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Wear due to fatigue initiation

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ABSTRACT

Persson and coauthors have recently proposed an extension of the Rabinowicz idea for fatigue wear at different scales of roughness, where Paris' crack growth law is applied to "potential" wear particles. However, Persson's theory suffers from the fact that initial size of defects is unknown and fatigue life is not entirely due to propagation, so we investigate a different formulation, where a law for initiation of cracks is used for a specimen with initial roughness of engineering interest. We find that results (in particular dependence on amplitude of roughness, and on friction coefficient) are qualitatively similar to the original Persson and coworkers' theory, but may differ substantially quantitatively. As the assumption of a constant fatigue threshold may be incorrect for short cracks, both fatigue limit and fatigue threshold are made dependent of crack size, using the Murakami formulation as one of the possible alternatives. This makes wear rate be sensitive to the fine scale details of the roughness spectrum, which has an effect on increasing wear rate and small particles emission. The model seems to have qualitative trends in agreement with experiments, but obviously wear is a very complex phenomenon and many factors may be not captured.

1. Introduction

Wear is generally approached with the Archard's law [1] which relates the worn volume V in a nominal area A_0 slid of an amount L to the nominal pressure p and the (Brinnell) Hardness [MPa]

$$\frac{V}{A_0 L} = K\frac{p}{3H} = \frac{k_a}{3}p \tag{1}$$

where K is dimensionless wear coefficient varying between 10^{-7} to 10^{-3} for most materials as reported in Fig. 1 adapted from [2], whereas k_a[1/MPa] spans 10^{-9} to 10^{-5}[1/MPa] orders of magnitude for metals, technical ceramics and polymers and elastomers. The circumstance that K is very low induced Archard to interpret K as the probability of asperity encounters, looking at the process as a static mechanism. More recently, other authors are looking at this small number as coming from a fatigue process which requires a large number of contacts to make damage progress into wear particles. In particular, Persson and coworkers [3,4] have very recently proposed an interesting theory which attempts to include a full description of roughness writing strain energy in an asperity as driving a certain crack advancement per contact. Before we discuss more in depth the theory, we notice that from Fig. 1, k_a for a given class of materials clearly tends to decrease roughly with hardness, and this is compatible also with fatigue models, since it must be recollected that fatigue properties also scale with hardness.

In particular, it is know that under cyclic loading (see [5]), most materials fail at stresses below the static monotonic limit. This is due to inevitable damage mechanisms which degrade progressively the materials and eventually nucleate cracks. In many metals, there is a distinct fatigue "limit" stress $\sigma_{\lim}$ (typically corresponding to 1 to 10 million cycles) below which there seems to be no damage and life can be infinite (this does not occur for light alloys). In unnotched specimens, the plastic deformations occur on micro and mesoscale and therefore are not easily detected at macroscopic level (which is why fatigue limit is of the same order of cyclic yield strength). The signature of damage processes, however, can be obtained from thermographic careful measurements, showing that below the fatigue limit, temperature increase is minimal, while beyond the fatigue limit, there are clearly distinct mechanisms: this permits rapid measurements of fatigue properties [6] which otherwise require long and expensive testing, particularly as fatigue properties depend sensitively on initial defect sizes and therefore are highly scattered. Another fatigue property emerges for specimen which are cracked. In this case the microscopic interpretation of the fatigue threshold ΔK_{th} (range of stress intensity factors at the tip of cracks) are various (see [5], again). For example, that we need to reach the Griffith condition at the peak stress or that the crack opening is of the order of the failure extension of an atomic bond, or that the crack growth has fallen below the atomic spacing per cycle.

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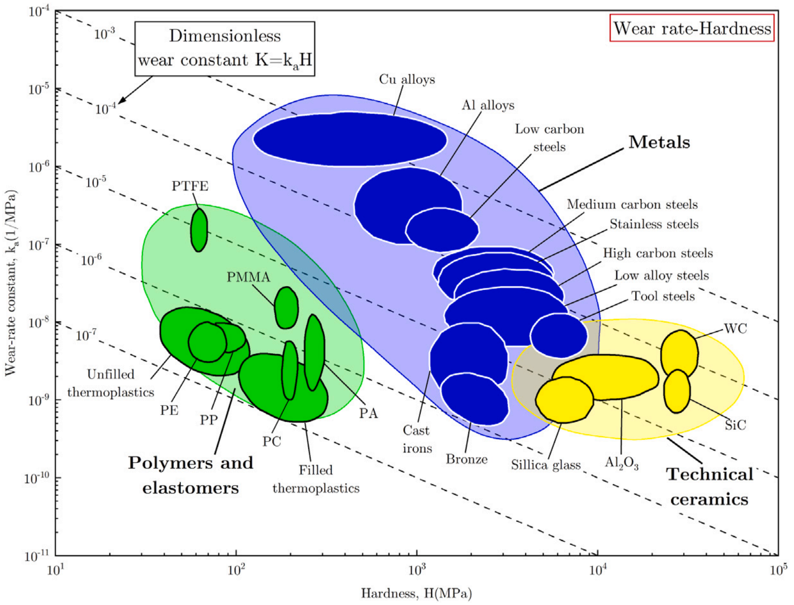

Fig. 1. Wear-rate constant $k_a = K/H$ where K is dimensionless wear rate coefficient, adapted from [2] for many materials. Notice that the k_a for a given class of materials clearly tends to decrease with hardness. .

