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Influence of surface roughness on the tangential coefficient of restitution in particle-particle collisions

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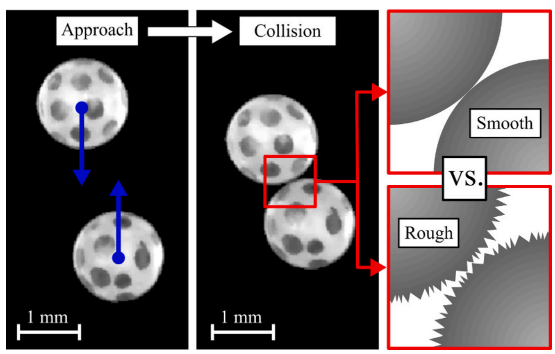
HIGHLIGHTS

- Collision experiments between two spherical particles.
- Adjustment of surface roughness of collision partners.
- Increased scatter in the tangential restitution coefficient with higher roughness.
- General dependence of tangential restitution on collision angle is not influenced.

GRAPHICAL ABSTRACT



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ABSTRACT

In this experimental study, two highly spherical 1.5 mm sized zirconia particles are brought into collision in free space at moderate impact velocities and at different collision angles. The collision is recorded by two synchronized high-speed cameras and the particle movements are analyzed in three dimensions. The focus of the investigation is on the tangential restitution coefficient, which depends on the collision angle. Several series of experiments are carried out with varying surface roughness of the particles adjusted by mechanical treatment. It is shown that even at relatively low surface roughness levels, the tangential restitution coefficient tends to exhibit strong inconsistency in the form of scattering, which increases with increasing surface roughness. Nevertheless, the fundamental dependence on the collision angle remains unaffected by the parameter variations carried out in this study.

1. Introduction

In bulk solids processes, such as mixers, fluidized beds, or spray dryers, collision events occur continuously between individual particles or between particles and the walls of the apparatus, tools, or fixtures. The knowledge of the normal and tangential restitution coefficients is crucial for the correct design and simulation of such equipment and processes. The parameters enable the description of particle collisions, in particular the prediction of particle motion after impact, as well as the dissipation of kinetic energy that occurs [1].

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Numerous studies on the collision dynamics of particles can be found, whereby the focus of most investigations has been set on the normal restitution coefficient, e.g. [2–4]. It is known that the normal restitution coefficient depends on both the material properties and the system and collision conditions, such as surface roughness and collision velocity [5–14]. The tangential restitution coefficient, on the other hand, is much less studied so far. This is partly due to the fact that, in addition to translational motion, it also includes rotational motion, whereas the normal restitution coefficient only considers translational motion components. The exact quantification of rotation is comparatively complicated [15]. Although there are experimental studies, e.g., [15–21], and simulative studies as well as modeling approaches, e.g., [22–26], that include or focus on the tangential restitution coefficient in their analysis for spherical or non-spherical particles, the state of knowledge regarding the influence of material, system, and collision conditions on the tangential restitution is still limited.

This study aims to experimentally investigate the effect of surface roughness on the tangential coefficient of restitution. As already mentioned, to date, experimental studies on the influence of roughness on collision dynamics have focused primarily on the normal coefficient of restitution. They have been investigated almost exclusively through collisions of particles with a flat plate having various surface roughness [7,8]. Since the normal coefficient of restitution is treated as independent of the collision angle, this approach is logical and sufficient. However, this is not the case for the tangential coefficient of restitution. Tangential collision dynamics depend on the collision angle, i.e., how oblique the collision is [24]. Experimental collision studies involving variations in the collision angle exist both between particles and a plate [16,17] and between two particles in a pendulum setup [27] or in free space [15,18], which is by far the rarest and most complex, but also the most realistic form of investigation. However, these studies have not yet included a systematic variation of surface roughness, which the present study aims to address. To this end, two spherical particles are brought into collision in free space at different collision angles at moderate speed, which is characteristic for, e.g., fluidized bed systems. The surface of the particles has varying degrees of roughness. For the experiments, hard material with almost ideal elastic collision behavior is used in order to exclude additional influencing factors.

2. Experimental setup and procedure

In this study, two spherical particles are shot against each other with the experimental setup shown in Fig. 1, causing them to collide. The collision is achieved by dropping one particle using vacuum tweezers while a second particle is accelerated upwards toward the first particle using a compressed air-powered particle accelerator. The vacuum tweezers and particle accelerator are mounted on positioning systems that enable the particle trajectories to be aligned with each other. The vacuum tweezers and particle accelerator are triggered by solenoid valves, which are coordinated in time by a control system. This ensures that the spheres collide in the field of view of two high-speed cameras. The cameras are arranged horizontally at a 90° offset to enable three-dimensional evaluation of the particle movement before and after the collision. The cameras are synchronized and record at a frame rate of 5100 fps. A powerful, DC-powered lighting system arranged around the collision zone provides the necessary intense, uniform illumination for high-speed recordings. Fig. 2 shows an example of a collision between two particles from both camera perspectives. At the moment of collision,

