323 | 1975

SCHRIFTENREIHE SCHIFFBAU

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Basic Theory of Wave Analysis

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K. Eggers, 1. Auflage, Hamburg, Technische Universität Hamburg-Harburg, 1975

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http://www.tuhh.de/vss

Basic Theory of Wave Analysis
(State of the Art 1975)

Hauptvortrag auf dem
International Seminar on Wave Resistance
in Tokio und Kansai
3. bis 9. Februar 1976

ABSTRACT

The paper provides an evaluative survey on mathematical models which are underlying the current methods for determining wave resistance through wave pattern measurements. Emphasis is on elucidating implicit assumptions rather than analytical derivations. Special weight is given to such problems which are still open for future research. Some comments are made to questions related to the decay of the 'local' wave field.

NOMENCLATURE

- A,B,C,D = control surface for energy flux
- a = wave amplitude
- b = tank width
- C = waterline along a ship
- C(w),S(w) = Fourier transforms for longitudinal cut method
- $C_2(w),S_2(w)$ = Fourier transforms for longitudinal cut method
- F(u),G(u) = sine, cosine component of free wave spectrum
- $f(\theta),g(\theta)$ = alternative definition of free wave spectrum
- $G(x,y,z;x',y',z')$ = Green's Function of point source of unit strength
- g = acceleration due to gravity
- $H(k,\theta)$ = complex Kochin function
- $\bar{H}$ = complex conjugate of H
- h = water depth
- J(u,w) = alternative form of Kochin function
- $\bar{J}$ = complex conjugate of J
- k = dimensionless circular wave number in direction θ
- k_o = fundamental wave number
- $l = \varkappa\cos\theta$ = cosine component of wave number $\varkappa$
- $m = \varkappa\sin\theta$ = sine component of wave number $\varkappa$
- $\bar{n}$ = unit vector normal to the ship
- P(w),Q(w) = Fourier transform of p(x) q(x), respectively
- p(x),q(x) = two measurable quantities
- R_w = wave resistance
- R,α = polar coordinates with respect to origin
- r = distance of flow point from source point
- r_1 = distance of flow point from image point of source
- S = control surface
- s = s(u) = function of u
- t = time
- U = ship speed
- u,v,w = flow component in horiz., lateral and vertival dir.
- $u = k\sin\theta$ = dimensionless circular wave number induced in y-direction
- u_ν = discrete value of u in tank of finite width
- $w = k\cos\theta$ = dimensionless circular wave number induced in x-direction
- X = longitudinal component of horizontal wave force
- Y = transverse component of horizontal wave force
- x = coordinate in direction of motion of ship
- y = horizontal coordinate positive to portside
- z = vertical coordinate positive upwards
- (x,y,z) = flow point
- α_c = Kelvin angle
- ζ = wave elevation
- ζ_x,ζ_y = partial derivatives of ζ
- θ = direction of wave propagation
- $\varkappa = k_o k$ = wave number
- ϱ = fluid density
- σ = wave frequency
- φ = perturbation potential
- $\varphi_x,\varphi_y,\varphi_z$ = components of perturbation velocity

As long as man has used naval transportation, he must have observed that moving floating bodies make waves, which reflect in some way part of the effort needed to keep a vessel advancing in the desired direction. But it was not earlier than 25 years ago that interest awakened to determine the power supplied to the wave pattern from the ship quantitatively through evaluating the wave pattern geometry.

Exploratory enterprise came up in this country [1] and in France [2] around 1950, but it was the truly pioneering work of Prof. Inui which cleared the soil for the second, more rational phase of research. He presented his achievements at the - I may now say historical - 18th meeting of the "H-5 panel" - that is the ship-wave committee of the American Society of Naval Architects - and this worked like an ignition spark for world wide simultaneous research.

The timing for the first international seminar on wave resistance theory in 1963 was just appropriate to have the attendance confronted with about ten different methods of wave analysis already. This set signal for the next phase, which meant synoptical analysis and generalisation together with systematic assessment of practical experience. Thus the variety of different approaches could be tied to and re-derived from common mathematical models.

It would be premature if I claim that this phase is facing its completion already, but I feel that meanwhile so much progress is evident that we are under an almost compulsary need to have the state of the art presented here and to be discussed cooperatively by potential experts. This is in particular so with respect to rapidly increasing demands facing the naval architect for most accurate prediction of powering requirement of ships.

My role to give an overall report on wave analysis theory within this seminar is made easier in so far as four well-arranged surveys on current theoretical development are already available: One [3] by Eggers, Sharma and Ward (1967, referred to as ESW in the sequel), one (1969) by Ikehata [4], one (1973) by Wehausen [5] and finally one (1975) by Gadd [6] within the report of the ITTC Resistance Committee. I take the liberty to assume that you had opportunity to study at least one of above papers so that I can concentrate on certain particular aspects which - I feel - have not been treated with adequate weight so far. Apart from reporting what has been achieved, I intend to compile material which still has to be evaluated and completed, in order to find cross-correlations and to show gaps where further research is needed. For this purpose, I have built up a bibliographical appendix. Its aim is to update the literature given in ESW to cover all papers somehow related to problems of wave analysis since 1967. References to this appendix are made in standard form, i.e. by author and year of publication as is used in the bibliographical work of Wehausen and in ESW.

One basic assumption of all wave analysis methods is that viscosity is neglected in the following sense:

(i) There should be no vorticity where measurements are taken.
(ii) Any attenuation of waves off the ship due to viscosity is neglected (this is justified through investigations of Nikitin (1965), Cumberbatch (1965) and Brard (1970)).

Another tacit implication is that wave-breaking resistance, though originating from gravity effects, can neither be measured nor detected by wave analysis. This question together with interaction of resistance components has been analysed by Wehausen [5] and in more special works of Baba [7], Brard [8], Landweber [9], Sharma [10] and Weinblum [11] .

1. MODELS OF WAVE FLOW

Any deviation of the water surface from its state-of-rest position represents some positive amount of potential energy, which may be evaluated directly from the wave pattern geometry. Due to the associated fluid m o t i o n , however, there must be also kinetic energy. And the wave resistance, in which we are finally interested, results from the lengthwise component of total energy transport. The group velocity, which governs this process, depends on wave length of individual components. Hence it can not be expected as suggested by Korvin-Kroukovsky [12] that there is a direct relation between potential energy of some area under obseration and the resistance of the ship creating it.