These all give $\Delta K_{th}/E \simeq 10^{-6} - 10^{-5}$ where E is elastic modulus. Notice that from Fig.9 of [5] we observe that $\Delta K_{th}/K_{Ic}$ (where K_{Ic} is static toughness) is close to 1 for engineering ceramics (for which fatigue design becomes not very convenient) but could go down to almost 10^{-2} for some steels or filled elastomers for different reasons. In elastomers it could be simply that E is small for slow propagation, but becomes perhaps 1000 times higher for conditions of fast propagation near the static toughness. In metals, the behaviour should be interpreted in terms of plasticity mechanisms. Kitagawa and Takahashi [7] suggested that below a size related to a crossover between fatigue limit and fatigue threshold, namely

$$a_0 \simeq \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\sigma_{\lim}}\right)^2 \tag{2}$$

$\sigma = \sigma_{\lim}$ rather than $\Delta K < \Delta K_{th}$ became the critical condition for propagation of very small flaws. In addition, El Haddad et al. [8] suggested to use as size of a fictitious crack $a + a_0$ in both the threshold condition and the Paris crack growth equation, and found reasonable agreement, which is why the El Haddad simple proposal is now widely used. Other authors consider that the reduced treshold of short cracks may be due to the loss of the positive effect of crack closure, which is a mechanism induced by residual plastic stresses near the crack tip due to cyclic loading for which the crack opens not when the remote stress is positive, but for a positive stress. Therefore, there could be a true (reduced) threshold for short cracks [9]. However, there is large consensus that for short enough cracks the Stress Intensity Factor (SIF) eventually cannot govern the stress–strain environment of the process zone and hence the El Haddad proposal for which short crack threshold is

$$\Delta K_{th,sc} \simeq \Delta K_{th}\sqrt{\frac{a}{a+a_0}} \tag{3}$$

is largely adopted (see [10]) even by a NASA software.[1] Perhaps this is a conservative approach. Indeed, Appendix X3 in the fatigue test standard ASTM E647-15el[2] notes that it is not clear if a fatigue threshold exists for small naturally occurring cracks. Notice that in rubber wear, for which the original Persson's theory was developed, it is less clear if short cracks should have a different threshold, because the argument of plasticity and crack closure for example is unlikely to happen, although there remains the second issue that for small cracks the Linear Elastic Fracture Mechanics singular solution looses any sense. We just notice that [11] measured Paris curves for different initial crack sizes in rubber, and we find by best fitting his data that they correspond to very different power laws for initial crack sizes $a_i = 0.5$ mm than for $a_i = 1, 1.5$ mm. For long cracks the behaviour seems to us similar to the value used in [3], but the value for short crack seems very different, so the difference short vs long cracks may be similar to what happens in metals, for different reasons.

Yet another approach which we tentatively follow is for example the use of very well known equations from [12] who reports fatigue threshold for cracks and fatigue limit for specimen containing defects or inclusions of linear dimension $\sqrt{area}[\mu\text{m}]$ (for $\sqrt{area} < 1000\ \mu$m)

$$\Delta K_{th} = 3.3\times10^{-3}\,(HV+120)\left(\sqrt{area}\right)^{1/3} \tag{4}$$

$$\sigma_{\lim} = 1.43\,(HV+120)\,/\left(\sqrt{area}\right)^{1/6} \tag{5}$$

where HV is Vickers hardness, ΔK_{th} is in MPa$\sqrt{m}$ and $\sigma_{\lim}$ is in MPa. For $\sqrt{area} > 1000\ \mu$m, we can assume ΔK_{th} stay constant, and that the fatigue limit is decaying as in a more standard fracture mechanics prediction $\sigma_{\lim} = \Delta K_{th}/\left(0.65\sqrt{\pi\sqrt{area}}\right)$.

Returning to wear, [13] formulated an energy balance concept where there is a critical stress needed for a particle of diameter r_0 formation/detachment which is really a global form of the Griffith fracture mechanics concept, found imposing that the elastic strain energy release rate $G = \gamma$ where γ is a material constant tearing energy, and hence $\mu^2\sigma_c^2 r_0 = 2\gamma E$ or

$$\sigma_c = \frac{1}{\mu}\sqrt{\frac{2E\gamma}{r_0}} \tag{6}$$

where E is elastic modulus and μ is friction coefficient. Rabinowicz used this, together with the information on yield strength of material, to estimate particle sizes.

