536 | November 1993

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On a Consistent Linearized Theory of the Wave-making Resistance of Ships

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Abstract

There are few discussions on the uniqueness in the theory of the wave-making resistance of ships. Moreover, a line integral term,singularity distribution around a periphery of the water plane area, appearing in the theory casts a shadow on the uniqueness of the boundary value problem.

There is the only one well-known consistent theory, that is, two-dimentional theory of planing on the water surface in which a line integral term does not appear explicitly. In this theory, the sinkage and trim vary with speed and also the wetted length changes to fulfill Kutta's condition. However,in a usual ship,displacement ship, having a nearly vertical stem, the wetted length could not vary as in the planing ship. In the present paper, introducing a new singularity just before the bow, we try to obtain a consistent linearized theory for a displacement ship. We solve numerically the boundary value problem,investigating the properties of solutions and then calculate the sinkage and trim when a barge is running freely or is being towed without any external force or moment except a towing force.

Then, it is found that this free running becomes unstable over the speed Fr.=.61 regardless of the bottom shape. The resistance consists of three components,namely, the wave-making, the spray and the water head resistance. The former two components are well-known and the last one is a component introduced and named so here temporarily. This component resembles a wave-breaking resistance but we have no direct explanation.

'A la verité, et ne crains points de l'advouer, je porterois facilement,au besoing, une chandelle à Saint Michel, l'autre à son serpent.[†] *Michel de Montaigne,1533-1592*

1. Introduction

Many researchers have studied the theory of the wave-making resistance of ships nearly for the past one century. Such studies now enables us to design ship forms with the least wave-making resistance on the theoretical basis and the usefulness of the theory has been well recognized(Wehausen 1973). However,we have not yet verified the uniqueness concerning the boundary value problem. Moreover the existence of the so-called line integral term casts a shadow on the uniqueness of such solution(Bessho 1959,1976b,Brard 1972,Wehausen 1973). It seems that this question relates closely to the behaviour

[†]This is from Montaigne's essay *De l'utile et de l'honnête*(Of usefulness and honesty) and may be freely translated as 'In truth, and I am not afraid to admit it,I would, in need, light a candle to Saint Michael and another to his dragon'. (Murray,J.D.'Asymptotic analysis'Preface,Clarendon Press,Oxford,1974.)

of the water along the crossline(or the cross points in the two dimensional case)between the water and the ship side wall.

The water surface is an open boundary mathematically, so that there may exist some arbitrariness. Let us consider this situation in Fig. 1.

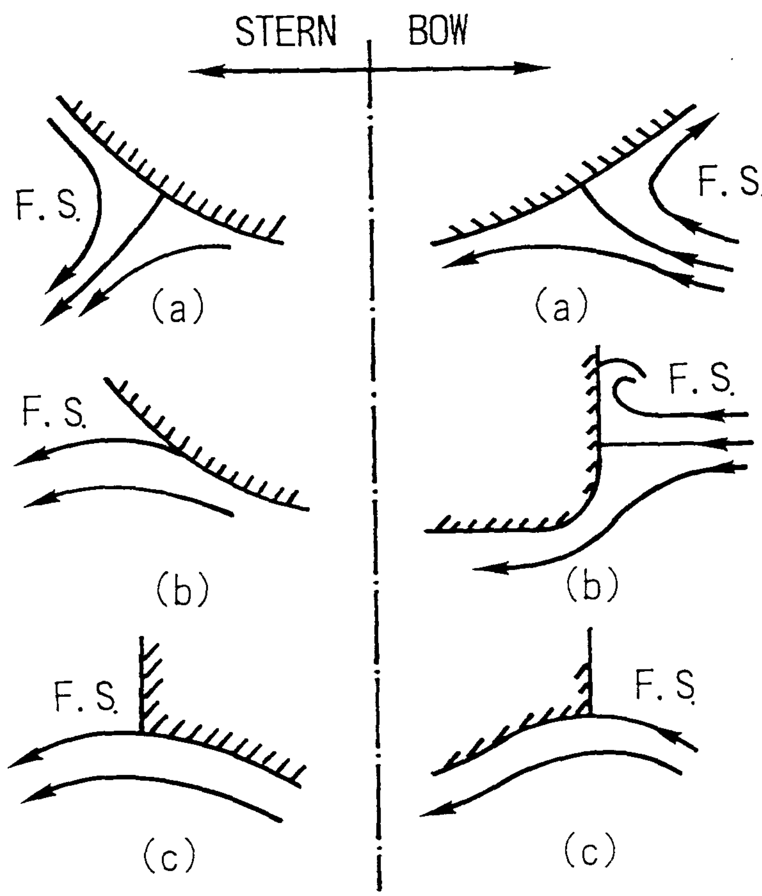

Fig. 1 Flow patern at the bow and stern

At first,when a ship is planing,the flow near the bow will become case (a) but when the stem of a ship is nearly vertical, then the flow around there will become case(b). The case (c) is a splash-free case found by Maruo 1949, and this is nearly the same flow as the shock-free entrance of an aerofoil. On the other hand,the flow (a) at the stern corresponding (a) or (b) at the bow can not be realized at first sight because the flow above the branch stream line can not be supplied from anywhere and the pressure at the stagnation point must rise very high and the flow must separate and the phenomenon can not be treated analytically. Therefore, the flow to be considered practically will be the cases (b) and (c) at the stern. The flow at the stern in the case (c) is flowing out smoothly from the bottom to the free surface and the separation condition will be Kutta's condition in the wing theory. The flow in the case (b) is known in the theory of cavity flow from the curved surface and its separation condition is the well-known Brillouin and Villat's(Birkoff et al. 1957). This condition is neither simple nor linear. However, in the linearized theory, the separation point is determined beforehand so that the separation condition might be substituted by Kutta's condition. In fact, a planing craft has flow patterns (a) at the bow and (c) at the stern. On the other hand, a displacement ship has flow patterns (b) at the bow and (b) or (c) at the stern, and clearly the case (c) at the stern represents the flow of a transom stern or destroyer stern. Now, in the theory of planing craft, we are making use of Kutta's condition at the stern and have a consistent theory, but practically its trim and wetted length at a given speed must vary to support the given displacement weight and balance with the center of gravity(Bessho 1970,1977b,Suzuki et al. 1986a,Bessho et al. 1992a). This is, of course, true of the planing craft but not the so-called displacement ship. Namely, the water plane area of the displacement type ship does not so change because they have almost vertical side wall. For such ships, this flowing-out condition could not be satisfied in the linearized theory.

The present report is a proposal to a new aproach avoiding this difficulty by introducing a source singularity at the bow(around the forward waterplane in the three dimensional case)known as a line integral term (Suzuki 1981,1985). We confine our discussion to the two dimensional case and a shallow draft ship represented by a surface pressure distribution,because we have few results in the three dimentional cases and the theory of the shallow draft ship is very simple and gives many fruitful and useful relations as shown in the following.

- Nomenclature -

Item	Dimensional	Dimensionless
Co-ordinates	x', y'	$x = 2x'/L, y = 2y'/L$
Ship length	L	2
Uniform Velocity	V	1
Density of Water	ρ	1
Gravity Constant	g	$K = gL/(2V^2)$
Froude Number	$F_r = V/\sqrt{gL}$	$F_r = 1/\sqrt{2K}$
Velocity Potential	ϕ'	$\phi = 2\phi'/(VL)$
Stream Function	ψ'	$\psi = 2\psi'/(VL)$
Velocity	u', v'	$u = u'/V = -\phi_x, v = v'/V = -\phi_y$
Surface elevation	η'	$\eta = 2\eta'/L$
Pressure	p'	$p = p'/(\rho V^2)$
Displacement Area	∇'	$\nabla = 4\nabla'/L^2$
Center of buoyancy	l'	$l = 2l'/L$
Draft	T'	$T = 2T'/L$
Camber	c'	$c = 2c'/L$
Rise(Negative Sinkage)	h'	$h = 2h'/L$
Trim	t'	$t = t'$
Lift	L'	$L = 2L'/(\rho V^2 L)$
Moment	M'	$M = 4M'/(\rho V^2 L^2)$
Wave-making,Spray Head and Total resistance	D'_W, D'_S, D'_H, D'_T	$D_W, D_S, D_H, D_T = 2D'/(\rho V^2 L)$
Resistance Quotient	$r = D'/(\rho g \nabla'^2/L^2)$	$r = 4D/(K\nabla^2)$

2. Formulation of the problem(Bessho 1991)

A ship advances with the speed V on the water surface and the co-ordinate system,and the unit system are shown in Fig.2 and the Nomenclature respectively.

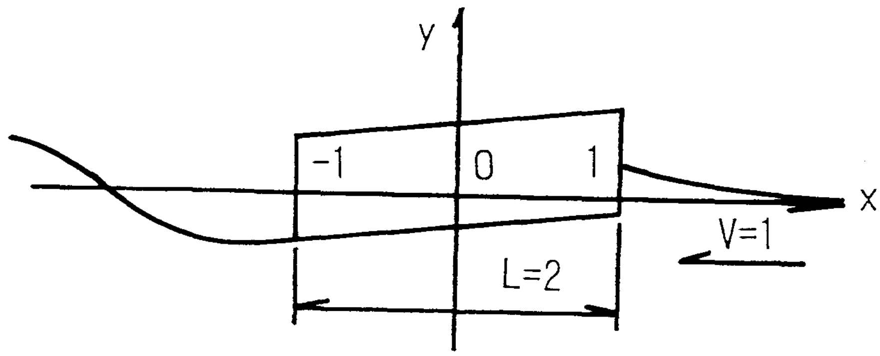

Fig. 2 CO-ordinate system

Let us define the complex potential as follows;

$$f(z) = \phi(x, y) + i\psi(x, y), \tag{1}$$

Then the linearized water surface condition becomes ;

$$\phi_x(x, 0) - K\psi(x, 0) = 0 \text{ for } |x| > 1, \ -p(x) \text{ for } |x| \leq 1, \tag{2}$$

where p(x) denotes the pressure on the ship bottom. The linearized surface elevation $\eta(x)$ will be

$$\eta(x) = -\psi(x, 0), \tag{3}$$

The usual boundary condition on the bottom surface of the ship becomes

$$\eta_x(x) = -\psi_x(x, 0) = \phi_y(x, 0), \text{ for } |x| \leq 1. \tag{4}$$

In the wing theory, this condition is sufficient to determine uniquely the solution with Kutta's condition,but it is not our case because the stream line of the free surface must go along the ship bottom. In other words, the equation (4) determines the inclination of the off-sets of the ship bottom but does not the sinkage. Hence, we introduce here a "swell-up potential", temporarily we call it so,which is a source singularity just before the bow and a so-called line integral term. This potential produces a logarithmically infinite vertical speed but a finite one horizontally. Its surface elevation $S^*(x, 0)$ is shown in Fig.3 and it has no jump at the origin, but the function S has a jump. This differs from the usual spray explained by Wagner and Maruo (Maruo 1949) which has an infinite horizontal speed but finite vertically. Making use of the well known kernel defined in the Appendix A, we obtain the velocity potential and stream function in the following forms;

$$f(z) = \int_{-1}^{1} p(\xi) W(z - \xi) d\xi + AW^*(z - 1), \tag{5}$$

$$\phi(x, y) = \int_{-1}^{1} p(\xi) S(x - \xi, y) d\xi + AT^*(x - 1, y), \tag{6}$$

$$\psi(x,y) = \int_{-1}^{1} p(\xi)T(x-\xi,y)d\xi - AS^*(x-1,y), \tag{7}$$

and the usual boundary integral equation without the swell-up potential term becomes for the inclination of the bottom off-set of a given ship;

$$\eta_x(x) = \phi_y(x,0) = \frac{1}{\pi}\int_{-1}^{1}\frac{p(\xi)d\xi}{\xi - x} + K\int_{-1}^{1} p(\xi)S(x-\xi,0)d\xi, \tag{8}$$

This is a well known singular integral equation and has a unique solution except a homogeneous

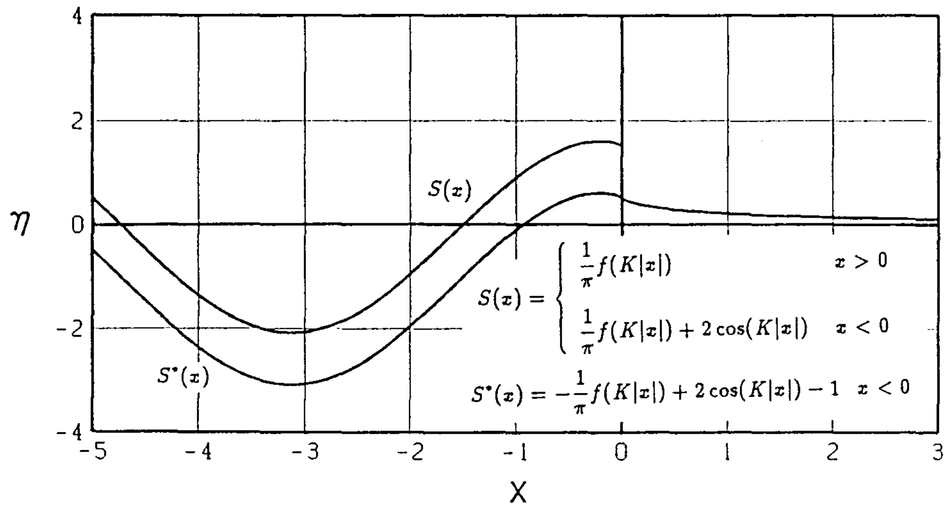

Fig. 3 Surface elevation of swell-up potential

solution which is determined by Kutta's condition. However, as described above, the sinkage or vertical position of the ship can not be designated arbitrarily. Inversely, if the sinkage is designated beforehand, Kutta's condition can not be satisfied. This is the reason why we introduce the swell-up potential so that we may designate the sinkage arbitrarily and also Kutta's condition may be satisfied.

