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Accurate integration of trimmed cells based on Bezier approximation

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In this work, a new adaptive integration method for simulation of two-dimensional linear elasticity problems is presented. The main benefit of the proposed method is the reduction of the computational cost by lowering the number of integration points required to reach a certain level of accuracy. The main concept of the proposed method is to calculate new weights for trimmed cells employing the advantage of Bezier parametric curves. Within this concept, it is possible to map a square to a triangle with one curved edge where any curved edge is approximated by a parametric Bezier curve. In this way, a new set of Gaussian quadrature points is introduced for each trimmed cell in a fast and robust way. Besides main mapping cases, the integration method includes supplementary cases as well to increase the robustness and generality of the method. In the next step, the proposed method is implemented in a two-dimensional fictitious domain code in MATLAB to solve structural problems. The results will be compared to those obtained through the commercial finite element code ABAQUS. It is shown that the proposed method is accurate and robust.

1 Introduction

The numerical integration of stiffness matrix and load vector of broken cells is a very important concept in fictitious domain methods aiming to simplify the mesh generation step of complicated geometries (see Fig. 1). It comes at the cost of complexity of numerical integration. Within this context, the most popular integration methods which are widely used by researchers are space-tree [1, 2] (quadtree in 2D and octree in 3D) and moment fitting [3, 4] approaches. Although the space-tree methods are very robust, they are computationally expensive due to a high number of integration points required for a certain level of accuracy. Therefore in this paper, a new two-dimensional boundary conforming integration method is presented. The main concept of the proposed scheme is to calculate the new positions and weights of integration points for a broken cell by employing the benefit of Bezier parametric curves. Consequently, a new set of Gaussian quadrature points will be introduced for each broken cell in a fast and robust way.

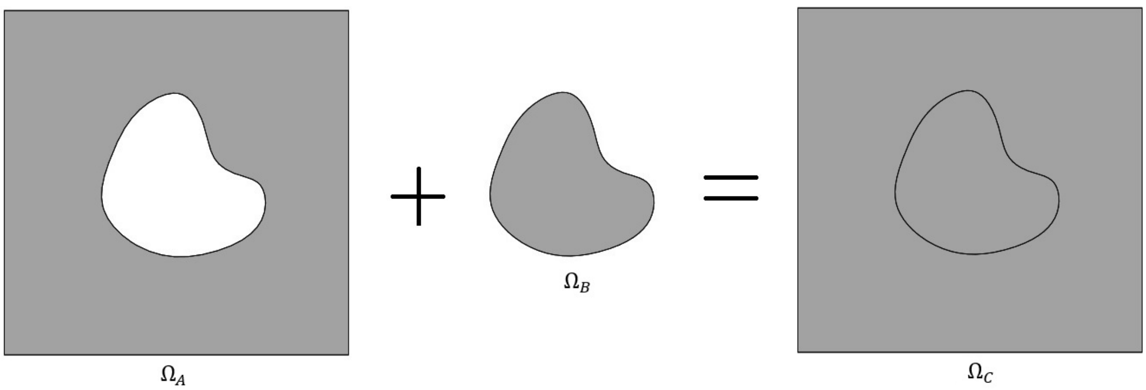

Fig. 1: The concept of fictitious domain approaches. Ω_A, Ω_B and Ω_C refer to physical domain, fictitious domain and embedded domains respectively

2 Proposed integration scheme

There are two main cases to be addressed in this section: 1-corner cutting and 2-edge cutting cases. A comprehensive description of these cases along with related mapping formulations are shown in Fig. 2 where the green parts are the physical portions of the domain. Please note that the mapping formulation for other corners/edges can be obtained by employing the rotation matrix.

