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The O.N.R. - N.S.F. Sympsium on Wave Resistance Theory in Ann Arbor held 1963 made clear that current research is focussed around the following items:

(A) Determination of quantities from the wave pattern representative for wave resistance.

(B) Formal and semi-empirical corrections to the classical linearized theory.

(C) More refined techniques for optimizing ship forms within linear theory.

Within the present paper I shall report on work done since then which might provide material to enforce pro-ress in any of these directions. The "pièce de résistance" of this contribution is the gradual evolution of a computer programm which in a rationalized way gives the basic information of flow and wave components due to typical singularities, - (as discrete doublets, doublet struts, continuous parabolic distributions on submerged lines, infinite and truncated vertical planes) - all within linearized approach. This information lends itself readily for application to item (A). Any method proposed for determination of energy flow from characteristics of the flow, in peculiar from the geometry of wave pattern, can be tested for accuracy and for consistency on such a theoretical wave field available numerically before entering into expensive experimental work which provides in general too little reliable information on optimal choice of region where to perform measurements. The overwhelming part of the methods proposed for (A) is implicit ly based on validity of certain asymptotic representations for the wave pattern. Only numerical calculations can tell what distances are already large enough, especially regarding decay of the so called "local flow components" in order that such representations may be applied.

For (B), the theories of wave resistance used nowadays are of second order, based on linearized flow models. For calculation of wave resistance, only a far field component of this flow has to be known explicitely; for any consistent approach to third order resistance contributions, however, the knowledge of the entire first order flow is essential. Aside from some semi-empirical approaches to alternate formulations of linear theory, which we shall submit to some critical examen, and aside from indirect approach as successfully carried out by Kajitani recently, the tool for a systematic perturbation attack to the higher order flow components has been provided by Wehausen [1], [2], [3] in a series of papers starting with that read before this audience in 1956 up to his contribution to the Ann Arbor conference. As, however, the step to formulate resistance expressions was not performed, credit is generally given to Sisov [4] for first dealing with these. We should, nevertheless, be aware, that expressions given by Sisov so far essentially contain divergent integrals due to selection of improper radiation condition for Green's function of pressure point. In our present investigation, we will rederive some of Sisov's results from a Green's theorem approach essentially following Wehausen. We will, in particular, show up some simplifications which make calculations straightforward once a Fourier representation of first order flow components is given. It will become evident that integration over undisturbed free surface has to be performed only in a small domain where local flow is significant; third order wave resistance is, therefore, much more tractable to numerical evaluation than is apparent from what was formulated by Sisov, provided we decide on an appropriate definition of wave resistance.

We decided to deviate from Wehausen's approach by some simplifications regarding the actual flow boundaries. However, the resulting expressions found for third order resistance depend in a simple manner only on ship's offsets and on first order velocity components. We, therefore, feel that these deviations at least have not introduced artificial complications against results still to be found from more refined analysis.

Regarding the third problem, i.e. ships of minimum resistance within lowest order theory, our investigation should throw light on the question to what degree third order contributions might counteract the tendencies predicted. At the present stage, however, our calculations are limited to a two-parameter class of hull forms having parabolic waterlines. This is mainly due to the fact that we prefered analytical evaluation of integrals over the geometry of the ship. An extension of our programm for local flow, to include contributions from empirical surface elements is feasible, but loss of closed integration would probably increase time for computation and weaken control of accuracy. Moreover, the necessary degree of hull-subdivision will in general depend on Froude number and is not known beforehand. Even for analytical ship forms, the development of formal expressions for closed integration cannot be done by the computer and provides many opportunities for errors in evaluation of singular regions of integrands for local flow.

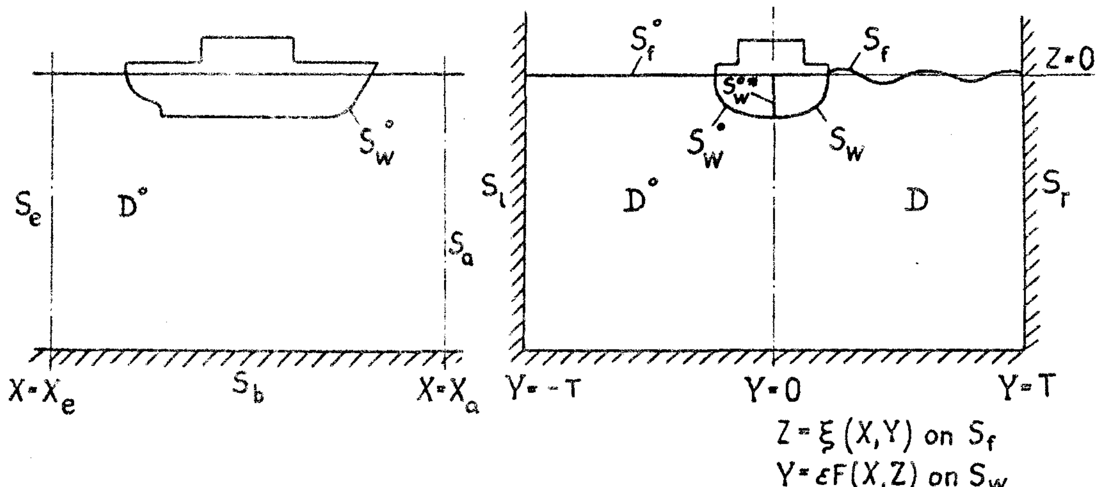
Description of analysis to derive potential and wave resistance.

We shall essentially follow the approach of Wehausen [3], but modify it for flow in a tank of rectangular cross section. This will simplify the formulation of radiation conditions for the flow and allows the use of a Green's function in a Fourier series representation regarding the ordinate y chosen in direction perpendicular to the vertical tank walls. The ship's motion is in the +x-direction with speed c, the z coordinate is taken vertically upwards to conform with earlier work [6]. As far as possible we otherwise use notation consistent with [3]. However, direction of normal vectors is reversed resulting from our definition of Green's function with an opposite sign. Extension of results to unrestricted water is straightforward.

1. Derivation of second order potential.

We introduce dimensionless coordinates as $X = 2x/L$, $Y = 2y/L$, $Z = 2z/L$, where L is the ship's length. The velocity potential is nondimensionalized as $\psi = 2\varphi/Lc$. As speed parameter we use $\gamma_0 = gL/2c^2$.

Let $Y = \pm\epsilon F(X,Z)$ be the dimensionless representation of the hull geometry, where B is the ship's breadth. $\epsilon = B/L$ will serve as a perturbation parameter and is considered as a small quantity. Let $X = X_a$ and $X = X_e$ be the equations of two vertical control planes S_a and S_e ahead of and behind the ship. Let S_b stand for the tank bottom plane $Z = -H$. Let $Y = \pm T$ be the equations of the vertical tank walls S_r and S_l, where $T = b/L$, b = tank width. Let S_f stand for the free surface $Z = \zeta(X,Y)$ for $X_e < X < X_a$, $T < Y < T$; let S_f° stand for the undisturbed free surface $Z = 0$ with the waterplane area of the ship excluded. Let S_w stand for the wetted surface of the ship and S_w° stand for the part of the surface up to $Z = 0$. Let D stand for the domain of the complete flow, bounded by S_w, S_f, S_a, S_e, S_r, S_l and S_b. Let D° describe the corresponding domain if S_w and S_f are replaced by S_f° and S_w°. Let $\psi^{(1)}$ stand for the Michell type first approximation to the exact potential ψ, let P stand for a point in D or D° with coordinates X, Y, Z and let P' represent a point on a boundary surface with coordinates ξ, η and ζ. Let G(P,P') stand for the potential of a source of output 4π as defined in the Appendix.



The functions ψ, $\psi^{(1)}$ and G of the variables X, Y, Z are subject to the following set of conditions:

A. Laplace equation: $\Delta\psi = 0$ in D, $\Delta\psi^{(1)} = 0$ in D°,
$\Delta G = 4\pi\cdot\delta(P-P')$ in D°
where Δ stands for $\delta^2/\delta X^2 + \delta^2/\delta Y^2 + \delta^2/\delta Z^2$ and δ means the Dirac delta function, which is zero if P is unequal P'; (condition **A.** implies that G becomes singular as $-1/|P-P'|$).