We can consider some models for the flow manifested through the wave pattern and derive wave resistance as function of the flow components. Inserting then relations between flow and wave elevation, we are aiming to express wave resistance through characteristics of the wave geometry only. This is actually possible for several models of wave flow, which in turn give reasonable approximations to the actual flow related to the wave pattern, at least away from certain domains close to the ship, where "local waves" can not be disregarded. This restriction comes up because these flow models can be accepted only under special assumptions. We mentioned already that there should be no vorticity; hence a velocity potential φ can be introduced such that flow components u,v,w may be expressed as its partial derivatives. It is further necessary to require the smallness of wave elevation ζ and flow components $\varphi_x, \varphi_y, \varphi_z$ in such a sense that a linearised free surface condition can be assumed to hold on the undisturbed free surface.

1.1 Expression for the Total Wave Pattern (including Near-Field Effects)

Let us take a standard coordinate system to describe waves and the flow. The plane z=0 is the undisturbed free surface, the positive x axis is in the direction where the ship advances steadily with speed U, and z is positive upwards. We shall not commit ourselves to take a specific choice of origin regarding y, unless a symmetry plane y=0 can be found. The position of the coordinate origin regarding x is arbitrary in so far as it will not influence in general the quality of approximations used; there is a special situation, however, in connection with Kelvin wave patterns 1.3.

Any arbitrary wave pattern $z = \zeta(x,y)$ with sufficient decay at infinity will admit then a global double Fourier integral representation regarding x and y. Anticipating later insight, we shall separate some particular denominator from the Fourier spectral function and set

$$\zeta(x,y) = \frac{1}{4\pi^2 k_0} \operatorname{Im} \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{J(u,w)\, e^{ik_0 wx + ik_0 uy}}{w^2 - \sqrt{w^2+u^2}}\, w\, dw\, du \right\} \tag{1a}$$

with $k_0 = g/U^2$.

Thus the wave field is governed by some "Kochin function" J(u,w) which is in general complex. It is convenient - or more instructive - to express ζ through polar coordinates in the x-y and u-w plane equivalently as

$$\zeta(R,\alpha) = \frac{1}{4\pi^2 k_0} \operatorname{Im} \left\{ \int_0^{\infty}\int_0^{\pi} \frac{H(k,\theta)\, k\cos\theta\, e^{ik_0 k R\cos(\theta-\alpha)}}{k - \cos^{-2}\theta}\, dk\, d\theta \right\} \tag{1b}$$

(1a) and (1b) are then interlinked through the system of relations,

$$x = R\cos\alpha \qquad w = k\cos\theta \qquad du\,dw = k\,dk\,d\theta$$
$$y = R\sin\alpha \qquad u = k\sin\theta \qquad J(u,w) = H(k,\theta).$$

(1b) is in close conformity with the notation introduced by Havelock [13] and by Kochin [14] . We shall write down our results in both notations simultaneously if considered helpful for better understanding.

The evaluation of (1a) near the line $w = \pm s(u) \equiv \left((1+\sqrt{1+4u^2})/2\right)^{1/2}$ (or the evaluation of (1b) near the line $k = \cos^{-2}\theta$) must be performed in terms of complex integration in such a manner that no far-field waves appear for x positive. No special considerations would be required if $J(u,s(u))$ resp. $H(\cos^{-2}\theta, \theta)$ were identically zero. In this case $\zeta(x,y)$ degenerates to a "wave free" wave pattern. In fact, it is only the one-variable "degenerated Kochin function" $J(u,s(u))$ which determines the far-field and thus the wave resistance R_w. But there is no method available to determine R_w from a wave field described through (1). It is a fortunate circumstance that already not too far from the ship (1) may be fairly well replaced by much simpler expressions found from (1).

1.2 Wave Patterns represented through Single-Integral Expressions or through a Series

For regions sufficiently far behind the ship (i.e. for $x = x_b \ll 0$) we may deduce from (1) up to terms of order $O(x_b^{-1})$

$$\zeta(x_b,y) = \frac{1}{4\pi k_0} \int_{-\infty}^{\infty} \left\{ F(u) \sin k_0\left(s(u)x+uy\right) + G(u)\cos k_0\left(s(u)x+uy\right) \right\} du \tag{2a}$$

with $G(u) + iF(u) = \frac{4s^2(u)}{2s^2(u)-1} J(u,s(u))$

In polar notation this leads to

$$\zeta(R_b,\alpha) = \frac{1}{k_0} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ f(\theta)\sin\left(k_0\cos^{-2}\theta\, R_b\cos(\theta-\alpha)\right) + g(\theta)\cos\left(k_0\cos^{-2}\theta\, R_b\cos(\theta-\alpha)\right) \right\} d\theta \tag{2b}$$

with $g(\theta) + if(\theta) = \frac{1}{\pi}\cos^{-3}\theta\, H(\cos^{-2}\theta, \theta)$

(2b) makes clear that this is a representation of ζ through a continuous system of plane waves ("free waves") with wave number k ranging from k_0 to infinity and angle of propagation θ against x axis between $-\pi/2$ and $\pi/2$.

If the ship is moving in a canal with vertical walls along $y = \pm b/2$, the expression analogue to (2a) - as well as to (1a) - can be derived by evaluating the u-integration in the sense of a trapezoidal rule with a step width $\Delta u = 2\pi/k_0 b$. We then obtain (up to $O(x_b^{-1})$)

$$\zeta(x_b,y) = \frac{1}{k_0} \sum_{\nu=-\infty}^{\infty} \left\{ F(u_\nu)\sin k_0\left(s(u_\nu)x + u_\nu y\right) + G(u_\nu)\cos k_0\left(s(u_\nu)x+u_\nu y\right) \right\} \Delta u \tag{3a}$$

with $u_\nu = \nu \cdot \Delta u$

Inserting $G(u_\nu) + iF(u_\nu) = \frac{4\pi \cos^2\theta_\nu}{1+\sin^2\theta_\nu} \left\{ g(\theta_\nu) + if(\theta_\nu) \right\}$;

$$u_\nu = \sin\theta_\nu \cos^{-2}\theta_\nu$$

we have the polar representation

$$\zeta(R_b,\alpha) = \frac{1}{k_0} \sum_{\nu=-\infty}^{\infty} \left\{ f(\theta_\nu)\sin\left(k_0\cos^{-2}\theta_\nu R_b\cos(\theta_\nu-\alpha)\right) + g(\theta_\nu)\cos\left(k_0\cos^{-2}\theta_\nu R_b\cos(\theta_\nu-\alpha)\right) \right\} \cdot \frac{8\pi^2\cos^2\theta_\nu}{k_0 b\,(1+\sin^2\theta_\nu)} \tag{3b}$$

We obtain (2a), (2b) from (3a), (3b) reversely with b tending to infinity, hence $\Delta u \to du$.