In the far field, the surface elevation becomes asymptotically ;

$$\left.\begin{aligned} \eta(x) &\longrightarrow \frac{A}{\pi K x} && ,\text{for } x \gg 1 \\ &\longrightarrow -A + 2\mathrm{Im}[F(K)e^{-iKx}] && ,\text{for } x \ll -1 \end{aligned}\right\} \tag{9}$$

where

$$F(K) = H(K) - iAe^{iK}, \tag{10}$$

$$H(K) = \int_{-1}^{1} p(x)e^{iKx}dx, \tag{11}$$

That is, the water level at far down stream falls down by A in the mean. Therefore, the flux there decreases by A but this is cancelled by the flux flowing from the free surface,namely,

$$-\int_{-\infty}^{\infty} v(x,0)dx = \int_{-\infty}^{\infty} \eta_x(x)dx = [\eta(x)]_{-\infty}^{\infty} = A \tag{12}$$

Therefore, the mean horizontal velocity must increase. In fact, when we consider the shallow water case, we can find the corresponding increment of the velocity. In actual cases, the mean water level rises higher than the up-stream's. Making use of this swell-up potential, we may solve the following boundary integral equation to obtain the solution which satisfies Kutta's condition and a given surface elevation.

$$\eta(x) = -\int_{-1}^{1} p(\xi)T(x-\xi)d\xi + AS^{*}(x-1,0), \tag{13}$$

$$\text{with} \qquad p(-1) = 0 \tag{14}$$

The kernel T has a logarithmic singularity so that this may have a unique solution.The boundary condition may be written as follows;

$$\eta(x) = -f(x) + tx + h, \qquad \text{for } |x| < 1 \tag{15}$$

where f(x) is the off-set of the ship's bottom surface at still water, t trim by the stern and h the rise or negative sinkage. The trim and rise, of course, are determined after the problem is solved. The lift and moment are calculated as follows;

$$\left.\begin{matrix} L \\ M \end{matrix}\right\} = \int_{-1}^{1} p(x) \left\{\begin{matrix} 1 \\ x \end{matrix}\right\} dx \tag{16}$$

and the displacement area and the center of buoyancy are

$$\nabla = \int_{-1}^{1} f(x)dx, \tag{17}$$

$$l = \frac{1}{\nabla}\int_{-1}^{1} f(x)xdx. \tag{18}$$

The resistance could be calculated by usual pressure integration over the bottom , but there must be some questions for the contribution of the swell-up potential. However, since it is also represented by a constant pressure distribution KA from the bow to the infinite down stream, the resistance due to the swell-up potential may be calculated as follows;

$$KA\int_{-\infty}^{1} \eta_x(x)dx = KA\{\eta(1) + A\}, \tag{19}$$

Then, the total resistance becomes as follows;

$$D = \int_{-\infty}^{1} p(x)\eta_x(x)dx + KA\{\eta(1) + A\}, \tag{20}$$

3. Reciprocity Theorems(Hanaoka 1959,Bessho 1977a,1991)

At first,the boundary integral equation (13) is solved numerically for the conditions and items shown in Table 1 , putting $x = -\cos\theta$ and dividing the interval of θ 0 to π into an appropriate number of elements. Solutions with star mark do not satisfy Kutta's condition at the stern. Now, before next discussions, we should like to see mutual relations between such various solutions. It is well known that we have two reverse flow theorems by Hanaoka extended from Flax's principle in the wing theory to our case (Flax 1952,Hanaoka 1959). These theorems may be applied in the present case too. In deducing such theorems, they made use of the idea of reverse flow which is a flow around the ship when the direction of the uniform velocity is reversed. However, for simplicity sake, we show the theorems extended in the present case as follows, reversing the sign of x co-ordinate instead of introducing their idea. The first theorem of Hanaoka may be extended for two solutions p and p′ and corresponding inclination of the surface off-sets η_x and η'_x.

$$\int_{-1}^{1} p(-x)\eta'_x(x)dx + KA'\eta(-1) = \int_{-1}^{1} p'(-x)\eta_x(x)dx + KA\eta'(-1), \tag{21}$$

because A-term is the same as a constant pressure distribution from the bow to the infinite down stream. This theorem holds only when the pressure satisfies Kutta's condition at the trailing edge. His second theorem holds for the solution not satisfying Kutta's condition and it becomes;

$$\int_{-1}^{1} p^*(-x)\eta'^*(x)dx = \int_{-1}^{1} p'^*(-x)\eta^*(x)dx, \tag{22}$$

and this is not much useful for the present case because this formula containing A-term in general form, including A-term, includes the potential values at trailig edge instead of the surface elevation in the first theorem, and,of course, the former can not be known until after solving the problem but the latter is given before.

Table 1 Basic solutions

η_x	η	Suffix	Kutta's Condition	A	Remarks
0	1	h	Yes	Yes	Rise or of a flat plate
0	1	h*	No	No	The same as the above
1	x	t	Yes	Yes	An inclining flat plate
1	x	t*	No	No	The same as the above
x	$\frac{x^2-1}{2}$	c	Yes	Yes	The plate with negative camber
x	$\frac{x^2-1}{2}$	c*	No	No	The same as the above
$-\frac{\sin n\theta}{\sin\theta}$	$\frac{\cos n\theta}{n}$	s	Yes	Yes	Spray solution for n infinity
0	0	s*	No	1	Homogeneous solution
e^{-iKx}	$\frac{i}{K}e^{-iKx}$	d	Yes	Yes	Diffraction solution
Do	Do	d*	No	No	Diffraction without Kutta's condition

Table 2-1 The basic solutions for rise with Kutta's condition

K	L_h	M_h	A_h	σ_h	τ_h	H_h	$\frac{phs(H_h)}{\pi}$	F_h	$\frac{phs(F_h)}{\pi}$
0.100	0.0748	0.0995	1.6917	0.0320	0.0	0.0751	0.0422	1.6910	1.5460
0.200	-0.1702	0.0832	1.9514	0.0583	0.0	0.1713	0.9692	1.9084	1.5363
0.300	-0.5781	0.0340	2.2721	0.0827	0.0	0.5736	0.9945	2.1629	1.5144
0.400	-1.1346	-0.0374	2.6207	0.1018	0.0	1.1133	1.0043	2.4306	1.4879
0.500	-1.8387	-0.1269	2.9878	0.1171	0.0	1.7802	1.0111	2.7066	1.4591
0.600	-2.6931	-0.2325	3.3693	0.1264	0.0	2.5653	1.0168	2.9895	1.4292
0.700	-3.7019	-0.3528	3.7631	0.1325	0.0	3.4585	1.0217	3.2785	1.3987
0.800	-4.8697	-0.4871	4.1679	0.1384	0.0	4.4478	1.0262	3.5736	1.3677
0.900	-6.2010	-0.6347	4.5828	0.1444	0.0	5.5187	1.0304	3.8742	1.3365
1.000	-7.7005	-0.7951	5.0070	0.1508	0.0	6.6545	1.0343	4.1804	1.3051
2.000	-32.7955	-3.0226	9.6493	0.1511	0.0	16.6883	1.0719	7.4957	0.9882
3.000	-78.2072	-6.1973	14.7908	-0.1687	0.0	9.2623	1.1837	11.1435	0.6701
4.000	-145.6614	-10.1156	20.2327	-0.6085	0.0	22.0436	0.0025	14.9957	0.3518
5.000	-236.0889	-14.6318	25.8661	-1.1729	0.0	44.6701	0.0670	18.9857	0.0335
10.000	-1046.2454	-43.2913	55.3707	-5.9627	0.0	38.0150	1.9388	39.8475	0.4422
15.000	-2466.4741	-78.3594	85.7652	-14.1375	0.0	116.6884	1.1039	61.3788	0.8507
20.000	-4503.8395	-117.4702	116.4930	-25.7476	0.0	163.8365	1.0250	83.0400	1.2593
25.000	-7161.3925	-159.5172	147.3921	-40.8114	0.0	91.1943	0.2079	104.9817	1.6681
30.000	-10441.0815	-203.8692	178.3986	-59.3392	0.0	304.9937	0.0605	126.8613	0.0763
35.000	-14343.2857	-250.1013	209.4657	-81.3312	0.0	104.7244	1.8730	148.7655	0.4854
40.000	-18870.1406	-297.9509	240.5942	-106.8002	0.0	356.4414	1.0946	171.0323	0.8936
45.000	-24020.4700	-347.1777	271.7479	-135.7377	0.0	343.6186	1.0129	192.6417	1.3023
50.000	-29796.0400	-397.6409	302.9355	-168.1543	0.0	245.0103	0.1671	215.1979	1.7112

Table 2-2 The basic solutions for an inclining flat plate with Kutta's condition

K	L_t	M_t	A_t	σ_t	τ_t	H_t	$\frac{phs(H_t)}{\pi}$	F_t	$\frac{phs(F_t)}{\pi}$
0.100	2.0363	0.9800	-2.4397	1.5287	0.0	2.0337	0.0153	3.0942	0.3039
0.200	1.7231	0.8385	-1.1005	1.3171	0.0	1.7141	0.0310	1.9399	0.2219
0.300	1.4227	0.7305	-0.3450	1.1624	0.0	1.4064	0.0492	1.3988	0.1277
0.400	1.0712	0.6265	0.2160	1.1084	0.0	1.0517	0.0750	1.1077	0.0133
0.500	0.6438	0.5180	0.6899	1.0520	0.0	0.6375	0.1285	0.9834	1.8824
0.600	0.1258	0.4019	1.1196	1.0420	0.0	0.2405	0.4007	0.9909	1.7525
0.700	-0.4936	0.2763	1.5257	0.9537	0.0	0.5456	0.8938	1.0932	1.6406
0.800	-1.2236	0.1407	1.9195	0.9137	0.0	1.1830	0.9736	1.2552	1.5504
0.900	-2.0718	-0.0054	2.3075	0.9177	0.0	1.9103	1.0024	1.4525	1.4775
1.000	-3.0459	-0.1621	2.6938	0.9060	0.0	2.7054	1.0187	1.6718	1.4164
2.000	-20.9068	-2.3073	6.7488	0.7755	0.0	10.8926	1.0878	4.3472	1.0115
3.000	-56.2718	-5.3998	11.2789	0.4430	0.0	7.3952	1.2270	7.4863	0.6795
4.000	-111.6243	-9.2760	16.1834	0.0214	0.0	17.0125	1.9860	10.9157	0.3567
5.000	-188.3824	-13.7984	21.3527	-0.5060	0.0	35.7716	0.0693	14.5482	0.0365
10.000	-915.3364	-43.1111	49.2560	-4.9657	0.0	33.7834	1.9271	34.2111	0.4429
15.000	-2237.6916	-79.5286	78.6698	-12.6883	0.0	106.1744	1.1068	55.0185	0.8510
20.000	-4168.3959	-120.3975	108.7036	-23.7826	0.0	151.8906	1.0232	76.1789	1.2594
25.000	-6713.0646	-164.4672	139.0695	-38.2886	0.0	86.2489	0.2128	97.7283	1.6682
30.000	-9875.0529	-211.0268	169.6446	-56.2277	0.0	288.6907	0.0605	119.2988	0.0763
35.000	-13655.6810	-259.6038	200.3508	-77.6070	0.0	100.6211	1.8690	140.9456	0.4855
40.000	-18057.6288	-309.9033	231.1689	-102.4430	0.0	341.3962	1.0953	162.9787	0.8936
45.000	-23080.2573	-361.6646	262.0512	-130.7308	0.0	330.5119	1.0119	184.4087	1.3023
50.000	-28725.6122	-414.7304	292.9975	-162.4828	0.0	236.8032	0.1689	206.7746	1.7112

