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Numerical analysis of shear crack propagation in a concrete beam without transverse reinforcement

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Abstract

Crack formation and growth in reinforced concrete members is, in many cases, the cause for the collapse of buildings. Such serious failures impair the structural behavior and can also damage property and persons. An intensive investigation of the crack propagation is indispensable. Numerical methods are being developed to analyze crack growth in a concrete member and to detect fracture failure at an early stage. For reinforced concrete components, however, further research and action is required in the analysis of shear cracks. This paper presents numerical simulations and continuum mechanical modeling of bending shear crack propagation in a concrete beam without transverse reinforcement. As numerical method to map discrete cracks the extended finite element method (XFEM) is applied. The crack propagation is compared with the smeared crack approach using concrete damage plasticity material model. For validation, the crack patterns of a real beam tests are compared with the results of the different finite element models. The numerical analysis will provide further understanding of crack growth and redistribution of inner forces in concrete members. The XFEM makes it possible to predict the fracture behavior of concrete members.

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Keywords: crack propagation; fracture mechanics; extended finite element method; concrete damage plasticity

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1. Introduction

The numerical simulation of crack propagation and the analysis of crack growth in reinforced concrete members is still an unsolved problem and an important part of current research. Mostly, the traditional finite element method (FEM) is used. Discontinuities like cracks can't be modelled, as the FEM is based on a continuum approach. Cracks are mapped by regions of high strain rates (smeared crack method). This effect of smeared crack formation is due to the division of the crack opening into an equivalent element length of a finite element. However, this approach doesn`t represent the real crack pattern because the location of the distortion and the discontinuity in the displacement field are not mapped. Alternatively, in the discrete crack approach discontinuities are introduced at the element edges. This method is associated with a high numerical effort, due to the continuous re-meshing in each iteration step. To avoid this disadvantage, several numerical approaches have been developed in recent decades. Numerical mechanics offers the possibility to embed the approaches as extensions into the FE or to formulate certain approach functions in such a way that they are no longer bound to the finite elements. Even if there are possible advantages in methods of the last possibility regarding simulations accuracy and adaptability, the computational effort is relatively high, so that for practical applications the extensions of the conventional FE are often used. The extended finite element method (XFEM) offers one of these promising analysis methods. In contrast to other approaches, XFEM offers the advantage of a much shorter calculation time, the simplicity of the initial crack definition and generation of the required FE mesh, and a simplified application. Using the Partition of Unity Method (PUM) and considering additional degrees of freedom, discontinuities can be described mesh-independently. The numerical method offers the possibility to model cracks as strong discontinuities within the finite elements. Concerning to reinforced concrete members, however, the prediction of crack propagation is still largely limited by the analysis method mentioned.

In this paper the numerical crack simulation, using XFEM, is briefly explained and the method is validated exemplarily with the crack pattern of beam tests from the test series of Nghiep (2011), which was conducted in the framework of his research at the Hamburg University of Technology (TUHH). Particularly with regard to the fracture and failure behavior of reinforced concrete members without shear reinforcement, analyses are also performed with the elastic-plastic material model „Concrete Damage Plasticity (CDP)“ for concrete. The results of this approach and the XFEM-simulations are compared with the test results. The CDP model assumes an isotropic damage based on a combination of plasticity and damage theory (Lee and Fenves (1998)). The software package Abaqus (Dassault Systèmes (2012)) is used for the numerical simulations. The results show a good agreement among each other.

2. Basic formulations of the XFEM

In conventional FEM, discontinuities are mapped by refining the element mesh or by increasing the polynomial degree of the used form function. However, this refinement approach is associated with a high numerical effort. To avoid this effort and to avoid permanent re-meshing during simulation, Belytschko and Black (1999) introduced the XFEM. Based on the Partition-of-Unity Method (PUM), developed by Babuska and Melenk (1997), it is possible to integrate discontinuous geometries into the classic FE form function and to perform a mesh independent crack propagation analysis. Sections 2.1 to 2.4 briefly explain the concept of XFEM in conjunction with the PUM and the Level-Set-Method (LSM) defined for locating cracks.

