The Total Collapse of the Twin Towers: What It Would Have Taken to Prevent It Once Collapse Was Initiated

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Abstract: It is generally taken as a given that there is no reasonable design concept that could have prevented the collapse of the Twin Towers, once it was initiated, from progressing all the way down to the ground. This view is rooted in the idea that the force generated during the inevitable impact between what may be called the intact upper section (IUS) and the intact lower section (ILS)—meaning the building sections above and below the initially lost columns, respectively—will exceed by at least one order of magnitude the capacity of the latter. On closer inspection, this turns out to be only partially correct—it is correct with regard to the topmost floor plate of the ILS but not with regard to the columns below this floor plate. This paper shows that if the ILS in the Twin Towers had been topped by a stronger-than-ordinary floor plate allowing the columns below to respond properly, rather than be bypassed, these columns—and with them the ILS—would likely have survived. The paper subsequently proposes a building design concept consisting in the insertion of strengthened floor plates in intervals of 10–20 stories. **DOI: 10.1061/(ASCE)ST.1943-541X.0003244.**

Introduction

The relatively young research field of progressive collapse, naturally receiving a thrust each time a prominent structural collapse occurs, has undoubtedly received its strongest thrust yet due to the terrorist attacks on September 11, 2001. Significant progress has been made since then in the research on progressive collapse of buildings. However, given the nature of the disaster that prompted this research—a disaster that carried with itself the painful message, so deceptively tempting to discard in hindsight, that there are scenarios that we cannot anticipate and/or specifically design against—the direction taken in the attempts to tackle the problem cannot be called entirely satisfactory. With most of the attention falling on the design methods of providing increased local resistance and alternative load paths, the research efforts since 2001 have been mainly concentrated on what may be called the outermost defense lines. Comparatively very little attention has been given to the question of what can be done if these outermost defense lines are broken and collapse is initiated after all.

Must everything be considered lost in such a case? If the explanation for the total collapse of the World Trade Center (WTC) Twin Towers proposed by Bažant and Zhou (2002) is correct, as currently widely accepted among engineering professionals, then it really would seem that given the various practical restrictions in the design of high-rise buildings, the endeavor of developing measures to arrest a collapse once initiated is very challenging, if not hopeless. The “if” at the beginning of this last sentence, however, must not be forgotten; it is among the issues addressed in this paper.

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The objective of this paper is to follow one of the recommendations from the World Trade Center building performance study released by FEMA in 2002, and to “determine, given the great size and weight of the two towers, whether there are feasible design and construction features that would permit such buildings to arrest or limit a collapse, once it began” (FEMA 2002, p. 40).

Employed Approach

The initial situation assumed for all following considerations is shown in Fig. 1. Bypassing the preceding events, we assume that collapse is initiated as all columns over the height of one or several neighboring intermediate stories of a high-rise building lose their entire axial capacity. Based on observations from video footages of the WTC Twin Towers’ collapses, this assumption is also supported by the findings of the official final report on these collapses released by NIST in 2005 [NIST NCSTAR 1-6 (NIST 2005)].

Fig. 1 leaves the cause of the assumed initial column loss unspecified because this is irrelevant for the following considerations. There is only one restriction regarding this cause: the initial column loss in Fig. 1 is assumed to occur due to an immediate action on the columns, rather than due to some extraordinarily high gravity load acting on the intact structure; in other words, the building is assumed to be under normal operation load at collapse initiation.

As shown in Fig. 1, three sections are distinguished in the considered building at collapse initiation. The following can be said regarding the survival chances of these sections. The damaged intermediate section (DIS) will inevitably be lost. This is a direct consequence of what we assume with regard to this section—namely, that all its columns lose their entire axial capacity thus initiating collapse. The intact upper section (IUS), which, like the DIS, finds itself in free fall, must be written off as well for the simple reason that we have no control over this section’s motion. In particular, we cannot prevent, but must rather expect, a rotation of the IUS during its free fall—a behavior that would result from a nonsimultaneous failure of the DIS columns across the building cross section and clearly observed in the case of the WTC Twin Towers. The intact lower section (ILS) is the only section whose saving we can begin to realistically consider.

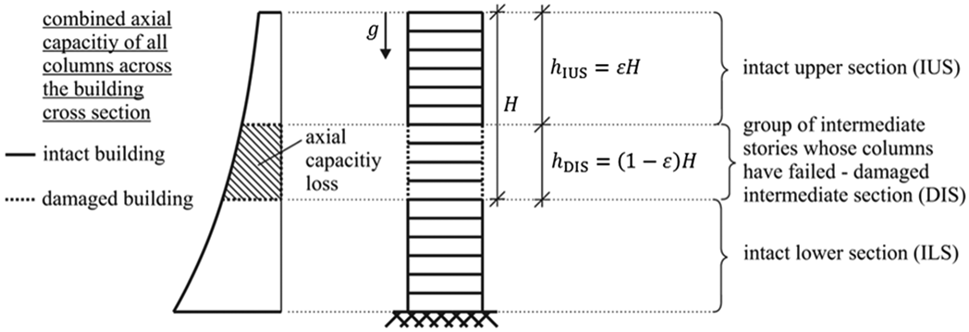

Fig. 1. Schematic representation of a high-rise building at collapse initiation.

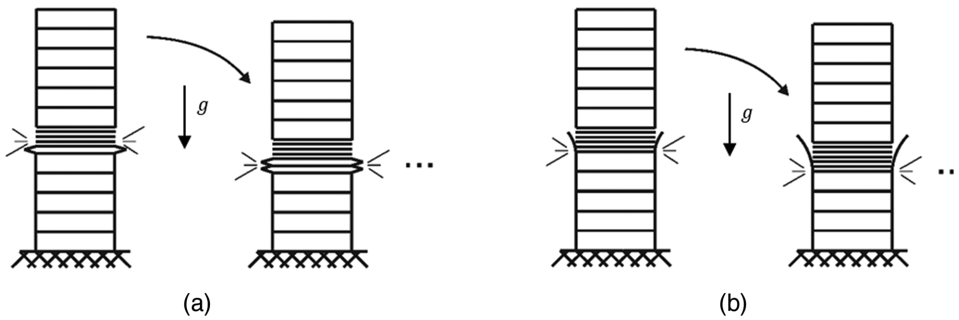

Fig. 2. Mechanisms of downward progression of a pancake-type collapse: (a) C-F-driven downward collapse progression; and (b) F-P-F-driven downward collapse progression.

Before we do this, it is necessary to take a brief side step and examine the phenomenon we endeavor to prevent—an overloading of the ILS followed by a downward collapse progression, or a "crush-down," as Bažant and Verdure (2007) fittingly called it (the terms downward collapse progression and crush-down are used interchangeably in the following). A collapse progressing downward from story to story and thereby collecting the floor plates into a stack is generally referred to as a pancake-type collapse—a term derived from the appearance of the collected stack (Starossek 2018, p. 18). This term, however, with the description just given, is by itself not sufficient to describe the way in which a crush-down can be initiated and will then unfold. The reason is that there are two possible and mutually exclusive ways for this to occur; these are shown in Fig. 2.

In what we here call a column-failure-driven (C-F-driven) downward collapse progression, the floor plates of the successively collapsing stories are stacked together due to the overloading and subsequent successive buckling of the columns in these stories [Fig. 2(a)]. This type of collapse progression is assumed by Bažant and Verdure (2007) (although those authors do not use the term C-F-driven) in developing their one-dimensional (1D) collapse model, an evolution of a simple model presented by Bažant and Zhou (2002). Bažant and Verdure (2007) employed the concept of the "mean crushing force"—a concept to which we will shortly return—to account for the force from the buckling columns acting at the collapse front and thereby slowing this collapse front down.

The second type of downward collapse progression, which we call floor-plate-failure-driven (F-P-F-driven), is shown in Fig. 2(b). Here, the floor plates of the successively collapsing stories are stacked together due to the shearing off of these floor plates at their connections to the columns; the columns themselves, resembling trees whose trunks are being disbranched, do not participate, a consequence being that the properties of these columns remain irrelevant with regard to the initiation and unfolding of the crush-down. The schematic representation in Fig. 2(b) focuses only on the F-P-F-driven crush-down, the topic at hand; in general, however, and for the same reasons explaining the F-P-F-driven crush-down, the IUS will be collapsing simultaneously in what may be called F-P-F-driven crush-up.

Now, in any conventionally designed building, a given floor plate and its connections to the columns are only designed to carry this plate's own dead and live loads with the corresponding safety factors; the columns supporting this floor plate, on the other hand, are designed for the loads from this and all above-lying floor plates, and can thus take a much greater load than the single floor plate. This obvious fact, combined with the realization that the IUS can never land onto the ILS with such precision that the columns of these two sections align perfectly and thus prevent activation of the topmost floor plate of the ILS (let alone the collapsing DIS mass, which will inevitably activate the topmost floor plate of the ILS), is all that is necessary to explain why the ILS in the WTC Twin Towers had no chance once the DIS and IUS were in motion: Any floor plate in the ILS was utterly unable to cope with the mere weight of the above-lying mass, not to mention the dynamic forces due to impact.