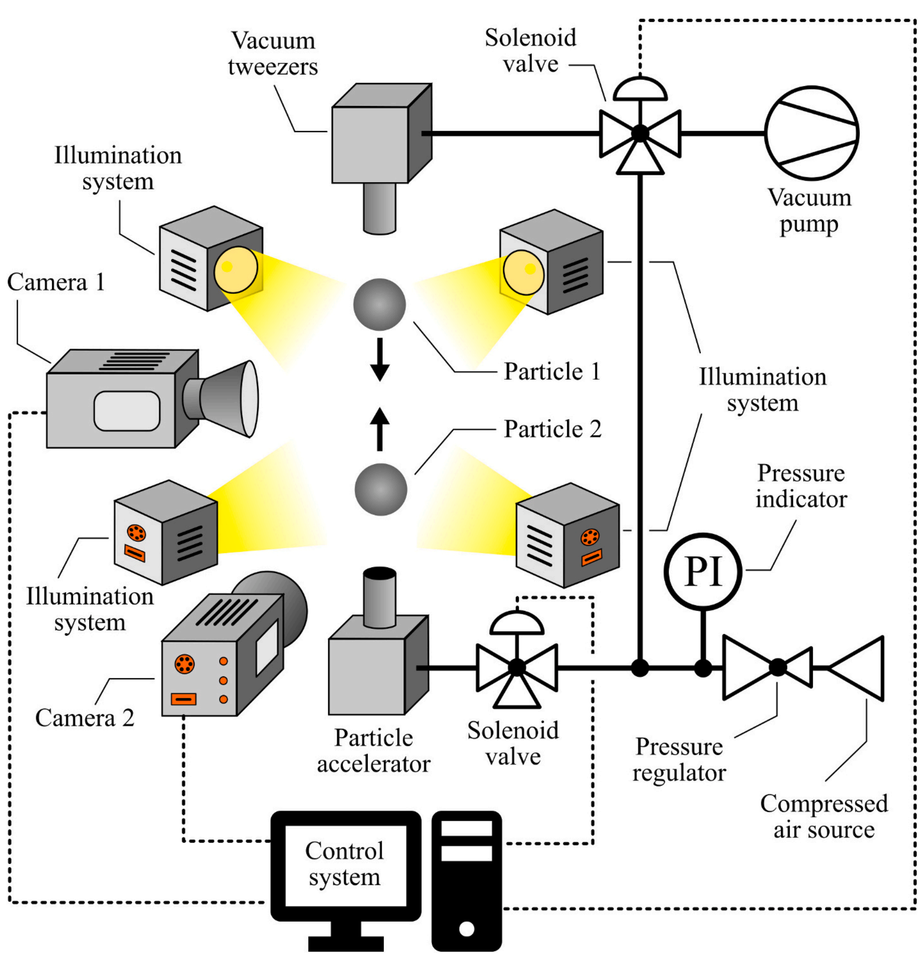

Fig. 1. Experimental setup for conducting and recording interparticle collisions. Details can be found in [15].

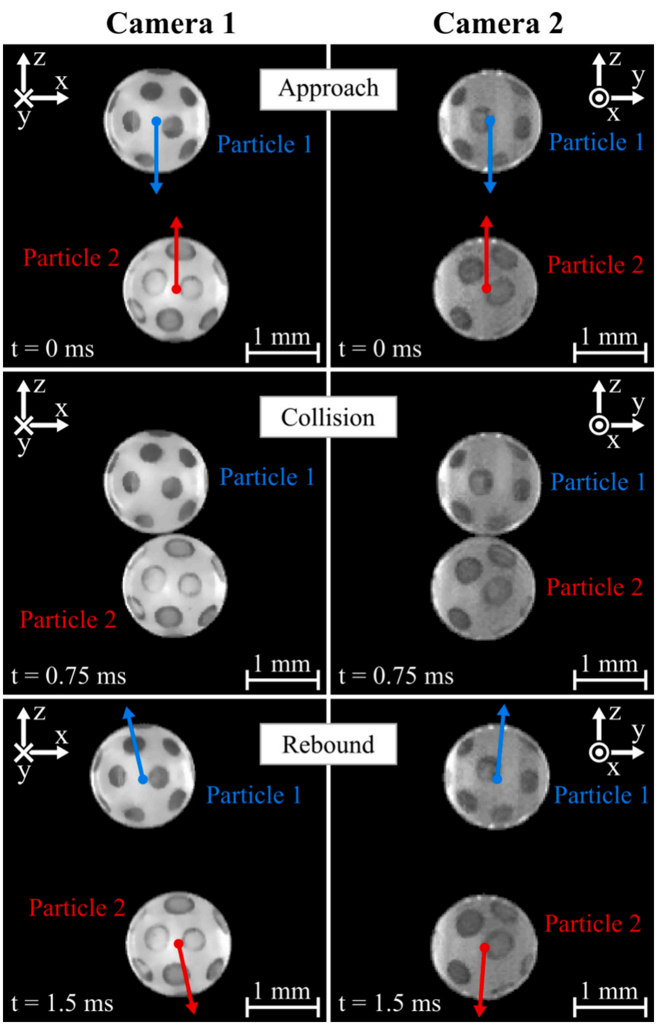

Fig. 2. Example of a recorded collision between two spherical particles with a diameter of 1.5 mm from both camera perspectives.

the spheres have a relative vertical velocity of 1.55 ± 0.18 m/s and have almost no initial rotation. The particle velocities were selected to correspond to typical velocities in fluidized and spouted beds [28].

