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Damage Stability and Subdivision

of

Semisubmersible Drilling Rigs

by

W. Abicht

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ABSTRACT

Damage stability calculations are carried out for column stabilized drilling units. Following types of rigs are considered: multi-legged rigs with circularly and rectangularly arranged columns, with and without footings, and with subdivided and non-subdivided columns.

Formulas are set up showing the influence of the geometric characteristics on the angle of inclination in the final stage of flooding. From the results conclusions are drawn as to the type of rig to be preferred from the standpoint of damage stability.

Finally, it is demonstrated that rigs with favourable characteristics can attain a probability of survival of P = 1 without any subdivision of the columns. Rigs which are not primarily designed with respect to damage stability must be subdivided. It will be shown how subdivision must be in order to get also for these rigs high probability values.

NOMENCLATURE

- a = half height of a compartment
- B = breadth of a rectangular rig (measured between the centers of two opposite columns)
- B_o = center of buoyancy of the remaining intact part of the rig after parallel sinkage
- B_φ = center of buoyancy of the remaining intact part of the rig after parallel sinkage and inclination about the axis x'
- B_ψ = center of buoyancy of the remaining intact part of the rig after parallel sinkage and inclination about the axis y'
- $(\overline{BM})_o$ = transverse metacenter above center of buoyancy before damage
- $(\overline{BM_L})_o$ = longitudinal metacenter above center of buoyancy before damage
- $(\overline{BM})_R$ = transverse metacenter of the remaining intact part of the rig above center of buoyancy after flooding
- $(\overline{BM_L})_R$ = longitudinal metacenter of the remaining intact part of the rig above center of buoyancy after flooding
- b = B/2 = half breadth of a rectangular rig
- D = diameter of a column
- e = half vertical extent of damage
- F = center of the remaining intact waterplane area after parallel sinkage
- G = center of gravity
- $(\overline{GM})_o$ = transverse metacentric height before damage
- $(\overline{GM_L})_o$ = longitudinal metacentric height before damage
- $(\overline{GM})_R$ = transverse metacentric height of the remaining intact part of the rig
- $(\overline{GM_L})_R$ = longitudinal metacentric height of the remaining intact part of the rig
- g = acceleration due to gravity
- H = height of a column
- J_{ox} = moment of inertia of the waterplane area of the rig before damage, related to the longitudinal principal axis
- J_{oy} = moment of inertia of the waterplane area of the rig before damage, related to the transverse principal axis
- J_x ($J_{x'}$) = moment of inertia of the remaining intact waterplane area after parallel sinkage, related to axis x (x')
- J_y ($J_{y'}$) = moment of inertia of the remaining intact waterplane area after parallel sinkage, related to axis y (y')
- J_{xy} = centrifugal moment of the remaining intact waterplane area after parallel sinkage, related to axes x and y ($J_{x'y'}$ = 0)
- $(\overline{KB})_o$ = center of buoyancy above bottom before damage
- $(\overline{KB})_R$ = center of buoyancy of the remaining intact part of the rig above bottom after flooding
- $\overline{KG}$ = center of gravity above bottom before and after flooding (lost buoyancy method)

- L = length of a rectangular rig (measured between the centers of the outer columns of a row)
- L = force of buoyancy
- ℓ = $L/2$ = half length of a rectangular rig
- m = projection of r on axis x'
- n = projection of r on axis y'
- N = number of columns
- p = distance of B_o from axis y_o (measured in a horizontal plane at the level of B_o)
- q = distance of B_o from axis x_o (measured in a horizontal plane at the level of B_o)
- R = distance of a column from the vertical centerline of a radial symmetric rig
- r = distance between the forces of buoyancy and weight acting on the remaining intact part of the rig after parallel sinkage ($r^2 = p^2 + q^2 = m^2 + n^2$)
- T_o = draft before damage
- ΔT = parallel sinkage
- u = distance of F from axis y_o (measured in the waterplane after parallel sinkage)
- v = distance of F from axis x_o (measured in the waterplane after parallel sinkage)
- ∇_o = volume of displacement before and after flooding (lost buoyancy method)
- W = force of weight
- x_o = longitudinal principal axis of inertia of the waterplane area before damage
- x = axis running through F in parallel direction to axis x_o
- x' = principal axis of inertia of the remaining intact waterplane area after parallel sinkage (about this axis the moment of inertia comes to a minimum value)
- y_o = transverse principal axis of inertia of the waterplane area before damage
- y = axis running through F in parallel direction to axis y_o
- y' = principal axis of inertia of the remaining intact waterplane area after parallel sinkage (about this axis the moment of inertia comes to a maximum value)
- α = angle between the directions of the axis x' and the longitudinal axis (applicable to rectangular rigs if $N = 4$ and $\eta \leq 1$ or if $N = 6$ and $\eta \leq \sqrt{(12-5\mu_s)/(18-6\mu_s)}$) or transverse axis of the rig (applicable if $N = 4$ and $\eta \geq 1$ or if $N = 6$ and $\eta \geq \sqrt{(12-5\mu_s)/(18-6\mu_s)}$)
- β = angle of inclination about axis x' at which the center of the upper flat of a compartment immerges
- γ = angle between axis x_o and the line connecting the centers of the diagonally opposite corner columns of a rectangular rig
- η = B/L = breadth-length ratio
- ϑ_o = angle of inclination of the platform against sea level in the final stage of flooding
- μ_S = surface permeability
- μ_V = volume permeability
- ρ_W = mass density of water
- φ_o = angle of inclination about axis x' due to loss of buoyancy ("angle of heel")
- ψ_o = angle of inclination about axis y' due to loss of buoyancy ("angle of trim")

1. INTRODUCTION

This paper is concerned with the influence of the geometric characteristics of semisubmersible rigs on damage stability. Only damages to the columns caused by collisions will be considered. The intention is to find out what types of rigs can survive damages even without any watertight subdivision of the columns and what types must be subdivided in order to obtain a certain amount of survival probability.

Contrary to ships, semisubmersible offshore units have several buoyant bodies contributing to the buoyancy of the rig. In general, spacing of columns is wide enough for excluding damage to two or more adjacent columns. Hence, for a high degree of ability to survive collisions, it only must be observed that the rig can withstand the total flooding of one column. This method of providing survivability, which is only applicable to multihull units, is much more effective than the well known method of subdividing a floating body into watertight compartments. Of course, a watertight subdivision of the columns of a multi-legged rig is also possible and contributes additionally to its survivability.

From these simple considerations we may conclude that it is much more easier to attain high survival probability values for rigs than for ships. In spite of this fact there exists a number of semisubmersibles which evidently were designed without closer regard to the influence of hull configuration on damage stability. For example, three-legged rigs without a well-considered subdivision will hardly survive a damage to one of the legs.

Modern drilling rigs meet damage stability and subdivision requirements of the classification societies and of IMCO [1]. They all have a minimum amount of survival probability. But the rules are not as effective as they could be. Especially, they do not give practical instructions to the designer what type of rig should be preferred from the damage stability point of view. After the capsizing of the rig "Alexander Kielland" the Norwegian Maritime Directorate (NMD) introduced amendments to the existing national stability rules for floatable drilling units [2]. In future, semisubmersible rigs operating off the Norwegian coast must be able to withstand heeling moments even if the buoyant force of one of the legs will be lost. This requirement is an important step forward to more safety on rigs and we only can hope that IMCO will follow the Norwegian Maritime Directorate in adopting this amendment to IMCO's "MODU CODE" [1].

2. DAMAGE STABILITY OF DIFFERENT TYPES OF RIGS

2.1 Multi-legged rig of radial symmetric type with non-subdivided columns and without footings

Fig.1 of Appendix 1 illustrates a column stabilized rig without footings (N = number of columns). All columns have the same diameter D, the same distance R from the vertical centerline of the rig and are spaced equidistantly. The columns are not subdivided by watertight bulkheads or flats. If one column is damaged the center of gravity G - according to the lost buoyancy method - is the same as before flooding. The center of buoyancy, however, changes its location because of the fact that the form of the remaining intact part of the rig differs from the original configuration. The resultant force of buoyancy L acts vertically upward through the new center of buoyancy B_o . It is equal to the resultant force of weight W , but of opposite direction. In the upright position, the distance between these two forces is r .

By reasons of symmetry B_o is located on the connecting line drawn at the level of B_o between the center of the column being flooded and the center of the rig. In this special case where no footings are arranged the center of buoyancy B_o and the center F of the remaining intact water plane area are lying on the same vertical line. The principal axis of inertia x', about which the moment of inertia attains its minimum value, runs through F and has a normal direction to the connecting line mentioned above.

The forces of buoyancy and weight cause a list of the rig about the axis x' . As shown in Appendix 1 the angle of inclination about x' - called angle of heel φ_o - can be calculated from the remaining transverse metacentric height $\overline{GM}_R$ and the leverarm r of the couple of forces. For small angles of heel the result is

$$\sin\varphi_o = 2\left(\frac{\frac{\mu_V}{N}}{1-\frac{\mu_V}{N}}\right)\cdot\frac{R}{T_o}\ \times \tag{1}$$

$$\frac{1}{\left(\frac{1-3\frac{\mu_S}{N}}{1-\frac{\mu_S}{N}}\right)\left(\frac{R}{T_o}\right)^2+\frac{1}{1-\frac{\mu_V}{N}}+\frac{1}{8}\left(1-\frac{\mu_S}{N}\right)\left(\frac{D}{T_o}\right)^2-2\,\frac{\overline{KG}}{T_o}}$$

(where μ_S = surface permeability, μ_V = volume permeability, T_o = draft before flooding, $\overline{KG}$ = center of gravity above base). This formula is only applicable if the final waterline is below the lower edge of the platform and above the bottoms of the columns.

The second principal axis of inertia y' of the remaining intact waterplane area (about this axis the moment of inertia attains its maximum value) intersects the axis x' in F and runs through the center of the flooded column. Because of the radial symmetry of the rig the forces of buoyancy and weight have no component moment causing an inclination about the axis y' . Thus, the inclination of the rig in this direction - designated as angle of trim ψ_o - is zero.

In a rough sea, rigs can withstand damages the more likely, the smaller the angle of inclination of the platform against sea level in the final stage of flooding is. Therefore, it can be taken as a safety standard. Generally this angle, denoted by ϑ_o , must be calculated from φ_o and ψ_o (for details see Section 2.4). Only if flooding does not cause a trimming moment, ϑ_o and φ_o are identical. This is true for all radial symmetric types of rigs.