(2a) is the basis for the so called "transverse cut" wave analysis methods. (3a) is the starting point for the matrix method of Hogben (1972) and for the "multiple longitudinal cut" method of Moran and Landweber (1972). We have for simplicity assumed here that the wave pattern is symmetric to the plane y=0, accordingly we must take $F(\nu\Delta u) = F(-\nu\Delta u)$, $G(\nu\Delta u) = G(-\nu\Delta u)$. The velocity potential associated to (2a) is

$$\varphi(x,y,z) = \frac{U}{4\pi k_0}\int_{-\infty}^{\infty}\Big\{-F(u)\cos k_0\big(s(u)x+uy\big) + G(u)\sin k_0\big(s(u)x+uy\big)\Big\}\frac{du}{s(u)} \quad (4)$$

from which the expression for finite tank width can be derived following above rules.

Considering now unrestricted water again, but taking $y=y_c$ sufficiently large, we may approximate $\zeta \sim \zeta_1+\zeta_2$ up to order $O(y_c^{-1})$ by

$$\zeta_1(x,y_c) = \frac{1}{\pi k_0}\int_1^{\infty}\frac{1}{w\sqrt{w^2-1}}\Big\{C(w)\cos k_0 w\big(x+\sqrt{w^2-1}\,y_c\big) + S(w)\sin k_0 w\big(x+\sqrt{w^2-1}\,y_c\big)\Big\}dw \quad (5a)$$

with $C(w)+iS(w) = e^{-ik_0 y(2w^2-1)w}\big(G(u)+iF(u)\big)/4u\,;\; u = w\sqrt{w^2-1}$

$$\zeta_2(x,y_c) = \frac{1}{\pi k_0}\,\mathrm{Im}\Big\{\int_0^1\frac{C_2(w)+iS_2(w)}{w\sqrt{1-w^2}}\,e^{ik_0 wx + w\sqrt{1-w^2}\,|y_c|}\,dw\Big\} \quad (5b)$$

We shall give a corresponding polar representation only for ζ_1 as it displays an analogy to (2b), though with different upper limit of integration,

$$\zeta_1(R_c,\alpha) = \frac{1}{k_0}\int_{-\frac{\pi}{2}}^{\theta}\Big\{f(\theta)\sin\big(k_0\cos^{-2}\theta\,R_c\cos(\theta-\alpha)\big) + g(\theta)\cos\big(k_0\cos^{-2}\theta\,R_c\cos(\theta-\alpha)\big)\Big\}d\theta \quad (5c)$$

It was outlined in ESW that for each method of wave analysis just one particular wave flow model is pertinent. The approximate basis for "longitudinal cut" wave analysis is (5a). It is certainly true that (2a) could serve as an approximation as well for x sufficiently large, and Havlock's "variable integration limit" model (ESW p. 143) is valid in a formal sense under roughly the same restrictions as (5a). But only (5a) leads to a consistent result, perhaps due to being "uniformly valid" in a certain sense along a cut $y=y_c$.

It needs some pondering to understand the joint action of the two components ζ_1 and ζ_2. Whereas ζ_1 again is a system of free waves, the components of ζ_2 display non-oscillatory decay with increasing $|y_c|$. With (1) given, $C(w)$, $S(w)$, $C_2(w)$, $S_2(w)$ depend clearly on the choice of y-coordinate origin, as does the value of y_c. But for $y = 0$, ζ_2 - though convergent - can not be considered part of some "local disturbance" in so far as it contributes to far-field waves with x tending both to plus and minus infinity; so does ζ_1 in this case. An asymptotic analysis shows that for x positive, ζ_2 finally cancells ζ_1 - the range of value $C_2(w)$ and $S_2(w)$ for w less than 1 is irrelevant - whereas for x tending to minus infinity, ζ_2 duplicates ζ_1. If, however, y_c is nonzero, no far-field contributions from ζ_2 can be found with $|x|$ approaching infinity (Stoke's phenomenon of mathematical physics).

1.3 Slowly Varying Wave Trains, Kelvin Patterns in particular

If to (2) - or to (5) in case $y_c \neq 0$ - a second asymptotic evaluation is performed, a representation of ζ is found with no waves outside some wedge-shaped region $x < 0$, $|y/x| \leq 1/\sqrt{8}$, whereas for each point of this region two systems of wave ("transverse" and "divergent") are present, whose characteristics $\varkappa$ and Θ depend on space coordinates x and y, or R and α if we use polar notation. Explicitly we obtain

$$\zeta(R,\alpha) = \frac{1}{k_0}\sqrt{\frac{2}{\pi R\sqrt{1-9\sin^2\alpha}}}\sum_{j=1}^{2}\cos^{-3/2}\theta_j\,\overline{H}\big(\cos^{-2}\theta_j,\theta_j\big)\,e^{ik_0 k(\theta_j)R\cos(\theta_j-\alpha)} \quad (6a)$$

degenerating to

$$\zeta(R,\pi) = \frac{1}{k_0}\sqrt{\frac{2}{\pi R}}\,\mathrm{Re}\Big\{H(1,0)\,e^{i(k_0 k(0)R+\pi/4)}\Big\} \quad (6b)$$

for $\alpha = \pm\pi$, $x = -R$, with Θ_1, Θ_2 as roots of

$$\frac{\partial}{\partial\theta}\Big\{\varkappa(\theta)\cos(\theta-\alpha)\Big\} = 0 \quad (7)$$

This is the condition of stationary phase. In case of deep water, we have

$$\varkappa(\theta) = k_0\cos^{-2}\theta \quad (8)$$

so that Θ_1, Θ_2 are roots of $\cot\alpha + 2\tan\Theta + \cot\Theta = 0$, i.e.

$$\tan\theta_{1,2} = \frac{1}{4}\Big\{-\cot\alpha \pm \sqrt{\cot^2\alpha - 8}\Big\} \quad (9)$$

and thereby $\tan\Theta_1 < 1/\sqrt{2} < \tan\Theta_2$. In case that (8) does not hold, e.g. for finite depth h, factors of (6a), (6b) will be different.