Persson's fatigue wear theory with a rough surface looks at various magnifications $\zeta = q/q_0$ (where q is wavevector in roughness, and q_0 is the smallest in the spectrum, usually around 10^3 m^{-1}) which define the radius of asperity $r_0 = \pi/q$ and seems to be the extension

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[1] Fatigue crack growth computer program "NASGRO" version 3.0: reference manual, JSC-22267b, March; 2002.

[2] ASTM E647-15e1; Standard Test Method for Measurement of Fatigue Crack Growth Rates. ASTM International: West Conshohocken, PA, USA, 2015.

of the Rabinowicz idea that one can suppose a wear particle size and then a posteriori examine the conditions under which the latter can form.[3] Persson writes that we need a number of contacts/cycles for a particle formation — which we also identify as a probability, written as $N(\gamma) = r_0/\Delta x(\gamma)$ where $\Delta x(\gamma)$ is the Paris crack propagation growth rate per cycle at energy γ is required to create the crack of size r_0. Considering that lowest stress to have some crack propagation is the threshold value $\sigma_{th}(\zeta)$

$$\sigma_{th}(\zeta) = \frac{1}{\mu}\sqrt{\frac{2E\gamma_{th}}{r_0}} \tag{7}$$

where γ_{th} is the threshold tearing energy ($\gamma_{th} = \Delta K_{th}^2/E$) and writing a Paris law for the crack advancement per cycle Δx for example as suggested by Xiulin and Hirt [14] and Hong and Xiulin [15], including the effect of the γ_{th} in the curve, to have some generality also for many metals

$$\Delta x \simeq \frac{16}{E}\left(\gamma - \gamma_{th}\right) \tag{8}$$

Persson and coauthors show that the wear rate is equal to

$$\frac{V}{A_0 L} \simeq \frac{1}{3\ln 2}\int_0^{\xi_1} d\xi \int_{\sigma_c(\zeta)}^{\infty} d\sigma \frac{P(\sigma,\zeta)}{1 + N(\zeta,\gamma)} \tag{9}$$

where we have integrated over the various contact pressures σ in the contact area at each magnification, and at the various magnifications (integration interval over ζ was changed in steps of factors of 2 in order not to compute particles twice using $q = q_0 e^{\xi}$). Here, $P(\sigma,\zeta)$ is the probability distribution of contact stresses σ in the contact area

$$P(\sigma,\zeta) = \frac{1}{\sqrt{4\pi G(\zeta)}}\left(e^{-\frac{(\sigma-p)^2}{4G(\zeta)}} - e^{-\frac{(\sigma+p)^2}{4G(\zeta)}}\right) \tag{10}$$

where p is the applied nominal pressure and $G = \frac{1}{2}V_{fc}/p^2$ where V_{fc} is the variance of the full contact pressures (see also [16]).

The trouble in this calculation is that Persson takes as initial size the size r_0 and uses this to estimate the number of cycles to failure of this crack. Hence, there is really no information about true initial defects in the material in the theory, which are *confused with the roughness wavelengths*. Moreover, r_0 being the wavelength of roughness at a given magnification, is really the final size of the particle and not the initial size of defect, and this cannot be used (contrary to the case of Rabinowicz) to estimate the driving force.[4]

Additionally, the initial cracks for a rough surface of engineering interest are generally below the size of application of Paris' law (ie. are not "long cracks"), and in any case are not related to the final size of the particle, but on initial size of defects. As we have partly discussed already in terms of short cracks, fatigue in a nominally uncracked specimen has a SN (stress vs number of cycles to failure) curve which is dominated by "initiation" processes [17] rather than fatigue crack propagation, particularly in metals. Integration of Paris' law for a material following (8) in a form simplified of the threshold $\Delta x \simeq \frac{16}{E}\gamma$ leads to

$$N = \frac{E^2 r_0}{16\pi\mu^2\sigma^2 r_0} = \frac{E^2}{16\pi\mu^2\sigma^2} \tag{14}$$

which is independent on the size of asperity. Hence, Persson's theory seems to suggest a SN curve independent (or nearly independent) on initial crack size

$$\frac{\mu\sigma}{E}N^{0.5} = \sqrt{\frac{1}{16\pi}} \tag{15}$$

which may suggest the theory is not so sensitive to the assumption of initial cracks equal to wavelengths in roughness which we find unjustified. However, even within these strong assumptions, this SN curve typically largely differ from a SN for failure of a material without initial defects. Indeed, a typical metal has a SN law (for shear stresses $\tau = \mu\sigma$)

$$\mu\sigma N^k = S_u \tag{16}$$

where S_u is amplitude of shear stress for failure in 1 cycle, which can be assumed as a first approximation $S_u = 2\tau_{\lim}$ [5]. Imposing that the law holds up to fatigue limit $\tau_{\lim}$ at $N = 10^7$ cycles leads to $k = \log_{10}(2)/7 = 0.043$. Notice the striking one order of magnitude difference in the exponents in (15) and (16). Although [18–20] attempted to find unified laws for initiation and propagation, we shall not make use of these more complex proposals.