B. On S_f° we have (linearized free surface condition):

$$\gamma_0\,\psi_z^{(1)} + \psi_{xx}^{(1)} = 0; \qquad \gamma_0\,G_z + G_{xx} = 0.$$

For the exact potential ψ, no such conditions holds. But we define a function $\delta(X,Y)$ by

$$\gamma_0\,\psi_z + \psi_{xx} = \delta(X,Y).$$

C. On S_l and S_r we have: $\psi_Y = 0$; $\psi_Y^{(1)} = 0$; $G_Y = 0$.

D. On S_b we have: $\psi_z = 0$; $\psi_z^{(1)} = 0$; $G_z = 0$.

E. On S_w we have: $\psi_n = \pm\varepsilon F_X/\sqrt{\varepsilon^2 F_X^2 + \varepsilon^2 F_Z^2 + 1}$,
where $\pm$ stands for η positive or negative and the index n stands for derivation in normal direction out of the fluid's domain D.
On $S_w^{\circ *}$, the projection of S_w° on the plane Y = 0, we have $\psi_Y^{(1)} = \pm\varepsilon F_X$ for the first order potential.

F. For fixed P' we have
$G = O(1)$ with $X\to -\infty$; $G = O(X^{-1})$ with $X\to +\infty$,
$G_X = O(1)$ with $X\to -\infty$; $G_X = O(X^{-1})$ with $X\to +\infty$.

Application of Green's theorem shows that $\psi^{(1)}$ and ψ can be defined subject to the same modes of asymptotic decay, provided the quantity $\delta(X,Y)$ will then turn out to be well behaved.
The symmetry of functions ψ and $\psi^{(1)}$ regarding the lateral coordinate Y will be taken for granted by the symmetry of ship sections and tank profile.

We should bear in mind that [3] for the function ψ existence as a harmonic function is, - if at all, - guaranteed only in domain D, but not necessarily in D°. However, low order approximations have been derived [3] which are found to exist in the whole interior of D°. As for the moment we seek terms up to second degree only, we shall in the following formulate the problem for the domain D° with boundaries known a priori in favor of a less intricate analysis, and derive approximate solutions to this auxiliary problem by perturbation techniques. Then, for point P within D°, we may apply Green's theorem to functions ψ and G to find a representation of $\psi(P)$ as

$$\psi(P) = \frac{1}{4\pi}\iint\left\{\psi_n(P')\,G(P,P') - \psi(P')\,G_n(P,P')\right\} ds' \qquad (1.1)$$

where the closed boundary S' is composed of S_w°, S_f°, S_r, S_l, S_b, S_a, S_e and the subscript n stands for normal derivative outward in P' space.

From conditions C. and D. we may conclude that the contributions of S_l, S_r and S_b can be omitted on the right hand side.

The integral over S_f°, where the normal derivative is in Z direction, may be transformed by integration regarding ξ and use of B.:

$$\frac{1}{4\pi}\int\limits_{S_f^\circ}(\psi_z G - \psi G_\xi)\,dS = \frac{1}{4\pi}\cdot\frac{1}{\gamma_o}\int_{-T}^{T}\left|(\psi_x G - \psi G_\xi)\right|_{\xi = X_e}^{\xi = X_a} d\eta \; +$$

$$+ \frac{1}{4\pi\gamma_o}\int\limits_{S_f^\circ}\delta(\xi,\zeta)\,G\,dS - \frac{1}{4\pi\gamma_o}\oint\limits_{L_p}(\psi_x G - \psi G_\xi)\,d\eta \qquad (1.2)$$

where the line integral around the ship's load waterline L_p has to be understood in counterclockwise sense when viewed from above (compare (19)[3]).

The line integral has been thoroughly investigated by Yim [5]. We shall find, however, that it is pertinent to merge it with a similar term from the wetted surface S_w°. - If now we assume the functions ψ and ψ_x uniformly bounded for $X < X_e$, then, due to the finite size of S_e with conditions F. we may infer that the contribution of S_e becomes insignificant as we let X_e tend to $-\infty$. Similary: If ψ and ψ_x tend to zero with $X \to \infty$, then, due to boundedness of G and G_X, the contribution of S_a may be neglected with X_a becoming large. But the contributions of S_a and S_b must be independent of position X_a, X_e in as much as the contribution of the defect $\delta(X,Y)$ may be neglected. Considering higher order terms, however, we will see that independence from X_e cannot be assumed in general.

For the first integral in (1.2) over the wetted surface S_w°, we shall make the assumption that condition E. for ψ holds even for parts of the hull not included in S_w up to the undisturbed free surface, so that we may substitute:

$$\psi_n \, dS = \varepsilon F_X \Big/ \sqrt{\varepsilon^2 F_X^2 + \varepsilon^2 F_Z^2 + 1} \;\; dS_w^{\circ *} \tag{1.3}$$

observing that

$$dS_w^\circ = \sqrt{\varepsilon^2 F_X^2 + \varepsilon^2 F_Z^2 + 1} \;\; dS_w^{\circ *} \,. \tag{1.4}$$

For the second integral over S_w° we substitute the actual components of the normal vector as

$$\left\{ -F_X, \pm 1, -F_Z \right\} \Big/ \sqrt{1 + \varepsilon^2 F_X^2 + \varepsilon^2 F_Z^2} \quad \text{(compare [3] (18))}$$

and thereby have

$$\frac{-1}{4\pi} \int_{S_w^\circ} \psi G_n \, dS' = \frac{-1}{4\pi} \iint_{S_w^{\circ +}} \psi\left(\xi, \pm\varepsilon F(\xi,\zeta), \zeta\right) \cdot \Big(\varepsilon F_X \cdot (G_\xi^+ + G_\xi^-) +$$

$$+ \varepsilon F_Z (G_\zeta^+ + G_\zeta^-) - G_\eta^+ + G_\eta^- \Big) \, d\xi \, d\zeta \tag{1.5}$$

where $\pm$ stands for η positive or negative.
By partial integration regarding ξ and ζ, observing Laplace equation for G as stated in A. and making use of the fact that F = 0 at the integration limits if $\zeta < 0$, we then have:

$$-\frac{1}{4\pi} \int_{S_w^\circ} \psi G_n \, dS' = \frac{\varepsilon}{4\pi} \iint_{S_w^{\circ +}} F(\xi,\zeta) \Big(\psi_X (G_\xi^+ + G_\xi^-) + \psi_Z (G_\zeta^+ + G_\zeta^-) \Big) \, d\xi \, d\zeta -$$

$$-\frac{\varepsilon}{4\pi} \iint_{S_w^{\circ +}} F(\xi,\zeta) \cdot \psi \cdot (G_{YY}^+ + G_{YY}^-) \, d\xi \, d\zeta + \frac{1}{4\pi} \iint_{S_w^{\circ +}} \psi (G_\eta^+ - G_\eta^-) \, d\xi \, d\zeta - \frac{\varepsilon}{4\pi} \int_{\xi=-1}^{1} \psi F(\xi,0) \left[G_\zeta^+ + G_\zeta^- \right] d\xi \tag{1.6}$$

We can now transform the line integral, obtained previously, as

$$-\frac{1}{4\pi\gamma_o}\oint_{L_p}(\psi_X G - G_\xi \psi)\,d\xi = -\frac{1}{4\pi\gamma_o}\int_{-1}^{1}\left(\psi_X(G^+ + G^-) - \psi(G_\xi^+ + G_\xi^-)\right) F_X(\xi,0)\,d\xi$$

$$= -\frac{1}{4\pi\gamma_o}\int F(\xi,0)\left\{\psi_{XX}(G^+ + G^-) - \psi(G_{\xi\xi}^+ + G_{\xi\xi}^-)\right\} d\xi \tag{1.7}$$

and then combine all components, observing B. for G, as:

$$\psi = -\frac{\varepsilon}{4\pi}\iint_{S_W^\circ} F(\xi,\zeta)(G_\xi^+ + G_\xi^-)\,d\xi\,d\zeta + \frac{1}{4\pi\gamma_o}\iint_{S_f^\circ}\delta(\xi,\eta)\,G\,d\xi\,d\eta +$$

$$+\frac{\varepsilon}{4\pi\gamma_o}\int_{-1}^{+1} F(\xi,0)\,\psi_{XX}\,(G^+ + G^-)\,d\xi - \frac{\varepsilon}{4\pi}\int_{S_W^\circ} F(\xi,\zeta)\left\{\psi_X(G_\xi^+ + G_\xi^-) + \psi_Z(G_\zeta^+ + G_\zeta^-)\right\} d\xi\,d\eta$$

$$+\left|\frac{1}{\gamma_o}\int_{-T}^{T}\left\{\psi_X(\xi,\eta,0)\cdot G - \psi(\xi,\eta,0)G_\xi + \int_{-H}^{0}\left(\psi_{XX}(\xi,\eta,\zeta)\cdot G - \psi_X(\xi,\eta,\zeta)\cdot G_\xi\right)d\zeta\right\}d\eta\right|_{\xi=X_e}^{\xi=X_a}$$

$$+\frac{1}{4\pi}\iint_{S_W^\circ}\psi(\xi,\varepsilon F(\xi,\zeta),\zeta)\cdot\left((G_\eta^+ - \varepsilon F\cdot G_{\eta\eta}^+) - (G_\eta^- + \varepsilon F G_{\eta\eta}^-)\right)d\xi\,d\zeta \tag{1.8}$$

If we here neglect the contributions of S_a and S_e, the remaining expression, proper behavior of $\delta(X,Y)$ assumed, really makes this omission legitimate due to the properties stated under F. for the function G.