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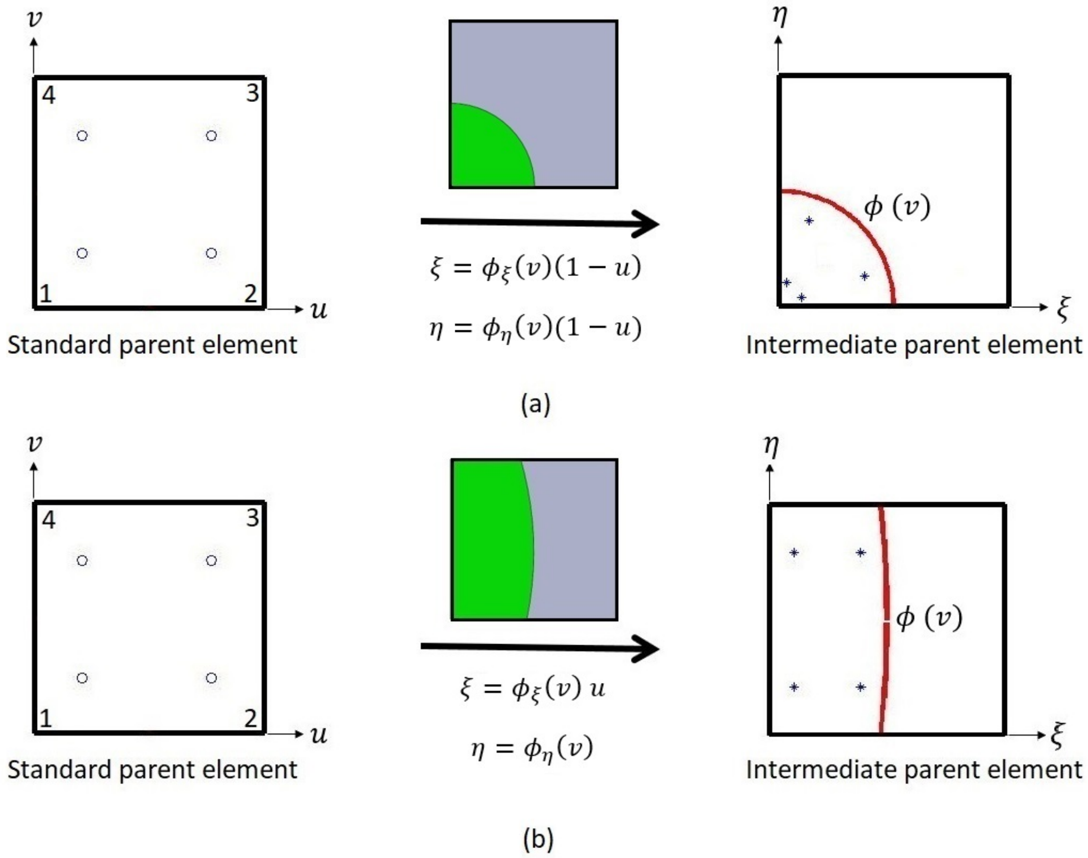


Fig. 2: Main mapping cases: (a) corner cutting and (b) edge cutting.

All other cases that cannot be strictly categorized within the main cases are defined as supplementary cases. Supplementary cases can be taken into account by the superposition of main cases. Some examples are shown in Fig. 3. Please note that if a hole is completely inside the element, a subdivision step is required before superposition.

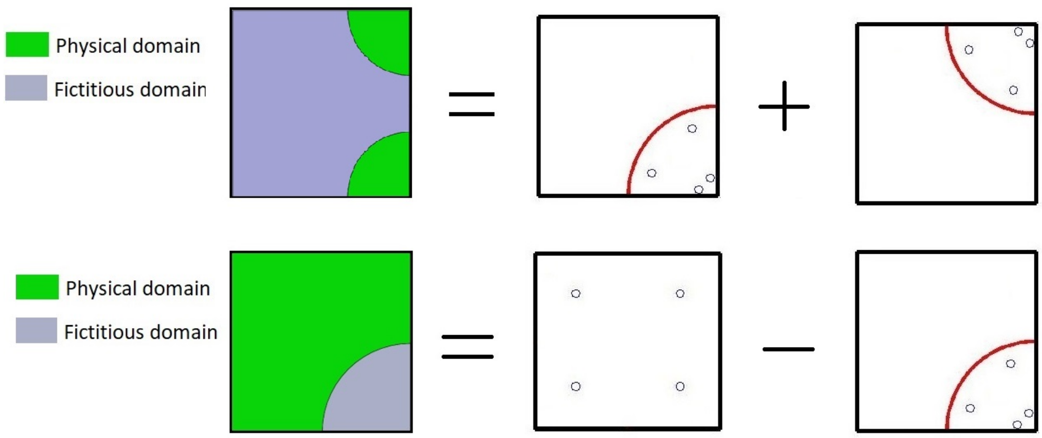


Fig. 3: Sample supplementary (superposition) cases.