Table 2-3 The basic solutions for a plate with negative camber with Kutta's condition

K	L_c	M_c	A_c	σ_c	τ_c	H_c	$\frac{phs(H_c)}{\pi}$	F_c	$\frac{phs(F_c)}{\pi}$
0.100	-1.0795	0.2186	0.9947	0.0574	0.0	1.0789	0.9936	1.3770	1.2481
0.200	-0.9217	0.2467	0.4160	0.0941	0.0	0.9209	0.9830	0.9106	1.1289
0.300	-0.7644	0.2623	0.1132	0.1270	0.0	0.7656	0.9676	0.7288	1.0133
0.400	-0.5890	0.2762	-0.0936	0.1533	0.0	0.5978	0.9420	0.6539	0.9039
0.500	-0.3911	0.2909	-0.2538	0.1760	0.0	0.4204	0.8912	0.6327	0.8051
0.600	-0.1694	0.3072	-0.3873	0.1925	0.0	0.2571	0.7597	0.6408	0.7184
0.700	0.0764	0.3253	-0.5039	0.2069	0.0	0.2172	0.4468	0.6653	0.6428
0.800	0.3463	0.3452	-0.6088	0.2230	0.0	0.3720	0.2403	0.6990	0.5762
0.900	0.6399	0.3668	-0.7051	0.2409	0.0	0.5939	0.1676	0.7377	0.5168
1.000	0.9570	0.3901	-0.7949	0.2611	0.0	0.8343	0.1355	0.7792	0.4628
2.000	5.3290	0.6841	-1.5110	0.4832	0.0	2.6873	0.0981	1.2012	0.0497
3.000	11.5947	1.0382	-2.0653	0.4840	0.0	1.4632	0.1817	1.5671	1.7047
4.000	19.3869	1.4104	-2.5283	0.5000	0.0	2.6352	1.0101	1.8796	1.3748
5.000	28.4224	1.7818	-2.9255	0.5275	0.0	5.0658	1.0740	2.1507	1.0502
10.000	86.3655	3.4596	-4.3272	0.7723	0.0	2.9091	0.9387	3.1148	1.4486
15.000	157.7980	4.8249	-5.2209	1.1200	0.0	7.1339	0.1052	3.7367	1.8544
20.000	237.7002	5.9613	-5.8693	1.5263	0.0	8.2326	0.0260	4.1839	0.2618
25.000	323.7142	6.9348	-6.3752	1.9726	0.0	3.9708	1.2077	4.5410	0.6700
30.000	414.4977	7.7880	-6.7889	2.4491	0.0	11.6054	1.0611	4.8277	1.0777
35.000	509.1529	8.5484	-7.1378	2.9493	0.0	3.5505	0.8724	5.0695	1.4866
40.000	607.1220	9.2360	-7.4396	3.4695	0.0	11.0317	0.0950	5.2886	1.8946
45.000	707.9037	9.8636	-7.7046	4.0065	0.0	9.7303	0.0132	5.4618	0.3032
50.000	811.1999	10.4422	-7.9410	4.5584	0.0	6.4361	1.1672	5.6412	0.7120

Table 2-4 The basic solutions for rise without Kutta's condition

K	L_h^*	M_h^*	σ_h^*	τ_h^*	H_h^*	$\frac{phs(H_h^*)}{\pi}$
0.100	1.3533	-0.1926	0.0625	0.1089	1.3499	1.9955
0.200	1.9832	-0.5087	0.1050	0.2864	1.9647	1.9836
0.300	2.6775	-0.9284	0.1468	0.5845	2.6260	1.9666
0.400	3.4682	-1.4506	0.1828	1.0113	3.3614	1.9459
0.500	4.3700	-2.0775	0.2150	1.5941	4.1813	1.9227
0.600	5.3921	-2.8126	0.2396	2.3151	5.0921	1.8976
0.700	6.5417	-3.6596	0.2613	3.2181	6.0981	1.8711
0.800	7.8251	-4.6230	0.2847	4.3859	7.2034	1.8436
0.900	9.2481	-5.7070	0.3100	5.8876	8.4115	1.8153
1.000	10.8157	-6.9162	0.3399	7.8120	9.7257	1.7863
2.000	35.3697	-26.7687	0.6419	65.0102	29.2948	1.4812
3.000	78.2699	-63.0516	0.5548	168.5148	62.0128	1.1666
4.000	141.6588	-118.1924	0.5307	342.9897	109.3242	0.8498
5.000	226.8302	-193.7607	0.5422	603.6192	172.1083	0.5322
10.000	1001.0355	-905.8641	0.3729	3632.7884	732.8961	0.9417
15.000	2378.2461	-2206.2419	0.3218	10438.3856	1720.5021	1.3504
20.000	4358.0852	-4100.9638	0.5775	21945.2020	3134.3564	1.7589
25.000	6992.4086	-6641.5293	1.7456	39206.1662	5011.3860	0.1674
30.000	10228.0650	-9780.0851	2.5899	62644.5887	7313.7588	0.5760
35.000	14002.0957	-13456.3787	1.6312	92432.8082	9996.5296	0.9845
40.000	18539.2330	-17886.2340	0.2141	130602.6262	13219.5124	1.3930
45.000	23464.5478	-22710.6219	0.5145	175078.7494	16716.8059	1.8015
50.000	28813.3138	-27960.4475	3.7487	226334.1947	20512.6071	0.2100

Table 2-5 The basic solutions for an inclining flat plate without Kutta's condition

K	L_t^*	M_t^*	σ_t^*	τ_t^*	H_t^*	$\frac{phs(H_t^*)}{\pi}$
0.100	0.1926	1.4012	0.1811	-0.1570	0.2377	0.2004
0.200	0.5087	1.1723	0.2592	-0.1615	0.5547	0.1382
0.300	0.9284	0.8766	0.3321	-0.0887	0.9426	0.0889
0.400	1.4506	0.5100	0.3904	0.0834	1.4018	0.0455
0.500	2.0775	0.0676	0.4408	0.3681	1.9346	0.0053
0.600	2.8126	-0.4555	0.4765	0.7693	2.5439	1.9672
0.700	3.6596	-1.0644	0.5076	1.3048	3.2326	1.9305
0.800	4.6230	-1.7640	0.5427	2.0199	4.0041	1.8948
0.900	5.7070	-2.5594	0.5825	2.9645	4.8615	1.8598
1.000	6.9162	-3.4554	0.6286	4.2030	5.8081	1.8254
2.000	26.7687	-18.9157	1.1187	45.4689	20.8080	1.4952
3.000	63.0515	-48.7547	0.9947	128.5028	47.5090	1.1734
4.000	118.1923	-95.7226	0.9326	274.3445	87.6124	0.8537
5.000	193.7608	-161.6707	0.9098	498.2922	142.2125	0.5347
10.000	905.8607	-810.4290	0.6702	3231.6149	652.0302	0.9423
15.000	2206.2236	-2031.3708	0.5747	9574.8152	1578.2100	1.3506
20.000	4100.9665	-3837.5304	0.7823	20477.8122	2924.8077	1.7591
25.000	6641.5317	-6280.4677	1.8654	36992.3613	4728.4406	0.1675
30.000	9779.8597	-9317.3382	2.6624	59570.6219	6954.8941	0.5760
35.000	13456.2587	-12891.2137	1.7453	88410.6193	9561.5506	0.9845
40.000	17886.2198	-17209.1594	0.3789	125486.2235	12701.6478	1.3930
45.000	22710.3697	-21927.1220	0.6595	168831.4771	16120.3181	1.8015
50.000	27961.0228	-27073.3209	3.7808	218909.1510	19839.6875	0.2100

Table 2-6 The basic solutions for a plate with negative camber without Kutta's condition

K	L_c^*	M_c^*	σ_c^*	τ_c^*	H_c^*	$\frac{phs(H_c^*)}{\pi}$
0.100	-0.3278	0.0469	0.0753	0.0640	0.3289	0.9955
0.200	-0.4627	0.1206	0.1040	0.0610	0.4656	0.9836
0.300	-0.6021	0.2144	0.1302	0.0291	0.6063	0.9666
0.400	-0.7533	0.3266	0.1504	-0.0361	0.7569	0.9459
0.500	-0.9184	0.4566	0.1677	-0.1354	0.9189	0.9227
0.600	-1.0989	0.6038	0.1794	-0.2661	1.0932	0.8976
0.700	-1.2952	0.7681	0.1897	-0.4309	1.2800	0.8711
0.800	-1.5079	0.9493	0.2016	-0.6406	1.4792	0.8436
0.900	-1.7370	1.1473	0.2154	-0.9059	1.6909	0.8153
1.000	-1.9827	1.3619	0.2311	-1.2403	1.9151	0.7863
2.000	-5.3454	4.4027	0.4064	-10.1804	4.8181	0.4812
3.000	-10.2552	8.9771	0.3830	-23.5307	8.8292	0.1666
4.000	-16.5165	14.9156	0.3577	-42.8598	13.7965	1.8498
5.000	-23.9352	22.0418	0.3335	-68.2713	19.5787	1.5322
10.000	-73.6299	70.8698	0.2772	-283.9031	57.3378	1.9417
15.000	-137.1215	134.3585	0.2398	-635.4307	104.7773	0.3504
20.000	-208.7899	206.6615	0.1999	-1105.6644	157.9509	0.7589
25.000	-288.4872	287.3045	0.1319	-1695.8040	216.7865	1.1674
30.000	-372.0645	372.2099	0.0924	-2383.9332	278.3466	1.5760
35.000	-456.7503	458.5683	0.1223	-3149.7599	340.6631	1.9845
40.000	-549.6347	553.0934	0.1605	-4038.4390	408.7850	0.3930
45.000	-638.3856	643.9092	0.1435	-4963.8106	473.9679	0.8015
50.000	-725.1632	732.9630	0.0522	-5933.0377	537.7232	1.2100

Table 2-7 The homogeneous solutions without Kutta's condition

K	L_s^*	M_s^*	A_s^*	σ_s^*	τ_s^*	H_s^*	$\frac{phs(H_s^*)}{\pi}$	F_s^*	$\frac{phs(F_s^*)}{\pi}$
0.100	-0.7557	0.1726	1.0000	-0.0180	-0.0644	0.7541	0.9927	1.1763	1.3123
0.200	-1.1035	0.3033	1.0000	-0.0239	-0.1467	1.0945	0.9824	1.2827	1.2545
0.300	-1.4328	0.4236	1.0000	-0.0282	-0.2573	1.4074	0.9716	1.3829	1.2048
0.400	-1.7563	0.5392	1.0000	-0.0309	-0.3859	1.7021	0.9604	1.4807	1.1591
0.500	-2.0781	0.6529	1.0000	-0.0327	-0.5335	1.9792	0.9490	1.5778	1.1159
0.600	-2.3997	0.7658	1.0000	-0.0336	-0.6871	2.2373	0.9373	1.6749	1.0743
0.700	-2.7221	0.8787	1.0000	-0.0342	-0.8552	2.4743	0.9252	1.7722	1.0340
0.800	-3.0459	0.9923	1.0000	-0.0351	-1.0523	2.6877	0.9127	1.8702	0.9947
0.900	-3.3711	1.1068	1.0000	-0.0362	-1.2847	2.8754	0.8996	1.9689	0.9562
1.000	-3.6981	1.2225	1.0000	-0.0378	-1.5602	3.0354	0.8859	2.0686	0.9183
2.000	-7.0643	2.4609	1.0000	-0.0509	-6.7373	3.0425	0.6649	3.1173	0.5613
3.000	-10.5794	3.8439	1.0000	-0.0489	-11.3932	3.5675	0.1636	4.2517	0.2233
4.000	-14.2008	5.3417	1.0000	-0.0563	-16.9522	6.3896	1.8749	5.4492	1.8932
5.000	-17.8967	6.9252	1.0000	-0.0663	-23.3363	6.6895	1.6148	6.6912	1.5672
10.000	-36.9741	15.5782	1.0000	-0.1144	-65.6085	13.9227	1.9415	13.2545	1.9590
15.000	-56.4882	24.8106	1.0000	-0.1686	-121.7089	19.1116	0.3662	20.0725	0.3617
20.000	-76.0726	34.1951	1.0000	-0.2260	-188.3822	27.8684	0.7708	26.9146	0.7673
25.000	-96.0283	43.9780	1.0000	-0.2887	-265.9992	33.3868	1.1667	34.0063	1.1741
30.000	-115.8594	53.6788	1.0000	-0.3471	-351.1496	41.1151	1.5892	41.0022	1.5815
35.000	-135.3223	63.0475	1.0000	-0.3961	-441.2790	48.1939	1.9833	47.7271	1.9892
40.000	-155.4874	73.1035	1.0000	-0.4448	-542.8335	54.0816	0.4000	54.9485	0.3971
45.000	-174.7392	82.2948	1.0000	-0.5014	-644.2690	62.5164	0.8054	61.5181	0.8051
50.000	-193.4714	90.9857	1.0000	-0.5675	-747.1366	66.9114	1.2105	67.7137	1.2133

