Multiple Criteria Decision Analysis Techniques in Aircraft Design and Evaluation Processes

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Abstract

Air transportation systems are complex, interdisciplinary integrated systems, because there are large numbers of components with different characteristics. It is challenging to assess new technology in complex, interdisciplinary integrated systems, such as air transportation systems. The focus of this research is one element in complex air transportation systems: aircraft. The success of an aircraft is no longer dominated by economic criteria. Several other criteria, such as environmental aspects and level of comfort, need to be taken into consideration. Therefore, aircraft design and evaluation are typical multi-criteria decision problems and need to be prudently conducted. One solution is to apply Multi-Criteria Decision Analysis (MCDA) techniques.

The goal of this research is to investigate how MCDA techniques can be applied in order to provide better decision aiding for stakeholders in air transportation systems.

First, an advanced approach to effectively select the most appropriate decision analysis method for a given decision making problem is formulated and presented in this research. This method selection approach is implemented and an intelligent multi-criteria decision support system is developed.

Second, a new approach is proposed for assessing the uncertainties propagated in the decision analysis process. The uncertainty assessment approach consists of four steps: uncertainty characterization, uncertainty analysis, local sensitivity analysis, and global sensitivity analysis. This novel approach for uncertainty assessment can be used to aggregate input data from tools with different fidelity levels and is capable of propagating uncertainties in an assessment chain.

Third, two proofs of concept are conducted to demonstrate the effectiveness of applying the most appropriate MCDA techniques in aircraft design and evaluation processes. In order to efficiently assess the uncertainties related to the subjective preference information in aircraft design process, surrogate models for design criteria in terms of weighting factors are developed. Furthermore, soft criteria quantification in aircraft evaluation process permits the synergy of *hard* technical criteria and additional *soft* criteria for the MCDA techniques.

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Glossary

- ACARE: Advisory Council for Aeronautical Research in Europe
- ACJ: Airbus Corporate Jet
- AHP: Analytical Hierarchy Process
- AI: Appropriateness Index
- ANP: Analytical Network Process
- ANSP: Air Navigation Service Provider
- ATM: Air Traffic Management
- ATS: Air Transportation Systems
- BBJ: Boeing Business Jet
- BCA: Business & Commercial Aviation
- CI: Consistency Index
- CL: Confidence Level
- CR: Consistency Ratio
- DLR: German Aerospace Center
- DM: Decision Maker
- DOC: Direct Operating Costs
- ELECTRE: Elimination and Choice Translation Reality
- EPNdB: Decibels of Effective Perceived Noise
- GA: Genetic Algorithms

- GUI: Graphical User Interface
- ICAO: International Civil Aviation Organization
- IFR: Instrument Flight Rules
- ITOPSIS: Improved TOPSIS
- LCA: Life Cycle Assessment
- LHS: Latin Hypercube Sampling
- MCDA: Multi-Criteria Decision Analysis/Aid
- MCDM: Multi-Criteria Decision Making
- NAIADE: Novel Approach to Imprecise Assessment and Decision Environments
- N/F: Non-Feasible
- NBAA: National Business Aviation Association
- OEM: Operating Empty Mass
- OR: Operational Research
- PN/F: Physically Non-Feasible
- PROMETHEE: Preference Ranking Organization METHod for Enrichment Evaluations
- RI: Random Consistency Index
- RMSE: Root Mean Square Error
- SAW: Simple Additive Weighting
- SMART: Simple Multi-Attribute Rating Technique
- SNR: Signal-to-Noise Ratio
- TOM: Take-off Mass
- TOPSIS: Technique for Order Preference by Similarity to Ideal Solution
- VAMPzero: Virtual Aircraft Multidisciplinary Analysis and Design Processes
- VIKOR: VlseKriterijumska Optimizacija I Kompromisno Resenje in Serbian, which means multi-criteria optimization and compromise solution.

1

Introduction

Air transportation systems are composed of infrastructure and service providers with the primary goal of safely transporting people and freight by air [51]. Air transportation systems are complex, interdisciplinary integrated systems, because there are large numbers of components with different characteristics. Main stakeholders in air transportation systems include manufacturers, airlines, airports, air navigation service provider (ANSP), government agencies, international organizations, and passengers.

The demands on air travel are increasing, not only regarding lower costs, but also better service quality, higher safety, and more environmental friendliness. The imperatives of air transport have evolved from *Higher, Further, Faster* to *More Affordable, Safer, Cleaner and Quieter* [1]. Vision 2020 set ambitious ACARE (Advisory Council for Aeronautical Research in Europe) goals for future air transportation systems, in terms of quality and affordability, environment, efficiency, safety, and security [1]. In order to sustain the growth of air transport in the long term, multiple stakeholders in air transportation systems such as manufacturers, airlines, and airports are involved to meet these ambitious goals. Multi-Criteria Decision Analysis (MCDA) techniques can provide decision aid for these stakeholders.

As an important field in Operational Research (OR), MCDA is a process that allows one to make decisions in the presence of multiple, potentially conflicting criteria [139]. Common elements in the decision analysis process are a set of design alternatives, multiple decision criteria, and preference information representing the attitude of a Decision Maker (DM) in favor of one criterion over another when choosing between alternatives. MCDA techniques can help the DM to evaluate the overall performance of the design alternatives. Furthermore, MCDA techniques can aid in the generation, analysis, and optimization of design solutions.

1.1 Motivation

It is challenging to assess new technology in complex, interdisciplinary integrated systems, such as air transportation systems. The focus of this research is one element in complex air transportation systems: aircraft. Aircraft are complex engineered systems which involve multiple disciplines, such as aerodynamics, structures, and disciplines involving human behavior which are extremely difficult to quantify and integrate into mathematical models and optimization problems [115].

Severe schedule delays and cost overruns are often encountered in complex engineered systems. For example, Boeing 787 program has suffered numerous production delays and huge cost overruns. Qantas Airways has canceled 35 B787 and this is the largest cancellation for B787 [95]. Air India has ordered 27 B787 and asked for the compensation package ranging between \$ 145 million and \$ 800 million because of delivery delays [125]. The cancellation of airliners and compensation to carriers for delays and cost overruns has constrained the profitability of the 787 program. It is estimated that Boeing currently loses \$ 100 million for each B787 it sells [95]. In order to address these severe problems, more advances are needed to improve the design process of complex engineered systems [115].

The single economic criterion, such as operating cost, is not the only metric for final technology evaluation as well as the figure of merit for design optimization. The success of an aircraft is no longer dominated by economic criteria, such as purchase price and operating costs [39]. Moreover, it is alerted that by applying classic Direct Operating Costs (DOC) comparison as the only yardstick in the evaluation of an aircraft, manufacturers run the risk of designing aircraft types and capabilities not fully suited to satisfy long term transportation needs [84].

In addition to the economic consideration, there are several other criteria need to be taken into account in aircraft design and evaluation processes. For instance, environmental impact and level of comfort. Continuous growth in passenger traffic and increasing public awareness of aircraft noise and emissions have made environmental considerations extremely critical in the design of future aircraft [10]. Besides, passengers are more concerned about crowded flights and airlines are criticized for increasing load factors to fully utilize the capacity [116]. Therefore, considering these multiple criteria simultaneously, aircraft design and evaluation are typical multi-criteria decision problems and need to be prudently conducted. However, it is often difficult to derive a reliable transfer function to convert these non-monetary into monetary values [117]. One solution is to apply Multi-Criteria Decision Analysis (MCDA) techniques.

Applying MCDA techniques in aircraft design and evaluation processes is one strategy to deal with multiple, potentially conflicting criteria. MCDA techniques can be utilized to aggregate multiple design criteria into one composite figure of merit, which serves as an objective function in the optimization process. MCDA techniques allow transparent trade-offs among criteria and support designers in quickly assessing the compromised design alternatives. Moreover, MCDA techniques have the ability to handle a large number of criteria in aircraft design and evaluation processes.

Preference information describes a DM's attitude in favor of one criterion over another when choosing between alternatives. There are many ways to represent a DM's preference information, such as weighting factors, utility function [65], loss function [99],[119], reference points consisting of desirable aspiration levels for the criteria [130], and fuzzy numbers. In this research, weighting factors are chosen to represent a DM's preference information, considering that most decision analysis methods require the preference information in the form of weighting factors.

1.2 Literature Review on MCDA in Air Transportation Systems

MCDA techniques have been used to solve multi-criteria decision problems in air transportation systems. This section reviews the research work of applying MCDA techniques in aircraft, airlines, airports, Air Traffic Management (ATM), and air cargo related multi-criteria decision problems, respectively.

Nowadays, more stringent societal, environmental, financial, and operational requirements have to be addressed in aerospace engineering designs [80]. MCDA techniques can facilitate the decisions regarding which concept to pursue in the conceptual design process. The Aerospace Systems Design Laboratory at the Georgia Institute of Technology pioneered the application of MCDA techniques in aerospace systems design. A probabilistic MCDA method for multi-objective optimization and product selection was developed [12]. However, it was pointed out that this method did not consider the absolute location of joint probability distribution [73]. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was utilized for the selection of technology alternatives in conceptual and preliminary aircraft design [66]. However, TOPSIS has limitations in that it assumes each criterion's utility is monotonic and it is rather sensitive to weighting factors. A modified procedure for applying MCDA techniques to large scale systems design problems with several requirements was presented in [21], where each criterion had a two-part relative importance model: a static portion quantifying basic relative importance and a dynamic portion to reduce the impact of the assumption that the utility of the criteria are monotonically increased. A multi-criteria interactive decision-making advisor for the selection of the most appropriate method was developed [72]. However, only few methods were implemented and uncertainty propagation was not addressed explicitly.

Lots of research has been conducted to aircraft evaluation using MCDA techniques. Four civil aircraft were evaluated by six criteria [29]: cost, performance, comfort, environmental influence, product support and family concept, and availability of aircraft. A 10-point ratio scale was employed to normalize the values of the six criteria, where the maximum value of the benefit criteria obtained 10 points and the minimum value was given 0 point. Simple Additive Weighting (SAW) was used to rank the candidate aircraft. However, the normalization of the six criteria was conducted by a linear relationship between the criteria values and the scale values, and SAW is very sensitive to the normalization method and the weighting factors. Besides, civil aircraft were assessed by three criteria: DOC, operational commonality, and added values [84], [39]. The added values were quantified by *equivalent DOC* based on weighting factors. However, inherent subjectivity and uncertainty of weighting factors detriments the usefulness of this approach.

Furthermore, seven initial training aircraft were evaluated by sixteen criteria using TOPSIS [128]. However, only technical criteria were considered because of the difficulty of collecting qualitative data. Four regional aircraft were assessed by using TOPSIS method, based on three groups of criteria: technological (aerodynamic efficiency, structural efficiency, fuel flow, cruise endurance and requested trip fuel for the fixed cruise range), operational (max range with max payload and ground efficiency), and climb capability [32]. Analytic Hierarchy Process (AHP) was used to obtain the weighting factors for these criteria. However, only one set of the weighting factors was studied in the evaluation process of four regional aircraft. Considering that the pairwise comparison of AHP is highly subjective, the ranking of the four regional aircraft will probably change with different sets of weighting factors. Thus, it is necessary to conduct uncertainty analysis for the weighting factors in the aircraft evaluation process. One MCDA method named NAIADE (Novel Approach to Imprecise Assessment and Decision Environments) was used to select an aircraft among eight alternatives for a regional charter company [52]. Three group criteria were considered and subdivided into eleven sub-criteria: financial group (acquisition costs, liquidity, and operating costs), logistics group (range, flexibility, cruising speed, replacement parts availability, landing and take-off distance), and quality group (comfort, avionics availability, and safety). This work showed that the NAIADE method was capable to aid DMs in the aircraft selection problem. However, it was pointed out that the NAIADE method acquired undesirable levels of complexity when the uncertainty of the variables was added, thus, reducing the transparency of the results for DMs.

MCDA techniques have also been used to solve multi-criteria decision problems related to airlines. Three MCDA methods: SAW, weighted product model, and TOPSIS, were used to evaluate the competitiveness of Taiwan's five major airlines [27]. Equal weighting factors were considered for five performance criteria: cost, productivity, service quality, price, and management. A fuzzy MCDA method was used to develop an airline safety index for Taiwan's major airlines, in terms of four criteria: management, flight operations, engineering and maintenance, and fleet planning [28]. Another fuzzy MCDA method was used to evaluate the financial performance of Taiwan airlines [128]. AHP with fuzzy numbers was applied to evaluate the competitiveness of five major Chinese airlines, in terms of five criteria: internationalization level, market competitiveness, scale competitiveness, asset operation competitiveness, and human resource competitiveness [132]. These five criteria were further decomposed into seventeen sub-criteria. The results showed that this approach could help to improve Chinese airline competitiveness in the market. Moreover, MCDA methodology was used to evaluate a set of generated line maintenance plan alternatives for an airline operator at the airports, concerning four criteria: cost, remaining useful life, operational risk, and flight delay [96]. This approach can help to achieve high fleet interoperability and low maintenance cost. A combined AHP, TOPSIS, and VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje in Serbian, which means multi-criteria optimization and compromise solution) was applied to the selection of a maintenance strategy for an aircraft system [3]. It was shown that the proposed combination of AHP, TOPSIS, and VIKOR was able to identify the most effective maintenance alternative.

Some research has been done on applying MCDA techniques in airport related multi-criteria decision problems. Three MCDA methods: SAW, TOPSIS, and AHP, were applied to an airport selection problem, where seven alternatives were evaluated in terms of twelve criteria [61]. The authors concluded that these three methods produced the same results if the same weighting factors were used, and they also suggested that the weighting factors should be considered more carefully. AHP was used to study the relative importance of the means to improve passenger security checks at the airports, among three major factors: human resources, equipment and facilities, and procedures and responsibility structures [134]. A questionnaire survey was conducted to gather the data for AHP in Incheon International Airport in South Korea. It was revealed that the factor of human resources was most important for the performance of passenger screening. A fuzzy method combing MCDA and gray relational analysis was used to evaluate the service quality of Northeast-Asian international airports [68]. The results showed that this method was able to tackle multi-criteria decision problems with qualitative attributes in a fuzzy environment.

MCDA techniques are also helpful for ATM in air transportation systems. ATM is defined by International Civil Aviation Organization (ICAO) as *the dynamic, integrated management of air traffic and airspace - safely, economically and efficiently - through the provision of facilities and seamless services in collaboration with all parties.* Compromises between multiple criteria have to be made for multiple stakeholders, for instance, the cost minimization for airline, capacity maximization for ANSP, and the concerning of local air quality for airport. With the increase of air traffic volume, the overwhelming data makes more difficult for air traffic controllers to make decisions rapidly and safely. In order to help en-route air traffic controllers to quickly share information and maintain good common situation awareness with adjacent sectors, MCDA methodology was applied in their decision making process [6], [7], [8]. The evaluation criteria and preferences between the criteria were extracted through interviews. This work showed that it was possible to develop multi-criteria cooperative decision aiding tools for conflict management in ATM.

MCDA techniques have also been applied to solve multi-criteria decision problems in air cargo transportation. AHP was applied to evaluate the competitiveness of air cargo express carriers in Korean market, in terms of six criteria: promptness, accuracy, safety, convenience, economic efficiency, and dependability [98]. This analysis showed that accuracy and promptness were the two most influential factors to competitiveness. Analytic Network Process (ANP) was applied to examine the trade-offs between costs, benefits, and risks in the selection of logistics service providers for air cargo [133]. Moreover, in order to resolve potential conflicts between safety, efficiency, and well-being in risk assessments for emerging technology in air transportation systems, AHP/ANP methodology was used to overcome the fragmentation perceived by risk, budget, quality, and schedule management [13].