This realization makes it clear that the crush-down in the WTC Twin Towers was F-P-F-driven, and that the ILS columns, much stronger than the governing floor plates, never had a say in the matter. Any possibly remaining doubt that this was the case is dispelled by the observations from available video footage of the collapses. Despite the dust cloud enshrouding the downward-traveling collapse front, it can be observed that "[a]s the floors collapsed, this left tall free-standing portions of the exterior wall and possibly central core columns" (FEMA 2002, p. 27). This is only possible in a F-P-F-driven crush-down [Fig. 2(b)]. If the crush-down had been C-F-driven [Fig. 2(a)], the ILS columns would have been crushed

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and would thus not have been left to stick above the downward-traveling collapse front.

This raises the following question: if the ILS in the WTC Twin Towers had been topped by a floor plate strong enough to fully mobilize the underlying columns, making these columns the ultimate arbiters on the initiation of crush-down, would these columns have been able to sustain the load from the collapsing DIS and IUS, thus saving the ILS? A floor plate as just described, designed not to fail before the columns supporting it, is referred to in the following as a strong floor plate.

As a practical side note, because the load-bearing capacity of a strong floor plate must be at least an order of magnitude above that of an ordinary floor plate, the depth of a strong floor plate must also be correspondingly larger, amounting to at least 1 story. Furthermore, in a building like the WTC Twin Towers, employing numerous closely spaced perimeter columns and a relatively large number of columns in the core region, a strong floor plate must include at least 1-story deep belt trusses coupling, respectively, the perimeter and the core columns. This is because the similarly deep trusses of this same strong floor plate that must span between perimeter and core, and that cannot be as many in number as the perimeter- and the core columns can only be attached to a correspondingly smaller number of these columns, say, only to every fourth or fifth of them; this smaller number of directly loaded columns must then communicate the information to the remaining columns and garner their participation, which is the task of said belt trusses.

Having clarified these basics, we now proceed to address the aforeposed question. To do this, the total force exerted by the collapsing DIS and IUS on the strong floor plate topping the ILS must be determined as a function of time. This problem has many similarities to determining the force exerted by an impacting aircraft on the reinforced concrete shell of a nuclear power plant—a problem treated by Riera (1968). The most important feature of Riera's (1968) approach is that it aims at uncoupling the problems of describing the collapse of the impacting body, in our case the DIS and IUS, and the response of the impacted structure, in our case the ILS. Riera (1968) recognized that such uncoupling is possible if, unlike the impacting body, the impacted structure remains elastic (and this can be only an initial assumption that is later to be confirmed). In such cases, the impacted structure can be treated as a rigid body.

The aforeposed question can thus be answered by first assuming that the ILS remains elastic during the collapse of the DIS and IUS. The force that the ILS must be able to sustain if it is indeed to remain elastic, can then be determined as a function of time by assuming that the DIS and IUS collapse against a fixed rigid horizontal surface, representing the strong floor plate topping the ILS.

A final point to clarify before beginning the derivations is the way in which we are to account for the resistance offered by the IUS columns during collapse. With the assumed fixed rigid horizontal surface modeling the top surface of the ILS, both the DIS and IUS collapse against this surface in what Bažant and Verdure (2007) called "crush-up" mode. A structure collapsing in crush-up mode is being eaten up from the bottom toward the top, with the collapse front moving up into the structure. The crush-up of the DIS, which is referred to in the following as the first phase of collapse, unfolds at the rate of free fall because there is no resistance to be offered by the DIS columns, which are assumed completely lost. Impacting the debris pile formed by the collapsed DIS mass at the end of the first and beginning of the second phase of collapse, the columns at the bottom of the IUS are activated and begin to get crushed (the term crushed column refers to a column whose ends are moved toward each other). Thus, the crush-up of the IUS, occupying the second phase of collapse, is C-F-driven.

As Bažant and Zhou (2002) and Bažant and Verdure (2007) noted, a crushed column of the type employed in the WTC Twin Towers begins to buckle, thereby developing into a plastic-hinge mechanism, at a very early stage of its crushing. The force needed to continue the crushing process decreases quickly with the onset of buckling, as shown by the load-displacement function $F^*(z)$ in Fig. 3(a) (asterisks are used in Fig. 3 to refer to forces developed by individual columns, as opposed to a group of columns). The sketches integrated in Fig. 3(a), showing a crushed column over the height of one story, explain the meanings of the employed variables: F^* is the force developed by the crushed column in the direction of crushing (i.e., along the line connecting the column's ends), and z is the crushing distance, equal to the distance that the two ends of the crushed column have traveled toward each other since the onset of crushing.

The total crushing distance of the column in Fig. 3(a) is $(1-\kappa)h_{\text{story}}$, where h_{story} denotes the height of a story, equal to the column's buckling length, and κ is what Bažant and Verdure (2007) called a "compaction ratio" (although those authors used a different notation), accounting for the presence of compressed story contents. Due to these contents, the force rises steeply at $z = (1-\kappa)h_{\text{story}}$ [rehardening branch in Fig. 3(a)], terminating the crushing in this story and initiating crushing in the next story [not shown in Fig. 3(a)].

Bažant and Verdure (2007) proposed approximating the actual function $F^*(z)$ by a constant function yielding the same integral over the total crushing distance $(1-\kappa)h_{\text{story}}$. The magnitude of this constant function, which Bažant and Verdure (2007) called "mean crushing force," is denoted here as F^*_{mean} [Fig. 3(a)]. A significant difference can be seen between F^*_{mean} and the column's axial capacity, denoted as F^*_c in Fig. 3(a). For columns as employed in the WTC Twin Towers, the ratio F^*_{mean}/F^*_c can be shown to be smaller than 0.25. This large difference between the mean crushing force and the axial capacity of a column, which was also strongly emphasized by Bažant and Zhou (2002) and later by Bažant and Verdure (2007), is an important point to which we will return later.

Now, would there be any qualitative change in this consideration if instead of a single column we consider the crushing of a group of columns, assuming for simplicity that these columns are of equal properties, the same as considered in Fig. 3(a)? According to Bažant and Verdure (2007), the answer is no: The combined force of the crushed column group, which we denote as $F(z)$ (note the missing asterisk), would simply be a scaled-up version of $F^*(z)$; hence, the combined mean crushing force F_{mean} would be no better an approximation of $F(z)$ than F^*_{mean} is of $F^*(z)$. But is this realistic for the case at hand? Can we expect that the IUS columns are actually crushed in sync, so that they all reach their respective maxima F^*_c simultaneously, story by story, during the crush-up of the IUS? Obviously, this would only occur if the IUS collapses without developing any tilt and the surface of the debris pile formed by already collapsed mass below the IUS is perfectly even and horizontal, all of which is practically impossible.

In reality, the various hills and valleys in the surface of the debris pile below the IUS, as well as the tilt, be it even a small one, that the IUS is bound to develop will at any moment cause the columns at the bottom of the IUS to find themselves at different stages of their crushing; thus the force in one crushed column may just be reaching its maximum F^*_c while the force in another crushed column is at its lowest value. The effect of such asynchronicities among the contributions of individual columns on the combined force $F(z)$ of the crushed column group is visualized in Fig. 3(b), which considers the collapse of a generic 4-bay frame against a tilted surface.

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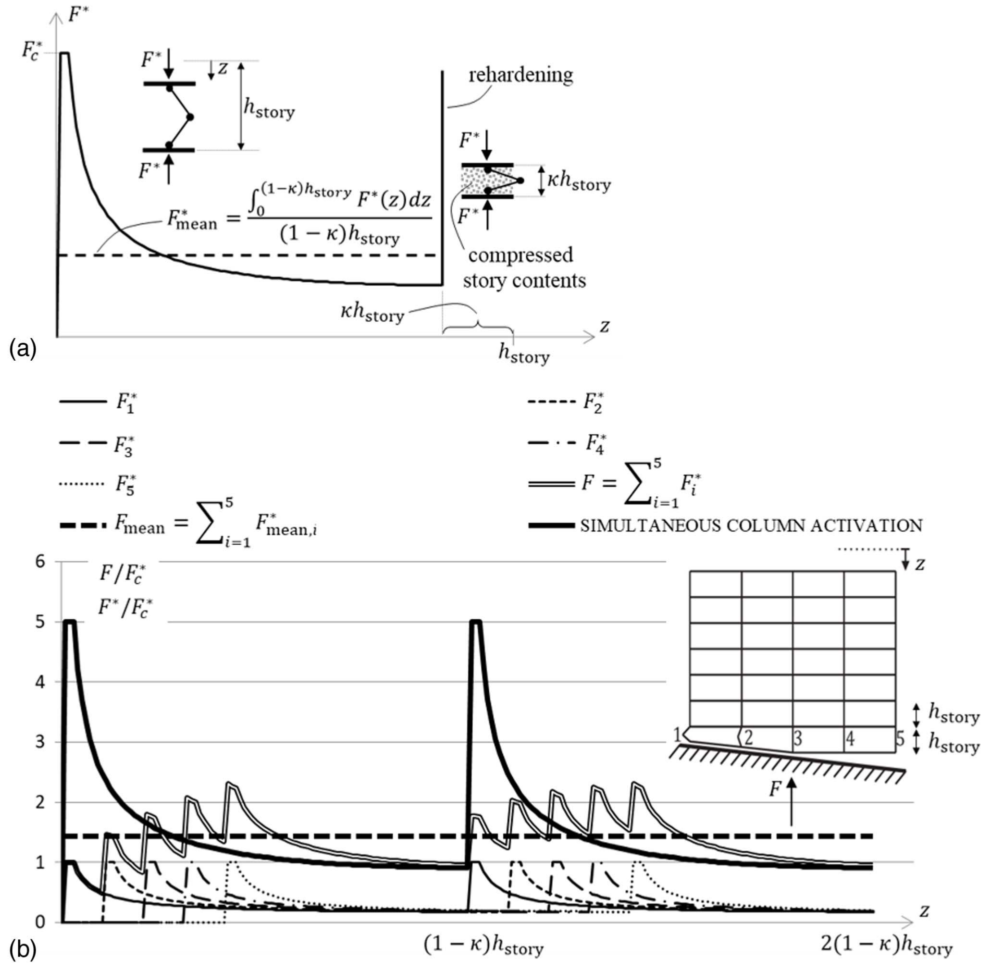