The camera recordings are evaluated using digital image analysis methods to determine the position, translational velocity, and rotational movement of both particles before and after the collision. For the analysis of rotation, gray dot-shaped markings are applied to the white particles, which provide good contrast against the black background. The image analysis procedure to determine the particle velocities and rotation, a thorough analysis of the uncertainties associated with the measurement methods as well as more detailed information on the experimental setup are described in our previous work [15].

The tangential restitution coefficient β is determined based on the motion data extracted from the images:

$$\beta = -sign\left(\vec{v}_t^{\,\prime} \bullet \vec{v}_t\right) \bullet \frac{\left|\vec{v}_t^{\,\prime}\right|}{\left|\vec{v}_t\right|}. \tag{1}$$

The tangential coefficient of restitution represents the sign-adjusted ratio of the tangential relative velocity of both spheres after the collision $\vec{v}_t^{\,\prime}$ to the tangential relative velocity before the collision $\vec{v}_t$. The tangential relative velocity is determined based on the slip velocity at the point of contact $\vec{q}$, which is calculated from the translational velocities $\vec{v}_i$, the rotational velocity $\vec{\omega}_i$, and the diameters d_i of the colliding particles:

$$\vec{q} = \vec{v}_1 - \vec{v}_2 - \left(\frac{d_1}{2} \bullet \vec{\omega}_1 + \frac{d_2}{2} \bullet \vec{\omega}_2\right) \times \vec{n}. \tag{2}$$

The normal unit vector $\vec{n}$ points from the center of the second particle toward the center of the first particle and is determined from the particle center positions $\vec{s}_i$:

$$\vec{n} = \frac{\vec{s}_1 - \vec{s}_2}{\left|\vec{s}_1 - \vec{s}_2\right|}. \tag{3}$$

The slip velocity at the contact point is divided into two components. The first component is the relative velocity in the normal direction $\vec{v}_n$. This velocity component consists exclusively of translational movements and is calculated by:

$$\vec{v}_n = \vec{n} \bullet (\vec{q} \bullet \vec{n}). \tag{4}$$

The second component of the slip velocity is the tangential relative velocity, which includes both translational and rotational motion and is determined by subtracting the normal relative velocity from the slip velocity:

$$\vec{v}_t = \vec{q} - \vec{v}_n. \tag{5}$$

Fig. 3 illustrates these correlations in a collision between two spherical particles.

The tangential restitution coefficient depends on the angle at which the particles collide with each other. To avoid confusion, it should be noted here that Walton [26] and Doménech-Carbó [25], for example, have defined different model parameters in their modeling approaches, which they also refer to as a tangential coefficient of restitution. Doménech-Carbó refers to his model parameter as the “restricted” tangential restitution coefficient, and Walton refers to his as the “maximum” tangential restitution coefficient. These model parameters do not depend on the collision angle but are subject to model assumptions. Ultimately, these model parameters must be fitted to an entire series of measurements in which the collisions occurred at various collision angles. The definition of the tangential coefficient of restitution used in the present study according to Eq. (1) is referred to as the “actual” tangential coefficient of restitution by Doménech-Carbó [25]. It is determined directly from the measured motion data of the particles for

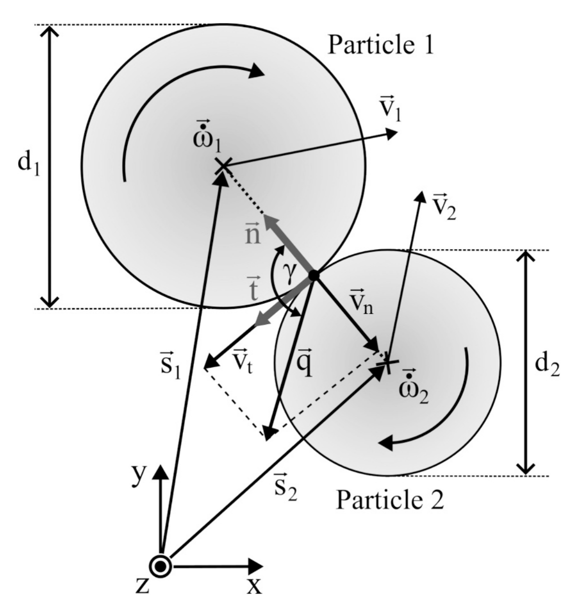

Fig. 3. Scheme of a collision between two spherical particles at collision angle γ. The sketch shows the translational and rotational motion components.

each individual collision measurement, is not subject to any limiting model assumptions, but exhibits a dependence on the collision angle [24]. The collision angle γ is defined as the angle enclosed by the normal unit vector $\vec{n}$ and the slip velocity vector $\vec{q}$:

$$\gamma = arccot\left(\frac{\vec{q} \bullet \vec{n}}{|\vec{q} \times \vec{n}|}\right). \tag{6}$$

The collision angle ranges from 90°, which corresponds to a maximum oblique collision, to 180°, which reflects a perfectly straight collision. The collision angle is varied within each series of measurements by moving the vacuum tweezers and particle accelerator relative to each other so that the entire spectrum of collision angles is covered. Fig. 4 shows examples of collisions at different collision angles from a series of measurements taken from one of the two camera perspectives. To investigate the influence of surface roughness on the tangential restitution coefficient, several series of measurements were carried with varying roughness levels, which were adjusted as described in the following chapter.

