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Nonlinear computation of cables with high order solid elements using an anisotropic material model

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Due to their complex structure cables exhibit an anisotropic behaviour and undergo large deformations in various applications. The large deformations are simulated using blended high order solid elements that can represent large deformations efficiently. To reduce the computational complexity the cross section consisting of many single wires and different layers of material is homogenized and represented by a transversely isotropic material model. Frictional effects and possible reordering of the parts are modelled through elastoplastic material behaviour with an anisotropic yield function. The simulations show that for a simple tension test and a free torsion test the material parameters can be satisfactorily identified.

1 Introduction & model assumptions

In the modern world we are surrounded by cables in various applications. For instance, they serve as structural elements (in bridges and ski lifts), feed lines in maritime applications, or as electrical conductors to transmit energy or signals. In some applications, like signal-cables in robotics, large displacements combined with small strains can occur. In other situations, e.g. the installation of cables, large local deformations can arise due to contact when pulled over edges.

Since electrical cables are a conglomerate of conductive wires (twisted or straight), an insulation, and a rubber coating, the mechanical behaviour is composed of hyper-elasticity, elastoplasticity, friction between the strings, geometrical effects, and reordering of the parts. The consideration of these effects leads to a huge computational effort, if simulated in detail. To decrease the computational cost, a single elastoplastic material model is used to describe the mechanical behaviour. To compensate for the simplifications of this approach, the material model is considered to be anisotropic in both the hyper-elastic as well as the elastoplastic regimes [4].

Since the model has to be able to deal with large strains a hyper-elastic model based on an orthotropic strain energy density function is applied. The orthotropy is imposed through structural tensors (${}_i\boldsymbol{M}_e = {}_i\boldsymbol{v}_e \otimes {}_i\boldsymbol{v}_e$), that are constructed by the privileged directions ${}_i\boldsymbol{v}_e$. Having defined the structural tensors, a set of invariants $J_i = \mathrm{tr}[{}_i\boldsymbol{M}_e\boldsymbol{C}_e]$ and $J_{i+3} = \mathrm{tr}[{}_i\boldsymbol{M}_e\boldsymbol{C}_e^2]$ for $i = 1, 2, 3$, where $\boldsymbol{C}_e$ is the right Cauchy green tensor of the elastic deformations, is used to describe the strain energy density function [4],

$$\Psi_e = \sum_{i=1}^{3}\left[\alpha_i J_i + \frac{1}{2}\sum_{j=1}^{3}\alpha_{(ij)}J_iJ_j + \alpha_{i+9}J_{i+3}\right], \tag{1}$$

with $\alpha_{(11)} = \alpha_4$, $\alpha_{(22)} = \alpha_5$, $\alpha_{(33)} = \alpha_6$, $\alpha_{(12)} = \alpha_{(21)} = \alpha_7$, $\alpha_{(13)} = \alpha_{(31)} = \alpha_8$, and $\alpha_{(23)} = \alpha_{(32)} = \alpha_9$ ($\alpha_{(ij)}$ is only used for a compact notation). The parameters $\alpha_1 - \alpha_{12}$ denote material constants, that can be computed from the set of Youngs moduli (E_i), shear moduli (G_{ij}), and Poisson ratios (ν_{ij}) for all three directions, hence 9 parameters.

In this work only cables with parallel wires and coaxial structures are considered. This leads to the assumption, that the cross section exhibits an isotropic behaviour. Whether this assumption is also feasible for twisted cables will be analysed in future work. This assumption leads to a transversely isotropic material model and reduces the amount of parameters to five in addition to the preferred direction, which is in this case always the axial direction of the cable. Hence the preferred direction is denoted by $i = 1$ and the remaining material parameters are E_1, $E_2(= E_3)$, $G_{12}(= G_{13})$, $\nu_{12}(= \nu_{13})$, and ν_{23} ($G_{23} = \frac{E_2}{2(1+\nu_{23})}$).

The kinematics for large inelastic strains are based on the concept of the multiplicative split of the deformation gradient $\boldsymbol{F} = \boldsymbol{F_e}\boldsymbol{F_p}$. The incompressibility criterion of the plastic deformation gradient $\det\boldsymbol{F_p} = 1$ can be fulfilled exactly by using the exponential integration scheme [3].