Fig. 3. Force developed during crushing of (a) a single column; and (b) a group of columns.

Again, the sketch integrated in Fig. 3(b) helps to explain the meanings of the employed variables; z is again the crushing distance, with $z = 0$ marking the onset of crushing of the column group, occurring as the first column in this group makes contact with the tilted surface. The five columns of the frame in Fig. 3(b), each of them having in every story the same properties as the single column considered in Fig. 3(a), are numbered in order to track their individual contributions, F_1^* to F_5^*. These contributions are seen to be shifted relative to each other along the abscissa, as a result of the nonsimultaneous activation of the columns caused by the tilted impact surface.

The actual combined force $F(z)$, sum of the contributions F_1^* to F_5^*, of the columns of this frame [double line in Fig. 3(b)] is seen to be much better approximated by the combined mean crushing force F_{mean} [thick dashed line in Fig. 3(b)] than the combined force that would occur if the columns were activated simultaneously and then crushed in sync [thick solid line in Fig. 3(b)]. This latter case of simultaneous column activation, removing the shifts among the individual contributions F_1^* to F_5^*, would occur if instead of tilted, the impact surface was horizontal and thus parallel to the frame's floor lines. All forces in Fig. 3(b) are normalized with respect to the axial capacity F_c^* of a single column [Fig. 3(a)] to facilitate comparison.

The important general point to be conveyed by Fig. 3(b) is that any practically present cause of nonsimultaneous activation of the columns of the impacting structure—be this cause a tilt of the floor lines of this structure relative to the impact surface, as exemplarily assumed in Fig. 3(b), or some unevenness in the form of hills and valleys of the impact surface, or, most generally and practically most likely, a randomly time-varying mixture of both—will have a smoothening effect on the function $F(z)$, reducing the fluctuations of this function about the mean value F_{mean}, and thus improving the quality of the approximation of $F(z)$ by F_{mean}. Also, this quality can be expected to improve further as the number of columns in the crushed column group increases.

Considering this on the one hand, and noting that the aforementioned causes of nonsimultaneous column activation will practically always be present, F_{mean} appears to be more than merely a very good approximation of $F(z)$, much better than what Bažant and Verdure (2007) recognized when they employed this approximation; in fact, using F_{mean} to account for the action of $F(z)$ appears the only reasonable thing to do. Having said that, this very realization, rather than a welcome improvement to the model of Bažant and Verdure (2007), turns out to pose a problem for this model—a problem that is to be explained at the end of the derivations in this paper.

This, then, is how we want to account for the resistance offered by the IUS columns during crush-up: by using the combined mean crushing force of these columns. The term used from now on to refer to this combined mean crushing force is mean crushing force of the IUS.

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Crush-Up Collapse of a One-Dimensional Tower with Uniform Continuous Mass Distribution and Mean Crushing Force Linearly Decreasing to Zero at the Top: Closed-Form Solution

We begin by considering a situation identical with that occurring during the second phase of collapse, after the IUS, falling with a certain initial velocity, meets resistance from below. It is pertinent to begin with this problem because its solution also contains the solution for the first phase of collapse (the collapse of the DIS) as a special case.

Fig. 4(a) shows the considered 1D tower of initial height h, constant mass per unit height $\bar{m}$, and a mean crushing force linearly decreasing toward the top, described by the function $F_{\text{mean}} = \psi g \bar{m} h'$, where h' is a coordinate measured always from the (moving) tower top downward, and ψ is a factor equal to the ratio between the mean crushing force at level h' and the weight of the tower part above that level. The tower has a compaction ratio κ [Fig. 3(a)] constant along the height. An underlying assumption in the concept of a compaction ratio is that no further compaction of already-collapsed material can occur; in other words, the collapsed compact mass resting on top of the rigid horizontal surface at any time t during the collapse is treated as rigid. At the beginning of its crush-up collapse, the tower, acted upon by gravity, is moving with initial downward velocity $\dot{z}_0$ against a rigid horizontal surface.

Before moving on, let us briefly analyze the background of these assumptions and how well they match up to the actual WTC Twin Towers, whose IUS (and DIS, with $\psi = \dot{z}_0 = 0$) the 1D tower in Fig. 4(a) should represent. The WTC Twin Towers were prismatic buildings whose largest mass portion was contained in their floor plates, all designed for and standing under the load of similar floor contents (office furniture) of similar texture. This makes the assumptions of constant mass per unit height $\bar{m}$ and constant compaction ratio κ reasonable. It is also reasonable to assume that the mean crushing force, depending among other things on the columns' cross-section dimensions, decreases toward the tower top.

Although it is true that in an actual building, F_{mean} does not decrease to zero at the top, this deviation from reality is an acceptable price to be paid for what the assumption $F_{\text{mean}} = \psi g \bar{m} h'$, combined with the assumption of constant $\bar{m}$, offers: a closed-form solution. As Bažant and Verdure (2007) noted when deriving the crush-up differential equation, during a crush-up collapse the mass above the collapse front is generally in a state of fall under variable gravity acceleration. The assumptions in Fig. 4(a) make this acceleration constant, and thus remove the obstacle to a closed-form solution.

Collapsing Tower Kinematics

At the beginning of its collapse ($t = 0$), the considered 1D tower is moving toward a fixed horizontal surface with velocity $\dot{z}_0$ while also being under the action of g. Of main interest to us is the force needed to hold the horizontal surface in place during collapse; we designate this support force as F_s. It is instructive to make the following derivations using a reference frame in which at $t = 0$, the tower itself is stationary while the horizontal surface at its bottom has an upward velocity $\dot{z}_0$ and an upward acceleration g. The driving force needed to move the surface in this prescribed fashion is the same force $F_s(t)$ that we seek.

Moving upward with the prescribed velocity $gt + \dot{z}_0$, the horizontal surface starts compacting the mass of the tower at $t = 0$.

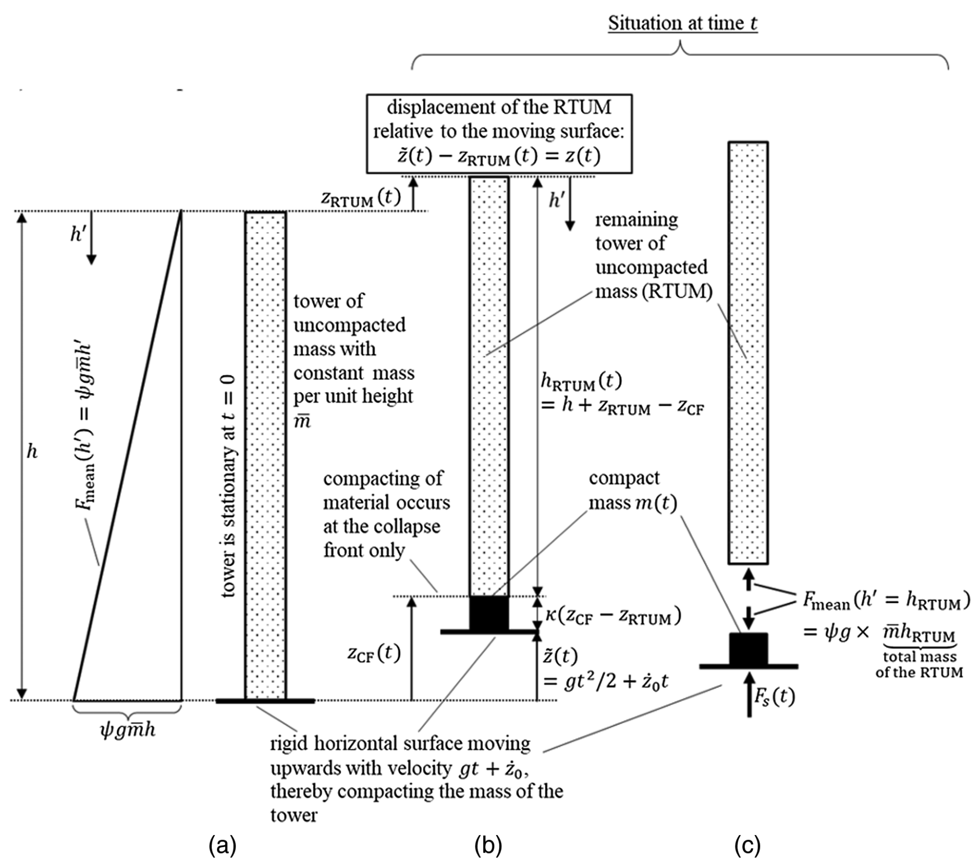

Fig. 4. Model of a 1D tower collapsing in crush-up mode: (a) tower of uncompacted mass at $t = 0$; (b) displacements; and (c) forces.

