

# Towards neural network-based numerical friction models

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Dynamics

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Technology

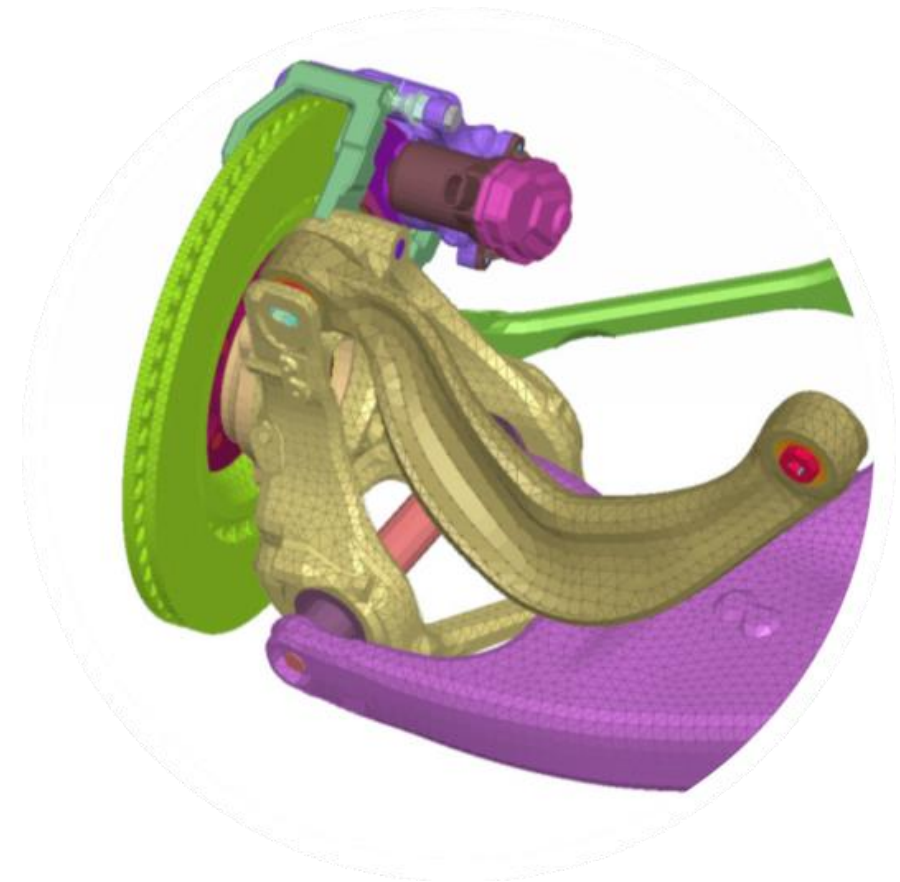
**Prof. Norbert Hoffmann**

Head of the Dynamics Group

Hamburg University of  
Technology

# Motivation

- Frictional contact behavior is hard to describe
- Large variety of parameters (velocity, pressure, temperature, humidity, load history,...)
- Difficult to represent contact behavior in numerical simulations properly
- Analytical models often highly simplified
- Data-driven approaches open up new opportunities

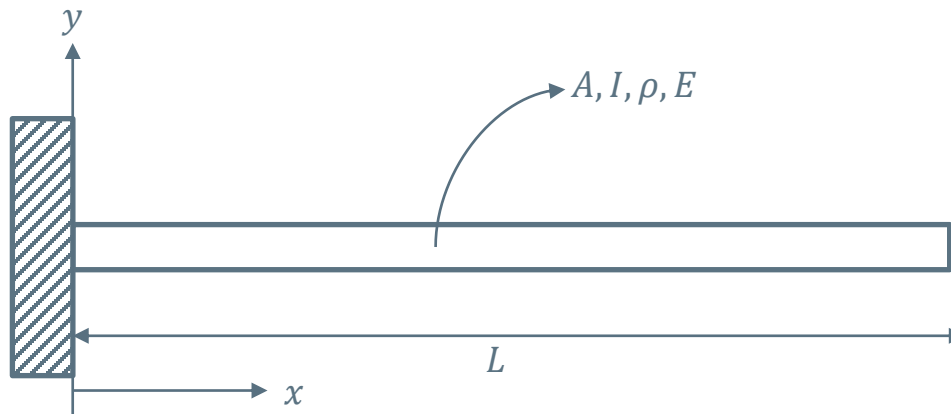


# Proceeding

- I. Setup of a transient 2-D plane-stress Finite Element model
- II. Training and validation data generation
- III. Selection of a neural network for regression
- IV. Network training and performance assessment
- V. Neural network model deployment within FE simulation

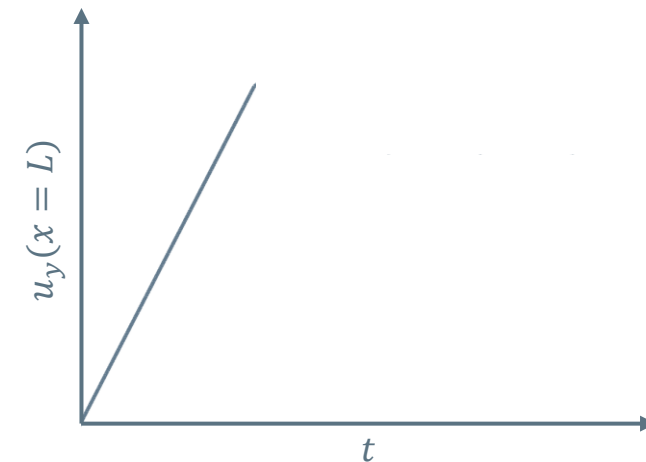
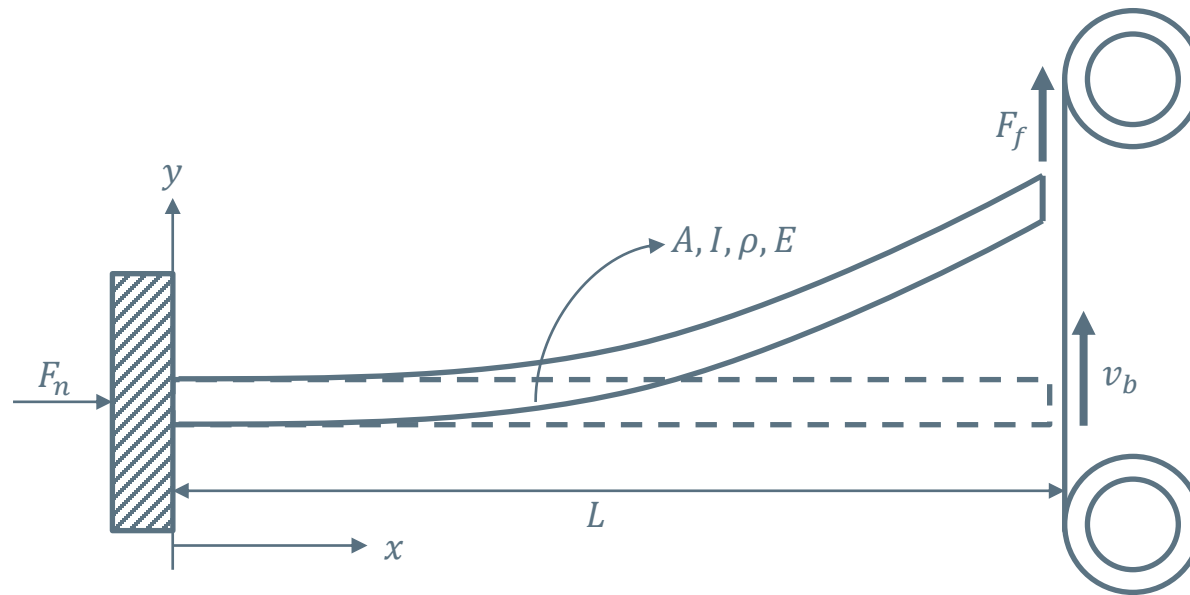
# Cantilever beam model

- Slender cantilever beam obeying Euler-Bernoulli beam theory



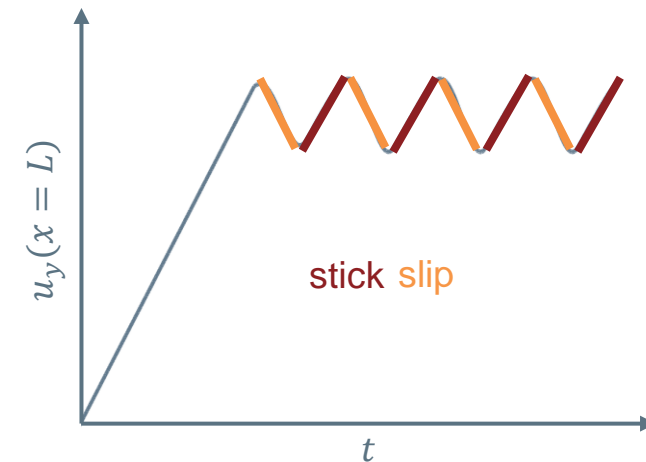
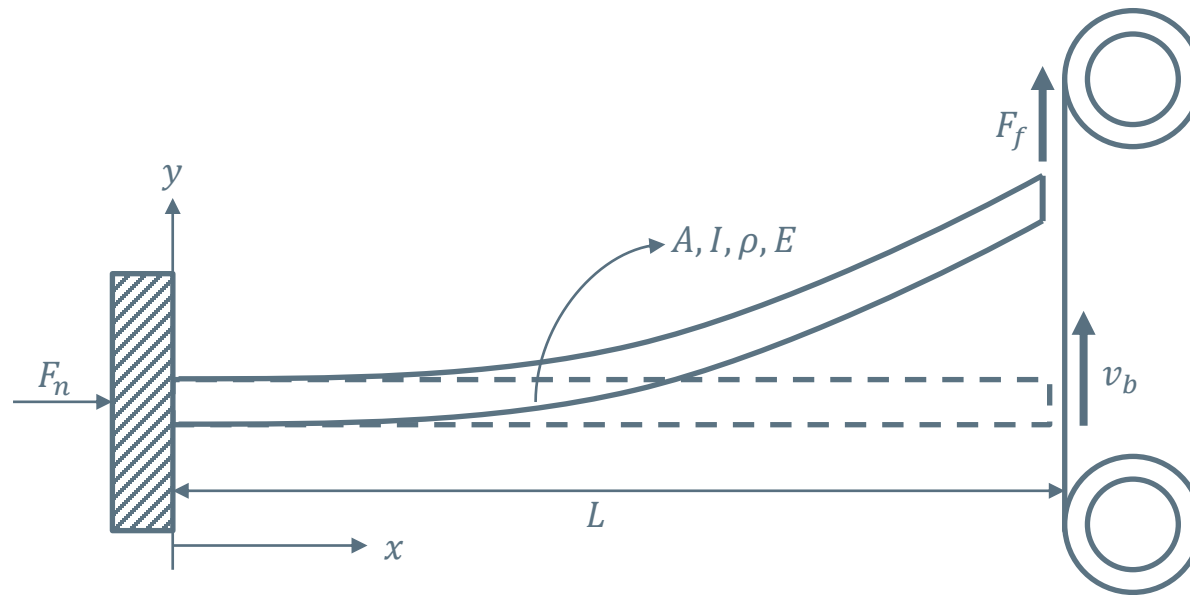
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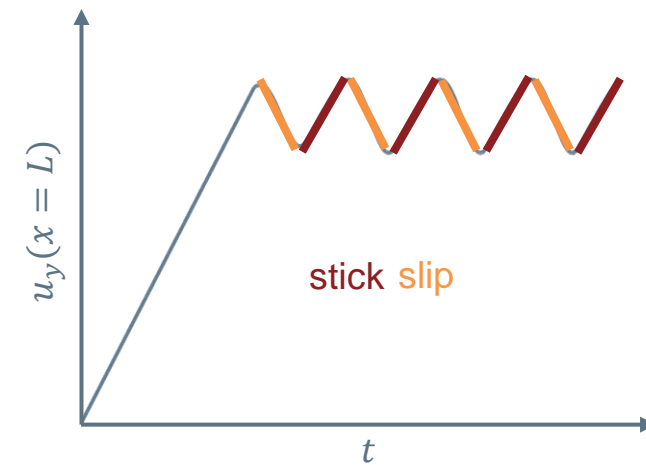
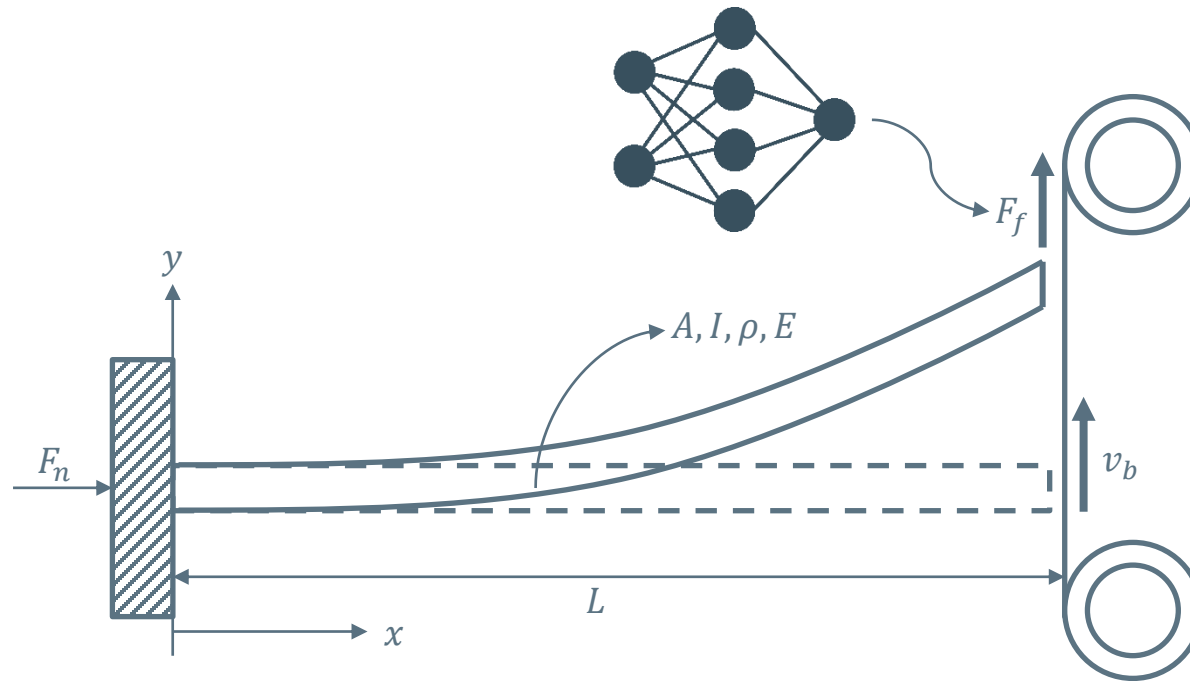
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- Slender cantilever beam obeying Euler-Bernoulli beam theory
- Frictional contact with moving belt induces stick-slip vibration at the free end