While the elastoplastic behaviour is described by a non-linear isotropic hardening function Y, the von Mises like yield function ϕ introduces anisotropy, which leads to different yield stresses for different load situations,

$$\phi = \sqrt{\frac{2}{3}}[\sigma_{11}^y\sqrt{\chi} - (\sigma_{11}^y - Y)], \tag{2}$$

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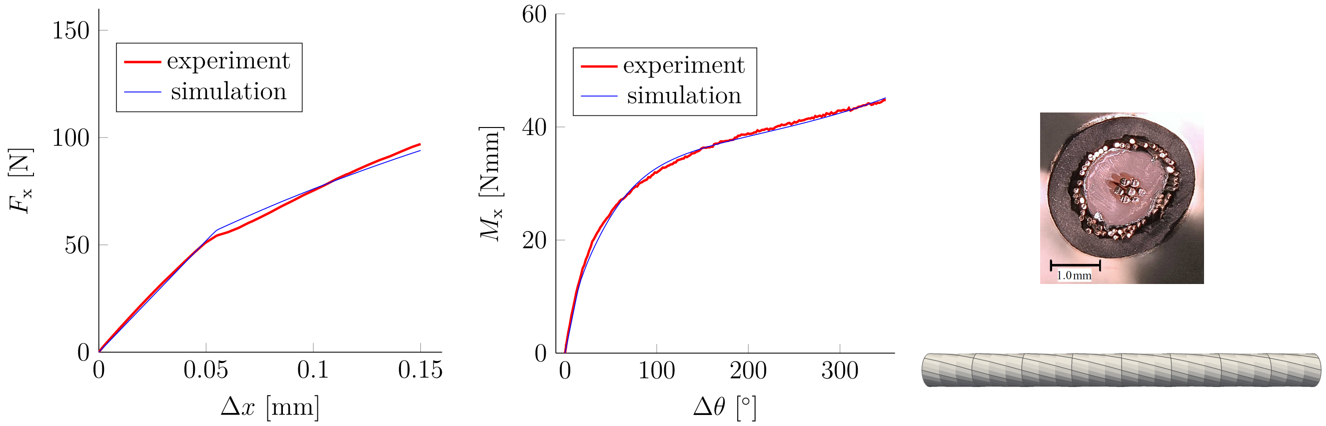

Fig. 1: Experimental data of a coaxial cable (top right) [1] and simulation results for the anisotropic material model: tension test on the left, torsion test in the middle, and deformation of FE model under torsion on the right bottom.

where σ_{11}^y is the yield stress in direction 1 and χ denotes the equivalent stress, which is an anisotropic function of a suitable deviatoric stress tensor,

$$\chi = \sum_{i=1}^{3} \left[\beta_i I_i^2 + \beta_{i+6} I_{i+3} + \frac{1}{2} \sum_{j=1}^{3} \beta_{i+j+1} I_i I_j \right] . \tag{3}$$

$I_1 - I_6$ are invariants of the deviatoric stress tensor combined with the structural tensors ${}_i\boldsymbol{M}_e$ and $\beta_1 - \beta_9$ are material constants derived from six independent yield stresses (three independent yield stresses for transverse isotropy). The structural tensors for the yield function are chosen to be the same as for the hyper-elastic anisotropy, but could be chosen independently in theory. A detailed description of the model including the non-linear isotropic hardening function, the associative flow function for the plastic strain, and how to compute all parameters can be found in [4, 5].

2 Experiments and simulations

In Fig. 1 the results of two experiments with a coaxial cable (top right) [1] and the results of corresponding simulations can be seen. On the left-hand side the force-displacement curve of a simple tension experiment is pictured. The cable length was $L = 32$ [mm] and the diameter $D = 2.8$ [mm]. On the middle plot the moment-rotation curve for a torsional experiment with free torsion is shown. The diameter is the same as in the tension experiment and the length is $L = 33$ [mm]. Free torsion means that one side of the cable can freely move in axial direction, such that only negligible axial forces appear in this experiment. For the simulations blended solid hexahedral elements of order $p = 3$ with hierarchic shape functions [2] were used to compute the load displacement curves for 200 equidistant load steps.
The identified material parameters are $E_1 = 5363.3$ [MPa], $G_{12} = 249.52$ [MPa], $\sigma_{11}^y = 9.262$ [MPa], and $\sigma_{12}^y = 2.329$ [MPa]. The difference between Youngs modulus in axial direction and the shear modulus G_{12} clearly indicates that this is not achievable with an isotropic material, since the Poisson ratio would have to be $\nu = \frac{E_1}{2G_{12}} - 1 \approx 9.75$. Since for an isotropic setting a Poisson ratio greater than 0.5 denotes an increasing volume when deformed, this is clearly not realistic. The results show, that the uniaxial tests can be captured by a transversely isotropic material model in an acceptable accuracy. For validation of the material parameters and identification of restrictions of the model additional simulations have to be conducted. For this purpose free bending experiments, as well as three point bending and combined load situations, as tension + torsion, tension + bending, and torsion + bending should be analysed. For torsion + bending though the buckling modes are likely to be dependent on small local imperfections, which leads to the assumption that for this combination local anisotropies in the cross-section can be crucial to reproduce accurate deformations.

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