After a time t, the surface has traveled the distance $\tilde{z}(t) = gt^2/2 + \dot{z}_0 t$ [Fig. 4(b)]. The thickness of the compact-mass layer resting on top of the moving surface is continually growing with time. It follows that the border between the compact mass layer and the remaining tower of uncompacted mass (RTUM), i.e., the collapse front, is moving upward faster than the horizontal surface. The distance traveled by the collapse front is given by the coordinate $z_{\mathrm{CF}}(t)$ [Fig. 4(b)].

The RTUM is accelerating upward under the action of the mean crushing force at the current collapse front $F_{\mathrm{mean}}(h' = h_{\mathrm{RTUM}})$ [Fig. 4(c)]. At any time t during the collapse, $F_{\mathrm{mean}}(h' = h_{\mathrm{RTUM}})$ has the same ratio ψg to the mass of the RTUM, meaning that this RTUM has a constant upward acceleration of ψg. The (upward) displacement of the RTUM is then $z_{\mathrm{RTUM}}(t) = \psi g t^2/2$ [where $\dot{z}_{\mathrm{RTUM}}(0) = 0$ in the reference frame used here]. At time t, the collapse front has engulfed a certain part of the original tower. The height of this part is given by $h - h_{\mathrm{RTUM}}(t) = z_{\mathrm{CF}}(t) - z_{\mathrm{RTUM}}(t)$ [Fig. 4(b)]. Multiplying this height by the compaction ratio κ gives the thickness of the compact mass layer at time t. The coordinate $z_{\mathrm{CF}}(t)$ can be expressed as the sum of the coordinate $\tilde{z}(t)$ and the thickness of the compact-mass layer

$$z_{\mathrm{CF}}(t) = \tilde{z}(t) + \kappa[z_{\mathrm{CF}}(t) - z_{\mathrm{RTUM}}(t)] \tag{1}$$

Substituting $\tilde{z}(t) = gt^2/2 + \dot{z}_0 t$ and $z_{\mathrm{RTUM}}(t) = \psi g t^2/2$, and solving for $z_{\mathrm{CF}}(t)$ yields

$$z_{\mathrm{CF}}(t) = \frac{(1 - \kappa\psi)gt^2 + 2\dot{z}_0 t}{2(1 - \kappa)} \tag{2}$$

The compact mass $m(t)$ is equal to the product of $\bar{m}$ and height $h - h_{\mathrm{RTUM}}(t) = z_{\mathrm{CF}}(t) - z_{\mathrm{RTUM}}(t)$. Substituting the expressions for $z_{\mathrm{CF}}(t)$ and $z_{\mathrm{RTUM}}(t)$ yields

$$m(t) = \bar{m}\frac{(1 - \psi)gt^2 + 2\dot{z}_0 t}{2(1 - \kappa)} \tag{3}$$

Force $F_s(t)$

Now consider the compact mass and the forces acting on it at time t [Fig. 4(c)]. Over an infinitesimal time interval dt, the compact mass $m(t)$, moving with velocity $\dot{\tilde{z}}(t)$, grows by the mass particle dm. At time t, just before it becomes part of the compact mass $m(t)$, dm is still part of the RTUM, and as such has the velocity $\dot{z}_{\mathrm{RTUM}}(t) = \psi g t$. The resultant upward force $F_s(t) - F_{\mathrm{mean}}(h_{\mathrm{RTUM}})$ acting over the time interval dt leads to a change in the combined momentum $P(t)$ of the compact mass $m(t)$ and the collected mass particle dm. Applying the principle of conservation of momentum, we obtain

$$\begin{aligned}&[F_s(t) - F_{\mathrm{mean}}(h_{\mathrm{RTUM}})]dt\\ &\quad = \underbrace{[m(t) + dm][\dot{\tilde{z}}(t) + d\dot{\tilde{z}}]}_{P(t+dt)} - \underbrace{[m(t)\dot{\tilde{z}}(t) + dm\,\dot{z}_{\mathrm{RTUM}}(t)]}_{P(t)}\end{aligned} \tag{4}$$

where the one-dimensional vectors are already replaced by their single components. Further developing this expression yields (after leaving out the higher-order terms)

$$\begin{aligned}F_s(t) &= m(t)\ddot{\tilde{z}}(t) + \dot{m}(t)[\dot{\tilde{z}}(t) - \dot{z}_{\mathrm{RTUM}}(t)]\\ &\quad + \underbrace{\psi g\bar{m}\,\underbrace{[h + z_{\mathrm{RTUM}}(t) - z_{\mathrm{CF}}(t)]}_{h_{\mathrm{RTUM}}(t)}}_{F_{\mathrm{mean}}(h_{\mathrm{RTUM}})}\end{aligned} \tag{5}$$

where $F_s(t)$ is seen to be the sum of three forces: With $\ddot{\tilde{z}}(t) = g$, the first term on the right-hand side of Eq. (5) represents the weight of the compact mass $m(t)$; the second term, referred to in the following as the mass-flow force, results from the continuous flow of infinitesimal mass particles experiencing an abrupt velocity change at the collapse front; the third and last term is the mean crushing force acting at the collapse front.

The expressions for $z_{\mathrm{RTUM}}(t)$, $z_{\mathrm{CF}}(t)$, $\tilde{z}(t)$, and $m(t)$ were derived in explicit form in the preceding paragraphs. Substituting these expressions and their time derivatives in Eq. (5) yields, after some algebra, the following:

$$F_s(t) = \frac{\bar{m}}{2(1 - \kappa)}[3(1 - \psi)^2 g^2 t^2 + 6(1 - \psi)\dot{z}_0 g t + 2\dot{z}_0^2] + \psi g\bar{m}h \tag{6}$$

The properties of the function $F_s(t)$, and in particular the time when the maximum is reached, depend on the value of ψ. Consider first the special case $\psi = 1$, where the time-dependent terms in Eq. (6) vanish, and the force F_s is the time-independent sum of $\bar{m}\dot{z}_0^2/(1 - \kappa)$ and $g\bar{m}h$. The first term represents the force resulting from mass flow; the mass-flow velocity $\dot{z}_0$ is here constant because at any time t, the force $F_{\mathrm{mean}}(h_{\mathrm{RTUM}})$ is exactly equal to the weight of the RTUM. The second term, $g\bar{m}h$, represents the time-independent sum of the weight of the compact mass $m(t)$ and $F_{\mathrm{mean}}(h_{\mathrm{RTUM}})$.

Now consider the case when $\psi > 1$. In this case, the mass-flow velocity, and thus the mass-flow force, decrease as the collapse progresses (decelerated collapse) because at any time t, the force $F_{\mathrm{mean}}(h_{\mathrm{RTUM}})$ is greater than the weight of the RTUM. Furthermore, the sum of the other two forces, $gm(t)$ and $F_{\mathrm{mean}}(h_{\mathrm{RTUM}})$, which can be shown to be equal to $g\bar{m}h + (\psi - 1)g\bar{m}h_{\mathrm{RTUM}}$, also decreases during collapse because this same claim applies to h_{RTUM}, and because $(\psi - 1) > 1$. Hence, for $\psi > 1$ the force $F_s(t)$ has its maximum at $t = 0$, as in the case when $\psi = 1$. Following the same line of reasoning, it can be shown that when $\psi < 1$ the force $F_s(t)$ increases as the collapse progresses. In this case, the maximum of $F_s(t)$ is reached at the end of the collapse. To determine this maximum, the collapse duration must be known. This duration is determined next.