3. Materials

To ensure the best possible comparison with real particle systems, the sphere diameter was chosen to be as small as possible regarding material and measurement setup requirements. To rule out factors other than surface roughness that might affect collision dynamics, the spheres must also be manufactured with a high degree of precision. For this reason, high-precision ball bearings made of yttrium-stabilized zirconium oxide with a diameter of 1.5 mm were used for the investigation. This is the smallest size at which such balls are still commercially available in the highest accuracy class. At the same time, this ball size is still very well resolved in the high-speed images, which has a positive effect on measurement accuracy [15]. The zirconium oxide balls have a maximum roundness deviation of 0.13 μm, a density of 6000 kg/m^3, a Young's modulus of 210 GPa, and a hardness of 89 ± 2 HRa. When delivered, the surface of the balls is very smooth due to polishing. This surface condition is referred to as “untreated” in the following. Due to their well-defined geometry and material properties, they represent near-ideal model particles with predominantly elastic collision behavior. To investigate the influence of surface roughness on the tangential restitution coefficient, the surface was modified by roughening it with silicon carbide abrasive powder. Abrasive powders with three different grain sizes were used to create different roughness depths and profiles. The grain sizes are designated F8000, F1000, and F400, whereby a lower number corresponds to a coarser abrasive powder and, thus, a rougher final particle surface. To roughen the surfaces, the abrasive powder was mixed with distilled water to form a paste. The particles were processed together with the abrasive powder paste in a mortar with a pestle until a homogeneous surface roughness was achieved. The progress of the process was checked every five minutes under a light microscope. After the process, the particles were re-measured with a Series 293–821-30 micrometer screw (Mitutoyo, Japan) to ensure that the sphericity is unaffected and to determine their final dimensions.

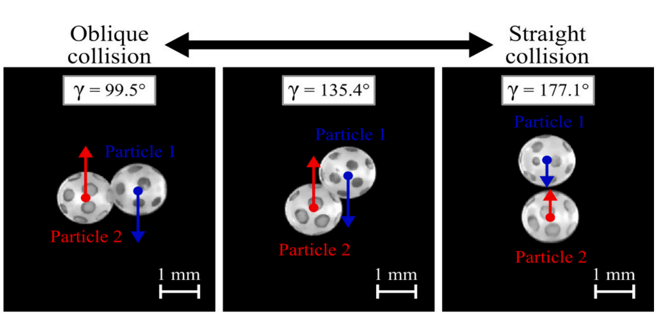

Fig. 4. Collisions at different angles γ, from very oblique (left) to very straight (right).

In the following, the surface of the particles is named according to the abrasive powder grain size used, i.e., F8000, F1000, and F400. Fig. 5 shows the surface structure of the spheres before and after mechanical treatment analyzed with VK-X160K 3D laser scanning confocal microscope (Keyence, Japan). Table 1 shows the different surface roughness values obtained with the corresponding analysis software. The roughness measurement was performed according to the multi-line principle, measured on three different particles at five different positions per particle. The arithmetic mean roughness value R_a and the mean roughness depth R_z were measured. R_a is defined as the arithmetic mean of the absolute deviation of the surface profile from the mean profile line within a measuring section and provides a good overview of the general smoothness of the surface. R_z, on the other hand, provides more of a picture of the extreme properties of the surface, as it analyzes the distances from the highest roughness peak to the lowest valley of five consecutive individual measurement sections [29]. The values in Table 1 represent the determined mean values with the corresponding standard deviation.

4. Results and discussion

Seven different collision combinations of particles with different surface roughness levels were investigated. The results of all individual combinations, as well as a superimposed representation of all measurement series, are shown in Fig. 6. The figure shows the tangential restitution coefficient β plotted against the collision angle γ. All combinations show a consistent, fundamental trend, as already demonstrated in our previous work [15] for interparticle collisions and, e.g., by Cross [16] for particle-wall collisions. For small collision angles close to 90°, i.e., for very oblique collisions, the tangential restitution coefficient tends toward minus one. As the collision angles increase, i.e., as the collisions become more and more straight, the tangential restitution coefficient rises steadily and eventually enters the positive value range. Negative values mean that the tangential relative velocity after the collision points in the same direction as before the collision. Conversely, positive values of the tangential restitution coefficient mean that the direction of the tangential relative velocity reverses and points in the opposite direction after the collision compared to before the collision.

The integration of all measurement data into one plot, illustrated in Fig. 6 H, shows that there are no significant differences in the curve behavior of the tangential restitution coefficient for the combinations of surface roughness of the colliding particles examined here. The difference lies primarily in the degree of scatter in the measurement data. As might be expected intuitively, the measurement series with the two smoothest particles (“Untreated – Untreated”) shows the smallest variability. In addition, this measurement series runs centrally through all other measurement series in the superimposed view of all combinations. The data from the other combinations exhibit significantly greater scatter around this central curve. The measurement data also show that the other extreme of the experiments, the roughest combination of F400 with F400 surface-treated particles, exhibits the strongest scattering. To quantify the scattering, the curve of the smoothest surface combination (“Untreated – Untreated”) and thus the measurement series closest to the ideal case is used as a benchmark. As illustrated in Fig. 7, the curve of the “Untreated – Untreated” experiment series is approximated with a high coefficient of determination (R^2) of just under 96% with a fourth-degree polynomial, according to the equation:

$$\beta_{approx} = A \bullet \gamma^4 + B \bullet \gamma^3 + C \bullet \gamma^2 + D \bullet \gamma + E, 90^\circ \leq \gamma \leq 180^\circ. \tag{7}$$