It should be observed that in case $\varkappa$ depends on Θ solely - in particular under (8) - for each of above wave systems the locus of constant Θ (and thus constant $\varkappa$), the so called "characteristic curves", comes out as straight lines, radiating from the origin within the wedge domain. A system of curved "wave crests" may be constructed by eliminating Θ from two equations (7) and

$$R\varkappa(\theta)\cos(\theta-\alpha) = n\pi \;;\quad n = 1,2,3\ldots \quad (10)$$

as shown by Hogner [15] in the special

case (8). Wave crests are thus envelopes to a parametric manifold of plane waves.

It is easily seen that the approximation (6) to the wave pattern depends essentially on the coordinate origin through which R and α are determined. If we replace x by $\tilde{x}=x+\Delta x$, y by $\tilde{y}=y+\Delta y$ and thus R by $\tilde{R}=\sqrt{\tilde{x}^2+\tilde{y}^2}$, α by $\tilde{\alpha}=\arctan \tilde{y}/\tilde{x}$, we can expect that the function (6a) may be of quite different character if $\sqrt{\Delta x^2+\Delta y^2}$ is not small compared to R - i.e. far away - and that this change cannot be compensated through selecting another function $H(\cos^2\theta,\theta)$.

Only intuitive arguments are at hand for optimal selection of Δx (and of Δy if no symmetry arguments can be used). For most practical applications, at least when considering waves generated along the fore body, the fore perpendicular is taken as origin. This may be justified through the fact that for a discrete pressure point located in the plane z=0 the peak of the wedge, wherein waves occur, coincides actually with the location of the pressure point. This is substantiated through numerical calculation (Ursell (1960a)). However, for submerged sources it is known that their wave pattern may be imagined as created by some distribution of pressure points over the entire plane z=0 with maximum above the singularity!

If we have to analyse a (symmetric) wave pattern which was obtained through experiment, it may be tempting-starting from a transverse profile at $x=x_b$ say - to find values $\pm y^*$ as limits of an interval $-y^*\leq y\leq y^*$ such that $\zeta(x_b,y)$ vanishes outside this range; then Δx could be defined in such way that $|y^*/x_b+\Delta x| = 1/\sqrt{8}$. But such a procedure is not only inaccurate for a measured profile: There is ample evidence that in general the inclination of boundaries to the area where waves are observed is excessive close to the fore body (Hogben (1972), Standing (1974)) with regard to the value $\pm 19°28"=\arctan 1/\sqrt{8}$ predicted by theory, which is actually observed behind the ship (Newman (1971)). This is an indication of obviously nonlinear mechanisms invalidating results of the classical linearised ship wave theory.

An analytical tool for explaining this phenomenon is available since Lighthill (1967) and Witham provided the theory of slowly varying wave trains. It applies to monochromatic wave fields - this term means that only one plane wave is felt in the vicinity of each point - where in general the relation between wave number $\varkappa$ and angle of propagation θ may be effected by inhomogeneities of the environment. In order that the wave train may be called "slowly varying", it is required that variations $\varkappa$, θ and the wave amplitude a can be detected only on a scale comparable to several distances between wave crests. Both the transverse and the divergent components of a Kelvin wave pattern (6) may be considered as special cases, where (8) holds. In this case no influence of the environment is considered, which otherwise may result from nonuniformities (i) of average wave amplitude (ii) of basic flow (for example due to the presence of wake) or (iii) of water depth h .

There is a direct formal analogy (Eggers (1974)) between stationary ship wave patterns and unsteady disperse wave system in one dimension; so that mutually corresponding sets of symbols can be freely exchanged. If unsteady wave systems are characterised through $(x,t,\sigma,\varkappa)$ with x as space coordinate, t as time, σ as frequency, $\varkappa$ as wave number, and $c_{gr}=\partial\sigma/\partial\varkappa$ as appropriate definition of group velocity, the corresponding parameter set is (x,y,m,ℓ) for a stationary ship wave pattern (Lighthill (1967)), with $m=\varkappa\sin\theta$, $\ell=\varkappa\cos\theta$. The "dispersion relation" $m=m(\ell)$ is given through $m^4=k_0^2(m^2+\ell^2)$ if (8) holds. Using this analogy, we may then say that regions of constant "frequency" m "travel" with "group velocity" along curved characteristic line in the x-y plane if the relation between m and ℓ is not dependent on x and y.

Both causes (i) and (ii) will be effective near the ship. The influence of finite wave amplitude a is treated in Lighthill's paper and in the work of Howe (1968) together with experiments. The action of basic flow non-uniformities has been investigated by Longuet Higgins (1961) by Ursell (1960b) and by Wijngaarden (1969). Their point is that small amplitude wave perturbation - and thus linearisation in particular - should be applied to the basic flow (arround the ship) rather than to parallel uniform flow relative to the ship. As far as non-uniformities of basic flow must be considered small themselves due to a thin ship assumption, this argument may be inconsistent.

Relation between $\varkappa$ and θ generalising (8) are subject to two requirements:

(i) The wave pattern must be stationary with regard to a system fixed to the ship

(ii) Waves travel with phase velocity c against the basic flow component normal to the wave front.

With basic flow components $\{-U+u,v,w\}$ and with $c(\varkappa,a)=\sqrt{g/\varkappa}(1+(\varkappa a)^2)$ up to higher order terms in a, we then must require that

$$\left|(-U+u)\cos\theta + v\sin\theta\right| = \sqrt{\frac{g}{\varkappa}}\left(1+(\varkappa a)^2\right) \tag{11}$$

instead of (8).

It is beyond the scope of this lecture to touch the question how the change of wave pattern can be found once a profile along the ship is given (see however the approach made by Inui, Kajitani and Okamura (1975)). From the viewpoint of wave analysis, referring to results derived later, the following insight is important:

(i) Even if (8) is not valid close to the ship, u, v and a will be small enough for sufficiently far away that this relation can be taken as a starting point for wave analysis. But functions $F(u)$, $G(u)$, $S(w)$, $C(w)$ need not to be derivable from some global spectral function $J(u,w)$.

(ii) It is only in case of one monochromatic wave train that the local flux of energy (or momentum) can be determined from the associated wave elevation directly. In general, only integral effects as the wave resistance R_w or the wave spectrum $H(\cos^{-2}\theta,\theta)$ can be determined. For the latter purpose, measurements for one single longitudinal wave profile will be found sufficient (see 2.3) if (8) holds.