For an uncracked specimen, at high loading amplitudes above fatigue limit, cracks will develop early, and main part of fatigue life is crack growth, although it remains to be seen if this phase is simple, because large plastic flow do not permit to use simple Paris' laws. However, at lower loading amplitudes, "measuring" the crack initiation stage is much more difficult and there is no reliable definition of the transition from crack initiation to crack growth. Some proposal consider initiation ended when we can start describing propagation by fracture mechanics (see [17]). We will assume however that a Basquin law of the type (16) will describe the number of cycles until formation of a large crack.

Therefore, while in a recent note we have attempted to simplify the original theory from Persson and coworkers [21], finding in particular the role of factors affecting global wear rate as rms amplitude of roughness h_{rms} and friction coefficient μ, in the present note we shall explore the results of using as an alternative to Paris crack growth concept. In particular, the new wear theory becomes

$$\frac{V}{A_0 L} \simeq \frac{1}{3\ln 2}\int_0^{\xi_1} d\xi \int_{\sigma_c}^{\infty} d\sigma \frac{P(\sigma,\zeta)}{1 + N(\sigma)} \tag{17}$$

where now N now depends only on σ according to (16), and the lower end of integration interval for the stress is the highest between fatigue limit $\sigma_{\lim}$, and fatigue threshold $\sigma_{th}(\zeta)$ is a modification of (6)

$$\sigma_c = \max\left(\sigma_{th}(\zeta), \sigma_{\lim}\right) = \max\left(\frac{1}{\mu}\sqrt{\frac{2E\gamma_{th}}{r_0}}, \frac{1}{\mu}\tau_{\lim}\right) \tag{18}$$

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[3] We should note that there is a difference between the Rabinowicz criterion for particle formation in wear, and Griffith theory of cracks. Rabinowicz makes a global energy balance before and after formation of a particle of assumed spherical size: no mechanism is identified for this nucleation to occur, and no stability condition is given for a small crack to propagate up to final catastropic dimension: in a sense, Rabinowicz is therefore not very rigorous. Griffith criterion assumes a crack is pre-existing, and writes a condition for infinitesimal propagation to occur: anyway it is true that if one integrates energy release rate over a certain path, one should get the total energy in formation of crack, that is surface energy times the crack area. However, this is not a crucial observation.

[4] One could notice that $N = r_0/\Delta x$ is not really the result of an integration of a Paris curve. If one integrates Paris' law, for a crack of initial size a_i up to failure a_f under stress σ

$$\Delta x(\gamma) = \frac{da}{dN} = C\gamma^m = C\left(\frac{\sigma^2(\pi a)}{E}\right)^m = C\frac{\sigma^{2m}(\pi a)^m}{E^m} \tag{11}$$

so that

$$\int_{a_i}^{a_f}\frac{da}{a^m} = \frac{a_f^{-m+1} - a_i^{-m+1}}{-m+1} \simeq \frac{a_i^{-m+1}}{m-1} = C\frac{\sigma^{2m}\pi^m}{E^m}N_f \tag{12}$$

where we used that $a_f \gg a_i$ and this eliminate also the dependence on the static toughness. Since $a_i = r_0$ this gives therefore $N_f = \frac{r_0}{(m-1)C\frac{\sigma^{2m}\pi^m r_0^m}{E^m}} = \frac{r_0}{(m-1)C\gamma^m}$ which for $m = 2$ is exactly the number of cycles used by Persson $N(\gamma) = r_0/\Delta x(\gamma)$ since Persson identifies γ with the crack size r_0. Notice that for higher m there is a small error in Persson's calculation, whereas for $m = 1$

$$N_f = \frac{\log\frac{a_f}{a_i}}{C\frac{\sigma^2\pi}{E}} = \frac{r_0}{C\gamma}\log\frac{r_{0f}}{r_0} = N(\gamma)\log\frac{r_{0f}}{r_0} \tag{13}$$

and this more correct calculation corresponds to the Persson one only if we assume $\log\frac{r_{0f}}{r_0} = 1$ which means arbitrarily the final crack size is $r_{0f} = er_0$.

where notice that the fatigue limit is also corrected for the presence of roughness according to the Murakami (5) model (for $h_{rms} < 10^{-3}$ m)

$$\tau_{\lim} = \tau_{\lim,1\ \mu\text{m}} / \left(h_{rms}\right)^{1/6} \quad \text{for } h_{rms} < 10^3\ \mu\text{m} \tag{19}$$

$$\tau_{\lim} = \tau_{\lim,1\ \mu\text{m}} / \left(10^3\right)^{1/6} \left(h_{rms}/10^3\right)^{-1/2} \quad \text{for } h_{rms} > 10^3\ \mu\text{m} \tag{20}$$

where h_{rms} is in μm and $\tau_{\lim,1\ \mu\text{m}}$ is the fatigue limit with a roughness of 1 μm (we take 1 μm as reference as this is a common value for polished steel). However, this correction is not as important as may be the correction for fatigue threshold, as we shall discuss.