The reason for using a polynomial to describe the behavior of the tangential coefficient of restitution is that this approach allows any potentially limiting model assumptions to be excluded initially. The models by Walton [26] or Doménech-Carbó [25], for example, assume a

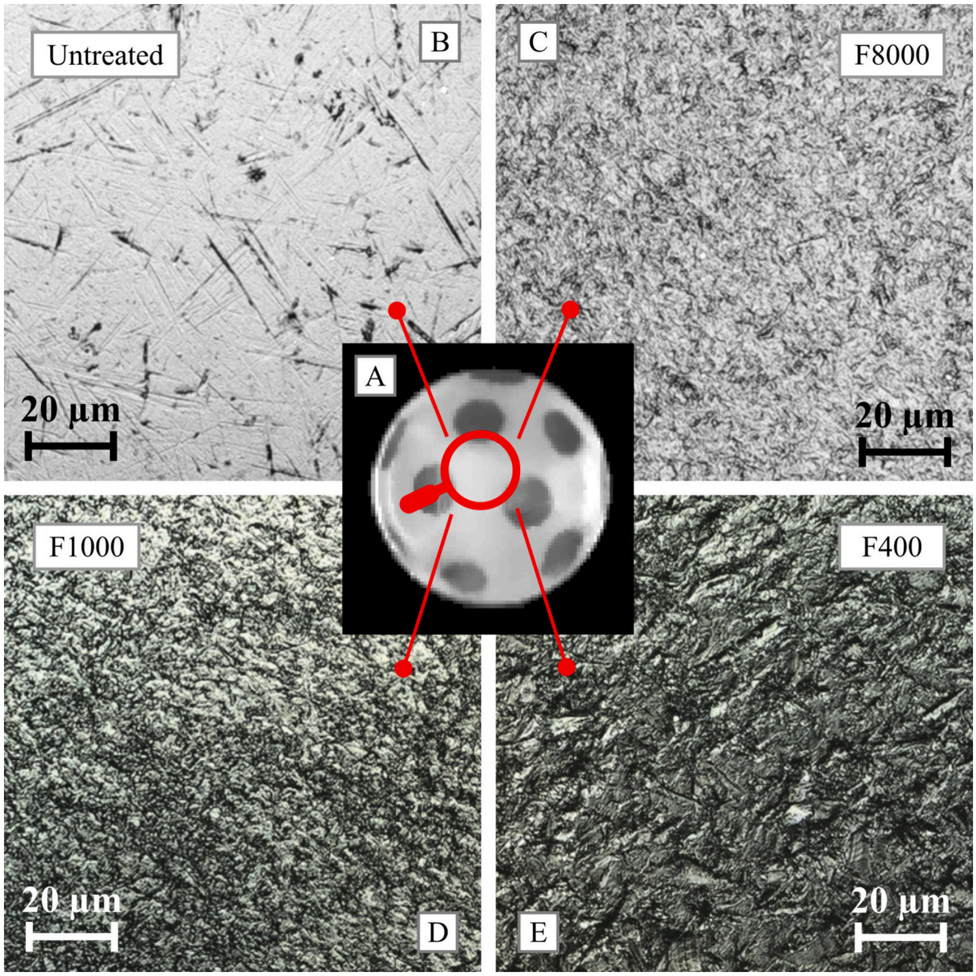

Fig. 5. Full view of the particle (A) and laser scanning microscope (VK-X160K, Keyence) images of the particle surface before and after treatment with silicon carbide abrasives of various grain sizes with increasing surface roughness level from B to E.

Table 1
Surface roughness values of particles before and after treatment with silicon carbide abrasives of various grain sizes.

Surface treatment	R_a [µm]	R_z [µm]
Untreated	0.026 ± 0.003	0.198 ± 0.045
F8000	0.097 ± 0.009	0.695 ± 0.049
F1000	0.190 ± 0.056	1.747 ± 0.509
F400	0.310 ± 0.038	2.576 ± 0.252

constant trend of the tangential coefficient of restitution in the so-called rolling or sticking contact regime. The rolling contact regime occurs in relatively head-on collisions and appears in our measurements in the range from approximately 155° to 180°. In the reference measurement, a tangential coefficient of restitution that rises gradually rather than remaining constant can be identified in this rolling contact regime. For more detailed information on the contact regimes and Walton's model curve, we refer to our previous work [15]. The polynomial approximation proposed here, on the other hand, is capable of accurately reproducing the entire curve of the tangential coefficient of restitution. It also adheres to the allowed interval of ±1 and known properties, such as the continuous increase in the sliding contact regime. In our measurements, the sliding contact regime occurs in the range from 90° to approximately 155°. The parameters for the polynomial are listed in Table 2.