Collapse Duration

The expression for $F_s(t)$ in Eq. (6) is, of course, only valid as long as the collapse front travels into the RTUM. Assuming that the tower collapses totally, the collapse comes to an end when h_{RTUM} [Fig. 4(b)] is reduced to zero. The time t^* when this occurs can be calculated from the equation $h_{\mathrm{RTUM}}(t^*) = h + z_{\mathrm{RTUM}}(t^*) - z_{\mathrm{CF}}(t^*) = 0$. Substituting the expressions for z_{RTUM} and z_{CF} yields

$$\frac{g(\psi - 1)}{2(1 - \kappa)}t^{*2} - \frac{\dot{z}_0}{1 - \kappa}t^* + h = 0 \tag{7}$$

This is a quadratic equation with two roots for t^*. The only root with physical meaning is

$$t^* = \frac{\dot{z}_0 - \sqrt{\dot{z}_0^2 - 2gh(1 - \kappa)(\psi - 1)}}{g(\psi - 1)} \tag{8}$$

For the special case $\psi = 1$, Eq. (8) yields an expression of the type $0/0$. Applying L'Hospital's rule yields

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$$\lim_{\psi\to 1} t^* = \lim_{\psi\to 1} \frac{\frac{\partial}{\partial\psi}\left[\dot{z}_0 - \sqrt{\dot{z}_0^2 - 2gh(1-\kappa)(\psi-1)}\right]}{\frac{\partial}{\partial\psi}[g(\psi-1)]} = \frac{h(1-\kappa)}{\dot{z}_0} \tag{9}$$

The collapse duration t^* as given by Eq. (8) is a real number only as long as the term under the radical sign does not become negative. This is fulfilled as long as ψ does not exceed a certain value, which we designate as $\bar{\psi}$. This condition can be expressed as follows:

$$\psi \leq \underbrace{\frac{2gh(1-\kappa)+\dot{z}_0^2}{2gh(1-\kappa)}}_{\stackrel{\text{def}}{=}\bar{\psi}} \tag{10}$$

This result points at a limitation of the assumption that the tower collapses totally. The matter is resolved by examining the velocity with which the collapse front propagates into the RTUM. This velocity is equal to $\dot{z}_{\text{CF}}(t) - \dot{z}_{\text{RTUM}}(t)$, which can be calculated

$$\dot{z}_{\text{CF}}(t) - \dot{z}_{\text{RTUM}}(t) = \frac{(1-\psi)gt + \dot{z}_0}{1-\kappa} \tag{11}$$

The collapse front propagates into the RTUM with constant or increasing velocity if $\psi = 1$ or $\psi < 1$, respectively. In both these cases, the collapse is total because the collapse front eventually reaches the top of the RTUM. This is in line with the inequality in Eq. (10)—the derived condition for the validity of the assumption of total collapse—which is obviously fulfilled when $\psi \leq 1$ (noting that $\bar{\psi} \geq 1$).

On the other hand, if $\psi > 1$, the velocity difference in Eq. (11) decreases with time, reaching zero at time $\bar{t}$, calculated as follows:

$$\bar{t} = \frac{\dot{z}_0}{g(\psi-1)} \tag{12}$$

It can now be established whether the tower collapses totally by calculating the height of the RTUM at time $\bar{t}$. If the result is positive, the collapse is not total, in which case the collapse duration t^* is equal to $\bar{t}$ according to Eq. (12). The condition for nontotal collapse of the tower can be expressed as follows:

$$\underbrace{h + \frac{\psi g\bar{t}^2}{2}}_{z_{\text{RTUM}}(\bar{t})} - \underbrace{\frac{(1-\kappa\psi)g\bar{t}^2 + 2\dot{z}_0\bar{t}}{2(1-\kappa)}}_{z_{\text{CF}}(\bar{t})} > 0 \tag{13}$$

After substituting the expression for $\bar{t}$, this inequality can be transformed to obtain a condition for ψ, yielding

$$\psi > \underbrace{\frac{2gh(1-\kappa)+\dot{z}_0^2}{2gh(1-\kappa)}}_{\bar{\psi}} \tag{14}$$

The right sides of both inequalities Eqs. (10) and (14) are, as they should be, equal to $\bar{\psi}$, the value of ψ that marks the transition between total and nontotal collapse. The collapse duration t^* for each case is given by a different expression: Eq. (8) for the case of total collapse, and by Eq. (12) for nontotal collapse

$$t^* = \begin{cases} \dfrac{\dot{z}_0 - \sqrt{\dot{z}_0^2 - 2gh(1-\kappa)(\psi-1)}}{g(\psi-1)}, & \psi \leq \bar{\psi} \\ \dfrac{\dot{z}_0}{g(\psi-1)}, & \psi > \bar{\psi} \end{cases} \tag{15}$$

To complete the solution of the problem in Fig. 4, the height of the RTUM after the collapse comes to an end—we designate this height as h_{rest}—is determined

$$h_{\text{rest}} = \begin{cases} 0, & \psi \leq \bar{\psi} \\ h_{\text{RTUM}}(t^*_{\psi>\bar{\psi}}) = h - \dfrac{\dot{z}_0^2}{2g(1-\kappa)(\psi-1)}, & \psi > \bar{\psi} \end{cases} \tag{16}$$

Capacity Requirement on the ILS

Using the expressions derived in the preceding section, the solutions for the first and second phases of collapse (collapse of the DIS and IUS, respectively) can now be readily obtained by substituting the respective quantities. The constant mass per unit height and the compaction ratio are the same for DIS and IUS ($\bar{m}_{\text{DIS}} = \bar{m}_{\text{IUS}} = \bar{m}$ and $\kappa_{\text{DIS}} = \kappa_{\text{IUS}} = \kappa$). The total force exerted by the collapsing DIS and IUS is referred to in the following as $F_{s,\text{tot}}$.

First Phase of Collapse: Collapse of the DIS, $0 \leq t < t^*_{\text{DIS}}$

The following quantities are to be substituted in the preceding general solution to obtain the solution for the first phase of collapse: $h = h_{\text{DIS}} = (1-\varepsilon)H$ (Fig. 1), $\psi = \psi_{\text{DIS}} = 0$, and $\dot{z}_0 = 0$.

The force $F_{s,\text{tot}}$ during the first phase of collapse is thus obtained from Eq. (6) as follows:

$$F_{s,\text{tot}}(t) = \frac{3g^2\bar{m}}{2(1-\kappa)}t^2 \tag{17}$$

The collapse duration t^*_{DIS} of the DIS is obtained from Eq. (15) for $\psi = 0 < \bar{\psi}$ as follows:

$$t^*_{\text{DIS}} = \sqrt{\frac{2H(1-\kappa)(1-\varepsilon)}{g}} \tag{18}$$

The force $F_{s,\text{tot}}(t)$ increases monotonically with time, reaching immediately before the beginning of the second collapse phase the following value:

$$F_{s,\text{tot}}(t \nearrow t^*_{\text{DIS}}) = 3g\bar{m}H(1-\varepsilon) \tag{19}$$

This force, equal to three times the weight of the DIS, is independent of the compaction ratio κ.

Second Phase of Collapse: Collapse of the IUS, $t^*_{\text{DIS}} \leq t \leq t^*_{\text{tot}}$

A change in the solution occurs at time t^*_{DIS} due to the nonzero mean crushing force of the IUS. Substituting the initial velocity $\dot{z}_0 = \sqrt{2gH(1-\kappa)(1-\varepsilon)}$ [where the free-fall height of the IUS is $H(1-\kappa)(1-\varepsilon)$], $h = h_{\text{IUS}} = \varepsilon H$ (Fig. 1), and $\psi = \psi_{\text{IUS}}$ in the general solution, the solution for the second phase of collapse can now be obtained.

To obtain the correct result for the force $F_{s,\text{tot}}(t)$ during the second phase of collapse, the weight of the now-collapsed DIS must be added to the force exerted by the collapsing IUS. The fact that the collapse of the IUS begins at $t = t^*_{\text{DIS}}$ must also be accounted for. Substituting the quantities for the IUS in Eq. (6), replacing t by $t - t^*_{\text{DIS}}$, and adding to the result the weight of the DIS, equal to $g\bar{m}H(1-\varepsilon)$, yields

$$F_{s,\text{tot}}(t) = \frac{\bar{m}}{2(1-\kappa)}\Big[3(1-\psi_{\text{IUS}})^2 g^2 (t-t^*_{\text{DIS}})^2 + 6(1-\psi_{\text{IUS}})\sqrt{2g^3 H(1-\kappa)(1-\varepsilon)}(t-t^*_{\text{DIS}})\Big] + g\bar{m}H[3(1-\varepsilon)+\psi_{\text{IUS}}\varepsilon] \quad (20)$$

At the beginning of the second phase of collapse, the force $F_{s,\text{tot}}$ rises suddenly from the value given by Eq. (19) to

$$F_{s,\text{tot}}(t^*_{\text{DIS}}) = g\bar{m}H[3(1-\varepsilon)+\psi_{\text{IUS}}\varepsilon] \quad (21)$$

Comparing this result with Eq. (19) shows that the jump in $F_{s,\text{tot}}(t)$ is equal to $g\bar{m}H\psi_{\text{IUS}}\varepsilon$, the value of the activated mean crushing force at the bottom of the IUS.