Table 2-8 The diffraction solutions with Kutta's condition

K	abs(L_d)	$\frac{phs(L_d)}{\pi}$	abs(M_d)	$\frac{phs(M_d)}{\pi}$	abs(A_d)	$\frac{phs(A_d)}{\pi}$	abs(σ_d)	$\frac{phs(\sigma_d)}{\pi}$
0.100	2.2026	0.1264	1.3770	0.2481	16.9096	0.5460	0.3851	0.3019
0.200	1.8245	1.8840	0.9106	0.1289	9.5419	0.5363	0.3888	0.2411
0.300	2.1275	1.7258	0.7289	0.0133	7.2096	0.5144	0.4029	0.1892
0.400	2.5891	1.6281	0.6540	1.9039	6.0765	0.4879	0.4068	0.1410
0.500	3.0922	1.5590	0.6328	1.8051	5.4132	0.4591	0.4087	0.0953
0.600	3.6110	1.5041	0.6409	1.7184	4.9824	0.4292	0.4019	0.0507
0.700	4.1397	1.4569	0.6654	1.6427	4.6836	0.3986	0.3959	0.0073
0.800	4.6772	1.4142	0.6991	1.5762	4.4669	0.3677	0.3963	1.9653
0.900	5.2236	1.3745	0.7378	1.5168	4.3047	0.3365	0.4022	1.9245
1.000	5.7792	1.3367	0.7793	1.4627	4.1804	0.3051	0.4130	1.8847
2.000	11.8342	0.9967	1.2013	1.0496	3.7476	1.9881	0.5566	1.5148
3.000	18.6262	0.6738	1.5673	0.7046	3.7142	1.6700	0.4553	1.1370
4.000	25.9122	0.3538	1.8801	0.3747	3.7492	1.3518	0.4087	0.7609
5.000	33.5295	0.0348	2.1511	0.0502	3.7968	1.0335	0.3913	0.3873
10.000	74.0691	0.4423	3.1166	0.4483	3.9854	1.4420	0.4957	0.6086
15.000	116.3366	0.8506	3.7370	0.8541	4.0899	1.8505	0.7038	0.9432
20.000	159.2881	1.2590	4.1888	1.2614	4.1539	0.2590	0.9345	1.3189
25.000	202.6003	1.6675	4.5424	1.6693	4.1971	0.6674	1.1725	1.7101
30.000	246.1390	0.0760	4.8322	0.0774	4.2284	1.0759	1.4135	0.1081
35.000	289.8150	0.4844	5.0769	0.4856	4.2520	1.4844	1.6561	0.5098
40.000	333.6147	0.8929	5.2890	0.8939	4.2708	1.8929	1.8997	0.9136
45.000	377.4786	1.3014	5.4755	1.3023	4.2859	0.3014	2.1437	1.3187
50.000	421.4124	1.7099	5.6421	1.7107	4.2983	0.7099	2.3883	1.7246

K	abs(τ_d)	$\frac{phs(\tau_d)}{\pi}$	abs(H_d)	$\frac{phs(H_d)}{\pi}$	abs(F_d)	$\frac{phs(F_d)}{\pi}$
0.100	0.0	0.0	2.1489	0.1451	19.0160	0.0853
0.200	0.0	0.0	1.6878	1.9071	10.9714	0.0720
0.300	0.0	0.0	1.9299	1.7454	8.1970	0.0411
0.400	0.0	0.0	2.3353	1.6476	6.7280	0.0030
0.500	0.0	0.0	2.7701	1.5808	5.7839	1.9605
0.600	0.0	0.0	3.2004	1.5289	5.1036	1.9148
0.700	0.0	0.0	3.6151	1.4850	4.5752	1.8662
0.800	0.0	0.0	4.0085	1.4458	4.1429	1.8149
0.900	0.0	0.0	4.3762	1.4095	3.7768	1.7608
1.000	0.0	0.0	4.7142	1.3750	3.4597	1.7037
2.000	0.0	0.0	5.7520	1.0694	2.1605	0.9724
3.000	0.0	0.0	2.0380	0.8567	2.8065	0.3073
4.000	0.0	0.0	3.9175	1.3635	2.8101	1.7265
5.000	0.0	0.0	6.2678	1.1042	2.4915	1.0724
10.000	0.0	0.0	2.7004	1.3858	2.9118	1.8881
15.000	0.0	0.0	5.4752	0.9552	2.8547	0.6952
20.000	0.0	0.0	5.7922	1.2852	2.9393	1.5237
25.000	0.0	0.0	2.5624	0.8759	2.9894	0.3319
30.000	0.0	0.0	7.1821	1.1370	2.9607	1.1539
35.000	0.0	0.0	2.1185	1.3593	3.0340	1.9701
40.000	0.0	0.0	6.2922	0.9878	3.0040	0.7848
45.000	0.0	0.0	5.4003	1.3149	3.0343	1.6059
50.000	0.0	0.0	3.4565	0.8772	3.0484	0.4191

Table 2-9 The diffraction solutions without Kutta's condition

K	$abs(L_d^*)$	$\frac{phs(L_d^*)}{\pi}$	$abs(M_d^*)$	$\frac{phs(M_d^*)}{\pi}$	$abs(\sigma_d^*)$	$\frac{phs(\sigma_d^*)}{\pi}$
0.100	13.4993	0.4955	2.3767	1.7004	0.6400	0.4089
0.200	9.8233	0.4836	2.7737	1.6382	0.5563	0.3474
0.300	8.7535	0.4666	3.1421	1.5889	0.5379	0.2938
0.400	8.4034	0.4459	3.5044	1.5455	0.5210	0.2445
0.500	8.3626	0.4227	3.8692	1.5053	0.5084	0.1980
0.600	8.4867	0.3976	4.2398	1.4672	0.4896	0.1535
0.700	8.7115	0.3711	4.6180	1.4305	0.4743	0.1105
0.800	9.0042	0.3436	5.0051	1.3948	0.4682	0.0687
0.900	9.3461	0.3153	5.4017	1.3598	0.4692	0.0278
1.000	9.7256	0.2863	5.8081	1.3254	0.4773	1.9886
2.000	14.6473	1.9812	10.4039	0.9952	0.6032	1.6168
3.000	20.6707	1.6666	15.8361	0.6734	0.4724	1.2619
4.000	27.3306	1.3498	21.9028	0.3537	0.4037	0.9282
5.000	34.4210	1.0322	28.4420	0.0347	0.3593	0.6034
10.000	73.2870	1.4417	65.2010	0.4423	0.2493	0.9761
15.000	114.6939	1.8503	105.2092	0.8506	0.2028	1.3746
20.000	156.7077	0.2589	146.2309	1.2590	0.1763	1.7964
25.000	200.4391	0.6674	189.1221	1.6675	0.1642	0.2656
30.000	243.7647	1.0759	231.8092	0.0760	0.1555	0.7055
35.000	285.5815	1.4844	273.1575	0.4844	0.1362	1.0630
40.000	330.4471	1.8929	317.5023	0.8929	0.1236	1.4032
45.000	371.4284	0.3014	358.1791	1.3014	0.1167	1.8241
50.000	410.1952	0.7099	396.7305	1.7099	0.1226	0.3534

K	$abs(\tau_d^*)$	$\frac{phs(\tau_d^*)}{\pi}$	$abs(H_d^*)$	$\frac{phs(H_d^*)}{\pi}$
0.100	1.0882	0.5460	13.6085	0.4910
0.200	1.4003	0.5363	9.9898	0.4680
0.300	1.8547	0.5144	8.9340	0.4357
0.400	2.3448	0.4879	8.5608	0.3971
0.500	2.8881	0.4591	8.4633	0.3540
0.600	3.4235	0.4292	8.4996	0.3076
0.700	4.0053	0.3986	8.6075	0.2586
0.800	4.7006	0.3677	8.7559	0.2074
0.900	5.5304	0.3365	8.9276	0.1544
1.000	6.5224	0.3051	9.1130	0.0997
2.000	25.2484	1.9881	11.3596	1.4904
3.000	42.3172	1.6700	15.2839	0.8367
4.000	63.5575	1.3518	20.4606	0.2012
5.000	88.6028	1.0335	25.3062	1.5694
10.000	261.4753	1.4420	52.7870	0.3834
15.000	497.7759	1.8505	81.9919	1.2011
20.000	782.5154	0.2590	111.8138	0.0179
25.000	1116.4229	0.6674	142.6679	0.8349
30.000	1484.8127	1.0759	173.3673	1.6520
35.000	1876.3378	1.4844	202.9271	0.4689
40.000	2318.3552	1.8929	234.6468	1.2859
45.000	2761.2435	0.3014	263.6691	0.1029
50.000	3211.4540	0.7099	291.0338	0.9199

Applying the first theorem and making use of basic solutions of Table 1, we may deduce the following relations corresponding to Munk's theorem in the wing theory(Flax 1952).

$$KA = -\int_{-1}^{1} p_h(-x)\eta_x(x)dx + KA_h\eta(-1), \tag{23}$$

$$L = \int_{-1}^{1} p_t(-x)\eta_x(x)dx - K\{A + A_t\eta(-1)\}, \tag{24}$$

$$M = -\int_{-1}^{1} p_c(-x)\eta_x(x)dx + KA_c\eta(-1), \tag{25}$$

$$F(K) = \int_{-1}^{1} p_d(-x)\eta_x(x)dx - KA_d\eta(-1), \tag{26}$$

and finally,when we define σ,the quantity of the horizontal spray,as follows;

$$\sigma = \lim_{x\to 1}\left[p(x)\sqrt{1-x^2}\right], \tag{27}$$

which is evaluated from the pressure of the front end element. On the other hand,it can be also represented as follows;

$$\sigma = \lim_{n\to\infty}\frac{2}{\pi}\int_{-1}^{1} p(x)\frac{\sin n\theta}{\sin\theta}dx = -\frac{2}{\pi}\int_{-1}^{1} p(x)\eta_{sx}(x)dx, \tag{28}$$

where η_s denotes the surface elevation of the spray function in table 1. The values in these two definitions must coincide with each other but the number n in the formula (28) must be selected carefully.
Then,we have

$$\sigma = -\frac{2}{\pi}\int_{-1}^{1} p_s(-x)\eta_x(x)dx + \frac{2K}{\pi}A_s\eta(-1), \tag{29}$$

This spray function becomes (D.3) at the high speed limit,namely, it is a highly oscillating function, so that its numerical calculation may be very much difficult. In fact, we can not assert that the present computed values is accurate. Therefore, the relation expressed in eq(29) does not coincide with the tabulated values in Table 2, and it may be understood as a qualitative relation. The solution for $\eta = 0$, however, is found easily, for example, when we subtract p_h^* from p_h of the basic solution in Table 1. Namely, the difference of two solutions having the same boundary condition except Kutta's condition is, a homogeneous solution, and if we define that it has unit A-term, it becomes ;

$$p_s^* = (p - p^*)/A, \tag{30}$$

Since p satisfies Kutta's condition,then we have

$$\tau_s^* = -\frac{\tau^*}{A} \quad \text{or} \quad A = -\frac{\tau^*}{\tau_s^*}, \tag{31}$$

where

$$\tau^* = \lim_{x\to -1}\left[p^*(x)\sqrt{1-x^2}\right], \tag{32}$$

These formulas, at first sight, show that the force and moment etc. are given by known basic solutions, inclination of the bottom surface and the off-set of the aft end. Since the pressure of these solutions is zero at the aft end and infinite at the bow, then these forces etc. are affected largely by the inclination of the bottom near the stern. It seems somewhat paradoxically; for example, the spray at the bow may be diminished by flattening near the aft end, but this fact may be understood when we consider the effectiveness of a flap of the wing. Some of actual relations are as follows;

$$\left.\begin{aligned} L_h &= -K(A_h + A_t), \\ M_h &= -KA_c, \\ L_c &= -M_t + KA_c, \end{aligned}\right\} \qquad (33)$$

$$\left.\begin{aligned} F_h(K) &= -KA_d, \\ F_t(K) &= L_d + KA_d, \\ F_c(K) &= -M_d, \end{aligned}\right\} \qquad (34)$$

$$\left.\begin{aligned} \sigma_h &= 2KA_s/\pi, \\ \sigma_t + \sigma_h &= -2L_s/\pi, \\ \sigma_c &= -2M_s/\pi, \\ \sigma_d &= -2F_s(K)/\pi, \end{aligned}\right\} \qquad (35)$$

These relations are ascertained by computed results, and they are also useful for the check of the computation.

