental results only and crude estimates based on hydraulic concepts are available 9 . Further, the resistance of systems of simple bodies like spheres has been investigated 10 .

To deal with ships we repeat now the well known basic classification. We distinguish

- I. displacement ships
- and II. hydrodynamic craft.

Unfortunately there are transitions between these two classes which so far are difficult to handle. Class I we divide into surface vessels I,1 and submerged vessels I,2. Again there are transitions between I,1 and I,2.

Class II is generally subdivided into planing craft and hydrofoil. GEMs (Hovercraft) do not fit into this scheme; but the investigation of wave resistance of these vehicles in which alone we are interested here can be carried out using the pressure point (or distribution) concept.

We agreed to eliminate hydrofoils and we shall mention occasionally only planing craft which deserves a more substantial treatment. Thus we concentrate our efforts on the displacement ship class.

In the most important 'subclass' the surface displacement ships we have to distinguish hull forms with geometrically smooth surfaces (rounded forms or forms with a continuous curvature) and those with corners (sharp edges) or even discontinuities. A similar distinction is commonly made when studying viscous effects of bodies.

With respect to wave phenomena our division into smooth and not smooth hulls is insofar physically founded as it

as it marks a characteristic difference between slower and faster ships (transom stern, chine!); displacement hulls with corners form the transition to hydrodynamic vessels, and a further development of steps finally results in planing craft hulls. Hull forms with corners of different character have been designed independently of speed considerations simply for economy in construction. In this case, edges in the longitudinal direction may have a minor influence on wave resistance. The influence of corners in waterlines on the wave formation has been investigated by Havelock. Although, in the light of our present knowledge his results refer to corners in singularity distributions, the assumption appears permissible that small discontinuities in waterline angles in general do not change the wave resistance heavily as compared with that of similar smooth forms. We shall not follow further this case of 'not essential' corners.

The study of influence on wave resistance of 'essential' corners (or even steps) will be an important task for the future.

Attempts have been made to systematize roughly hull forms of normal displacement ships in the following way:

- | | |
|-----------------|---|
| 1. narrow ship | B/L small |
| 2. flat ship | B/T large |
| 3. deep ship | B/T small |
| 4. thin ship | $B/L \ll 1$; B/T small (narrow and deep) |
| 5. slender ship | $\sqrt[3]{L}$ small; $\frac{\delta_{BT}}{L^2} = \phi \frac{\sqrt[3]{L}}{L^2}$ small |
| 6. fat ship | $\sqrt[3]{L} = c_{\sqrt[3]{L}}$ large |
| 7. fine ship | small prismatic $\phi = c_p$ (small $\delta = c_B$) |
| 8. full ship | large $\delta = c_B$ (large prismatic $\phi = c_p$) |

Unfortunately there is no clear agreement as to the concept 'fine ship'; while as criterium of the 'full ship' the large block coefficient is considered.

For theoretical reasons the concept 'elementary' ship has been introduced which, however, in the English literature appears to have been replaced by the designation 'simple ship'. It is characterized by the equation of the hull $y(x,z) = X(x) Z^*(z)$. It is advantageous to distinguish principal dimensions, proportions and a dimensionsless form. The latter can be roughly described by form coefficients (integral values) which are invariant with respect to affine transformations. Differential parameters propagated by D.W. Taylor become increasingly important in connection with approximate wave resistance calculations based on suitable expansions.

The enumeration of types 1 - 8 is based partially on proportions, partially on coefficients and partially on mixed items.

The form coefficients, so useful in the low and medium speed ranges and when dealing with smooth surfaces (curves), become less meaningful ^{when} hulls with corners or discontinuities are considered in the range of high or medium high Froude numbers. An adequate 'geometry' for the latter class of vessels has not yet been developed.

An analytic representation of smooth ship surfaces has been aimed at for rational as well as mystic reasons. In the field of wave resistance research especially, and in ship theory in general such a representation is useful; this applies especially to the backbone of ship design - the sectional area curve. In the future systematic model investigations in our field should be based

without exception an analytic representation of sectional area curves. Thus no more would be reached than a standard proclaimed 60 years ago by Taylor. The well known example which will be discussed once more in chapter VI illustrates the need for an 'exact' determination of lines (and surfaces).

Various kinds of simple functions lend themselves to an 'exact' representation of the ship surface. The polynomial has advantages deeply rooted in the art of the profession since it pictures adequately the 'spline curve'. D.W. Taylor has developed, used and recommended a consistent system of parabolic ship lines claiming as the principal merits of analytic expressions the possibility to fix definitely a ship form and thus to reproduce it at will. The usefulness of similar simple equations in evaluating Michell's integral and generally in wave resistance research has been demonstrated.

Equations of simplified ship surfaces were constructed in an inductive way based on experience in naval architecture. More recently, using high speed computers empirically given ship surfaces have been approximated by polynomials with numerous terms [11]. When no attempts were made to embody empiric knowledge the expressions became cumbersome; nonetheless, straightforward application of the least square method did not lead to an accuracy of results sufficient for actual building purposes. From our present point of view the formal procedure just described is not too promising. Probably those simple algebraic expressions will gain in value which are suitable for a systematic variation of basic features of hull forms.

II. Singularities

The direct solution of our boundary problems is cumbersome. The method of images (singularities) proves to be efficient in obtaining results which can be improved step by step. We list pressure, source-sink and doublet systems (including points as well as distributions) and vortex systems (lines and distributions). The usefulness of higher order singularities in wave resistance research has not been investigated. In an important note Havelock ~~has~~ has shown the equivalence of the source-sink and pressure representation [12]. The former is now more popular and especially suitable for volume generation; the pressure concept fits rather the requirements presented by studies on planing craft. Earlier the pressure concept was universally dominating, but later became less fashionable almost to a point of oblivion - which in our opinion is not justified. Essentially, the different singularities are closely interrelated.

Until recently the problem of generating bodies by singularities has been most frequently restricted to a motion in an uniform flow (liquid at rest) and to the case of unbounded fluid. Let us first consider this 'indirect' method which consists in constructing bodies from prescribed singularity systems. It is simple and efficient in the case of the plane and the axially symmetric problem, where discrete and line-singularities are the most important working tools. Already the generation of 'Michell ships' derived by singularity distributions located on the longitudinal center plane is cumbersome as demonstrated originally by Inui 4. These Inuids are almost the first truly three-dimensional bodies designed by hydrodynamic means except for the general ellipsoid. The amount of work involved in

constructing an Inuid is considerable. Although the characteristic features of such bodies could have been guessed from the ellipsoid; we were surprised by the hull shapes generated by singularities distributed following simple laws over a rectangle. For reasons pointed out below the application of Inuids is slightly losing importance in future research. That does not mean that Inui's pioneering work has 'blocked progress' in our field; on the contrary, it has decisively clarified the situation and contributed tools for solving more complicated bodies. We do not go into details of the generating problem, for instance into the limitations of forms which can be produced by certain line singularities and into methods which admit a generalization (e.g. singularities distributed over a disc normal to the axis of symmetry,, vortex rings etc.). Wide use has been made of the slender body approximation for bodies of revolution

$$V = \frac{1}{2} \rho \omega^2 R^2 \quad (1)$$

and the thin ship approximation (Havelock)

or cylinders degenerate (2)

into

(3)

By mirroring many problems have been solved like those connected with cylinders moving in the vicinity of a rigid wall (or walls) and with bodies of revolution, located axially in an axially symmetrical duct. But in general rough approximations only have been used to satisfy 'additional' boundary conditions. The statement applies primarily to the free surface problem.