From these applications of MCDA techniques in multi-criteria decision problems in air transportation systems, two observations can be formulated:

Observation 1: There are various decision analysis methods which have been developed for solving multi-criteria decision problems. Different methods have different underlying assumptions, analysis models, and decision rules that are designed for solving a certain class of decision making problems. For example, SAW chooses the most preferred alternative which has the maximum weighted criteria values, while TOPSIS ranks the alternatives based on the Euclidean distance. This implies that it is critical to select the most appropriate method to solve a given problem, since the use of inappropriate methods is often the cause of misleading design decisions. However, most researchers use one method without a formal method selection process, thus, the research area of decision analysis method selection has not drawn enough attention.

Observation 2: Due to different preferences and incomplete information, uncertainty always exists in the decision analysis process. When MCDA methods are used to solve decision problems, the values of decision criteria and weighting factors are main input data. It is observed that there are always uncertainties existing in decision criteria due to incomplete information or limited knowledge, while weighting factors are often highly subjective, considering that they are elicited based on the DM's experience or intuition [14], [44]. The inherent uncertainties associated with the input data have significant impacts on the final decision solution. This implies that it is critical to effectively address these uncertainties in the decision analysis process in order to get more accurate results.

In this research, a three-step framework for solving decision making problems is proposed and implemented: definition of a decision making problem, selection of the most appropriate MCDA method for the given problem, and uncertainty assessment in the decision analysis process. This three-step framework provides a general guideline on how to structure and solve any given decision making problems.

1.3 Research Statement

The goal of this research is to investigate how MCDA techniques can be applied in order to provide better decision aiding for stakeholders in air transportation systems, by investigating how existing MCDA techniques can be improved to solve complex decision problems, and how to implement the improved MCDA techniques in aircraft design and evaluation processes. The following research objectives are considered critical to achieve the overall research goal:

1. Select the most appropriate MCDA method in a problem oriented way to solve the decision making problem under consideration effectively.
2. Assess the uncertainties propagated in the decision analysis process when applying the MCDA techniques.
3. Demonstrate the capabilities of the MCDA techniques with uncertainty assessment in aircraft design and evaluation processes.

The research objectives of this study can be best introduced through a series of research questions as follows:

- **Question 1:** How to select the most appropriate MCDA method for a given decision making problem under consideration?
- **Question 2:** How to capture and assess the uncertainties propagated in the decision analysis process when solving decision making problems?
- **Question 3:** How to effectively implement the MCDA techniques in aircraft design and aircraft evaluation processes?

In order to answer the research questions described above, several hypotheses are proposed:

- **Hypothesis 1:** The goodness of the MCDA methods for a given decision making problem can be quantified mathematically. (Question 1)
- **Hypothesis 2:** Statistical techniques can effectively deal with the uncertainties propagated in the decision analysis process. (Question 2)
- **Hypothesis 3:** MCDA techniques facilitate compromised decision solutions in aircraft design and evaluation processes. (Question 3)

1.4 Thesis Outline

The outline of the thesis is illustrated in Figure 1.1. In Chapter 2, an overview of the MCDA techniques is provided. An advanced approach to facilitate the selection of the most appropriate MCDA method is presented and an intelligent multi-criteria decision support system is developed in Chapter 3. Chapter 4 introduces a new uncertainty assessment approach in the decision analysis process. In Chapter 5, the implementation of an improved MCDA technique with uncertainty assessment in aircraft conceptual design is presented as the first proof of concept. In Chapter 6, business aircraft evaluation using an appropriate MCDA technique with uncertainty assessment is presented as the second proof of concept. The thesis is summarized and some recommendations for future work are given in Chapter 7.

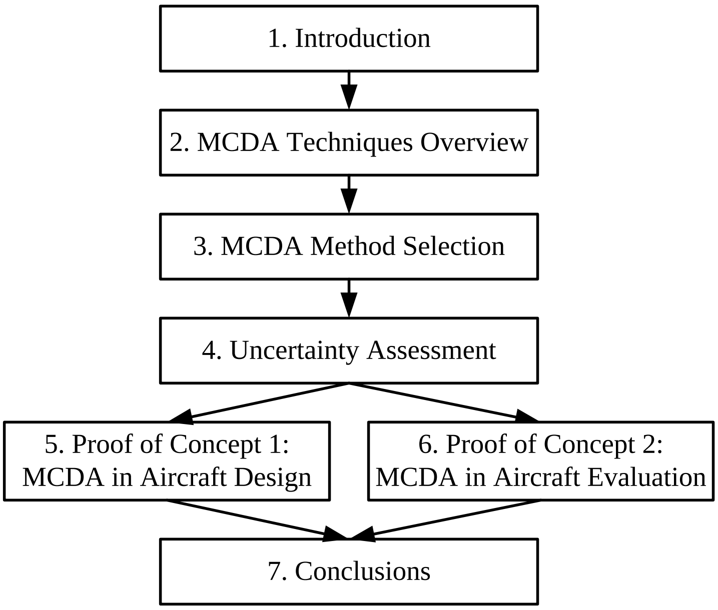


Figure 1.1: Thesis Outline

2

Multi-Criteria Decision Analysis Techniques Overview

As an important and active discipline in Operational Research (OR), Multi-Criteria Decision Analysis (MCDA) has a long history. In 1896, Vilfredo Pareto proposed the concept of dominance [97]. The dominance concept is the foundation of modern MCDA theory. In the 1940s, von Neumann and Morgenstern introduced utility theory [93]. The utility theory lays the foundation for one major stream of MCDA methodology: multi-attribute utility theory. In the 1960s, Roy introduced the concept of outranking relation [104]. The outranking relation concept sets the grounds of the European school of MCDA.

Nowadays, over 70 different MCDA techniques have been developed in order to facilitate the decision making process in complex and ill-structured problems, focusing on the resolution of multiple and conflicting criteria, preferences modeling, and identification of compromised decision solutions [139]. MCDA discipline continues its growth in the development of new approaches and methodologies, the interactions with other disciplines such as problem structuring methods and evolutionary optimization algorithms, and innovative applications in new areas such as transportation systems [139].

This chapter provides an overview of several widely used decision analysis techniques. There are essentially two approaches to solve decision making problems: non-compensatory and compensatory methods [58]. Non-compensatory methods do not permit trade-offs among criteria, while compensatory methods permit trade-offs among criteria. According to this classification, several widely used decision analysis methods are summarized in Table 2.1. It is noted that ELECTRE is classified as one non-compensatory method [22], considering that the role of criteria weights in ELECTRE are *coefficients of importance* [106], [31]. Besides, a poor criterion is judged irrespective to other good criteria, which distinguishes ELECTRE from compensatory methods [87].

Table 2.1: Typical Non-compensatory and Compensatory Decision Analysis Methods [58]

Non-compensatory methods	Compensatory methods
Conjunctive method	Analytic hierarchy process
Disjunctive method	Expected utility theory
Dominance method	Multi-attribute utility theory
ELECTRE	Multiplicative weighting method
Elimination by aspects	PROMETHEE
Lexicographic method	Simple additive weighting
Maximin method	TOPSIS
Maximax method	

2.1 Concepts and Terminologies

In order to have a universal understanding of the MCDA techniques, several important concepts and terminologies are introduced in this section.

MCDM and MCDA

There are two schools of decision analysis methods: Multi-Criteria Decision Making (MCDM) developed by the American school [137], and Multi-Criteria Decision Analysis/Aid (MCDA) created by the European school [105]. Most researchers use MCDM and MCDA interchangeably [14], [139], [44]. In this research, the European school (MCDA) is followed.

Criteria, Attributes, and Objectives

The distinctions among *criteria*, *attributes*, and *objectives* are made as follows [58].

- **Criteria**: A criterion is a measure of performance for the evaluation of an alternative.
- **Attributes**: An attribute is an inherent characteristic of an alternative.
- **Objectives**: An objective is something to be pursued to its fullest. It indicates the desired direction of change.

The relationships among *criteria*, *attributes*, and *objectives* are illustrated in Figure 2.1. As shown in Figure 2.1, criteria are emerging as a form of attributes or objectives, and attributes with directions are objectives. For example, level of comfort is a criterion when evaluating an aircraft; cabin volume and noise are attributes of the aircraft which can be used to measure the level of comfort; while the maximization of cabin volume and the minimization of noise are objectives in the aircraft design process.

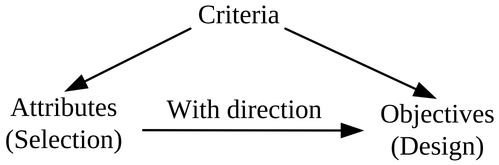


Figure 2.1: The Relationship among Criteria, Attributes, and Objectives [111]

Decision Matrix

At the heart of the MCDA techniques is the concept of decision matrix. Let A_i be the i-th alternative ($i = 1, 2, ..., m$) and x_j be the j-th criterion ($j = 1, 2, ..., n$). Suppose x_{ij} stands for the value of criterion x_j with respect to alternative A_i. Then, a quantitative MCDA problem can be represented using decision matrix, as shown in Table 2.2.

Table 2.2: Decision Matrix

Alternatives	Criteria			
A_1	x_{11}	x_{12}	$\cdots$	x_{1n}
A_2	x_{21}	x_{22}	$\cdots$	x_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	x_{m1}	x_{m2}	$\cdots$	x_{mn}

Pareto Frontier

Pareto frontier is introduced to find the best compromised solution which has the maximum overall performance [57]. In the feasible solution space, a solution is dominated if there is another solution which excels it in one or more criteria and equals it in the remainder [26]. A non-dominated solution is one which no criteria can be improved without a simultaneous detriment to at least one of the others. A two-dimensional Pareto frontier for the minimization of two criteria is illustrated in Figure 2.2.

2.2 Preference Information Elicitation Techniques

Preference information represents a DM's attitude in favor of one criterion over another when choosing between alternatives. This section introduces typical preference information elicitation techniques: direct assignment method [58], eigenvector method [108], entropy method [58], Simple Multi-Attribute Rating Technique (SMART)[43], Kano's model [140], and distance-to-target method [86].

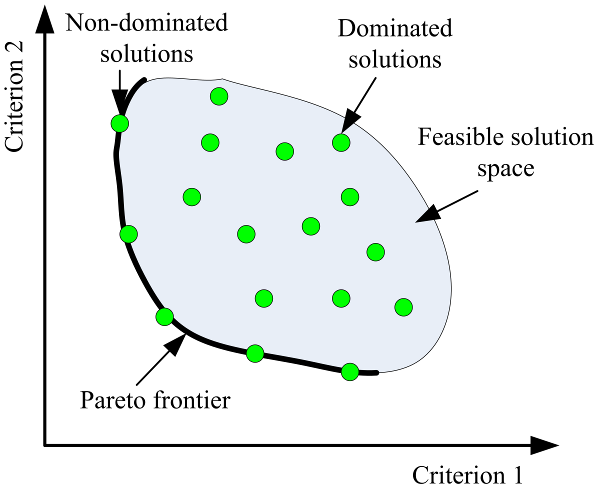


Figure 2.2: Pareto Frontier in Two Dimensions

2.2.1 Direct Assignment Method

In this method, the DM directly assigns numbers to represent the relative importance of one criterion over others. For instance, a ten-point scale can be chosen with calibration that 0 stands for extremely unimportant criterion, while 10 stands for extremely important one, as shown in Table 2.3.

Table 2.3: Direct Assignment Method with a Ten-point Scale

Criterion evaluation	Value
Extremely low	0
Very low	1.0
Low	3.0
Average	5.0
Very high	9.0
Extremely high	10.0

This method is popular because of its simplicity. However, it should be noted that the numerical assignment is arbitrary, and this type of scaling assumes that a scale value of 9.0 is three times as favorable as a scale value of 3.0. Besides, it also assumes that the difference between *low* and *average* is the same as the difference between *average* and *very high*. In complex decision making problems, it is rather difficult even for an experienced DM to precisely assign weights for all criteria directly.

2.2.2 Eigenvector Method

The eigenvector method is an analytical way of eliciting preference information in Analytical Hierarchy Process (AHP) [108]. This method uses pairwise comparisons between criteria represented by a comparison matrix M, the relative weights of criteria can be obtained by solving the eigenvalue function, as shown in Equation 2.1 [108].

$$M * W = \lambda_{max} * W \tag{2.1}$$

where λ_{max} is the maximum eigenvalue of the comparison matrix M, the weights of criteria are the normalized eigenvector $W = [w_1, w_2, ..., w_n]^T$ corresponding to the maximum eigenvalue.

In most decision making problems, the eigenvalue function is solved to evaluate the priorities of different criteria. In AHP, the consistency of the weights is assessed by Consistency Ratio (CR), as shown in Equation 2.2.

$$CR = \frac{CI}{RI} \tag{2.2}$$

where Consistency Index (CI) is calculated by Equation 2.3.

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{2.3}$$

Random Consistency Index (RI) is an average value derived from a large sample of reciprocal matrices having all elements varying from 1/9 to 9. Table 2.4 lists RI for up to ten elements [108].

Table 2.4: Random Consistency Index [108]

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.89	1.11	1.25	1.35	1.40	1.45	1.49

In general, CR of 0.1 or less is considered acceptable. In order to maintain reasonable consistency when deriving weights from pairwise comparisons, it is suggested that the number of elements being considered should be less than nine.

2.2.3 Entropy Method

The entropy method provides an alternative way of assigning weights when the input data of a decision making problem is represented by decision matrix, the weights of criteria w_j can be calculated by Equation 2.4 [58].

$$
\begin{aligned}
w_j &= \frac{d_j}{\sum\limits_{j=1}^{n} d_j}, \ \ \forall j \\
d_j &= 1 - E_j, \ \ \forall j \\
E_j &= -\frac{1}{\ln m}\sum_{i=1}^{n} p_{ij}\ \mathrm{ln} p_{ij}, \ \ \forall j \\
p_{ij} &= \frac{x_{ij}}{\sum\limits_{i=1}^{n} x_{ij}}, \ \ \forall i, j
\end{aligned}
\tag{2.4}
$$

where p_{ij} is the value of the j-th criterion $(i = 1, 2, ..., m, j = 1, 2, ..., n)$, E_j is the entropy of the j-th criterion, d_j is the degree of diversity of the information involved in the j-th criterion.

The entropy method helps to investigate contrasts between sets of data, that is, the weight of a criterion is small when all the alternatives have similar values on the criterion. In other words, a criterion does not contribute much when the criterion has similar values for all alternatives.

2.2.4 Simple Multi-Attribute Rating Technique

Simple Multi-Attribute Rating Technique (SMART) was originally developed as a whole process of rating alternatives and weighting criteria [43]. The weights are obtained in two steps:

- Firstly, the DM ranks the importance of the changes in the criteria from the worst criterion levels to the best criterion levels;
- Then, the DM makes ratio estimates of the relative importance of each criterion relative to the one ranked lowest in importance.

The second step usually begins with assigning ten points to the least important criterion. Relative importances of other criteria are then evaluated by giving them points from ten upwards.