Table 3 The asymptotic expressions for the basic solutions

Function	C_0	C_1	C_2	C_3
L_h/K^2	-12.4339	2.7400	-0.7480	0
L_t/K^2	-12.3333	4.4950	-1.2773	0
L_c/K^2	0.09870	1.2295	-0.4495	0
L_s	0.0003371	0.001180	-0.007901	0
L_d	0.005631	0.8352	-0.09733	0
M_h/K^2	-0.04760	-0.6048	0.2124	0
M_t/K^2	-0.05160	-0.6245	0.2370	0
M_c/K^2	-0.002448	0.03181	0.004890	0
M_s	0.0002780	-0.001320	0.003981	0
M_d	-0.004880	0.02305	0.01215	0
∇/K^3	8.2613	-4.3381	2.2491	-0.6565
A_h/K	6.1787	-0.6619	0	0
A_t/K	6.0740	-1.1909	0	0
A_c/K	-0.09830	-0.3548	0	0
KA_s	-0.0002191	-0.0001220	0.001990	0
KA_d	0.001913	0.4264	-0.02875	0
σ_h/K^2	-0.06920	0.009811	-0.00009110	0
σ_t/K^2	-0.06850	0.01753	0.001751	0
σ_c/K^2	-0.0006400	0.005690	0.001522	0
σ_s/K	0.001091	0.1222	0	0
σ_d/K^2	0.0001480	0.003831	0.001171	0
F_h/K	4.2617	-0.2852	0	0
F_t/K	4.0858	-0.6837	0	0
F_c/K	0.1499	0.1713	0	0
F_s/K	-0.008171	0.03103	0	0
F_d/K^2	-0.0002523	0.002051	0.002621	0

Remarks;

Absolute value of Function
$= C_0 + C_1(10/K) + C_2(10/K)^2 + C_3(10/K)^3$, for $5 < K < 50$

Function	α	ϵ
L_d	-1	1.623
M_d	-1	1.623
A_d	-1	0.627
σ_d	-1	0.627
F_h	-1	1.623
F_t	-1	1.623
F_c	-1	0.627
F_s	-1	1.130
F_d	-2	0.255

Remarks;
Phase of Function
$= \alpha K + \pi\epsilon$

The formulas (34) and the last equation of (35) are the same as Haskind's relation well known in the theory of oscillating floating body(Bessho 1977a). Namely, Kotchin function or the amplitude of wave left in the down stream and the bottom off-set is calculated from the diffraction pressure and the bottom off-set. The diffraction of the wave does not appear explicitly in our problem, but the boundary value problem contains the phenomena implicitly(see Appendix B).

4. Resistance Components(Bessho 1991)

The resistance is given by the formula (20) and another expression of the formula is obtained making use of the representation:

$$\eta_x(x) = \frac{1}{\pi}\int_{-1}^{1}\frac{p(\xi)d\xi}{\xi - x} + K\int_{-1}^{1} p(\xi)S(x-\xi,0)d\xi + KAT(x-1,0), \tag{36}$$

multiplying the pressure and integrating, paying attention to the integral(Flax 1952);

$$\frac{1}{\pi}\int_{-1}^{1} p(x)dx\int_{-1}^{1}\frac{p(\xi)}{\xi - x}d\xi = \frac{\pi}{4}(\sigma^2 - \tau^2), \tag{37}$$

we have finally the following formula for the solution satisfying Kutta's condition.

$$D = D_S + D_W + D_H, \quad D = \frac{2D'}{\rho V^2 L}, \quad r = \frac{D'}{\rho g \nabla'^2/L^2}, \tag{38}$$

$$D_S = \frac{\pi\sigma^2}{4}, \quad r_S = \frac{4D_S}{K\nabla^2}, \tag{39}$$

$$D_W = K|F(K)|^2, \quad r_W = \frac{4D_W}{K\nabla^2}, \tag{40}$$

$$D_H = \frac{KA^2}{2}, \quad r_H = \frac{4D_H}{K\nabla^2}, \tag{41}$$

The first term D_S of the formula (38) is a well-known Wagner-Maruo spray drag and represented by the equation (39). The second term D_W is the wave resistance and the third D_H is a new component and we should like to call a water head resistance. It seems to be the same one as the wave-breaking resistance from its derivation but we have now no clear reasoning to say so(Baba 1969).

Lastly, it is worthwhile to consider a resistance integral of solutions without Kutta's condition. For example, the resistance of the solution p_h^* in Table 1 becomes;

$$D_h^* = \int_{-1}^{1} p_h^*(x)\eta_{hx}^*(x)dx = 0, \tag{42}$$

because the inclination vanishes in this case. However, this is false because this solution does not satisfy Kutta's condition so that the term such as the integral (37) might appear and the right answer becomes;

$$D_h^* = \frac{\pi}{4}\left[(\sigma_h^*)^2 - (\tau_h^*)^2\right] + K|H_h^*(K)|^2, \tag{43}$$

At lower speed, the first term becomes very small and the second term originated from the aft spray becomes negative and nearly double to the third wave resistance term . Namely, the resistance becomes negative and this is the other reason why we give up the solution without Kutta's condition. In the same way, we have the formula:

$$D_t^* = \int_{-1}^{1} p_t^*(x)\eta_{tx}^*(x)dx \neq L_t^*, \tag{44}$$

$$D_t^* = \frac{\pi}{4}\left\{(\sigma_t^*)^2 - (\tau_t^*)^2\right\} + K|H_t^*(K)|^2, \tag{45}$$

5. Free running solution(Bessho 1970,1977b,Bessho et al.1992a)

When a ship is running freely with a constant speed on a still water surface, she trims and sinks from the still water line and her displacement weight and moment balance to her dynamic lift and moment. Usually, this phenomenon is not taken into account in the theory of the wave-making resistance except in higher speed region. Then, the almost only one case which has been analyzed is of a planing flat plate. In this case, as described the above, the wetted length changes as to satisfy the flowing out condition, that is, Kutta's condition. Then, the balance equations are making use of the solutions including the swell-up potential,

$$K\nabla = hL_h + tL_t, \quad K\nabla l = hM_h + tM_t, \tag{46}$$

$$hA_h + tA_t = 0, \tag{47}$$

where ∇ denotes the displacement volume and l the position of the center of gravity. Eliminating the rise up h , we have

$$K\nabla = tL_G, \quad L_G = L_t - L_h\frac{A_t}{A_h}, \quad K\nabla l = tM_G, \quad M_G = M_t - M_h\frac{A_t}{A_h}, \tag{48}$$

Eliminating once more the trim from the above equations, we have

$$l = \frac{M_G}{L_G}, \tag{49}$$

On a still water, this flat plate or a ship floats with trim $t = T/L_o$, the aft draft T and wetted length L_o , then we have

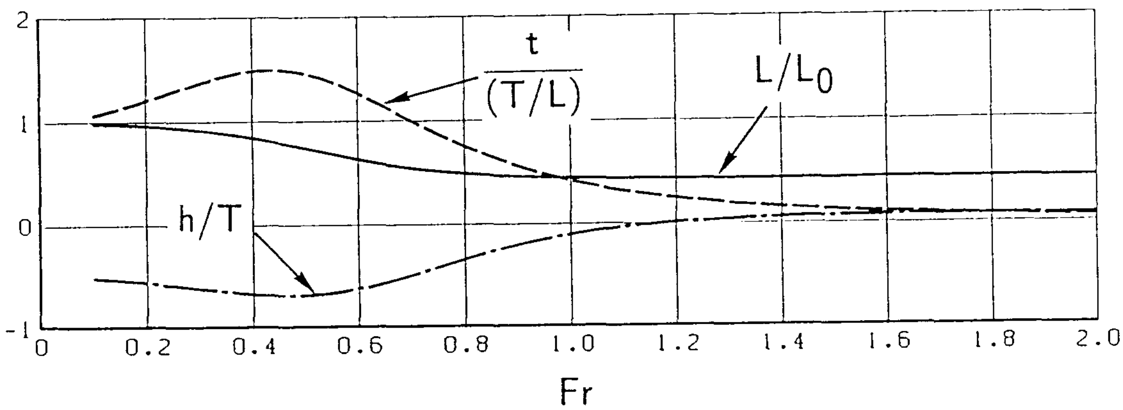

Fig. 4 t, h, L/L_0 of a planing flat plate

$$\nabla = \frac{TL_o}{2}, \ l = \frac{2}{3}\left(\frac{L_o}{L}\right) - 1, \tag{50}$$

Equating the second one to the fomula (49), a relation between Lo and the wetted length L(=2) is obtained in freely running and, putting this into the first equation of (48), we have the trim from the first one of the equation (48) and the trim means, of course, the drag-lift ratio in this case.

Table 4 Hydrodynamic property of a planing flat plate

K	L_G	M_G	l	L/L_0	t/(T/L)	D_S/D_T	σ_G	h/T
0.000	3.1416	1.5708	0.5000	0.4444	0.0000	1.0000	2.0000	0.0000
0.100	2.1442	1.1235	0.5240	0.4375	0.0408	0.9080	1.5750	0.0673
0.500	1.0684	0.5474	0.5123	0.4408	0.4126	0.7721	1.0251	-0.1081
1.000	1.0971	0.2657	0.2421	0.5367	0.9784	0.4872	0.8253	-0.4904
5.000	6.5107	-1.7198	-0.2641	0.9060	1.3915	0.0258	0.4622	-0.6340
10.000	15.3708	-4.6006	-0.2993	0.9514	1.2380	0.0059	0.3385	-0.5787
30.000	53.6853	-17.1614	-0.3197	0.9799	1.0952	0.0006	0.1997	-0.5314
50.000	92.9489	-30.1345	-0.3242	0.9865	1.0613	0.0002	0.1550	-0.5203

Namely,

$$\text{Drag} - \text{Lift ratio} = t = \left(\frac{2K}{L_G}\right)\left(\frac{L}{L_o}\right)\left(\frac{T}{L}\right), \tag{51}$$

Table 4 and Fig.4 show these values. The experimental results show a good agreement with the theory(Suzuki et al. 1986a,b). At very high speed , we have (see Appendix D)

$$L_G \to \pi, \ \frac{M_G}{L_G} \to \frac{1}{2}, \ \frac{L}{L_o} \to \frac{4}{9}, \ \frac{t}{T/L} \to \frac{8K}{9\pi} \tag{52}$$

and they almost coincide with the result of wing theory. On the other hand, at very low speed, the lift and moment increase almost proportionally with the wave number K but the rise and trim tend to finite values.

$$L_o \to L, \ t \to \frac{T}{L}, \ h \to -0.5T, \tag{53}$$

These values are nearly equal to the values on a still water except sinkage.