The reason for including the fatigue threshold is that we want to make sure that the fatigue process, after initiation can lead also to propagation of the crack, so the stress should be sufficient to propel also the crack towards the final size of the crack (the particle size r_0). This is profoundly different from the original Persson's theory (Xu et al. 2024, [3]), where Paris' crack growth curve was used estimating the crack driving energy directly from the final size of the crack, even when the crack is in the initiation phase, and is of unknown dimensions.

2. Results for the new wear theory

We apply now the new theory based on initiation Basquin law for various cases exploring the dependence on the main parameters. We consider as parameters for the original Persson's theory a reference steel having $\gamma_{th} = 500$ J/m^2 and $E = 200$ GPa with crack propagation law following (8). For the roughness, we consider a typical power law spectrum of roughness $C(q) = C_0 q^{-2(1+H)}$, where H is Hurst exponent (which we assume as the typical value of $H = 0.8$, see [22]), where $V_{fc} = \frac{1}{2}E^2 h_{rms}'^2 \simeq \frac{1}{2}E^2 \frac{\pi C_0}{2-2H} q^{2-2H}$ while the rms roughness is $h_{rms} = \sqrt{\pi C_0/H} q_0^{-H}$ and h_{rms}' is rms slope, $q_0 = 10^3$ m^{-1}. For the first comparison, we use $h_{rms} = 1$ μm, and vary the friction coefficient μ. Also, the SN Basquin law is used for the shear stress SN curve where $\tau_{\lim,1\ \mu\text{m}} = 400, 600, 800$ MPa and $\tau_U = 2\tau_{\lim,1\ \mu\text{m}}$.

As it can be seen in Fig. 2, we obtain the dimensional wear rate coefficient k_a as a function of the friction coefficient μ for four level of increasingly strong metal, having $\tau_{\lim,1\ \mu\text{m}} = 400, 600, 800$ MPa respectively with black, blue and red curve, while green line is the standard Persson's theory using Paris' crack growth curve with $\gamma_{th} = 500\ \frac{\text{J}}{\text{m}^2}$ and the "universal" crack propagation law (8). Dashed black line is a reference Raboniwicz power five fit dependence of wear rate on friction coefficient for large range of experiments on metals (lubricated and unlubricated), supposing the data were relative to $H = 1000$ MPa (this is perhaps a lower bound estimate, but no better data are available). Finally, the figure assumes $\xi_1 = 10$, i.e. $\zeta_1 = \exp(10) = 22026$ so that the spectrum roughness is truncated at $q_1 = 2.2 \times 10^7$ m^{-1} (we shall discuss the effect of this choice later, showing that the data are converged, i.e. do not depend on fine scale tail of the roughness PSD). As it can be seen, some conclusions are:-

- new and standard theory are qualitatively predicting similar dependence on friction coefficient, with both showing a threshold for friction needed for some wear rate which probably indicates a transition to another wear mechanism, like adhesion wear which we have neglected;
- in both new and standard theory just changing friction coefficient from a low value to 1 changes the wear coefficient by about *six* orders of magnitude, which is more than what reported by Rabinowicz [23] across the same range of friction coefficient. Also, Rabinowicz [23] reports data in terms of dimensionless K from 10^{-6} to 10^{-2} : as we said, this leads supposing hardness is of the order of 1000 MPa, to the black dashed line in Fig. 2, which seems to indicate that the standard Persson theory tends to underpredict the wear rate, whereas the new theory underpredicts at low friction coefficients, and perhaps overpredicts at large friction coefficients (but only for low fatigue limits). These comparisons are however very qualitative, since we have made strong assumptions about Paris curve of metals, SN curves, and fatigue threshold.