For each measurement series, the standard deviation σ is calculated relative to the polynomial approximation curve. To quantify the roughness in the overall system, a definition of a dimensionless roughness value Λ_z is suggested. The here suggested dimensionless roughness relates the roughness depths $R_{z,i}$ of the colliding particles i to the reduced radius r^* of the collision system:

$$\Lambda_z = \frac{R_{z,1} + R_{z,2}}{r^*}. \tag{8}$$

The reduced radius is determined from the radii r_i of the colliding particles:

$$r^* = \frac{r_1 \bullet r_2}{r_1 + r_2}. \tag{9}$$

The rationale behind using the sum of the R_z roughness values for dimensionless roughness is that, in the context used here, dimensionless roughness is intended to be a measure of the maximum potential deviation from an ideally smooth surface. Since R_z is a measure of the extreme values of surface irregularities – and can thus be interpreted as an approximation of the maximum deviation from a spherical shape - the sum of the R_z values of both particles represent the maximum deviation of the collision system from the ideally smooth surface. It should be emphasized here that the definition of Λ_z is a suggestion. Depending on the application, it may be appropriate to use the arithmetic mean roughness R_a, as well as any other possible mathematical combination of the values of both collision objects. To answer the research question at hand, the exact definition of Λ_z is of minor importance.

Fig. 8 shows the standard deviation plotted against the dimensionless roughness parameter. It confirms the expected trend that a collision system with rougher particles comes along with a higher standard

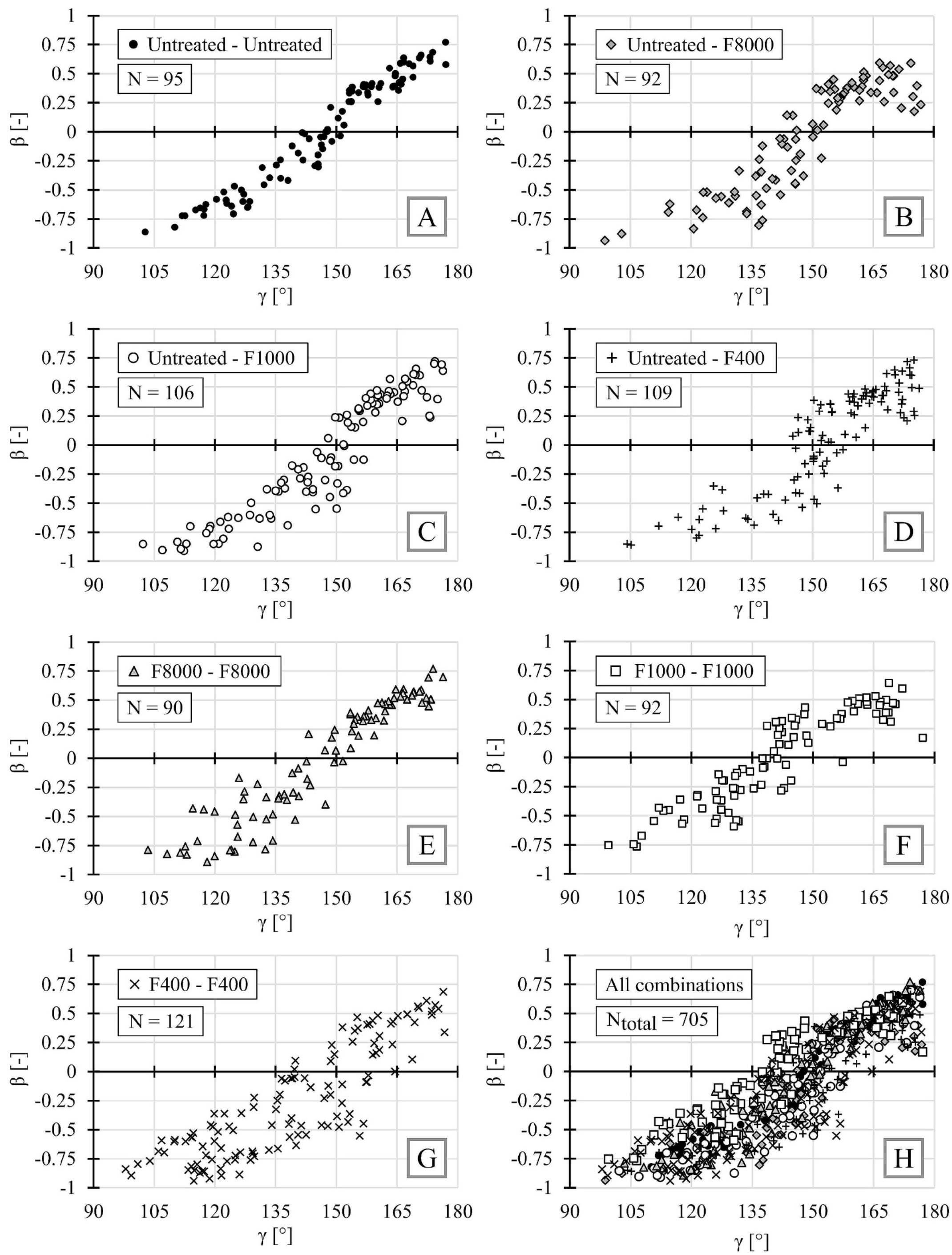

Fig. 6. Trend of the tangential restitution coefficient β as a function of the collision angle γ for collision events with partners of varying surface roughness levels (A-G) and a plot showing the overlay of all measurement series (H). N denotes the number of collision experiments per measurement series.