Depending on the value of ψ_{IUS}, the collapse of the IUS is total or nontotal. Substituting the height $h = \varepsilon H$ and initial velocity $\dot{z}_0 = \sqrt{2gH(1-\kappa)(1-\varepsilon)}$ of the IUS in the expression for $\bar{\psi}$ defined in Eq. (10) yields

$$\bar{\psi}_{\text{IUS}} = \frac{1}{\varepsilon} \quad (22)$$

For $\psi_{\text{IUS}} \leq (1/\varepsilon)$, the collapse of the IUS is total, and the collapse duration t^*_{IUS}, equal to the duration of the second phase of collapse, is calculated from Eq. (15) for $\psi \leq \bar{\psi}$ as follows:

$$t^*_{\text{IUS},\psi_{\text{IUS}}\leq\frac{1}{\varepsilon}} = \sqrt{\frac{2H(1-\kappa)}{g}} \times \frac{\sqrt{1-\varepsilon}-\sqrt{1-\psi_{\text{IUS}}\varepsilon}}{\psi_{\text{IUS}}-1} \quad (23)$$

With this result, the total collapse duration for $\psi_{\text{IUS}} \leq (1/\varepsilon)$, equal to the sum of t^*_{DIS} and $t^*_{\text{IUS},\psi_{\text{IUS}}\leq(1/\varepsilon)}$, is

$$t^*_{\text{tot},\psi_{\text{IUS}}\leq\frac{1}{\varepsilon}} = \sqrt{\frac{2H(1-\kappa)}{g}} \times \frac{\psi_{\text{IUS}}\sqrt{1-\varepsilon}-\sqrt{1-\psi_{\text{IUS}}\varepsilon}}{\psi_{\text{IUS}}-1} \quad (24)$$

The value reached by $F_{s,\text{tot}}(t)$ at the end of a total collapse can now be calculated using Eqs. (20) and (24) as follows:

$$F_{s,\text{tot}}(t^*_{\text{tot},\psi_{\text{IUS}}\leq\frac{1}{\varepsilon}}) = g\bar{m}H(3-2\psi_{\text{IUS}}\varepsilon) \quad (25)$$

Now consider the case of nontotal collapse of the IUS, occurring when $\psi_{\text{IUS}} > (1/\varepsilon)$. For this case, the collapse duration t^*_{IUS} is calculated using Eq. (15) for $\psi > \bar{\psi}$ as follows:

$$t^*_{\text{IUS},\psi_{\text{IUS}}>\frac{1}{\varepsilon}} = \sqrt{\frac{2H(1-\kappa)}{g}} \times \frac{\sqrt{1-\varepsilon}}{\psi_{\text{IUS}}-1} \quad (26)$$

The total collapse duration for $\psi_{\text{IUS}} > (1/\varepsilon)$, equal to the sum of t^*_{DIS} and $t^*_{\text{IUS},\psi_{\text{IUS}}>(1/\varepsilon)}$, is now

$$t^*_{\text{tot},\psi_{\text{IUS}}>(1/\varepsilon)} = \sqrt{\frac{2H(1-\kappa)}{g}} \times \frac{\psi_{\text{IUS}}\sqrt{1-\varepsilon}}{\psi_{\text{IUS}}-1} \quad (27)$$

The value reached by $F_{s,\text{tot}}(t)$ at the end of a nontotal collapse is calculated using Eqs. (20) and (27) as follows:

$$F_{s,\text{tot}}\left(t^*_{\text{tot},\psi_{\text{IUS}}>(1/\varepsilon)}\right) = g\bar{m}H\psi_{\text{IUS}}\varepsilon \quad (28)$$

Finally, using Eq. (16), the height of the part of the IUS remaining intact after the collapse comes to an end is obtained as follows:

$$h_{\text{IUS,rest}} = H_{\text{rest}} = \begin{cases} 0, & \psi_{\text{IUS}} \leq \frac{1}{\varepsilon} \\ H\frac{\psi_{\text{IUS}}\varepsilon-1}{\psi_{\text{IUS}}-1}, & \psi_{\text{IUS}} > \frac{1}{\varepsilon} \end{cases} \quad (29)$$

Summary of the Results for the First and Second Phases of Collapse

The results obtained in the preceding section are summarized in Fig. 5, which shows the function $F_{s,\text{tot}}(t)$ in normalized form. $F_{s,\text{tot}}(t)$ is only shown for the time of the collapse; before and after that, $F_{s,\text{tot}}(t)$ is a constant function, equal to $g\bar{m}H$, the total weight of the DIS and IUS.

Most important for us is the maximum of the force $F_{s,\text{tot}}$, which we designate as $F_{s,\text{tot,max}}$. Depending on the value of ψ_{IUS}, $F_{s,\text{tot,max}}$ is reached either at the beginning or at the end of the second collapse phase. The results for $F_{s,\text{tot,max}}$ can be summarized as follows (Fig. 6):

$$\frac{F_{s,\text{tot,max}}}{g\bar{m}H} = \begin{cases} 3-2\psi_{\text{IUS}}\varepsilon, & \psi_{\text{IUS}} < 1 \\ 3(1-\varepsilon)+\psi_{\text{IUS}}\varepsilon, & \psi_{\text{IUS}} \geq 1 \cup \varepsilon < 1 \\ 1, & \psi_{\text{IUS}} > 1 \cup \varepsilon = 1 \end{cases} \quad (30)$$

No collapse occurs in the special case where $\psi_{\text{IUS}} > 1$ and $\varepsilon = 1$; hence, $F_{s,\text{tot,max}} = g\bar{m}H$ in this case. Another interesting, if only theoretical, special case that is also covered by the results in Eq. (30) and Fig. 6 is the case where the IUS behaves as a rigid body. In this case, $\psi_{\text{IUS}} \to \infty$, which causes $F_{s,\text{tot,max}}$ to go toward infinity.

Eq. (26) further shows that the IUS collapse time $t^*_{\text{IUS},\psi_{\text{IUS}}>(1/\varepsilon)} \to 0$ for $\psi_{\text{IUS}} \to \infty$. Hence, for $\psi_{\text{IUS}} \to \infty$ the force exerted by the IUS during the second phase of collapse is described by a Dirac impulse, whose integral can be obtained using Eqs. (20) and (26) to the product of the velocity gained by the IUS during the first phase of collapse and the mass of the IUS equal to $\sqrt{2gH(1-\kappa)(1-\varepsilon)}$ and $\varepsilon\bar{m}H$, respectively. All this is consistent with what we know about the impact of a rigid body against a fixed rigid surface.

With the results just obtained, we return to the key question posed a while ago, here repeated in a more concise form.

Could the ILS in the WTC Twin Towers Have Been Saved If It Had Been Topped by a Strong Floor Plate?

Consider first the following conclusion reached by FEMA (2002, pp. 32–33) after an approximate structural analysis of WTC 2 for the state immediately preceding the aircraft impact:

> At the 80th floor level, exterior columns were found to be approximately uniformly loaded with an average utilization ratio (ratio of actual applied stress to ultimate stress) of under 20 percent. This low utilization ratio is due in part to the unusually close spacing of the columns in this building, which resulted in a very small tributary area for each column. It reflects the fact that wind and deflection considerations were dominant factors in the design. Core columns were more heavily loaded with average calculated utilization ratios of 60 percent, which would be anticipated for these columns, which were designed to resist only gravity loads.

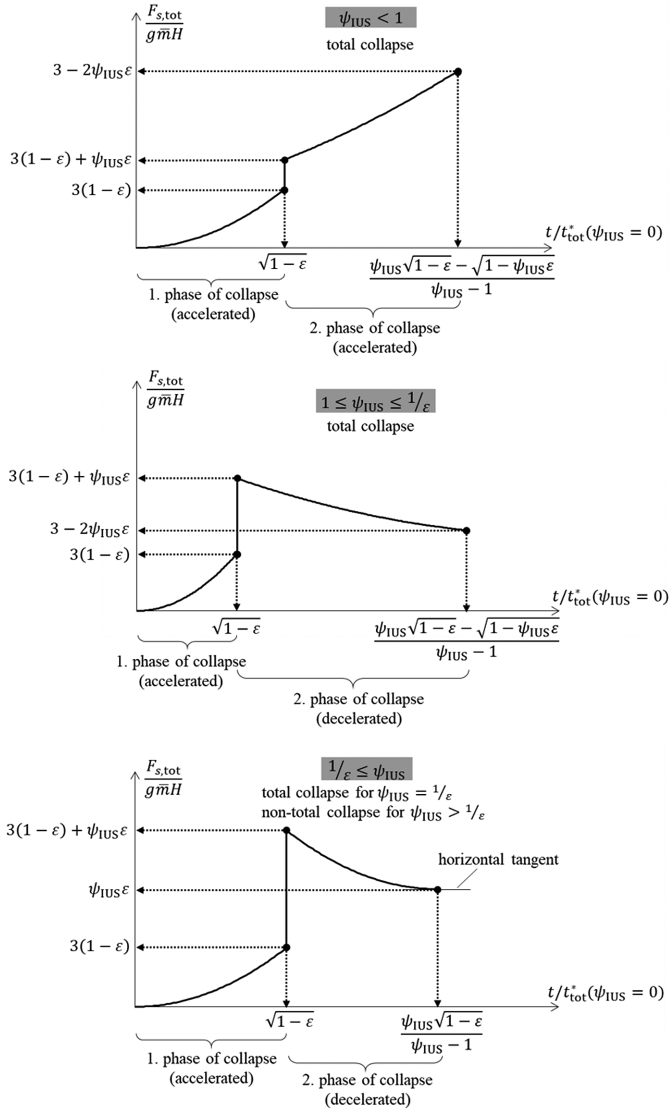

Fig. 5. Summary of the results for the function $F_{s,\text{tot}}(t)$.