For submerged bodies and even for bodies moving at the free surface (Michell's integral!) it is frequently assumed as first approximation that the generating sin-

gularity system is identical with that determined for a body advancing in an unbounded liquid (respectively for the corresponding double model).

Earlier Havelock has suggested to generate bodies by locating sources and sinks outside to the midship section ¹²; pertinent resistance calculations were performed by Lunde ¹³. To my knowledge no attempt has been made to determine the resulting bodies although by integrating the differential equation of the streamline one should be enabled to enforce explicit results. This leads to the concept of volume singularities, which has been applied to our problem first by Hogner ¹⁴. Eggers has contributed a fundamental study on such singularities ¹⁵. So far it appears that this generalization does not present an advantage when solving the direct problem i.e. the determination of images for a given body, but no attempts have been made to handle the indirect problem with which we are here concerned i.e. to construct the shape of the body generated by volume singularities.

The indirect method became so popular because the direct approach to determine the flow around a given body (even in an uniform stream of unbounded liquid) is difficult. Several methods are available by which generating line images can be found for cylinders or bodies of revolution moving at a constant speed of translation ¹⁶. Again, it is well known that a solution does not exist in all cases.

^a
In/few cases singularity systems located on an axis have been found for a cylinder or a body of revolution moving in a non uniform flow (circle, sphere, spheroid

theorem). No attempts have been made to determine the image distribution over a center plane for a given hull form. Very probably solutions exist under much more restricted conditions only as in the case of the body of revolution.

Obviously, the procedure of searching images for a given hull can here ^{be} overstrained. The mentioned methods of generating bodies will retain their value in simple cases. We remember that wave resistance calculations are based primarily on the singularity distribution rather than on the actual hull form. A similar reasoning applies to the problem of finding shapes of least or low resistance which we consider as basic in our research. Therefore we expect that attempts will continue to develop procedures for constructing bodies from singularities; such procedures are especially needed in case when fixed walls (a bottom) and a free surface are present.

When, however, the ship surface is given, the direct methods so far sketched are limited in its scope. It appears that because of its generality for some purposes the classical approach of potential theory is superior following which singularity sheets are distributed over the surface of the body. This well known method has been avoided because of tedious computations involved but it experiences now a splendid revival.

The solution of the problem depends on a Fredholm equation of second kind (twodimensional in the threedimensional case). It has been formulated and studied in connection with the wave resistance problem by Kochin some 25 years ago [3]. Apparently the method became fruitful, only after high speed computers were available. We must admit, however,

that mental laziness also has hampered the application of general methods due to Havelock and Kochin.

We distinguish two or three steps in the development of the problem dealing with the determination of the singularity distribution. The first two apply actually to the wholly submerged body and results are extended more or less legitimately to surface ships.

The simplest case deals with the deeply submerged body or better the body in infinite liquid. Distributions obtained under this condition for a double model are used for the corresponding surface form. We find splendid presentation of our problem in Koemmer's book. Kochin's integral equation (4)

$$q(x,y,z) = -\frac{1}{2\pi} \int q(x,y,z) \frac{\cos(r_2^n)}{r^2} dS + 2v \cos(n,x)$$

$$r^2 = (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2$$

is firstly discussed for the general ellipsoid.

The solution

$$q(x,y,z) = (1 + \mu_x)v \cos(n,x) \quad (5)$$

with μ_x - hydrodynamic inertia coefficient in the x direction indicates, that this simple expression can be considered as a first approximation to $q(x,y,z)$ for other elongated (slender) bodies also.

Let us consider the motion of a submerged body close to the free surface at small F-numbers. Replacing the water surface by a rigid cover the appropriate integral equation is obtained by mirroring

$$q(x,y,z) = \frac{1}{2\pi} \int \frac{\cos(r,n)}{r^2} + \frac{\cos(r',n)}{r'^2} q(x,y,z) dS \quad (6)$$

again the first approximation yields

$$q_0(x, y, z) = 2v \cos(n, x) \quad (7)$$

Formally, the solution $q(x, y, z)$ can be written as

$$q = \chi q_0 = (1 + \frac{1}{2}\chi_1 + \frac{1}{4}\chi_2 + \dots) q_0 \quad (8)$$

and for estimates we put following (5)

$$2\chi \doteq \text{const} \doteq 1 + \mu_x.$$

To my knowledge, the first exact explicit result for a double model of an actual ship form (series 60), moving in an unbounded fluid has been obtained by Hess and Smith; their work may mark a turning point in the application of wave resistance theory also [18].

We quote following general statements made by these authors

- 1) fixed walls or other bodies can be introduced
- 2) non-uniform basic flow can be treated
- 3) although the method applies to bodies with continuous curvature only, it remains practically valid for convex bodies with corners; concave corners cause difficulties.

The distribution corresponding to high Froude numbers can be dealt with in a similar way.

The question arises (touched upon earlier) as to how far distributions calculated for the body in unbounded fluid (or close to a fixed cover) are influenced by the presence of a free surface when the Froude number is finite. Ginsburg has proved the existence of a solution [19]. Reference is made to a substantial study on submerged bodies by Bessho based on a slightly different approach [4]. Impressive examples are calculated for the spheroid and the sphere; the free surface effect demonstrated here is large. When dealing with surface

ships the determination of the singularities at finite (usual) Froude numbers appears to be awkward. We have to consider the change of the ship attitude and the corrugation of the free surface. Perhaps some estimates can be derived from results obtained for immersed bodies. The solution of this problem appears indispensable from a physical point of view; possibly, 'sheltering effect' of the body on waves cannot be properly determined without the knowledge of the actual image distribution. Rigorous investigations will be needed to determine as to how approximations suggested by physical reasoning are legitimate within linear theory and as to how nonlinearity can be considered. The study of sidewise and oblique motions involves the use of dipol distributions the axes of which are normal to the symmetry plane of the ship. Reference is made to the classic investigation by v.Karman in the case of deeply submerged bodies of revolution. The application of surface singularities yields in principle a solution for all kinds of translation.

Fortunately, the same method promises to furnish useful results in an important and rather neglected field - the hydrodynamic description of bodies (ship hulls) moving in shallow or restricted water. As long as the draft-depth ration T/h and the beam-breadth (width) ratio B/b are small the influence of walls on the velocity distribution around a moving body is small; hence it is legitimate to use as approximation the singularity distribution valid for the motion in an unbounded liquid and to estimate corrections by elementary procedures. However, frequently the influence of wall interference is neglected on singularity distribution for a given shape or on shape for a given distribution even when T/h is close to unity. This, obviously, leads to unacceptable errors.