Nextly, when the wetted length does not change and the solution does not satisfy the flowing-out condition,then we have a unrealistic solution as seen in Fig.5 and 6. Hence,let us introduce the swell-up potential to satisfy Kutta's condition then balance equations become;

$$K\nabla = L_f + hL_h + tL_t, \ K\nabla l = M_f + hM_h + tM_t, \tag{54}$$

where L_f and M_f are the lift and moment of the pressure corresponding to the boundary condition f(x) of the equation (15), that is, the off-set of the proper hull bottom surface. At first, let us consider a box barge, namely L_f and M_f are zero, then we have her trim and rise(negative sinkage) as follows;

$$\frac{h}{\nabla} = \frac{K(M_t - lL_t)}{\Delta}, \ \frac{t}{\nabla} = \frac{K(lL_h - M_h)}{\Delta}, \tag{55}$$

$$\Delta = M_t L_h - M_h L_t, \tag{56}$$

$$\nabla = 2T, \tag{57}$$

Fig.7 shows the pressure and surface elevation at representative speeds and Fig.8 shows the rise, trim, A-value and resistance. As seen in Fig.7,8 at a first glance,this barge can not run freely over the speed Fr.=0.61 at which speed the determinant (56) vanishes to zero(Bessho et al.1992a). This instability has been confirmed also by a dynamical criterion and we can easily find that the vanishing of the determinant (56) is a quasi-statical criterion(Bessho et al.1987). It may be remarked that this instability does not depend upon a shape of the bottom but only upon a wetted length(water plane area in three dimensional case). A well-known dynamical instability for a ship is porpoising but it occurs at higher speed for a planing boat. Moreover, in that case, the variation of its wetted length is very important(Bessssho 1977b). Therefore, the present instability is not the same as porpoising. In fact, we experience a difficulty to tow a blunt and broad barge over some speed in the towing test. The barge trims by the stern largely just before this critical speed and then the bow dives suddenly and the towing test is interrupted. At the lower speed, we have also approximations from Table 3 as follows;

$$\left.\begin{array}{l} \frac{h}{T} \rightarrow 3l - \frac{2.4}{K} \rightarrow -\frac{2.4}{K}, \\ \frac{t}{T/L} \rightarrow -6l + \frac{4.4}{K} \rightarrow \frac{4.4}{K}, \\ \frac{A}{T} \rightarrow -.86 + .14Kl \rightarrow -.86 \end{array}\right\} \tag{58}$$

$$\left.\begin{array}{l} r \rightarrow -6l + \frac{4.4}{K} + (-.86 + .14Kl)^2 \rightarrow .74 + \frac{4.4}{K}, \\ r_W \rightarrow \frac{(-.86 + .14Kl)^2}{2} \rightarrow .37, \\ r_H \rightarrow r_W, \\ r_S \rightarrow 0, \end{array}\right\} \tag{59}$$

These values are not very accurate but they show the dependency upon the wave number ,that is, the speed and the position of the center of gravity. If the center of buoyancy lies on the midship, that is l equals zero, the sinkage and trim are proportional to the velocity head times the draft, but A-value is negative and a little less than the draft. This means that the mean water level at far down stream rises up a little less than the draft,so that the statical buoyancy from this water level may be nearly equal to the statical one. It may seem somewhat curious and this is the reason why we are hesitating to assert this mathematical model strongly. The resistance quotient r defined by (38)~(41) becomes constant at such low speed, the spray resistance becomes zero and the water head resistance nearly equals the wave resistance and the wave resistance does not have distinctive hump and hollow. This property is very much interesting and is caused by the fact that the phase of the trailing wave has a simple form as seen in Table 3. The stern wave may become negligible because the pressure becomes zero there owing to Kutta's condition. If the center of buoyancy moves forwards,it may be concluded that the sinkage, trim and A-value decrease and eventually the resistance decreases. Lastly, let us consider a case in which the bottom has a negative camber c.

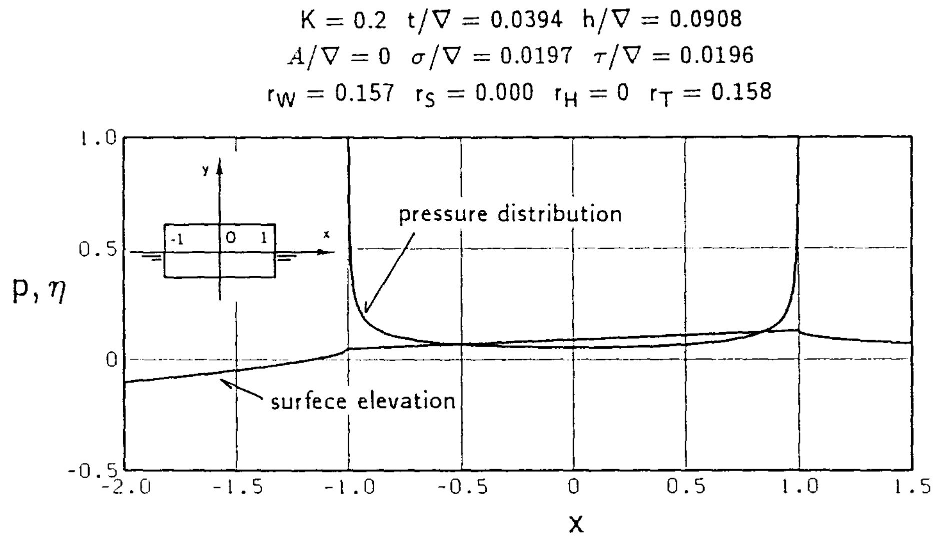

Fig. 5-1 Pressure and surface elevation of a flat bottom barge without Kutta's condition

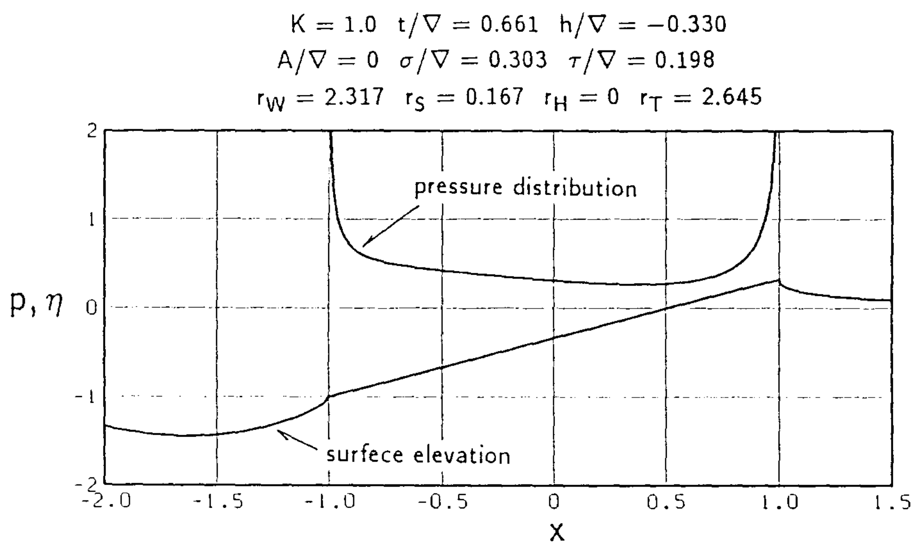

Fig. 5-2 Pressure and surface elevation of a flat bottom barge without Kutta's condition

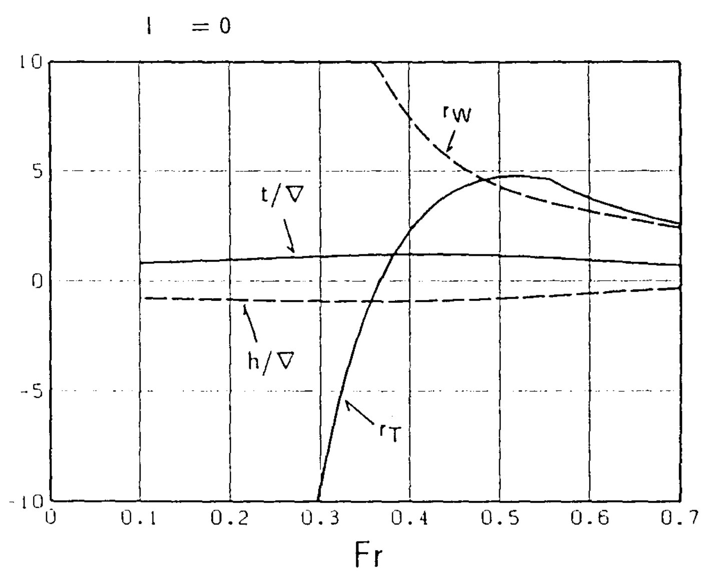

Fig. 6 h, t, r of a flat bottom barge without kutta's condition

$K = 1.0 \quad t/\nabla = -0.678 \quad h/\nabla = 0.138$

$A/\nabla = -1.134 \quad \sigma/\nabla = -0.336 \quad \tau/\nabla = 0$

$r_W = 1.550 \quad r_S = 0.355 \quad r_H = 2.570 \quad r_T = 4.875$

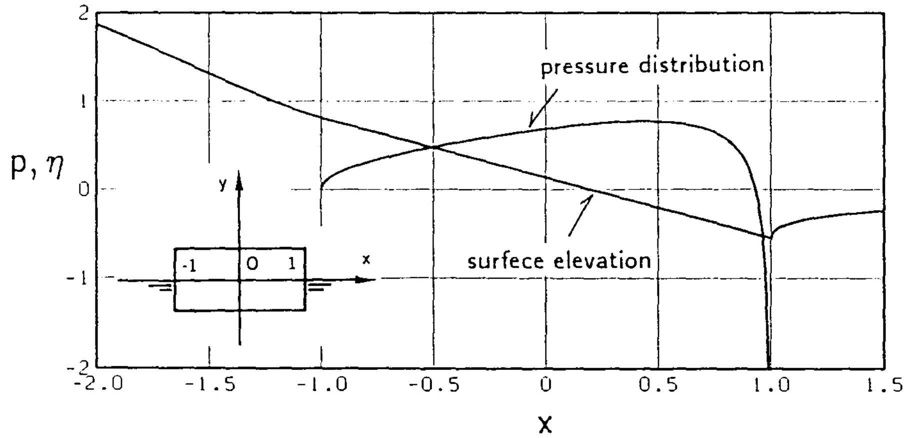

Fig. 7-1 Pressure and surface elevation of a flat bottom barge with Kutta's condition

$K = 2.0 \quad t/\nabla = 0.485 \quad h/\nabla = -0.370$

$A/\nabla = -0.299 \quad \sigma/\nabla = 0.320 \quad \tau/\nabla = 0$

$r_W = 1.900 \quad r_S = 0.161 \quad r_H = 0.179 \quad r_T = 2.158$

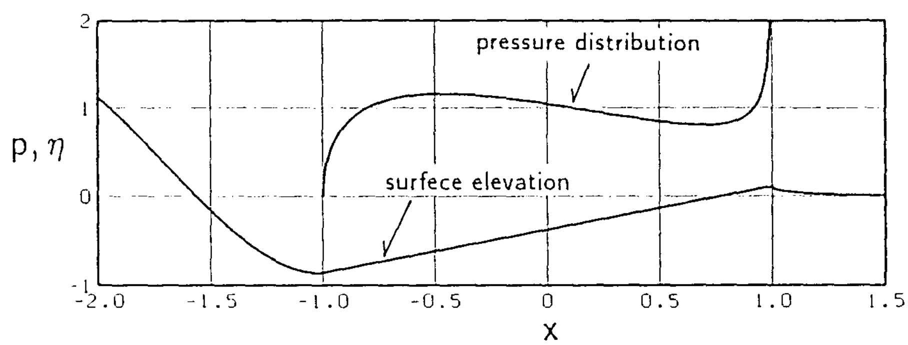

Fig. 7-2 Pressure and surface elevation of a flat bottom barge with Kutta's condition

I = 0

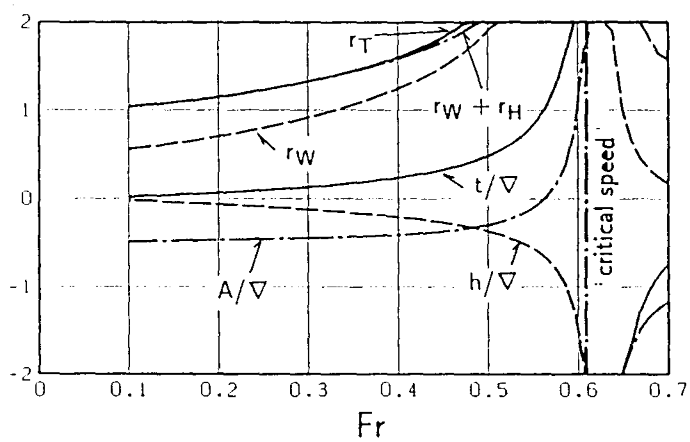

Fig. 8 h, t, A, r of a flat bottom barge with kutta's condition

$K = 1.0 \quad t/\nabla = -1.155 \quad h/\nabla = 0.358 \quad \nabla = 4c$

$A/\nabla = -1.518 \quad \sigma/\nabla = -0.489 \quad \tau/\nabla = 0$

$r_W = 3.231 \quad r_S = 0.753 \quad r_H = 4.606 \quad r_T = 9.430$

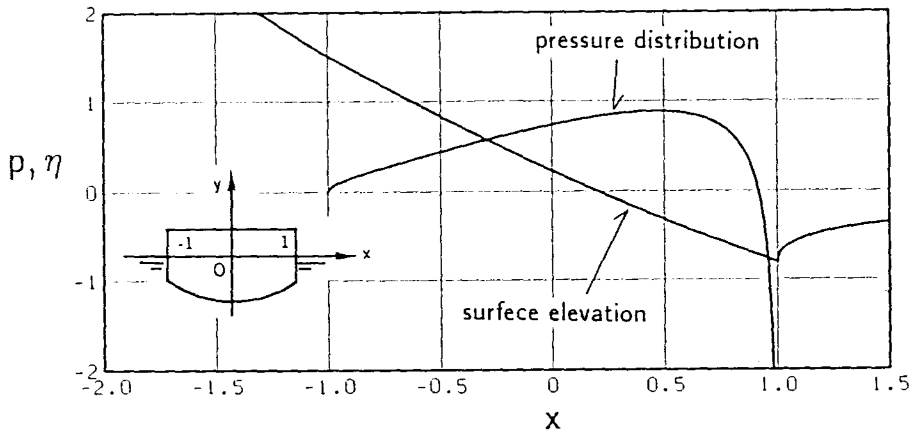

Fig. 9-1 Pressure and surface elevation of a camberd bottom barge with Kutta's condition