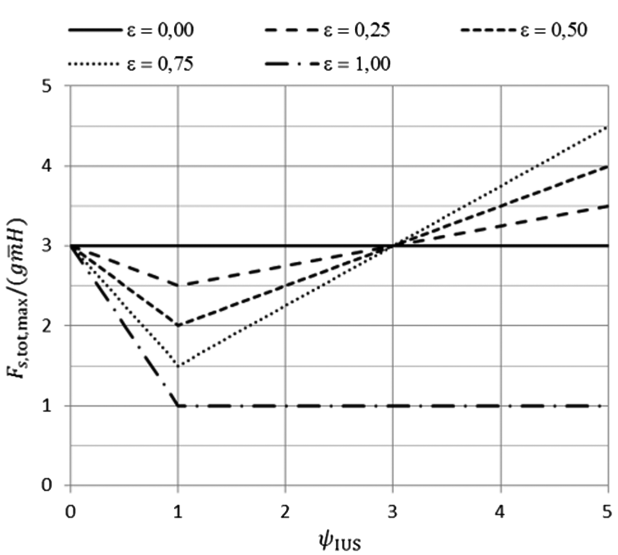

Fig. 6. Normalized force $F_{s,\text{tot,max}}$ as function of ψ_{IUS} for several discrete values of ε.

Now consider also that according to FEMA (2002, p. 20), the core- and perimeter columns in the WTC Twin Towers carried, respectively, 60% and 40% of the gravity loads. Based on these two pieces of information, we can conclude that core columns at the 80th floor level in WTC 2, utilized to 60%, were able to take a force of $(1/0.6) \times 0.6W = W$, whereas the perimeter columns there, utilized to 20%, were able to take $(1/0.2) \times 0.4W = 2W$, or in total $3W$, where W is the weight of the structure above the considered columns.

Furthermore, the combined mean crushing force F_{mean} of the considered columns was smaller—in fact significantly smaller—than their combined axial capacity F_c of $3W$. This can be understood by returning to the discussion related to Fig. 3 and remembering the mention made there on the smallness of the ratio F^*_{mean}/F^*_c for a single crushed column; this point applies equally to the ratio of the respective combined properties F_{mean} and F_c of a column group, where $F_{\text{mean}} = nF^*_{\text{mean}}$ and $F_c = nF^*_c$, with n being the number of columns in the group.

Now, because both the bottom of the IUS and the top of the ILS in WTC 2 were close to the 80th floor level, whose columns we have just considered, we can establish the following: (1) the combined mean crushing force of the columns at the bottom of the IUS in WTC 2 was smaller than 3 times the weight of the IUS, which means that $\psi_{\text{IUS}} < 3$; and (2) the provided combined axial capacity of the columns at the top of the ILS in WTC 2 was about $3g\bar{m}H$ (where $g\bar{m}H$ is the weight of the DIS and IUS). Now consider the results in Fig. 6, representing the capacity demand on the columns at the top of the ILS. Without entering a debate on the precise value of ψ_{IUS} in WTC 2, we can merely note that, as just established, this value was smaller than 3, which according to Fig. 6 means a force $F_{s,\text{tot,max}}$ smaller than $3g\bar{m}H$. Similar relationships can be expected for the other tower, WTC 1.

It must be acknowledged that the aforementioned results for the provided capacity and the capacity demand of the columns at the top of the ILS are both based on certain approximate assumptions, which is why it cannot be claimed for certain that strong floor plates topping the ILS would have prevented downward collapse progression in the WTC Twin Towers. The claim that we can make, however, is already remarkable enough: the capacity demand that would have been posed on the aforementioned columns, had they only been given a chance to properly respond, would have been similar to their provided capacity, and hence the situation of these columns would have been far from hopeless.

Yet, on the other hand, "hopeless" is the adjective first brought to mind by the result of Bažant and Zhou (2002), who obtain a force of about 31 times the weight of the IUS (which those authors called the upper part) acting on the ILS (i.e., the lower part) on initial impact. Now, where is the root of this staggering discrepancy of one order of magnitude between the result of Bažant and Zhou (2002) and the results in Fig. 6? How did Bažant and Zhou (2002) and later Bažant and Verdure (2007), who considered a C-F-driven crush-down, come to the conclusion that the initiation of such crush-down in the WTC Twin Towers was inevitable once the IUS was set in motion? A detailed answer to this question has been provided by Lalkovski (2021).

A concise formulation of this answer is the following. The factor of 31 derived by Bažant and Zhou (2002) is based on an assumption that they leave unjustified: the assumption that the IUS behaves as a rigid body as it impacts the ILS. In a later paper by Bažant et al. (2008), it is acknowledged that the IUS does not actually behave as a rigid body on impact with the ILS; yet Bažant et al. (2008) still insisted that the initiation of C-F-driven crush-down in the WTC Twin Towers was inevitable once the IUS was set in motion and that treating the IUS as a rigid body is generally correct.

The attempt of Bažant et al. (2008) to "rigorously justify" this claim is based on the following ideal assumptions. Firstly, it is assumed that on impact with the ILS all the IUS columns are activated simultaneously, so that when these columns begin to get crushed in the lowermost story of the IUS, they simultaneously reach their respective maxima F_c^*, making it possible for the combined force of these columns to reach the sum of these maxima.

Secondly, Bažant et al. (2008) assumed that the downward-moving lowermost floor plate of the IUS and the initially stationary topmost floor plate of the ILS are both perfectly even, horizontal, and rigid. Due to this second assumption, when the two said floor plates meet, contact is developed instantaneously in a perfectly plastic impact, the result of which is a downward velocity imposed at the top of the ILS. This imposed velocity, resulting in yielding, and thus crushing, of the columns in the topmost story of the ILS, is what initiates the C-F-driven crush-down that Bažant et al. (2008) envisioned. The situation of these now crushed ILS columns is made still worse by the downward force coming from the IUS columns above—a force that, as mentioned, is allowed to reach the sum of the axial capacities of the IUS columns. This is what allowed Bažant et al. (2008) to conclude that although crushing of the columns in the lowermost IUS story actually also occurs after initial contact with the ILS, simultaneously with the initiated C-F-driven crush-down, this IUS column crushing is bound to last only a short time, after which only the C-F-driven crush-down remains. As the C-F-driven crush-down continues, the IUS remains elastic, and can thus be treated as a rigid body.

Now, the predictions of a model like that of Bažant et al. (2008) can only be of practical value if it can be shown that practically inevitable deviations from the assumed ideal conditions have no significant effect on the results. Precisely the opposite is the case for said model, as can be easily observed. For instance, if the IUS is tilted by only a few degrees, its lowermost floor plate, also tilted, will develop contact with the horizontal topmost floor plate of the ILS over an extended period of time, rather than instantaneously, and hence there will be no imposed velocity at the top of the ILS as in the model of Bažant et al. (2008). The IUS tilt will furthermore have consequences for the force developed by the crushed IUS columns, which, as explained in Fig. 3(b), will not be able to reach the sum of the axial capacities but will remain significantly below that. This strong dependence on unrealistic assumptions makes the model of Bažant et al. (2008) unfit for its intended purpose, at any rate when it comes to real buildings. The conditions for the initiation of C-F-driven crush-down can only be adequately established by a model that accounts for the practically inevitable imperfections left aside by Bažant et al. (2008) and that recognizes the necessity of placing a strong floor plate on top of the ILS before considering the possibility of C-F-driven crush-down.

A number of other questions, related to the results of this paper, can now also be asked. Some of these questions are as follows:

- How would a mean crushing force that does not decrease to zero at the top of the IUS affect the time history and the maximum of $F_{s,\mathrm{tot}}(t)$?
- Is it possible for the IUS to disintegrate during its crush-up collapse (for instance, due to shearing off of the floor plates at their connections to the columns), and, if yes, how would this affect the time history and the maximum of $F_{s,\mathrm{tot}}(t)$?
- How would a possible eccentricity of $F_{s,\mathrm{tot}}(t)$ relative to the cross-section centroid of the ILS viewed as ground-clamped cantilever (with the ILS columns acting as the fibers in this cantilever's cross section) affect the capacity demand on the ILS columns?
- How would this demand be affected by the dynamic response of the ILS to the time-varying force $F_{s,\mathrm{tot}}(t)$ acting at its top?
- What about the effect of a possible large tilt of the IUS—so large that the IUS columns are impacted obliquely and thus not actually crushed when the IUS meets resistance from below?
- Given that most of the mass of the DIS and IUS is contained in the floor plates of these sections—floor plates that can be viewed as discrete masses placed equidistantly along the height—how can we justify the smearing of these discrete masses to obtain the continuous mass distribution assumed for the 1D tower in Fig. 4?