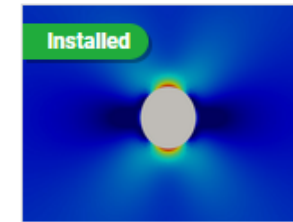
# Cantilever beam model

- Slender cantilever beam obeying Euler-Bernoulli beam theory
- Frictional contact with moving belt induces stick-slip vibration at the free end
- Kinetic friction force estimated by neural network model



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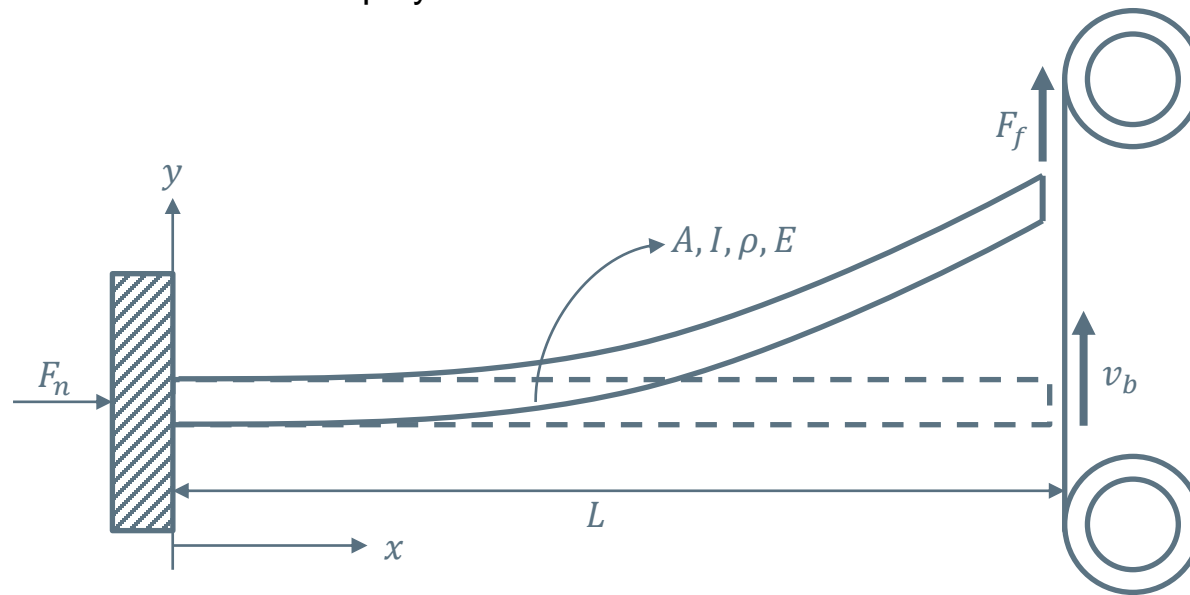


## Partial Differential Equation Toolbox

R2022a by MathWorks

Solve partial differential equations using finite element analysis

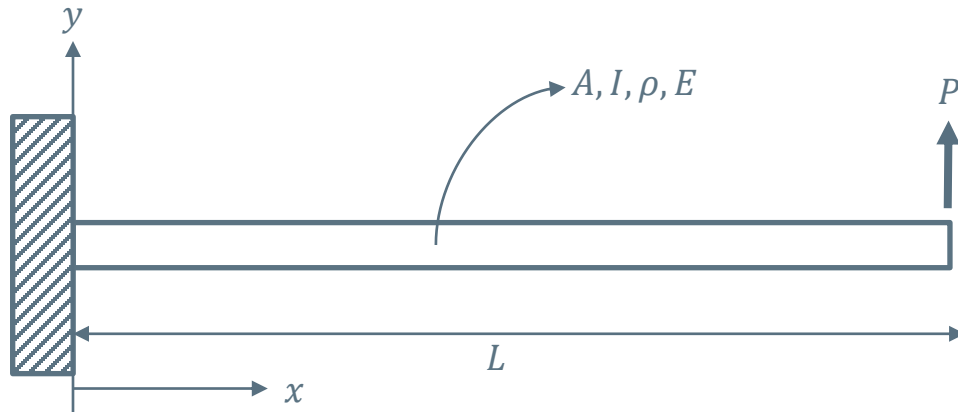
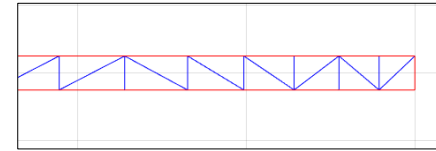
MathWorks Toolbox





# Mesh convergence study

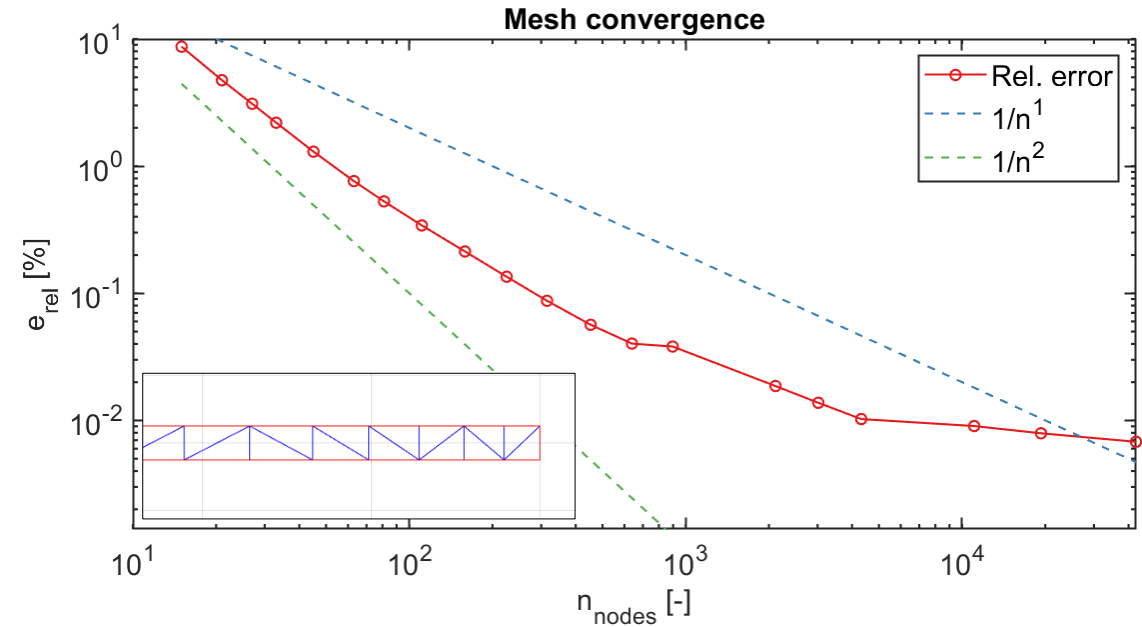
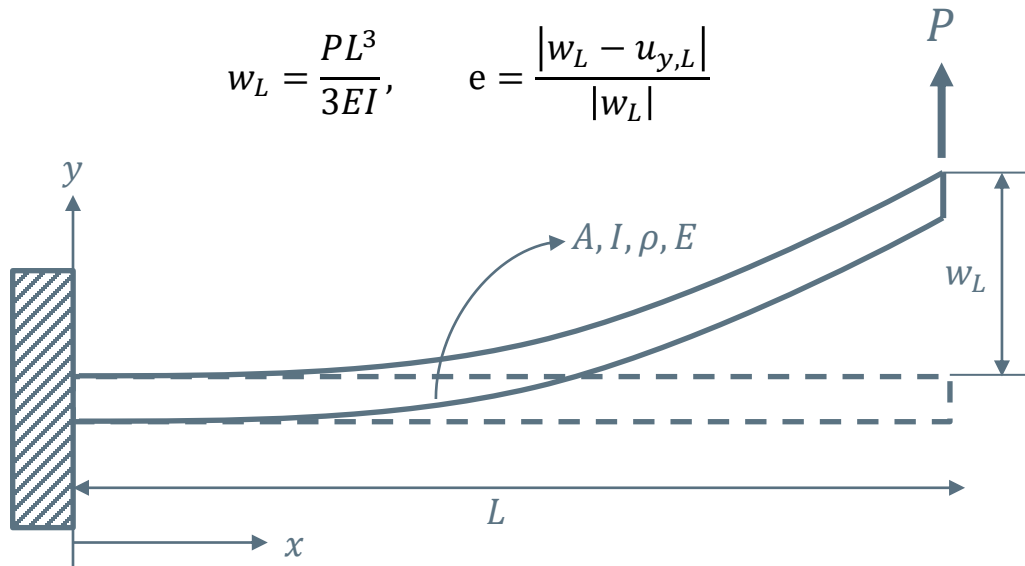
- Static Finite Element analysis of a 2-D plane-stress problem
- Triangular mesh of quadratic Finite Elements
- Cantilever beam subjected to a point load at the free end



# Mesh convergence study

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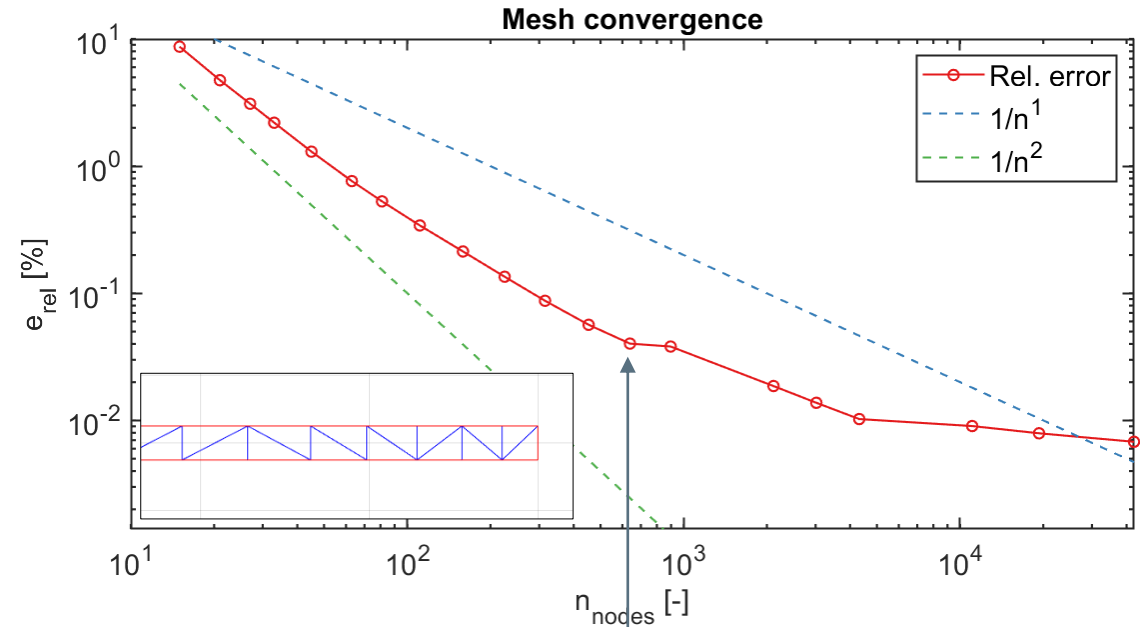
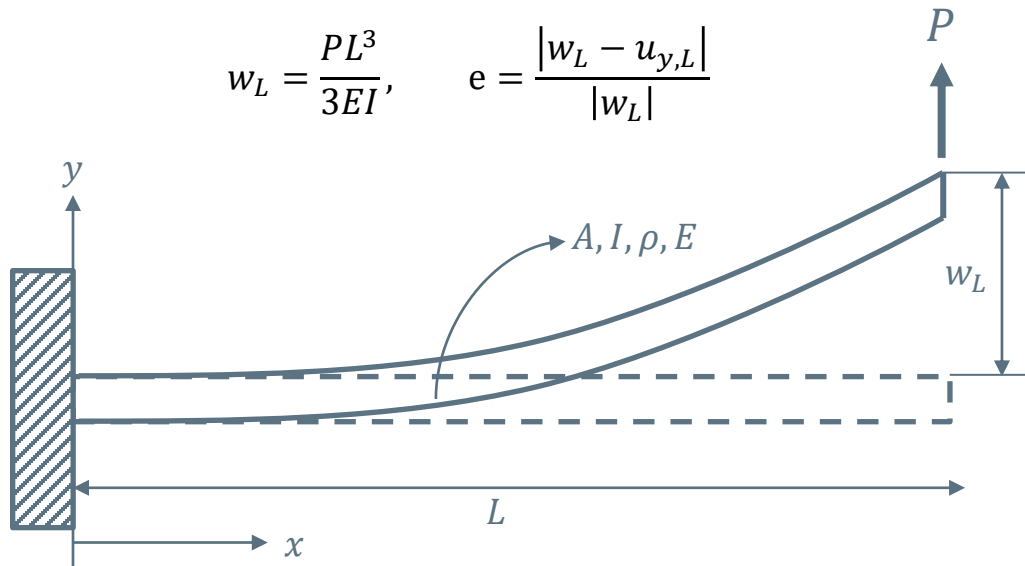
$$w_L = \frac{PL^3}{3EI}, \quad e = \frac{|w_L - u_{y,L}|}{|w_L|}$$