Quantitative information on increase of velocity around a given body moving in shallow water was so scarce, that difficulties arose to estimate the magnitude of the frictional resistance.

In addition, change of vertical position and trim of the ship and deformation of the water surface generally are still more serious effects than in the unrestricted deep water condition. This is one aspect why resistance etc. formulae based on linearized theory must fail in the critical speed range $v \approx \sqrt{gh}$.

Summarizing we see that a lot remains to be done in the field of restricted water.

The concept of moving pressure systems has lost its popularity as a victim of fashion except in the case of planing phenomena. Here again the problem arises of the correspondence between given bottom forms and pressure systems. Quite a bit has been done in this respect in the two-dimensional case [6]; attempts have been made to deal with the three-dimensional case [20].

III. Wave Pattern

The observation of model waves is a popular method in resistance research. Experienced designers are even able to make use of the numerous wave profile pictures which are regularly furnished by model basins. Concepts like wave making length (length of wave separation) were important criteria with earlier investigators. Simple interference calculations were used to establish favorable and unfavorable ranges of the Froude number. Such information should be used with caution only (including charts in DWTaylor's 'Speed and power...').

Well known are attempts by Yourkevitch to minimize resistance in a systematic way by studying wave contours along models. His patent referring to the optimum position of an inflection point in the waterline rests on a reasonable foundation for medium Froude numbers.

The theoretical determination of wave resistance is widely based on wave pattern research. We note especially the work by Guilloton and Inui which in an explicit manner makes use of wave contours. It is worth remembering that Guilloton improved the physics of Michell's resistance calculation by considering the wave profil above the load water line. We shall mention beneath modern attempts to determine wave resistance from the measured wave field and the theoretical reasoning involved.

The study of wave pattern presents a useful method of comparing theoretical and experimental results in wave resistance research. We note first a difficulty in the experimental field: Wave profiles along a model determined by marking the wetted contour may differ considerably from those obtained by photography, since the wave surface (especially at the bow) may display a steep gradient in a direction approximately normal to the hull (s.sketch).

Earlier the calculation of the wave profile along the model (ship) was considered tedious; therefore a temptation existed to simplify computations by assuming infinite draft of the ship. Obviously, a comparison with measurements based on such an approximation is meaningful for low Froude numbers only. At present there is no excuse to avoid the threedimensional case.

In principle, within linear theory the wave pattern can be calculated for any reasonable system of singularities (sources and sinks, doublets) moving uniformly parallelly to the free surface and a vertical wall. The most complicated case is that of an eccentric motion in a rectangular channel.

The general expression is of the type

$$\zeta_w = -\frac{v}{4\pi g} \int_S (A_1 + A_2) q(x_1, y_1, z_1) dS \quad (9)$$

where A_1, A_2 are integrals derived from the potential for a unit source ϕ_0 by

$$\zeta_w = \frac{v}{g} \frac{\partial \phi_0(x, y, 0)}{\partial x} \quad (10)$$

Reference is made to the book by Kocm ~~okob~~ where a lot of useful information can be found. Expressions are given for the wave surface at high and small Froude numbers and asymptotic formulae valid far away from the ship (on deep, shallow and 'restricted' water), especially behind the ship. Attention is drawn to a formula for the wave formation generated by a sink system distributed over a vertical circular disc (waves created by a propeller) which to my knowledge was not known in the Western literature. ~~Fig.~~

Havelock has presented the formula

$$R_w = \pi g v^2 \int_0^{\pi/2} [a^2(\theta) + b^2(\theta)] \cos^3 \theta d\theta$$

which expresses the wave resistance by the wave pattern far behind the ship. This information is not yet sufficient to handle an old cherished idea - the determination of the wave resistance experimentally by measuring the wave field behind the body. Quite a lot of thinking

was needed to establish the theoretical foundation, even beyond what had been accomplished by our Japanese colleagues. 21 , 22 . The discussion of pertinent methods represents an important part of this meeting's work.

Attempts have been made to reach better approximations within linear theory and to cope with the nonlinear problem. We quote Jinnaka^{who} investigated the difference in the wave profiles computed at the centerline and at the actual position of the hull 4 . We refer once more to Guilloton's assiduous attempts to introduce improvements in the calculation of the wave pattern 23 , to some calculations by Kocmiskob 8 in which the wave profile (pressure lines) were corrected by the quadratic terms in the Bernoulli equation (although the surface boundary condition is linearized) and finally some solutions in the non linear field by Bessho and Jinnaka 4 . So far no consistent results have been obtained, i.e. no agreement has been reached on the influence of non linearity with the meager experimental evidence as e.g. presented by Inui.

IV. Resistance formulae

A synopsis of methods has been given by Havelock for determining the wave resistance 2 . Within the validity of linear theory the wave resistance can be calculated for any singularity system moving rectilinearly parallelly to the free surface (and to fixed vertical walls, including a rectangular channel) with uniform speed. Investigations centered around the thin ship and the body of revolution. Only recently explicit

expressions have been obtained for the slender ship. An excellent collection of formulae so far available is to be found in Kocmankob's book of which ample use will be made in what follows [8].

Probably the most urgent problem is at present the evaluation of Havelock's integral representing the wave resistance of source-sink sheets $q(x, y, z)$ distributed over the surface of the hull

$$R = 8\pi \kappa^2 g \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |H(\theta)|^2 \sec^3 \theta d\theta \quad (11)$$

with

$$H(\theta) = \iint_S q(x, y, z) \exp [\kappa \sec^2 \theta (z + ix \cos \theta + iy \sin \theta)] dS \quad (12)$$

or another form based on the surface equation $y(x, z)$ as given by Kocmankob.

$$R = \frac{4\pi g \kappa}{\pi} \int_0^{\frac{\pi}{2}} (I^2 + J^2) \sec^3 \theta d\theta \quad (13)$$

$$\begin{aligned} I &= \iint \exp\left(\frac{\kappa z}{\cos^2 \theta}\right) \cos\left(\frac{\kappa x}{\cos \theta}\right) \cos\left(\frac{\kappa y \sin \theta}{\cos^2 \theta}\right) \chi \frac{\partial y}{\partial x} dx dz \\ J &= \iint \exp\left(\frac{\kappa z}{\cos^2 \theta}\right) \sin\left(\frac{\kappa x}{\cos \theta}\right) \sin\left(\frac{\kappa y \sin \theta}{\cos^2 \theta}\right) \chi \frac{\partial y}{\partial x} dx dz \end{aligned} \quad (14)$$