So far, we have not used any considerations regarding smallness of $\varepsilon = B/L$. We should note that G is defined even for $\eta = 0$, i.e. for P' within the ship's hull, and is well behaved there, if P is not too close to P'. By development in Taylor series regarding εF we therefore may infer that

$$G_\eta^+ = -\varepsilon F \cdot G_{\eta\eta}^+ + O(\varepsilon^2)$$

and

$$G_\eta^- = +\varepsilon F \cdot G_{\eta\eta}^- + O(\varepsilon^2) \tag{1.9}$$

which shows that the factor of ψ in the last integral is small at least of order ε^2.

In general we have

$$G_\zeta^+ + G_\zeta^- = G_\zeta(\xi,0,\zeta) + O(\varepsilon^2)$$

$$G_\xi^+ + G_\xi^- = G_\xi(\xi,0,\zeta) + O(\varepsilon^2) \tag{1.10}$$

If we now assume an expansion

$$\varphi = \varepsilon\varphi^{(1)} + \varepsilon^2\varphi^{(2)} + O(\varepsilon^3)$$
$$\delta = \varepsilon\delta^{(1)} + \varepsilon^2\delta^{(2)} + O(\varepsilon^3) \qquad (1.11)$$

inserting into Green's formula and collecting terms of equal order in ε, we find that $\psi^{(1)}$ is just the expression from Michell's theory with $\delta^{(1)}=0$ and no contributions from S_a and S_e.

For examination of $\psi^{(2)}$ we must go into the nature of $\delta^{(2)}(X,Y)$. From [2] page 464, (10.12) we find with p = const. and replacing ∂/∂_t by $-\partial/\partial_X$ and g by γ_0 for our nondimensional representation:

$$\gamma_0\psi_Z^{(2)} + \psi_{XX}^{(2)} = \delta^{(2)}(X,Y) = (\operatorname{grad}\psi^{(1)})_X^2 + \frac{1}{\gamma_0}\psi_X^{(1)}(\gamma_0\psi_Z^{(1)} + \psi_{XX}^{(1)})_Z \qquad (1.12)$$

(It should be observed that only the local component of $\psi^{(1)}$ contributes to the expression in brackets in the second term due to structure of G, see Appendix). Sisov's expression corresponding to (1.12) is incorrect.

From the decay of G and its derivatives as $O(X^{-1})$ for $X\to+\infty$, causing the same mode of decay for $\psi^{(1)}$, it may be seen that $\delta^{(2)} = O(X^{-2})$ for $X\to+\infty$ and this means that $\psi^{(2)} = O(X^{-1})$ ahead of the ship and the contribution of S_a to the Green's formula expression may be neglected. If now we can assume that the potential

$$\psi_\delta = \frac{1}{4\pi}\int_{S_f^\circ}\delta\cdot G\cdot dS' \qquad \text{and its X-derivative are}$$

uniformly bounded for $X < X_e$, - and to prove this for not too peculiar $\psi^{(1)}$ should be possible with moderate effort, - then we may drop the contribution of S_e as well as X_e tends to infinity and may finally write:

$$\psi^{(2)} = \psi_1^{(2)} + \psi_2^{(2)}$$

$$\psi_1^{(2)}(X,Y,Z) = \frac{2}{4\pi}\iint_{S_W^{\circ *}} F(\xi,\zeta)\left\{\psi_X^{(1)}(\xi,\varepsilon F,\zeta)\,G_\xi(\xi,0,\zeta) + \psi_Z^{(1)}(\xi,\varepsilon F,\zeta)\,G_\zeta(\xi,0,\zeta)\,d\xi\,d\zeta + \right.$$
$$+ \frac{2}{4\pi\gamma_0}\int_{-1}^{1} F(\xi,0)\,\psi_{XX}^{(1)}\,G(\xi,0,0)\,d\xi \qquad (1.13)$$

$$\psi_2^{(2)}(X,Y,Z) = \frac{1}{4\pi\gamma_0} \iint\limits_{S_f^\circ} \delta^{(2)}(\xi,\eta)\, G\, d\xi\, d\eta\,. \qquad (1.14)$$

The second order potential is thus produced by

1. A distribution of doublets with moment corresponding to deviation of local first order flow relative to the ship from uniform parallel flow (giving rise to the Michell-distribution) times local volume of the ship.
2. A distribution of sources over the plane Z = 0 the density of which is essentially the time-derivative in an inertial system of dynamic pressure (save a contribution of the local flow components in the vicinity of the ship).
3. A line distribution of sources around the ship's load waterline of output corresponding to local breadth times wave slope in X-direction along the ship's contour according to linear theory.

One should observe that the potential appears only in derivative form. On the other hand, no differentiability of the hull surface function F is required to make the expression for $\psi^{(2)}$ meaningful.

For numerical evaluation, the following approximations are made:

(i) calculating the potential from a distribution $\delta(X,Y)$ extending over the entire undisturbed free surface Z = 0 including the waterplane area,

(ii) inserting the flow components calculated for the plane $\eta = 0$ rather than on the hull surface.

Both these steps require continuation of the flow potential and some of its derivatives into the domain occupied by the ship. This is achieved by extension of the corresponding Fourier series in η. In so far as these series would not converge for $\eta = 0$ in the usual sense,-i.e. for example the series for ψ_Y,-we treat them as generalized functions. The second term of $\delta(X,Y)$ in (1.12) will in general become singular at bow and stern; however, for a symmetric hull we will find that it can be left out for calculation of wave resistance.

The error involved with above modifications will in general be of higher order in ε than the terms to be determined, nevertheless, it should be checked for components not uniformly bounded in the extended domain. It should be noted that the expression (13) for $\psi_1^{(2)}$ can be retransformed by partial integration in ξ and ζ to a representation by a source distribution over $S_w^{\circ *}$, - eliminating the line integral, - so that

$$\psi^{(2)} = \frac{-1}{2\pi} \iint_{S_w^{o*}} G \left\{ (F \cdot \psi_x^{(1)})_x + (F \cdot \psi_z^{(1)})_z \right\} d\xi d\zeta + \frac{1}{4\pi\gamma_o} \int_{-T}^{T} \int_{-\infty}^{\infty} \delta(\xi,\eta) \cdot G \cdot d\xi d\eta . \tag{1.15}$$

We should observe that the source density corresponds to the change of internal flow with coordinate rather than to the normal velocity.

Formula (1.15) may be compared with [3] (43), where the influence of trim and sinkage is included. We see that all line integrals presented there can be eliminated under validity of our assumptions.

2. Determination of wave resistance.

Having thus selected a model for the approximation of the flow, there is a decision to be made for definition of wave resistance to the corresponding degree of approximation. Three different approaches may be considered:

(a) Integrate pressure components over the wetted part of the hull bounded by the calculated wave profile, retain only terms up to third order.

(b) Start with expressions for the energy flow through a vertical plane behind the ship, as given in [2] (8.6) page 460, evaluate these for approximate flow, using wave contour from this approximate flow.

(c) Consider the approximate second order flow to be physically real in the domain D°.
Consider a closed surface, part of which is the wetted hull; from the fact that momentum in the enclosed volume D should not change with time, we infer that action of pressure on the hull can be expressed through flow of generalized momentum across the rest of the surface.

It can be shown easily that for the linearized flow model one and the same expression for the resistance can be derived by either approach - (see however the objections raised by Sharma [9]). Up to third order, however, (a) and (c) should only give equivalent expressions R_a and R_c, if the boundary condition on the hull is already met exactly by the approximate flow, as otherwise we may have substantial flux of momentum into the ship's interior. The formula for (b) was derived under assumption of a free surface under constant pressure and composed of streamlines. For a second order flow, it is unrealistic to maintain this assumption. We should therefore expect that resistance R_b, calculated by this formula applied to the approximate flow, could, even in a nonmonotonic way, depend on the location X_e of the vertical control plane where data are taken.