# Mesh convergence study

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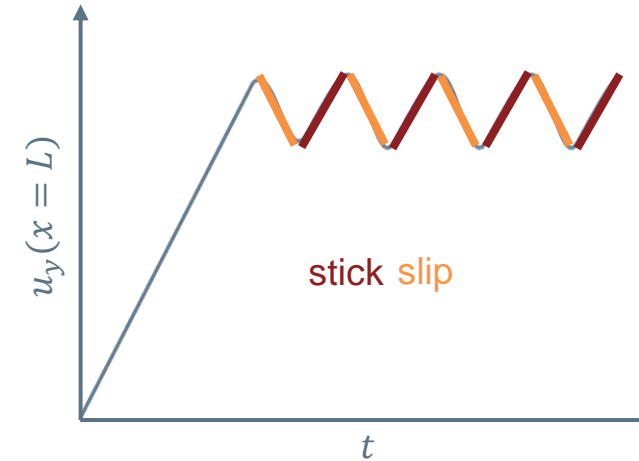
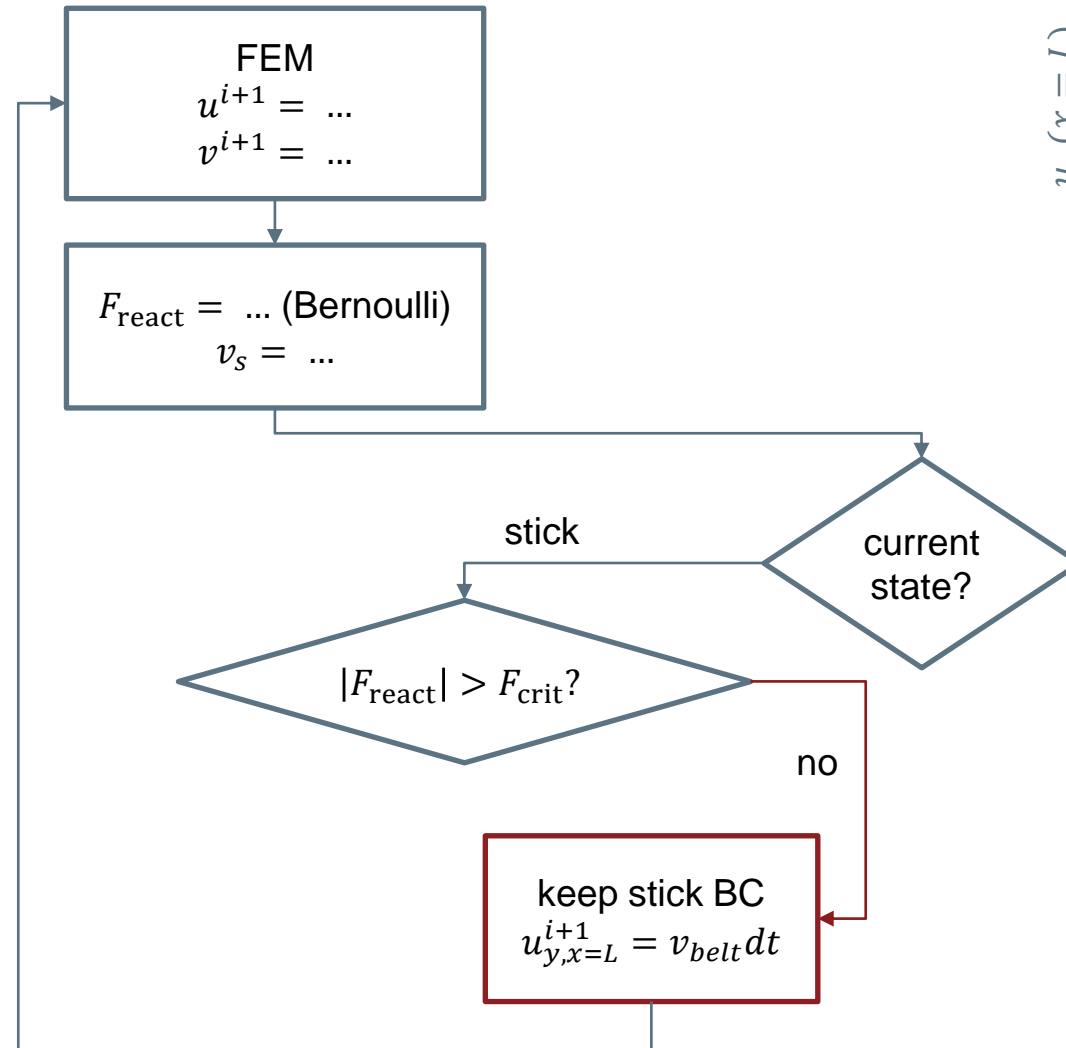
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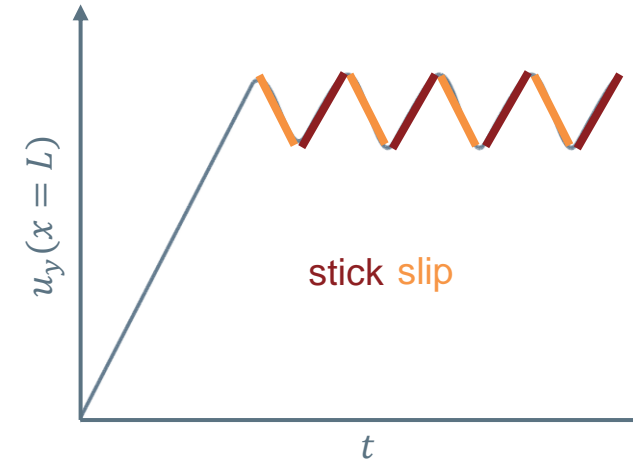
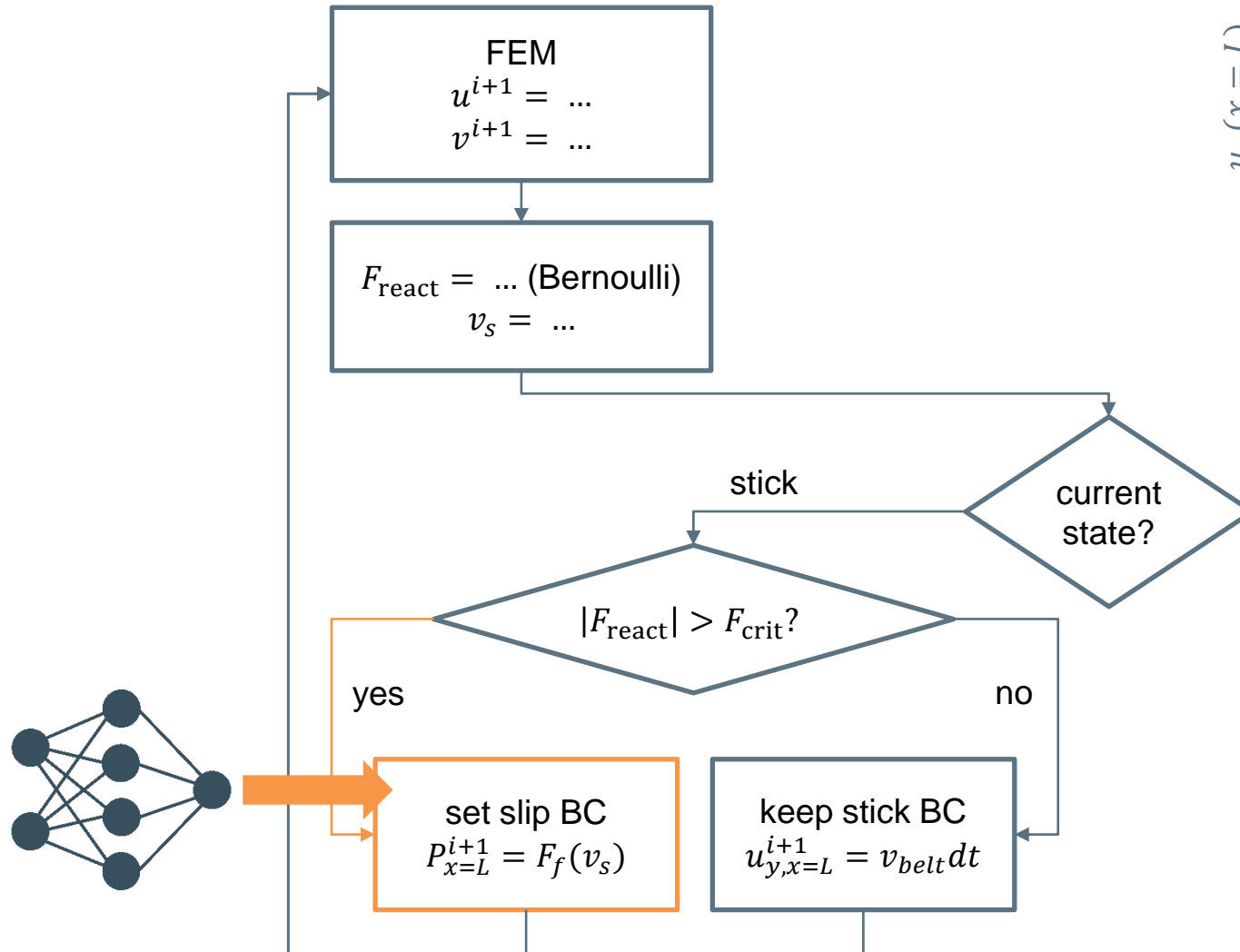
- Choose  $h_{max} = 0.02$
- Very low relative error
- Number of nodes still manageable

$h = 0.1153$	nodes =	111	$e = 0.3417 \%$
$h = 0.0807$	nodes =	159	$e = 0.2133 \%$
$h = 0.0565$	nodes =	225	$e = 0.1350 \%$
$h = 0.0395$	nodes =	315	$e = 0.0873 \%$
$h = 0.0277$	nodes =	453	$e = 0.0565 \%$
$h = 0.0194$	nodes =	639	$e = 0.0402 \%$
$h = 0.0136$	nodes =	897	$e = 0.0381 \%$
$h = 0.0095$	nodes =	2113	$e = 0.0186 \%$
$h = 0.0066$	nodes =	3019	$e = 0.0138 \%$