deviation in the measurement data. It should be emphasized that this is a rough trend without predictive power for the scatter at given surface roughness of the collision partners. Simple correlations are not to be expected due to the complexity of the contact mechanics and the many possible surface structures. In addition, the measurements are random samples and the result naturally depends heavily on the sample size. This could also explain the significant outlier in the "Untreated – F8000" measurement series, which has a significantly higher standard deviation even though it has a lower system roughness than the "F8000 – F8000" measurement series, which has a lower standard deviation. Nevertheless, it can be concluded that surface roughness levels in the investigated range have a clear influence on the scattering, even with very low surface roughness. A review of the existing literature provides some possible explanations. Chang et al. [30] identified the fracture or plastic deformation of asperities as an energy sink, which can consequently lead to random fluctuations in collision behavior. Based on experiments and simulations of head-on collisions between rough particles and a smooth plate, Montaine et al. [4] concluded that asperities lead to energy transfer between translational and rotational degrees of freedom due to eccentric impacts. The particle-plate collision setup used by Montaine et al. [4] differs from the particle-particle collision setup used in the present study. However, there are no obvious reasons why the findings

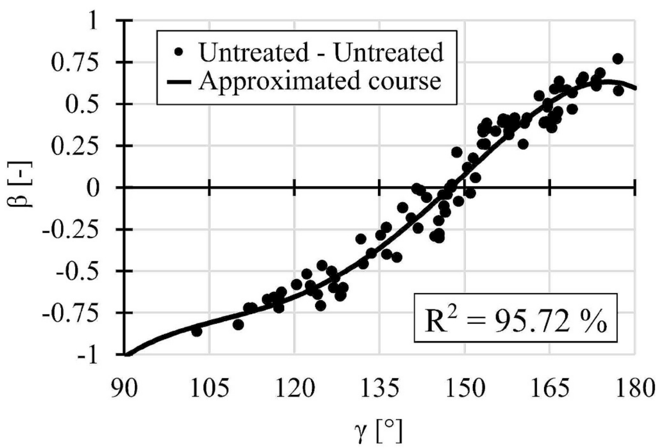

Fig. 7. Approximated curve of the tangential restitution coefficient using a fourth-degree polynomial, adjusted to the data of the smoothest collision pairing ("Untreated - Untreated").

Table 2
Table of parameters for the fourth-degree polynomial for approximating the tangential restitution coefficient from Eq. (7).

Parameter	Value
A	$-1.43 \bullet 10^{-7}$
B	$7.36 \bullet 10^{-5}$
C	$-1.38 \bullet 10^{-2}$
D	1.14
E	−35.66

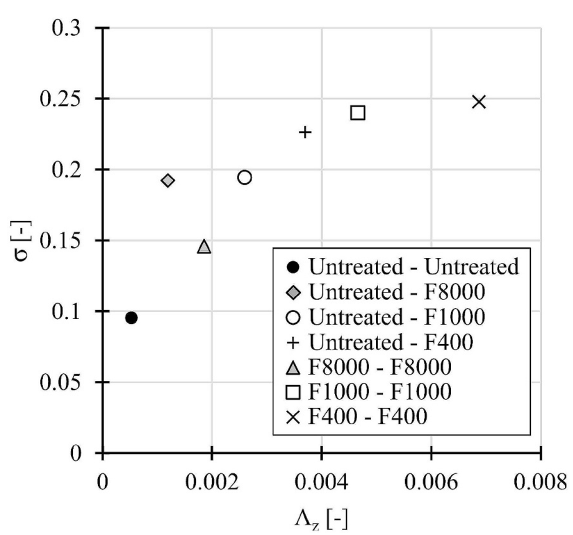

Fig. 8. Representation of the intensity of scatter in the data of the tangential restitution coefficient as a function of the dimensionless surface roughness Λ_z of the collision system.

of Montaine et al. [4] should not also be valid for particle-particle collisions. Unfortunately, because Montaine et al. [4] exclusively investigated head-on collisions, they do not provide any direct measurements of the tangential coefficient of restitution that could be used for comparison.

The course of the tangential restitution coefficient is significantly influenced by the friction between the particle surfaces [15,24]. A change in the friction between the colliding partners would lead to a shift in the zero crossing in the plot of the tangential restitution coefficient over the collision angle, which is displayed in Fig. 6. For the reference case of "Untreated-Untreated" collisions, the zero-crossing point is quite clearly at a collision angle of approximately 150°, or slightly less. The next coarser, but still comparatively smooth, collision combinations "Untreated-F8000", "F8000-F8000" and "Untreated-F1000" still show a very clear zero-crossing in the same angular range as the "Untreated-Untreated" reference measurement. The three roughest measurement series, "Untreated-F400", "F1000-F1000" and "F400-F400" are increasingly less clear regarding the exact zero-crossing due to the increasingly intense scattering, particularly in the zero-crossing region. Although not entirely clear, a slight trend toward a larger collision angle at the zero-crossing point can be discerned in the third-roughest "Untreated-F400" combination. This would imply a reduction in the coefficient of friction between the particle surfaces compared to the smooth reference case. A contrary trend, although also not entirely clear, is observed in the second-roughest system, the "F1000-F1000" collisions. Here, a zero-crossing could be interpreted as occurring somewhere between 140° and 150°, i.e., tending toward a slightly smaller collision angle. If that is the case, there would be a higher coefficient of friction between the particle surfaces compared to the smooth reference case. The roughest combination, "F400-F400" does have a zero-crossing of approximately 150° on average again, like the reference measurement. However, the data scatter here is so massive that the interpretable zero-crossing range is enormous, lying somewhere between 135° and 165°. Thus, it is concluded, that no clear trend can be identified for the material, particle size and collision velocity under investigation, regarding the influence of surface roughness on the fundamental behavior of the tangential coefficient of restitution, and consequently on the coefficient of friction.