Here χ is the function, which is obtained by solving the integral equation for the singularity (surface) distribution $q(x, y, z)$

$$q(x, y, z) = \chi(x, y, z) q_0 = \chi(x, y, z) 2v \cos(n, x)$$

Michell's integral may be expressed by the same formula as (13)

but with

$$\begin{aligned} I_m &= \iint \exp\left(\frac{\kappa z}{\cos^2 \theta}\right) \cos\left(\frac{\kappa x}{\cos \theta}\right) \frac{\partial y}{\partial x} dx dz \\ J_m &= \iint \exp\left(\frac{\kappa z}{\cos^2 \theta}\right) \sin\left(\frac{\kappa x}{\cos \theta}\right) \frac{\partial y}{\partial x} dx dz \end{aligned} \quad (15)$$

The comparison with the ellipsoid suggests that 2χ for elongated ships may be close to unity. Thus the essential difference between Havelock's (general) and Michell's integral is due to the factor $\cos \frac{\kappa y \sin \theta}{\cos^2 \theta}$

We are comparing now results derived by the complete formula and by the simplified expression

$$\frac{J}{J} = 2\chi \iint_S \exp\left(\frac{\kappa z}{\cos^2 \theta}\right) \frac{\cos}{\sin}\left(\frac{\kappa x}{\cos \theta}\right) \cos\left(\frac{\kappa y \sin \theta}{\cos^2 \theta}\right) \frac{\partial \eta}{\partial x} dx dz \quad (16)$$

and are hoping to present numerical results in the near future. At Ann Arbor at least one lecture is announced which deals with the resistance of surface distributions.

Corresponding resistance formulae of more general character have been derived for the motion of the ship on shallow water and in a rectangular channel [8]; explicit expressions are available for the vertical force Z (in case of asymmetrical ships the side force Y) and the trimming moment M_y also for the shallow water case. These formulae are based on the pertinent expressions for the unit source or dipole.

The most general resistance formula is that for a body (asymmetrical with respect to the xz plane), moving in a rectangular channel excentrically [8]. It is valid for a surface ship as well as for a submerged vessel. The essential difficulty consists in the determination of the appropriate surface distribution of singularities. While the formalism for submerged bodies has been established it is difficult to develop similar methods for the ship floating at the interface. The work is greatly simplified when instead of the body shape discrete singularities

and singularity distributions are presented; reference is made to earlier work by Havelock and Lunde [2], [13]. A large amount of evaluating and computing will be required to obtain results useful for application in practice.

Thus, in principle, ample information is available to check the thin ship theory. The question arises as to how far second order effects must be considered and up to which limits linear theory remains meaningful.

Earlier investigations by Havelock [24] and more recent ones by Bessho [4] indicate that these second order effects can be large. Reference is made to the chapter by Bessho [4] in which to my knowledge has not yet been properly acknowledged. Impressive pictures show the influence on the magnitude of resistance by considering second order approximation and non-linearity. We expect that much will be done in this field.

Maruo [25] has reconsidered conditions under which Michell's linearisation is permissible; it appears that the speed parameter $\frac{gB}{U^2}$ must remain small. Some indirect information on non-linear effects can be derived from Hess and Smith calculations of the velocity distribution around deeply submerged double models as a function of the beam [8].

Several attempts have been made to relax in a more or less intuitive way restrictions on which Michell's theory is based. Once more we mention Guilloton's original ideas to improve assumptions which are almost prohibitive from the point of view of practice. His oeuvre has become so impressive that it deserves a thorough critical

review by a mathematician who is familiar with naval architecture; it should be an interesting and useful subject to investigate and eventually justify the introduction of partial departures from linearity into linear theory.

Finally, the solution of the nonlinear problem becomes urgent at least for simple cases which admit to appraise errors committed^m by linearisation as was done by Bessho.

While in the field of engineering sciences the bookish wisdom in general lags behind research activities a reversal existed in our studies insofar as the ample information stored in the books by Koc^m kob, Bessho etc. has not yet been widely used. Obviously, this anomaly will be corrected by our seminar.

So far we have based our treatment upon the singularity concept. Difficulties in principle arise e.g. when we try to determine adequately the surface distribution of surface ships. Here Michell's method of solving the boundary problem appears superior; it can be generalized in such a way that nonlinear effects are considered. J.J. Stoker and especially Wehausen have shown how by a how by appropriate expansions further approximations can be constructed. The latter has obtained some elegant simplifications in the process of developing a second approximation. Possibly the most elaborate investigation in this direction has recently been given by SSisov 45. Expansions have been introduced in powers of a parameter for all important items including trim and sinkage. It is shown that the method can be reduced to the determination of singularities located on the longitudinal symmetry plane. Herefrom, obviously, results a restriction in the class of ships which can be treated by the method.

3 The scientific value of this procedure appears to be high; it fills a gap in our earlier reasoning. But notwithstanding the generality of physical assumptions the practical use may ^{be} limited when dealing with vessels operated at high Froude numbers because of the geometric restrictions. It is assumed that this problem will be treated thoroughly during our seminar.

5 After these generalities we point out some special problems the solutions of which should be better known or formed. From Kocmykob's book we quote the formula for the resistance of a tandem arrangement of n identical Michell ships located at an equal distance L_x between their midship sections

4

$$R_w = - \frac{4\rho g x}{\pi} \int_1^{\infty} \frac{\sin^2\left(\frac{1}{2} n x \lambda L_x\right)}{\sin^2\left(\frac{1}{2} x \lambda L_x\right)} \left(J_1^2 + J_2^2 \right) \frac{\lambda^2 d\lambda}{\sqrt{\lambda^2 - 1}}.$$

From this expression the optimum distance L_x and various asymptotic properties can be derived.

Another complex of interest is the determination of the wave resistance caused by local peculiarities like bulbs, bossings, damping plates, hadrofoils etc. Some ideas were developed on this subject already by Michell; the investi-

gation of the bulb is now a fashionable topic; some general ideas on local effects have been mentioned by Shor. But to be definite, an explicit solution is lacking e.g. for a system consisting of a vertical strut and a horizontal body of revolution i.e. the scheme of the normal facility for testing the (wave) resistance of submerged bodies. We may anticipate here that because of this gap in theory (plus deficiencies in the experimental setup) the comparison of experimental and theoretical results for these bodies so far made was subject to considerable errors or even in some cases almost meaningless - a deplorable fact since the submerged body lends itself to better theoretical treatment.

A short remark on pressure systems. Although, Hogner's integral has been communicated almost at the same time when Michell's formula was rediscovered and applied. Modest use only has been made of the former important solution. Reference is made to Sedov's and Keldysh's work dealing with the resistance in a rectangular channel and a thorough evaluation by Newman 29 !

Some physical aspects of the problem of resistance in a rectangular channel and ~~to a thorough evaluation arising~~ in restricted water will be touched upon. It is obvious that the limits of linearised theory are reached earlier than in the deep water case. We cannot expect valid resistance results in the critical speed range when the depth h is small (when dealing with ships 'small h ' means small $\frac{h}{L}$); further difficulties arise when T/h is close to unity. An open problem is that dealing with phenomena occurring in channels with constant but not rectangular cross sections.