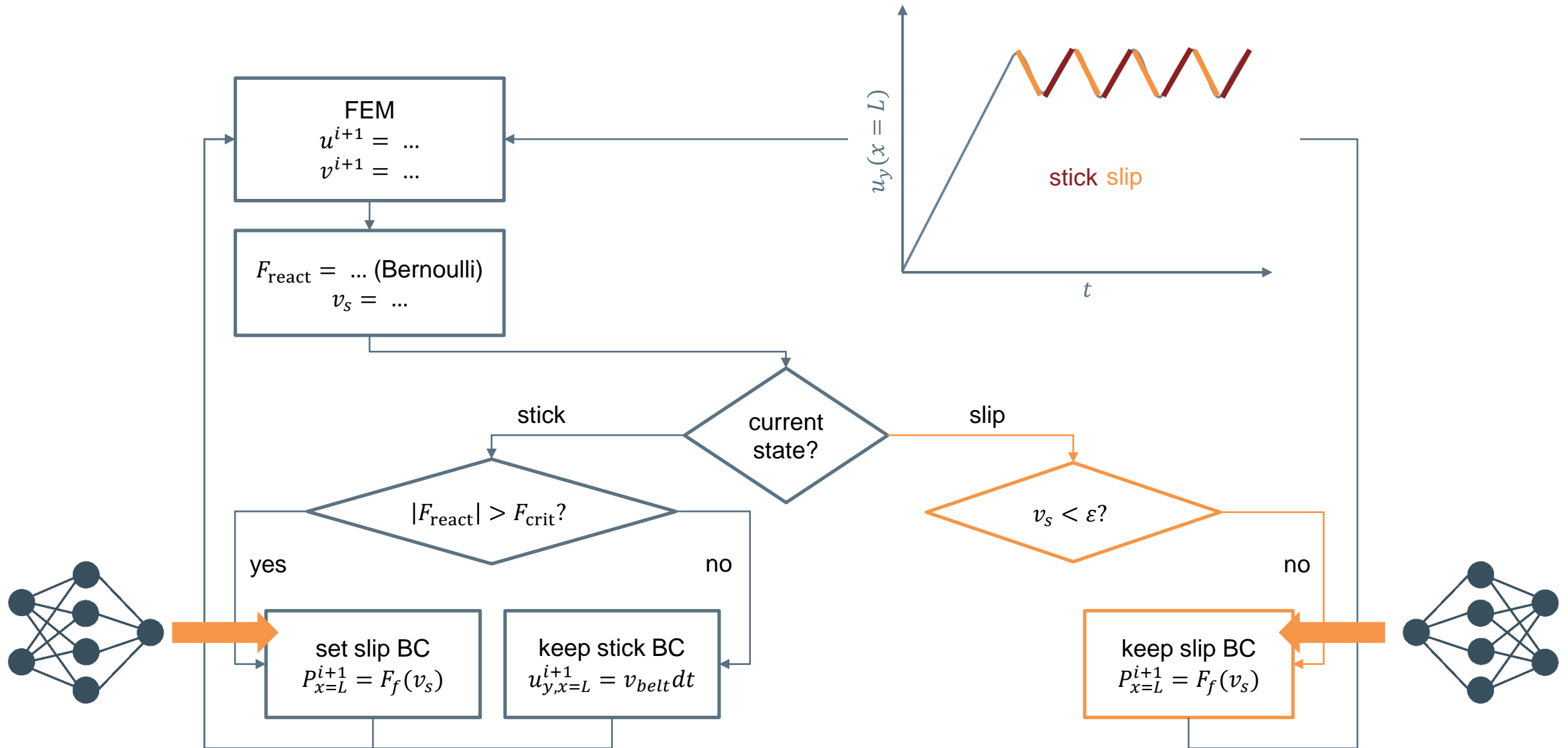
# Transient Finite Element algorithm



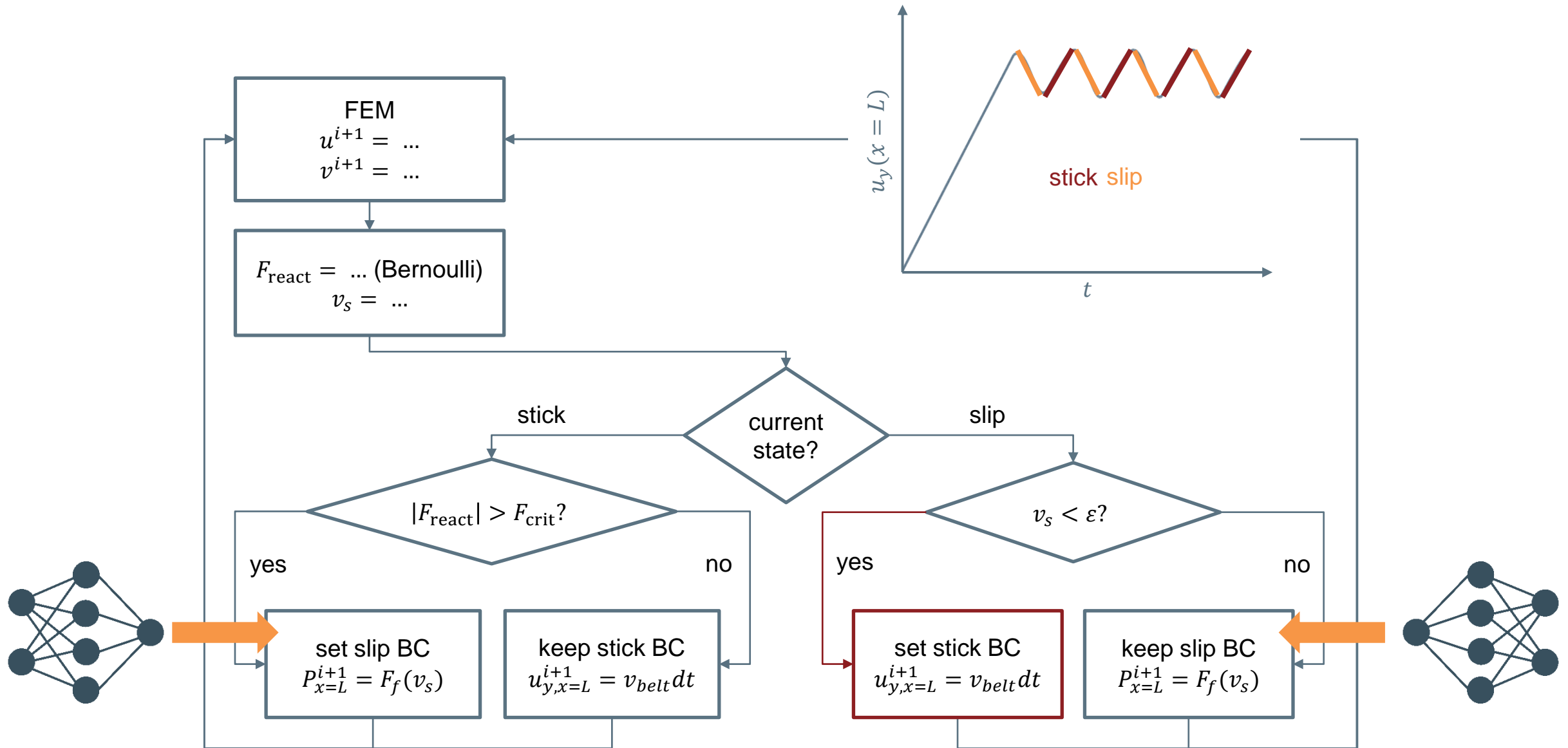
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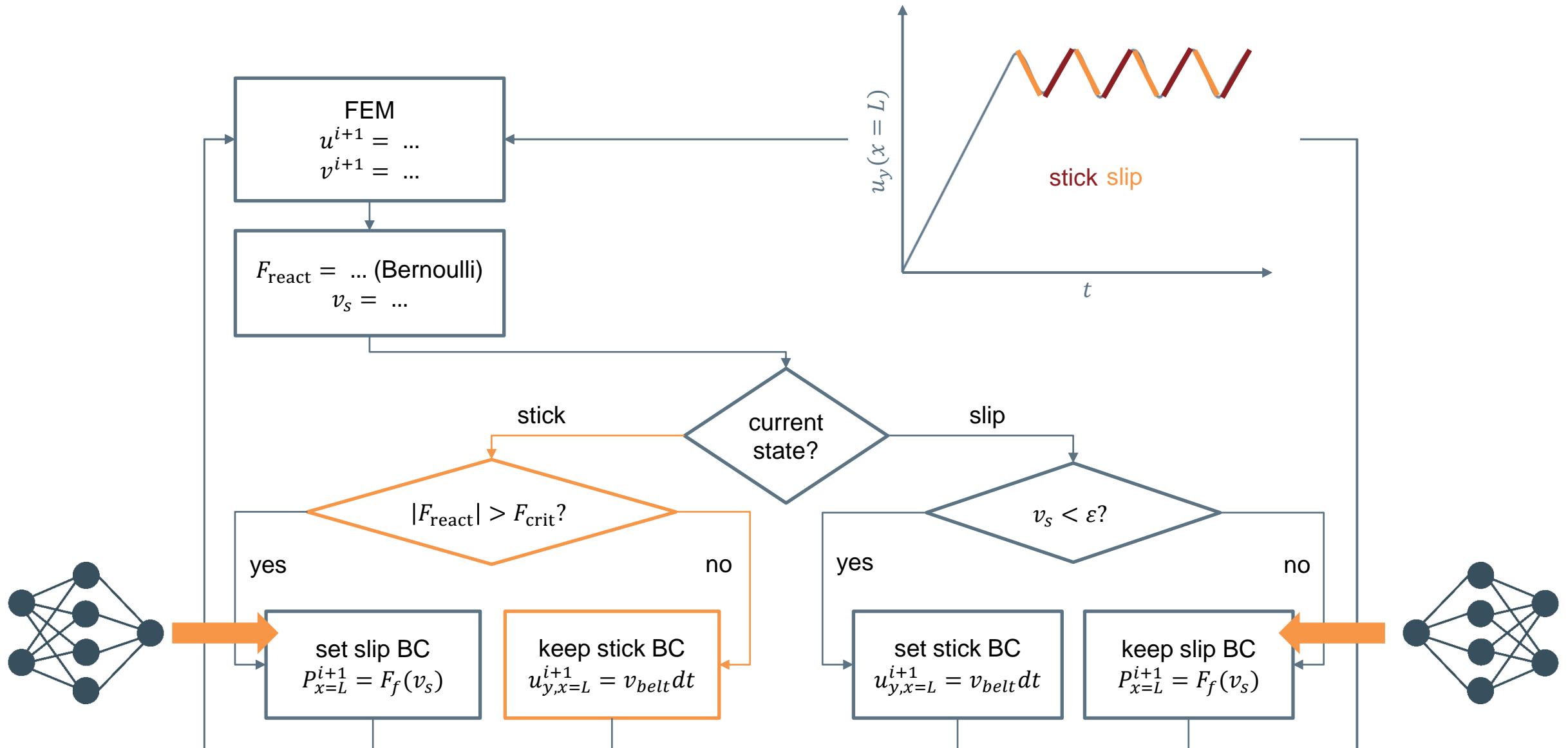
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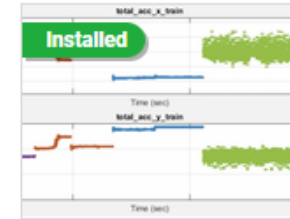
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# Proceeding

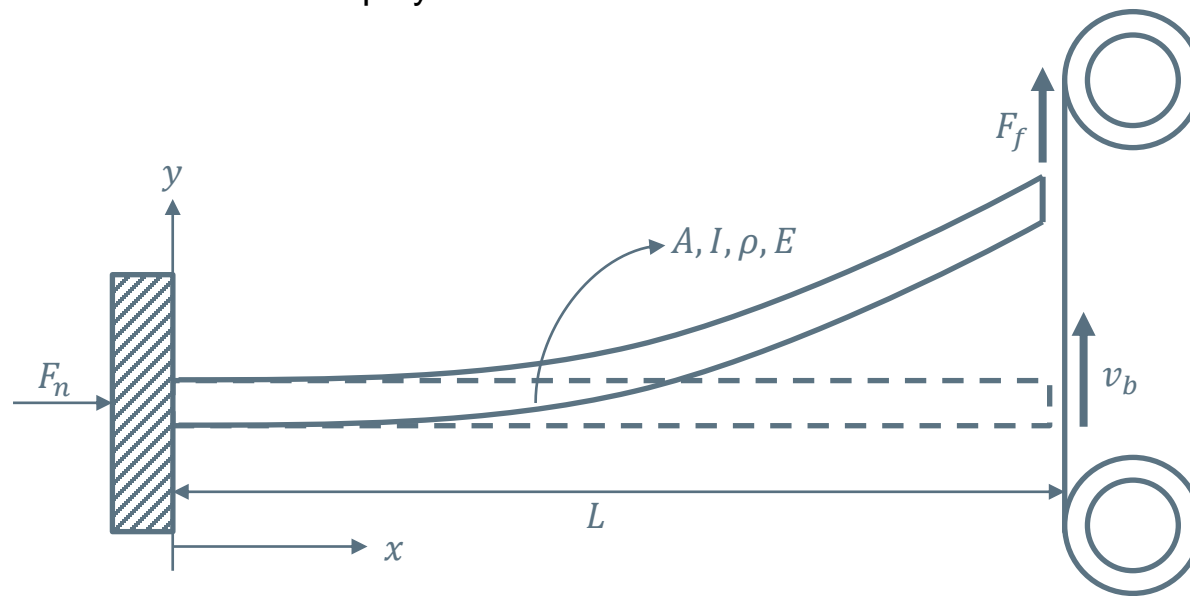
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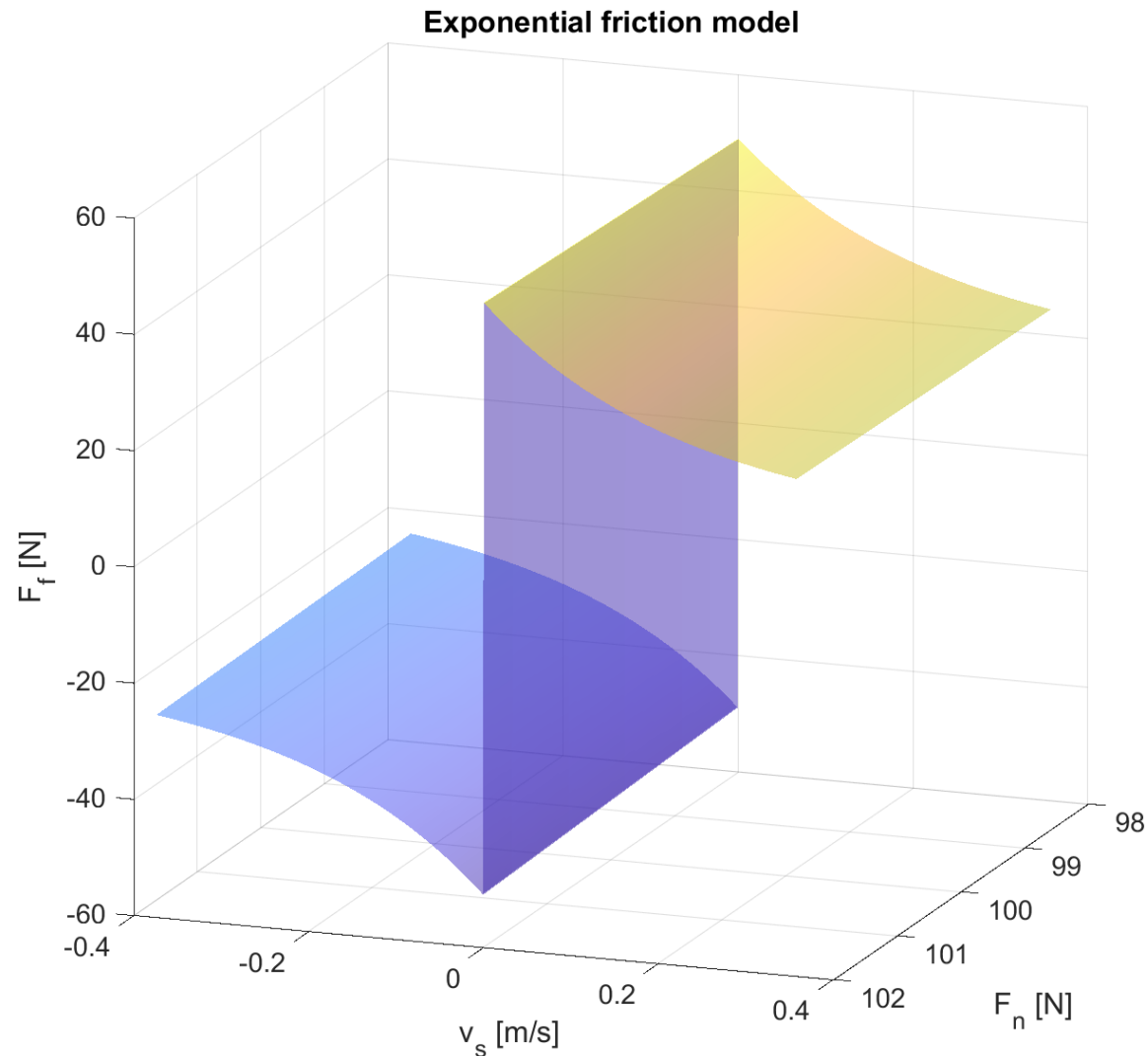
## Statistics and Machine Learning Toolbox