2.2.5 Kano's Model

Kano's model provides a way of classifying importance among the attributes of alternatives [63]. There are three types of product attributes in Kano's model: must-be attributes, one-dimensional attributes, and attractive attributes.

- **Must-be attributes**: The must-be attributes are the basic requirement of the product. The consumer regards these attributes as prerequisites. Their fulfillment will not increase consumer's satisfaction; however, if the product does not have these attributes, the customer will become extremely dissatisfied.

- **One-dimensional attributes**: The one-dimensional attributes have proportional satisfaction degree with regard to their fulfillment level. The consumer has more satisfaction with better attributes.

- **Attractive attributes**: The attractive attributes are unique selling points of the product. The consumer will not feel dissatisfaction without them, however, their fulfillment greatly enhances the consumer's expectation and satisfaction.

Each attribute type described above influences customer satisfaction in a different way, as shown in Figure 2.3. As time passes by, the attractive attributes will evolve into one-dimensional ones, and the one-dimensional attributes will evolve into must-be ones, and new attractive attributes will emerge.

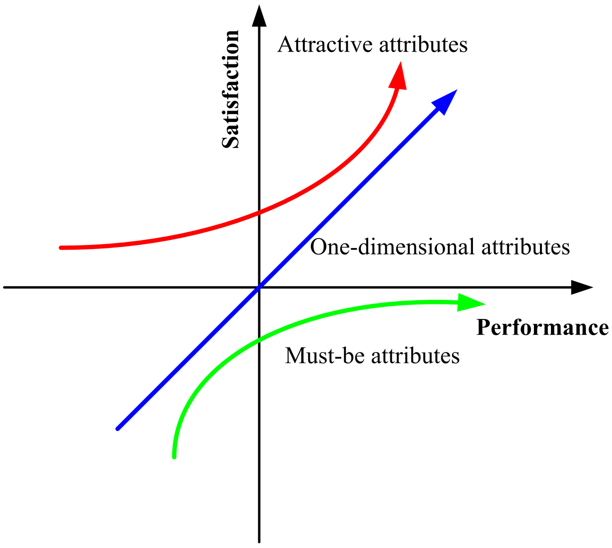


Figure 2.3: Attributes Classification in Kano's Model [16]

2.2.6 Distance-to-target Method

The distance-to-target method is widely applied in the field of Life Cycle Assessment (LCA), which describes the environmental impacts associated with a product, process, or service by multi-attribute product evaluations [86]. The distance-to-target method derives the weights from the distance between the current levels of the criteria and the future target values [2]. This method ranks impacts as being more important the further away from achieving the desired standard [112]. The total environmental impact caused by a product system a is calculated by

Equation 2.5 [112].

$$I(a) = \sum_{j=1}^{m} \frac{L_j^N}{L_j^T} * \frac{1}{L_j^T} * c * L_j(a) \tag{2.5}$$

where $I(a)$ represents the total environmental impact result caused by product system a, L_j^N stands for the actual level of environmental intervention j related to a given region, L_j^T represents the target level of environmental intervention j related to a given region, $L_j(a)$ is the amount of environmental intervention j caused by product system a, and c is a constant.

However, it has been pointed out the setting of the targets is seldom transparent, they may vary between countries and may be obtained more politically rather than scientifically [74]. It was suggested to use a correction factor which indicates the relative significance of an impact category regarding other impact categories within a given region [70].

2.3 Typical Non-compensatory Decision Analysis Methods

Non-compensatory decision analysis methods do not permit trade-offs among criteria, that is, a disadvantage in one criterion cannot be offset by an advantage in other criterion. The non-compensatory methods are credited for their simplicity. As summarized in Table 2.1, typical non-compensatory methods are explained in detail in the following subsections.

2.3.1 Conjunctive Method

The DM sets up the acceptable minimal criteria values. Any alternative which has a criterion value less than the standard level will be rejected [58]. When bigger criteria values are preferred, the i-th alternative A_i $(i = 1, 2, ..., m)$ is classified as an acceptable alternative only if

$$x_{ij} \geq x_j^0, \quad j = 1, 2, ..., n \tag{2.6}$$

where x_j^0 is the standard level of the j-th criterion x_j. The cutoff values play a key role in eliminating the alternatives; if too high, none is left; if relatively low, several alternatives are left after filtering. Hence, increasing the minimal standard levels in an iterative way, the alternatives can be narrowed down to a single choice.

The Conjunctive method does not require the criteria to be in numerical form, and the relative importance of the criteria is not needed. This method is usually used for dichotomizing alternatives into acceptable and unacceptable categories.

2.3.2 Disjunctive Method

In the Disjunctive method, an alternative is evaluated on its greatest value of a criterion [58]. When bigger criteria values are preferred, the i-th alternative A_i $(i = 1, 2, ..., m)$ is classified as an acceptable alternative only if

$$x_{ij} \geq x_j^0, \quad j = 1 \text{ or } 2 \text{ or } ... \text{ or } n \tag{2.7}$$

where x_j^0 is the desirable level of the j-th criterion x_j.

As with the Conjunctive method, the Disjunctive method does not require the criteria to be in numerical form, and it does not need information on the relative importance of the criteria.

2.3.3 Dominance Method

The Dominance method can be used to screen the alternatives in order to obtain a set of non-dominated solutions before the final choice. The procedures of the Dominance method are described as follows [26].

- Compare the first two alternatives and if one is dominated by the other, discard the dominated one.
- Next, compare the retained alternative with the third alternative and discard any dominated alternative.
- Then, compare the fourth alternative and so on.
- After all the alternatives are compared, the non-dominated set is determined.

The Dominance method does not require any assumption or any transformation of criteria. The non-dominated set usually has multiple alternatives, hence, the Dominance method is mainly used for initial filtering.

2.3.4 ELECTRE

ELECTRE (Elimination and Choice Translation Reality) methods use the concept of outranking relation introduced by Benayoun [15]. For instance, suppose there are m alternatives based on n evaluation criteria, with weighting factors $[w_1, w_2, ..., w_n]$, x_{ij} stands for the value of criterion x_j with respect to alternative A_i. An outranking relation between alternative A_k and alternative A_l $(k, l = 1, 2, ..., m, k \neq l)$ is defined as: A_k is preferred to A_l when A_k is at least as good as A_l with respect to a majority of criteria and when A_k is not significantly poor regarding any other criteria. After the assessment of the outranking relations for each pair of alternatives, dominated alternatives can be eliminated and non-dominated alternatives can be obtained for further consideration.

There are several different versions of ELECTRE methods, including ELECTRE I, IS, II, III, IV and TRI [106], [33]. ELECTRE I is the first decision analysis method using the concept of outranking relation, the other versions of ELECTRE methods are extensions of ELECTRE I. In this subsection, the stepwise calculations of ELECTRE I are described in detail and the other ELECTRE methods are briefly introduced.

ELECTRE I is composed of the following nine steps [58].

1. Normalize the decision matrix

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}, r_{ij} = \frac{x_{ij}}{\sqrt{\sum\limits_{i=1}^{m} x_{ij}^2}}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.8}$$

2. Calculate the weighted normalized decision matrix.

$$V = RW = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \begin{bmatrix} w_1 & & & \\ & w_2 & & \\ & & \ddots & \\ & & & w_n \end{bmatrix} \tag{2.9}$$

3. Determine the concordance and discordance sets.

   For each pair of alternatives A_k and A_l, the set of decision criteria $J = (j \mid j = 1, 2, ..., n)$ is divided into two disjoint subsets. The concordance set C_{kl} of A_k and A_l is composed of all criteria which support that A_k is preferred to A_l. The discordance set D_{kl} is the complementary subset of the concordance set C_{kl}.

$$\begin{array}{l} C_{kl} = \{ j \left| x_{kj} \ge x_{lj} \right. \}, \ (k, l = 1, 2, ..., m, \text{and } k \ne l) \\ D_{kl} = \{ j \left| x_{kj} < x_{lj} \right. \} = J - C_{kl} \end{array} \tag{2.10}$$

4. Calculate the concordance matrix C.

   Each element of the concordance matrix C is calculated by the sum of the criteria weights which are contained in the concordance set. For example, the element c_{kl} between A_k and A_l is calculated by Equation 2.11.

$$C = \begin{bmatrix} - & c_{12} & \dots & c_{1n} \\ c_{21} & - & \dots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \dots & - \end{bmatrix}$$

$$c_{kl} = \sum_{j \in C_{kl}} w_j \tag{2.11}$$

5. Calculate the discordance matrix D.

   Each element of the discordance matrix D reflects the degree to which one alternative is worse than the other. For instance, the element d_{kl} between A_k and A_l is calculated by Equation 2.12.

$$D = \begin{bmatrix} - & d_{12} & \dots & d_{1n} \\ d_{21} & - & \dots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \dots & - \end{bmatrix}$$

$$d_{kl} = \frac{\max\limits_{j \in D_{kl}} |v_{kj} - v_{ij}|}{\max\limits_{j \in J} |v_{kj} - v_{ij}|} \tag{2.12}$$

   It should be noticed that differences among weighting factors are contained in the concordance matrix C, while differences among criteria values are reflected in the discordance matrix D.

6. Determine the concordance dominance matrix.

   A concordance threshold c needs to be chosen to perform the concordance test. Alternative A_k possibly dominates alternative A_l, if the element c_{kl} exceeds at least a certain threshold c, that is, $c_{kl} \geq c$.

   In ELECTRE I, a Boolean matrix is used to convert the concordance test into numerical values (0 or 1). If the concordance test is passed ($c_{kl} \geq c$), then the element is 1. Otherwise, if the concordance test is failed ($c_{kl} < c$), the element is 0.

7. Determine the discordance dominance matrix.

   A discordance threshold d needs to be chosen to perform the discordance test. Alternative A_k possibly dominates alternative A_l, if the element d_{kl} is smaller than a certain threshold d, that is, $d_{kl} \leq d$.

As with the case of the determination of the concordance dominance matrix, the discordance test is converted into numerical values (0 or 1) by a Boolean matrix. The element is 1 when the discordance test is passed ($d_{kl} \leq d$), and it is 0 when the discordance test is failed ($d_{kl} > d$).

8. Aggregate the dominance matrix.

   An outranking relation can be justified only if both the concordance test and the discordance test are passed. That is, $c_{kl} \geq c$ and $d_{kl} \leq d$. The aggregated dominance matrix is calculated by an element-to-element product of the concordance dominance matrix and the discordance dominance matrix.

9. Eliminate the dominated alternatives.

   The aggregated dominance matrix gives the partial preference of the alternatives. In the aggregated dominance matrix, the element 1 in the column indicates that this alternative is dominated by other alternatives. Thus, any alternative which has at least one element of 1 in the column can be eliminated.

ELECTRE I is widely used because of its simple logic and refined computational procedures. However, the two concordance and discordance threshold values have significant impact on the final results. Additionally, the calculation procedures will become more complex as the size of decision matrix increases.

An Aircraft Selection Example using ELECTRE I

An aircraft selection example is presented to show how to use ELECTRE I in this subsection. Suppose that the DMs of an airline consider to purchase an aircraft among three competing aircraft, with the consideration of three criteria: comfort, cost, and environmental friendliness. Smaller value of cost is preferred, while bigger values of comfort and environmental friendliness are preferred. A ten-point score is assigned to the three criteria for each alternative, respectively. The weighting factors among the three criteria are [0.3 0.4 0.3]. The decision matrix is summarized in Table 2.5.

Given the decision matrix shown in Table 2.5, going through the described nine-step calculations of ELECTRE I, the aggregated dominance matrix is shown in matrix M.

$$M = \begin{array}{cc} & \text{is dominated by} \\ \text{dominates} & \begin{bmatrix} - & 0 & 1 \\ 0 & - & 0 \\ 0 & 0 & - \end{bmatrix} \end{array}$$

Table 2.5: Decision Matrix of an Aircraft Selection Example using ELECTRE I

	Criteria		
	C_1: Comfort	C_2: Cost	C_3: Environmental friendliness
Alternatives	w_1: 0.3	w_2: 0.4	w_3: 0.3
Aircraft A	8	7	10
Aircraft B	9	6	5
Aircraft C	6	7	8

In the aggregated dominance matrix M, the element 1 in the column indicates that this alternative is dominated by other alternatives. Thus, Aircraft C is dominated by Aircraft A and Aircraft B. In another words, Aircraft A and Aircraft B are non-dominated alternatives. Therefore, in this aircraft selection example using ELECTRE I, Aircraft C should be eliminated from the candidate alternatives, Aircraft A and Aircraft B can be recommended for further consideration.

ELECTRE IS

ELECTRE IS is similar to ELECTRE I, except that in Step 6 (Determine the concordance dominance matrix), instead of Boolean numbers (0 or 1), interval values between 0 and 1 are used [106], [33]. In order to discriminate the alternatives, two thresholds have to be defined for each criterion: indifference threshold and strict preference threshold.

ELECTRE II

ELECTRE II is also similar to ELECTRE I. The main difference is the definition of two outranking relations: strong outranking and weak outranking [106]. For each criterion, two strong outranking thresholds and one weak outranking threshold have to be defined.

ELECTRE III

ELECTRE III uses the same principle of ELECTRE II. For each criterion, an indifference threshold, a preference threshold, and a veto threshold have to be defined in order to compare the alternatives. Both the concordance dominance matrix and discordance dominance matrix are constructed by interval values between 0 and 1. The aggregation of the concordance dominance matrix and discordance dominance matrix is obtained by a credibility matrix. The final classification of alternatives is based on ascending and descending distillations [106], [33].

ELECTRE IV

Unlike the previously described ELECTRE methods, ELECTRE IV does not require criteria weights in the calculation procedures. Instead, it uses the number of criteria in different preference areas [106]. For each criterion, an indifference threshold, a preference threshold, and a veto threshold are required in order to compare the alternatives. Similar to ELECTRE III, a credibility matrix is calculated, and the classification of alternatives is based on ascending and descending distillations.

ELECTRE TRI

In ELECTRE TRI, some reference alternatives are introduced, all alternatives are compared to these reference alternatives [106]. Similar to ELECTRE III, a credibility matrix is computed with respect to reference alternatives. The outranking relations between candidate alternatives and reference alternatives are established using the credibility matrix and a veto threshold. ELECTRE TRI can reduce the computational cost of alternative comparisons when the number of alternatives is large.

Summary of ELECTRE Methods

The main characteristics of all versions of ELECTRE methods were summarized by Roy [106], as shown in Table 2.6. Considering different problem statements, some guidelines on how to choose among ELECTRE methods were also suggested. For instance, if it is truly essential to work with a very simple method and it is realistic to have no information on the indifference threshold and preference threshold, ELECTRE I should be selected in order to eliminate the non-dominated alternatives, while ELECTRE II should be used in order to build a partial preorder of alternatives. ELECTRE IV would be convenient only if there exists a good reason to refuse the introduction of importance coefficients. In general, ELECTRE IS, II, III, IV, and TRI do provide powerful support for the classification of the alternatives. However, they require too many threshold definitions from DMs, thus, it is rather complex to implement these methods in real world problems [87].

2.3.5 Elimination by Aspects Method

In this method, the DM is assumed to have minimum cutoffs for each criterion. A criterion is selected, and all alternatives which do not pass the cutoff on that criterion are eliminated. Then another criterion is selected, and so forth. The process continues until all alternatives but one are eliminated [58].