$K = 2.0 \quad t/\nabla = 0.611 \quad h/\nabla = -0.410 \quad \nabla = 4c$

$A/\nabla = -0.209 \quad \sigma/\nabla = 0.533 \quad \tau/\nabla = 0$

$r_W = 2.139 \quad r_S = 0.446 \quad r_H = 0.087 \quad r_T = 2.452$

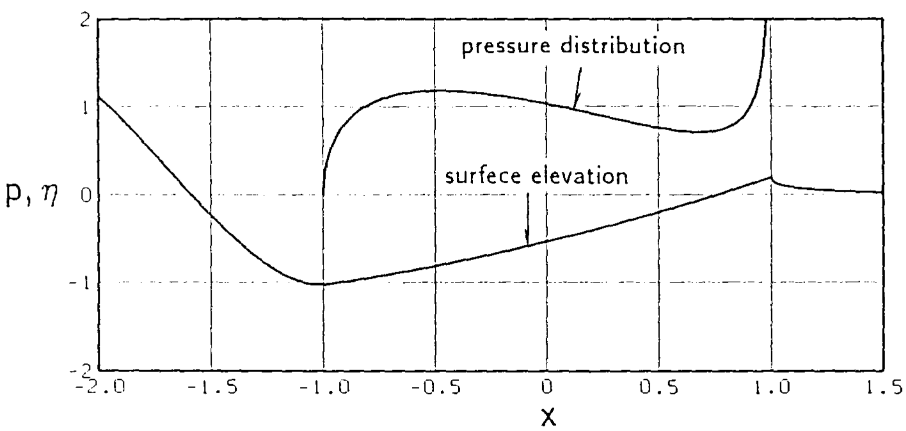

Fig. 9-2 Pressure and surface elevation of a camberd bottom barge with Kutta's condition

$\mathrm{l} = 0 \quad \nabla = 4c$

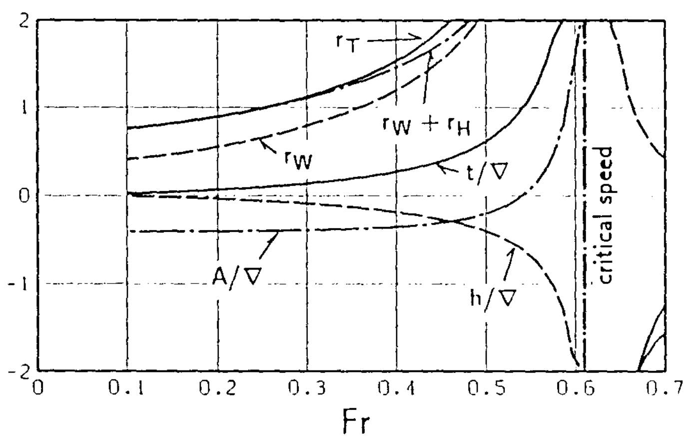

Fig. 10 h, t, A, r of a camberd bottom barge with kutta's condition

The results are shown in Fig.9 and 10. In this case,the balance equations (54) become, by making use of the solution p_c in Table 2,

$$K\nabla = 2cL_c + hL_h + tL_t, \;\; K\nabla l = 2cM_c + hM_h + tM_t, \tag{60}$$

Now,let us define the equivalent draft T_E from the given displacement volume(area here)by the following formula.

$$\nabla = 2T_E, \tag{61}$$

Moreover ,putting

$$\alpha = 1 - c\frac{L_c}{KT_E}, \;\; \beta = l - c\frac{M_c}{KT_E}, \tag{62}$$

the balance equations (60) become as follows;

$$K\alpha\nabla = hL_h + tL_t, \;\; K\beta\nabla = hM_h + tM_t, \tag{63}$$

Then, the solutions become;

$$\frac{h}{\nabla} = \frac{K(\alpha M_t - \beta L_t)}{\Delta}, \;\; \frac{t}{\nabla} = \frac{K(\beta L_h - \alpha M_h)}{\Delta}, \tag{64}$$

Since L_c and M_c are positive at lower speed, α becomes negative for some c/T_E(say > 0.1) but β does not change so much. This causes a large sinkage and trim by the bow and the resistance increases but the share of the water head resistance decreases. It is interesting to consider here a case in which the camber is positive. All formulas may be applied only to change sign of c. It is easily understood that the effect of camber is reversible.

6. Conclusion

In order to construct a consistent linearized theoy of the wave-making resistance of displacement ships, we have introduced a source singularity which is known as a line integral term up to the present. Then, we have developed the theory for two-dimensional pressure distribution for simplicity sake. We have solved the boundary value problems numerically for many basic conditions and verified mutual relations lying between these solutions. Namely, we have extended Hanaoka's reciprocity theorems in our case, by which theorems and some basic solutions we may estimate a lift, moment , Kotchin function etc. as Munk's theorem in the wing theory. Analizing the resistance integral, we have a new resistance component, we would like to call the water head resistance here, in addition to another well-known two components, that is, the wave-making and spray resistance. Making use of this theory, we have calculated the trim and sinkage of a box barge and a barge having a negative cambered bottom when they are running or towed freely. Although the trim and sinkage have not played main roles in the theory of wave-making resistance up to the present,they must be determined intrinsically by the balance equations of the displacement weight and its moment with the hydrodynamical force and moment. Carrying out this calculation, we find an instability at the speed Fr.=.61. A barge couldnot be towed freely over this critical speed which depends only on the wetted length. The water head resistance becomes as large as the wave-making resistance at very low speed and resembles very much the wave-breaking one, but we can not explain their mutual relation. Thus, it seems that the present theory may dissolve difficulties of the linealized theory and the reciprocity theorems may be usefull to estimate hydrodynamical properties of an arbitrarily given ship. Lastly, we would like to point out that the reduction of the resistance is a simple in this case as follows. Firstly, the spray resistance becomes zero by Maruo when the pressure at both ends stays finite. This spray-free condition is the same as a shock-free entrance of an aerofoil section. Secondly, a new water head resistance does not appear when we do not put the new singularity there. Thirdly and lastly, the wave-making resistance can be also reduced to zero in this two dimensional case as shown in Appendix C.

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Appendix A. Kernel function(Bessho 1991,Bessho et al.1992a)

The kernel function is defined as follows;

$$S(x,y)+iT(x,y)=W(z)=\frac{1}{i\pi}\lim_{\mu\to+\infty}\int_0^\infty \frac{e^{-ikz}dk}{k-K-i\mu},\quad z=x+iy,\ \ y\geq 0, \tag{A.1}$$

$$T^*(x,y)-iS^*(x,y)=W^*(z)=\int_z^\infty W(z)dz, \tag{A.2}$$

Since W(z) satisfies the following diffrential equation:

$$\left(iK+\frac{d}{dz}\right)W(z)=\frac{i}{\pi z}, \tag{A.3}$$

we have by integration;

$$W^*(z)=-iW(z)-\frac{1}{\pi}\log z, \tag{A.4}$$

where the imaginary part of logarithmic term in the right hand side is selected as zero at far upstream.Hence we have;

$$S^*(x,y)=S(x,y)+\frac{\theta}{\pi},\ \ T^*(x,y)=T(x,y)-\frac{1}{\pi}\log r,\ \ z=re^{i\theta},\ \ -\pi\leq\theta\leq 0 \tag{A.5}$$

When y tends to zero from the negative side, S and T are represented by sine and cosine integral and we may write them as follows, making use of the definition of Abramowitz et al.,(1970),

$$\begin{aligned} S(x,0)+iT(x,0) &= \frac{1}{\pi}[f(Kx)-ig(Kx)] &&, \text{for } x>0 \\ &= \frac{1}{\pi}[-f(-Kx)-ig(-Kx)]+2e^{-iKx} &&, \text{for } x<0 \end{aligned} \tag{A.6}$$

Near the origin, they behave as ;

$$\begin{aligned} S(x,0)+iT(x,0) &\to \frac{1}{2}+\frac{i}{\pi}(\gamma+\log Kx) &&, \text{for } x>0, \\ &\to \frac{3}{2}+\frac{i}{\pi}(\gamma+\log Kx) &&, \text{for } x<0, \end{aligned} \tag{A.7}$$

$$S^*(x,0)+iT^*(x,0)\to\frac{1}{2}+\frac{i}{\pi}(\gamma+\log K)\quad ,\text{for } x\to 0, \tag{A.8}$$

where γ means Euler's constant 0.5772 . In far field, we have the following asymptotic approximations;

$$\begin{aligned} S(x,0)+iT(x,0) &\to \frac{1}{\pi Kx} &&, \text{for } x\gg 0, \\ &\to \frac{1}{\pi Kx}+2e^{-iKx} &&, \text{for } x\ll 0 \end{aligned} \tag{A.9}$$

$$\begin{aligned} S^*(x,0)+iT^*(x,0) &\to -\frac{i}{\pi}\log x+\frac{1}{\pi Kx} &&, \text{for } x\gg 0, \\ &\to -\frac{i}{\pi}\log(-x)+\frac{1}{\pi Kx}-1+2e^{-iKx} &&, \text{for } x\ll 0, \end{aligned} \tag{A.10}$$

Appendix B. Diffraction Problem(Bessho 1976b,1977a)

The boundary integral equation(13) becomes;

$$\eta(x) = -\int_{-1}^{1} p(\xi)T(x-\xi,0)d\xi, \tag{B.1}$$

where we consider only solutions without Kutta's condition, for simplicity sake,and abbreviate the star mark in this appendix. Then, the usual solutions are given by the formula (30), that is, by adding a homogeneous solution.

Now, since the kernel T has the following relation;

$$T(x,0) = T(-x,0) - 2\sin Kx, \tag{B.2}$$

changing the sign of x, we have;

$$\begin{aligned} \eta(-x) &= -\int_{-1}^{1} p(\xi)T(-x-\xi)d\xi, \\ &= -\int_{-1}^{1} p(-\xi)T(x-\xi)d\xi + i\left[e^{iKx}H^{+}(K) - e^{-iKx}H^{-}(K)\right], \end{aligned} \tag{B.3}$$

where

$$H^{\pm}(K) = \int_{-1}^{1} p(x)e^{\pm iKx}dx, \tag{B.4}$$

$$H^{-}(K) = \overline{H^{+}}(K), \tag{B.5}$$

Subtracting or adding the formula (B.1) and (B.3) on both sides according to the surface elevation is even or odd, we have

$$-\int_{-1}^{1} \{p_e(\xi) - p_e(-\xi)\} T(x-\xi)d\xi = i\left\{e^{iKx}H_e(K) - e^{-iKx}\overline{H_e}(K)\right\}, \tag{B.6}$$

$$-\int_{-1}^{1} \{p_o(\xi) - p_o(-\xi)\} T(x-\xi)d\xi = -i\left\{e^{iKx}H_o(K) - e^{-iKx}\overline{H_o}(K)\right\}, \tag{B.7}$$

where suffix e or o stands for the quantities for the odd or even surface elevation respectively and the bar over the leter the complex conjugate. Then, if we introduce the diffraction solution without Kutta's condition as;

$$-\int_{-1}^{1} p_d(\xi)T(x-\xi,0)d\xi = \frac{i}{K}e^{-iKx}, \tag{B.8}$$

solutions of (B.6) and (B.7) are obtained directly as follows;

$$p_e(x) - p_e(-x) = K\left\{p_d(x)\overline{H_e}(K) + \overline{p_d}H_e(K)\right\}, \tag{B.9}$$

$$p_o(x) + p_o(-x) = K\left\{p_d(x)\overline{H_o}(K) + \overline{p_d}H_o(K)\right\}, \tag{B.10}$$

Namely, the odd or even part of the pressure for the even or odd surface elevation can be represented by the diffraction pressure. Integrating (B.9) and (B.10), we obtain the lift and moment M as follows;

$$\frac{\overline{H_e}(K)}{H_e(K)} = -\frac{\overline{L_d}(K)}{L_d(K)}, \quad \frac{\overline{H_o}(K)}{H_o(K)} = -\frac{\overline{M_d}(K)}{M_d(K)}, \tag{B.11}$$

$$\frac{2M_e}{KH_eM_d} = \frac{2L_o}{KH_oL_d} = \left\{\frac{\overline{L_d}}{L_d} - \frac{\overline{M_d}}{M_d}\right\}, \tag{B.12}$$