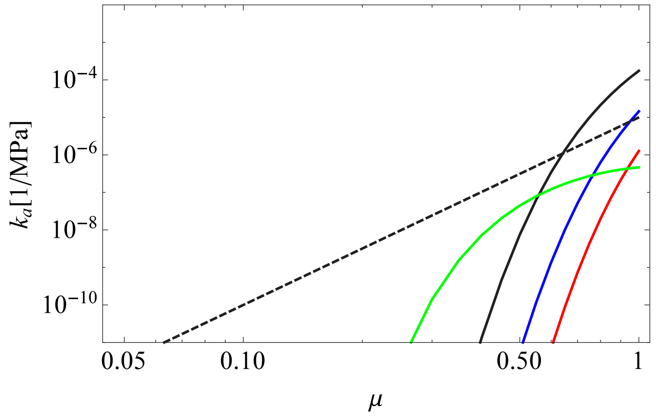

Fig. 2. Wear-rate constant k_a as function of friction coefficient μ for our example case steels with $E = 200$ GPa, $h_{rms} = 1$ μm, $q_0 = 10^3$ m^{-1}. The new theory is reported with black, blue, red curves for $\tau_{\lim,1\ \mu\text{m}} = 400, 600, 800$ MPa respectively, while green line is the standard Persson's theory as described in the text with $\gamma_{th} = 500\frac{\text{J}}{\text{m}^2}$ and the "universal" crack propagation law as described in the text. Dashed black line is a reference Raboniwicz power five dependence of wear rate on friction coefficient, supposing the data were relative to $H = 1000$ MPa.

We now use friction coefficient $\mu = 1$ and change the rms amplitude of roughness, see results in Fig. 3, all other parameters being the same as Fig. 2. Again, black, blue and red curves are relative to $\tau_{\lim,1\ \mu\text{m}} = 400, 600, 800$ MPa (baseline values for $h_{rms} = 1$ μm, further corrected for roughness), while the green line is the standard Persson's theory. We observe that again the trend in the new theory and in the standard one are qualitatively very similar, showing first a very sharp increase of wear rate with increasing roughness, and then after reaching a peak, a slow decrease. It turns out that the initial increase is actually stronger for the new theory, and reaches a higher peak after which the decay is almost independent on SN curve of the material, and tends eventually to the same line of the standard Persson's theory.

Experimental literature reports generally a trend of increased wear with roughness ([24–27]), although hardly as strong as predicted.

Finally, Fig. 4 shows the effect of the choice of truncation wavevector ξ_1 on wear-rate constant k_a for $h_{rms} = 1$ μm and other parameters are in Fig. 2. Black, blue and red curve are for $\xi_1 = 1, 2, 5$ showing that for $\xi_1 = 5$ the curves are converged and wear rate does not further increase. This effect is prevalently due to the choice of integrating the stresses in the formulation only above the threshold stress, which becomes increasingly high for small wavevectors (i.e. asperities). In other words, as higher stress is needed to propagate small crack asperities, at least as long as we believe the existence of a long crack fatigue threshold, this gives a limitation to the effect of small scale features of the roughness PSD on wear rate. This will also give a strong effect on particle distributions, which however we do not investigate.

Indeed, if we remove the effect of fatigue threshold and only consider fatigue limit as the inferior limit of integration in the theory, we obtain results of Fig. 5, where every other constant is like Fig. 4. It is clear that the difference with Fig. 4 is dramatic, and indeed there is no convergence with truncating wavevector, with wear rates increasing of orders of magnitude just changing the upper wavevector. The deviation with respect to known data would also become too large.

3. Discussion

It appears that despite the initial assumptions in Persson's theory are rather questionable, and particularly the treatment of crack growth data, with the confusion made between final size of the crack and initial size when computing the energy to propel crack growth, the results using an initiation model are not too different, and of course

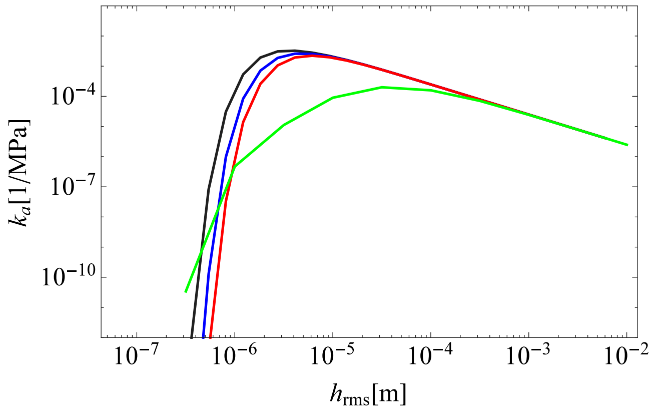

Fig. 3. Wear-rate constant k_a as function of rms amplitude of roughness h_{rms} with $\mu = 1$ and other constants as Fig. 2.