When in the subcritical range $0,6 < F_h$ wave resistance on shallow water begins to increase; conditions arise which are similar to those on deep water but at higher Froude numbers. (O. Schlichting's hypothesis). An important effect is the trim. While at low and medium Froude numbers trim cannot serve as useful criterium for resistance, excessive trim at high F . or at F_h close to unity is detrimental. The advantage of the transom stern in this range can be thus 'explained'.

As dynamic lift is involved in these phenomena many authors borrow the concept 'induced drag' to explain some of the effects mentioned. Obviously, this concept is legitimate e.g. in the case of finite span hydrofoils when energy is actually dissipated by a process which can be described by the action of a vortex or vortex sheet. However, it is erroneous to introduce 'induced drag' as antithesis to 'wave resistance' when such a vortex scheme is not appropriate. Dynamic vertical force and moment of trim of ships can be explained by wave action ; bodily sinkage and trim calculated by Michell's integral already checks nicely ^{with} experimental values.

This reasoning applies especially to fast rounded form craft (with transom stern). Here the design rule has been proposed to reduce dynamic lift to zero (better to reduce trim); the explanation, however, appears erroneous following which by such means the total resistance consisting of induced, wave and viscous resistance should decrease because the first component - the induced drag - disappears. Actually the wave resistance can be reduced by aiming at a moderate trim. These summary remarks show how far our state of knowledge is when dealing with ships in the range of transition between floating and planing.

Possibly, however, consideration of pressure distribution will lead to a reasonable resistance theory for high speed craft. A first step in this direction we see in the work on gliding surfaces especially for those with low aspect ratio .

Beside the linearized hydrodynamic approach there exists an elementary hydraulic method for treating resistance of ships moving in a (rectangular) channel. This theory promoted especially by Krey and Kreitner is based on the continuity and Bernoulli's equations and makes wide use of the concept of supercritical flow; notwithstanding its simplicity it yields a good physical understanding of important phenomena. The value of the method decreases when the distance of sidewalls increases. It is inapplicable for infinite breadth of water. Nonetheless attempts have been made to make qualitative use of such parameter as $\frac{h}{A_x}$ for

determining local critical speeds (analogous to problems in compressible flow). Obviously, these items can now be calculated with comparative ease; further, the augmentation of viscous resistance can be determined and the attitude of the model.

Actual local depth h' due to changes in the water level must be considered rather than the undisturbed depth h .

We mentioned the limitations within which linear theory presents useful information. Nonlinear treatment is required in the range in which shallow water effects become strong. It must be remembered, however, that non steady effects become important when the local critical velocity is reached; this fact complicates the situation further. Reference is made to investigations by Laitone and Schuster on supercritical around hydrofoil moving close to the free surface.
flow

O. Schlichting's well known hypothesis on the shallow water resistance is valuable as a first orientation: it is noteworthy as one of the few cases where in our field useful results have been obtained by simple means.

V.

We mentioned that two basic practical needs -

- 1) determination of magnitude of resistance in concrete cases and
- 2) development of good hull forms -
influence in a slightly different way theoretical investigations in our field.

These two problems are fundamental for the work of model basins also. We may subsume under 'development of good hull forms', development of means designed with the purpose to influence wave formation (resistance) locally like hydrofoils, damping-plates etc.

Roughly speaking in routine work of towing tanks emphasis is shifted to the 'exact' determination of the resistance magnitude rather than to the development of optimum forms although, obviously, improving lines is considered an important task.

The development of good hull forms by experimental methods was based on intuition, hypotheses, observations, experiences and on systematic variations of hull geometry. These systematic series work proved to be extremely efficient in the hands of such a highly gifted systematical spirit like D.W. Taylor.

Attempts have been made to establish experimentally dependency of resistance upon main dimensions, proportions and basic coefficients. Notwithstanding the large amount of

experimenting reliable, empirical information on some fundamental relations is surprisingly scarce.

In principle, the development of theory should change the picture. The determination of hulls of minimum resistance may become a central problem in ship theory both from a practical as well as a scientific standpoint although resistance is only one basic (detrimental) property of the ship and the wave resistance in general a minor part of the total drag only. The high variability and intricate relations involved make these investigations so important. The necessity of complying with other fundamental ship properties and the existence of severe restrictions under which optimisation is valid suggest ~~to~~ speak on form of low resistance rather, when practical viewpoints are at stake. When dealing with such more modest aims the method of systematic geometric variations gains in value for theoretical research also.

In what follows we shall treat as far as possible simultaneously surface ships and submerged bodies.

VI. Methods of evaluation and results

The arrival of high speed computers has solved in principle the evaluation of such standard solutions as Michell's integral and the resistance integral for bodies of revolution. Nonetheless, even here the development of appropriate methods remains important especially from the viewpoint of systematizing results and finding dependences of resistance upon form.

Thus we mention two solutions of the problem suggested by Birkhoff following which Michell's integral should be evaluated by splitting it up into a permanent part

and a hull function. The first solution has been given by Michelsen [30] using hypergeometric series. Another method is due to Michailov [8]. Here the permanent part has been tabulated as a function,

$(F(a,b))$, based upon expansions by K and Y (Bessel) functions.

The wave resistance is given by the relation

$$R = C \iint_{S \cup S_c} \frac{\partial \eta}{\partial x} \frac{\partial \eta}{\partial z} F(a, b) dS_c dS \quad ()$$

For the determination of the resistance of elementary ships at low (and even medium Froude numbers up to $F = 0.3$), Inui's expansion has been used with best success. A similar development has been proposed by Haskind.

Asymptotic expansions are available for very low and very high Froude numbers. Well known important results are

$$R \sim t^2 \quad \text{for } R \rightarrow 0$$

$$R \sim V^2 \quad \text{for } F \rightarrow \infty$$

Tables are available for the computation of wave resistance of forms based on polynomial distributions..Most refer to Michell ships, some to submerged bodies of revolution. Although the availability of high speed computers reduces quite generally the need for tabulation, it appears advisable to complete such tables e.g. in the range of bulb forms since they enable a good survey.

An impressive amount of numerical work will now be required for evaluating more complicated integrals, in the first line Havelock's formula. We expect from these results information on the dependency of wave-resistance upon main dimensions - an important problem which can

be solved by Michell's integral in a qualitative manner only. Probably, it will be necessary to consider the influence of the free surface on the surface singularity distribution for a large range of (except low) Froude numbers.

Beside upon proportions the dependency of wave resistance has been investigated upon

- 1) the shape of the sectional area curve
- (2) " " " " waterline area curve)
- 3) " " " " load waterline
- 4) " " " " main section.

So far results are obtained for thin ships and bodies of revolution. By far the most important research deals with the sectional area curve $A(x)$ since it yields the most fruitful results which theory can contribute. Special resistance formulae derived from Michell's integral are almost trivial since they coincide essentially with the expressions derived for elementary ships. Actually the elementary ship concept has been introduced because of the preponderance of interest in the longitudinal distribution of displacement. More interesting are recently developed formulae for the slender ship which again are based on the sectional area curve $A(x)$ [31] [25].