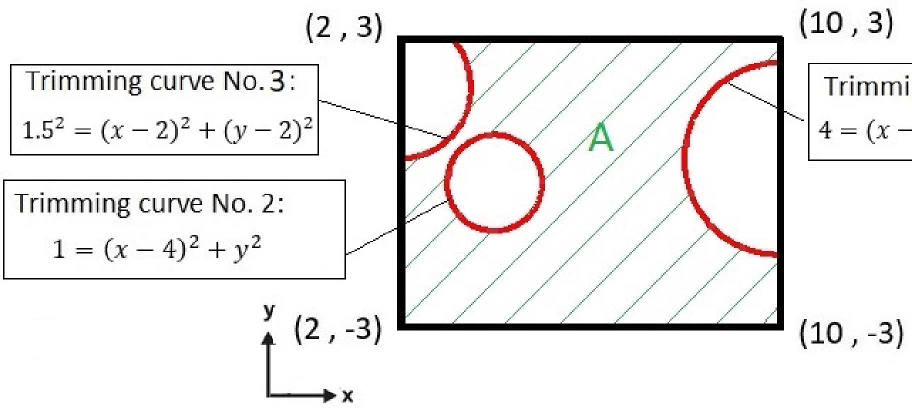
In order to satisfy the requirements of mapping from standard parent element domain to the intermediate parent element domain, it is essential to include a parametric description of the trimming curve (the red curves in Figs. 2 and 3). Even though any parametric description would be acknowledged, our attention in this study is placed on Bezier curves. The advantage of this choice is that the Bezier approximation algorithm is very efficient. Also, Bezier curves can take the form of any given complex set of trimming points. For further information about Bezier curves, interested readers may refer to [5].

3 Numerical examples

In this section, two numerical examples are provided. The first example only takes a single cell integration into account while the second example implements the integration method into a real structural example with multiple cells.

3.1 Integration over a complex domain

The provided integration example is a mixture of fully/partly inside holes as well as a corner cutting case as shown in Fig. 4. The equations shown in this figure are used to find the trimming points as well to determine the exact value of the integral I, which is provided in the picture, over the green area A. The parametric forms of the trimming curves are attained through Bezier approximation to be implemented into the mapping equations.



$$I = \iint_A (x^2 + 5y^2 - 3xy + 4)\, dxdy$$

Fig. 4: Input data for the integration example.

The integration errors with respect to the reference values are depicted in Fig. 5 for different numbers of 1D integration points (n) and Bezier degrees (p_1). It should be noted that the reference solution is an over-kill solution obtained by MATLAB.

According to these results, the error values are relatively low for even low numbers of n and p_1. All curves reach to a plateau specially after n = 16 which means that the remaining error is only related to the Bezier curve approximation. Relatively low

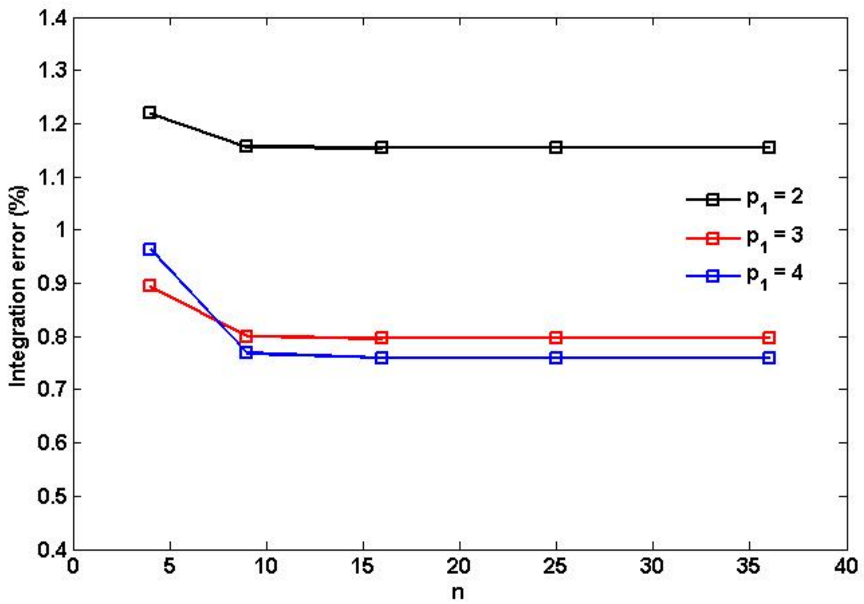


Fig. 5: Convergence study of the integration example.

error values for this complicated hybrid case demonstrates the superiority and robustness of the proposed approach to be used in full structural fictitious domain examples.