From formula (1) conclusions may be drawn how the characteristics of this type of rig should be in order to minimize the angle of inclination. φ_o depends on six parameters: $\varphi_o = f\ (\mu_S, \mu_V, \overline{KG}/T_o, N, R/T_o, D/T_o)$. Parameters N, R/T_o and D/T_o are geometric factors which are of main interest because they can be freely chosen in the early design stage. Permeability and location of center of gravity, however, can only be varied within a limited range of values.

To get a comprehensive graphical representation of the results obtained for this and the following types of rigs, several sets of curves were plotted by computer. They will be published in [3]. In this paper some interesting partial results are given in tabular form. They clearly show the influence of the geometric characteristics on damage stability.

Table I presents for the multi-legged rig without footings values of $\overline{KG}/T_o$ which must not be exceeded if after flooding of one column the angle of inclination shall be limited to $\vartheta_o = 8^o$. Permeability is assumed to be $\mu_S = \mu_V = 1$.

TABLE I. Radial symmetric rig with non-subdivided columns and without footings:

Maximum permissible values of $\overline{KG}/T_o$ if inclination shall be limited to $\vartheta_o = 8^o$ ($\mu_S = \mu_V = 1$).

		$R/T_o = 2$	$R/T_o = 3$	$R/T_o = 4$	$R/T_o = 6$
N=4; D/T_o =	0.25	-	-	-	-
	0.50	-	-	-	-
	1.00	-	-	-	-
N=5; D/T_o =	0.25	-	-	-	-
	0.50	-	-	-	-
	1.00	-	-	-	-
N=6; D/T_o =	0.25	-	-	-	2.80
	0.50	-	-	-	2.80
	1.00	-	-	-	2.83
N=7; D/T_o =	0.25	-	0	1.14	> 5
	0.50	-	0	1.14	> 5
	1.00	-	0	1.18	> 5
N=8; D/T_o =	0.25	0	0.70	2.17	> 5
	0.50	0	0.70	2.18	> 5
	1.00	0	0.75	2.23	> 5
N=9; D/T_o =	0.25	0.27	1.23	2.98	> 5
	0.50	0.28	1.25	2.99	> 5
	1.00	0.32	1.30	3.01	> 5

In those cases where values of $\overline{KG}/T_o$ are given, $\vartheta_o = 8^o$ can be attained without emersion of the bottom of any column. Values $\overline{KG}/T_o < 0$ and $\overline{KG}/T_o > 5$ are of no practical interest. The platform is assumed to be arranged at such a height that in the final stage of flooding its lower edge does not immerge. Results which were achieved for other types of rigs will be presented in the same way.

2.2 Multi-legged rig of radial symmetric type with subdivided columns and without footings

Rigs that cannot withstand flooding of one column must be subdivided in order to reduce the angle of inclination to an acceptable value. The effect of subdivision is similar to that of reduction of permeability. This is especially true in the case of vertical subdivision. If the columns are subdivided horizontally the free-surface effect cannot be neglected, except the upper flat of the flooded compartment is below the final waterplane.

For this reason, separate damage stability calculations were carried out for rigs, the columns of which are subdivided by watertight flats. As can be seen in Fig.2 of Appendix 2, the center of the flooded compartment is assumed to be located at the level of the waterplane in the initial stage of flooding. The height of the compartment (=2a), however, can vary.

Separate calculations are necessary for two different regions: In region I the upper flat of the flooded compartment is above and in region II below the final waterplane. These two regions are approaching each other if the final waterline intersects the center of the upper flat. In that case the angle of inclination is $\varphi_o = \beta_I$ or β_{II} respectively. The difference between β_I and β_{II} follows from the different waterplane areas and the different positions of their centers. As can be seen in the lower illustration of Fig.2, the center of the waterplane area moves back to the center of the rig as soon as the flooded compartment is fully submerged. For details of damage stability calculations see Appendix 2. Following results are obtained:

Region I

$$\sin\varphi_o = 2\left(\frac{\frac{\mu_v}{N}}{1-\frac{\mu_v}{N}}\right)\cdot\frac{a}{T_o}\cdot\frac{R}{T_o} \times \frac{1}{\left(\frac{1-3\frac{\mu_s}{N}}{1-\frac{\mu_s}{N}}\right)\left(\frac{R}{T_o}\right)^2 + 1 + \left(\frac{\frac{\mu_v}{N}}{1-\frac{\mu_v}{N}}\right)\left(\frac{a}{T_o}\right)^2 + \frac{1}{8}\left(1-\frac{\mu_s}{N}\right)\left(\frac{D}{T_o}\right)^2 - 2\,\frac{\overline{KG}}{T_o}} \tag{2}$$

In Region I the angle of inclination must satisfy the inequation $\varphi_o < \beta_I$ where

$$\tan\beta_I = \left(1-2\frac{\mu_v}{N}\right)\cdot\left(\frac{1-\frac{\mu_s}{N}}{1-\frac{\mu_v}{N}}\right)\cdot\frac{a}{R} \tag{3}$$

Region II

$$\tan\varphi_o = \frac{4\,\frac{\mu_v}{N}\cdot\frac{a}{T_o}\cdot\frac{R}{T_o}}{\left(\frac{R}{T_o}\right)^2 + 1 + 4\left(\frac{\mu_v}{N}\right)^2\left(\frac{a}{T_o}\right)^2 + \frac{1}{8}\left(\frac{D}{T_o}\right)^2 - 2\,\frac{\overline{KG}}{T_o}} \tag{4}$$

In Region II the angle of inclination must satisfy the inequation $\varphi_o > \beta_{II}$ where

$$\tan\beta_{II} = \left(1-2\frac{\mu_v}{N}\right)\cdot\frac{a}{R} \tag{5}$$

Formulas (2) and (4) are applicable provided that the angles of inclination are small, the lower edge of the platform does not immerge and the bottom of a column does not emerge.

The positive effect of subdivision on damage stability becomes evident in Table (2). The limits of $\overline{KG}/T_o$ which must be observed if the angle of inclination shall not be greater than 8^o, are considerably higher than for the rig without subdivision of the columns. The values are calculated for a homogeneous permeability of $\mu_s = \mu_v = 1$.

TABLE II. Radial symmetric rig with subdivided columns and without footings:

Maximum permissible values of $\overline{KG}/T_o$ if inclination shall be limited to $\vartheta_o = 8^o$ ($\mu_s = \mu_v = 1$).

a) $a/T_o = 0.5$

		$R/T_o = 2$	$R/T_o = 4$	$R/T_o = 6$
N = 3;	D/T_o = 0.25	-	-	4.30
	0.50	-	-	4.40
	1.00	-	-	4.40
N = 4;	D/T_o = 0.25	-	1.42	> 5
	0.50	-	1.45	> 5
	1.00	-	1.48	> 5
N = 5;	D/T_o = 0.25	-	2.82	> 5
	0.50	-	2.83	> 5
	1.00	-	2.90	> 5
N = 6;	D/T_o = 0.25	0.29	3.78	> 5
	0.50	0.30	3.80	> 5
	1.00	0.33	3.82	> 5
N = 7;	D/T_o = 0.25	0.66	4.43	> 5
	0.50	0.67	4.45	> 5
	1.00	0.70	4.50	> 5
N = 8;	D/T_o = 0.25	0.91	4.95	> 5
	0.50	0.93	4.97	> 5
	1.00	0.98	5.01	> 5
N = 9;	D/T_o = 0.25	1.11	> 5	> 5
	0.50	1.12	> 5	> 5
	1.00	1.17	> 5	> 5

b) $a/T_o = 0.25$

		$R/T_o = 2$	$R/T_o = 4$	$R/T_o = 6$
N = 3;	D/T_o = 0.25	0.13	3.77	> 5
	0.50	0.15	3.79	> 5
	1.00	0.20	3.82	> 5
N = 4;	D/T_o = 0.25	0.73	4.96	> 5
	0.50	0.74	4.98	> 5
	1.00	0.80	5.01	> 5
N = 5;	D/T_o = 0.25	1.09	> 5	> 5
	0.50	1.10	> 5	> 5
	1.00	1.15	> 5	> 5

N = 6; D/T_o =	0.25	1.33	> 5	> 5
	0.50	1.34	> 5	> 5
	1.00	1.39	> 5	> 5
N = 7; D/T_o =	0.25	1.50	> 5	> 5
	0.50	1.51	> 5	> 5
	1.00	1.55	> 5	> 5
N = 8; D/T_o =	0.25	1.61	> 5	> 5
	0.50	1.62	> 5	> 5
	1.00	1.67	> 5	> 5
N = 9; D/T_o =	0.25	1.71	> 5	> 5
	0.50	1.73	> 5	> 5
	1.00	1.77	> 5	> 5

Of course, the results given in Table II are only correct if flooding is limited to the compartment under consideration. Much smaller or even negative $\overline{KG}/T_o$-values may be obtained if location and extent of damage are of such kind that adjacent compartments are involved too. Taking this into account it can be concluded that, in reality, the effectiveness of watertight subdivision is smaller than may be assumed from Table II. In Section 3 it will be demonstrated that in the case of watertight subdivision a true judgement of the ability to survive flooding can be made by including the randomness of damage dimensions and calculating the probability of survival.

2.3 Multi-legged rig of radial symmetric type with non-subdivided columns and a ringlike lower hull

For reasons of minimizing motion in waves the columns of a multi-legged rig shall not be too big. Therefore, normal semisubmersibles have additional displacement bodies. They are arranged at the bottoms of the columns (e.g. one small footing at each column, one common ringlike footing for the columns of radial symmetric rigs, two longitudinal parallel footings for the columns of rectangular rigs).

From the stability standpoint, however, the additional buoyant hull should be as small as possible. Its unfavourable effect on intact stability follows from the lower height of the center of buoyancy [4]. It is to be expected that in most cases footings will also reduce damage stability. In order to get an idea to what extent the angle of inclination φ_o will change if footings are fitted, damage stability calculations were carried out for a radial symmetric rig with a ringlike footing (Fig.3 of Appendix 3). The dimensions of the columns are the same as in Fig.1 (Appendix 1). All columns are connected by a ring with a box beam section; breadth and height are identical with the diameter D of the columns. The bottoms of the ring and the columns are lying in the same horizontal plane.