2.1. Partition of unity Method (PUM)

The PUM extend the conventional finite element form function with additional enrichment functions (see eq. 1) to consider discontinuities at the crack boundaries. Eq. (1) consists of the standard FE approximation $N_I(x)$ combined with a discontinuity function, called Heaviside function $H(x)$ to represent displacement jumps across the crack faces and asymptotic function $F_a(x)$ to model singularities at the crack tip.

$$u^h(x) = \sum_{I \in N} N_I(x) \cdot \left[u_I + \underbrace{H(x) \cdot a_I}_{I \in N_\Gamma} + \underbrace{\sum_{a=1}^{4} F_a(x) \cdot b_I^{\,a}}_{I \in N_\Lambda} \right] \quad (1)$$

For the XFEM displacement interpolation the respective degrees of freedom (DOF) have to be considered in the equation. u_I describes the vector of the nodal degrees of freedom of the standard FE function $N_I(x)$. The vector a_I consist of the node variable with enriched degrees of freedom for the jump discontinuity. b_I^a are the nodal degrees of freedom for the crack tip enrichment.

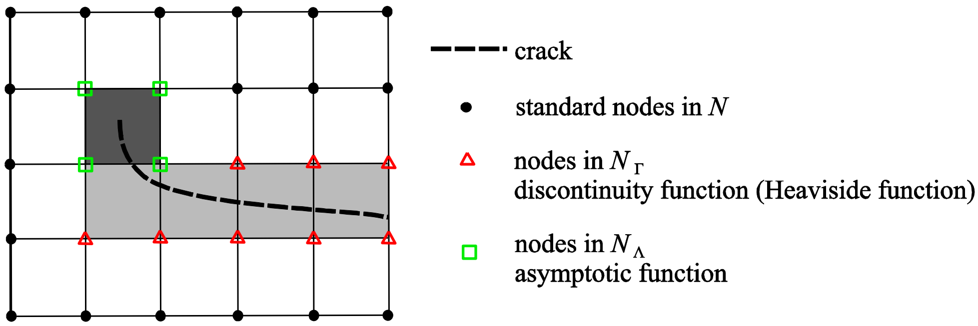

Fig. 1. Crack representation with associated XFEM nodes

Fig.1 shows a crack through an element mesh with the sets described in equation (1). N is a set which contains all nodes in the FE-model. N_Γ, on the other hand, considers all enriched nodes on the crack edges. N_Λ belongs to the elements at the crack tip. The crack tips enrichment term is only considered for stationary cracks. The focus of this paper is on propagating cracks, the Heaviside enrichment term, and the final crack growth is discussed in more detail.

2.2. Heaviside enrichment term

To map the completely cracked elements, the Heaviside function is selected as a jump function along the crack geometry. Above the crack, the discontinuity function $H(x)$ takes the value 1 and below it the value -1 (Fig. 2).

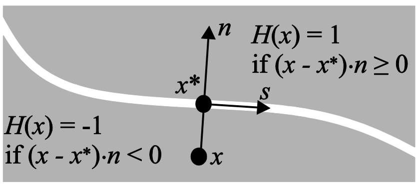

Fig. 2. Coordinate representation of a crack with the integration point x for $H(x) = -1$

The point x represents an integration point in a finite element and x^* is the point on the crack with the shortest distance to x. The normal vector to x^* is declared with n. In order to discretely describe the displacement jump over the crack surfaces, the phantom node method is used which is valid for growing cracks and therefore only considers the Heaviside enrichment over the crack surfaces. The enrichment at the crack tip is not taken into account.

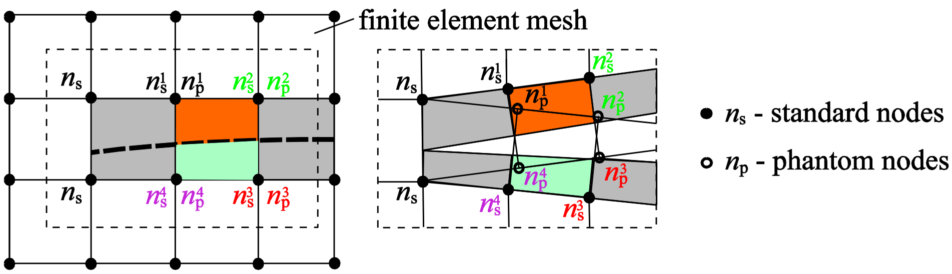

Fig. 3. Principle of phantom nodes

As shown in Fig. 3, the discontinuous elements receive a second set of nodes n_p (phantom nodes) over the real standard node n_s instead of additional nodal degrees of freedom in the conventional discrete crack approach. If an element is now separated by a crack, two separate elements are created, which have independent displacement fields and thus describe the discontinuity. In this context, the Heaviside function and the phantom node method can be used to discreetly map a crack. However, before the position of the discontinuity must be estimated. These properties are provided by the Level-Set-Method (LSM).