The current available information regarding experimental comparative data from the existing literature is very limited. However, data from modeling approaches and simulations are available, albeit to a limited extent. Li et al. [7] calculated the theoretical coefficient of friction for their exclusively straight-line collision experiments between microspheres and rough plates using a model by Zhang et al. [31,32], but did not measure it directly. They conclude that the coefficient of friction between two colliding surfaces depends heavily on surface roughness. Brilliantov et al. [24] simulated collisions between two spheres and included a descriptive approach for surface roughness. Brilliantov et al. [24] conclude that surface roughness has a strong influence on the behavior of the tangential coefficient of restitution. As previously mentioned, the coefficient of friction and the tangential coefficient of restitution are directly related. Both Li et al. [7] and Brilliantov et al. [24] thus come to the same conclusion. The experimental results shown here contradict the model-based results of Li et al. [7] and Brilliantov et al. [24]. It should be noted that the experimental results presented here do not exclude a fundamental influence. However, it has been shown that, in the collision system investigated here, the influence of surface roughness on the variation of the tangential coefficient of restitution is significantly greater than any possible influence on the fundamental behavior. It is therefore feasible that scattering could mask potential effects of roughness on friction. Conversely, a direct comparison of the results shown here with those of Li et al. [7] and Brilliantov et al. [24] is only possible to a limited extent. While Li et al. [7] investigated comparable roughness depths on their plates, their spheres were approximately two orders of magnitude smaller, which is likely to significantly increase the influence of the irregularities. Furthermore, the model by Zhang et al. [31,32], which was used by Li et al. [7] to determine the theoretical friction coefficients, is designed for quasi-steady-state sliding. The extent to which this assumption is justified for a highly dynamic process such as a collision is, at the very least, open to critical analysis. In their simulations, Brilliantov et al. [24] used the material properties of supercooled ice and a particle size that is one order of magnitude larger than that experimentally investigated here.

5. Conclusion

It has been shown that increased surface roughness in collisions between spherical, hard particles of equal size at moderate collision velocities results in a significant increase in the scatter in the tangential coefficient of restitution. It is therefore concluded that greater irregularities on the particle surfaces lead to increasingly random energy transfer during a collision. No influence of surface roughness on the friction between the particle surfaces and thus on the tangential coefficient of restitution could be detected. However, the strong, roughness-induced increase in scattering may mask any systematic changes in the trend of the tangential coefficient of restitution. It needs to be noted, that these findings are, in fact, only confirmed for the parameters considered in this study, such as particle size, collision velocity, and material. At this point, it cannot be ruled out that a change in the parameters, such as a change in collision velocities, could significantly influence the intensity of the observed roughness-induced effects on the tangential coefficient of restitution. Within the scope of this experimental study, the underlying mechanisms of contact mechanics that lead to the behavior observed here cannot be conclusively explained. However, the results presented enable a rough estimation of the possible deviations from the ideal model of a perfect sphere, as is often assumed in Discrete Element Method (DEM) simulations.

List of symbols

Symbol	Description, Unit
A	Fourth-degree polynomial parameter, [−]
B	Cubic polynomial parameter, [−]
C	Quadratic polynomial parameter, [−]
D	Linear polynomial parameter, [−]
d	Particle diameter, [m]
E	Constant polynomial parameter, [−]
i	Particle number index, [−]
$\vec{n}$	Normal unit vector, [−]
N	Number of measurements, [−]
$\vec{q}$	Slip velocity in contact point, [m/s]
R_a	Arithmetic mean roughness, [m]
R_z	Mean roughness depth, [m]
r	Particle radius, [m]
r^*	Reduced radius, [m]
$\vec{s}$	Particles center position vector, [m]
$\vec{t}$	Tangential unit vector, [−]
$\vec{v}$	Translational velocity vector, [m/s]
$\vec{v}_n$	Normal relative velocity vector, [m/s]
$\vec{v}_t$	Tangential relative velocity vector, [m/s]
β	Tangential restitution coefficient, [−]
γ	Collision angle, [°]
Λ_z	Dimensionless surface roughness, [−]
σ	Standard deviation of β, [−]
$\vec{\omega}$	Rotation velocity vector, [rad/s]

CRediT authorship contribution statement

Falk Bunke: Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Stefan Heinrich:** Writing – review & editing, Supervision, Resources, Project administration, Funding acquisition, Conceptualization.

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Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Given his role as associate editor, Stefan Heinrich had no involvement in the peer-review of this article and has no access to information regarding its peer-review. Full responsibility for the editorial process for this article was delegated to another journal editor. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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