As shown in Appendix 3 the angle of inclination φ_o after flooding of one column is as follows:

$$\sin\varphi_o = \frac{2\left(\frac{\mu_v/N}{1-\mu_v/N}\right)\cdot\frac{R}{T_o}}{\left(\frac{1-3\mu_s/N}{1-\mu_s/N}\right)\left(\frac{R}{T_o}\right)^2+\frac{8}{N}\left(\frac{R}{T_o}\right)\left(\frac{D}{T_o}\right)+\frac{1}{1-\mu_v/N}-\frac{1}{8}\left(7+\frac{\mu_s}{N}\right)\left(\frac{D}{T_o}\right)^2-2\left(1-\frac{D}{T_o}+\frac{8R}{NT_o}\right)\frac{\overline{KG}}{T_o}} \qquad (6)$$

This formula can be applied if the angle of heel is small and neither the footing nor the platform will reach the surface of the water.

The values of $\overline{KG}/T_o$ being necessary in order to obtain an angle of inclination of $\varphi_o = 8^o$ are presented in Table III. Permeability is again assumed to be $\mu_s = \mu_v = 1$. For conclusions which may be drawn from these results see Section 3.

TABLE III. Radial symmetric rig with non-subdivided columns and a ringlike lower hull:

Maximum permissible values of $\overline{KG}/T_o$ if inclination shall be limited to $\vartheta_o = 8^o$ $(\mu_s = \mu_v = 1)$.

		R/T_o = 2	R/T_o = 3	R/T_o = 4	R/T_o = 6
N=4; D/T_o =	0.25	-	-	-	-
	0.50	-	-	-	-
N=5; D/T_o =	0.25	-	-	-	0
	0.50	-	-	-	0.11
N=6; D/T_o =	0.25	-	-	0.04	0.42
	0.50	-	-	0.14	0.55
N=7; D/T_o =	0.25	-	0.09	0.31	0.82
	0.50	0	0.19	0.42	0.95
N=8; D/T_o =	0.25	0.06	0.28	0.56	1.18
	0.50	0.13	0.38	0.68	1.33
N=9; D/T_o =	0.25	0.18	0.45	0.78	1.53
	0.50	0.26	0.57	0.92	1.70

Large diameter columns with $D/T_o = 1$ are not considered as the lower hull contributes to the displacement and, accordingly, the columns must be made more slender in comparison to a rig without footings.

2.4 Four-legged rig of rectangular type with non-subdivided columns and two longitudinal parallel footings

This type of rig is often preferred because of its low towing resistance and its cost-saving construction. The section of each footing is quadratic; its breadth, its height and the diameters D of the four columns are equally large. An illustration is given in Fig.4 of Appendix 4.

As compared with the radial symmetric type, rectangular rigs have an additional geometric parameter, the breadth-length ratio $\eta = B/L$. From Table III it may be concluded that in the special case $\eta = 1$ a four-legged rig will hardly withstand the flooding of a column. In case the breadth-length ratio is smaller ($\eta < 1$), the final angle of inclination, ϑ_o, will presumably even be greater. This assumption follows from the strong influence of breadth on intact stability.

In Appendix 4 the angles of inclination

about the two axes x' and y' are calculated. The results are applicable to rigs with $\eta \leq 1$ and with columns of sufficient length in order that after flooding footings are still below and the platform still above water surface. Furthermore, the angles φ_o and ψ_o must be small. The formulas obtained are as follows:

Angle of heel:

$$\sin\varphi_o = \frac{\pi\mu_v \frac{l}{T_o}\cdot\left[\eta+\left(\frac{2-\mu_s}{\mu_s\eta}\right)(\varsigma+\eta^2-1)\right]}{\sqrt{2}\,(4-\mu_v)\cdot\sqrt{1-\left(\frac{2-\mu_s}{\mu_s\eta}\right)^2(1-\eta^2)(\varsigma+\eta^2-1)}\cdot\left[2\frac{lD}{T_o^2}+\frac{2\pi}{4-\mu_v}+\pi\left(\frac{2-\mu_s}{4-\mu_s}\right)\left(\frac{l}{T_o}\right)^2(1+\eta^2-\varsigma)-\frac{\pi}{64}(12+\mu_s)\left(\frac{D}{T_o}\right)^2-\frac{\overline{KG}}{T_o}\left(4\frac{l}{T_o}+\pi-\frac{\pi D}{2T_o}\right)\right]} \tag{7}$$

Angle of trim:

$$\sin\psi_o = \frac{\pi\mu_v \frac{l}{T_o}\cdot\left[1-\left(\frac{2-\mu_s}{\mu_s}\right)(\varsigma+\eta^2-1)\right]}{\sqrt{2}\,(4-\mu_v)\cdot\sqrt{1-\left(\frac{2-\mu_s}{\mu_s\eta}\right)^2(1-\eta^2)(\varsigma+\eta^2-1)}\cdot\left[2\frac{lD}{T_o^2}+\frac{2\pi}{4-\mu_v}+\pi\left(\frac{2-\mu_s}{4-\mu_s}\right)\left(\frac{l}{T_o}\right)^2(1+\eta^2+\varsigma)-\frac{\pi}{64}(12+\mu_s)\left(\frac{D}{T_o}\right)^2-\frac{\overline{KG}}{T_o}\left(4\frac{l}{T_o}+\pi-\frac{\pi D}{2T_o}\right)\right]} \tag{8}$$

New symbols are

l = L/2 and $\varsigma = \sqrt{(1-\eta^2)^2+\left(\frac{\mu_s\cdot\eta}{2-\mu_s}\right)^2}$

As shown in Fig.4 the principal axes x' and y' have their origin in F ; their directions are given by α . If $\eta \leq 1$, α is obtained as follows:

$$\tan 2\alpha = \left|\frac{2J_{xy}}{J_y - J_x}\right| = \frac{\mu_s\cdot\eta}{(2-\mu_s)\cdot(1-\eta^2)} \tag{9}$$

After parallel sinkage, heeling about axis x' and trimming about axis y', the final angle of inclination of the platform against sea level is

$$\tan\vartheta_o = \sqrt{\tan^2\varphi_o + \tan^2\psi_o} \tag{10}$$

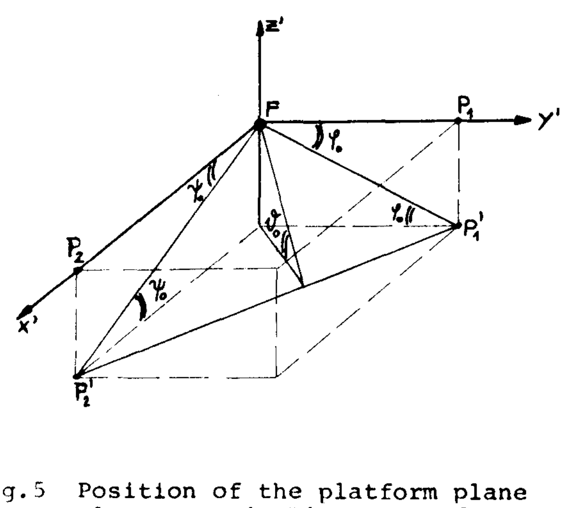


Fig.5 Position of the platform plane element F P'_1 P'_2 after flooding

This formula can directly be derived from Fig.5. Let P_1 be a point on the axis y' which will move down by heeling from P_1 to P'_1, then a point P_2 exists on the axis x' which will move down by trimming to the same level: $\overline{P_2P'_2} = \overline{P_1P'_1}$. After flooding, the plane marked by the points F, P'_1 and P'_2 is parallel to the platform deck. The final angle of inclination, ϑ_o , lies within a vertical plane being perpendicular to the line $P'_1P'_2$.

Systematic calculations which were made for different diameter-draft ratios (D/T_o = 0.25 and 0.50), different length-draft ratios (l/T_o = 4 and 6) and different breadth-length ratios (η = 0.8, 0.9 and 1.0) proved the correctness of the assumption that, without subdivision, the rig will hardly survive the flooding of a column. In each case a final angle of inclination of $\vartheta_o \leq 8^o$ is not attainable if permeability is $\mu_s = \mu_v = 1$ and $\overline{KG}/T_o \geq 0$. In order to demonstrate the influence of η , additional calculations were made for $\mu_s = \mu_v = 0.5$. Here, ϑ_o is considerably smaller. Table IV presents maximum values of $\overline{KG}/T_o$ that are just allowable if the final angle of inclination shall be $\vartheta_o = 6^o$.

TABLE IV. Four-legged rectangular rig with non-subdivided columns and two longitudinal parallel footings:

Maximum permissible values of $\overline{KG}/T_o$ if inclination shall be limited to $\vartheta_o = 6^o$ ($\mu_s = \mu_v = 0.5$) .

		l/T_o = 4	l/T_o = 6
η = 1.0 ;	D/T_o = 0.25	0.81	1.83
	0.50	0.91	*)
η = 0.9 ;	D/T_o = 0.25	0.69	1.59
	0.50	0.80	1.70
η = 0.8 ;	D/T_o = 0.25	0.52	1.27
	0.50	0.64	1.39

*) formulas not applicable because footings become awash

In case of a breadth-length ratio of $\eta < 1$ heeling as well as trimming occurs. Trimming can only be avoided if $\eta = 1$: from equation (9) follows $\alpha = 45^o$

(axis y' runs through the center of the damaged column). As can be seen from Table IV, $\eta = 1$ should be preferred because of the higher allowable center of gravity.

2.5 Six-legged rig of rectangular type with non-subdivided columns and two longitudinal parallel footings

A six-legged rig with a catamaran hull is a compromise between considerations of economy and of damage stability. Furthermore, columns can be made smaller in diameter and, as a consequence, excitation by waves will be less severe compared to a four-legged rig.

Damage may occur to a central column or a corner column. In the latter case the effects of flooding will be more serious. For this reason, damage stability calculations, as carried out in Appendix 5, start out from a damage to a corner column.

The footings of the six-legged rig are of the same shape as the footings of the four-legged rig in Section 2.4 . The box beam section is of equal breadth and height. Each column is assumed to be floodable right down to the bottom of the hull.

The directions of the heel axis x' and the trim axis y' are obtainable from

$$\tan 2\alpha = \left|\frac{2J_{xy}}{J_y - J_x}\right| = \left|\frac{6\mu_s\eta}{12-5\mu_s-6\eta^2(3-\mu_s)}\right| \qquad (11)$$

If $\eta \leq \sqrt{\frac{12-5\mu_s}{18-6\mu_s}}$, the difference between the moments of inertia J_y and J_x is positive. The angle of direction, α, must then be measured as shown in the upper illustration of Fig.6 . For values $\eta > \sqrt{\frac{12-5\mu_s}{18-6\mu_s}}$, the difference $J_y - J_x$ will be negative and α must be established as demonstrated in the lower illustration.