2.3. Level-Set-Method (LSM)

Discontinuities (here the crack) can be described with the LSM. The crack is represented by a zero-level-set ($\phi(x) = 0$ and $\psi(x) < 0$). The nodes surrounding the crack are assigned displacement values which are interpolated with the Heaviside enrichment term for the crack propagation description and describe the crack in relation to the crack position ($\phi(x) = 0$ und $\psi(x) < 0$) and to the crack tip ($\phi(x) = 0$ und $\psi(x) = 0$). Fig. 4 illustrates a crack through an element mesh with the respective level set functions $\phi(x)$ and $\psi(x)$.

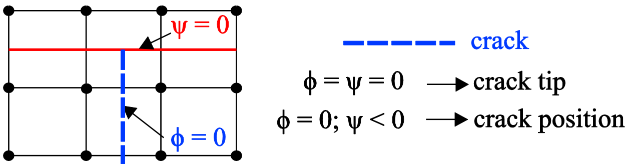

Fig. 4 Level-Set-Functions ϕ(x) and ψ(x)

2.4. Basic settings of the numerical simulations

The FE-software Abaqus offers two different methods to model the fracture propagation using XFEM, the cohesive zone model (CZM) and the virtual crack closing technique (VCCT). The latter is based on the concept of linear elastic fracture mechanics. In VCCT, the energy absorbed by material fracture is assumed as the work required to close the crack surface. The CZM is based on damage mechanics and uses traction-separation relations. Fracture is initiated when a damage criterion is met. Using the CZM in this analysis, the crack progresses when the maximum principal stress (MAXPS) reaches the critical value $f = 1$. For the damage propagation an energy damage evolution approach is used which is based on a power law fracture criterion. The relevant material data for concrete are taken from the Abaqus Analysis User´s Guide (2012) and are estimated as follows: Maximal principal stress σ_{max} = 10.45 MPa, normal mode fracture energy G_{IC} = 19.58 N/m, shear mode fracture energy first direction G_{IIC} = 19.58 N/m and shear mode fracture energy second direction G_{IIIC} = 19.58 N/m.

For the CDP material model, the default settings of the FE program are used. The simulation for the beam described in chapter 4 is started with a dilatant angle of ψ = 30°, an eccentricity ε = 0.1, a compression plastic strain ratio σ_{b0}/σ_{c0} = 1.16 and an invariant stress ration K_c = 0.667.

3. Numerical simulation

3.1. Beam tests

The numerical simulation is conducted for beams no. 1L-2, 5L-2, 1K-1 and 4K-1 from Mr. Nghiep´s (2011) test series. The beams differ in their dimensions and in contrast to the beams no. 1L-2 and 1K-1, the beams no. 5L-2 and 4K-1 are haunched. Table 1 summarizes the associated dimensions and material properties. Fig. 5&6 shows the test specimen no. 1L-2 with geometries often used in practice. The effective span is l_{eff} = 3.0 m. The beam has a height of h = 340 mm and the effective depth is d = 302 mm. The shear span to depth ratio is thus a/d = 5.0. All beams are reinforced with three bars diameter Ø20 mm. In order to prevent an anchorage fracture, stirrup reinforcement was arranged in the area behind the support. The girder is supported on both sides on rollers. It was loaded in mid-span up to fracture via a steel plate with the dimensions 100 × 200 × 50 mm.