But R_c, derived by approach (c), should be independent of choice of domain D. We shall select $D = D^\circ$, the domain bounded above by the undisturbed free surface as described before. - Due to conservation of momentum we have for surface integrals enclosing any domain D of the flow (compare [2], [7]):

$$\rho \iint \left\{ (\vec{v} \cdot \vec{v})/2 \cdot \vec{n} - (\vec{v} \cdot \vec{n}) \cdot \vec{v} \right\} ds = 0 \qquad (2.1)$$

where $\vec{v}$ may be the flow vector in any system of reference either at rest or in uniform translatory motion, $\vec{n}$ be the unit normal vector directed outwards.
If we now select

$$\vec{v} = \left\{ \varphi_X, \varphi_Y, \varphi_Z \right\} = \left\{ \psi_X, \psi_Y, \psi_Z \right\} \cdot |c|$$

and <u>define</u> R_c as X-component of

$$\vec{R} = \rho \int_{S_W^\circ} \left\{ (\vec{v} \cdot \vec{n}) \vec{v} - (\vec{v} \cdot \vec{v})/2 \cdot \vec{n} \right\} dS, \qquad (2.2)$$

where integration has to be performed over S_W°, the hull surface up to Z = 0, then we have from (1.15), returning to nondimensional quantities, with $\vec{e}_X$ as unit vector in X-direction,

$$R_c = \frac{\vec{R}_c \cdot \vec{e}_X}{\rho c^2 (L/2)^2} = \gamma_o \iint_{S_f^\circ} \psi_X \psi_Z \, d\xi d\eta - \gamma_o \left\{ \iint_{S_r} - \iint_{S_l} \right\} \psi_X \psi_Y \, d\xi d\zeta$$
$$+ \gamma_o \iint_{S_b} \psi_X \psi_Z \, d\xi d\eta + \gamma_o \left\{ \iint_{S_e} - \iint_{S_a} \right\} \frac{\psi_Y^2 + \psi_Z^2 - \psi_X^2}{2} \, d\eta d\zeta \qquad (2.3)$$

Reference to conditions <u>C</u>. and <u>D</u>. shows, that the surface S_r, S_l and S_b may be left out. The integral over S_f° may be transformed to line integrals along the boundaries and an integral containing the function $\delta(X,Y)$ in a similar way as was done for the potential (1.2). The contribution from S_a, including the line integral from S_f°, tends to zero with $X_a \to \infty$ due to <u>F</u>. Thus, we are left with

$$R_c = \iint_{S_f^\circ} \delta(\xi,\eta) \, \psi_X(\xi,\eta,0) \, d\xi d\eta - \oint_{L_p} \frac{\psi_X^2(\xi, \varepsilon F(\xi,0), 0) \, dL}{2} +$$
$$+ \int_{-T}^{T} \left\{ \frac{\psi_X^2(X_e,\eta,0)}{2} + \gamma_o \int_{-H}^{0} \frac{\psi_Y^2(X_e,\eta,\zeta) + \psi_Z^2(X_e,\eta,\zeta) - \psi_X^2(X_e,\eta,\zeta)}{2} d\zeta \right\} d\eta \qquad (2.4)$$

where the line integral over the load waterline L_p is again in counterclockwise direction when viewed from above.

Now, there is no reason to assume that the contribution from S_f°, though bounded in magnitude, should tend to a definite limit with $X_e \to -\infty$, nor can we postulate this for the contribution of the region $X = X_e$. It is only by some property of the Green's function G involved that we shall be able to evaluate the contribution of $\delta(X,Y)$ to the resistance by an integration on S_f° in the vicinity of the ship only. - We shall now look for a relation between the quantities defined as R_c and R_a. To achieve this, we add an expression to the integrand of (2.2) which has no component in direction of $\vec{e}_x$. We set

$$R_c = R_e + \left(\vec{e}_1 \rho \int ((\vec{v} \times \vec{n}) \times \vec{c})\, dS\right) = R_c + \rho\left(\vec{e}_1 \int \left\{(\vec{v}\cdot\vec{c})\vec{n} - (\vec{c}\cdot\vec{n})\vec{v}\right\} dS\right) \tag{2.5}$$

and therefore, with $(\vec{c}\cdot\vec{n}) = (\vec{v}\cdot\vec{n})$ assumed even for the first order flow,

$$R_c = \int \rho(\vec{e}_1\cdot\vec{n})\cdot\left\{(c\cdot\vec{v}) - (\vec{v}\cdot\vec{v})/2\right\} dS. \tag{2.6}$$

But this is just R_a, the resistance defined from pressure integration over the hull, for the nonstatic pressure is $\rho((\vec{v}\cdot\vec{c}) - (\vec{v}\cdot\vec{v})/2)$ and we thus have

$$R_a = R_c + \rho \int_{S_w^\circ} \varphi_x \left((\vec{c}\cdot\vec{n}) - (\vec{v}\cdot\vec{n})\right) dS \tag{2.7}$$

To R_c as defined above, we now add an appropriate correction for the influence of the wave profile along the waterline, as only the wetted part of the hull can experience pressure from the fluid, and then define the quantity obtained as "third order wave resistance". Now, up to second order, pressure is atmospheric pressure plus hydrostatic pressure due to the wave elevation ζ. Integrating the last quantity over dZ dY, the projection of the surface element on the plane vertical to X axis, we find a correction as

$$\Delta R = \rho g \int\oint_{L_p} (Z - \zeta)\, dZ\, d\eta = \rho g \oint_{L_p} \frac{\zeta^2(\xi, \varepsilon F)}{2}\, d\eta. \tag{2.8}$$

As now the perturbation procedure gives the first order wave elevation [2] as

$$\zeta^{(1)} = \varepsilon \frac{c}{g} \varphi_x \qquad \text{i.e.} \qquad \frac{\zeta^{(1)}}{L/2} = \frac{1}{\gamma_0} \varphi_x^{(1)}(X,Y,0), \tag{2.9}$$

adding this in nondimensional form to expression (2.4), we see that this correction just cancels the line integral around L_p, which we, therefore, can happily discard [8]. -

A further simplification will be made by extending the integration of δ over the whole plane $\zeta = 0$, $-T < \eta < T$, $\xi < X_e$ which means an error of order ε^4, as the waterplane area is of order ε. Inserting now $\psi = \varepsilon\psi^{(1)} + \varepsilon^2\psi^{(2)}$, $\delta = \varepsilon^2\delta^{(2)}$, $R = \varepsilon^2 R^{(2)} + \varepsilon^3 R^{(3)}$ we have

$$R^{(2)} = \int_{-T}^{T} \left\{ \frac{1}{2\gamma_o} \psi_X^{(1)^2}(X_e, \eta, 0) + \int_{-H}^{0} \frac{\psi_Y^{(1)^2} + \psi_Z^{(1)^2} - \psi_X^{(1)^2}}{2} d\zeta \right\} d\eta \qquad (2.10)$$

$R^{(3)} = R_1^{(3)} + R_2^{(3)}$

(corresponding to the partition $\psi^{(2)} = \psi_1^{(2)} + \psi_2^{(2)}$) with

$$R_1^{(3)} = \int_{-T}^{T} \left\{ \frac{1}{\gamma_o} \psi_X^{(1)} \cdot \psi_{2X}^{(2)} + \int_{-H}^{0} \left(\psi_Y^{(1)} \psi_{2Y}^{(2)} + \psi_Z^{(1)} \psi_{2Z}^{(2)} - \psi_X^{(1)} \psi_{2X}^{(2)} \right) d\zeta \right\} d\eta \qquad (2.11)$$

$$R_2^{(3)} = -\frac{1}{\gamma_o} \int_{-T}^{T} \int_{X_e}^{\infty} \delta^{(2)}(\xi, \eta)\, \psi_X^{(1)} d\xi d\eta + \int_{-T}^{T} \left\{ \frac{1}{\gamma_o} \psi_X^{(1)} \psi_{1X}^{(2)} + \int_{-H}^{0} \left(\psi_Y^{(1)} \psi_{1Y}^{(2)} + \psi_Z^{(1)} \psi_{1Z}^{(2)} - \psi_X^{(1)} \psi_{1X}^{(2)} \right) d\xi \right\} d\eta \qquad (2.12)$$

3. Resistance due to additional singularities within the hull.

Let us now, only to save labor in writing down formulas, assume that the depth of the tank is large enough that we may put $H = \infty$. If $\psi_X^{(1)}$ can at $X = X_e$ be represented by a system of free waves - we omit terms nonsymmetric in Y for reasons of simplicity - as

$$\psi_X^{(1)\,\text{free}} = \sum_{\nu=-\infty}^{\infty} \left(A_\nu^{(1)} \cos(W_\nu \gamma_o X) + B_\nu^{(1)} \sin(W_\nu \gamma_o X) \right) \cdot e^{K_\nu \gamma_o Z} \cos(U_\nu \gamma_o Y) \qquad (2.13)$$

where $\Delta U = \pi/(\gamma_o T)$ and $U_\nu = \nu \cdot \Delta U = \sec^2\theta_\nu \sin\theta_\nu$; $M_\nu = 1 + 4U_\nu^2$;

$K_\nu = (1 + M_\nu)/2 = \sec^2\theta_\nu$;

$W_\nu = \sqrt{K_\nu} = \sec\theta_\nu$; $A_\nu^{(1)} = A_{-\nu}^{(1)}$; $B_\nu^{(1)} = B_{-\nu}^{(1)}$,

(where θ_ν stands for the angle of wave propagation against X axis), then [6] we can evaluate the integrals in closed form as

$$R^{(2)} = T \cdot \sum_{\nu=-\infty}^{\infty} \frac{2-\cos^2\theta_\nu}{2} \left(A_\nu^{(1)^2} + B_\nu^{(1)^2} \right) / \gamma_0 \tag{2.14}$$

(This formula reflects the fact that resistance is essentially equal to average energy in wave components times difference between ship's speed c and X-component of group-velocity, divided by c.)