Details of calculation are given in Appendix 5. It must be observed that in the case of $\eta \leq \sqrt{\frac{12-5\mu_s}{18-6\mu_s}}$ the damaged column always lies within the first quadrant of the x'-y'-system of coordinates. In the case of $\eta > \sqrt{\frac{12-5\mu_s}{18-6\mu_s}}$ the damaged column lies - with the exception of a very small range of η-values - within the second quadrant of the x'-y'-system. The narrow limits of the aforementioned range are as follows:

$$\sqrt{\frac{12-5\mu_s}{18-6\mu_s}} < \eta < \sqrt{\frac{2}{3}} \ .$$

This range is of some importance in so far as there is a change in the direction of inclination about the axis y'. Accordingly, the point of emersion of the hull is situated on the opposite side.

Depending on whether η is below or above $\eta = \sqrt{\frac{12-5\mu_s}{18-6\mu_s}}$, different formulas are obtained for φ_0 and ψ_0 :

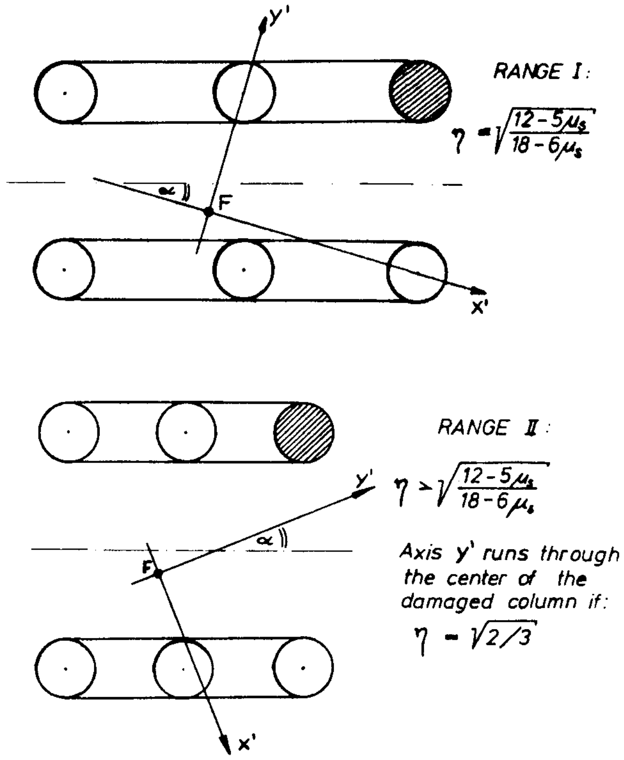


Fig.6 Six-legged rectangular rig with two longitudinal parallel footings: Location of axes x' and y' for different breadth-length ratios η

Angle of heel; $\eta \leq \sqrt{\frac{12-5\mu_s}{18-6\mu_s}}$:

$$\sin\varphi_0 = \frac{\frac{3}{2}\pi\mu_v\frac{\ell}{T_0}\cdot\left(\eta + [12-5\mu_s-6\eta^2(3-\mu_s)]\frac{\xi-1}{6\mu_s\eta}\right)}{(6-\mu_v)\cdot\sqrt{1+[12-5\mu_s-6\eta^2(3-\mu_s)]^2\left(\frac{\xi-1}{6\mu_s\eta}\right)^2}} \times \qquad (12)$$

$$\times \frac{1}{\frac{2\ell D}{T_0^2} + \frac{9\pi}{2(6-\mu_v)} + \frac{\pi(\ell/T_0)^2}{4(6-\mu_s)}\left(12-5\mu_s+6\eta^2(3-\mu_s) - [12-5\mu_s-6\eta^2(3-\mu_s)]\xi\right) - \frac{\pi}{64}(26+\mu_s)\left(\frac{D}{T_0}\right)^2 - \left(4\frac{\ell}{T_0}+\frac{3}{2}\pi - \pi\frac{D}{T_0}\right)\frac{\overline{KG}}{T_0}}$$

Angle of heel; $\eta > \sqrt{\frac{12-5\mu_s}{18-6\mu_s}}$:

$$\sin\varphi_0 = \frac{\frac{3}{2}\pi\mu_v\frac{\ell}{T_0}\cdot\left(1+\left[6\eta^2(3-\mu_s)+5\mu_s-12\right]\frac{\xi-1}{6\mu_s}\right)}{(6-\mu_v)\cdot\sqrt{1+\left[6\eta^2(3-\mu_s)+5\mu_s-12\right]^2\left(\frac{\xi-1}{6\mu_s\eta}\right)^2}} \times$$
$$\times \frac{1}{\frac{2\ell D}{T_0^2}+\frac{9\pi}{2(6-\mu_v)}+\frac{\pi(\ell/T_0)^2}{4(6-\mu_s)}\left(6\eta^2(3-\mu_s)-5\mu_s+12-\left[6\eta^2(3-\mu_s)+5\mu_s-12\right]\xi\right)-\frac{\pi}{64}(26+\mu_s)\left(\frac{D}{T_0}\right)^2-\left(4\frac{\ell}{T_0}+\frac{3}{2}\pi-\pi\frac{D}{T_0}\right)\frac{\overline{KG}}{T_0}} \tag{13}$$

Angle of trim; $\eta \leq \sqrt{\frac{12-5\mu_s}{18-6\mu_s}}$:

$$\sin\psi_0 = \frac{\frac{3}{2}\pi\mu_v\frac{\ell}{T_0}\cdot\left(1-\left[12-5\mu_s-6\eta^2(3-\mu_s)\right]\frac{\xi-1}{6\mu_s}\right)}{(6-\mu_v)\cdot\sqrt{1+\left[12-5\mu_s-6\eta^2(3-\mu_s)\right]^2\left(\frac{\xi-1}{6\mu_s\eta}\right)^2}} \times$$
$$\times \frac{1}{\frac{2\ell D}{T_0^2}+\frac{9\pi}{2(6-\mu_v)}+\frac{\pi(\ell/T_0)^2}{4(6-\mu_s)}\left(12-5\mu_s+6\eta^2(3-\mu_s)+\left[12-5\mu_s-6\eta^2(3-\mu_s)\right]\xi\right)-\frac{\pi}{64}(26+\mu_s)\left(\frac{D}{T_0}\right)^2-\left(4\frac{\ell}{T_0}+\frac{3}{2}\pi-\pi\frac{D}{T_0}\right)\frac{\overline{KG}}{T_0}} \tag{14}$$

Angle of trim; $\eta > \sqrt{\frac{12-5\mu_s}{18-6\mu_s}}$:

$$\sin\psi_0 = \frac{\frac{3}{2}\pi\mu_v\frac{\ell}{T_0}\cdot\left(\eta-\left[6\eta^2(3-\mu_s)+5\mu_s-12\right]\frac{\xi-1}{6\mu_s\eta}\right)}{(6-\mu_v)\cdot\sqrt{1+\left[6\eta^2(3-\mu_s)+5\mu_s-12\right]^2\left(\frac{\xi-1}{6\mu_s\eta}\right)^2}} \times$$
$$\times \frac{1}{\frac{2\ell D}{T_0^2}+\frac{9\pi}{2(6-\mu_v)}+\frac{\pi(\ell/T_0)^2}{4(6-\mu_s)}\left(6\eta^2(3-\mu_s)-5\mu_s+12+\left[6\eta^2(3-\mu_s)+5\mu_s-12\right]\xi\right)-\frac{\pi}{64}(26+\mu_s)\left(\frac{D}{T_0}\right)^2-\left(4\frac{\ell}{T_0}+\frac{3}{2}\pi-\pi\frac{D}{T_0}\right)\frac{\overline{KG}}{T_0}} \tag{15}$$

Symbol ξ in equations (12) to (15) was introduced for the purpose of contracting the formulas: $\xi = \sqrt{1+\left[\frac{6\mu_s\eta}{12-5\mu_s-6\eta^2(3-\mu_s)}\right]^2}$

From these formulas the final angle of inclination against sea level can be determined by using equation (10). Systematic calculations carried out for different values of μ_s, μ_v, η, ℓ/T_0, D/T_0 and $\overline{KG}/T_0$ clearly show the favourable effect of the arrangement of the two additional legs. Compared to the four-legged rig the final angle of inclination, ϑ_0, is smaller. Some results are given in Table V.

TABLE V. Six-legged rectangular rig with non-subdivided columns and two longitudinal parallel footings:

a) Maximum permissible values of $\overline{KG}/T_o$ if inclination shall be limited to $\vartheta_o = 8^o$ ($\mu_S = \mu_V = 1$)

		$l/T_o = 4$	$l/T_o = 6$
$\eta = 1.0$;	$D/T_o = 0.25$	< 0	0.52
	0.50	0	*)
$\eta = 0.9$;	$D/T_o = 0.25$	< 0	0.44
	0.50	< 0	*)
$\eta = 0.7$;	$D/T_o = 0.25$	< 0	0.21
	0.50	< 0	*)
$\eta = 0.6$;	$D/T_o = 0.25$	< 0	0.10
	0.50	< 0	0.20
$\eta = 0.5$;	$D/T_o = 0.25$	< 0	0
	0.50	< 0	0.09

*) formulas not applicable because footings become awash

b) Maximum permissible values of $\overline{KG}/T_o$ if inclination shall be limited to $\vartheta_o = 6^o$ ($\mu_S = \mu_V = 0.5$)

		$l/T_o = 4$	$l/T_o = 6$
$\eta = 1.0$;	$D/T_o = 0.25$	1.27	2.47
	0.50	1.41	*)
$\eta = 0.9$;	$D/T_o = 0.25$	1.20	2.37
	0.50	1.35	*)
$\eta = 0.7$;	$D/T_o = 0.25$	0.90	1.80
	0.50	1.04	1.97

*) formulas not applicable because footings become awash

Comparing the results presented in Sections 2.3, 2.4 and 2.5 it can be deduced that the damage stability values of rectangular rigs with a catamaran hull and a breadth-length ratio of $\eta = 1$ do not differ much from the damage stability values of a radial symmetric rig with a ringlike lower hull and a radial distance of columns of $R \approx l$. This statement is approximately true on condition that number and diameter of the columns are the same. For instance, Table III may be used for a rough assessment of damage stability of a eight-legged rectangular rig. Formulas for this type of rig are not given in this paper but can be derived in a similar way as demonstrated in Appendix 5.