Table 2.6: Main Characteristics of ELECTRE Methods [106]

ELECTRE methods	I	IS	II	III	IV	TRI
Require indifference and preference thresholds	no	yes	no	yes	yes	yes
Require criteria weights	yes	yes	yes	yes	no	yes
Outranking relations	binary	binary	strong and weak	interval values	strictly, weakly, hardly preferred, or indifferent	interval values

The elimination by aspects method eliminates alternatives which do not satisfy some standard level, and it continues until all alternatives except one have been eliminated. However, only small part of the information is used when comparing the alternatives.

2.3.6 Lexicographic Method

In the Lexicographic method, the DM compares the alternatives on the most important criterion. If one alternative has a better criterion value than all the other alternatives, the alternative is chosen and the decision process ends. However, if some alternatives are tied on the most important criterion, the subset of tied alternatives is then compared on the second most important criterion. The process continues sequentially until a single alternative is chosen or until all the criteria have been considered.

The Lexicographic method does not require comparability across criteria, and the preference information on the criteria is not necessarily in numerical values. However, it only utilizes a small part of the available information in making a final decision.

2.3.7 Maximin Method

In the Maximin method, the overall performance of an alternative is determined by the weakest or poorest criterion. The DM examines the criteria values for each alternative, notes the worst value for each alternative, and then selects the alternative with the most acceptable value in its worst criterion. It is the selection of the maximum (across alternatives) of minimum (across criteria) values [58]. Mathematically speaking, the alternative A^* is selected such that

$$A^* = \left\{ A_i \middle| \max_i \min_j r_{ij} \right\}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.13}$$

where r_{ij} are normalized criteria values, and bigger criteria values are preferred.

2.3.8 Maximax Method

In contrast to the Maximin method, the Maximax method selects an alternative by its best criterion value rather than its worst criterion value. In this method, the best criterion value for each alternative is identified, then these maximum values are compared in order to select the alternative with the best value [58]. Mathematically speaking, the alternative A^* is selected such that

$$A^* = \left\{ A_i \middle| \max_i \max_j r_{ij} \right\}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.14}$$

where r_{ij} are normalized criteria values, and bigger criteria values are preferred.

The Maximin method and the Maximax method utilize only one criterion per alternative in making a final choice. The two methods are widely used in game theory, however, their applicability in other fields is relatively limited.

2.4 Typical Compensatory Decision Analysis Methods

Compensatory decision analysis methods permit trade-offs among criteria, that is, small changes in one criterion can be offset by opposing changes in any other criteria. As summarized in Table 2.1, typical compensatory methods are explained in detail in the following subsections.

2.4.1 Analytic Hierarchy Process

Analytic Hierarchy Process (AHP) was proposed to deal with decision making problems that have hierarchical structures of attributes [108]. AHP is based on the idea of translating the hierarchical decision making problem into a series of pairwise comparison matrices and obtaining the preference information for the attributes using eigenvector method.

As one popular preference information elicitation technique, the eigenvector method is explained in Subsection 2.2.2. The first part of this subsection introduces pairwise comparison matrix, followed by computational steps of AHP.

Pairwise Comparison Matrix

The pairwise comparison concept originated from an experiment considering the subject of stimuli and responses performed by Weber in 1846. Weber stated that change in sensation was noticed when the stimulus was increased by a constant percentage of the stimulus itself. A nine-point scale based on Weber's law was created and shown in Table 2.7.

Suppose there are m alternatives and n criteria in a given problem. A pairwise comparison matrix is a m by m matrix, whose element y_{ij} indicates the DM's preference information of

Table 2.7: Pairwise Comparison Scale [108]

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective.
3	Moderate importance of one over another	Experience and judgment slightly favor one activity over another.
5	Strong importance	Experience and judgment strongly favor one activity over another.
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice.
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation.
Reciprocals of above	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i.	A reasonable assumption.

alternative i over alternative j for a given criterion. In total, there are n $m \times m$ comparison matrices, as shown in matrix M.

$$M = \begin{bmatrix} 1 & y_{12} & \cdots & y_{1m} \\ y_{21} & 1 & \cdots & y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & 1 \end{bmatrix}$$

Computational Steps of AHP

1. Establish the decision making problem in a hierarchy structure.
2. Formulate the pairwise comparison matrix for elements at a single level of the hierarchy, with respect to each of the elements at a level immediately above.
3. Generate the weights of elements using the eigenvector method, as described in Subsection 2.2.2.
4. The alternative with a larger relative value is more favorable.

AHP provides a simple way to formulate a decision making problem and to elicit preference information, as it only requires pairwise comparisons between criteria or alternatives. However, it has some limitations. The preference independence among all elements at any level except for the bottom level is assumed. It would be problematic to use AHP if the criteria at the same level have correlated dependence. Another limitation is that the pairwise comparison matrix is required with each element describing the relative importance of a criterion over all other criteria, or the relative preference of an alternative over all other alternatives. The complete pairwise comparison is not a trivial task for the DM and may trigger inconsistency problems. These problems will become worse with increasing size of pairwise comparison matrix.

2.4.2 Expected Utility Theory

Expected utility can be dated back to Daniel Bernoulli's resolution to the St. Petersburg paradox in 1738 [35], [38]. The rule of the St. Petersburg game is that the player tosses a fair coin until *head* shows up for the first time, if this occurs at the k-th toss, the payoff is 2^k guilders. The expected monetary value is $\sum\limits_{i=1}^{n}(\frac{1}{2})^k 2^k = 1+1+1+...... = \infty$. The people were asked how much they would pay for the game? However, the paradox is that no reasonable people would want to pay even small amount of money for the game with infinite expected value.

Bernoulli used a logarithmic utility index defined over wealth to compute a finite price for a gamble with an unbounded expected value, with the argumentation that the people estimate the game in terms of the utility of money outcomes, and the marginal utility is diminishing. For a person with present wealth a, the expected utility of the game is calculated by Equation 2.15 [38].

$$\sum_i p_i \log(a + x_i) \tag{2.15}$$

where p_i is the probability of the i-th game, and x_i is the outcome of the i-th game.

The value of the game with fixed amount v is calculated by $\log(a+v) = \sum\limits_i p_i \log(a+x_i)$ and is shown in Equation 2.16 [38].

$$v = \prod_i (a + x_i)^{p_i} - a \tag{2.16}$$

Expected utility theory states that the DM chooses between risky prospects by comparing their expected utility values, which are calculated by the weighted sum of utility values of outcomes multiplied by their probabilities, as shown in Equation 2.17.

$$E(u|p, X) = \sum_{x \in X} p(x)u(x) \tag{2.17}$$

where x is a particular outcome from the set of all possible outcomes X, $p(x)$ is the probability of the particular come, $u(x)$ is its utility function.

Expected utility theory is suitable for decision making problems with risk and uncertainty. However, it is difficult to obtain an accurate utility function for each criterion, and the consistency of the utility functions among different criteria is hard to maintain.

2.4.3 Multi-Attribute Utility Theory

This method is based on the concept of utility function, which represents a mapping from the DM's preference into a mathematical function [65]. The most widely used form is the additive multi-attribute utility method given by Equation 2.18, with two assumptions stating that utility functions of all attributes are independent and the weighting factor of an attribute can be determined regardless of the weighting factors of other attributes.

$$U\left(x_1, x_2, ..., x_n\right) = \sum_{i=1}^{n} w_i u_i\left(x_i\right) \tag{2.18}$$

where w_i is the weighting factor of the i-th attribute x_i, $u_i(x_i)$ is its utility function.

The additive multi-attribute utility theory provides utility function to represent the DM's preference information. However, the two assumptions including the independence of utility function and weighting factors do not hold true for many practical decision making problems, which limits the use of this method.

2.4.4 Multiplicative Weighting Method

In this method, weighting factors $[w_1, w_2, ..., w_n]^{\mathrm{T}}$ are assigned to the criteria by the DM, the criteria values for each alternative are multiplied, with the weighting factors as exponents. This method chooses the most preferred alternative which has the biggest value, as shown in Equation 2.19, when bigger criteria values are preferred.

$$A^* = \left\{ A_i \left| \max_i \prod_{j=1}^{n} x_{ij}^{w_j} \right. \right\}, i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.19}$$

Considering the exponentiation property, all criteria values should be greater than one in order to assure its monotonicity. When criteria values are smaller than one, 10^k should be multiplied, where k is an exponent which make the smallest criterion value bigger than one.

2.4.5 PROMETHEE

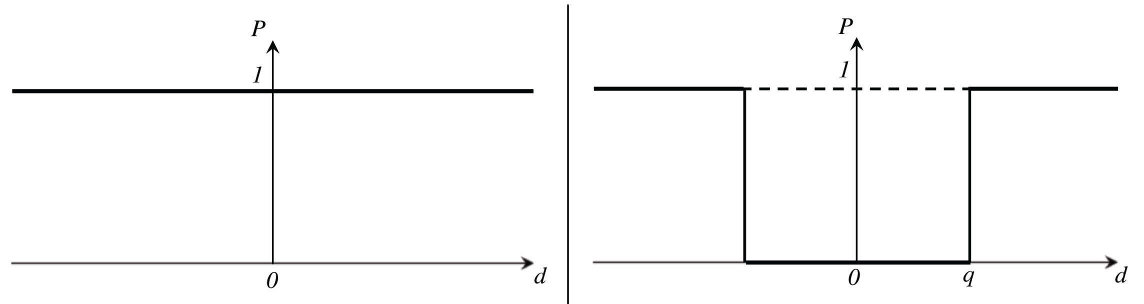
In PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations) method [24], [25], a valued preference relationship based on a generalization of the notion of criteria is constructed first, and a preference index is defined, then a valued outranking graph is obtained. According to the preference index, PROMETHEE I provides a partial preorder and PROMETHEE II offers a complete preorder on all actions (alternatives).

Criteria Generalization

The valued preference relationship between two actions a and b is defined as follows [25].

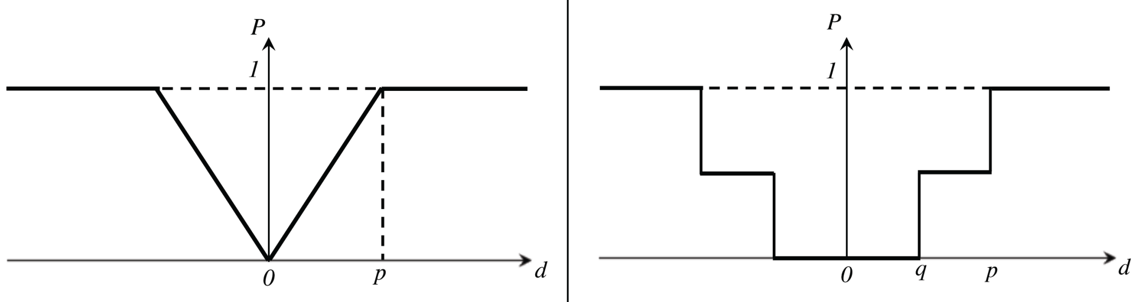
- $P(a, b) = 0$ means an indifference between a and b.
- $P(a, b) \approx 0$ means weak preference of a over b.
- $P(a, b) \approx 1$ means strong preference of a over b.
- $P(a, b) = 1$ means strict preference of a over b.

For each criterion, a generalized criterion and a corresponding preference function are considered. In PROMETHEE, six types of generalized criteria are provided, as illustrated in Figure 2.4, where d is the difference between two criteria, p is the strict preference threshold, and q is the indifference threshold, s is the standard deviation in Gaussian distribution.



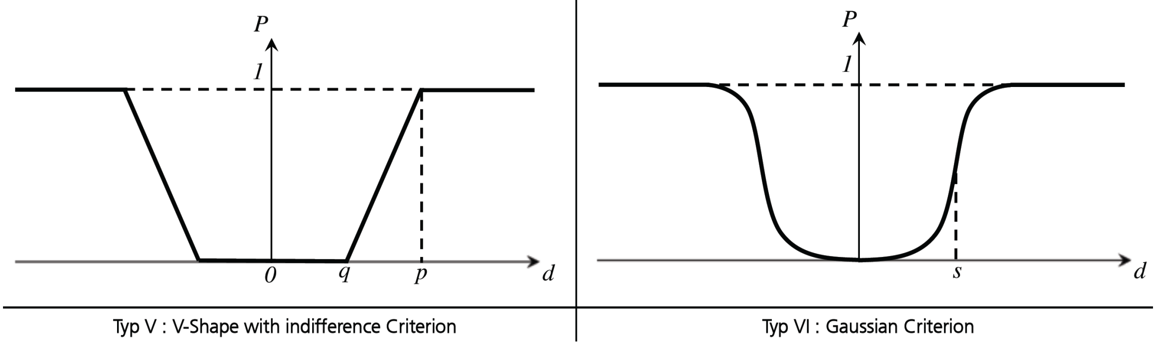
Typ I : Usual Criterion

Typ II : U-Shape Criterion



Typ III : V-Shape Criterion

Typ IV : Level Criterion



Typ V : V-Shape with indifference Criterion

Typ VI : Gaussian Criterion

Figure 2.4: Six Types of Generalized Criteria [25]

Multi-Criteria Preference Index

The multi-criteria preference index of action a over action b, denoted by $\Pi(a, b)$, is defined as in Equation 2.20.

$$\Pi(a, b) = \sum_{i=1}^{n} w_i P_i(a, b) \tag{2.20}$$

where n is the number of criteria, w_i is the weighting factor of the i-th criterion, and P_i is the preference function of the i-th criterion. The multi-criteria preference index ranges from 0 to 1, with $\Pi(a, b) \approx 0$ represents a weak preference of action a over action b, and $\Pi(a, b) \approx 1$ represents a strong preference of action a over action b.

PROMETHEE Rankings

A positive outranking flow is defined by Equation 2.21 and a negative outranking flow is defined by Equation 2.22, respectively. Besides, a net outranking flow is calculated by Equation 2.23.

$$\Phi^+(a) = \sum_{b \in A} \Pi(a, b) \tag{2.21}$$

$$\Phi^-(a) = \sum_{b \in A} \Pi(b, a) \tag{2.22}$$

$$\Phi(a) = \Phi^+(a) - \Phi^-(a) \tag{2.23}$$

Based on Equation 2.21 and Equation 2.22, PROMETHEE I provides a partial preorder by considering the intersection of the positive outranking flow and negative outranking flow, which is listed as follows.

- Action a outranks action b, if $\Phi^+(a) \geq \Phi^+(b)$ and $\Phi^-(a) \leq \Phi^-(b)$.
- Action a is indifferent from action b, if $\Phi^+(a) = \Phi^+(b)$ and $\Phi^-(a) = \Phi^-(b)$.
- Otherwise, action a and action b are incomparable.

Based on Equation 2.23, PROMETHEE II considers action a outranks action b if $\Phi(a) > \Phi(b)$, and action a is indifferent from action b if $\Phi(a) = \Phi(b)$.

The six types of preference functions and the partial or complete preorder in PROMETHEE provide the DM more insights in solving the given problem. However, in order to define the preference function, it requires too many threshold parameters. Moreover, these threshold parameters are rather subjective and different DMs often have different threshold values, which increases the complexity of the problem significantly.