R2022a by MathWorks

Analyze and model data using statistics and machine learning



# Friction data sampling and partitioning

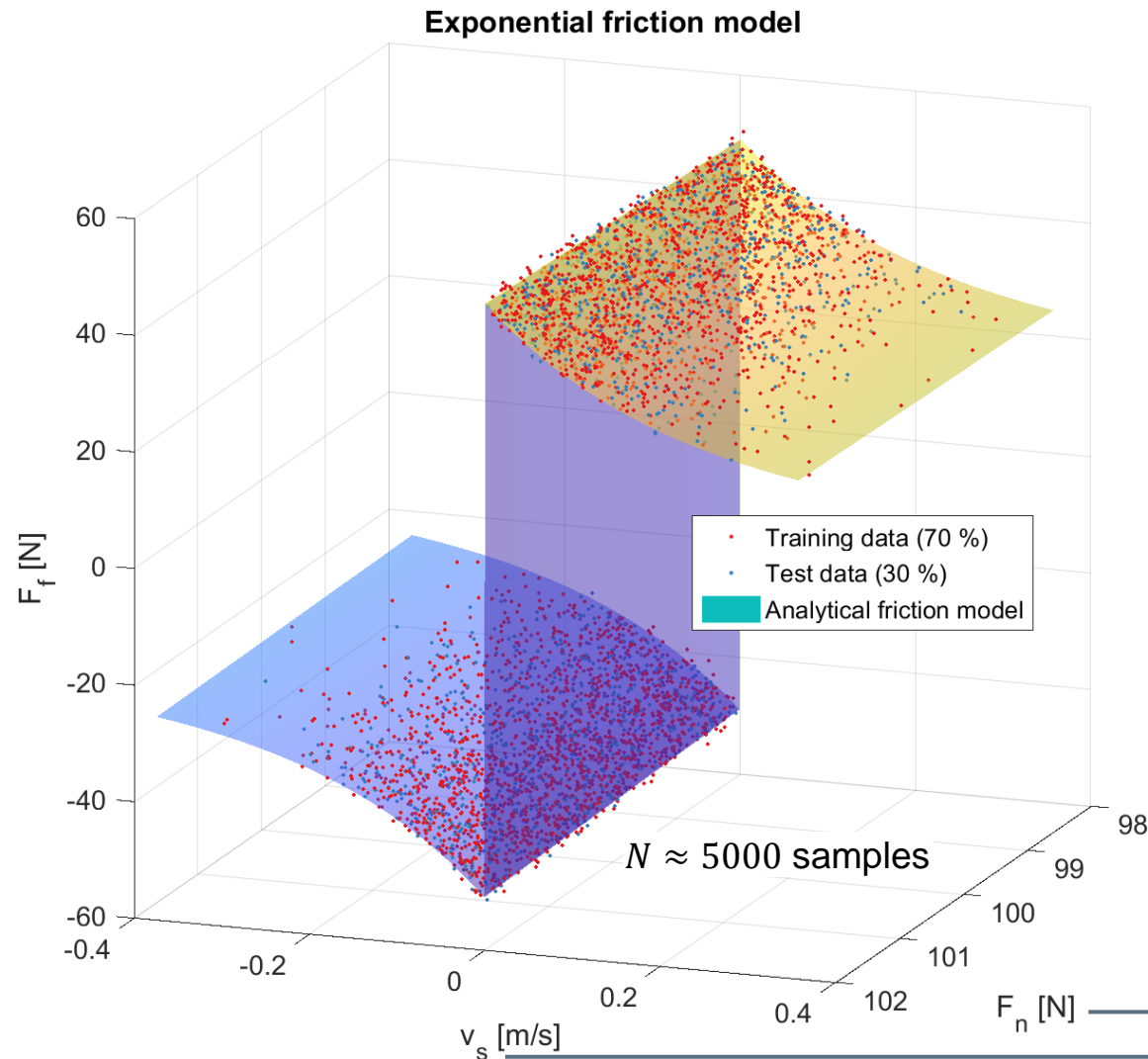


Exponential-type friction model:

$$\mu(v_s) = \mu_k + (\mu_s - \mu_k)e^{-\alpha|v_s|}$$

$$F_f(v_s) = \text{sgn}(v_s) \times \mu(v_s) \times F_n$$

# Friction data sampling and partitioning



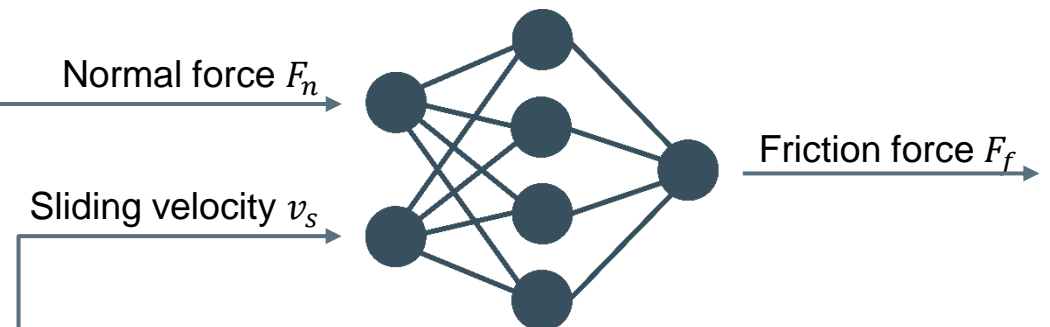
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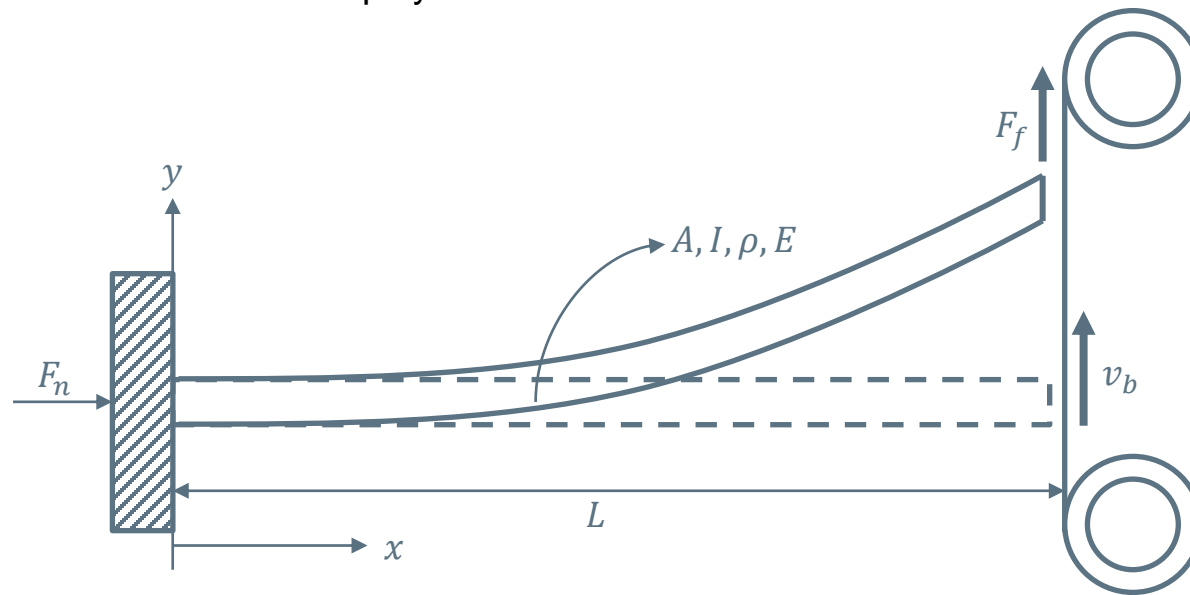
+ artificial noise

→ Mimic real measurement data



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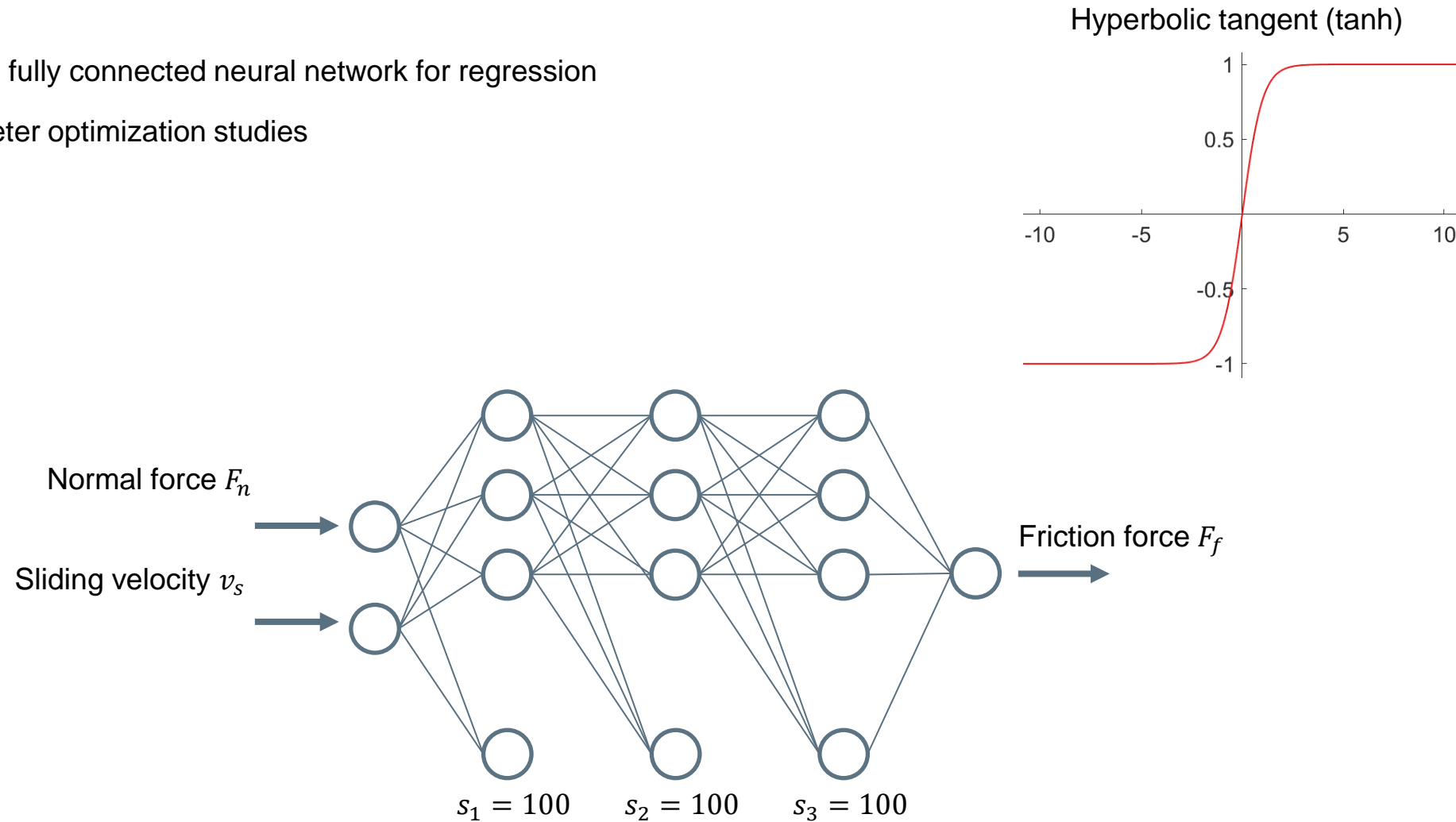
# Neural network selection

- Feedforward, fully connected neural network for regression
- Hyperparameter optimization studies



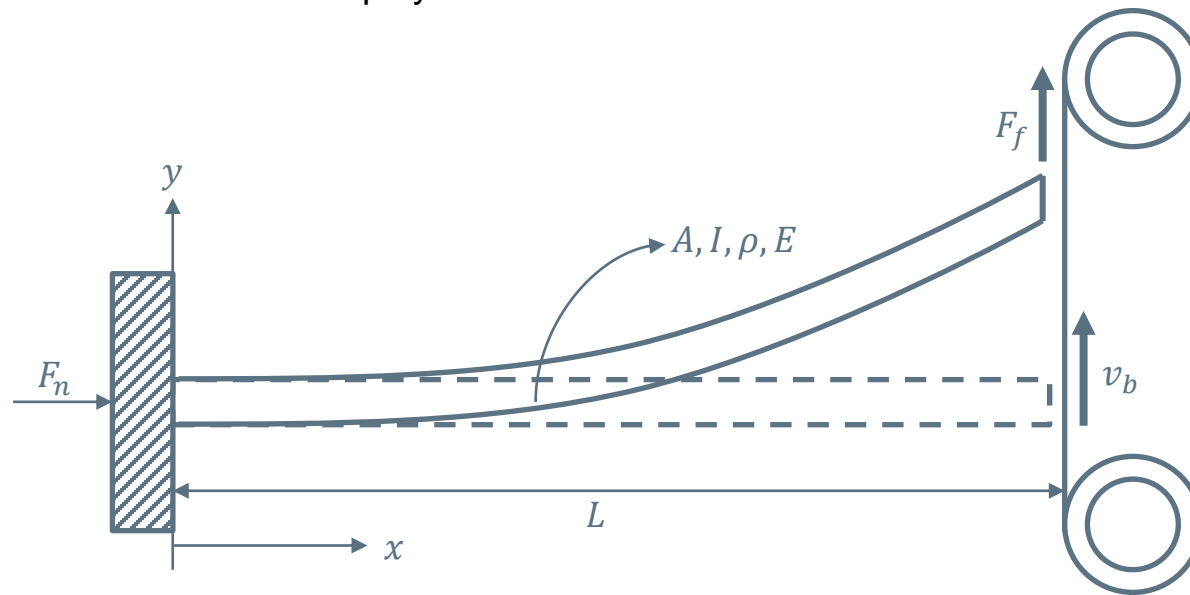
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# Neural network training and performance

- Limited-memory BFGS optimization algorithm
- Minimizing MSE loss function

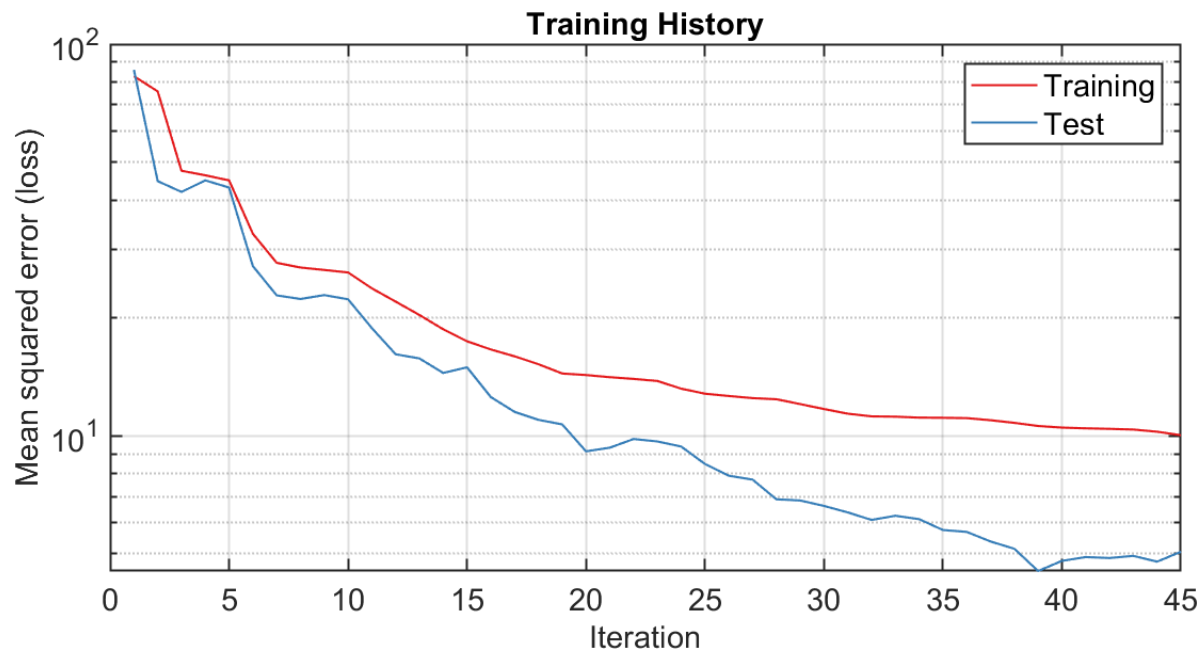
```
rnet = fitrnet(dataTrain, "Ff", ...  
  "ValidationData", dataTest, ...  
  "Activations", "tanh", ...  
  "Lambda",      0.004, ...  
  "LayerSizes",  [100 100 100], ...  
  "Standardize",  true, ...  
  "StoreHistory", true, ...  
  "Verbose",      true);
```