These and a number of other questions that emerge after delving deeper into the matter are also addressed by Lalkovski (2021). Although the effects addressed by these questions are generally not negligible, the aforemade claim about the similarity between provided capacity and capacity demand of the columns at the top of the ILS does not change.

Proposed Design Concept

As stated in the "Introduction," the objective of this paper is to follow one of the recommendations from the World Trade Center building performance study released by FEMA in 2002, and to "determine, given the great size and weight of the two towers, whether there are feasible design and construction features that would permit such buildings to arrest or limit a collapse, once it began" (FEMA 2002, p. 40). The aforementioned findings show that there is in fact one such feature: a strong floor plate topping the ILS. However, it is still an open question of how this feature can be successfully incorporated into the design of a building of the Twin Towers' type, especially noting that only a fraction of the building's floor plates (say, only every 10th or 20th plate) can be realized as strong, whereas column loss according to Fig. 1 can occur anywhere. In other words, how can we ensure, given the limited number of strong floor plates that a building can practically contain, that no matter where initial column loss occurs, the ILS is always topped by a strong, rather than an ordinary, floor plate?

To appreciate the problem that can emerge if no answer is found to this question, consider a building in which every 15th floor plate is designed strong and assume that collapse is initiated in this building due to the loss of all columns in a story immediately below one of the strong floor plates. The DIS in this example is thus only one story high, and the topmost 14 floor plates of the ILS are ordinary, which makes the initiation of F-P-F-driven crush-down inevitable. When the crush-down front reaches the next lower strong floor plate in the ILS, this floor plate gets impacted by a considerable compact mass (the collected mass of the 14 ILS stories with ordinary floor plates) moving at a considerable velocity. This impacting compact mass (which, being compact, behaves as a rigid body) can be shown to pose a serious capacity demand on the ILS columns below the impacted strong floor plate—a demand even higher than what Bažant and Zhou (2002), treating the IUS as a rigid body, determined for the columns of the impacted ILS. Clearly, a way must be found to prevent the problematic development of a compact mass in a case as just considered. A design concept achieving this is presented in Fig. 7.

This concept ensures that in the event of the loss of all columns somewhere between two strong floor plates, or at the level of a strong floor plate, the entire building mass below the lost columns down to the next lower strong floor plate is immediately and fully released from its structural bounds and thus made to behave as part of the DIS; the de facto DIS—that is, the building section whose columns' supporting function to the attached floor plates is eliminated, thus initiating collapse—is thus extended down to the next lower strong floor plate. The topmost floor plate of the ILS is thus guaranteed to be a strong one, according to the assumption underlying the results in Figs. 5 and 6. It should be clear, however, that what has just been said

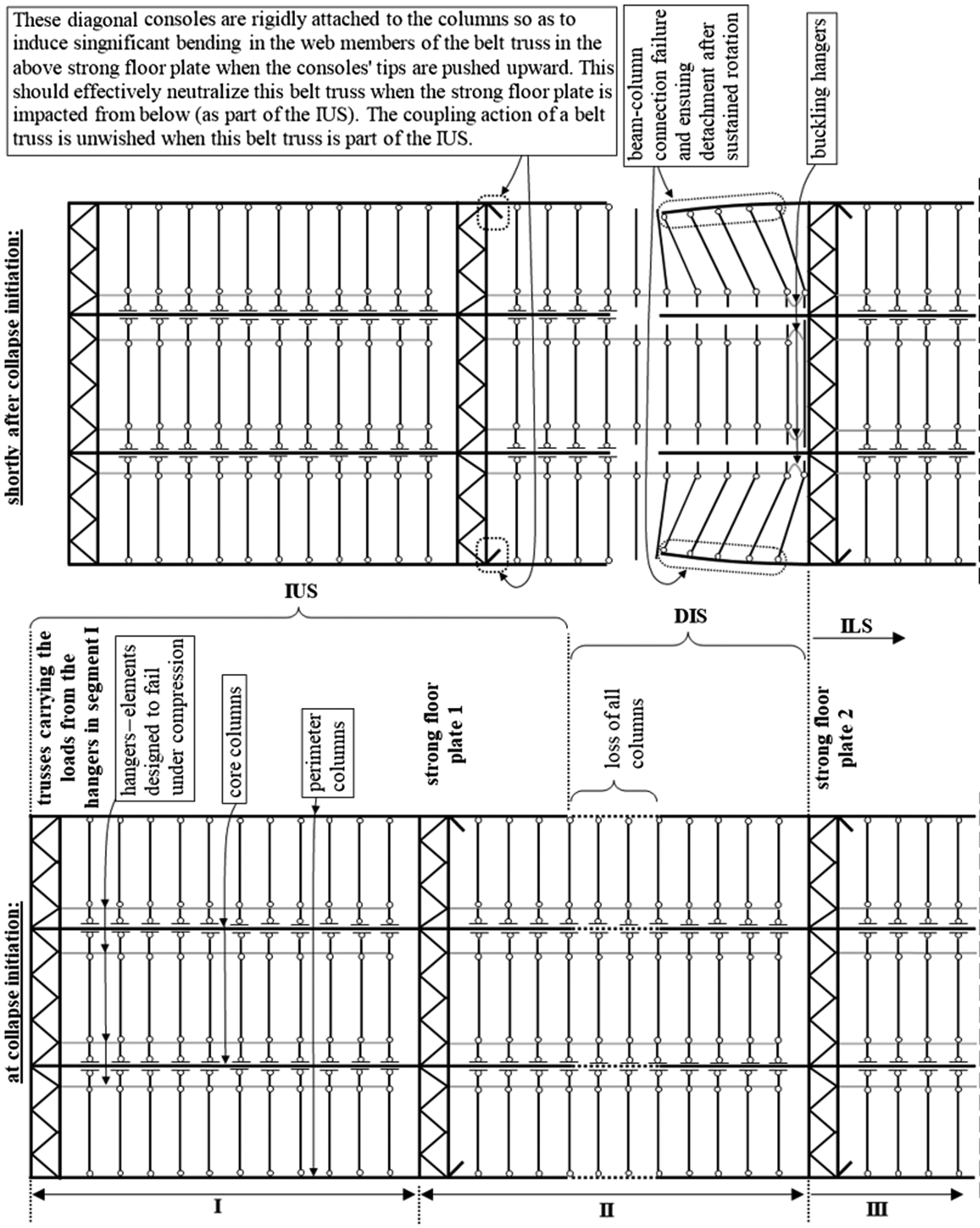

Fig. 7. Proposed design concept ensuring that the ILS is always topped by a strong floor plate.

applies as long as initial column loss occurs above the lowermost strong floor plate in the building; if, on the other hand, it occurs at or below the level of this strong floor plate, everything above and below the lost columns—that is, the entire building—will be lost.

The effect of the longer vertical load paths coming along with the proposed use of hangers, as well as certain practical aspects like the realization of the apparently exotic connections in Fig. 7, or the most proper choice of the spacing between the strong floor plates, which should be neither too small nor too large, have been discussed by Lalkovski (2021).

Conclusions

Two are the main questions that can be asked from an engineering standpoint with regard to the Twin Towers' collapse:

- How did the intact building section above the aircraft impact zone—the intact upper section (IUS)—begin its descent?
- Once the IUS was set in motion, how did it manage to destroy the entire intact lower section (ILS) of the building?

This paper provides an answer to the second question, as well as to the more important question that naturally follows: how could the ILS have been saved? Making the distinction between two possible and mutually exclusive types of downward collapse progression, referred to as column-failure-driven and floor-plate-failure-driven, the paper has shown that the total collapse of the ILS in the WTC Twin Towers was clearly the result of F-P-F-driven downward collapse progression. Based on this insight, showing that the ILS columns were simply bypassed, the paper went on to consider the question whether these columns, and with them the ILS, could have survived if they had been topped by a floor plate strong enough to fully mobilize them—a plate referred to as a strong floor plate.

Assuming a strong floor plate at the top of the ILS, the time history of the force exerted on this plate by the mass collapsing from above was then determined. It was shown that the maximum of this force, and thus the capacity demand on the columns at the top of the ILS, lies at about three times the total weight of the collapsing mass. Based on the findings of FEMA (2002), it turns out that the combined axial capacity of said columns in the Twin Towers' case was also about three times the weight of the above lying mass. This shows that these columns, and with them the entire ILS, could potentially have survived in the presence of a strong floor plate designed accordingly. Based on this result, a design concept is finally presented that incorporates a strong floor plate every approximately 15 stories and that can ensure that irrespective of where initial column loss occurs—as long as it occurs above the lowermost strong floor plate in the building—the ILS is always topped by a strong floor plate.

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article.

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