2.4.6 Simple Additive Weighting

In Simple Additive Weighting (SAW) method [58], weighting factors $[w_1, w_2, ..., w_n]^{\mathrm{T}}$ are assigned to the criteria by the DM. The multiple criteria values together with their weighting factors are aggregated into a single performance metric. SAW chooses the most preferred alternative A^* which has the maximum weighted outcome, as shown in Equation 2.24, where bigger criteria values are preferred.

$$A^* = \left\{ A_i \,\middle|\, \max_i \sum_{j=1}^{n} w_j x_{ij} \right\}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.24}$$

SAW is one of the most widely used decision analysis methods because of its simplicity. However, it also has some limitations. SAW requires all criteria values to be both numerical and comparable, which will trigger the quantification problem for the qualitative criteria and normalization problem for all the elements in decision matrix. The quantification methods and normalization methods have a significant influence on the final decision results. Moreover, SAW is sensitive to the weighting factors.

2.4.7 TOPSIS

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is based on the idea that the chosen alternative should have the shortest distance to the positive ideal solution A^* and the furthest distance from the negative ideal solution A^-. The distance is in the form of Euclidean distance [58], as shown in Figure 2.5.

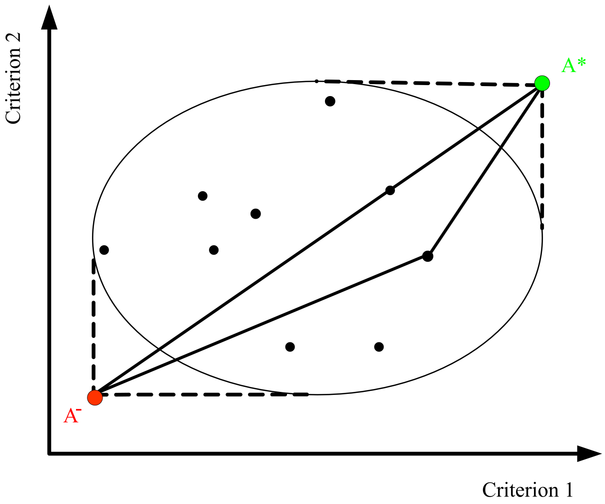


Figure 2.5: TOPSIS Method [58]

TOPSIS requires a decision matrix and weighting factors as input data, its computational steps are summarized as follows.

1. Normalize the decision matrix.

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.25}$$

2. Calculate the weighted normalized decision matrix.

$$r_{ij} = w_j z_{ij}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.26}$$

3. Identify the positive ideal solution A^* and the negative ideal solution A^-.

$$A^* = \left\{ \left(\max_i r_{ij} | j \in J \right), \left(\min_i r_{ij} | j \in \hat{J} \right) | i = 1, 2, ..., m \right\} = \{x_1^*, x_2^*, ..., x_n^*\} \tag{2.27}$$

$$A^- = \left\{ \left(\min_i r_{ij} | j \in J \right), \left(\max_i r_{ij} | j \in \hat{J} \right) | i = 1, 2, ..., m \right\} = \left\{x_1^-, x_2^-, ..., x_n^-\right\} \tag{2.28}$$

where J is the benefit criteria set (bigger criterion value is preferred), and $\hat{J}$ is the cost criteria set (smaller criterion value is preferred). Thus, the positive ideal solution is composed of the maximum values of benefit criteria and the minimum values of cost criteria; while the negative ideal solution is composed of the minimum values of benefit criteria and the maximum values of cost criteria.

4. Calculate the distance for each alternative to the positive ideal solution and the negative ideal solution, respectively.

$$S_i^* = \sqrt{\sum_{j=1}^{k} \left(r_{ij} - x_j^* \right)^2}, \quad i = 1, 2, ..., m \tag{2.29}$$

$$S_i^- = \sqrt{\sum_{j=1}^{k} \left(r_{ij} - x_j^- \right)^2}, \quad i = 1, 2, ..., m \tag{2.30}$$

5. Calculate the relative closeness for each alternative to the ideal solutions.

$$C_i^* = \frac{S_i^-}{S_i^- + S_i^*}, \quad i = 1, 2, ..., m \tag{2.31}$$

6. Rank the alternatives according to the value of C_i^* .

   TOPSIS suggests the best alternative which has the furthest distance from the negative ideal solution (biggest value of S_i^-) and shortest distance to the positive ideal solution (smallest value of S_i^*). Thus, the increase of numerator and the decrease of denominator will lead to a bigger value of C_i^* in Equation 2.31. In other words, the alternative which maximizes the value of C_i^* ranks first.

Furthermore, in addition to Equation 2.31, the relative closeness of each alternative to the ideal solutions could be also aggregated by Equation 2.32.

$$C_i^- = \frac{S_i^*}{S_i^* + S_i^-}, \quad i = 1, 2, ..., m \tag{2.32}$$

where the decrease of numerator and the increase of denominator will result in a smaller value of C_i^-. Thus, the alternative which minimizes the value of C_i^- ranks first. Besides, since the sum of C_i^* and C_i^- is one, it is sufficient to compute one of them, and the other one can be inferred easily.

Another approach is to visualize the relative closeness of each alternative to the ideal solutions via Pareto frontier, as illustrated in Figure 2.6, where the horizontal axis represents the distance to the positive ideal solution (S_i^*), while the vertical axis stands for the distance to the negative ideal solution with minus signal ($-S_i^-$). The minus signal is used to convert the preference direction of S_i^- for the convenience of displaying Pareto frontier.

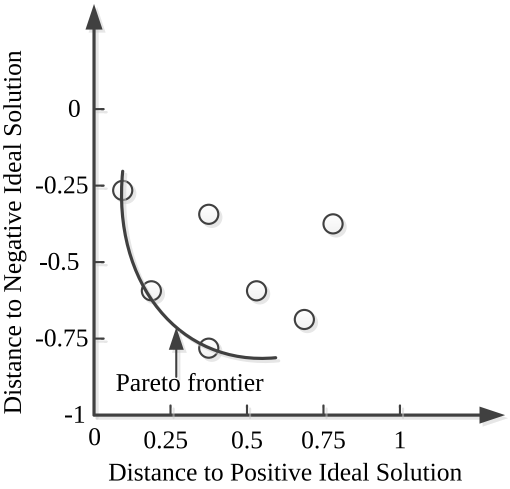


Figure 2.6: Pareto Frontier for Relative Closeness to Ideal Solutions in TOPSIS

The Pareto frontier approach does not need to aggregate the relative closeness, however, instead of one best alternative, a set of non-dominated alternatives is often obtained.

TOPSIS is one of the widely used compensatory decision analysis methods considering its simplicity and systematic calculation procedures. However, TOPSIS assumes that each criterion's utility is monotonic, which is not appropriate for problems where a particular criterion value is desired to be achieved [58]. TOPSIS is also rather sensitive to the weighting factors.

An Aircraft Selection Example using TOPSIS

In this subsection, TOPSIS is used in an aircraft selection example, as described in Subsection 2.3.4. The decision matrix is repeated in Table 2.8 for the convenience of calculation.

Table 2.8: Decision Matrix of an Aircraft Selection Example using TOPSIS

	Criteria		
	C_1: Comfort	C_2: Cost	C_3: Environmental friendliness
Alternatives	w_1: 0.3	w_2: 0.4	w_3: 0.3
Aircraft A	8	7	10
Aircraft B	9	6	5
Aircraft C	6	7	8

Given the decision matrix summarized in Table 2.8, going through the described six-step calculations of TOPSIS, the relative closeness aggregated by Equation 2.31 is shown in vector C^*. Considering that the alternative which maximizes the value of C^* ranks first, Aircraft A is recommended as the best alternative.

$$C^* = \begin{bmatrix} 0.5175 \\ 0.4866 \\ 0.5043 \end{bmatrix}$$

Furthermore, the relative closeness aggregated by Equation 2.32 is shown in vector C^-. In this case, the alternative which has the smallest value of C^- ranks first. Therefore, Aircraft A is ranked as the best alternative.

$$C^- = \begin{bmatrix} 0.4825 \\ 0.5134 \\ 0.4957 \end{bmatrix}$$

The Pareto frontier for the relative closeness to the ideal solutions is illustrated in Figure 2.7. It can be observed that Aircraft A is a non-dominated alternative.

In summary, in this aircraft selection example using TOPSIS, three approaches of representing the relative closeness for each alternative to the ideal solutions (aggregation by Equation 2.31 and Equation 2.32, and visualization via Pareto frontier), generate the consistent result that Aircraft A is the best alternative for the DMs to consider among the three candidate aircraft.

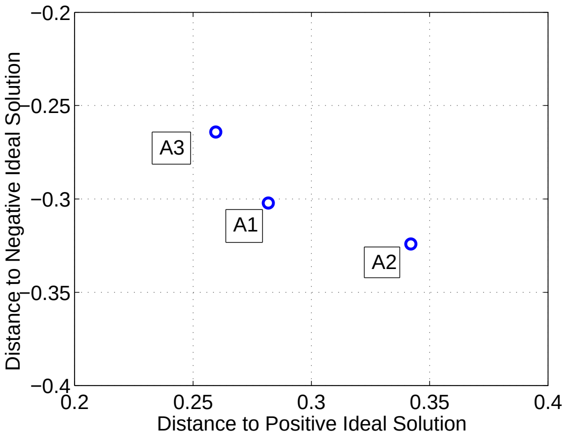


Figure 2.7: Pareto Frontier for Relative Closeness to Ideal Solutions in an Aircraft Selection Example

2.5 Emerging MCDA Techniques Interacting with Other Disciplines

MCDA discipline continues its growth in the development of new approaches and methodologies, especially through the interactions with other disciplines, such as problem structuring methods and evolutionary optimization algorithms [139]. This section introduces these new emerging MCDA techniques. More comprehensive review of the new trends in MCDA can be found in [44].

2.5.1 Problem Structuring Methods and MCDA

Most MCDA literature focused on analysis methods and took a well-structured problem as a starting point, with the assumptions that the alternatives have been well-defined and a coherent set of evaluation criteria has been identified [14]. It is an erroneous impression that arriving this point is a relatively trivial task, while in reality this is not so simple [44]. In order to enable effective multi-criteria analysis and appropriately formulate the multi-criteria decision problem, the problem structuring methods can be applied to provide a rich representation of a problematic situation and conceptualize a decision which is initially simplistically presented.

Lots of research has been conducted on problem structuring for MCDA. One approach is to implement problem structuring within the existing MCDA framework, such as value focused thinking proposed by Keeney [64]. Before the evaluation of candidate alternatives and the selection of a preferred one, value focused thinking stresses the importance of understanding the DM's values and objectives and using them as the basis for creative generation of alternatives. The decision frame, objectives, and alternatives are three components need to be coherently specified. Keeny also compared value focused thinking against alternative focused thinking.

Alternative focused thinking starts from a specified set of alternatives and identifies values based on these alternatives. Corner et al. proposed a dynamic decision problem structuring framework to advocate a continuing process of iteration between value focused thinking and alternative focused thinking [34], as illustrated in Figure 2.8. The consideration of values and objectives prompts new alternatives, while in turn the reflection of alternatives contributes to new values and objectives. This iterative process helps DMs to learn about the problem context and reflect on their values.

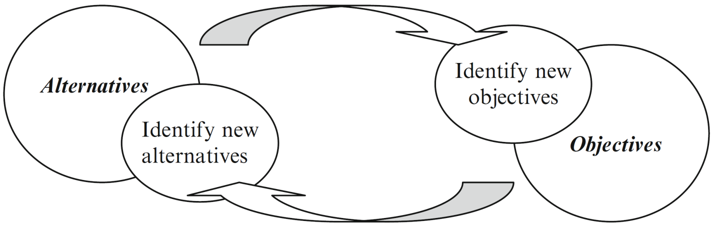


Figure 2.8: The Framework of Dynamic Decision Problem Structuring [34]

Rosenhead and Mingers outlined five principal methods for problem structuring: strategic options development and analysis, soft systems methodology, strategic choice approach, robustness analysis, and drama theory [103]. These five methods are all based on the UK communities of system thinking. The key features of the five principal methods are summarized as follows [103], [44] .

- **Strategic options development and analysis**: begins with a process of idea generation, seeks to capture and structure the complexity of an issue reflected by multiple perspectives.
- **Soft systems methodology**: uses rich pictures, root definitions and conceptual models to explore the issue from a number of different perspectives.
- **Strategic choice approach**: four modes: shaping, designing, comparing, choosing. This method focuses on key uncertainties (about related areas, environment and values) and analysis of interconnected decision options.
- **Robustness analysis**: focuses on identifying options which perform well in all possible futures.
- **Drama theory**: appropriate in multi-party contexts where the outcome is dependent on the inter-dependent actions of the parties. This method seeks to identify stable options.

Another approach is to integrate cognitive/causal mapping with MCDA, such as reasoning maps developed by Montibeller and Belton [90]. Reasoning maps introduce a focused casual map in order to enable the qualitative analysis of the alternatives within the structure of the map directly.

2.5.2 Evolutionary Optimization Algorithms and MCDA

Optimization techniques perform the task of searching for one or more solutions in order to minimize or maximize one or more specified objectives, while satisfying all the constraints. There are at least two equally important tasks: an optimization task for finding Pareto optimal solutions, and a decision making task for choosing a single most preferred solution [23].

Evolutionary algorithms are widely used optimization techniques which are inspired by biological evolution, they operate on a population of candidate solutions and apply the principle of survival of the fittest to evolve the candidate solutions towards the desired optimal solutions [36]. Continuous and discrete variables can be included in evolutionary algorithms simultaneously, where the continuous variables are discretized with a reasonable resolution. Additionally, evolutionary algorithms consider the whole design space, thus, the risk of convergence to a local optimum can be avoided. However, evolutionary algorithms suffer from expensive computation, and different optimization runs may result in different optimal solutions.

The decision making task usually involves the elicitation of the preference information from a DM. There are typically three strategies to incorporate the DM's preference information with optimization techniques [85]: a priori approach, a posteriori approach, and an interactive approach. In the a priori approach, the DM's preferences are utilized to aggregate the multiple objectives into one figure of merit. Then, optimization techniques are applied to search for the most preferred design solution, with the composite figure of merit as a single objective function. In the a posteriori approach, optimization techniques are applied firstly to search for a set of non-dominated solutions, usually in terms of a Pareto front. Then, the DM's preferences are used to select the most preferred design solution among several design alternatives from the Pareto front, taking multiple objectives into consideration simultaneously. In the interactive approach, the DM iteratively specifies the preference information during the optimization process.

The link between evolutionary algorithms and MCDA can be built, considering that MCDA techniques have been developed for solving decision problems with discrete solution alternatives. For instance, one a posteriori approach is to use MCDA techniques to analyze the final population of evolutionary optimization algorithms [23], [123].

3

MCDA Method Selection

The first objective of this research is the development of an intelligent multi-criteria decision support system in order to facilitate the selection of the most appropriate MCDA method for the problem under consideration effectively. In this chapter, with the perspective that the method selection itself is a complicated MCDA problem, twelve evaluation criteria are proposed to assess sixteen widely used MCDA methods. An Appropriateness Index (AI) is used to quantify the goodness of a method for solving the problem under consideration. This method selection approach is implemented and an intelligent multi-criteria decision support system is developed in MATLAB.

The framework of MCDA method selection was originally developed by [72]. In this research, the framework has been successfully improved in order to yield more accurate and reliable solutions [118]. Three major improvements are listed as follows.