3. ASSESSMENT OF THE ABILITY TO SURVIVE DAMAGES AND CONCLUDING REMARKS

An assessment of the ability to survive damages can be made by calculating the probability of survival. Regulations based on this principle exist for some ten years for passenger ships [5]. Contrary to ships, rigs can attain a survival probability of P = 1 under good weather conditions and of P = 1 or nearly 1 under bad weather conditions. Assuming that no heeling moments are acting, a rig can be made "unsinkable" even without any watertight subdivision if following points are observed:

- high number of columns (N>8 is to be preferred)
- columns arranged as far as possible from the vertical centerline of the rig (R/T_o or $l/T_o>3$ is to be preferred)
- no footings
- breadth-length ratio of rectangular rigs not smaller than $\eta = 1$
- big columns (compared to other parameters the influence of D/T_o is rather small)
- center of gravity as low as possible

Example: Nine-legged radial symmetric rig without footings (N=9, $R/T_o=4$, $\mu_S = \mu_V = 1$). From equation (1) follows:

$$\left.\begin{array}{l}\vartheta_o = 5.7^o \text{ if } \overline{KG}/T_o = 1.5\\ \vartheta_o = 8.0^o \text{ if } \overline{KG}/T_o = 3.0\end{array}\right\} \text{influence of } D/T_o \text{ can be neglected}$$

It is quite evident that this rig can withstand any damage to a column. Therefore, under good weather conditions, the probability of survival will be P = 1.

For "unsinkability" also under bad weather conditions, additional factors are important:

- ability to withstand large heeling moments in damaged condition (ϑ_o should be as small as possible; large righting moments over a wide range of angles are to be aimed at)
- minimization of wind heeling moment*) (in heeled condition the total projected area exposed to wind should be as small as possible; structural members are to be arranged and shaped with a view to a minimum wind force coefficient)
- minimization of wave heeling moment (the rig, considered to remain in its position, will be affected by a heeling moment which can be kept small if the volume of displacement is concentrated at the bottoms of the columns. Hence, footings also have a positive effect on survivability)
- avoidance of large amplitudes of wave-induced motions (small column diameters are to be preferred because the special quality of semisubmersibles, namely small amplitudes of motion, will be reached all the more the smaller the waterplane is. Another positive effect is that parametric excitation cannot occur)

If for reasons of economy and practicability the conditions stated above cannot be fulfilled, a probability of survival P < 1 must be accepted. Before calculating survival probability some definitions must be made. First of all, it must be tried to define the boundary between survival and non-survival. It may be assumed that a rig

*) The percentage of collisions occurring at severe storm conditions will be higher for rigs than for ships. Ships chiefly collide when sailing at bad visibility conditions and, according to nature, wind velocity will then be low [6].

will survive a damage if the angle of inclination caused by flooding and a resulting heeling moment does not exceed a critical value (wind heeling moment based on a wind velocity of 50 knots [1]; wave steepness according to the stability regulations for the German Navy = $(10+0.05\ m^{-1}\ \lambda)^{-1}$, where λ = wave length; the most unfavourable wave length may be taken). This critical angle may be determined by the location of lowest opening through which progressive flooding may take place or by an absolute value of 25 degrees, whichever is less.

In those cases in which the boundary between survival and non-survival will be almost reached, no reserve stability exists which enables the rig to survive a larger resulting heeling moment than the assumed one. This may be taken into account by introducing a factor which will reduce survival probability if the rig cannot withstand additional moments (in [5] this is done by the factor "s" which evaluates the effect of freeboard, stability and heel in the final flooded condition).

A rig which cannot withstand flooding, must be subdivided by watertight decks or bulkheads. Subdivision, however, will be only effective, if in case of collision at least some watertight decks or bulkheads remain undamaged. In order to calculate the probability that flooding will be limited to a compartment or a group of adjacent compartments it must be derived from damage statistics how location and dimensions of damage are distributed. As for rigs a sufficient quantity of damage data does not exist, realistic assumptions must be made for the frequency functions.

Collisions will mainly occur near the level of the waterplane. The vertical extent of damage will vary from very small values to large values. It can be deduced from [7] that the half vertical extent of damage, e , follows a lognormal frequency function:

$$f(e) = \frac{0.4343}{\sqrt{2\pi}\cdot\sigma\cdot e}\exp\left(-\frac{[\log e-\mu]^2}{2\sigma^2}\right) \tag{16}$$

where μ and σ are parameters which must be determined from damage statistics.

In Fig.7 the lognormal function (broken line) is replaced by a linear function (solid line):

$$f(e) = f_o\ (1-\frac{e}{e_o}) \tag{17}$$

If e_o is estimated at $e_o = \frac{H}{6}$ (H = height of column) *) , f_o follows from

$$\int_0^{e_o} f(e)\,de = f_o\int_0^{e_o}\left(1-\frac{e}{e_o}\right)de = 1 \tag{18}$$

The solution is $f_o = \frac{12}{H}$ and equation (17) becomes

$$f(e) = \frac{12}{H}\ (1-\frac{6}{H}\ e) \tag{19}$$

*) For comparison: in [5] a maximum total damage length of 0.24 L is assumed for passenger ships

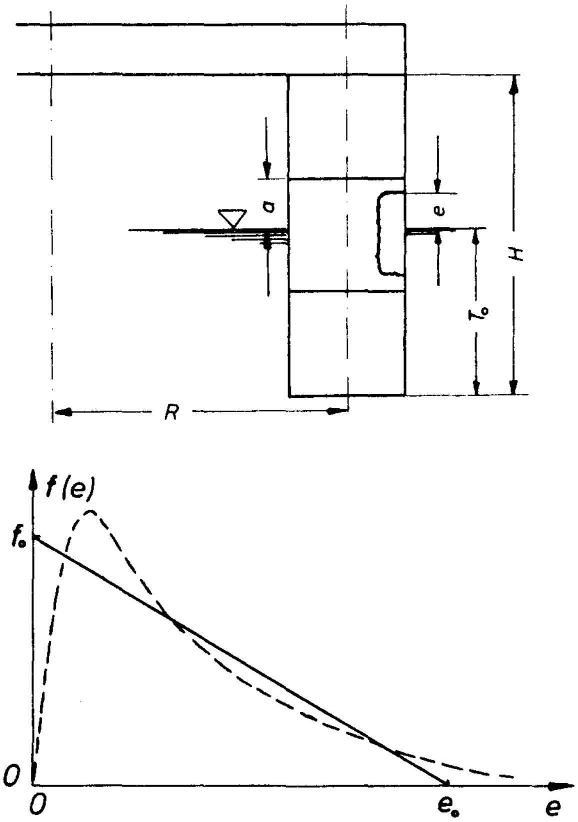


Fig.7 Horizontally subdivided damaged column and frequency function f(e) of vertical extent "e" of damage

In view of the lacking knowledge of the correct function this simplification may be accepted. Furthermore, it may be assumed that the center of damage is located at the level of T_o . Then, in case of horizontal watertight subdivision, a most simple formula can be derived for the probability P that damage extent "e" will be smaller than the distance "a" of a watertight flat from the initial waterplane (see Fig.7):

$$P\{e\leq a\} = \int_0^a f(e)\,de = 12\left(\frac{a}{H}\right) - 36\left(\frac{a}{H}\right)^2 \tag{20}$$

Of course, this formula is only applicable if $\frac{a}{H} \leq \frac{1}{6}$. If $\frac{a}{H} \geq \frac{1}{6}$, maximum damage extent e_o does not exceed flat distance "a" and thus $P\{e \leq a\} = 1$.

<u>Example:</u> Seven-legged radial symmetric rig <u>without</u> footings (N=7, $R/T_o=2$, $T_o/H=0.5$, $\overline{KG}/T_o=1.5$, $\mu_s = \mu_v = 1$). The influence of D/T_o on damage stability is rather small and will therefore be ignored. Heeling moments are assumed to cause capsizing if the angle of inclination due to loss of buoyancy exceeds $\vartheta_o = 10^o$.

a) $\frac{a}{T_o} = 1$ or $\frac{a}{H} = \frac{1}{2}$: $P\{e \le a\} = 1$ $\quad \vartheta_o > 10^o$

b) $\frac{a}{T_o} = \frac{1}{2}$ or $\frac{a}{H} = \frac{1}{4}$: $P\{e \le a\} = 1$ $\quad \vartheta_o > 10^o$

c) $\frac{a}{T_o} = \frac{1}{4}$ or $\frac{a}{H} = \frac{1}{8}$: $P\{e \le a\} = 0.9375$

$\vartheta_o = 8^o$ if $e < a$;
$\vartheta_o > 10^o$ if $e \ge a$.

From this example it can be seen that also rigs with non-optimum geometric characteristics (e.g. small values R/T_o as in this case) can survive damages if they are subdivided effectively. In this example, a probability of survival of P = 0.9375 can be attained by the arrangement of watertight flats at a distance a = H/8 from the level of T_o . As $\vartheta_o = 10^o$ will not be reached, the damaged rig can withstand some additional moments and, accordingly, the reduction of P , as mentioned afore, will be very small or even unnecessary.

Considering that the correct distribution of damage data may differ from the assumed one, the real probability of survival may be somewhat higher or lower than P = 0.9375. For the purpose of judging survivability, however, these differences are not problematic if P , as calculated in this paper, will be taken as a criterion.

The question of the minimum amount of P that should be required is a point of discussion and cannot be answered in this paper. From the technical point of view even a regulation prescribing P = 1 for all semisubmersible rigs could be satisfied. If also values P < 1 shall be permitted the minimum values can be set a good deal higher than those which are required in [5] for passenger ships.

ACKNOWLEDGEMENT

The author wishes to acknowledge the assistance of Mr. P. Kröger in developing computer programs for ϑ_o-plots. The results given in Table I to V are taken from these plots.

REFERENCES

1. Code for the Construction and Equipment of Mobile Offshore Drilling Units (MODU CODE). IMCO Sales Number 80.04, London 1980.

2. Regulations for Mobile Drilling Platforms (with amendments of February 3rd,1982). Norwegian Maritime Directorate, Oslo.

3. Abicht, W. and Kröger, P., Diagrams of Damage Stability of Drilling Rigs. IfS-Schrift, Hamburg 1983 (will be published in spring 1983).

4. Macy, R.H., Drilling Rigs. Chapter XVI of "Ship Design and Construction". Editor: A. D'Arcangelo. SNAME, New York 1969.