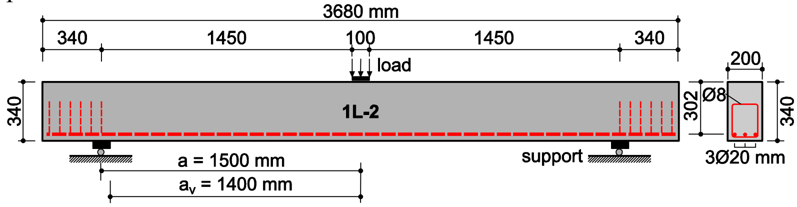

Fig. 5 Geometrical dimensions of test beam 1L-2

Fig. 6 Test setup of test beam 1L-2

Table 1. Main parameters of the concrete beam

beam	d (mm)	b (mm)	a/d	ρ_l (%)	f_{ck} [MPa]	$f_{ct,sp}$ [MPa]	E_c [MPa]	f_y [MPa]	F_u [kN]	failure
1L-2	300-300	200	5	1.5-1.5	49.2	3.6	31,294	550	158.4 kN	shear
5L-2	150-300	200	5	1.5-3.1	53.2	3.6	31,294	550	207 kN	flexure
1K-1	300-300	200	3	1.5-1.5	53.8	3.6	31,294	550	151.3 kN	shear
4K-1	150-300	200	3	1.5-3.1	54.8	3.6	31,294	550	169.5 kN	shear

with:
- d is the effective depth
- b is the overall width of a beam
- a/d is the shear slenderness
- ρ_l is the reinforcement ratio
- f_{ck} characteristic compressive cylinder compressive strength
- $f_{ct,sp}$ is the tensile strength in split tensile test
- E_c is the modulus of elasticity of concrete
- f_y is the yield strength of reinforcement
- F_u is the load at failure

3.2. FE- beam model

Fig. 7 shows a sketch of the finite element model. Taking the symmetry condition into account, only half of the beam is discretized. A rigid bond is assumed between the reinforcement bars and the concrete. The reinforcement bars are modelled as embedded truss elements.

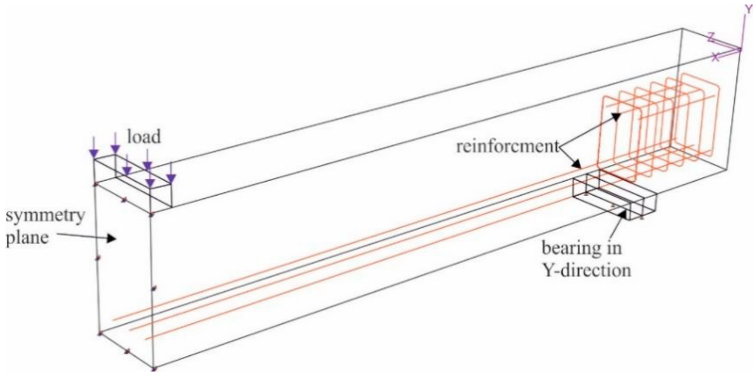

Fig. 7 FE-Model of test beam 1L-2

4. Results of numerical crack propagation in comparison with real beam test

At first it was investigated how the numerical crack propagation of the beams develops under specification with only one crack initiation. Subsequently, a second crack initiation is introduced and the final fracture behavior of the different beams are compared. Fig. 8 shows the opening of a crack from the XFEM simulation of the test beams 1L-2 at a load just before failure. For illustration a higher scaling was used. The inclined shear crack can be seen clearly.