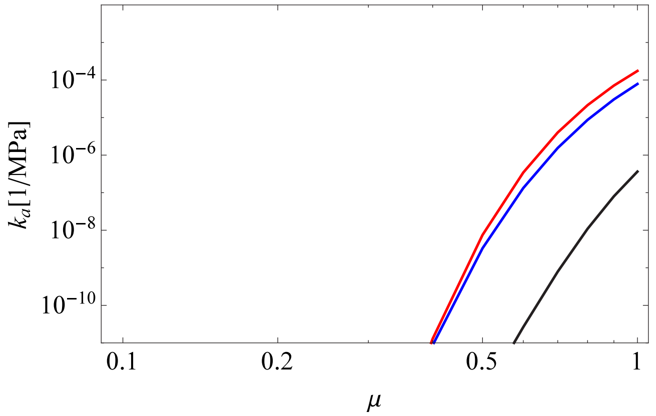

Fig. 4. Effect of truncation wavevector ξ_1 on wear-rate constant k_a for $h_{rms} = 1$ μm and varying μ and other constants as Fig. 2. Black, blue and red curve are for $\xi_1 = 1, 2, 5$.

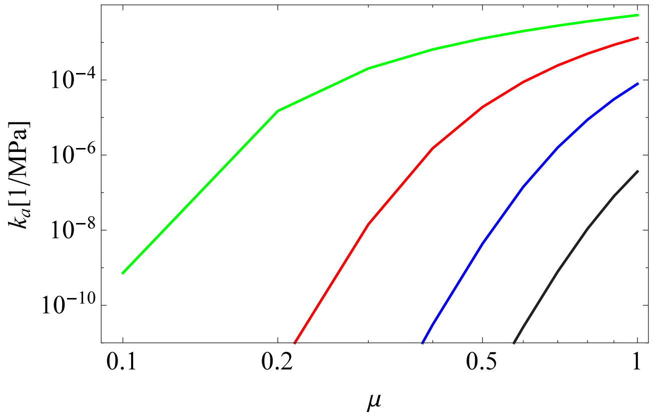

Fig. 5. Effect of truncation wavevector ξ_1 on wear-rate constant k_a for $h_{rms} = 1$ μm and varying μ and other constants as Fig. 2, but for a model where $\sigma_c = \sigma_{\rm lim}$ (no effect of fatigue threshold). Black, blue red and green curve are for $\xi_1 = 1, 2, 5, 10$.

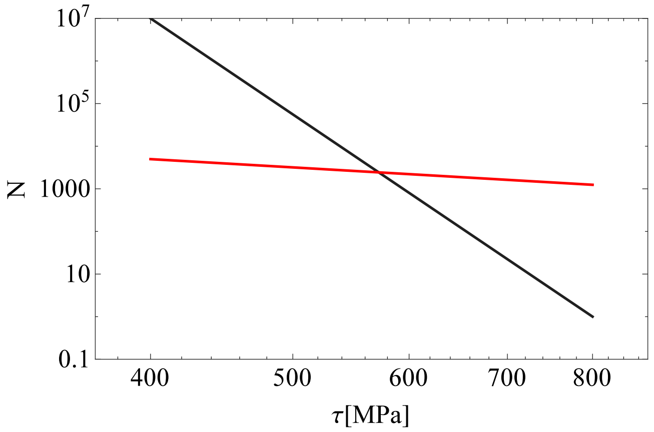

Fig. 6. SN curves used in the present theory (black curve, $\tau_{\rm lim} = 400$ MPa) and (in a simplified form) in the original Persson's theory (red curve).

the details matter and only an accurate comparison with actual wear data and disposing of enough information of roughness, friction, and material properties may shed more light. In particular, the use of crack growth curve of the form suggested for metals and rubber and PMMA near threshold (8) leads to a $r_0/\Delta x$ which does not depend strongly on the particle dimension (we have derived in the introduction $N = \frac{E^2 r_0}{16\pi\sigma^2 r_0} = \frac{E^2}{16\pi\sigma^2}$ which is independent on the size of asperity). This may explain why assuming as initial crack size the final crack size does not lead to serious errors. The key factor in the theory (which we have maintained in the present version) therefore remains that for the particle real detachment to occur, at the final stages of propagation there must be a stress above that corresponding to the fatigue threshold. It is correct that this could be estimated during the final stages of propagation rather than at the beginning.

The difference between the original and the new variant of the theory therefore remains only in the SN curve, which are plotted in an example case ($\tau_{\rm lim} = 400$ MPa) in Fig. 6 below, where for the standard Persson theory we assume $N = \frac{E^2 r_0}{16\pi\sigma^2 r_0} = \frac{E^2}{16\pi\sigma^2}$ derived in the introduction. It is clear that the original theory gives smaller N for low stresses, but higher N for higher stresses, and hence the two effects partly compensate. Results may differ more strongly when taking into account of plasticity at asperity contacts [3], where the new theory would give higher N only, and hence lower wear rates in general.