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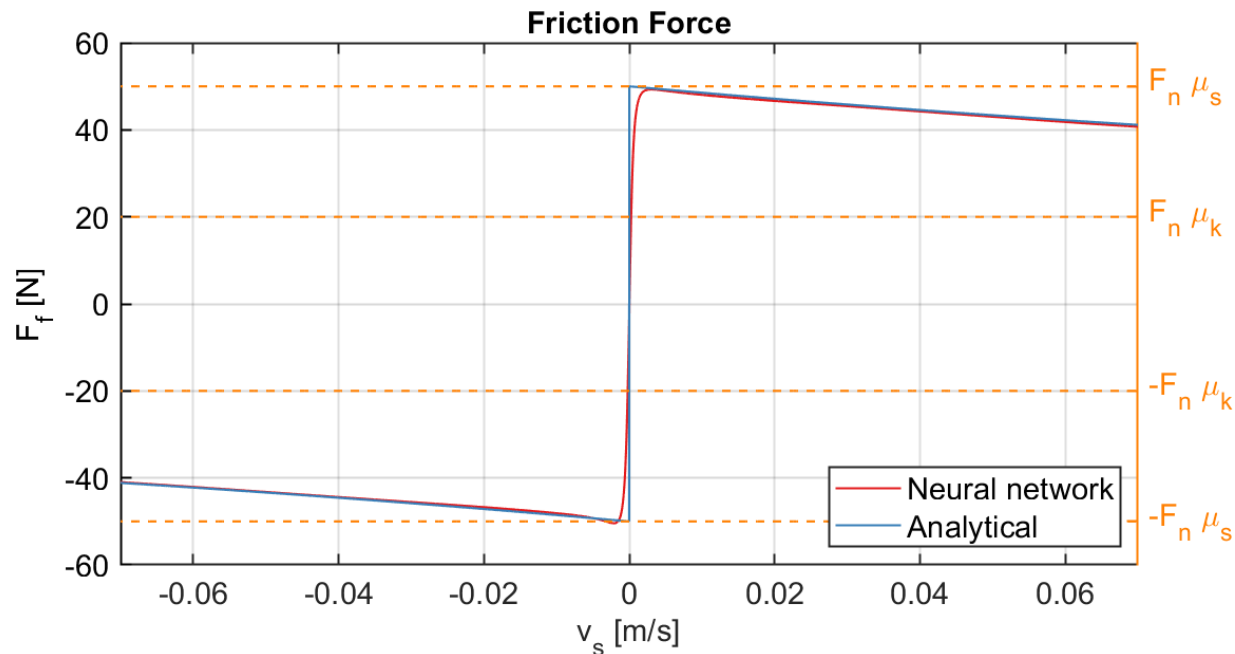
Training data	
Min. value	-52.4010
Median	27.9940
Max. value	52.5140

Training	
Test MSE	4.5217
R-squared	0.9974

5-fold cross-validation	
Max. loss	9.0095
Mean loss	5.8957
Std. deviation	3.1606
Validation RMSE	2.4281

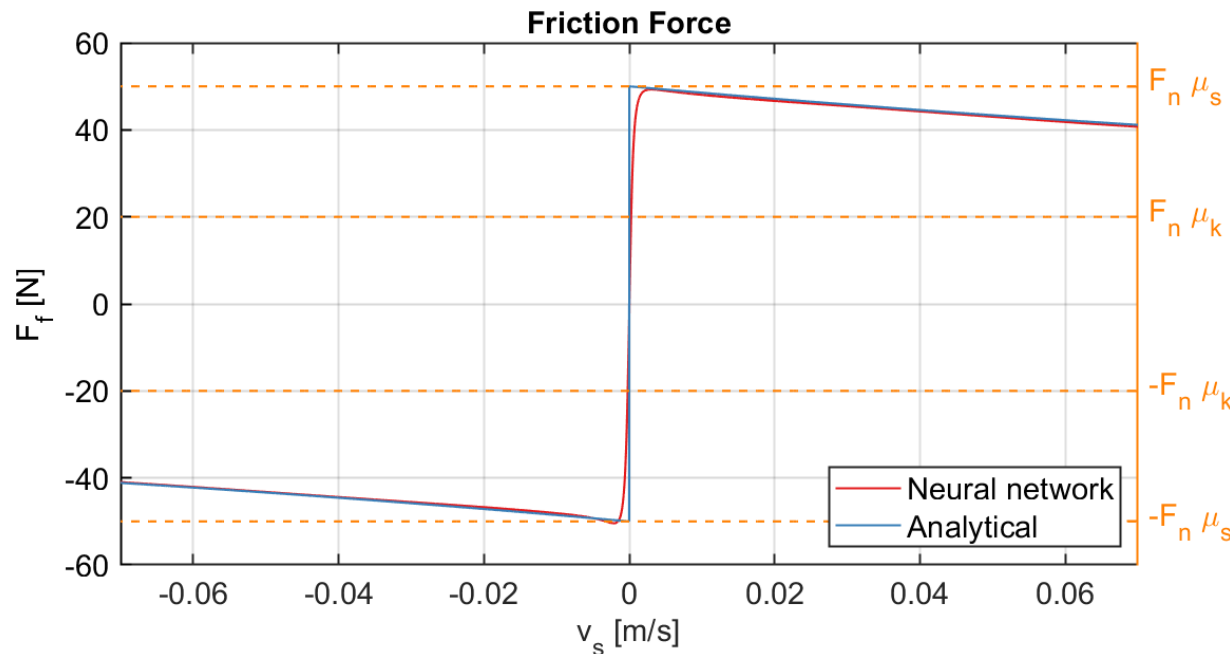
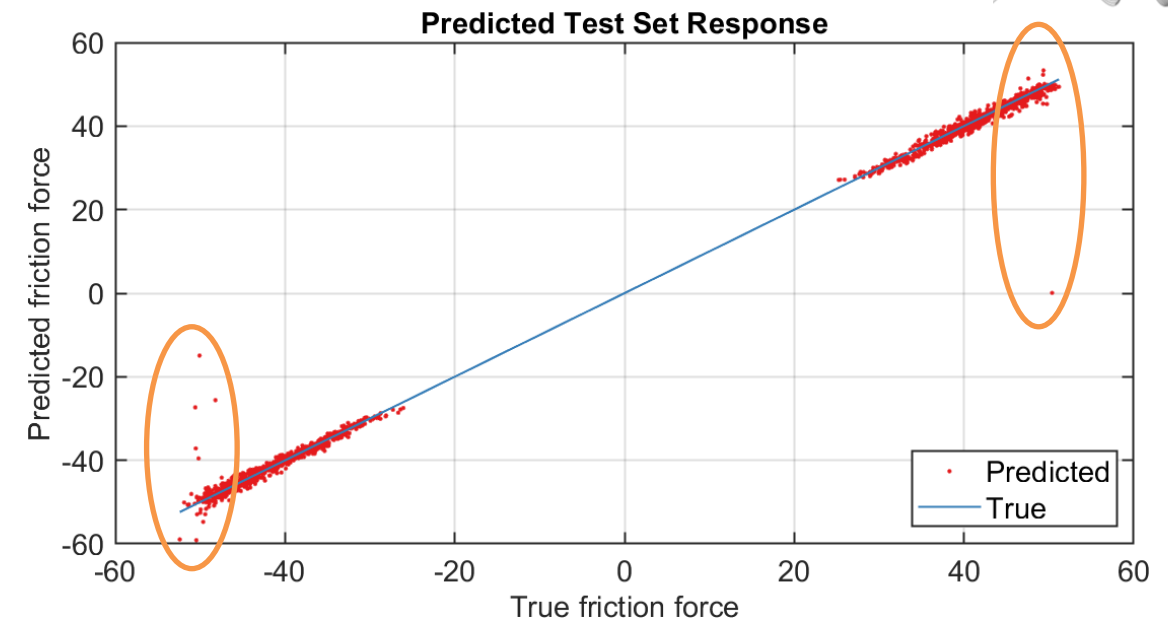
# Regression neural network friction model

- Good overall performance



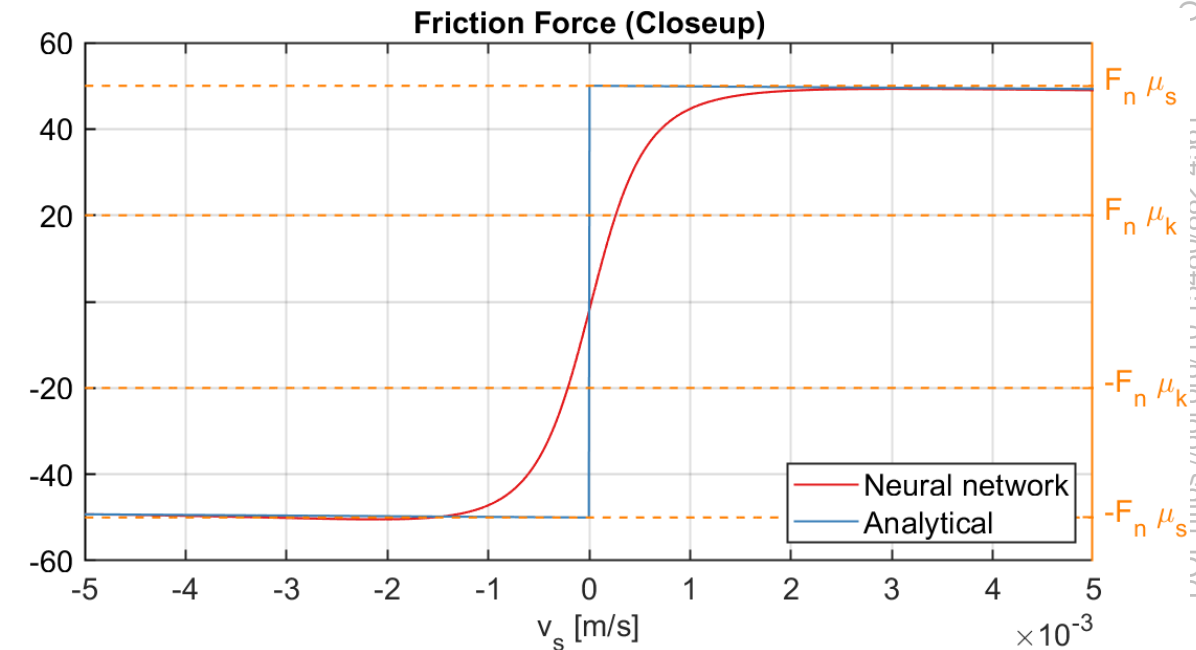
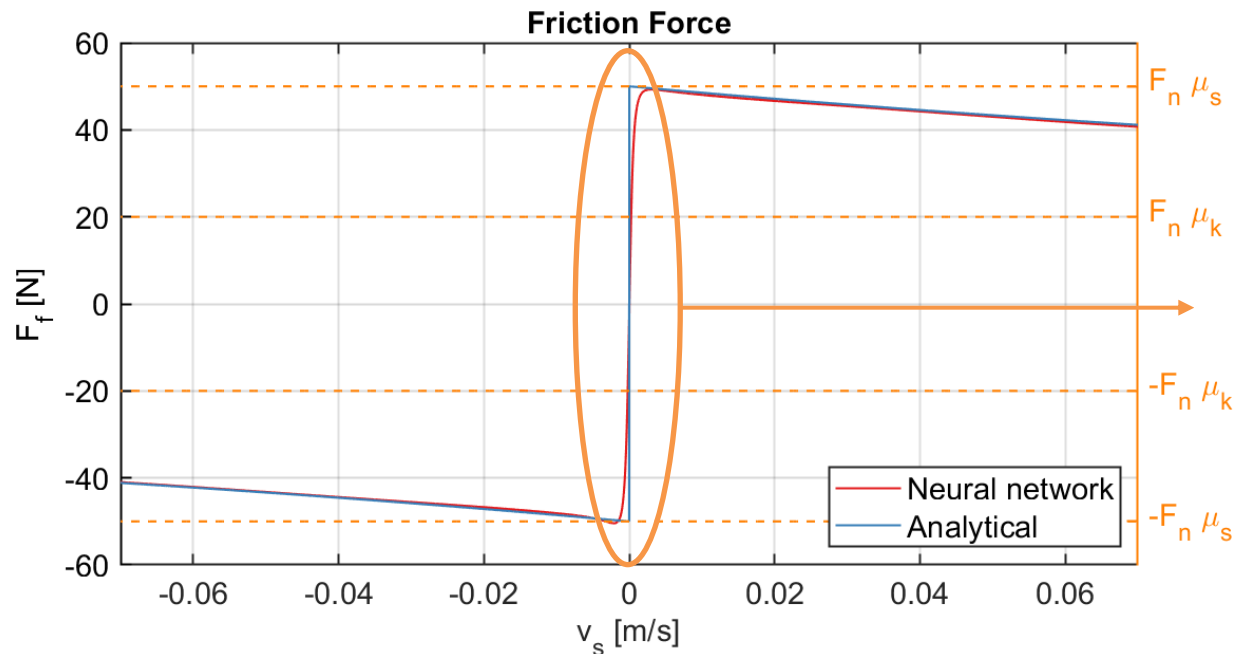
# Regression neural network friction model