A large amount of wave resistance diagrams have been published which are based on systematic variations of elementary hulls or $A(x)$ curves. Of special interest are those where comparisons are made with experimental results.

Much less investigations have been devoted to hull forms which depart from elementary type. Here the influence

of the load waterline is the most interesting item. For a reasonably wide class of ships explicit results are readily obtained by auxiliary tables so far published.

The dependency of wave drag upon the vertical displacement distribution can be easily estimated for simple ships. The well known rule results that it is favorable to arrange the displacement as far away from the free surface as possible. This simple rule does not hold, however, universally when complicated interference effects are involved.

A considerable amount of information is available on the wave resistance of ships and pressure systems moving in a rectangular basin ('tank corrections') 29 29a

VII. Ships of minimum resistance

We start with a synopsis on methods for determining forms of minimum wave resistance and on some obtained results. The literature is increasing rapidly. We quote specially a recent memoir by Shor [32]. Its title is promising being free from propaganda effects. The amount of information presented is impressive, still more the mature judgment of the author both in the scientific and technical field although he is a newcomer in our family. ~~national~~ front. We hope that the mathematical approach (the method of steepest descent) suggested by the author will be as fruitful in our case as he anticipates.

The status of a newcomer causes some errors when treating the history of the subject. No adequate credit is given to Havelock's work. Since similar slips are typical for the situation, I shall give a more extended review of the develop-

ment of our subject than necessary. On the other hand some problems especially those investigated by Inui and Hogner have been dealt with so lucidly by Shor, that there is no need to dwell on them.

1. The priority in treating a problem in our field belongs to Joukovsky [8]. He solved it for a wall-sided vessel with prescribed displacement advancing at a supercritical speed $F_h = \frac{V}{\sqrt{g h}} > 1$; he assumed further that the draft T is almost equal to the uniform depth of water h . Thus he investigated essentially a two-dimensional problem. As optimum waterline form $f(x)$ a common parabola is obtained.

The pertinent expression for the wave resistance is given by:

$$R_w = \frac{2 \rho g h^2 \bar{f}_h^2}{\sqrt{F_h^2 - 1}} \int_0^L \left(\frac{\partial \eta}{\partial x} \right)^2 dx, \quad T = h, \quad \eta = 2h \int_0^L \eta dx \quad (18)$$

Clearly, the physical content of this theory is meager.

2. The present writer's first attempts are based on Michell's integral [33]. The limitation due to the hydrodynamic theory and to Ritz's method have been clearly stated already in the original paper. A variety of isoperimetric problems which are useful from a technical point of view have been discussed. There are two weak spots in the investigation. Although the need has been recognized for introducing terms which picture a bulb form explicit results including the latter have not been aimed at. In the light of our present knowledge the omission of the bulb element (which may be an essential feature in optimum forms!) can lead to solutions which depart from orthodox ship lines in an unfavorable manner. Further, in the first publication no distinction has been made between ship form and

singularity distribution. This obvious and far reaching confusion was corrected when dealing with bodies of revolution .

3. By applying the exact method of the calculus of variations to Michell's integral Pavlenko obtained the integral equation [8]

$$\int_0^T \int_{-l}^l f(x, z) dx dz \int_0^\infty \exp[k\lambda^2(z+z_1) \cos(k\lambda(x-x_1))] \frac{\lambda^4 d\lambda}{\sqrt{\lambda^2-1}} = \text{const} \quad (19)$$

$$2 \int_0^T \int_{-l}^l f(x, z) dx dz = V = \text{const.}$$

which underlies most of the later investigations. Results have been obtained for the infinitely deep vertical cylinder (pile) by solving the simplified equation:

$$\int_{-l}^l f(x) dx \int_0^\infty \cos[k\lambda(x-x_1)] \frac{\lambda^3 d\lambda}{\sqrt{\lambda^2-1}} = \text{const}$$

or

$$\int_{-l}^l f(x) dx = \text{const}$$

$$\int_{-l}^l f(x) Y_0[k\lambda(x-x_1)] dx = \text{const} \quad (20)$$

numerically. Although the author found that in a certain range of the Froude number no 'reasonable' forms were obtained he presented a collection of almost orthodox waterlines which, however, at high Froude numbers became extremely full. In the light of our present knowledge the shape of most curves is decisively influenced by the approximations used in the numeric solution.

4. Further progress is essentially based on Pavlenko's equation. Exemption should be made for a less known investigation by *Basin* who optimized the waterline forms of ships moving on finite water depth. For $F_h > 1$ Joukovsky's result is again obtained - the best waterline is a common parabola [8].

5. Independently from Pavlenko, v. Karman derived as optimum condition for a pile the integral equation (35). He stated that most solutions so obtained violated Michell's assumptions, since the 'waterlines' ended with infinite 'horns' at the bow and stern. The manuscript presented before the IV. Int. Congress of Appl. Mech. (England) and some letters by v. Karman to the present writer contained a lot of information. Unfortunately the manuscript never was published (except for a tiny synopsis); ~~only~~ thus most results were lost and rediscovered later only. The paper stirred up a lot of (partly fruitless) discussions. It is a deeply felt duty to devote these short remarks to the memory of the genius v. Karman. Sretensky stated also that the optimisation of Michell's integral did not lead to functions which are square integrable. At this point the development slowed down for sometime.

Interesting investigations were due to Hogner who indicates means of improving given shiplines (Hogner's original approach has been highly praised by Shor), and later to Guilloton 37,. Further reference should be made to a paper by Lisov 8, he minimized the wave resistance as well as the thrust deduction. The present writer is acquainted with a short abstract only in Kostyukov's book.

The line of investigation aimed at immediate application has been continued by the present writer using Ritz's method. Following earlier ideas an attempt has been made to minimize wave resistance plus a term which in simplified form takes care of frictional resistance. The distinction between optimum doublet distribution and optimum hull form has been carried out for surface ships as had been done earlier for submerged bodies.

Neubausen has tried to obtain forms which are closer to shipbuilding practice; he fixed the shape of the afterbody with the purpose of avoiding detrimental viscosity effects (separation). [38].

The exact calculus of variations problem ('Favlenko's problem') for Michell's integral has been studied by Kotik and others and has resulted in a solution presented by Karp, Lurye and Kotik which for the deep immersed part of a pile can be considered as final [5]. Although similar results have been found earlier we consider as decisive achievement the clearcut distinction made between optimum distribution and hull (waterline) form and explicit examples of both for various Froude numbers.

Inspired by the awkward optimum distribution forms (with horns at the ends) the present writer together with Eggers and Sharma reconsidered the problem of optimum line doublet distributions for submerged bodies of revolution, using again Ritz's method. Contrary to the earlier attempt, beside the continuous doublet distribution concentrated sources and doublets were admitted at the bow and stern [5]. Such a doublet pictures reasonably well the singularities in the optimal distributions mentioned above. Following new results were obtained:

- 1) by adding a concentrated source at the bow (sink at the stern) to a continuous doublet distribution a bulb effect is achieved, i.e. the resistance is **appreciably** decreased in a wide range of the Froude number.