5. IMCO Resolution A.265 (VIII): Equivalent Regulations on Subdivision and Stability of Passenger Ships.

6. Robertson, J.B., Notes for Hamburg 1963 Meeting, Nov. 1963 (unpublished).

7. Krappinger, O., Über Leckverteilungen. Schiffstechnik, vol. 11, no. 57, 1964.

APPENDIX 1 :

Calculation of damage stability of a multi-legged radial symmetric rig with non-subdivided columns and without footings

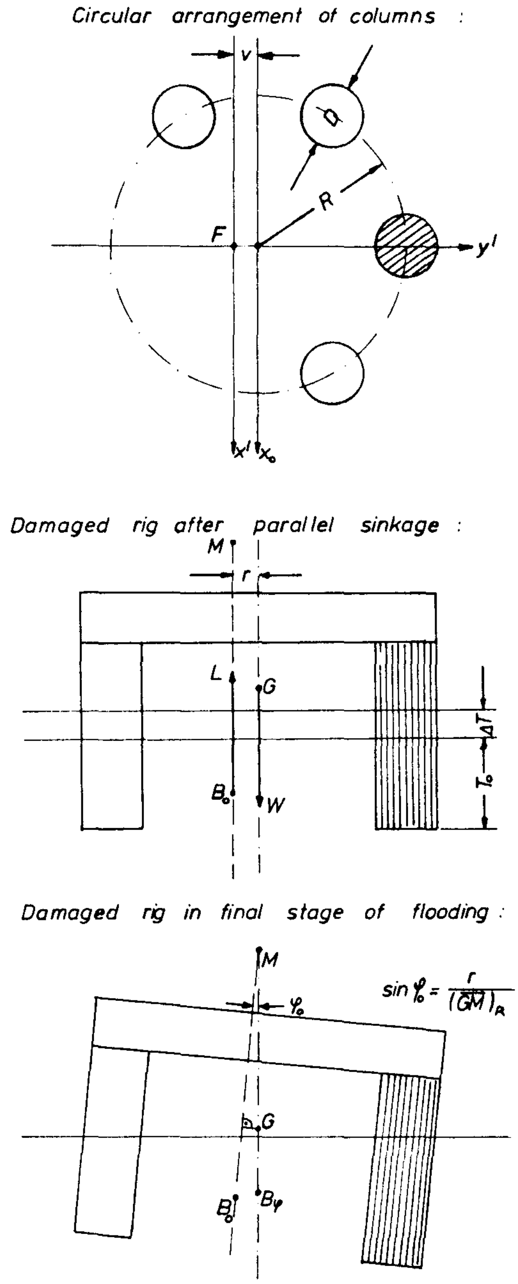


Fig.1 Radial symmetric rig without footings in damaged condition

a) Intact Stability

All symbols are listed and defined in the nomenclature.

$$\nabla_0 = \frac{\pi}{4} N D^2 T_0 \qquad \frac{(\overline{KB})_0}{T_0} = \frac{1}{2}$$

$$\frac{(\overline{BM})_0}{T_0} = \frac{J_{0x}}{\nabla_0 \cdot T_0} = \frac{1}{2}\left(\frac{R}{T_0}\right)^2 + \frac{1}{16}\left(\frac{D}{T_0}\right)^2$$

$$\frac{(\overline{GM})_0}{T_0} = \frac{1}{2}\left(\frac{R}{T_0}\right)^2 + \frac{1}{16}\left(\frac{D}{T_0}\right)^2 + \frac{1}{2} - \frac{\overline{KG}}{T_0}$$

b) Damage Stability

$$\frac{v}{R} = \frac{\mu_s/N}{1-\mu_s/N} \qquad \frac{r}{R} = \frac{\mu_v/N}{1-\mu_v/N}$$

$$\frac{\Delta T}{T_0} = \frac{\mu_v/N}{1-\mu_v/N} \qquad \frac{(\overline{KB})_R}{T_0} = \frac{1}{2}\left(\frac{1}{1-\mu_v/N}\right)$$

$$\frac{(\overline{BM})_R}{T_0} = \frac{J_{x'}}{\nabla_0 \cdot T_0} = \frac{1}{2}\left(\frac{1-3\mu_s/N}{1-\mu_s/N}\right)\left(\frac{R}{T_0}\right)^2 + \frac{1}{16}\left(1-\mu_s/N\right)\left(\frac{D}{T_0}\right)^2 .$$

$$\frac{(\overline{GM})_R}{T_0} = \frac{1}{2}\left(\frac{1-3\mu_s/N}{1-\mu_s/N}\right)\left(\frac{R}{T_0}\right)^2 + \frac{1}{16}\left(1-\mu_s/N\right)\left(\frac{D}{T_0}\right)^2 + \frac{1}{2}\left(\frac{1}{1-\mu_v/N}\right) - \frac{\overline{KG}}{T_0} .$$

From Fig.1 : $\sin \varphi_0 = \frac{r}{(\overline{GM})_R} = \frac{\frac{r}{R} \cdot \frac{R}{T_0}}{(\overline{GM})_R / T_0}$

The final formula for the angle of inclination is obtained by using the foregoing expressions for r/R and $(\overline{GM})_R/T_0$: see Section 2.1, equation (1) .

APPENDIX 2 :

Calculation of damage stability of a multi-legged radial symmetric rig with subdivided columns and without footings

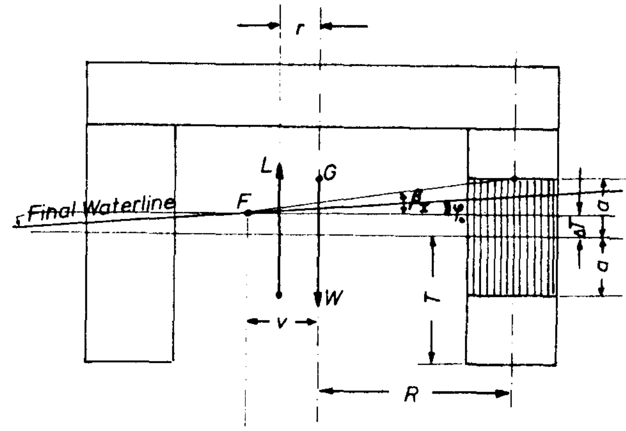
a) Intact Stability

See Appendix 1

b) Damage Stability

Depending on whether the upper flat of the damaged compartment is located above or below final waterline, damage stability values will be different.

REGION I : Upper flat above final waterline



REGION II : Upper flat below final waterline

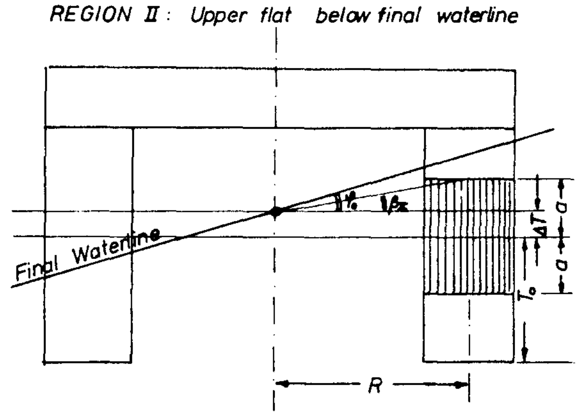


Fig.2 Radial symmetric rig with subdivided columns: upper flat of the damaged compartment above and below final waterline

$\boxed{\text{Region I : } \varphi_0 < \beta_I}$

$$\frac{v}{R} = \frac{\mu_s/N}{1-\mu_s/N} \qquad \frac{r}{R} = \left(\frac{\mu_v/N}{1-\mu_v/N}\right)\frac{a}{T_0}$$

$$\frac{\Delta T}{T_0} = \left(\frac{\mu_v/N}{1-\mu_v/N}\right)\frac{a}{T_0}$$

$$\frac{(\overline{KB})_R}{T_0} = \frac{1}{2}\left[1 + \left(\frac{\mu_v/N}{1-\mu_v/N}\right)\left(\frac{a}{T_0}\right)^2\right]$$

From Fig.2 : $\tan \beta_I = \frac{a - \Delta T}{v + R} =$

$$= \frac{(1-\mu_s/N)(1-2\mu_v/N)}{(1-\mu_v/N)}\left(\frac{a}{R}\right)$$

$$\frac{(\overline{BM})_R}{T_0} = \frac{J_{x'}}{\nabla_0 \cdot T_0} = \frac{1}{2}\left(\frac{1-3\mu_s/N}{1-\mu_s/N}\right)\left(\frac{R}{T_0}\right)^2 + \frac{1}{16}\left(1-\mu_s/N\right)\left(\frac{D}{T_0}\right)^2 .$$

$$\frac{(\overline{GM})_R}{T_o} = \frac{1}{2}\left(\frac{1-3\mu_s/N}{1-\mu_s/N}\right)\left(\frac{R}{T_o}\right)^2 + \frac{1}{16}\left(1-\mu_s/N\right)\left(\frac{D}{T_o}\right)^2 + \frac{1}{2}\left[1+\left(\frac{\mu_v/N}{1-\mu_v/N}\right)\left(\frac{a}{T_o}\right)^2\right] - \frac{\overline{KG}}{T_o} \quad .$$

The angle of inclination can be calculated as shown in Appendix 1. The result is given in Section 2.2, equations (2) and (3).