1. The distinction between filtering questions and scoring questions. The filtering questions are used to screen out inappropriate methods in the initial step of selection, and scoring questions are used to derive the attributes of a MCDA formulation.
2. The method library is extended to include all sixteen widely used MCDA methods.
3. Most importantly, a newly developed uncertainty assessment module, which is discussed in detail in Chapter 4.

3.1 Method Selection Background

Although MCDA has a relatively short history of about 40 years, over 70 MCDA techniques have been developed for facilitating the decision making process [111],[126],[139]. Among these developed MCDA methods, different methods have different underlying assumptions, analysis models, and decision rules that are designed for solving a certain class of decision making problems. It is critical to select the most appropriate method to solve the problem under consideration since the use of unsuitable methods might lead to misleading decisions. It can be seen that the selection of MCDA methods itself is a complicated MCDA problem [58] and needs to be prudently performed.

Over the past decades, considerable research has been conducted to deal with the selection of the most appropriate MCDA method for a given decision making problem. MacCrimmon firstly recognized the importance of MCDA method selection. He proposed a taxonomy of MCDA methods, created a method specification chart in the form of a tree diagram and provided an illustrative application example [78]. Hwang developed another tree diagram, which consists of nodes and branches connected by choice rules that can be used for selecting the decision making method for a specified problem [58]. Sen and Yang developed similar tree diagrams to help the DM with selecting the appropriate MCDA methods, and the selection was based on the type of preference information elicited [111]. The tree diagram approach provides reasonable classification schemes and is easy to utilize. However, this approach has its own disadvantages: it usually gives two or more MCDA methods rather than the most appropriate method, and it only considers limited types of decision problems, preference information, and available methods. These limitations stop the tree diagram approach from being an effective solution to the method selection problem [102].

Possible criteria for evaluating MCDA methods were proposed as an alternative solution to the method selection problem. Tecle and Duckstein developed an approach based upon a composite programming algorithm which aided in selecting an appropriate MCDA method [121]. They proposed four categories of criteria: DM-related characteristics, method-related characteristics, problem-related characteristics, and solution-related characteristics to evaluate an method. The independent criteria categories enable the DM to conduct the evaluation in a specified order. However, it is difficult to quantify all MCDA methods in terms of these four criteria categories. Besides, by using this approach, different users may get totally different results because the users' knowledge about the MCDA methods has a strong impact on the final results.

Artificial intelligence techniques were employed by Poh and Lu et. al. [100],[77] to help the DM select the most suitable method based on a series of user inputs. Poh suggested a knowledge-based system, which allowed the DM to select the most appropriate method among available eleven multi-attribute decision making methods. Lu et al. proposed an intelligent system, which facilitated selecting the most suitable method among seven multi-objective decision making methods. The knowledge-based intelligent system simplifies the method selection problem with simple questions by allowing direct selection or automated selection based on DM's inputs. However, they do not clearly state the limitations or failure modes of the systems [102].

Although the tree diagrams approach, the criteria approach, and the artificial intelligent systems provide some capabilities to find the suitable decision making methods for a given problem, they have their own disadvantages. Therefore, an advanced approach with more capabilities needs to be developed to facilitate the MCDA method selection.

3.2 An Advanced Approach for Method Selection

In order to effectively select the most appropriate MCDA method for a given decision making problem, a systematic framework is proposed in this study. The proposed approach consists of eight steps: define the problem, define the evaluation criteria, perform initial screening, define the preferences on evaluation criteria, calculate the Appropriateness Index, evaluate the MCDA methods, choose the most suitable method, and conduct sensitivity analysis. This framework is illustrated in Figure 3.1. Each step of the proposed approach to method selection is discussed in detail in the following subsections.

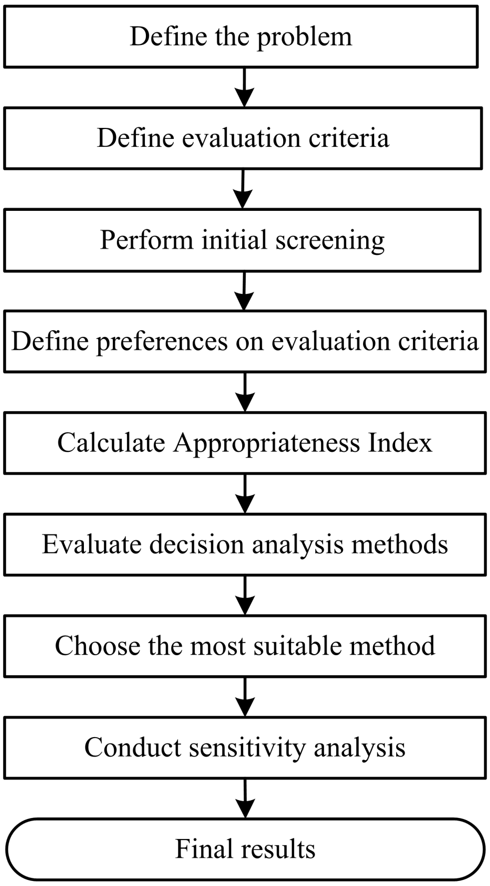


Figure 3.1: An Advanced Approach to MCDA Method Selection

3.2.1 Step 1: Define the Problem

The characteristics of the decision making problem under consideration are addressed in the problem definition step, such as identifying the number of alternatives, attributes, and constraints. The available information about the decision making problem is the basis on which the most appropriate MCDA techniques will be selected and utilized to solve the problem.

3.2.2 Step 2: Define the Evaluation Criteria

The proper determination of the applicable evaluation criteria is important because they have great influence on the outcome of the MCDA method selection process. However, simply using every criterion in the selection process is not the best approach because the more criteria used, the more information is required, which will result in higher computational cost. Therefore, a trade-off has to be made between the accuracy of the results and computational cost. In this study, the characteristics of the MCDA methods are identified by the relevant evaluation criteria in the form of a questionnaire. Twelve questions are defined to capture the advantages, disadvantages, applicability, and computational complexity of each MCDA method.

- **Filtering questions**

  1. Is the method able to handle selection or optimization problems?
  2. Does the method allow trade-offs among criteria?
  3. What input data is required by the method?

- **Scoring questions**

  4. What preference information does the method need?
  5. What decision rule does the method use to rank or sort the alternatives?
  6. Does the method evaluate the feasibility of the alternatives?
  7. Can the method handle any subjective attribute?
  8. Does the method handle qualitative or quantitative data?
  9. Does the method deal with discrete or continuous data?
  10. Can the method handle the problem with hierarchy structure of attributes?
  11. Is the method able to capture uncertainties existing in the problem?
  12. Can the method support visual analytics?

It should be noted that the first three filtering questions are used to screen out inappropriate methods in the initial step of selection, the other nine scoring questions are used as the attributes of a MCDA formulation and as the input data of decision matrix for method selection.

3.2.3 Step 3: Perform Initial Screening

In the initial screening step, the first three filtering questions are utilized to screen out inappropriate methods. For the first filtering question, only scoring MCDA methods are suitable for solving optimization problems since the scores aggregated by MCDA methods can serve as objective functions in the optimization process, while classification MCDA methods, such as ELECTRE, are not suitable since they cannot offer objective functions for optimization.

For the second filtering question, if trade-offs among criteria are allowable, all non-compensatory methods will be removed, and only compensatory methods remain as the candidate methods for further selection.

For the third filtering question, different decision analysis methods require different input data. For example, most MCDA methods require a decision matrix as input, while AHP needs pairwise comparison matrix. Thus, when the DM can provide pairwise comparison matrix, then AHP will be the only left method to solve the decision making problem. AHP and its extended version Analytical Network Process (ANP) are implemented in *Super Decisions* software (*www.superdecisions.com*). Thus, only methodology instructions for AHP are integrated in the multi-criteria decision support system.

3.2.4 Step 4: Define the Preferences on Evaluation Criteria

Usually, after the initial screening step is completed, more than one MCDA methods are expected to remain, otherwise the DM can directly choose the only one left to solve the decision making problem. With the nine scoring questions defined in Step 2, the DM's preference information on the evaluation criteria is defined. This will reflect which criterion is more important to the DM in the method selection process.

In this study, weighting factors are assigned to evaluation criteria to describe the DM's preference information. The weighting factors must be carefully elicited in order to accurately capture DM's preferences. A subjective scale of 0 to 10 recommended by Hwang [58] is used in this study, with calibration that 0 stands for extremely unimportant while 10 represents extremely important.

3.2.5 Step 5: Calculate the Appropriateness Index

In this study, sixteen widely used MCDA methods are identified and stored in the knowledge base as candidate methods for selection. The evaluation criteria are captured by answering twelve questions relevant to the characteristics of the methods. AI is used to rank the methods, given by Equation 3.1 [72], [118].

$$
\begin{aligned}
AI_j &= \frac{\sum\limits_{i=1}^{n} w_i * b_{ji}}{\sum\limits_{i=1}^{n} w_i * \mathbf{1}_i} * 100\% \\
b_{ji} &= \begin{cases} 1 \text{ if } c_{ji} = a_i \\ 0 \text{ if } c_{ji} \neq a_i \end{cases} \quad i = 1, 2, ..., n; j = 1, 2, ..., m
\end{aligned} \tag{3.1}
$$

where n is the number of evaluation criteria used to examine the methods with respect to the given problem, and m is the number of methods stored in the knowledge base, $\{w_1, w_2, ..., w_n\}$ are the weighting factors for the evaluation criteria, a_i is the value of the i-th characteristic of the decision problem, and c_{ji} is the value of i-th characteristic of the j-th method, b_{ji} is a Boolean number depending on the match of the i-th characteristic of the decision problem and the i-th characteristic of the j-th method. If the i-th characteristic of the decision problem matches the i-th characteristic of the j-th method, then $b_{ji} = 1$; otherwise, $b_{ji} = 0$. $\mathbf{1}_i$ denotes one.

With one set of weighting factors, the numerator of AI ($\sum\limits_{i=1}^{n} w_i * b_{ji}$) calculates the weighted score for each method, while the denominator ($\sum\limits_{i=1}^{n} w_i * \mathbf{1}_i$) calculates the maximum value if the characteristics of one method match completely with the characteristics of the decision problem. For each method, AI is calculated by the weighted score normalized by the maximum value. AI ranges from 0 to 100%, higher value of AI indicates the method is more appropriate to solve a given decision problem.

Table 3.1 shows one example of the AI calculation process for TOPSIS technique. At first, the DM identifies the key characteristics of the decision making problem by defining relative weights for the evaluation criteria. In this example, the decision rule, input data, and uncertainty analysis are considered as most important criteria, so high weights are assigned to these evaluation criteria. The other evaluation criteria are assigned relative weights in the same way, thus, the weighting factors of the nine evaluation criteria are defined as [5 8 4 4 6 4 3 6 5]. Second, the characteristics of the decision making problem are obtained from the answers to the questionnaire, while the characteristics of the MCDA methods can be obtained from the knowledge base. Then, the characteristics of the problem and method are compared pairwise in order to see if they match with each other. Finally, AI can be calculated for TOPSIS by using Equation 3.1 and the result is given by Equation 3.2.

Table 3.1: The Appropriateness Index Calculation Process for TOPSIS

Evaluation criteria	Criteria weights w_i	Problem criteria values a_i	Method criteria values c_{ji}	Match scores b_{ji}
Filtering questions				
1. Selection/Optimization	-	-	-	-
2. Allow trade-off	-	-	-	-
3. Input data	-	-	-	-
Scoring questions				
4. Preference information	5	Relative weight	Relative weight	1
5. Decision rule	8	Min. closeness	Min. closeness	1
6. Feasibility evaluation	4	Yes	No	0
7. Subjective	4	No	No	1
8. Qualitative/quantitative data	6	Quantitative	Quantitative	1
9. Discrete/continuous data	4	Discrete	Discrete	1
10. Single/hierarchy	3	Single	Single	1
11. Capture uncertainties	6	Yes	No	0
12. Visualization	5	Yes	Yes	1

$$\begin{aligned} AI_{TOPSIS} &= \frac{\sum\limits_{i=1}^{9} w_i * b_{ji}}{\sum\limits_{i=1}^{9} w_i * \mathbf{1}_i} * 100\% \\ &= \frac{[5\ 8\ 4\ 4\ 6\ 4\ 3\ 6\ 5] * [1\ 1\ 0\ 1\ 1\ 1\ 1\ 0\ 1]^{\mathrm{T}}}{[5\ 8\ 4\ 4\ 6\ 4\ 3\ 6\ 5] * [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]^{\mathrm{T}}} * 100\% \\ &= \frac{35}{45} * 100\% = 78\% \end{aligned} \tag{3.2}$$

3.2.6 Step 6: Evaluate the MCDA Methods

In order to compare the appropriateness of the methods with respect to the given decision making problem, each method is evaluated based on the nine scoring questions and AI for the MCDA methods are obtained. Based on the AI calculation, the MCDA method with the highest score will be chosen as the most appropriate method to solve the original decision making problem.

3.2.7 Step 7: Choose the Most Suitable Method

As noted in Step 6, the method with the highest AI will be recommended as the most appropriate method to solve the given problem. The developed decision support system is utilized to guide the user to reach the final decision when solving evaluation decision making problems. After one MCDA method is identified as the most appropriate method, the user can simply click the name of the method, and the methodology instructions will be displayed to guide the user to solve the given problem. The mathematical calculation steps are built in the MATLAB-based decision support system, thus, the user can just simply follow the instructions, such as inputting necessary data, to get the final results.

3.2.8 Step 8: Conduct Sensitivity Analysis

It is observed that different DMs often have different preference information on the nine scoring questions, thus, sensitivity analysis should be performed on the method selection algorithm to analyze its robustness with respect to the variations of weighting factors.

In order to accommodate different preference information from different DMs, weighting factor of each characteristic is treated in a parametric manner. In our integrated user interface, DMs can adjust criteria weights by moving the corresponding slide bars. It is worth noting that there is no absolute *best* decision analysis method that can solve any decision problem, since the method selection is problem specific. The selection of the most suitable decision analysis method depends on the problem under consideration.

3.2.9 Two Particular Scenarios During the Method Selection Process

There are two scenarios of particular interest which need to be considered during the method selection process: (1) the case when there are two or more methods whose appropriateness scores are the highest. (2) the case when there is no method which can be considered suitable for the problem under consideration. These two particular scenarios were not addressed in the previous research in [72]. In this study, these two particular scenarios are explicitly addressed and formulated as follows.

For the first scenario, when there are more methods that can be considered as the best ones to solve a given decision problem, the DM can perform uncertainty analysis of the weighting factors for the nine evaluation criteria. The method which has the highest probability to be ranked first is recommended as the most suitable method for the decision problem under consideration. In the developed multi-criteria decision support system, the DM can adjust the weighting factors of the nine evaluation criteria by moving the corresponding slide bars.

For the second scenario, when there is no method can be considered as the suitable one for a given decision problem, new methods or hybrid methods need to be used to solve the given problem. During the process of method selection, more insights on the characteristics of the methods can be obtained. For example, by combining two or more decision analysis methods, DMs may get one hybrid method which is more effective for solving the given problem. Moreover, the definition of a threshold value for the appropriateness index of the decision analysis method can be helpful to identify the occurrence of the second scenario.