3.2 Fictitious domain example

The adaptive integration scheme applied in this study can be straightforwardly implemented into finite element codes. To do so, a supervisory code must determine if a broken cell is of main or supplementary trimming case, then the stiffness matrix and force vector can be integrated using the relevant mapping formulations. The overview of the supervisory code is as follows:

- **Input**: Cells, Nodes, Loads, BCs, Material data, Sets of trimming points
- Pick a cell from the list, **loop** over set of trimming point sets to see which sets are trimming the cell and which points of each set are within the cell. Then store the points as well as set identification numbers. **End loop**.
- **If** no trimming point exists within a cell, the cell is untrimmed, then standard Gaussian Quadrature will be aoolied, **Else**, **loop** over the number of trimming sets which are trimming the cell, map the trimming points of each set from the physical domain back to the parent domain, then fit a Bezier curve to these points. After that, decide on the mapping case using inside/outside checks on cell corners and calculate the (positive/negative, ref. to Boolean operations) stiffness matrices and load vectors. **End loop**. **End if**.
- **If** all stiffness matrices are negative, calculate the stiffness matrix of the untrimmed cell as a positive stiffness matrix. **End if**
- Make a summation of all stiffness matrices to get the final stiffness matrix of the cell (superposition step).

In the fictitious domain example provided here, a linear elastic material model with elastic modulus of 210 GPa and poisson's ratio of 0.3 is considered. The displacement results are compared to the finite element results computed by ABAQUS commercial software. The geometry, loading and boundary conditions of this example are shown in Fig. 6. The Bezier degree is

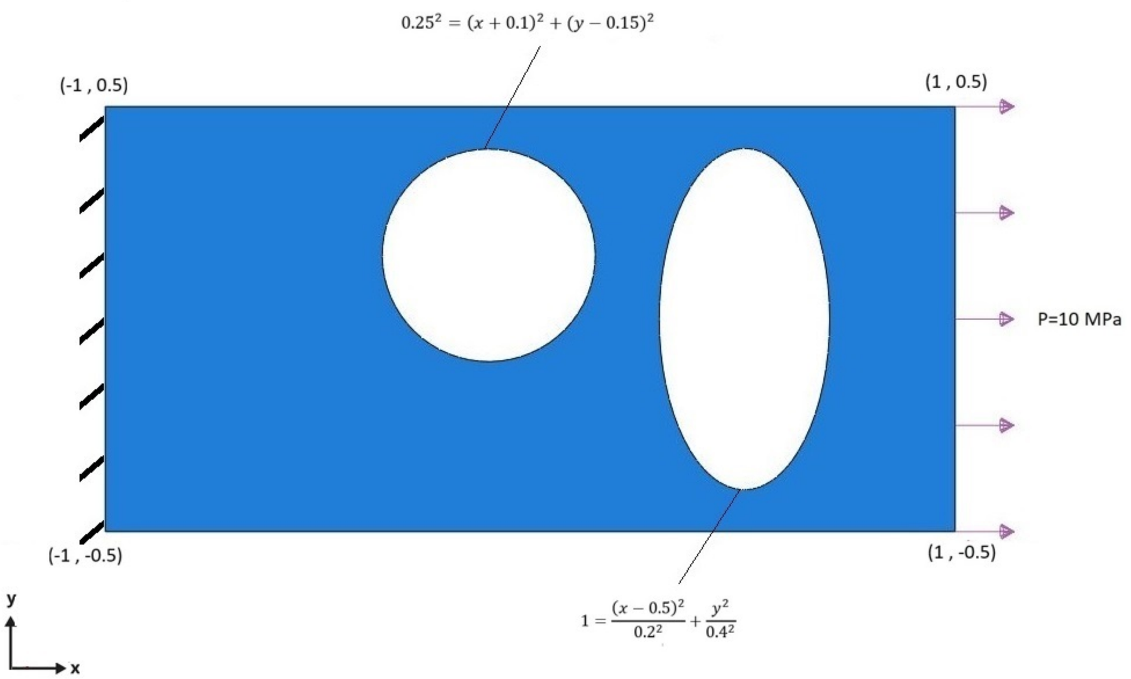


Fig. 6: Input data for the example of a plate with two holes.

fixed to be 3 and the number of 1D integration points is set to be 2 for all elements. Fig. 7 represents the location of integration points for the 50-cell discretization case. Asterisks are displaced integration points while circles denote the position of standard integration points. It should be noted that in cells which contain both asterisks and circles, superposition is applied.