Looking back at Fig. 5, in both the original theory and in the new one presented here, the role of the fatigue threshold appears crucial in truncating the stress: without consideration for fatigue threshold, there is virtually no convergence of wear rate which increase very wildly and evidently also the particle distribution would be strongly affected by the choices regarding fatigue threshold. It becomes therefore important to assess if fatigue threshold really depends on size of particles. In particular, it is known that fatigue threshold for short cracks is not the same as fatigue threshold for long cracks (see eg. the review [28]), and one may conservatively take as short crack threshold a value very close to zero, at least for metals. Under these conditions, wear rate would depend strongly on the contribution of small particles, which incidentally are important because they are the most harmful for health.

We tentatively assume the fatigue threshold to hold the Murakami scaling (4) for the threshold with defect size (assumed here as the particle size $r_0 = \pi/q$), and in particular assume $\gamma_{th} = 500$ J/m^2 is the correct value at size q_0 (which as we said, corresponds to the long crack threshold suggested by Murakami [12] anyway i.e. near 1000 μm): hence we now use

$$\sigma_c = \max\left(\sigma_{th}(\zeta), \sigma_{\rm lim}\right) = \max\left(\frac{1}{\mu}\sqrt{\frac{2E\gamma_{th}\left(q_0/q\right)^{1/3}}{\pi/q}}, \frac{1}{\mu}\tau_{\rm lim}\right) \tag{21}$$

Fig. 7 shows that with this modification, while the wear rate still converge, they converge more slowly and to higher values (but less than one order of magnitude increase on the wear rate coefficient). Hence, the values remain reasonable with respect to known experimental data. However, this also suggests more small particle contribution, as expected.

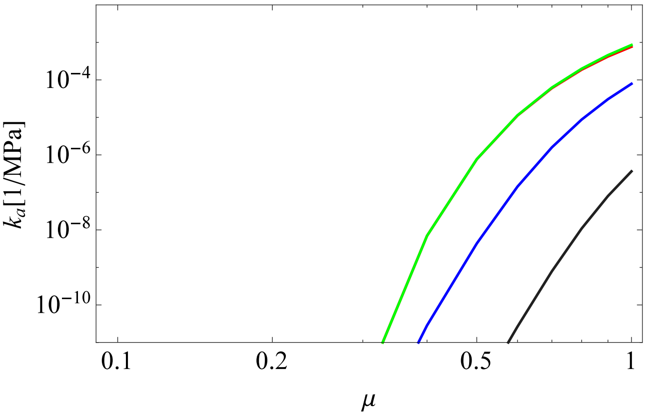

Fig. 7. Effect of a crack size-dependent fatigue threshold. Black, blue red and green curve are for $\xi_1 = 1, 2, 5, 10$. All constants as Fig. 5.

We should mention that fatigue wear is just one possible mechanism of a very complex problem, where changes in speed, in lubrication, in environment, may dramatically affect results (see [29] for so called wear maps) changing regimes from mild to severe and mechanisms. Also, wear may be very different depending on material, or even formation of third body particles and their elimination from the contact area is very important. We hope that with the development of the present theory, which is a follow-up of the theory first suggested by Persson and coauthors, which emphasizes the role of roughness in wear, experiments may be deviced with largely varying initial or steady state roughness, or friction coefficient, to compare with the theory prediction.

A few remarks should be addressed regarding the phenomenon of “running in” which may involve change of roughness, of third-bodies, of interface layers, etc., and which may result in a different wear rate (and even friction coefficient) with respect to a steady state one [30]. It is clear that this is not taken presently into account neither by the original Persson and coworkers model which assume as initial defects the roughness wavelengths, nor by the present modification. Similarly, there is some simplification in the original Persson and coworkers model and the present variant in applying stresses induced at given magnification as if they were nearly uniform stresses as in specimen used for SN curves or Paris curves determination. However, unfortunately this simplification is needed at present to make a simple fully analytical theory.

4. Conclusion

Given there seems to be some confusion in Persson’s wear theory of what is the size of initial cracks, and in particular fatigue life is arbitrarily assumed to be entirely that of propagation, we attempted an initiation theory of wear, based on SN curves (as an example, of typical metals). With the consideration that sufficient stress is however needed for the final stages of propagation of cracks, we have maintained a fatigue threshold in the formulation. We have obtained that results are not qualitatively very different from the original Persson’s theory, which perhaps depends on the actual typical crack growth curves used so far, but we have provided only rough estimates for wear rates based on simple material constants assumptions, and qualitative comparison with experiments, and therefore more investigation is needed to understand which of the two theories can lead to more accurate predictions under controlled conditions. A crucial role in both theories seems to be that of fatigue threshold: we have suggested this is known to decrease for small cracks, and including Murakami’s kind of dependence on defect size, it has been shown that this would quite significantly change the contribution of the fine scale parts of roughness to the global wear rate, and even more the fine particle distributions. Convergence to the “fractal limit” i.e. with upper wavevector implies that the theories suggest the main role of roughness is the amplitude h_{rms}. More investigations requires full information on material properties and on roughness, and detailed comparison with experiments.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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