3.3 An Intelligent Multi-Criteria Decision Support System

The proposed approach to method selection is implemented and an intelligent multi-criteria decision support system is developed in MATLAB. Its architecture is illustrated in Figure 3.2. For a given decision making problem, the DM needs to define the requirements of the problem and the preference information on these requirements. Then the intelligent multi-criteria decision support system will utilize the information provided in the knowledge base, and rank the methods. The method with the highest score will be selected as the most appropriate MCDA method to solve the given problem. The user guide of the intelligent multi-criteria decision support system can be found in Appendix A.

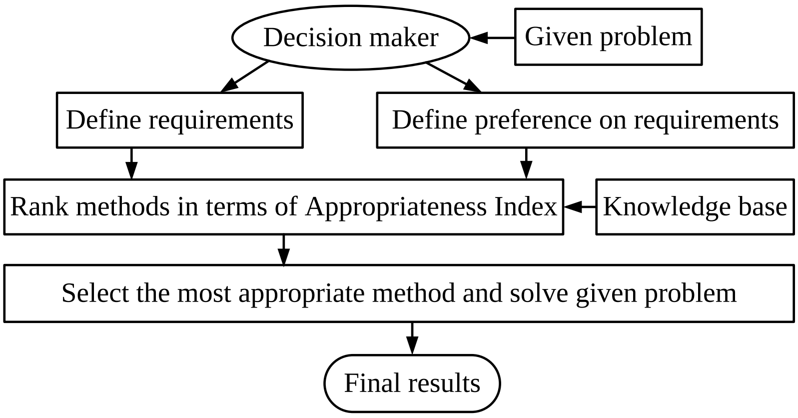


Figure 3.2: The Architecture of an Intelligent Multi-Criteria Decision Support System

3.4 Chapter Summary

An advanced approach to effectively select the most appropriate MCDA method for a given decision making problem was formulated and presented in this chapter. Twelve evaluation criteria were proposed to assess sixteen widely used MCDA methods. This method selection approach was implemented and an intelligent multi-criteria decision support system was developed in MATLAB. The capabilities of the developed intelligent multi-criteria decision support system will be demonstrated and evaluated in Chapter 5 and Chapter 6.

4

Uncertainty Assessment in the Decision Analysis Process

The second objective of this research is to assess the uncertainties propagated in the decision analysis process. The values of decision criteria and weighting factors are main input data when solving a decision problem. The inherent uncertainties associated with the input data have significant impacts on the final decision solution. In this chapter, a new approach for uncertainty assessment in the decision analysis process is proposed. This approach consists of four steps: uncertainty characterization by percentage uncertainty with confidence level, uncertainty analysis using error propagation techniques, local sensitivity analysis based on iterative binary search algorithm, and global sensitivity analysis using partial rank correlation coefficients. The proposed new approach is implemented and an uncertainty assessment module is developed and integrated into the intelligent multi-criteria decision support system.

4.1 Uncertainty Assessment: State of the Art

Considerable research has been conducted to assess the uncertainties propagated in the decision analysis process. Durbach and Stewart provided a review of uncertainty modeling for conducting multi-criteria decision analysis with uncertain attribute evaluations [42]. The review included models using probabilities, quantiles, variances, fuzzy numbers, and scenarios. Aschough et al. discussed the incorporation of uncertainty in environmental decision making process [11]. Especially, the authors asserted the importance of developing innovative methods for quantifying the uncertainty associated with human input.

A conceptual framework for the systematic treatment of uncertainty in model-based decision support was developed by Walker [127]. Another approach to incorporate uncertainty management with engineering systems design was proposed by de Neufville et al. [92]. A generic classification of uncertainty from the perspective of a generic product development and manufacturing firm was provided by de Weck and Eckert [129]. An exploratory of MCDA was developed by Van der Pas to address deep uncertainty [4]. However, the quantification of the uncertainty propagated into the final decision results was not fully addressed.

The analysis of multi-criteria problems with uncertainty using simulation based techniques has been studied by many researchers [40], [122], [41], [45]. An uncertainty analysis program for SAW (Simple Additive Weighting [58]) and PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations [24]) was developed in Excel by Hyde [60], [59]. A new surrogate modeling method for propagating uncertainty from model inputs to model outputs was developed by Allaire and Willcox [5]. However, the uncertainties of the criteria were directly defined in the form of probability distributions, the DM's confidence levels regarding these uncertainties were not explicitly captured. In other words, the accuracy of the uncertainty modeling has not been quantified. Accordingly, the quality of the final decision made under these uncertainties cannot be guaranteed.

The analytical error propagation technique was employed to measure the composite errors of multiplicative weighting method [135], where only two simple algebraic calculation steps were involved: the attribute values for each alternative were multiplied by the weights as exponentiation. However, for complicated decision analysis methods, there would be heavy calculation burden to infer the analytical error derivatives.

The uncertainty assessment approach proposed in this study overcomes the aforementioned limitations, through capturing the confidence levels of a DM, while utilizing simulation-based numerical error propagation technique to calculate the propagated error for complex decision analysis methods. This approach consists of four steps: uncertainty characterization, uncertainty analysis, local and global sensitivity analysis. Each step of the uncertainty assessment approach is discussed in detail in the following sections.

4.2 Uncertainty Characterization

The uncertainties are represented by percentage uncertainties with confidence levels. These uncertainties are transferred into standard deviations through the utilization of inverse error function. In this section, the relationship between normal distribution and error function is introduced first, then the uncertainty transformation using inverse error function is presented.

4.2.1 Relationship between Normal Distribution and Error Function

For a normal random variable X with $N\left(\mu, \sigma^2\right)$ distribution, the probability of a random sample value falling within the interval $[\mu - n\sigma, \mu + n\sigma]$ can be calculated by Equation 4.1.

$$P\left(\mu - n\sigma < X < \mu + n\sigma\right) = \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu-n\sigma}^{\mu+n\sigma} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)} dx \tag{4.1}$$

The error function is shown in Equation 4.2 [89], with the substitution $z = \frac{X-\mu}{\sigma}$, Equation 4.1 can be converted into Equation 4.3.

$$y = erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{\left(-t^2\right)} dt \tag{4.2}$$

$$P\left(\mu - n\sigma < X < \mu + n\sigma\right) \;=\; \frac{1}{\sqrt{2\pi}} \int_{-n}^{n} e^{\left(-\frac{z^2}{2}\right)} dz = erf\left(\frac{n}{\sqrt{2}}\right) \tag{4.3}$$

In other words, the probability of a normal random variable X falling within its interval $[\mu - n\sigma, \mu + n\sigma]$ can be calculated by the error function $erf\left(\frac{n}{\sqrt{2}}\right)$. Some typical numbers of standard deviation are plotted in Figure 4.1.

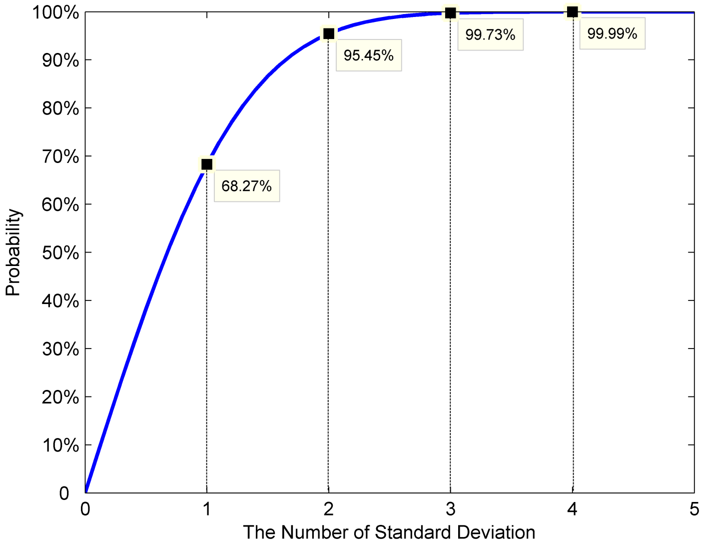


Figure 4.1: Typical Numbers of Standard Deviation

4.2.2 Uncertainty Transformation using Inverse Error Function

When the probability (confidence level) of a normal random variable X falling within certain confidence interval has been given, the numbers of standard deviation can be calculated by the inverse error function, as described in Equation 4.4.

$$n = \sqrt{2} erf^{-1}(\text{Confidence level}) \tag{4.4}$$

The relationship between mean μ and standard deviation σ is shown in Equation 4.5.

$$\text{Relative error}(\%)\mu = n\sigma \tag{4.5}$$

Note that relative error here is equivalent to percentage uncertainty, thus, the conversion of percentage uncertainty into standard deviation is shown in Equation 4.6.

$$\sigma = \frac{\text{Percentage uncertainty}(\%)\mu}{n} \tag{4.6}$$

4.3 Uncertainty Analysis

The process of uncertainty analysis using error propagation techniques is illustrated in Figure 4.2. In the first part of this section, the background of error propagation techniques is introduced. Robustness measurement using Signal-to-Noise Ratio (SNR) is presented in the second part.

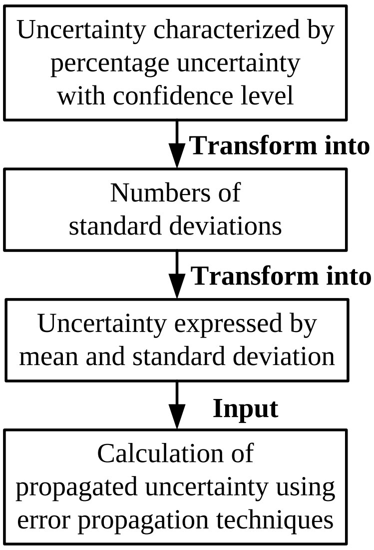


Figure 4.2: The Process of Uncertainty Analysis using Error Propagation Techniques

4.3.1 Background of Error Propagation Techniques

Error propagation techniques answer the question: how the uncertainties of input variables will be propagated to some predefined functions involving these variables and lead to the final result [17]. There are two classes of error propagation techniques: analytical and simulation-based numerical error propagation techniques.

The analytical error propagation technique relies on a linearized Taylor series expansion of the function about the mean of each variable, the total error of the function is obtained by combining the linearized individual error in quadrature. For a function

$$y = f(x_1, x_2, ..., x_n) \tag{4.7}$$

where $x_1, x_2, ..., x_n$ are input variables, $\delta_{x_1}, \delta_{x_2}, ..., \delta_{x_n}$ refer to the relatively small uncertainties in $x_1, x_2, ..., x_n$, respectively. The small uncertainties can be identified as Gaussian distribution provided that their magnitudes are not too large [17]. Small uncertainties of the variables $\delta_{x_1}, \delta_{x_2}, ..., \delta_{x_n}$ can be used with their standard deviation $\sigma_{x_1}, \sigma_{x_2}, ..., \sigma_{x_n}$ interchangeably. Based on Taylor series expansions, the propagated errors of input variables $x_1 \pm \delta_{x_1}, x_2 \pm \delta_{x_2}, ..., x_n \pm \delta_{x_n}$ can be analytically described by Equation 4.8 [17].

$$\sigma_y^2 = \sum_{j=1}^{n} \left(\frac{\partial f}{\partial x_j} \right)^2 \sigma_{x_j}^2 + 2 \sum_{j=1}^{n} \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_j} \right) \left(\frac{\partial f}{\partial x_i} \right) \sigma_{x_j x_i} (i \neq j) \tag{4.8}$$

where σ_y^2 is the total variance of the function, $\frac{\partial f}{\partial x_j}$ is a partial derivative of the function f with respect to variable x_j, when treating other variables $x_1, x_2, ..., x_{j-1}, x_{j+1}, ..., x_n$ as constants, $\sigma_{x_j}^2$ is the variance of variable x_j, and $\sigma_{x_j x_i}^2$ is the cross-product covariance when variables x_j and x_i are correlated. If the variables $x_1, x_2, ..., x_n$ are independent, we can omit the cross-product covariance term, Equation 4.8 reduces to

$$\sigma_y^2 = \sum_{j=1}^{n} \left(\frac{\partial f}{\partial x_j} \right)^2 \sigma_{x_j}^2 \tag{4.9}$$

The contribution due to the uncertainties in $x_1, x_2, ..., x_n$ is considered separately through Equation 4.9, provided that the errors of the input variables could be seen as normally distributed and there is no strong nonlinearity associated with the function in its evaluation range.

While analytical error propagation technique is appropriate for simple calculation processes, simulation-based numerical error propagation technique is more suitable for dealing with complex models, where trade-off has to be made between results accuracy and computation time.

4.3.2 Robustness Measurement using Signal-to-Noise Ratio

Robustness is an important performance measurement when uncertainty exists. Taguchi pioneered the application of robust design methods in product design and manufacturing process [120]. Robustness reflects product's ability to withstand uncontrollable variations in production and usage. The Signal-to-Noise Ratio (SNR) is one way to measure the robustness in Taguchi's method. The SNR in terms of mean and standard deviation is defined as Equation 4.10.

$$\mathrm{SNR} = 20\log_{10}(\frac{\mu}{\sigma}) \tag{4.10}$$

The SNR is expressed in decibel (dB). For instance, 40 (dB) means that the magnitude of mean is $10^{\frac{40}{20}} = 100$ times the magnitude of standard deviation. A larger SNR value indicates more robustness against uncertainty.

Moreover, linearity also influences the SNR value. When the relationship between the input and output of a system is not linear, deviation from linearity is taken as the error after the decomposition of variation and the SNR becomes smaller [120].

Uncertainty Analysis for an Aircraft Selection Example

Uncertainty analysis for an aircraft selection example, as described in Subsection 2.3.4, is conducted in this subsection. The decision matrix is repeated in Table 4.1 for the convenience of calculation.

Table 4.1: Decision Matrix of an Aircraft Selection Example for Uncertainty Analysis

	Criteria		
	C_1: Comfort	C_2: Cost	C_3: Environmental friendliness
Alternatives	w_1: 0.3	w_2: 0.4	w_3: 0.3
Aircraft A	8	7	10
Aircraft B	9	6	5
Aircraft C	6	7	8

Assume that the DM states that there are 15% uncertainties existing in criteria values with 80% confidence level, and there are 30% uncertainties existing in weighting factors with 90% confidence level. Following the uncertainty analysis process shown in Figure 4.2, percentage uncertainties with confidence levels are transferred into means and standard deviations, Monte Carlo-based error propagation technique is used to calculate the propagated uncertainties.

When SAW is used to solve the aircraft selection example, the probabilistic ranking of the three candidate aircraft is summarized in Table 4.2. The largest number in each row indicates the most likely ranking. It can be observed that Aircraft A has the highest probability to be ranked first, Aircraft B is most likely to be ranked second, and Aircraft C has the highest probability to be ranked in the last place.