If $\varphi_{1_X}^{(2)}$ has a corresponding far-field representation:

$$\varphi_{1_X}^{(2)(free)} = \sum_{\nu=-\infty}^{\infty} \left\{ A_{1_\nu}^{(2)} \cos(W_\nu \gamma_0 X) + B_{1_\nu}^{(2)} \sin(W_\nu \gamma_0 X) \right\} e^{K_\nu \gamma_0 Z} \cos(U_\nu \gamma_0 Y) \tag{2.15}$$

then $R_1^{(3)}$ as interference between both systems can be written down directly as

$$R_1^{(3)} = T \cdot \sum_{-\infty}^{\infty} (2-\cos^2\theta_\nu)(A_{1_\nu}^{(2)} \cdot A_\nu^{(1)} + B_{1_\nu}^{(2)} B_\nu^{(1)}) / \gamma_0 \tag{2.16}$$

For evaluation of (2.14) and (2.16) we have to keep in mind that for $\eta = 0$ the Green's function G has a representation for $\xi \gg X$ as system of free waves like

$$G_X \sim \sum_{\nu=-\infty}^{\infty} g_\nu \cdot \cos\left(W_\nu \gamma_0 (X-\xi)\right) e^{K_\nu \gamma_0 (Z+\zeta)} \cos(U_\nu \gamma_0 Y) \tag{2.17}$$

$$\text{with } g_\nu = g_{-\nu} = -8\pi \cdot K_\nu \cdot \gamma_0 / (M_\nu \cdot T)$$

(see Appendix) and that we have:

$$\varphi^{(1)} = -\frac{2}{4\pi} \iint_{S_W^{o*}} F(\xi,\zeta) \cdot G_\xi \, d\xi \, d\zeta \tag{2.18}$$

$$\varphi_{1_X}^{(2)} = \frac{2}{4\pi} \iint_{S_W^{o*}} F(\xi,\zeta) \cdot \left\{ \varphi_X^{(1)} G_{\xi X} + \varphi_Z^{(1)} G_{\zeta X} \right\} d\xi \, d\zeta + \frac{2}{4\pi\gamma_0} \int_{-1}^{1} F(\xi,0) \varphi_{XX}^{(1)} G_X \, d\xi$$

$$= -\frac{1}{2\pi} \iint_{S_W^{o*}} \left\{ (F\varphi_X)_X + (F\varphi_Z)_Z \right\} G_X \, d\xi \, d\zeta \tag{2.19}$$

and therefore

$$A_\nu^{(1)} = \frac{g_\nu}{2\pi} \iint_{S_W^{o*}} F(\xi,\zeta) \sin(W_\nu \gamma_0 \xi) \cdot e^{K_\nu \gamma_0 \zeta} \cdot W_\nu \gamma_0 \, d\xi \, d\zeta \tag{2.20}$$

$$B_{\nu}^{(1)} = -\frac{g_{\nu}}{2\pi} \iint\limits_{S_W^{\circ *}} F(\xi,\zeta)\cdot\cos(W_{\nu}\gamma_{o}\xi)\cdot e^{K_{\nu}\gamma_{o}\zeta}\cdot W_{\nu}\gamma_{o}\, d\xi\, d\zeta \qquad (2.21)$$

$$A_{1_{\nu}}^{(2)} = \frac{g_{\nu}}{2\pi} \iint\limits_{S_W^{\circ *}} F(\xi,\zeta)\left\{\psi_{x}^{(1)}\cdot W_{\nu}\gamma_{o}\cdot\sin(W_{\nu}\gamma_{o}\xi) + \psi_{z}^{(1)}\cdot K_{\nu}\gamma\cdot\cos(W_{\nu}\gamma_{o}\xi)\right\} d\xi\, d\zeta$$

$$+\frac{g_{\nu}}{2\pi\gamma_{o}}\int_{-1}^{1} F(\xi,0)\cdot\psi_{xx}^{(1)}\cdot\sin(W_{\nu}\gamma_{o}\xi)\, d\xi \qquad (2.22)$$

$$B_{1_{\nu}}^{(2)} = -\frac{g_{\nu}}{2\pi} \iint\limits_{S_W^{\circ *}} F(\xi,\zeta)\left\{\psi_{x}^{(1)}\cdot W_{\nu}\gamma_{o}\cdot\cos(W_{\nu}\gamma_{o}\xi) - \psi_{z}^{(1)}\cdot K_{\nu}\gamma_{o}\cdot\sin(W_{\nu}\gamma_{o}\xi)\right\} d\xi\, d\zeta$$

$$-\frac{g_{\nu}}{2\pi\gamma_{o}}\int_{-1}^{1} F(\xi,0)\cdot\psi_{xx}^{(1)}\cos(W_{\nu}\gamma_{o}\xi)\, d\xi\,. \qquad (2.23)$$

The above expressions can in general be evaluated in closed form for mathematical elementary hulls, save the contributions of local flow $\psi^{(1)}$ to the integrands, where however the V-integration may be interchanged with closed-form ξ, ζ integration. -

4. Resistance due to additional singularities at undisturbed free surface.

Consider a strip of width $d\xi$ extending from $\eta = -T$ to $\eta = T$ at $\zeta = 0$ with ordinate $X = \xi$. Assume that a Fourier expansion for $\delta(\xi,\eta)$ holds as

$$\overset{(2)}{\delta}(\xi,\eta) = \sum_{\nu=-\infty}^{\infty} \delta_{\nu}(\xi)\cdot\cos(U_{\nu}\gamma_{o}\eta) \qquad (2.24)$$

If $X_e \ll \xi$, this strip will contribute to $\psi_{2_X}^{(2)}$ by

$$d\psi_{2_X}^{(2)} = \int_{-T}^{T} \frac{\delta}{4\pi\gamma_{o}} G_{X}^{(free)}(\xi,\eta,0,X,Y,Z)\, d\eta\, d\xi \qquad (2.25)$$

with

$$G_{X}^{(free)} = \sum_{\nu=-\infty}^{\infty} g_{\nu}\cdot e^{K_{\nu}\gamma_{o}Z}\cos(W_{\nu}\gamma_{o}(X-\xi))\cos(U_{\nu}\gamma_{o}Y)\cos(U_{\nu}\gamma_{o}\eta) \qquad (2.26)$$

(+ terms odd in η not needed here)
where

$$g_{\nu} = g_{-\nu} = 4\pi K_{\nu}\gamma_{o}^{2}/(M_{\nu}\cdot T) \qquad (2.27)$$

(compare (A.1)). We then have:

$$d\psi_{2_X}^{(2)} = 2T\sum_{\nu=-\infty}^{\infty} \delta_\nu \cdot K_\nu/(M_\nu \cdot T)\cdot e^{K_\nu \gamma_0 Z}$$

$$\times\left(\cos(W_\nu\gamma_0\xi)\cos(W_\nu\gamma_0 X)+\sin(W_\nu\gamma_0\xi)\sin(W_\nu\gamma_0 X)\right)\cdot\cos(U_\nu\gamma_0 Y)\,d\xi \qquad (2.28)$$

If now for $\psi_X^{(1)}$ as well only free waves are significant at $X = X_e$, i.e. if we have:

$$\psi_X^{(1)} = \psi_X^{(1)(free)} = \sum_{\nu=-\infty}^{\infty}\left\{A_\nu^{(1)}\cos(W_\nu\gamma_0 X)+B_\nu^{(1)}\sin(W_\nu\gamma_0 X)\right\}e^{K_\nu\gamma_0 Z}\cos(U_\nu\gamma_0 Y) \qquad (2.29)$$

then $d\psi_{2_X}^{(2)}$ will make up a contribution to $R_2^{(3)}$ as

$$dR_2^{(3)} = T\sum_{\nu=-\infty}^{\infty}\delta_\nu(2-\cos^2\theta_\nu)\frac{K_\nu}{M_\nu}\left(A^{(1)}\cos(W_\nu\gamma_0\xi)-B^{(1)}\sin(W_\nu\gamma_0\xi)\right)d\xi \qquad (2.30)$$