The tip displacement values are obtained and compared to those computed by the finite element commercial code ABAQUS with the same element type and number of integration points. Quantitative and qualitative comparisons of the results are shown in Figs. 8 and 9 respectively.

A very good agreement between the results of the two methods can be observed both quantitatively and qualitatively which shows the applicability of the proposed integration method for solving full structural problems.

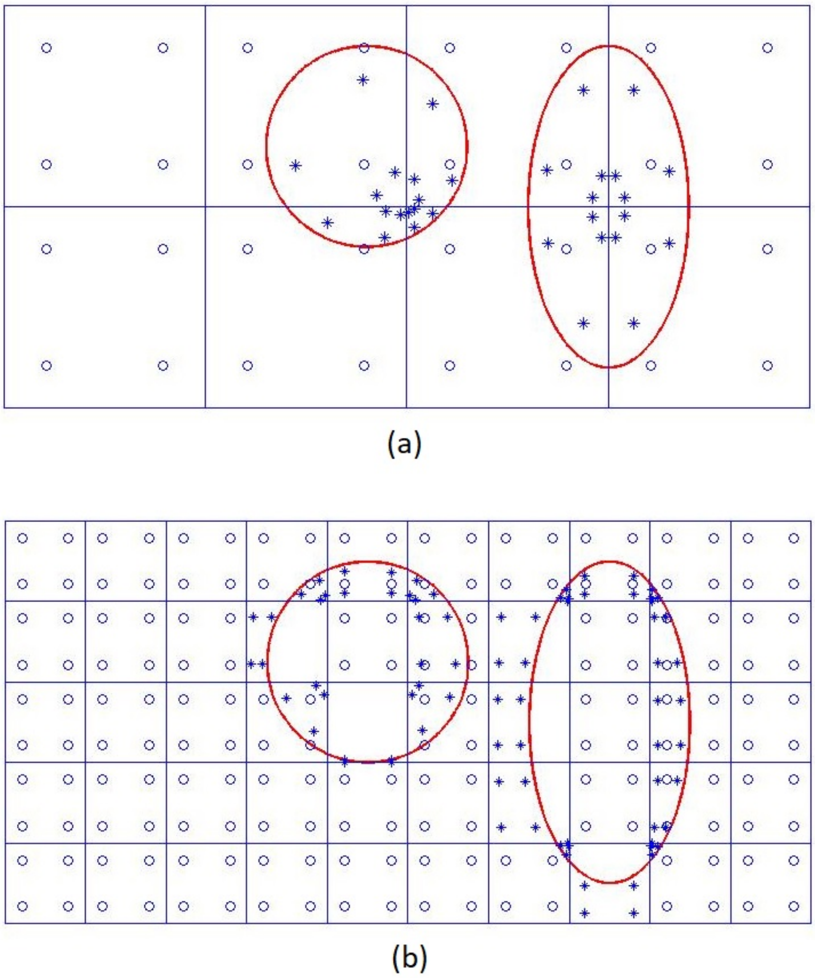


Fig. 7: Relocated position of integration points for: (a) 8-cell and (b) 50-cell discretization case.