- Good overall performance
- Discontinuity hard to capture by NN regression model



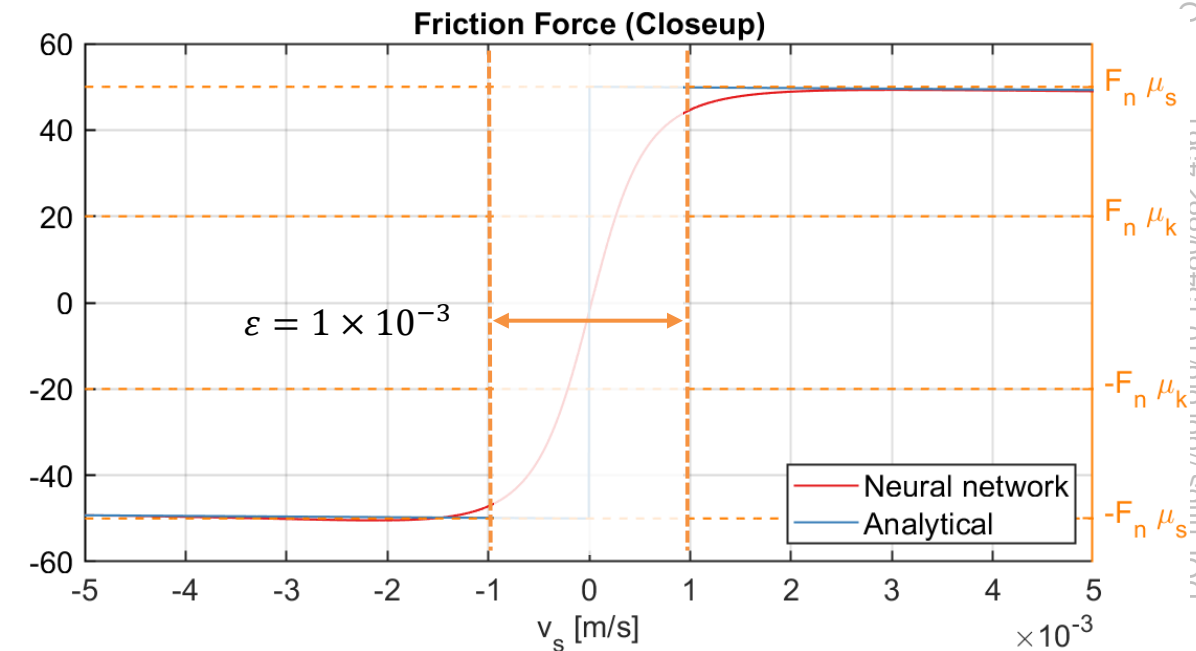
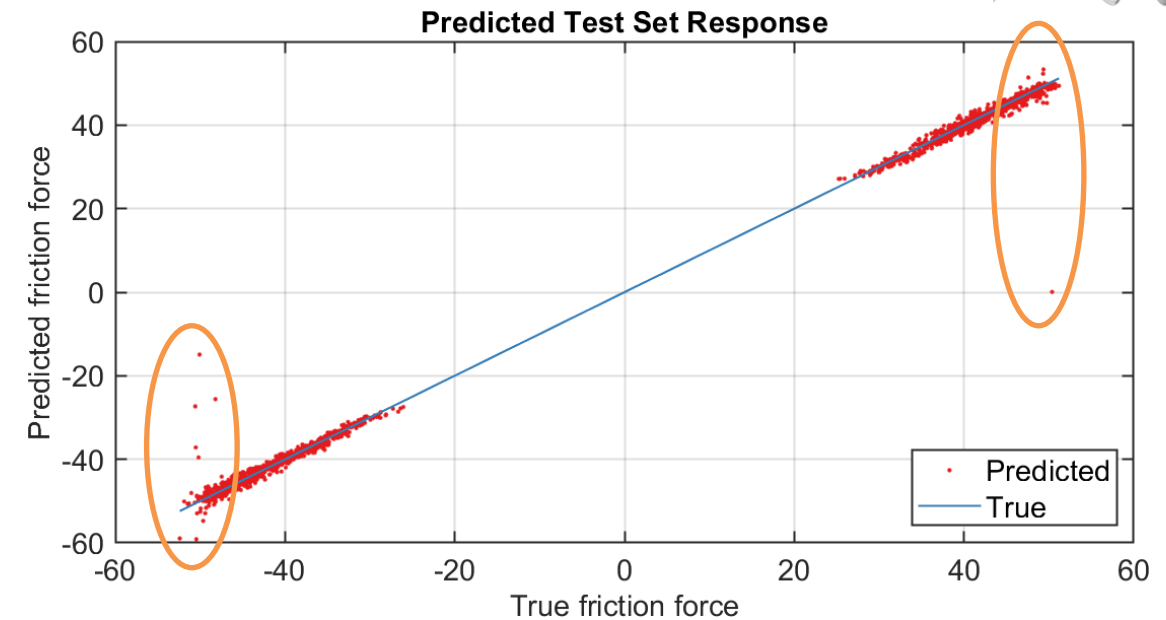
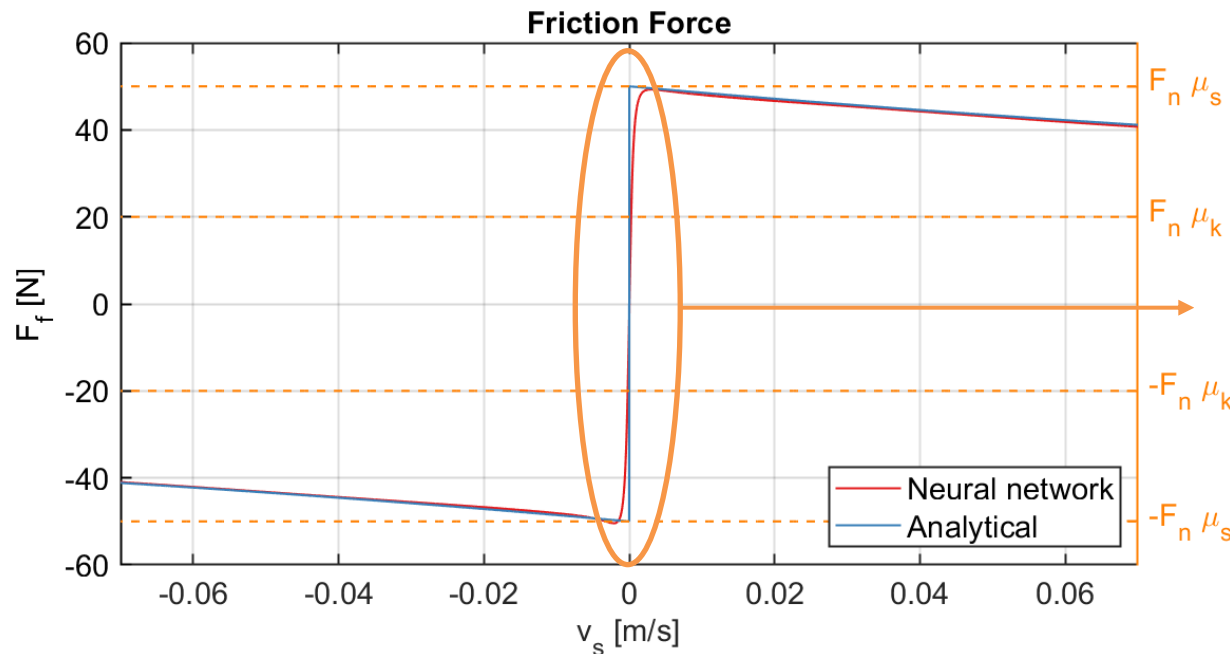
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- Good overall performance
- Discontinuity hard to capture by NN regression model
- Jump at the origin is smoothed out



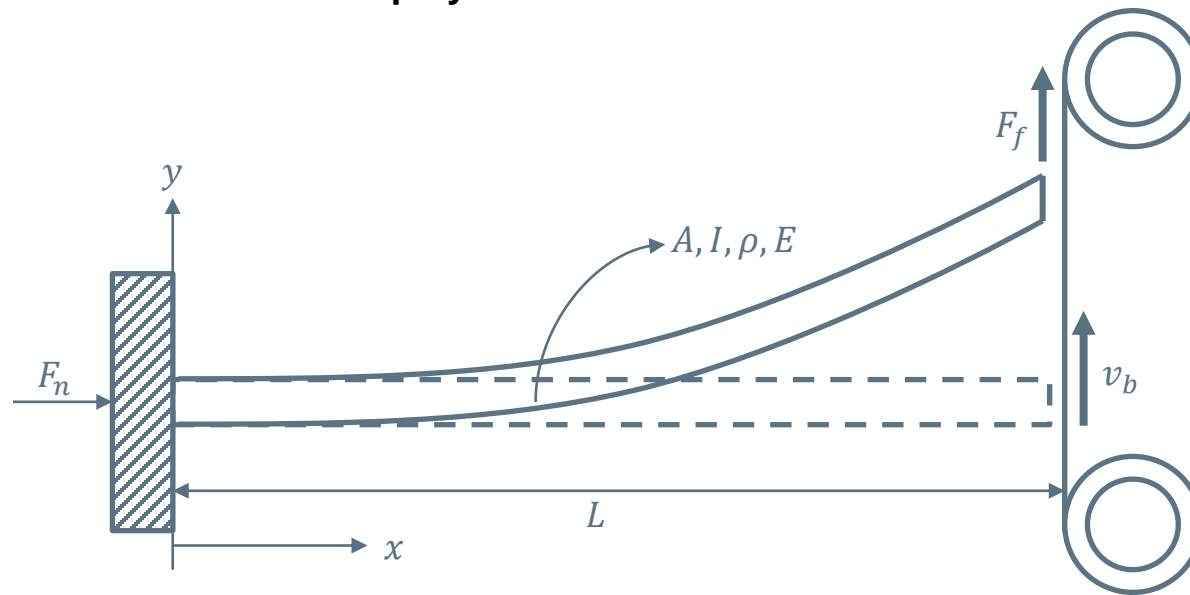
# Regression neural network friction model

- Good overall performance
- Discontinuity hard to capture by NN regression model
- Jump at the origin is smoothed out
- Threshold value for sliding velocity  $\varepsilon$  mitigates this problem

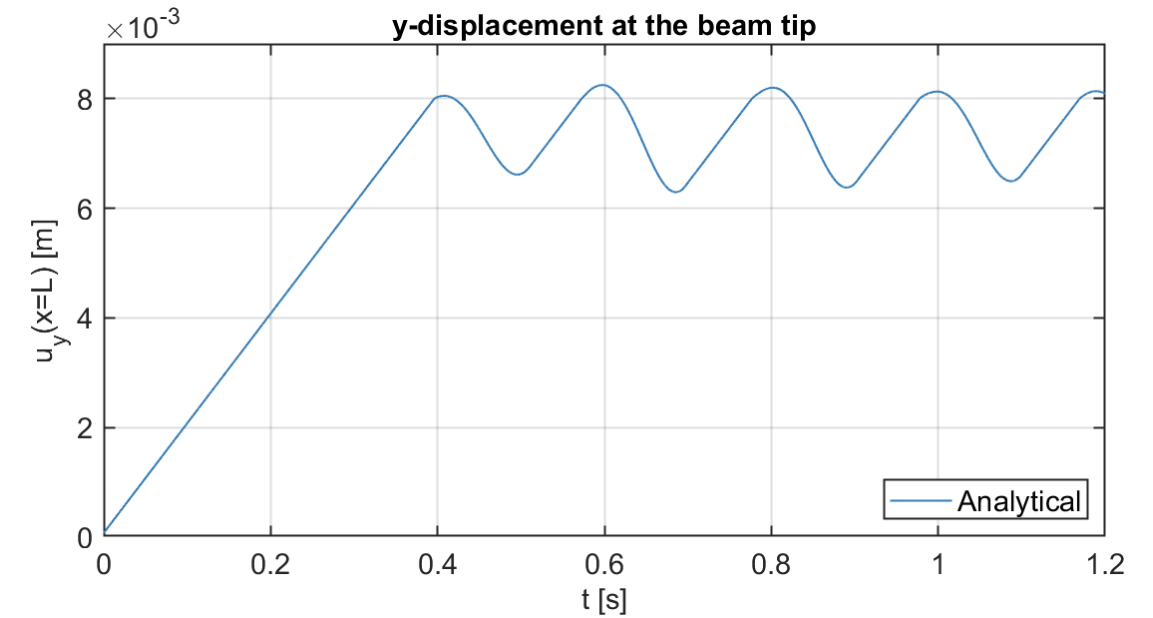


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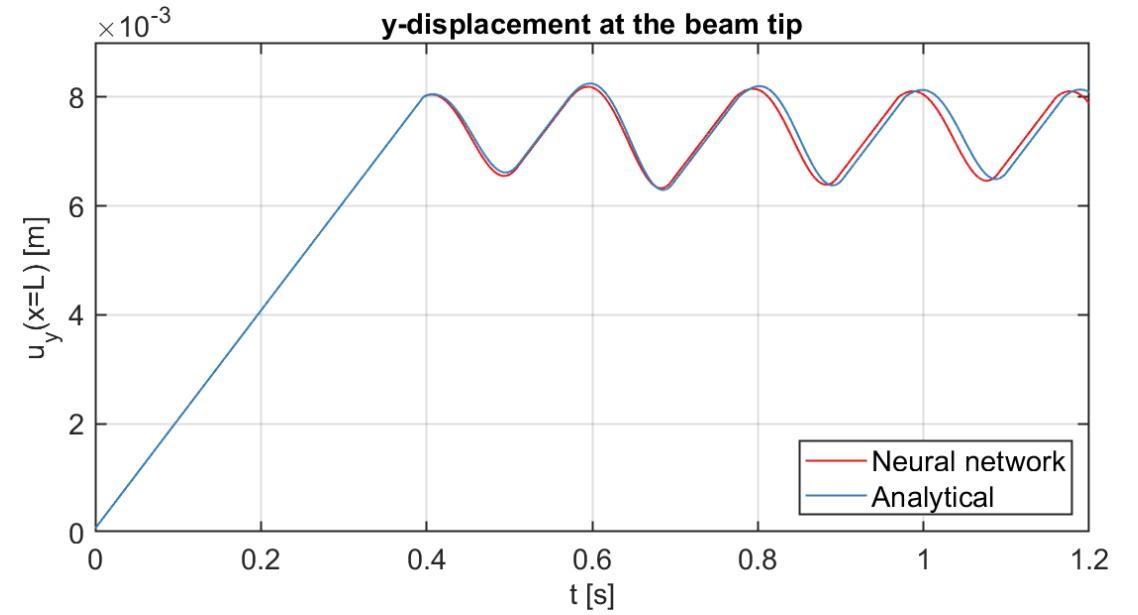
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# Cantilever beam Finite Element analysis