(iii) It will be shown that the wave analysis method of Roy and Millard (1971) is justified under assumption of one monochromatic wave train subject to (8) rather than the more general wave model (5).

(iv) If $H(\cos^{-2}\theta,\theta)$ is not changing rapidly with θ for θ close to 0, then sufficiently far behind the ship, ζ is a slowly varying function of x for y constant and the truncation correction proposed by Tanaka and Adachi (1967) can be applied.

1.4 Other Wave Flow Models

Methods for predicting ship wave patterns based on ideas of Guilloton [16] and refined recently by Gadd [17], Standing (1974), Dagan [18] and Noblesse [19] are not explicity mentioned within the program of this seminar. But it seems that the distortion of wave patterns found through this type of approach - compared to our models (1), (2), (3) - has something in common with the distortion of characteristic lines due to non-uniform basic flow as considered under 1.3. We should, however, observe that so far we have rather artificially split up the flow into a basic component and a wavy one. Resulting from the Guilloton's method - as well as from Wehausen's [20] treatment of the ship wave problem by Lagrangian formulation - there is a distortion of the total flow field from time-integrated action of total flow. And no use is made of semi-heuristic concepts of wave spreading over non-uniform flow. A quantitative comparison of both types of approach seems desirable, but has not been achieved so far.

It has been found out experimentally by Adachi (1974) that at least for a ship with very long parallel middle body waves generated near the bow fade off much faster with increasing x distance than predicted by the Kelvin wave pattern model (6). Using the mathematical technique of matched asymptotic expansions, Adachi derived a flow model which explains this behavior. But it has obviously still to be clarified under what conditions and in which region his model has to be applied. And it is not clear how any type of wave analysis can be performed for determinimg resistance under these circumstances.

2. SOME METHODS OF WAVE ANALYSIS

2.1 Wave Resistance defined via Momentum Flux Integrals

There is a very general formula to express R_w in terms of flow components and wave profile at some control surface S enclosing the ship and cutting the undisturbed free surface vertically along a closed curve C,

$$R_w = \frac{\rho g}{2}\oint_C \zeta^2\,dy + \frac{\rho}{2}\iint_S \left(\varphi_x^2+\varphi_y^2+\varphi_z^2\right) n_x\,dS - \rho\iint_S \varphi_x\varphi_n\,dS \qquad (12)$$

The orientation should be taken such that both dy along C and n_x on S is positive behind the ship, i.e. the normal vector $\vec{n}=\{nx, ny, nz\}$ should be directed towards the ship in our case. Thus it seems to be desirable to express $\varphi_x, \varphi_y, \varphi_z$ on S in terms of ζ, ζ_x, and ζ_y along C. Even if S is a vertical cylinder, this is possible only in case of a single monochromatic wave field under (8); for the other flow models considered so far even the contribution to (12) from vertical integration cannot be found that way.

Let us take S as a rectangular cylinder generated through the intersection of four lines

A: $x=x_a>0$, B: $x=x_b<0$, C: $y=y_c>0$, D: $y=y_d<0$

Equation (12) simplifies then to

$$R_w = -\frac{\rho}{2}\int_{B-A} \zeta^2\,dy + \frac{\rho}{2}\iint_{B-A} \left(\varphi_y^2+\varphi_z^2-\varphi_x^2\right) dx\,dy + \rho\iint_{C-D} \varphi_x\varphi_y\,dz\,dx \qquad (13)$$

where in the double integrals vertical integration ranges from lower fluid boundaries up to the wave profile.

2.2 Transverse Cut Methods

Let us now in (13) either have y_c and y_d tend to infinity - or let $y_c=b/2$, $y_d=-b/2$ in case of a tank with width b. Then there is no contributions from C and D to (13), and as that from A must vanish for x_a sufficiently large, it must be zero for any x_a ahead of the ship. We are left with

$$R_w = \frac{\rho g}{2}\int \zeta^2(x_b,y)\,dy + \frac{\rho}{2}\iint\left(\varphi_y^2(x_b,y,z)+\varphi_z^2-\varphi_x^2\right)dy\,dz \qquad (14)$$

To our degree of approximation, it is pertinent to drop the contribution of the wave elevation to the z-integral. It is then possible for flow models (2) and (3) to express R_w in terms of the functions F(u), G(u) which in turn can be determined from the wave profile along B. Inserting (2) or (3) into (14) we obtain

$$R_w = \frac{\rho U^2}{32\pi k_o^2}\int_{-\infty}^{\infty}\left\{F^2(u)+G^2(u)\right\}\frac{2S^2(u)-1}{S^2(u)}\,du \qquad (15a)$$

$$R_w = \frac{\rho U^2}{32\pi k_o^2}\sum_{\nu=-\infty}^{\infty}\left\{F^2(u_\nu)+G^2(u_\nu)\right\}\frac{2S^2(u_\nu)-1}{S^2(u_\nu)}\,\Delta u \qquad (15b)$$

with $\Delta u=2\pi/k_o b$ in case of a ship amidst a tank. (15) may be writen in sligtly different notation, valid then even for finite water depth h. It reflects then a generalisation to three dimensions of Lamb's [21] explanation of wave resistance as increase of wave energy ahead of B minus energy transport with group velocity component normal to B:

$$R_w = \frac{\rho g b}{2}\sum_{\nu=-\infty}^{\infty}\left(a_\nu^2+b_\nu^2\right)\left(1-r_\nu\cos^2\theta_\nu\right) \qquad (16)$$

with $a_\nu=F(\nu\cdot\Delta u)\Delta u/k_o$; $b_\nu=G(\nu\cdot\Delta u)\Delta u/k_o$; $\cos\theta_\nu=1/s(\nu\cdot\Delta u)$

$$r_\nu = \frac{\sinh(2\varkappa_\nu h)+(2\varkappa_\nu h)}{2\sinh(\varkappa_\nu h)} = \text{ratio of group velocity to phase velocity}$$

where $\varkappa_\nu$ is the root of $\varkappa_\nu s^2(\nu\Delta u)=k_o\tanh(\varkappa_\nu h)$.

From (16) it can be concluded that the wave resistance associated with some wave pattern (3) is smaller if the tank has shallow depth! Expressions for sidewise unrestricted water follow through analogy between Fourier integrals and Fourier series. It is possible to find F(u) and G(u) from ζ and wave slope ζ_x along B. In praxi, several cuts are taken and only wave height is measured. Redundant data serve to smooth out errors in the sense of least square fit.