But from (2.13) we may derive that

$$K_\nu/M_\nu = K_\nu/(2K_\nu-1) = 1/(2-\cos^2\theta_\nu), \qquad (2.31)$$

thus

$$dR_2^{(3)} = 2T\sum_{\nu=-\infty}^{\infty}\delta_\nu\left\{A_\nu^{(1)}\cos(W_\nu\gamma_0\xi)+B_\nu^{(1)}\sin(W_\nu\gamma_0\xi)\right\} \qquad (2.32)$$

$$= \int_{\eta=-T}^{T}\delta(\xi,\eta)\,\psi_X^{(1)(free)}(\xi,\eta,0)\,d\eta\,d\xi$$

This is not yet the whole contribution of the strip to $R_2^{(3)}$, however; from (2.12) we have to add:

$$dR_2^{(3)} = -\int_{\eta=-T}^{T}\delta^{(2)}(\xi,\eta)\,\psi_X^{(1)}(\xi,\eta,0)\,d\eta\,d\xi \qquad (2.33)$$

This leads to the simple result

$$R_2^{(3)} = -\int_{-\infty}^{\infty}\int_{-T}^{T}\delta^{(2)}(\xi,\eta,0)\,\tilde{\psi}_X(\xi,\eta,0)\,d\eta\,d\xi \qquad (2.34)$$

with $\tilde{\psi}_X = \psi_X^{(1)} - \psi_X^{(1)(free)}$ (2.35)

The potential $\tilde{\psi}$ would correspond to the solution of the first order boundary value problem if we had postulated waves traveling ahead of the ship instead of aft the ship. For a ship symmetrical to the midship section we may insert $\tilde{\psi}(X) = \psi(-X)$.

The above integral (2.34) will have significant contributions to resistance only from the vicinity of the ship, as $\tilde{\delta}$ has strong decay ahead and the factor Ψ_X shows a decay aft. The overwhelming contribution should therefore come from the rhombe-shaped region bounded by a Kelvin angle drawn from the bow and an opposite angle from the stern.

The expression for $R_2^{(3)}$ could have been derived directly as the Lagally force of the wave field due to the surface disturbance $\delta(\xi,\eta)$ acting on the singularities creating the first order flow field of the ship. It would, therefore, have been found by Sisov under use of proper radiation condition. For the case of a nonsubmerged body, we felt that formal application of Lagally's law even for higher order contributions deserved caution.- Inserting (1.13) we have

$$R_2^{(3)} = -\int_{-T}^{T}\int_{-\infty}^{\infty}\left\{\left(\operatorname{grad}\varphi^{(1)}\right)_X^2 + \varphi_X^{(1)}\left(\gamma_o\varphi_Z^{(1)} + \varphi_{XX}^{(1)}\right)_Z\Big/\gamma_o\right\}\left\{\varphi_X^{(1)} - \varphi_X^{(1)(free)}\right\} dX\,dY \tag{2.36}$$

In peculiar for a symmetrical hull, where $\gamma_o\varphi_Z^{(1)} + \varphi_{XX}^{(1)}$ is odd and $\varphi_X^{(1)}\cdot\tilde{\varphi}_X$ is even, we have:

$$R_2^{(3)} = -\int_{-T}^{T}\int_{-\infty}^{\infty}\left(\operatorname{grad}\varphi^{(1)}\right)_X^2\,\varphi_X^{(1)}(-X)\,dX\,dY = \int_{-T}^{T}\int_{-\infty}^{\infty}\left(\operatorname{grad}\varphi^{(1)}\right)^2\varphi_{XX}^{(1)}\cdot(-X)\,dX\,dY$$

$$= -\frac{1}{\gamma_o}\int_{-T}^{T}\int_{-\infty}^{\infty}\left(\operatorname{grad}\varphi^{(1)}\right)^2\varphi_Z^{(1)}\cdot(-X)\,dX\,dY \tag{2.37}$$

If now for a symmetrical hull we have:

$$\varphi_X^{(1)}(X,Y,0) = \sum_{\nu=-\infty}^{\infty}\alpha_\nu(X)\cos(U_\nu\cdot\gamma_o\cdot Y) \qquad \text{with } \alpha_\nu = \alpha_{-\nu} \tag{2.38}$$

$$\varphi_Y^{(1)}(X,Y,0) = \sum_{\nu=-\infty}^{\infty}\beta_\nu(X)\sin(U_\nu\cdot\gamma_o\cdot Y) \qquad \text{with } \beta_\nu = \beta_{-\nu}$$

$$\varphi_Z^{(1)}(X,Y,0) = \sum_{\nu=-\infty}^{\infty}\gamma_\nu(X)\cos(U_\nu\cdot\gamma_o\cdot Y) \qquad \text{with } \gamma_\nu = \gamma_{-\nu}$$

where according to (2.18) and (A.1) the coefficients α_ν, β_ν, and γ_ν depend on hull geometry given by $Y = \pm\varepsilon F(X,Z)$ through the relations

$$\alpha_\nu = \partial H_\nu/\partial X, \qquad \beta_\nu = U_\nu \gamma_o H_\nu, \qquad \gamma_\nu = \partial H_\nu/\partial Z \qquad (2.39)$$

with the function $H_\nu(X,Y,Z)$ given by

$$H_\nu(X,Y,Z) = \frac{2K_\nu \gamma_o}{T\cdot M_\nu} \iint_{S_w^{**}} [\mathrm{sign}(X-\xi)-1]\, e^{K_\nu \gamma_o \xi} \cdot F(\xi,\zeta)\cdot \cos(W_\nu \gamma_o (X-\xi))\, d\xi\, d\zeta$$
$$- \frac{2\gamma_o}{\pi\cdot T} \iint_{S_w^{**}} \int_{V=0}^{\infty} \mathrm{sign}(X-\xi)\, F(\xi,\zeta)\, e^{-U\gamma_o|X-\xi|} \left\{V\cos(V\gamma_o\zeta) - U^2 \sin(V\gamma_o\zeta)\right\} \frac{U\,dV}{U^4+V^2}\, d\xi\, d\zeta \qquad (2.40)$$

and U_ν, M_ν, W_ν, K_ν and U as given by (A.4), then we can express

$$R_2^{(3)} = R_2^{(3)} \cdot 4L/(\rho B^3 c^2)$$

as

$$R_2^{(3)} = -\frac{T}{\gamma_o} \int_{-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} \sum_{\lambda=-\infty}^{\infty} \Big[\left\{\alpha_\mu(X)\cdot\alpha_\lambda(X) - \beta_\mu(X)\cdot\beta_\lambda(X) + \gamma_\mu(X)\cdot\gamma_\lambda(X)\right\}\cdot \gamma_{\lambda+\mu}(-X) +$$
$$+ \left\{\alpha_\mu(X)\cdot\alpha_\lambda(X) + \beta_\mu(X)\cdot\beta_\lambda(X) + \gamma_\mu(X)\cdot\gamma_\lambda(X)\right\} \gamma_{|\mu-\lambda|}(-X) \Big]\, dX \qquad (2.41)$$

due to the Fourier orthogonality relations for Y integration, where the X integral may be truncated soon after X exceeds 1 in absolute value, convergence of (2,41) assumed.

The above formula can easily be extended to the case of infinite tank width with $T\to\infty$; however, for practical evaluation it is recommended to consider T as inverse of spacing in integration by trapezoidal rule and let T be just large enough, dependent on X, that for $\xi = \pm X$ the actual wave pattern is well within $|Y| < T$, i.e. that no tank effect can be felt.

For actual calculations we have to reintroduce dimensions; we have

$$R^{(3)} = (B/L)^3 \cdot \rho \cdot c^2 \cdot (L/2)^2 \quad \cdot \quad R^{(3)} = \rho c^2 B^3/(4\cdot L)\cdot R^{(3)}$$
$$R^{(2)} = (B/L)^2 \cdot \rho \cdot c^2 \cdot (L/2)^2 \quad \cdot \quad R^{(2)} = \rho c^2 B^2/4 \cdot R^{(2)} \qquad (2.42)$$

-(compare (2.3), (2.10), (2.11) (2.12)). - where $R^{(3)}$ and $R^{(2)}$ are the actual third and second order resistance components.