That is, the phase of all Kotchin function for which surface elevation is even or odd is the same from the equation (B.11). Especially, since the diffraction pressure consists of two parts correspondig to the even or odd surface elevation, that is;

$$p_d(x) = p_{do}(x) + ip_{de}(x), \quad \eta_d = \eta_{do} + \eta de = \frac{1}{K}(\sin Kx + i\cos Kx), \tag{B.13}$$

then its Kotchin function becomes

$$H_d^+ = \int_{-1}^{1} p_d(x)e^{iKx}dx = H_{do}^+ + iH_{de}^+, \tag{B.14}$$

$$\overline{H_d^-} = \int_{-1}^{1} \overline{p_d}(x)e^{iKx}dx = H_{do}^+ - iH_{de}^+, \tag{B.15}$$

and there is a relation deduced from the second Hanaoka's theorem ;

$$H_d^-(K) + \overline{H_d^-}(K) = 0, \tag{B.16}$$

Integrating (B.9) and (B.10) to obtain Kotchin functions, we have equalities;

$$e^{-2i\alpha} = -\frac{1 - K\overline{H_d^-}}{1 - KH_d^+}, \quad e^{-2i\beta} = \frac{1 + K\overline{H_d^-}}{1 - KH_d^+}, \tag{B.17}$$

where we define α and β as the phase of two Kotchin functions as follows;

$$H_{do}^+ = Oe^{i\alpha}, \quad H_{de}^+ = Ee^{i\beta}, \tag{B.18}$$

Then, putting these into the equation (B.16), we have finally the formulas;

$$E = \frac{\cos\alpha}{K\sin(\alpha-\beta)}, \quad O = -\frac{\sin\beta}{K\sin(\alpha-\beta)}, \tag{B.19}$$

$$1 - KH_d = -i\frac{e^{i(\alpha+\beta)}}{\sin(\alpha-\beta)}, \quad 1 + K\overline{H_d^-} = -i\frac{e^{i(\alpha-\beta)}}{\sin(\alpha-\beta)}, \tag{B.20}$$

Thus the amplitude of Kotchin function of diffraction potential without Kutta's condition is determined by its phase. The diffraction pressure with Kutta's condition, of course, relates to the present one by the formula (30), that is,

$$p_d(x) = p_d(x) + A_d p_s^*(x), \tag{B.21}$$

where we have discussed the one with the star mark in this Appendix.

Appendix C Optimization problems(Bessho 1966,1967)

This problem is very much simple in two dimension because we can get rid of all components of resistance considered here. At first, it has been pointed out by Maruo in 1949, that the spray vanishes and the resistance becomes zero, if the pressure is finite through the bottom.

Secondly, the water head resistance becomes zero, if there is no swell-up potential. Thirdly and lastly, we can obtain the pressure distribution having no trailing wave in the down-stream as follows. Let us consider an auxiliary function m(x) by the following differential equation;

$$m(x) + \frac{1}{K^2}\frac{d^2m(x)}{dx^2} = p(x), \tag{C.1}$$

where p(x) denotes the pressure distribution. Then, m(x) must be uniquely determined by the given pressure under appropriate boundary conditions, so that we may discuss m(x) instead of p(x) without losing the generality.

Now, Kotchin function may be integrated by part as follows;

$$H(K) = \int_{-1}^{1} p(x)e^{iKx}dx = \frac{1}{K}\left[\frac{dm(x)}{Kdx} - im(x)\right]_{x=-1}^{x=1}, \tag{C.2}$$

Hence, if we choose the boundary conditions of m(x) as;

$$m(x) = \frac{dm(x)}{dx} = 0 \qquad \text{for } x = \pm 1, \tag{C.3}$$

Then we have

$$H(K) = 0, \tag{C.4}$$

This means that there is no trailing wave. We may easily find such functions, for example, following functions are all wave-free.

$$m_n(-\cos\theta) = \frac{1}{2n}\left[\frac{\sin(n-1)\theta}{n-1} - \frac{\sin(n+1)\theta}{n+1}\right], \ \text{,for } n \geq 2, \tag{C.5}$$

$$p_n(-\cos\theta) = m_n(-\cos\theta) + \frac{1}{K^2}\frac{\cos n\theta}{\sin\theta}, \tag{C.6}$$

$$\left.\begin{aligned} \nabla_n &= \frac{1}{K}\int_{-1}^{1} p_n(x)dx = \frac{\pi}{8} && \text{for } n = 2 \\ &= 0 && \text{for } n > 2 \end{aligned}\right\} \tag{C.7}$$

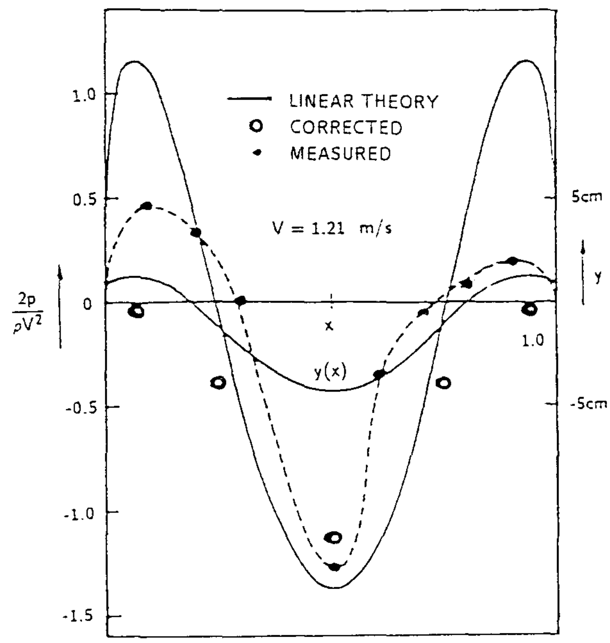

Fig. 11 Wave-free pressure distribution

Moreover, putting (C.1) into the equation (13) we have the surface elevation as follows;

$$K\eta_n(x) = -m_n(x) - \frac{1}{\pi K}\int_{-1}^{1}\frac{dm_n(\xi)}{x-\xi} = -m_n(-\cos\theta) - \frac{\cos n\theta}{nK}, \tag{C.8}$$

$$V_n = -\int_{-1}^{1}\eta_n(x)dx = \nabla_n + \frac{4}{n(n^2-1)K^2}, \tag{C.9}$$

where V_n denotes a virtual displacement volume (area here) and ∇_n the true one. Namely $K\nabla_n$ is the lift or the displacement weight. These pressure becomes infinite at both ends and there exists the spray. Therefore, if we choose a section , for example, such as;

$$\eta(x) = \eta_2(x) - \eta_4(x), \tag{C.10}$$

we may have a spray-free and wave-free pressure distribution. This section form was tested at the circulating water tunnel as shown in Fig.11.

The agreement between the calculated pressure with the experimental pressure is not good but the wave and spray vani-shed virtually at the designed speed K=1(Fr.=.7071)(Bessho 1967). The true displacement volume (C.7) is very small and almost zero practically. In fact, such wave-free pressure distribution in three dimentional case has no displacement weight(Bessho 1966).

Appendix D. On numerical results

Basic solutions shown in Table 1 are calculated and their force and moment etc. are shown in Table 2. At high speed and when K is sufficiently small, the kernel of the boundary integral equation(13) nearly equals the one of the wing theory as in Appendix A and we have approximately;

$$T(\cos\theta' - \cos\theta, 0) \to -\frac{1}{\pi}\left[C + 2\sum_{n=1}^{\infty}\frac{\cos n\theta\cos\theta'}{n}\right] + O(K), \tag{D.1}$$

$$S^*(-1-\cos\theta, 0) \to \frac{1}{2} + \frac{KC}{\pi} + \frac{K}{\pi}\left[(C-\frac{1}{2})\cos\theta - \frac{1}{3}\cos 2\theta + \frac{1}{12}\cos 3\theta - \frac{1}{30}\cos 4\theta + \cdots\right] + O(K^2), \tag{D.2}$$

where $C = \log(2/K) - \gamma$. Putting these into the integral equation (13), we obtain basic solutions in Table 1 as follows;

$$p_h^*(-\cos\theta) = \frac{1}{C}\mathrm{cosec}\theta, p_t^*(-\cos\theta) = -\cot\theta, p_c^*(-\cos\theta) = \frac{1}{4C}\mathrm{cosec}\theta(-1 + 2C\cos\theta),$$

$$p_s^*(-\cos\theta) = -\mathrm{cosec}\theta\left[\frac{1}{2C} + \frac{K}{\pi}\left\{1 + (C-\frac{1}{2})\cos\theta - \frac{2}{3}\cos 2\theta + \frac{1}{4}\cos 3\theta - \frac{2}{15}\cos 4\theta\right\}\right],$$

$$p_s(-\cos\theta) = p_s^*(-\cos\theta) - \tau_s^*\lim_{n\to\infty}\frac{\sin n\theta}{\sin\theta}, \tag{D.3}$$

$$\sigma_h^* = \tau_h^* = \frac{1}{C}, \sigma_t^* = -\tau_t^* = 1, \sigma_c^* = \tau_c^* = \frac{1}{2} - \frac{1}{4C},$$

$$\sigma_s^* = -\frac{1}{2C}\left(1 - \frac{2KC^2}{\pi}\right), \tau_s^* = -\frac{1}{2C}\left(1 + \frac{2KC^2}{\pi}\right), \tag{D.4}$$

$$L_h^* = \frac{\pi}{C}, L_t^* = 0, L_c^* = -\frac{\pi}{4C}, L_s^* = -\left(K + \frac{\pi}{2C}\right),$$

$$M_h^* = 0, M_t^* = \frac{\pi}{2}, M_c^* = 0, M_s^* = \frac{K}{2}\left(C - \frac{1}{2}\right), \tag{D.5}$$

$$A_h = \frac{2}{1+2KC^2/\pi}, A_t = -\frac{2C}{1+2KC^2/\pi}, A_c = \frac{C-1/2}{1+2KC^2/\pi},$$

$$A_s = -A_t, A_d = -2\frac{C-i/K}{1+2KC^2/\pi}, \tag{D.6}$$

$$L_h = -KA_h - KA_t, L_t = \frac{2KC+\pi}{1+2KC^2/\pi}, L_c = -\frac{\pi}{2} + \frac{K(C-1)(C-1/2)}{1+2KC^2/\pi},$$

$$L_s = -L_t, L_d = \frac{\pi + 2KC + 2i(C-1)}{1+2KC^2/\pi}, \tag{D.7}$$

$$M_h = -KA_c, M_t + M_h = -L_c, M_c = \frac{K(C-1/2)^2}{2(1+2KC^2/\pi)}, M_s = \frac{KC(C-1/2)}{1+2KC^2/\pi},$$

$$M_d = \frac{\pi}{2} + \frac{i(C-1/2) - KC(C-1/2)}{1+2KC^2/\pi}, \tag{D.8}$$

$$\sigma_h = \frac{2KAs}{\pi}, \sigma_h + \sigma_t = -\frac{2Ls}{\pi}, \sigma_c = -\frac{2M_s}{\pi}, \sigma_s = \pm 1 - \frac{1-2KC^2/\pi}{1+2KC^2/\pi},$$

$$\sigma_d = \frac{2+4iC/\pi}{1+2KC^2/\pi}, \tag{D.9}$$

$$F_h = -KA_d, F_t + F_h = L_d, F_c = -M_d, F_s = -\frac{\pi\sigma_d}{2}, F_d = \frac{2/K + (\pi + 4iC - 2i)}{1+2KC^2/\pi}, \tag{D.10}$$

These approximations are not very accurate but satisfy the reciprocities (33)-(35) and show a trend at high speed. Accurate numerical results obtained by solving boundary integral equations are shown in Table 3 as mentioned the above. It is to be remarked in this table that the lift for a unit rise of a flat plate or a barge becomes positive for $K \leqq 0.13$(Fr. $\geqq 2.0$) . Namely, the lift increases as the barge rises up. This phenomenon might mean another instability rather than the one described here. Although the relation between these instabilities is not clear, we might point out here that the instability of a planing flat plate when it oscillates in heaving or pitching mode is caused by the variation of the wetted length. At low speed, numerical results are all monotonic even Kotchin functions and we are able to represent by simple curve fittings as shown in Table 3 for the range $50 > K > 5$. The accuracy is less than one percent at $K = 50$.