2.3 Longitudinal Cut Methods

If in (13) we let x_b tend to minus infinity, assuming symmetry of flow regarding the plane y=0, we obtain

$$R_w = 2\rho\iint_C \varphi_x\varphi_y\,dz\,dx \qquad (17)$$

where the contribution of the wave elevation to z-integration may be also deleted. We shall show that (17) can be evaluated in terms of ζ or ζ_x or ζ_y along C.

If a global representation (1) holds for the wave pattern, it would do in principle to use (15) and determine F(u) and G(u) from the profile via (5a). However, though a longitudinal cut wave analysis should lead to the same R_w as (15), the Fourier transforms S(w) and C(w) must not necessarily be connected to F(u), G(u) obtained from a transverse cut if non-linear wave distortion effects near the ship must be expected. Discrepancies were actually found in ESW. We shall therefore rather start from (17). It is remarkable that (17) can be expressed through wave characteristics along C under two simple assumptions for the flow in the plane $y=y_c$, $z<0$ namely

$$\varphi_{xx}+k_o\varphi_z = 0 \quad \text{(heat flow differential equation)} \qquad (18a)$$

and

$$\varphi_{xx}+\varphi_{yy}+\varphi_{zz} = 0 \quad \text{(Laplace equation)} \qquad (18b)$$

Due to (18), under very general conditions φ and its derivatives may be expressed through values along z=0, namely

$$\varphi(x,y_c,z) = \sqrt{\frac{k_o}{\pi|z|}}\int_{-\infty}^{\infty}\varphi(x',y_c,0)\,e^{k_o\frac{(x-x')^2}{4z}}\,dx' - \frac{k_o}{\pi}\int_{-\infty}^{\infty}\varphi(x',y_c,0)\int_{-\infty}^{\infty}e^{k_o w^2 z}\cos w k_o(x-x')\,dw\,dx' \qquad (19)$$

This means that the x-Fourier transforms of φ and its derivatives depend on z just via a factor $e^{k_o w^2 z}$. If we define

$$X(w,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi_x(x',y_c,z)\,e^{ik_o w x'}\,dx'$$

$$Y(w,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi_y(x',y_c,z)\,e^{ik_o w x'}\,dx'$$

we must accordingly have

$$X(w,z) = X(w,0)e^{k_o w^2 z},$$

and

$$Y(w,z) = Y(w,0)e^{k_o w^2 z}, \qquad (20)$$

and through Parsevall's theorem we find from (17)

$$R_w = -2\rho \int_{-\infty}^{\infty} \int_{-\infty}^{0} q_x q_y \, dz \, dx$$

$$= 2\rho k_0 \operatorname{Re} \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{0} X(w,0)\, Y(-w,0)\, e^{2k_0 w^2 z} \, dz \, dw \right\}$$

$$= \rho \operatorname{Re} \left\{ \int_{-\infty}^{\infty} X(w,0)\, Y(-w,0)/w^2 \, dw \right\} \qquad (21)$$

At first glance it may appear that we have to determine X(w,O) <u>and</u> Y(w,O) in order to evaluate (21). However from (18b) (and from reasoning that R_w should come out positive) we may deduce that $Y(w,0) = \sqrt{w^2-1}\, X(w,0)$, or more general, as $X(w,0) = k_0 u \int_{-\infty}^{\infty} \zeta(x,y_c) \cdot e^{ik_0 wx} \, dx$:

$$XX(w,0) = k_0 U \int_{-\infty}^{\infty} \zeta_x(x,y_c)\, e^{ik_0 wx} \, dx = iwk_0 X(w,0).$$

$$XY(w,0) = k_0 U \int_{-\infty}^{\infty} \zeta_y(x,y_c)\, e^{ik_0 wx} \, dx = iwk_0 \sqrt{w^2-1}\, X(w,0) \qquad (22)$$

It can be further seen that there is no contribution to (20) from the range $-1 \leq w \leq 1$. From (5) it is apparent that X(w,z) should have a steep rise due to a factor $1/\sqrt{w^2-1}$ near w=1; this has been verified experimentally by Ikehata and Nowaza (1967). Chen (1973) observed the analogue in case of finite depth.

Now for optimum evaluation of (20) one may ask if ζ or ζ_x or ζ_y should be subject to measurement. This question was investigated by Michelsen and Uberoi (1971) who transformed (20) to include autocorrelation functions between these quantities (see Gadd [6]).

The requirement that the quantity to be measured should decay fast with x (to keep records short and to avoid tank wall reflection effects) is counteracted by the desire to have the Fourier transform intensity concentrated near w=1 (i.e. long waves). Lee (1969) concluded from numerical investigation that ζ should have be preferred.

2.4 Longitudinal Cut Methods for Tank Wave Systems

Transverse cut methods are permissible in quite narrow tanks in principle, provided there is ideal reflection along the walls. But there are two shortcomings of these methodes in compense:

(i) If not taken simultaneously, data must be collected in a reference system moving with the ship.

(ii) Part of the wave profiles extends over domain of the viscous wake where the basic assumption of no vorticity is invalided; furthermore the lengthwise basic flow component relative to the ship will markedly deviate from -U there.

For this reasons methods have been devised for determining the coefficients $a_\nu = F(u_\nu)\Delta u/k_0$ and $b_\nu = G(u_\nu)\Delta u/k_0$ from a wave pattern (3a) up to some adequate limit $|\nu| = \nu_{max}$ from one or more longitudinal wave records of finite length. (We should observe that even if a cut is nowhere crossed by wake flow, the wave profile will contain components which have passed the wake after being reflected at the opposite tank wall if the record is not short (Maruo and Hayasaki (1972)).

For one cut only, having a proper sidewise location $y=y_c$ such that for $\nu < \nu_{max}$ $\cos u_\nu y_c$ ($u_\nu = 2\pi\nu/k_0 b$) is not too small (in particular for $y_c = b/6$!) Eggers (1962) suggested a straightforward determination through

$$a_\nu = \lim_{T\to\infty} \frac{1}{T} \int_{x_0}^{\infty} \zeta(x,y_c) \frac{\sin k_0 w_\nu x}{\cos u_\nu y_c} \, dx$$

$$b_\nu = \lim_{T\to\infty} \frac{1}{T} \int_{x_0}^{\infty} \zeta(x,y_c) \frac{\cos k_0 w_\nu x}{\cos u_\nu y_c} \, dx$$

if the record begins at $x=x_0$. Such procedure should even eliminate local wave components of monotonic decay.