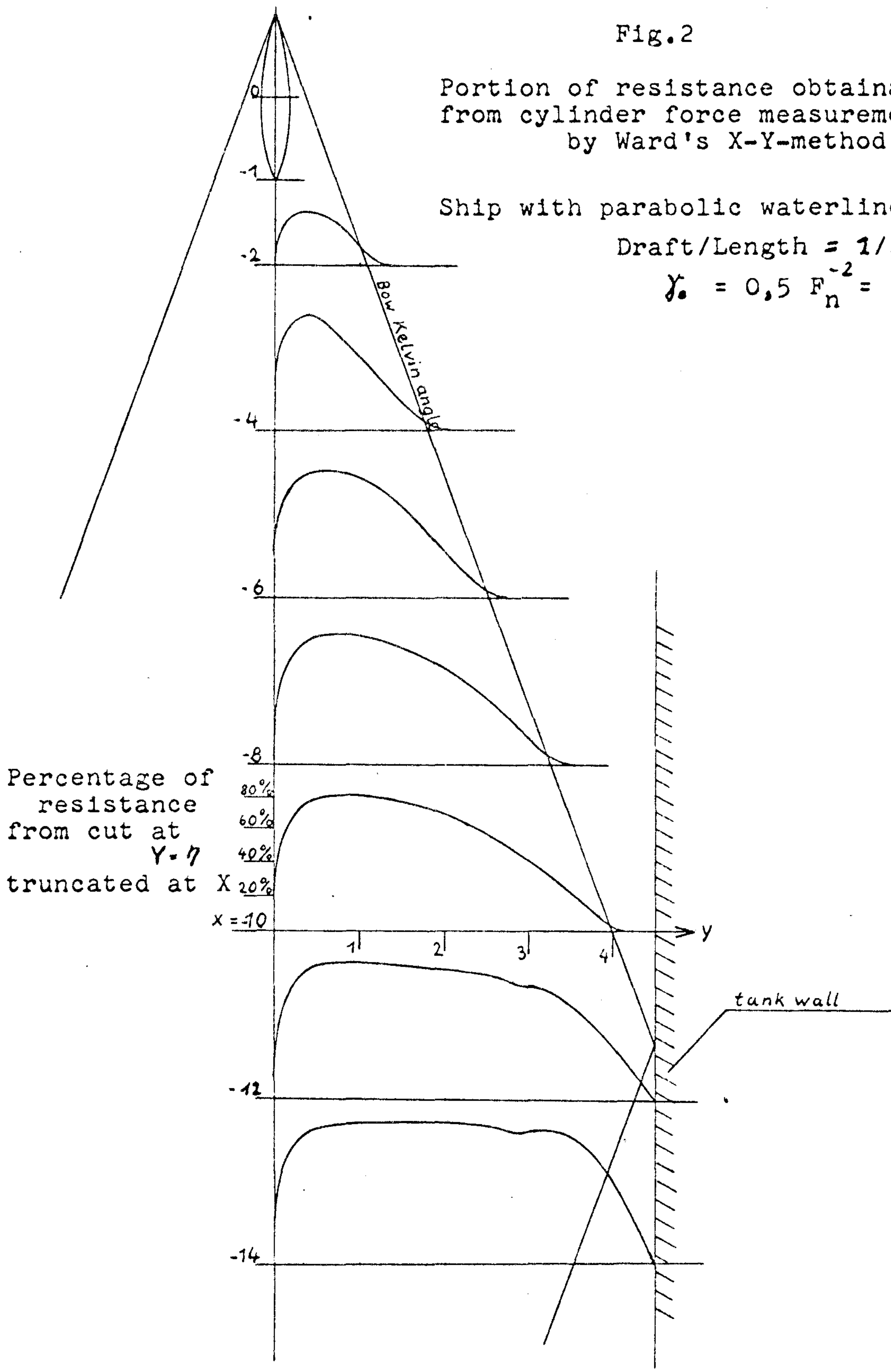
Fig.2

Portion of resistance obtainable from cylinder force measurements by Ward's X-Y-method.

Ship with parabolic waterlines

Draft/Length = 1/20

$\gamma_0 = 0{,}5\ F_n^{-2} = 10$



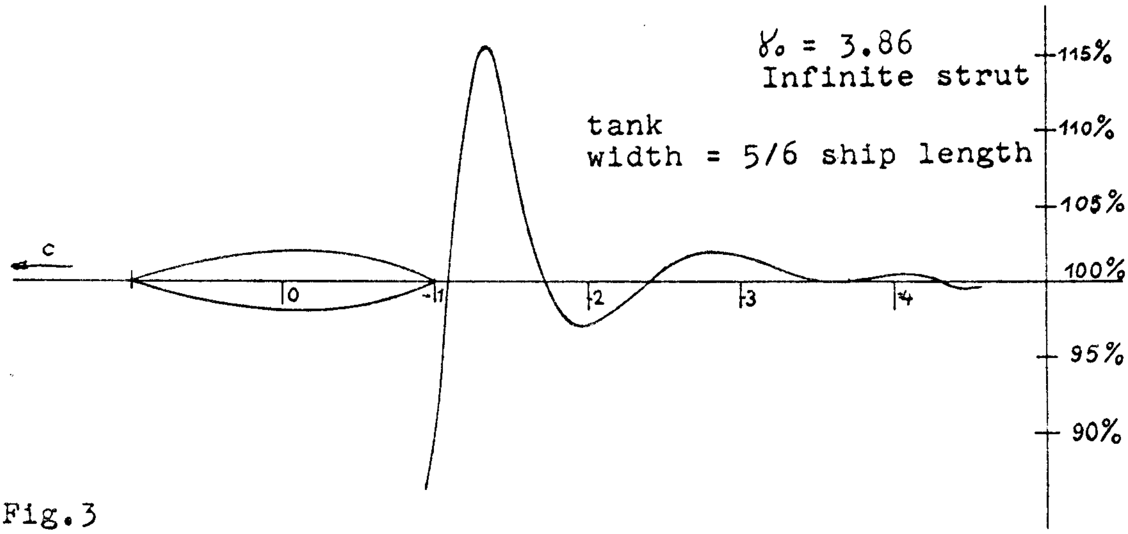


Fig.3

Ratio of Calculated Resistance $R_t^{(2)}(x)$ from Transverse Cut Analysis [6] (Wave Elevation and χ-slope) to Asymptotic Value $R^{(2)}$

Summary.

With the above analytical considerations, an attempt was made to coordinate the intuitive approach of Sisov with the rigorous procedure of Wehausen. Some simplifications allowed, we found that even the latter leads to a representation of the second order wave potential by sources only, located on the undisturbed free surface and on the longitudinal centerplane of the ship; - in particular all line integrals can be eliminated. Additional resistance can be expressed in terms of first order flow components which determine these singularities. Only a region of the free surface close to the ship need be considered.

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Appendix

A Fourier series representation of the source potential in a tank of finite width.

The expression to be presented here has essentially been derived in [6]. We shall confine ourselves to show certain properties which are needed in the foregoing applications.

The expression describes a wave potential of a source of output $+4\pi$, - i.e. a singularity like negative inverse distance, - in coordinates made dimensionless by ship's half length as introduced under (1). The expression is:

$$G = 2\gamma_o \sum_{\nu=-\infty}^{\infty} \left\{ g_\nu^{(free)}(X,\xi,Z,\zeta) + g_\nu^{(local)}(X,\xi,Z,\zeta) \right\} \left\{ \cos(U_\nu \gamma_o Y) \times \cos(U_\nu \gamma_o \eta)(1+(-1)^\nu) + \right.$$

$$\left. + \sin(U_\nu \gamma_o Y) \sin(U_\nu \gamma_o \eta)(1-(-1)^\nu) \right\} \Delta U \qquad (A.1)$$

with $\Delta U = \pi/(\gamma_o T)$ $\qquad U_\nu = \nu \cdot \Delta U = \sec^2\theta_\nu \sin\theta_\nu$

$$g_\nu^{(free)} = -g_{-\nu}^{(free)} = \left\{ \operatorname{sign}(X-\xi) - 1 \right\} \cdot \frac{W_\nu}{M_\nu} e^{K_\nu (Z+\zeta)\gamma_o} \sin W_\nu \gamma_o (X-\xi)$$

$$M_\nu = \sqrt{1+4U_\nu^2}\,; \qquad K_\nu = (1+M_\nu)/2 = \sec^2\theta_\nu$$

$$W_\nu = \sqrt{K_\nu} = \sec\theta,$$

and

$$g_\nu^{(local)} = g_{-\nu}^{(local)} = 1/\pi \int_{V=0}^{\infty} \left[e^{-U\gamma_o |X-\xi|} \cdot \left\{ \left(V\cos(V\gamma_o Z) - U^2 \sin(V\gamma_o Z) \right) \cdot \right.\right.$$

$$\left.\left. \times \left(V\cos(V\gamma_o \zeta) - U^2 \sin(V\gamma_o \xi) \right) \right\} - \delta_\nu^\circ(U) V^2 \right] / \left(V(U^4+V^2) \right) \cdot dV \qquad (A.2)$$

with $U = +\sqrt{V^2 + U_\nu^2}$ and $\delta_\nu^\circ(0) = 1$ for $\nu = 0$, else $\delta_\nu^\circ = 0$.
~~or small enough not to disturb convergence of series.~~
It is easy to find out by investigation of single terms of the series that the function G is subject to the following conditions:

A. $G_{XX} + G_{YY} + G_{ZZ} = 0$ provided the corresponding V integrals exist, which is guaranteed for $|X-\xi| > 0$

B. $\gamma_o G_Z + G_{XX} = 0$ for $Z = 0$

C. $G_Y = 0$ for $Y = \pm T$

D. $G_Z \to 0$ with $Z \to -\infty$

F. $G = O(X-\xi)^{-1}$ as $X \to +\infty$, $G = O(1)$ as $X \to -\infty$,
$G_X = O(X-\xi)^{-1}$ as $X \to +\infty$, $G_X = O(1)$ as $X \to -\infty$,
explicitely shown in [6].