- 2) By introducing a discrete doublet this bulb effect is increased. As compared with 1) the resistance is once more reduced in some regions of the Froude number by an order of magnitude. This indicates that bow (and stern) parties should be designed with utmost care. From a practical viewpoint we remark, however, that the resistance reduction obtained by 1) can be already so drastic that the gain by step 2) is no more decisive.

This result supports the conclusion derived from other more general considerations that frequently it does not pay to look for the optimum and that more orthodox good forms may be preferable.

- 3) By the additions of discrete singularities optimum distributions are obtained which frequently resemble more closely orthodox ship lines+ especially the tendency appears reduced to generate extreme swan neck forms.

We expect that similar procedures will be useful when minimizing more general resistance integrals.

A lot of useful isoperimetric problems can be formulated. While we have e.g. frequently kept the midship section (beam) constant Pavlenko admitted a variation in the beam. Quite recently Landweber made a similar investigation on bodies of revolution.

Resistance relations based on surface distributions lose their advantages when optimum problems are treated since it is difficult to imagine that such problems can be solved leaving the hull form undetermined. A witty suggestion has been made by Shor to admit additional singularities distributed e.g.

over the centerplane. The whole system is subject to an optimisation in such a way that the original hull is increased.

While we try to increase the generality of our solutions problems are meaningful especially from a practical viewpoint which are solved under very restricted conditions. Reference is made to a calculation by Maruo based on his slender body formula when $t = 0$ (cruiser stern and raked stern!)

Parallely to the work by Karp, Lurye and Kotik we quote a paper by Timman and Vossers [5] and earlier investigations by Krein [8]. The latter has obtained optimum distributions which agree with those by Karp etc., but a decisive step is lacking - the distinction between the distribution and the resulting form.

It is a sad fact that high abilities and lack of criticism can exist side by side.

An erroneous impression is created by Shor following which up till now optimisation has been treated in a style which indicated that just something should and could be done.

Firstly, one should remember that the problem of accuracy was a difficult task before the arrival of fast computers. Secondly, numerous attempts have been made to improve well developed forms e.g. of a racing boat, a destroyer, fast cargo vessels etc. Results were sometimes satisfactory but more frequently disappointing. When slight gains in resistance resulted they were in general not large enough to compensate possible disadvantages from the point of view of construction (e.g.

the case of swan neck forms). Occasionally results were inconclusive, sometimes even disastrous. A large amount of judgment is needed when applying results of approximate (Michell's) theory. It is expected that improvements in theory, development of semiempirical solutions and progress in numerical work will increase the efficiency of pertinent work.

When viscosity effects are considered by semiempirical methods it appears promising at present to start with the study of the wave formation rather than with the formal treatment of the minimum problem [41].

VI. Experimental methods, Theory and experiment.

Are such strenuous efforts justified in treating the ideal fluid case since actual phenomena depend heavily upon viscosity? In the neighboring field of hydraulic engines the opinion is widely spread following which concentrated attempts in applying methods of potential theory yield meager results only. In ship hydrodynamics we do not share this view; we consider it indispensable to treat the various sources of energy dissipation as thoroughly as possible and independently. Progress has been hampered by mixing results due to various effects which are neglected in simplified theory.

Exact theoretical investigations on the influence of viscosity on wave resistance are scarce. Well known is a first attempt by Sretenski in which the characteristic boundary condition on the body is neglected [42]. In general semiempirical methods are used. Havelock has introduced an effective hull shape by adding the displacement thickness of the boundary layer to the actual body. He has further described some effects of the frictional belt by changing the strength of the generating images in the

afterbody; Havelock's and Wigley's correction coefficient β can be interpreted in such a way.

The procedure was improved by Emerson who 43 introduced two empirical coefficients and Inui who refined the physical interpretation as well as the formalism and reached good agreement between calculated and measured wave resistance values 4 .

It is to be expected that the semiempirical procedure will be an important topic at our seminar. We shall therefore not dwell upon them.

Astonishingly, the experimental determination of the wave resistance has been neglected for a long time. We shall give here a short critical review of experimental methods. Some have been widely used by Horn and his school but only more recently internationally adopted and developed.

At the present state of knowledge it would be preferable to plan experiments with the purpose of increasing our physical insight rather than of obtaining results which are immediately applicable to practice. Actually the number of such fundamental experiments is still surprisingly small. The shortcomings in experimental procedures and in the interpretation of results are numerous.

The weakness of the classical Froude method is now universally recognized. Instead of the relation

$$R_t = R_{fo} + R_{rest} \quad (21)$$

attempts are being made to introduce the improved relation

$$R_t = R_v + R_w + R_{vw} \quad (22)$$

where R_v is the total viscous resistance and R_{vw} an interference term. The latter is generally omitted; we obtain

$$R_t = R_v + R_w \quad (23)$$

where R_v is sought for in the form

$$R_v = (1 + n) R_{fo} \quad (24)$$

with n , the form factor, independent of the Froude number.

The 'new' approach is physically better founded although neglecting the interference term R_{vw} may cause serious errors. A rough estimate of R_{vw} is frequently made by calculating the change of the wetted surface due to the wave formation (the effect is pronounced at $F \sim 0,35$). From a practical viewpoint the determination of the total viscous resistance appears as a problem of equal scientific weight as that of the wave resistance. We discuss the former as means for finding the wave drag.

Essentially four (six) methods are known for determining the viscous resistance. It would have been more consistent to formulate methods for determining wave resistance,

1. Double models (Föttinger's method)
2. Total resistance at small Froude numbers when $R_w \rightarrow 0$
3. Geosim experiments
4. Tulin's impulse method
5. As fifth method we may consider the reversal to the improved Froude method, i.e. finding from $R_v = R_t - R_w$. R_w is determined from wave configurations generated by the ship.
6. Finally the same equation is used but R_w is calculated (at small Froude numbers); this is a generalisation of 2.

Various objections have been raised against 1.. In principle, they refer to the neglect of viscosity-wave resi-

"interference"- an effect which may be stronger than anticipated. Well to the point is a criticism^{of}/so far experiments conducted. It is slightly disturbing that the evaluation of some experiments with deeply submerged bodies pushed to high Reynolds numbers yields constant coefficients $c_v = \frac{R_v}{\rho \frac{1}{2} V^2 S}$ rather than constant form factors. Errors due to neglecting wave action are possible in earlier experiments; further, at the comparatively high speeds involved, the model surface occasionally may be no more smooth hydraulically.

Results derived by method 2 are still more questionable. Large models are a prerequisite. Well known and justly feared are laminar effects which generally lead to a drop in the $c_v = c_t$ curve at very low speeds. Sometimes, however, the curve $c_t(F)$ increases with decreasing F in such a way that its gradient is much steeper than would correspond to the relation $c_v = (1 + n)c_t$. Reference is made to a substantial paper by O. Schlichting [44].... A meaningful determination of the form factor n is impossible in both cases mentioned.