Some exploratory numerical experiments made clear that the above limiting process shows poor convergence if applied to some mathematical wave profiles. This can be understood from the fact that the set of wave lengths $\lambda_\nu = 2\pi/s(u_\nu)$ is too "dense" asymptotically as to allow only one unique expansion of a given continuos profile in terms of free wave components (3a) over a semi-infinite domain [22].

Landweber and Moran (1972) took this problem up again and succeded in determining the set of a_ν, b_ν by the method of least square fit to Fourier-integrals over finite records. Their approach was modified by Tsai (1972) with the aim of reducing the sensibility of results regarding the choice of longitudinal cut location. He studied the influence of cut location, of record length and of ν_{max} within his numerical and experimental work.

An independent path has been followed by Hogben (1975) in refining his matrix method (1972). With ν_{max} given, the set of a_ν, b_ν is determined now by least square fit to a set of <u>pointwise</u> wave measurements along four longitudinal cuts. Both approaches have require of solving a system of linear equations so that their efficiency depends on numerical techniques as well as instrumentation. Tsai proposed a mode of combining iteration and elimination.

One may feel disturbed by one special feature of above methods. For a given set of more than $2\nu_{max}+1$ data, i.e. for measurements with redundancy, needed for a least square fit, any variation of ν_{max} will in principle affect the determination of all a_ν, b_ν including the long wave components, - this does not occur in case of transverse cut methods, where the data set is selected "orthogonal". This point has been raised by Sharma in a discussion to Hogben and Standing (1974). - We should admit, however, that the situation is not basically different in case of longitudinal cut methods described under 2.3 for sidewise unrestricted water. Any extension of the record length for improved approximation of long wave components will in reciprocity here affect the high frequency spectral range as well.

2.5 Ward's X-Y-Method and related Topics

If we could find two measurable quantities p(x) and q(x) along the cut $y=y_c$ with Fourier transforms

$$P(w) = 1/2\pi \int_{-\infty}^{\infty} p(x)\, e^{ik_o wx}\, dx \quad \text{and}$$

$$Q(w) = 1/2\pi \int_{-\infty}^{\infty} q(x)\, e^{ik_o wx}\, dx$$

such that we have

$$X(w,0)\; Y(-w,0) = w^2\, P(w)\, Q(-w) \tag{23}$$

then it is possible to express R_w in terms of p(x), q(x) directly without any use of Fourier transforms! Using Parsevall's theorem in opposite direction, we obtain from (21) under (23)

$$R_w = \rho \int_{-\infty}^{\infty} p(x)\, q(x)\, dx \tag{24}$$

One particular pair of such functions is

$$p(x) = \int_{-\infty}^{0} \varphi_{xx}(x,y_c,z)\, dz \qquad q(x) = \int_{-\infty}^{0} \varphi_{xy}(x,y_c,z)\, dz$$

$$P(w) = iw\, X(w,0)/w^2 \qquad Q(w) = iw\sqrt{w^2-1}\; Y(w,0)/w^2 \tag{25}$$

This is the rationale of Ward's X-Y-method, based on the approximation that forces acting on a vertical circular cylinder have components proportional to $\varphi_{xx}(x,y_c,z)$ and $\varphi_{xy}(x,y_c,z)$ in x- and in y-direction. It may be shown (private communications by Sharma) that in some average sense we even have

$$\int_{x_b}^{\infty}\int_{-\infty}^{0} \varphi_x \varphi_y\, dz\, dx \approx \int_{x_b}^{\infty}\int_{-\infty}^{0} \varphi_{xy}\, \varphi_{xx}\, dz\, dx \,/\, k_o^2 \tag{26}$$

if $-x_b$ is sufficiently large - however, this does not imply

$$\int_{-\infty}^{0} \varphi_x\, \varphi_y\, dz = \int_{-\infty}^{0} \varphi_{xx}\, \varphi_{xy}\, dz/k_o^2$$

under wave flow model (5) and (19). Such equivalence does, however, hold in a monochromatic wave field in the area behind the ship where $\cos\theta(x,y)$ is close to 1.

Sharma found that by one- and twofold integration of the wave elevation $\zeta = U/g\varphi_x$ and transverse wave slope $\zeta_y = U/g\varphi_{xy}$ another pair of functions satisfing (22) can be found, namely

$$p(x) = k_o^2 \int_{-\infty}^{x}\int_{-\infty}^{x} \varphi_x(x)\, dx\, dx$$

$$q(x) = k_o \int_{-\infty}^{x} \varphi_{xy}(x)\, dx\ ,$$

certainly without suggesting practical application.

But there is one practical conclusion which we may draw from our reasoning: If we want to determine wave resistance through flow measurements - and using Laser techniques this may be as simple as measuring wave profiles - there is no need to measure φ_x or φ_y at z=0, where we must expect disturbances through wave troughs where no fluid will be present. We can measure at one arbitrary depth $z=z_o<0$ and still find Fourier transforms for z=0 through (20); analogue reasoning holds for transverse cut analysis.

If the water depth h is finite, most of above conclusions remain valid, but in spite of (20) the influence of z on Fourier transform is through a factor $\cosh\kappa(z+h)\,/\,\cosh(\kappa h)$ instead of $e^{k_o w^2 z}$, where κ is the positive root of

$$k_o w^2 = \kappa \tanh(\kappa h).$$

But it is obvious that Ward's X-Y-method is no longer justified if h is small. Longitudinal cut wave analysis for finite water depth has been investigated in theory and experiment by Chen (1973).

As mentioned earlier, the quantity $\int_{-\infty}^{0} \varphi_x\, \varphi_y dz$ - i.e. the flux of momentum through a vertical line - cannot be expressed in terms of ζ, ζ_x and ζ_y in case of a flow model (19). But this is possible in case of one single slowly varying wave train under (8)! If we have $\kappa = k_o \cos^2\theta$, assuming

$$\varphi = \Big\{ A(x,y_c)\cos\kappa(x\cos\theta + y_c\sin\theta) + B(x,y_c)\sin\kappa(x\cos\theta + y_c\sin\theta)\Big\}\, e^{\kappa z}$$

where A, B and $\varkappa$ are slowly varying functions of x and y, we may approximate R_w as

$$R_w \sim \rho g \int_{-\infty}^{\infty} \zeta^2(x,y_c) \sin(2\Theta(x,y_c))\, dx \qquad (27)$$

This result may serve as a basis for the wave analysis method proposed by Roy and Millard (1971). It is, however, not evident how this formular could be justified for more general wave patterns (5) or at least for two monochromatic systems occuring simultaneously, as we must expect in case of a Kelvin pattern. Even if divergent waves could be disregarded at low Froude numbers, an interaction of waves from bow and from stern already invalidates the underlying assumption. (It is certainly true that sufficiently behind the ship Kelvin wave patterns from bow and from stern add up approximately to one single Kelvin pattern, for which the origin cannot be defind precisely).