As the structure of G is symmetric, corresponding relations can be obtained under exchange of X, Y, Z with ξ, η, ζ. It remains to be shown that

(i) the expressions for G and G_X match in a continous way at $X = \xi$.

(ii) for $|Y-\eta| \leq T$, $Z \leq 0$ and $\zeta \leq 0$, G and G_X become singular only for $X = \xi$, $Y = \eta$ and $Z = \zeta$, and that the functions

$$G^* = G + 1/r \quad \text{and}$$
$$G_X^* = G_X + (1/r)_X, \quad \text{with } r = \left\{(X-\xi)^2 + (Y-\eta)^2 + (Z-\zeta)^2\right\}^{-1/2} \tag{A.3}$$

remain finite here; - (convergence of the series for $\zeta \neq Z$ is shown in [6]). Statement (i) is evident for the function G. - Assume for simplicity $\xi = 0$.
Then it is sufficient to show that

$$\frac{K_\nu}{M_\nu} \cdot e^{K_\nu \gamma_o (Z+\zeta)} = \lim_{X \to 0} \frac{1}{\pi} \int_{V=0}^{\infty} e^{-K\gamma_o |X|} \left(V\cos(V\gamma_o Z) - U^2 \sin(V\gamma_o Z)\right)\left(V\cos(V\gamma_o \zeta) - U^2 \sin(V\gamma_o \zeta)\right) \frac{dV}{U^4+V^2} \tag{A.4}$$

for arbitrary $U_\nu \geq 0$ and $Z \neq \zeta$ with

$$U = \sqrt{V^2 + U_\nu^2}\,; \qquad M_\nu = \sqrt{1 + 4U_\nu^2}\,; \qquad K_\nu = (1 + M_\nu)/2\,.$$

On the right hand side we may substitute

$$= \frac{1}{2\pi} \lim_{X \to 0} \left\{ \int_{V=0}^{\infty} \left[V^2 \left\{\cos(V\gamma_o(Z+\zeta)) + \cos V\gamma_o(Z-\zeta)\right\} - U^4 \left\{\cos(V\gamma_o(Z+\zeta)) - \cos(V\gamma_o(Z-\zeta))\right\} + \right.\right.$$
$$\left. + 2U^2 V \sin V\gamma_o (Z+\zeta) \right] \frac{e^{-U\gamma_o X}\, dV}{U^4 + V^2}$$

$$= \frac{1}{4\pi} \lim_{X \to 0} \operatorname{Re} \left\{ \int_{V=-\infty}^{\infty} \frac{V + iU^2}{V - iU^2} e^{iV\gamma_o (Z+\zeta) - V\gamma_o |X|} dV + \int_{V=-\infty}^{\infty} e^{-U\gamma_o X + iV\gamma_o (Z-\zeta)} dV + \right.$$
$$\left. + \int_{V=-\infty}^{\infty} \frac{(e^{-U\gamma_o |X|} - e^{-V\gamma_o |X|})(V + iU^2) e^{iV\gamma_o (Z+\zeta)}}{V - iU^2} dV \right\} \tag{A.5}$$

The second term in the last expression is zero for any finite X. The last term is o(X). The first term may be written

$$= \frac{1}{4\pi} \lim_{X\to 0} \, Re\left\{ \int_{V=-\infty}^{\infty} \frac{(V-iK_\nu)(V+i(M_\nu+1)/2)}{(V+iK_\nu)(V-i(M_\nu+1)/2)} \, e^{iV\gamma_0(Z+\zeta)-V\gamma_0|X|} dV \right\} \qquad (A.6)$$

which shows poles of the integrand for $V = i(M_\nu-1)/2$ and $V=-i(M_\nu+1)/2=-iK_\nu$. By shifting the path of integration downward in the complex plane we can make the integral arbitrary small after splitting off the residuum at $V=-iK_\nu$, thus we finally get

$$= 2\pi i \operatorname*{Res}_{V=-iK_\nu} \left\{ \frac{(V-iK_\nu)(V+i(M_\nu+1)/2)}{(V+iK_\nu)(V-i(M_\nu+1)/2)} \, e^{V\gamma_0(Z+\xi)} \right\} \cdot \frac{1}{4\pi} = \frac{K_\nu \, e^{K_\nu\gamma_0(Z+\xi)}}{M_\nu} \qquad (A.7)$$

q.e.d.

To prove the statement (ii) we start with the representation

$$1/r = 1/(2\pi)\int_{u=0}^{\infty}\cdot\int_{\theta=0}^{2\pi} e^{-u|X-\xi|} \cos u\left\{(Y-\eta)\cos\theta+(Z+\zeta)\sin\theta\right\} du\, d\theta \qquad (A.8)$$

$$1/r_1 = 1/(2\pi)\int_{u=0}^{\infty}\cdot\int_{\theta=0}^{2\pi} e^{-u|X-\xi|} \cos u\left\{(Y-\eta)\cos\theta+(Z-\zeta)\sin\theta\right\} du\, d\theta$$

where r_1 corresponds to r with ζ under negative sign. - Introducing new variables of integration U, V, $\tilde{U}$ by

$$U = u/\gamma_0\, ; \qquad \tilde{U} = u\cos\theta/\gamma_0\, ; \qquad V = u\sin\theta/\gamma_0\, ; \qquad (A.9)$$

we have

$$1/r - 1/r_1 = \gamma_0/(2\pi)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-U\gamma_0|X-\xi|}\cdot 2\sin(V\gamma_0 Z)\sin(V\gamma_0\zeta)\cos(\tilde{U}\gamma_0(Y-\eta))/U\, d\tilde{U}\, dV \qquad (A.10)$$

$$(1/r-1/r_1)_X = -\gamma_0/(2\pi)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-U\gamma_0|X-\xi|}\, 2\sin(V\gamma_0 Z)\sin(V\gamma_0\zeta)\cos(\tilde{U}\gamma_0(Y-\eta))\, d\tilde{U}\, dV\cdot \mathrm{sign}(X-\xi) \qquad (A.11)$$

Now there is a general law in the theory of Fourier transforms [11]*) - essentially known as Poisson's summation rule - stating that if the function F(y) has a representation

$$F(y) = \int_{-\infty}^{\infty} G(\tilde{U})\, e^{i\tilde{U}y}\, d\tilde{U} \tag{A.12}$$

then $F^*(\delta, y) = \sum_{\nu=-\infty}^{\infty} F(y+\nu\delta)$, provided this series converges, has a representation

$$F^*(\delta, y) = \sum_{\nu=-\infty}^{\infty} G(U_\nu)\, e^{iU_\nu y} \cdot \Delta\tilde{U} \tag{A.13}$$

with $\Delta\tilde{U} = 1/\delta$, $U_\nu = \nu \cdot \Delta U$

With $\delta = T/\pi$, we therefore have the representation

$$\sum_{\nu=-\infty}^{\infty} \left\{ \left[(X-\xi)^2 + (Y+\nu T-\eta)^2 + (Z+\zeta)^2\right]^{-1/2} - \left[(X-\xi)^2 + (Y+\nu T-\eta)^2 + (Z-\zeta)^2\right]^{-1/2} \right\} =$$

$$= -2/\pi \int_{V=-\infty}^{\infty} \sum_{\nu=-\infty}^{\infty} e^{-U\gamma_0 |X-\xi|} \sin(V\gamma_0 Z) \sin(V\gamma_0 \zeta)/U \cdot dV \cdot \cos\left(U_\nu \gamma_0 (Y-\eta)\right) \tag{A.14}$$

with $U_\nu = 2\pi\nu/(\gamma_0 T)$; $U = \sqrt{V^2 + U_\nu^2}$.

But the terms under summation are equivalent to corresponding terms in the series for G (A.1), and it can be seen that after subtraction of these terms the integrals for the coefficients $g_\nu^{(local)}$ (A.1) converge even in the case $X = \xi$, $Z = \zeta$.

The argument for the function G_X is analogous.

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