An impressive number of Geosims have been tested, but it is doubtful if the ample material is always accurate enough for wave resistance research and if it has been efficiently evaluated. There are some recent experiments which disclose a high variability of the formfactor $n = n(F)$ when evaluated over a large range of the Froude number thus indicating a strong wave-viscosity interference effect.

Notwithstanding its generality Tulin's method has been applied in few cases only. Obviously, the experimental investment is considerable. The same applies to method 5 . The first attempt has been made to my knowledge some 50 years ago by the Berlin tank where Lacmann and later Eisner planned to derive the wave resistance of a moving vertical cylinder from stereophotogrammetric wave pictures. The task was a failure.

At this state of art it appeared urgent to determine the wave resistance of a simplified hull form based upon all experimental methods available and upon theoretical reasoning. Mr. Sharma, to whom the work was entrusted will report on his findings before this seminar. Similar research work has been conducted at other places.

Substantial experimental investigations on the resistance 'components' have been inspired some 30 years ago by Horn. Laute's 26 measurements indicated that the wave resistance of good ocean going cargo ships is small at economic speeds. This finding has been substantiated by theoretical work on forms of minimum resistance although because of limitations of theory quantitative results are not too reliable.

But astonishingly, in tank practice and when planning systematic series this basic fact has been almost neglected.

We refer to an ample literature dealing with a comparison of results obtained by measuring and calculating wave resistance. We wish to emphasize some points only.

1. There are some Russian investigations 27 dealing with the resistance of ships moving in a rectangular basin. For h/T and b/B large the qualitative and even the

quantitative agreement between theory and facts (except close to $F_h = 1$) is so satisfactory that the calculated correction factors can be used in tank work prediction [27].

2. Attention is drawn once more to the fact that comparative investigations on the wave resistance of fully submerged bodies suffer from inadequate experimenting and gaps in theory.
3. There are cases where (Michell's) theory failed to explain important phenomena in deep water resistance. We quote an experiment by D.W. Taylor in the range of $F = 0.38$ 39 ; here (contrary to theoretical predictions and well known systematic tests by Taylor himself) a hull characterized by a V-shaped bow and a sectional area curve with a low t -value was superior to orthodox U-forms based on a sectional area curve with a high t -value.

Another paradoxon is the independence of the wave resistance upon the B/L ratio (within a wide range of this parameter) in a domain of the Froude number close to $F = 0.30$. An explanation by Inui has been useful, but it is not exhaustive 4 .

Future work in this field will rest on more advanced theoretical work and will rely on crucial experiments rather than on systematic ones. To supply new information we quote a paper which is in the process of publication, although in the present state it does not meet the criteria just mentioned.

The first part of the study deals with the resistance of a family of analytically defined elementary ships,

part of which had been investigated earlier. It is the purpose to determine the viscous resistance more accurately ~~than~~ earlier (so that the comparison of experimental and theoretical wave resistance can be reasonably well founded), to establish some basic dependencies of R_w upon $B : T$, $C_p = \phi$ etc., using Michell's and Havelock's (general) integral and to check some simple rules of thumb much used in design work.

So far following results have been obtained:

The magnitude of the viscous drag determined by double models and by the normal model at low Froude numbers agrees to some extent but not too well; experiments are continued.

Calculations of the wave resistance indicate that in our case at $F = 0.1 - 0.125$ asymptotic conditions are approximately reached ($R \sim t^2$): the wave resistance becomes negligible.

So far computations have been completed for Michell's integral only. The comparison with experiment supports the fact that this formula exaggerates extremely favorable and unfavorable effects. (Figures ~~1-3, 6-9~~)

Although we do not stress quantitative coincidence the agreement is good for forms with $\phi = C_p = 0.56$ and 0.60 for all B/T ratios tested.

The original plan has been abandoned to construct and to test 'Inuids' (generated by distributions which are affine to our sectional area curves), since the evaluation of Havelock's integral fulfills the same purpose in a more satisfactory way.

The second part is devoted to an important conclusion mentioned before from Michell's integral, following which to small changes in the longitudinal displacement distribution may correspond large variations of the wave resistance, Figures 1, 4, 5... show the sectional areas of the two elementary ships from which the usefulness of **analytical** representation becomes obvious (it is difficult to describe the small differences by words): further results of resistance calculations and experiments are shown. The special case here discussed has been treated earlier but is now reconsidered in a much more substantial way because weak spots were detected in our original study when the resistance computations were some time ago extended to higher Froude numbers. Our recent investigation conducted with utmost care and extended to two more draft beam ratios has wholly endorsed the qualitative agreement between theory and facts found earlier.

Several years ago Inuids have been computed using our sectional area curves $\backslash 2, 4, 6; 0.56; 1$ and $2, 3, 4; 0.56; 1$ as distributions, The sectional area curves of both Inuids so generated differ less than the dipol distributions $2, 4, 6; 0.56; 1$ and $2, 3, 4; 0.56; 1$.

We are hoping that the evaluation of Havelock's integral based on the determination of the pertinent surface distributions $q(x, y, z)$ will lead to a closer quantitative agreement with experiments.

Conclusions

Our subject is a central problem in ship theory. Since no useful theoretical results are so far available for real fluids and not much progress can be expected in the near future because of the state of turbulence theory present efforts must be concentrated upon theoretical investigations of ideal fluid phenomena and upon an experimental and semiempirical approach when dealing with the influence of viscous effects. Different from experience in few other branches of engineering science the ideal fluid concept is valuable in wave resistance research from the point of practical shipbuilding also.

Beyond the standard solutions (Michell's integral, Havelock's integral for submerged bodies of revolution, Hogner's integral) more general resistance formulae are available, based on linear theory. They are promising to cope more efficiently with the properties of actual ship forms. We are confronted with two important auxiliary problems: the determination of the surface distribution for a given body and of the body shape generated by a given singularity system. **The process of solving both problems leads first to the necessity of second order effects and later to the necessity of treating non linear effects.** But very probably linear solutions will satisfy further practical needs in a large number of urgent cases some of which have been mentioned.

Difficulties in evaluation have hampered progress considerably but have been overcome to a large extent by computers. Systematic exploitation of form properties and optimisation of forms can now be handled with better success than before. A large amount of work will be required to

evaluate more advanced relations with the purpose of checking and correcting linearized results. As a basic task we consider the new trend to harmonize design postulates suggested by wave resistance research with other fundamental requirements presented by other branches of shipbuilding science.

Experimental wave resistance research was till recently in a rather crude or ossified state; theory has almost imperceptibly succeeded in pushing forward new methods although as stated before its influence on tank work in general has been unsatisfactory.

It is to be expected, however, that the combined application of theoretical and experimental methods will lead to satisfactory understanding of the complicated phenomena and to rational principles of design in our ample field.

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