$\boxed{\text{Region II : } \varphi_o > \beta_{II}}$

In this case, characterized by a constant amount of flooding water, preference is given to the "added-weight" method of calculating damage stability. From Fig.2 follows:

$$\mathrm{tg}\ \beta_{II} = \frac{a-\Delta T}{R} = \left(1-2\ \mu_V/N\right)\cdot\frac{a}{R}$$

With $\Delta T = 2\ (\mu_V/N)\cdot a$

the volume of displacement, including the weight of the flooding water, becomes

$$\nabla' = \frac{\pi}{4}\ N\left[1+2(\mu_V/N)\ \frac{a}{T_o}\right] D^2 T_o$$

The height of the center of gravity after flooding is

$$\frac{(\overline{KG})'}{T_o} = \frac{2(\mu_v/N)\frac{a}{T_o} + \frac{\overline{KG}}{T_o}}{1+2(\mu_v/N)\frac{a}{T_o}}$$

Further, one gets

$$\frac{(\overline{BM})'}{T_o} = \frac{\frac{1}{2}\left(\frac{R}{T_o}\right)^2 + \frac{1}{16}\left(\frac{D}{T_o}\right)^2}{1+2(\mu_v/N)\frac{a}{T_o}}$$

and

$$\frac{(\overline{KB})'}{T_o} = \frac{1}{2}\left[1+2(\mu_v/N)\frac{a}{T_o}\right] \quad .$$

The results of the "added-weight" method of calculation are:

$$\frac{(\overline{GM})'}{T_o} = \left[\frac{1}{1+2(\mu_v/N)\frac{a}{T_o}}\right] \times$$

$$\times\left[\frac{1}{2}\left(\frac{R}{T_o}\right)^2 + \frac{1}{16}\left(\frac{D}{T_o}\right)^2 + \frac{1}{2} + 2\left(\frac{\mu_v}{N}\right)^2\left(\frac{a}{T_o}\right)^2 - \frac{\overline{KG}}{T_o}\right]$$

Righting moment $M_R' = \rho_W\cdot g\cdot\nabla'\cdot(\overline{GM})'\cdot\sin\varphi_o$

Heeling moment $M_H' = \frac{\pi}{2}\ \rho_W\cdot g\cdot\mu_V\cdot a\cdot R\cdot D^2\cdot\cos\varphi_o$

From $M_R' = M_H'$ follows equation (4) in Section 2.2 which may be used for determining the angle of inclination φ_o .

APPENDIX 3 :

Calculation of damage stability of a multi-legged radial symmetric rig with non-subdivided columns and a ringlike hull

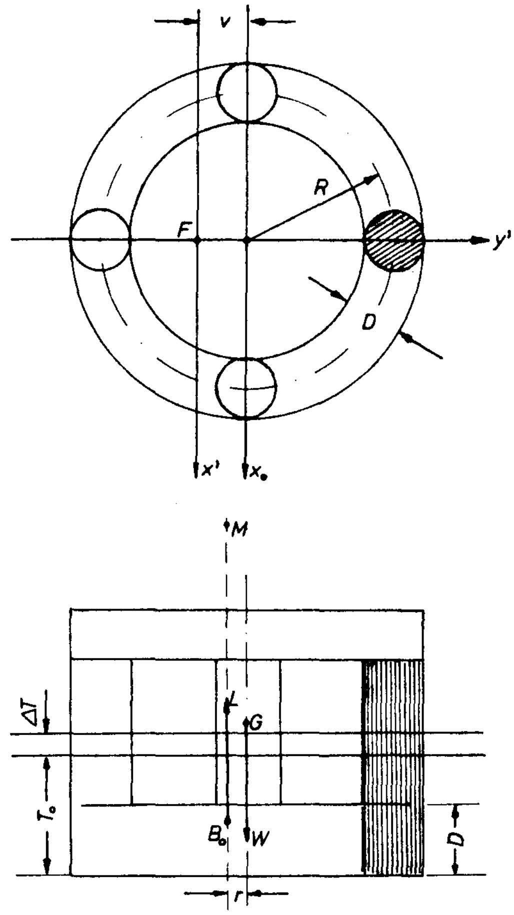


Fig.3 Radial symmetric rig with a ringlike hull in damaged condition

a) Intact Stability

$$\nabla_o = \frac{\pi}{4} N D^2 T_o \left(1-\frac{D}{T_o}+\frac{8R}{NT_o}\right)$$

$$\frac{(\overline{KB})_o}{T_o} = \frac{1-\left(\frac{D}{T_o}\right)^2 + \frac{8RD}{NT_o^2}}{2\left(1-\frac{D}{T_o}+\frac{8R}{NT_o}\right)}$$

$$\frac{(\overline{BM})_o}{T_o} = \frac{J_{ox}}{\nabla_o\cdot T_o} = \frac{\left(\frac{R}{T_o}\right)^2 + \frac{1}{8}\left(\frac{D}{T_o}\right)^2}{2\left(1-\frac{D}{T_o}+\frac{8R}{NT_o}\right)}$$

$$\frac{(\overline{GM})_o}{T_o} = \frac{\left(\frac{R}{T_o}\right)^2 + \frac{8RD}{N\cdot T_o^2} + 1 - \frac{7}{8}\left(\frac{D}{T_o}\right)^2}{2\left(1-\frac{D}{T_o}+\frac{8R}{NT_o}\right)} - \frac{\overline{KG}}{T_o}$$

b) Damage Stability

$$\frac{v}{R} = \frac{\mu_s/N}{1-\mu_s/N} \qquad \frac{r}{R} = \frac{\mu_v/N}{\left(1-\frac{\mu_v}{N}\right)\left(1-\frac{D}{T_o}+\frac{8R}{NT_o}\right)}$$

$$\frac{\Delta T}{T_o} = \frac{\mu_v/N}{1-\mu_v/N}$$

$$\frac{(\overline{KB})_R}{T_0} = \frac{\frac{8RD}{N \cdot T_0^2} + \frac{1}{1-\mu_v/N} - \left(\frac{D}{T_0}\right)^2}{2\left(1 - \frac{D}{T_0} + \frac{8R}{NT_0}\right)}$$

$$\frac{(\overline{BM})_R}{T_0} = \frac{J_{x'}}{\nabla_0 \cdot T_0} = \frac{\left(\frac{1-3\mu_s/N}{1-\mu_s/N}\right)\left(\frac{R}{T_0}\right)^2 + \frac{1}{8}\left(1-\mu_s/N\right)\left(\frac{D}{T_0}\right)^2}{2\left(1 - \frac{D}{T_0} + \frac{8R}{NT_0}\right)}$$

$$\frac{(\overline{GM})_R}{T_0} = \frac{\left(\frac{1-3\mu_s/N}{1-\mu_s/N}\right)\left(\frac{R}{T_0}\right)^2 + \frac{8RD}{N \cdot T_0^2} + \frac{1}{1-\mu_v/N}}{2\left(1 - \frac{D}{T_0} + \frac{8R}{N \cdot T_0}\right)} -$$

$$- \frac{(7+\mu_s/N) \cdot (D/T_0)^2}{16\left(1 - \frac{D}{T_0} + \frac{8R}{NT_0}\right)} - \frac{\overline{KG}}{T_0}$$

φ_0 follows from $\sin\varphi_0 = \frac{r/R \cdot R/T_0}{(\overline{GM})_R / T_0}$ (see equation (6) in Section 2.3).

APPENDIX 4 :

Calculation of damage stability of a four-legged rectangular rig with non-subdivided columns and a catamaran hull

a) Intact Stability

$$\nabla_0 = D^2 T_0 \left(4\frac{\ell}{T_0} + \pi - \frac{\pi D}{2T_0}\right)$$

$$\frac{(\overline{KB})_0}{T_0} = \frac{2\frac{\ell D}{T_0^2} + \frac{\pi}{2} - \frac{\pi}{4}\left(\frac{D}{T_0}\right)^2}{4\frac{\ell}{T_0} + \pi - \frac{\pi D}{2T_0}}$$

$$\frac{(\overline{BM})_0}{T_0} = \frac{\pi\eta^2\left(\frac{\ell}{T_0}\right)^2 + \frac{\pi}{16}\left(\frac{D}{T_0}\right)^2}{4\frac{\ell}{T_0} + \pi - \frac{\pi D}{2T_0}}$$

$$\frac{(\overline{BM_L})_0}{T_0} = \frac{\pi\left(\frac{\ell}{T_0}\right)^2 + \frac{\pi}{16}\left(\frac{D}{T_0}\right)^2}{4\frac{\ell}{T_0} + \pi - \frac{\pi D}{2T_0}}$$

$$\frac{(\overline{GM})_0}{T_0} = \frac{2\frac{\ell D}{T_0^2} + \frac{\pi}{2} + \pi\eta^2\left(\frac{\ell}{T_0}\right)^2 - \frac{3\pi}{16}\left(\frac{D}{T_0}\right)^2}{4\frac{\ell}{T_0} + \pi - \frac{\pi D}{2T_0}} - \frac{\overline{KG}}{T_0}$$

$$\frac{(\overline{GM_L})_0}{T_0} = \frac{2\frac{\ell D}{T_0^2} + \frac{\pi}{2} + \pi\left(\frac{\ell}{T_0}\right)^2 - \frac{3\pi}{16}\left(\frac{D}{T_0}\right)^2}{4\frac{\ell}{T_0} + \pi - \frac{\pi D}{2T_0}} - \frac{\overline{KG}}{T_0}$$

b) Damage Stability

$$\frac{u}{\ell} = \frac{v}{b} = \frac{\mu_s}{4-\mu_s} \qquad \frac{r}{\ell} = \frac{p}{\ell}\sqrt{1+\eta^2}$$

$$\frac{p}{\ell} = \frac{q}{b} = \frac{2\mu_v}{(4-\mu_v)\cdot\left(\frac{8\ell}{\pi T_0} + 2 - \frac{D}{T_0}\right)}$$

$$\frac{\Delta T}{T_0} = \frac{\mu_v}{4-\mu_v}$$

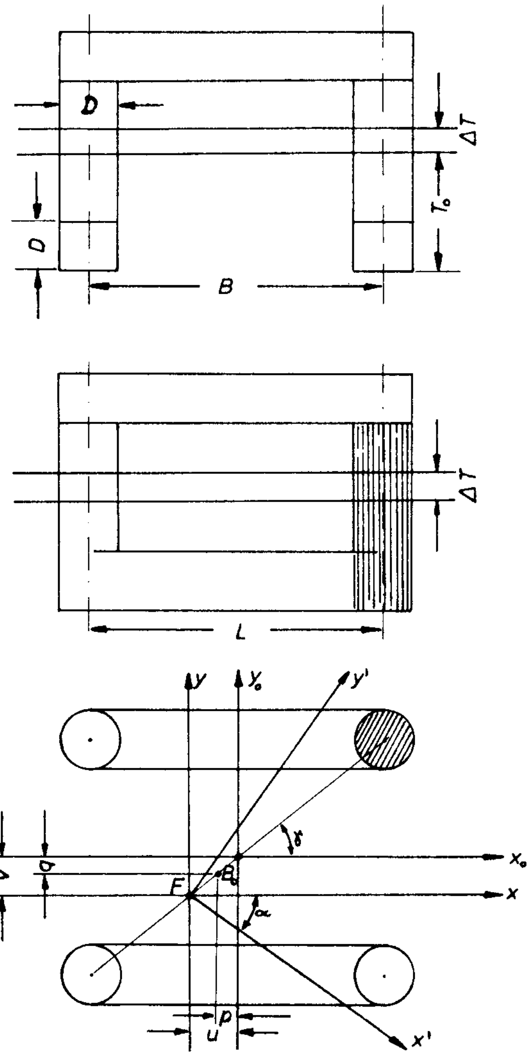


Fig.4 Four-legged rectangular rig with catamaran hull in damaged condition

$$\frac{n}{\ell} = \frac{r}{\ell}\cdot\sin(\alpha+\gamma) \qquad \frac{m}{\ell} = \frac{r}{\ell}\cdot\cos(\alpha+\gamma)$$

where $\tan 2\alpha = \left|\frac{2J_{xy}}{J_y - J_x}\right| = \frac{\mu_s \cdot \eta}{(2-\mu_s)\cdot(1-\eta^2)}$

and $\tan\gamma = \eta$

$$\frac{(\overline{KB})_R}{T_0} = \frac{2\frac{\ell D}{T_0^2} + \frac{2\pi}{4-\mu_v} - \frac{\pi}{4}\left(\frac{D}{T_0}\right)^2}{4\frac{\ell}{T_0} + \pi - \frac{\pi D}{2T_0}}$$