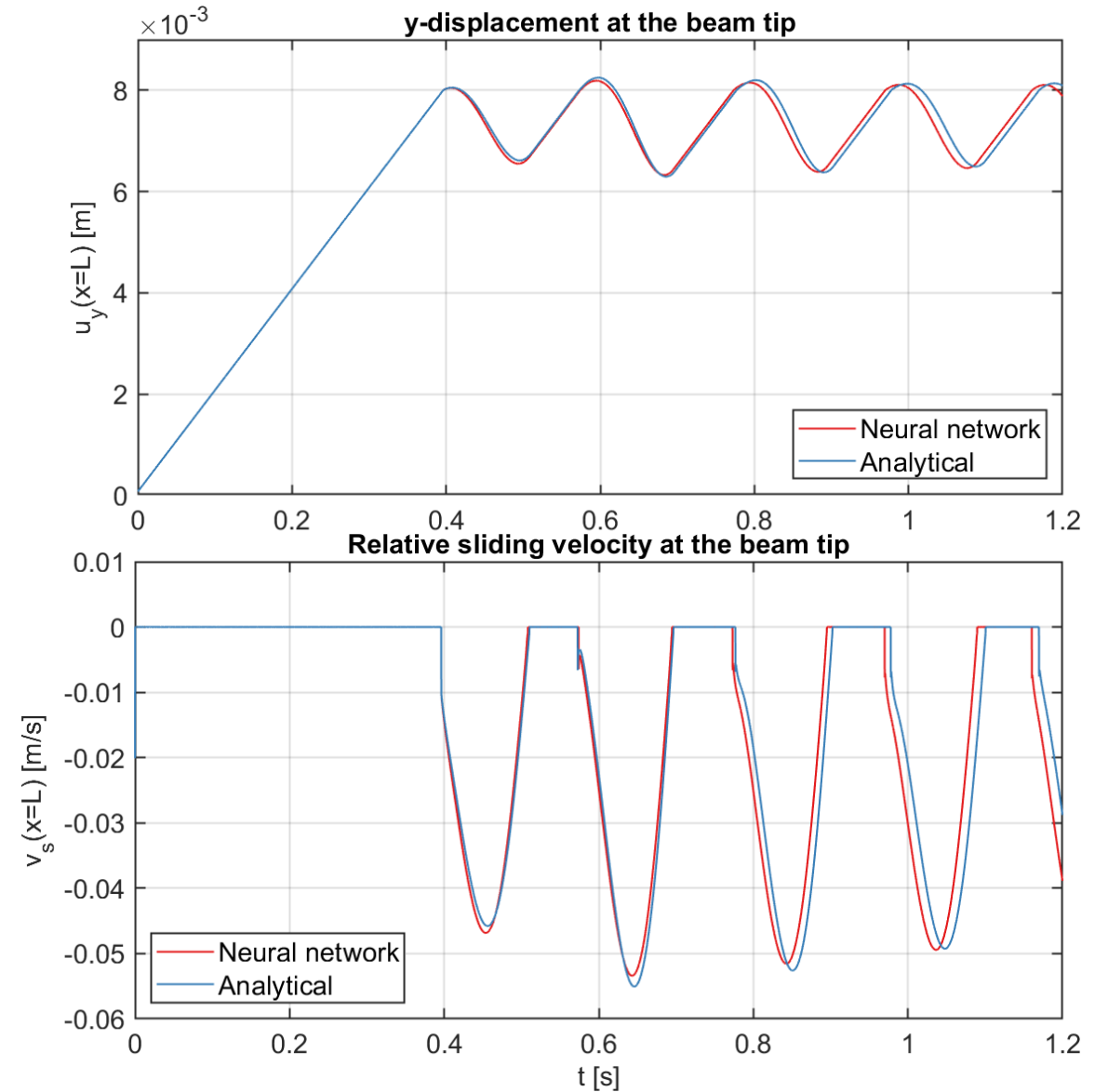
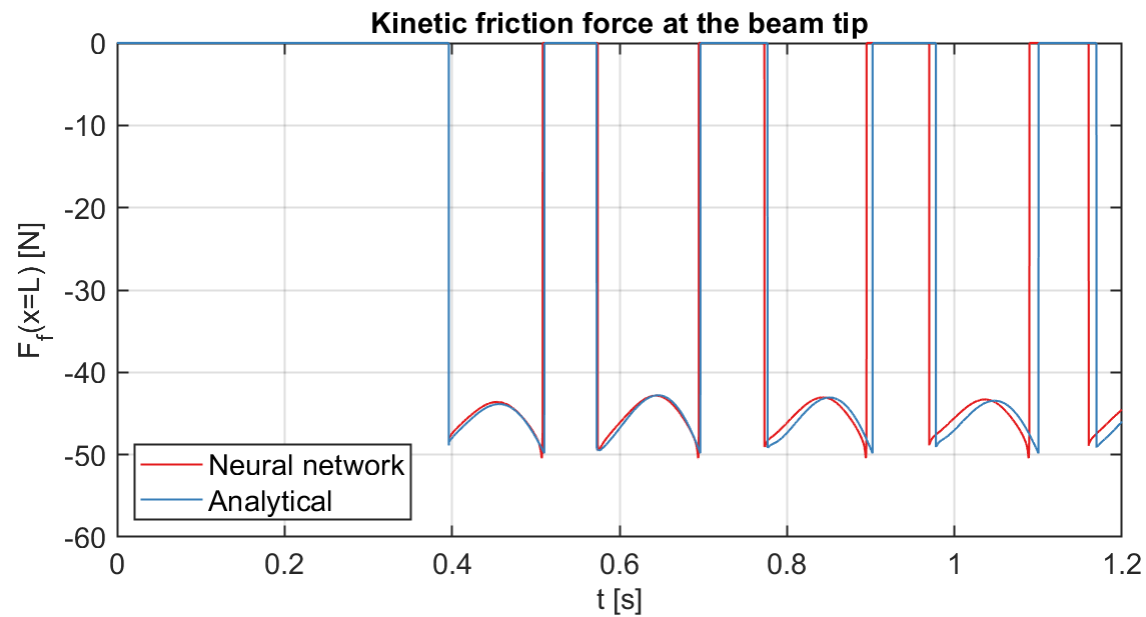


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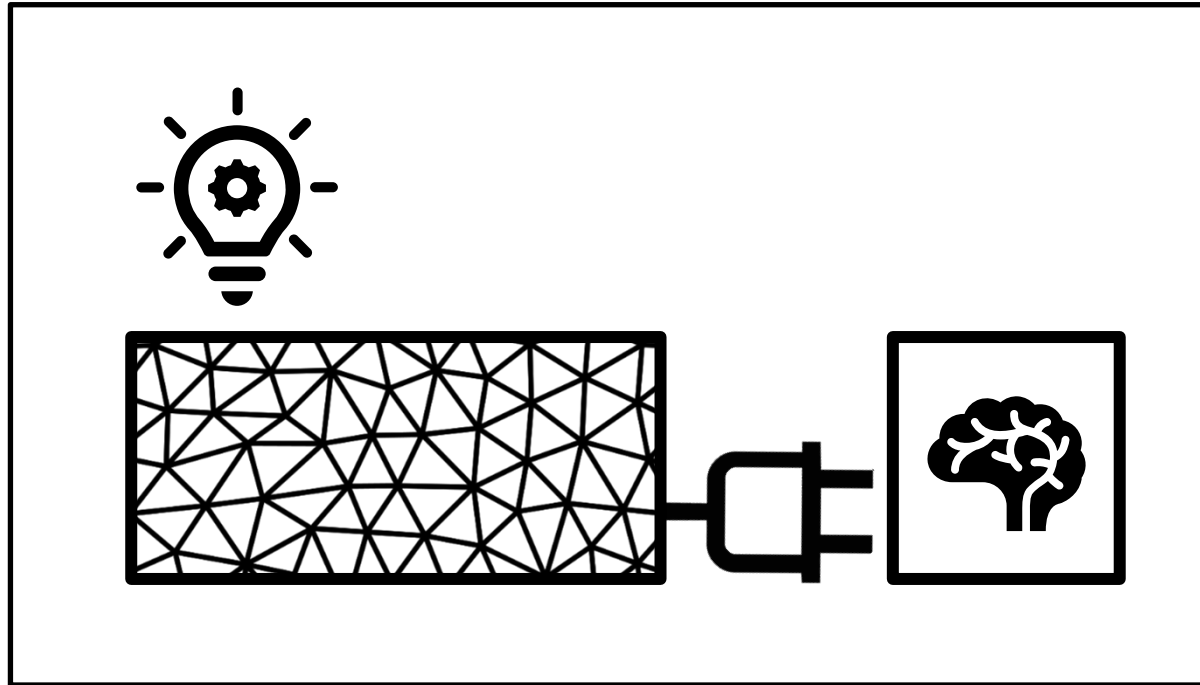
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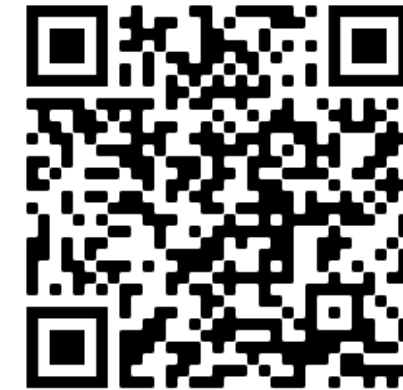
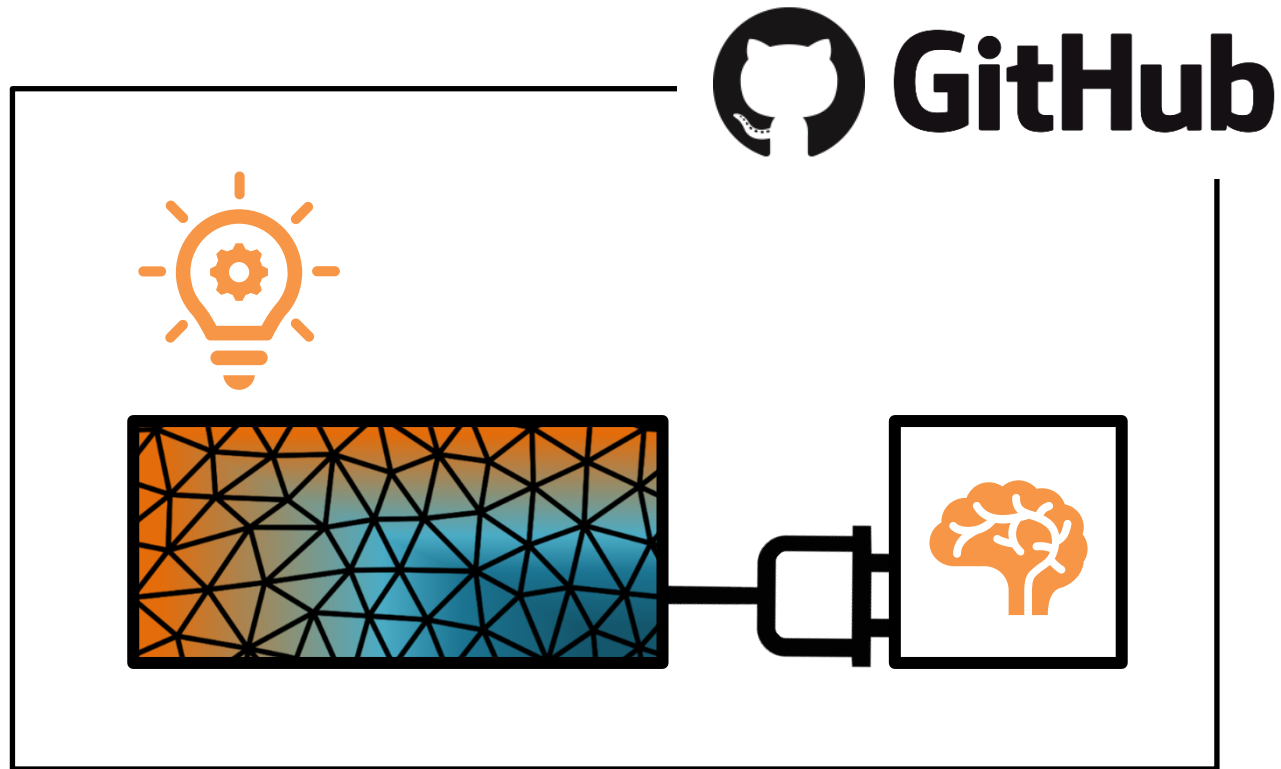
# Outlook

- Find better hyperparameters or network architectures
- Introduce physics awareness in the ML model
- Incorporate more features in the friction model
- Leverage data from real measurements

# Plug and play!



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[https://github.com/TUHH-DYN/NeuralNetworkFrictionModel\\_FEA](https://github.com/TUHH-DYN/NeuralNetworkFrictionModel_FEA)

Thank you!

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Phone.: +49 (0)40 42878 2993

# Parameter values

Cantilever beam model		
Beam length	$L$	2 m
Cross-sectional area	$A$	0.01 m <sup>2</sup>
Moment of inertia	$I$	$\frac{1}{12} \times 10^{-6} \text{ m}^4$
Mass density	$\rho$	1000 $\frac{\text{kg}}{\text{m}^3}$
Young's modulus	$E$	$2 \times 10^{11} \frac{\text{N}}{\text{m}^2}$
Belt velocity	$v_b$	0.02 $\frac{\text{m}}{\text{s}}$
Axial load	$F_n$	100 N

Computational parameters		
Damping	$\beta$	$1 \times 10^{-2}$
Threshold for sliding velocity	$\varepsilon$	$1 \times 10^{-3} \frac{\text{m}}{\text{s}}$
(Outer) time step size	$dt$	$1 \times 10^{-4} \text{ s}$
Abs. solver tolerance		$1 \times 10^{-6}$
Rel. solver tolerance		$1 \times 10^{-3}$

# Friction data sampling and partitioning