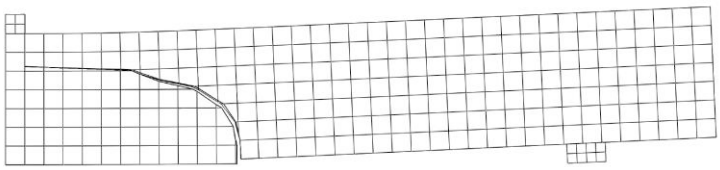

Fig. 8 Shear crack of test beam 1L-2 just before failure load

In order to compare the crack propagation of the numerical simulations with the crack pattern of the real test, Fig. 9 shows a comparison of the crack development for different load steps using CDP and XFEM. For a better contrast the numerical system is shown without the FEM mesh. For the XFEM analysis a crack initiation with a crack-plane is predefined in the FE-model similar to the real test. It can be seen that the crack pattern is in good agreement at all load steps. The horizontal crack along the reinforcement can't be modelled with XFEM, as the point and the orientation of crack initiation was fixed. The crack pattern at failure of the investigated beams is plotted in Fig. 10. Opposite are the crack patterns with only one crack-plane and with two crack-planes. The crack patterns also show good agreement independent of the number of crack initiations. The approaching of the two cracks near the load introduction, as observed in the experiment, is clearly visible in Fig 10(a). Fig. 10(b) and 10(d) show the shear failure and the rapid growth of the shear crack compared to the flexural crack. The cracks due to flexure failure are plotted in Fig. 10(c). Here a ductile failure occurred with the gradual widening of the critical flexural crack at the mid-span of the beam. The results confirm that the crack propagation analysis using XFEM reliably maps the crack pattern of the test beams.

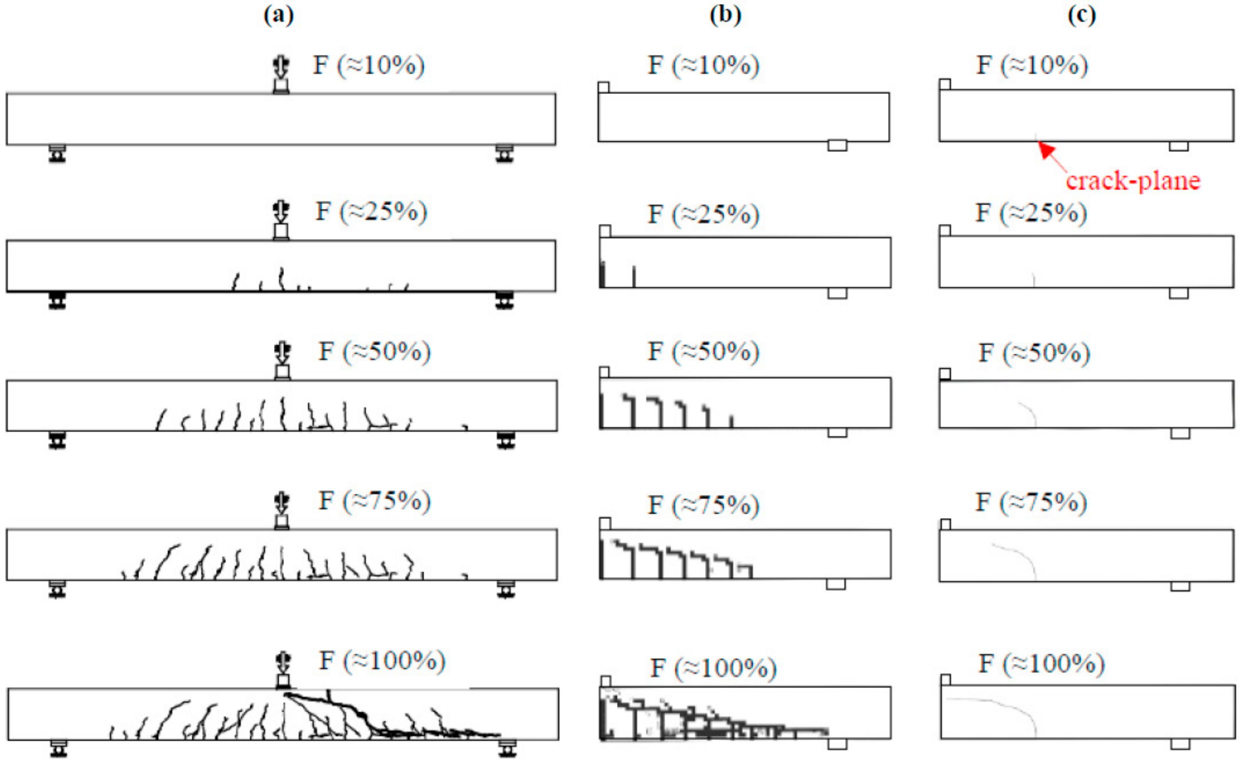

Fig. 9 Crack propagation of test beam 1L-2 from (a) test, (b) FEM with CDP and (c) XFEM