With $\zeta \equiv \sqrt{(1-\eta^2)^2 + \left(\frac{\mu_s \cdot \eta}{2-\mu_s}\right)^2}$

following formulas are obtained :

$$\frac{(\overline{BM})_R}{T_0} = \frac{\pi\left(\frac{2-\mu_s}{4-\mu_s}\right)\left(\frac{l}{T_0}\right)^2\left(1+\eta^2-\zeta\right) + \frac{\pi}{64}(4-\mu_s)\left(\frac{D}{T_0}\right)^2}{4\frac{l}{T_0}+\pi-\frac{\pi D}{2T_0}}$$

$$\frac{(\overline{BM}_L)_R}{T_0} = \frac{\pi\left(\frac{2-\mu_s}{4-\mu_s}\right)\left(\frac{l}{T_0}\right)^2\left(1+\eta^2+\zeta\right) + \frac{\pi}{64}(4-\mu_s)\left(\frac{D}{T_0}\right)^2}{4\frac{l}{T_0}+\pi-\frac{\pi D}{2T_0}}$$

$$\frac{(\overline{GM})_R}{T_0} = \frac{2\frac{lD}{T_0^2}+\frac{2\pi}{4-\mu_v}+\pi\left(\frac{2-\mu_s}{4-\mu_s}\right)\left(\frac{l}{T_0}\right)^2\left(1+\eta^2-\zeta\right)}{4\frac{l}{T_0}+\pi-\frac{\pi D}{2T_0}} - \frac{\frac{\pi}{64}(12+\mu_s)\cdot\left(\frac{D}{T_0}\right)^2}{4\frac{l}{T_0}+\pi-\frac{\pi D}{2T_0}} - \frac{\overline{KG}}{T_0}$$

$$\frac{(\overline{GM}_L)_R}{T_0} = \frac{2\frac{lD}{T_0^2}+\frac{2\pi}{4-\mu_v}+\pi\left(\frac{2-\mu_s}{4-\mu_s}\right)\left(\frac{l}{T_0}\right)^2\left(1+\eta^2+\zeta\right)}{4\frac{l}{T_0}+\pi-\frac{\pi D}{2T_0}} - \frac{\frac{\pi}{64}(12+\mu_s)\left(\frac{D}{T_0}\right)^2}{4\frac{l}{T_0}+\pi-\frac{\pi D}{2T_0}} - \frac{\overline{KG}}{T_0}$$

Inclination about axis x' caused by the corresponding component of the revolving couple of buoyancy- and weight-forces:

$$\sin\varphi_0 = \frac{n}{(\overline{GM})_R} = \frac{n/l\cdot l/T_0}{(\overline{GM})_R/T_0}$$

Inclination about axis y' caused by the corresponding component of the revolving couple of buoyancy- and weight-forces:

$$\sin\psi_0 = \frac{m}{(\overline{GM}_L)_R} = \frac{m/l\cdot l/T_0}{(\overline{GM}_L)_R/T_0}$$

Substituting the expressions given above into the equations for φ_0 and ψ_0, one obtains the full formulas (equations (7) and (8) in Section 2.4). If φ_0 and ψ_0 are known, ϑ_0 can be determined from equation (10).

APPENDIX 5 :

Calculation of damage stability of a six-legged rectangular rig with non-subdivided columns and a catamaran hull

a) Intact Stability

$$\nabla_0 = D^2 T_0\left(4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}\right)$$

$$\frac{(\overline{KB})_0}{T_0} = \frac{2\frac{lD}{T_0^2}+\frac{3}{4}\pi-\frac{\pi}{2}\left(\frac{D}{T_0}\right)^2}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}}$$

$$\frac{(\overline{BM})_0}{T_0} = \frac{\frac{3}{2}\pi\eta^2\left(\frac{l}{T_0}\right)^2+\frac{3\pi}{32}\left(\frac{D}{T_0}\right)^2}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}}$$

$$\frac{(\overline{BM}_L)_0}{T_0} = \frac{\pi\left(\frac{l}{T_0}\right)^2+\frac{3\pi}{32}\left(\frac{D}{T_0}\right)^2}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}}$$

$$\frac{(\overline{GM})_0}{T_0} = \frac{2\frac{lD}{T_0^2}+\frac{3}{4}\pi+\frac{3}{2}\pi\eta^2\left(\frac{l}{T_0}\right)^2-\frac{13}{32}\pi\left(\frac{D}{T_0}\right)^2}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}} - \frac{\overline{KG}}{T_0}$$

$$\frac{(\overline{GM}_L)_0}{T_0} = \frac{2\frac{lD}{T_0^2}+\frac{3}{4}\pi+\pi\left(\frac{l}{T_0}\right)^2-\frac{13}{32}\pi\left(\frac{D}{T_0}\right)^2}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}} - \frac{\overline{KG}}{T_0}$$

b) Damage Stability

$$\frac{u}{l} = \frac{v}{b} = \frac{\mu_s}{6-\mu_s} \qquad \frac{r}{l} = \frac{p}{l}\sqrt{1+\eta^2}$$

$$\frac{p}{l} = \frac{q}{b} = \frac{3\mu_v}{(6-\mu_v)\cdot\left(\frac{8l}{\pi T_0}+3-2\frac{D}{T_0}\right)}$$

$$\frac{\Delta T}{T_0} = \frac{\mu_v}{6-\mu_v}$$

$$\frac{(\overline{KB})_R}{T_0} = \frac{2\frac{lD}{T_0^2}+\frac{9\pi}{2(6-\mu_v)}-\frac{\pi}{2}\left(\frac{D}{T_0}\right)^2}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}}$$

With $\quad \xi = \sqrt{1+\left[\frac{6\mu_s\eta}{12-5\mu_s-6\eta^2(3-\mu_s)}\right]^2}$

following formulas are obtained :

$$\frac{(\overline{BM})_R}{T_0} = \frac{\frac{\pi(l/T_0)^2}{4(6-\mu_s)}\left(12-5\mu_s+6\eta^2[3-\mu_s]-\left|12-5\mu_s-6\eta^2[3-\mu_s]\right|\xi\right)}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}} + \frac{\frac{\pi}{64}(6-\mu_s)\cdot(D/T_0)^2}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}}$$

$$\frac{(\overline{BM}_L)_R}{T_0} = \frac{\frac{\pi(l/T_0)^2}{4(6-\mu_s)}\left(12-5\mu_s+6\eta^2[3-\mu_s]+\left|12-5\mu_s-6\eta^2[3-\mu_s]\right|\xi\right)}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}} + \frac{\frac{\pi}{64}(6-\mu_s)\cdot(D/T_0)^2}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}}$$

$$\frac{(\overline{GM})_R}{T_0} = \frac{\frac{2lD}{T_0^2}+\frac{\pi(l/T_0)^2}{4(6-\mu_s)}\left(12-5\mu_s+6\eta^2[3-\mu_s]-\left|12-5\mu_s-6\eta^2[3-\mu_s]\right|\xi\right)}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}} + \frac{\frac{9\pi}{2(6-\mu_v)}-\frac{\pi}{64}(26+\mu_s)(D/T_0)^2}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}} - \frac{\overline{KG}}{T_0}$$

$$\frac{(\overline{GM}_L)_R}{T_0} = \frac{\frac{2lD}{T_0^2}+\frac{\pi(l/T_0)^2}{4(6-\mu_s)}\left(12-5\mu_s+6\eta^2[3-\mu_s]+\left|12-5\mu_s-6\eta^2[3-\mu_s]\right|\xi\right)}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}} + \frac{\frac{9\pi}{2(6-\mu_v)}-\frac{\pi}{64}(26+\mu_s)(D/T_0)^2}{4\frac{l}{T_0}+\frac{3}{2}\pi-\frac{\pi D}{T_0}} - \frac{\overline{KG}}{T_0}$$

From the different definition of α (see Fig.6) follow different formulas for the component leverarms n and m :

if $\eta \leq \sqrt{\frac{12-5\mu_S}{18-6\mu_S}}$: $\frac{n}{l} = \frac{r}{l} \cdot \sin(\alpha+\gamma)$

$$\frac{m}{l} = \frac{r}{l} \cdot \cos(\alpha+\gamma)$$

if $\eta > \sqrt{\frac{12-5\mu_S}{18-6\mu_S}}$: $\frac{n}{l} = \frac{r}{l} \cdot \cos(\alpha-\gamma)$

$$\frac{m}{l} = \frac{r}{l} \cdot \left|\sin(\alpha-\gamma)\right|$$

where $\tan 2\alpha = \left|\frac{6\mu_S \cdot \eta}{12-5\mu_S-6\eta^2(3-\mu_S)}\right|$

and $\tan\gamma = \eta$.

Angles of inclination about axes x' and y' :

$$\sin\varphi_0 = \frac{n}{(\overline{GM})_R} = \frac{n/l \cdot l/T_0}{(\overline{GM})_R/T_0}$$

$$\sin\psi_0 = \frac{m}{(\overline{GM}_L)_R} = \frac{m/l \cdot l/T_0}{(\overline{GM}_L)_R/T_0}$$

φ_0 and ψ_0 are functions of μ_S, μ_V, η, l/T_0, D/T_0 and $\overline{KG}/T_0$. These functions are obtained after some transformations by using the above-written formulas (equations (12), (13), (14), (15) in Section 2.5). The final angle of inclination, ϑ_0 , can be calculated as shown in Section 2.4 (equation (10), Fig.5).