On the other hand, (27) may be generalised to the case that the relation between $\varkappa$ and Θ is affected through local inhomogeneities as presented in (11),provided the wave length and thus $\varkappa$ can be measured simultaneously with ζ and Θ . We obtain the more general formula

$$R_w \approx \rho k_0 g \int_{-\infty}^{\infty} \frac{\zeta^2(x,y_c) \tan\Theta(x,y_c)}{\varkappa(x,y_c)}\, dx \qquad (28)$$

if we still can assume that the relation between ζ and φ_x is

$$\zeta(x,y_c) = U/g\varphi_x(x.y_c,0).$$

3. CONSIDERATIONS ABOUT SOME SPECIAL TOPICS

3.1 On Decay of the Local Components in Wave Patterns

The error in wave analysis due to contamination of records through local-wave components has been investigated along two lines, both reported upon in ESW: .

(i) numerical experiments (recently extended by Lee (1969)).

(ii) by comparison of resistance obtained from analysis along different cuts (see also Ikehata and Nowaza (1967)).

The numerical evaluation of local flow components is admittedly tedeous, different Froude numbers require separate calculations. Not much has been done along another line, i.e. calculation of the amount by which the differential operator applied to φ leads to non-zero values. Applied to potentials (2), (3) and (5), this functional is a mapping on zero even for z unequal to 0, thus $\varphi_{xx}+k_0\varphi_z$ clearly is a measure for local-flow intensity.

Calculations are easy for that generated by source distribution, as for a single source potential G, singular like inverse distance 1/r, this operator leads to a rational function, dependent on speed only through a linear factor. We have [23]

$$G_{xx}+k_0G_{zz} = (1/r + 1/r_1)_{xx} + k_0(1/r-1/r_1)_z$$

where r_1 is the distance from the image point with regard to z=0.

The right hand side falls off like y^{-3} (the first term even like y^{-5}) sidewise, and in lengthwise direction the decay is like x^{-3} . For distibuted sources, integrating (29) through some quadrature formula will display the same rates of decay.

A very simple check on local-wave decay can be performed just by inspection, at least if the ship is symmetric to the main section. In this case linear theory predicts a fore and aft symmetry of the local wave elevation. This means that the local wave profile along a transverse cut must equal the total wave elevation along the image cut ahead of the ship, where no free waves occur. And the sidewise decay along the cut must be as well equal that of the total waves on the image cut. - It is true that for real ships part of their form is distributed in antimetry to the main section and the influence of Froude number on source distribution will spoil such symmetry anyway - but this means only a slight modification to above rule.

More elaborate numerical investigations of local-wave influence to transverse cut wave analysis were reported by Landweber and Tsou (1968).

3.2 Determination of "Equivalent Singularity Systems" from Wave Analysis Results

Classical methods for deriving wave flow and wave resistance for a given hull form have one common feature: As a first step - at least implicitely - a system of singularities (sources and sinks in general) has to be determined which has to stand on the ship's stead with regard to her wave making. Considering shortcomings of wave resistance theories, it may be found tempting to try determining these singularities from observed patterns rather than from the ship geometry data. Under thin-ship assumptions, such singularities depend on speed only through a linear factor and are not affected by change in water depth.

One merit of such investigations is that they show up limits of the range where the underlying hypotheses are sound (Everest and Hogben (1968)) and where not. In particular they may display invalidity of linear wave models (without distortion) if they lead to singularities outside (ahead of) the volume displaced by the ship (Hogben (1970)). Hogben evaluated free waves behind the ship for determining of equivalent source arrays along some line in lengthwise direction. In a broader sense we should also mention here measurements of waves (and flow) alongside the ship using more general wave model (1) and allowing greater freedom for the source distribution, namely determining sources all over the ship's center plane. This is the approach of Mori, Inui and Kajitani (1972) and Inui (1974).

Another motive for determining source distributions from wave pattern is to bypass the need to avoid local-flow effects in the region where measurements are taken (Sabuncu (1969) for transverse cuts, Baba (1973) for longitudinal cuts) and thus measure waves closer to the ship. For truncated longitudinal cuts this may lead to some analytic continuation of the wave profile to infinity (Ikehata and Nowaza (1968)) or of the spectral function towards $\theta = 0$ (Bessho (1969) derived therefrom).

With locus of singularities prescribed along a given line, their intensities may certainly be determined uniquely by postulating least square fit to a given set of data. Thus Sabuncu found that the wave profile of one single transverse cut is needed for determining equivalent (antisymmetric) sources along the axis of a submerged slender body of revolution.

However, we have seen earlier that even with local waves disregarded there is no redundancy left if we have measured two transverse cuts or one longitudinal cut. If waves are polluted by local flow components, one should therefore expect that more data must be at hand, at least if we could not anticipate that wave spectra should be smooth and well behaved.

In case of a ship wave pattern, there appears some arbitrariness in selecting z_o, the depth of submergence for singularities. However, z_o governs the rate of exponential decay of $F(u)$, $G(u)$ with large u, and from linear ship wave theory, no such decay can be expected for a floating ship. It may be argued, nevertheless, that the ultimate range of divergent waves affected is unimportant for wave analysis anyway.

A more fundamental caveat against such equivalent source arrays along one line may be based on postulates of one-to-one correspondance. Bessho [24] has shown that one and the same far-field wave pattern may result from very different systems of singularities. This is already obvious if we recall that a wave pattern (1) already under the weak restriction $J(u, s(u))=0$ does not display any far-field waves, but by no means needs to be small. - It is evident that promoters of 'equivalent source array' concepts are as well aware of all limitations mentioned so far, we want to make sure that their followers do.

ACKNOWLEDGEMENT

This paper was completed within the research programm of the Sonderforschungsbereich 98, Schiffstechnik und Schiffbau, sponsored by the Deutsche Forschungsgemeinschaft. The author feels happy to let know that Mr. Hang Shoon Choi gave valuable assistance in composing and editing this paper.

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