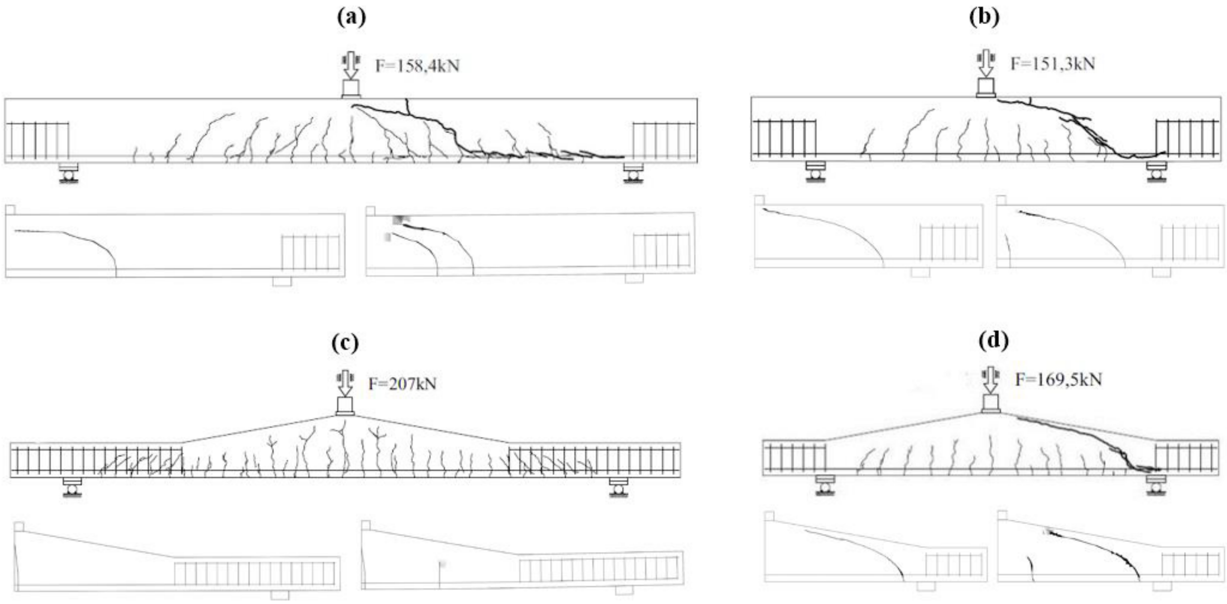

Fig. 10 Comparison of crack propagation at failure with one and two crack-planes of test beams (a) 1L-2, (b) 1K-1, (c) 5L-2 and (d) 4K-1

5. Conclusion

Numerical mechanics constantly provides new and improved discretization methods to describe the mechanical behavior of components as realistically as possible. In addition to the usual methods, which are based on the principle of virtual work and the formulation of the equilibrium condition in integral form as well as the discretization of deformations by extending local approach functions, more and more ‘unconventional’ discretization methods are being developed, such as the "Method of Virtual Elements" (VEM), the "Phase-Field Modelling" or the "Element-Free-Galerkin Method" (EFG). In terms of accuracy and adaptability, these methods provide possible advantages. If, however, these methods are considered to be suitable for practical applications for simple and quick analyses, they must still be considered with caution. The computational effort is also relatively high in this case. In addition, these topics are currently still being investigated and are themselves the focus of research. In order to obtain a realistic simulation of the component behavior and the crack prognosis which is practicable and realistic for the civil engineer, it is possible to embed the numerical approaches in the FE as extensions. The extended finite element method (XFEM) offers one of these promising analysis methods and offers an optimal solution for 2D and 3D structures. This paper compares the crack pattern of a real beam test with the results of two different numerical simulations. The results clearly demonstrate that both the FE-analysis with the CDP material model and the crack propagation analysis using XFEM show good agreement with the real test. This outcome is based on the fact that the numerical tool is suitable for a further study in which, i.e., crack velocity, crack displacement and crack tendency in connection with fracture energy and material parameters are suitable. For this study, however, further tests are necessary, where the above mentioned points will finally be recorded and validated. Also of interest are investigations of the crack frictional force. In this context, the basic formulations of XFEM and e.g. the programming language Python can be used to develop a formulation for determining the crack friction force in concrete and compare with various shear design models. However, the interpretation of the results requires an understanding of the flow of forces in a concrete member, which can be developed by observing real test beams.

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