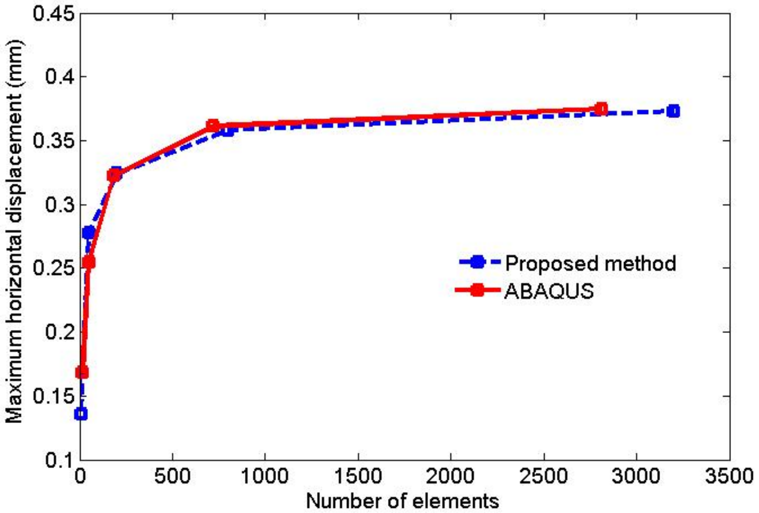


Fig. 8: Horizontal tip displacement results: proposed method vs. ABAQUS.

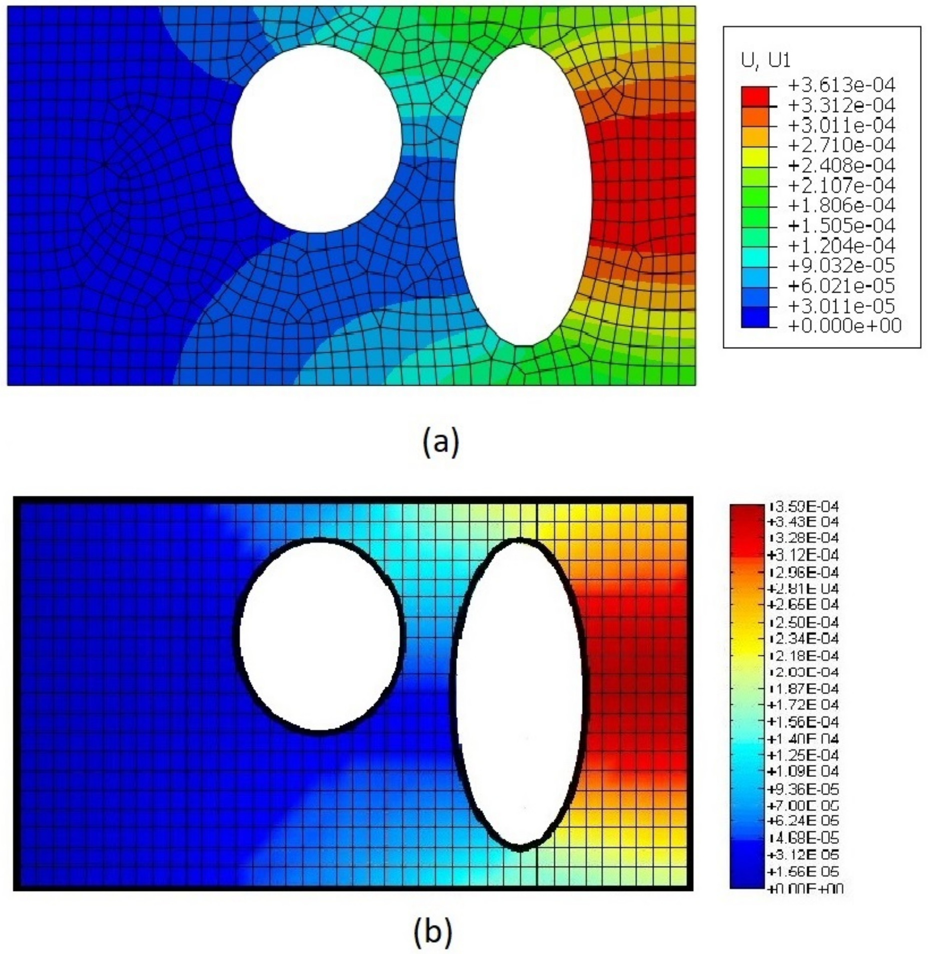


Fig. 9: Horizontal displacement contour for: (a) ABAQUS and (b) proposed method.

4 Conclusion

A new adaptive method for the numerical integration of trimmed cells was presented in this paper which includes main and supplementary cases. The integration example showed that the method can be accurately employed as an integration tool. In the next step, the method was successfully implemented into a finite element code with linear shape functions. Tip displacement values were compared to the finite element results obtained with ABAQUS and a very good agreement was observed.

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