Table 4.2: The Probabilistic Ranking in an Aircraft Selection Example

	Alternatives		
Ranking	Aircraft A	Aircraft B	Aircraft C
1st	72.00%	26.00%	2.00%
2nd	25.00%	56.00%	19.00%
3rd	3.00%	18.00%	79.00%

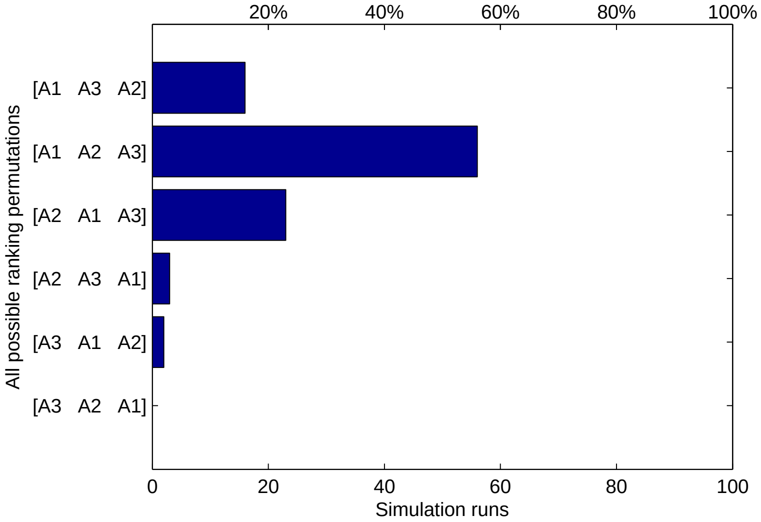


Figure 4.3: The Probabilistic Ranking Permutations in an Aircraft Selection Example

Table 4.3: Robustness Measurement using Signal-to-Noise Ratio in an Aircraft Selection Example

Alternatives	Mean	Standard deviation	SNR (dB)
Aircraft A	$\mu = 0.9041$	$\sigma = 0.1036$	18.8148
Aircraft B	$\mu = 0.8506$	$\sigma = 0.1096$	17.7978
Aircraft C	$\mu = 0.7741$	$\sigma = 0.0973$	18.0144

In addition to the probabilistic ranking of each alternative, the likelihood for alternatives permutation is also calculated and demonstrated in Figure 4.3, where the vertical axis represents all possible alternatives permutations, the lower horizontal axis stands for simulation runs, and the upper horizontal axis corresponds to the occurrence probability of each permutation. It can be seen that the alternative permutation $[A_1 \ \ A_2 \ \ A_3]$ ([Aircraft A Aircraft B Aircraft C]) has the highest probability of occurrence.

In order to compare the robustness of the three alternatives against uncertainties in weighting factors and criteria values, SNR for each alternative is calculated using Equation 4.10 and summarized in Table 4.3. Considering that a larger SNR value indicates more robustness against uncertainty, we can observe from Table 4.3 that Aircraft A is most robust against uncertainties in weighting factors and criteria values among the three alternatives.

4.4 Local Sensitivity Analysis via Iterative Binary Search Algorithm

Sensitivity analysis addresses the question how the variation of input variables influences model output [55]. There are two categories: local and global sensitivity analysis [110].

Local sensitivity analysis varies input variables one at a time to determine which variables have the greatest effect on model output, while holding the others fixed at nominal values. Local sensitivity analysis can provide an initial understanding of the sensitivity of an individual variable on model output over a small region around the nominal values of input variables, with efficient computation. However, local sensitivity analysis may not provide meaningful results when the model under consideration is nonlinear, or when input variables are perturbed simultaneously and by different amounts, and the effects of interactions among input variables on model output cannot be captured [46], [88].

Global sensitivity analysis varies all input variables simultaneously over the full range and investigates the influence of each variable averaged over all possible values of other input variables [110], [46]. Global sensitivity analysis can provide insights into model behavior over the full range of model output, taking into account the variable interactions [88]. However, computational cost of global sensitivity analysis is higher than local sensitivity analysis and may become prohibitive for large complex models.

In this research, we take the perspectives that different types of sensitivity analysis reveal model behaviors in different domains of the variables [138], and global sensitivity analysis should not precede local sensitivity analysis [50]. This section focuses on local sensitivity analysis when solving evaluation decision making problems, and global sensitivity analysis is investigated in the next section.

Local Sensitivity Analysis in the Decision Analysis Process

When the MCDA methods are utilized in evaluation decision making problems, local sensitivity analysis can be conducted to determine the sensitivity of alternatives' rankings to changes in input variables. A unified local sensitivity analysis approach for three MCDA methods including SAW, multiplicative weighting method, and AHP, was proposed [126], where two questions were addressed: (1) How sensitive the ranking of the best alternative or any alternative is to variations in the current weights or performance measures of decision criteria? (2) What is the smallest change in the current weights or performance measures of decision criteria which can alter the current ranking of two alternatives?

However, this sensitivity analysis approach is specific for these three MCDA methods and is not applicable to other MCDA methods. In addition, this approach was obtained through the analytical inferences of these three specified MCDA methods, which only involve simple mathematical calculation steps. For instance, SAW just has two calculation steps: multiplication and addition, multiplicative weighting method only involves multiplication, and AHP also merely involves multiplication and addition. Nonetheless, for other MCDA methods with complicated mathematical calculations, such as TOPSIS or ELECTRE, it is difficult to infer the sensitivity coefficient for each input variable analytically. Thus, this sensitivity analysis approach cannot be extended for general MCDA methods.

In this study, an iterative binary search algorithm is developed to investigate the sensitivity of alternatives' ranking to the variations of weighting factors or criteria values. The iterative binary search algorithm can overcome these drawbacks mentioned above, since it is a sampling-based method which will not be affected by the analytical calculation steps of MCDA methods. Additionally, it can be generalized to other MCDA methods.

4.4.1 Iterative Binary Search Algorithm

The binary search technique has been widely used to find a target value in a sorted (usually ascending) sequence efficiently [131], [82]. This technique compares the middle element of the sorted sequence to the target value, if the middle element is equal to the target value, then the search terminates. If the target value is less than middle element, then the algorithm eliminates the right half of the sorted sequence and conducts the same search for the left side. If the target value is bigger than the middle element, then the algorithm ignores the left half of the sorted sequence and performs the same search for the right side. Otherwise, we can conclude that the target value is not in the sorted sequence.

For example, given a sorted sequence [0 5 12 17 23 25 50 60 80], assume that we want to find the target value 25. The binary search technique works as follows.

- First iteration: [0 5 12 17 **23** 25 50 60 80]. The target value 25 is bigger than the middle element 23, ignore the left half of the sorted sequence, and perform the same search for the right side.
- Second iteration: [25 **50** 60 80]. The target value 25 is smaller than the middle element 50, ignore the right side of the sorted sequence, and perform the same search for the left side.
- Third iteration: [**25**]. The target value 25 equals the element 25, the target value is found.

When using the MCDA methods to solve a given problem, input parameters are decision criteria, weighting factors, the original ranking of the alternatives, and the number of iterations. The outputs of the iterative binary search algorithm are the minimum changes in decision criteria and weighting factors to alter the rankings of two alternatives. The iterative binary search algorithm varies one input variable at a time in order to find the minimum change in this input variable, which can alter the ranking of two alternatives.

The initialization of the iterative binary search algorithm is illustrated in Figure 4.4. The first step is to initialize input parameters: left lower bound `ll_bound`, left upper bound `lu_bound`, right lower bound `rl_bound`, and right upper bound `ru_bound`. In the next step, the left trial

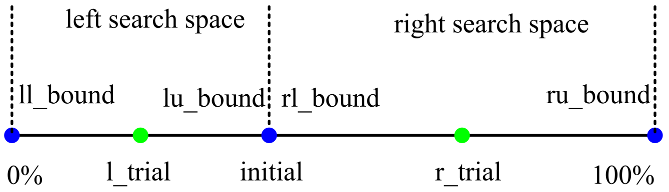


Figure 4.4: Initialization for the Iterative Binary Search Algorithm

value `l_trial` is calculated by the middle element in the left search space `(ll_bound+lu_bound)/2`, and the right trial value `r_trial` is calculated by the middle element in the right search space `(rl_bound+ru_bound)/2`.

The flow chart of the iterative binary search algorithm is shown in Figure 4.5, where `l` stands for left and `u` upper, `ll` stands for left lower, `lu` left upper, `rl` right lower, and `ru` right upper. `delta` is the minimum change in weights or decision criteria when two rankings are altered. The default setting is that it is non-feasible to change the current parameter to alter the ranking of two alternatives. The number of iteration `runs` determines the precision of the calculation [82]. For instance, when the iteration runs is set as `runs` $= 30$, the precision of the calculation is $log(2^{\texttt{runs}}) = log(2^{30}) \approx 9$.

The new trial values of the parameter under consideration are calculated and new rankings of alternatives are computed. The rankings in the left search space will be evaluated first. If the rankings using left new trial value change, then we will assign `true` to the judgment variable `isFeasible`, and calculate the relative quantity of the parameter under consideration `delta_decrement`, and the left new trial value `l_trial` is assigned to the left lower bound `ll_bound`. If the ranking using left new trial value does not change, then, the left new trial value `l_trial` is given to the left upper bound `lu_bound`. After the evaluation of the left search space, the similar procedure is performed to the right search space. The algorithm is terminated when the number of iteration is finished. Finally, if the judgment variable `isFeasible` is `true`, the absolute magnitude of the relative quantities `delta_decrement` and `delta_increment` is compared. The smaller quantity `delta` is the minimum change which can alter the ranking of two alternatives. Otherwise, we can conclude that it is not feasible to change the current parameter so that the ranking of two alternatives is altered.

4.4.2 Interactive Sensitivity Analysis for Weighting Factors

It is observed that weighting factors are often highly subjective considering that they are elicited based on DM's experience or intuition. The inherent uncertainties and subjectivities of weighting factors have significant impacts on the final result of a decision making problem. In this study,

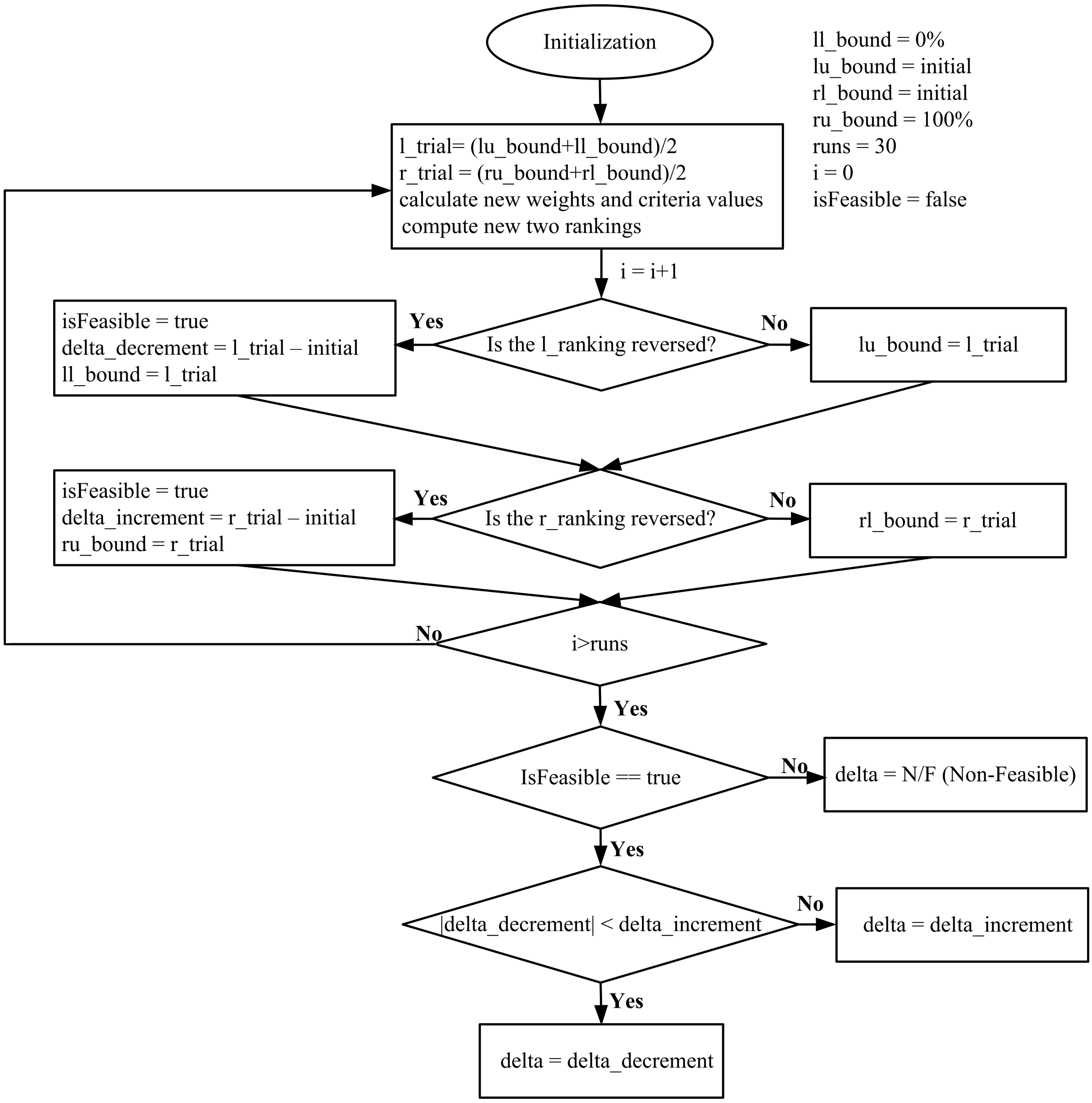


Figure 4.5: Flow Chart of the Iterative Binary Search Algorithm

an interactive sensitivity analysis for weighting factors is developed. The basic idea is to vary the weighting factor of one criterion from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting.

Local Sensitivity Analysis for an Aircraft Selection Example

Local sensitivity analysis for an aircraft selection example, as described in Subsection 2.3.4, is conducted in this subsection. The decision matrix is shown in Table 4.4.

When SAW is used to solve the aircraft selection example, the ranking of the three alterna-

Table 4.4: Decision Matrix of an Aircraft Selection Example for Local Sensitivity Analysis

	Criteria		
	C_1: Comfort	C_2: Cost	C_3: Environmental friendliness
Alternatives	w_1: 0.3	w_2: 0.4	w_3: 0.3
Aircraft A	8	7	10
Aircraft B	9	6	5
Aircraft C	6	7	8

tives is [Aircraft A Aircraft B Aircraft C]. The developed iterative binary search algorithm can answer the question: What is the smallest change in the weighting factors so that the ranking of the most preferred alternative or any alternative will be altered?

The absolute minimum changes in the weighting factors which can alter the ranking of the alternatives are summarized in Table 4.5. For the convenience of comparison, the relative minimum changes are also presented in Table 4.6. The relative minimum changes are the absolute minimum changes scaled against the original values of the weighting factors. In these two tables, N/F (Non-Feasible) means that it is not mathematically feasible to alter the ranking of the alternatives through the change of the current parameter.

The first two rows in Table 4.6 show that when the weighting factor of C_3 decreases -39.69%, Aircraft B becomes the most preferred alternative, and it is not possible to change the weighting factors so that Aircraft C ranks first. Moreover, it can be seen from the whole table that the weighting factor of C_3 is most sensitive to the ranking of the three alternatives.

Furthermore, following the proposed idea of varying the weighting factor of one criterion

Table 4.5: Absolute Minimum Changes in Weighting Factors to Alter the Rankings of Alternatives in an Aircraft Selection Example

Pairs of rankings	C_1	C_2	C_3
A_1:A_2	0.54	0.42	-0.12
A_1:A_3	N/F	N/F	N/F
A_2:A_3	-0.21	N/F	0.23

Table 4.6: Relative Minimum Changes in Weighting Factors to Alter the Rankings of Alternatives in an Aircraft Selection Example

Pairs of rankings	C_1	C_2	C_3
A_1:A_2	178.58%	104.17%	-39.69%
A_1:A_3	N/F	N/F	N/F
A_2:A_3	-67.15%	N/F	74.61%

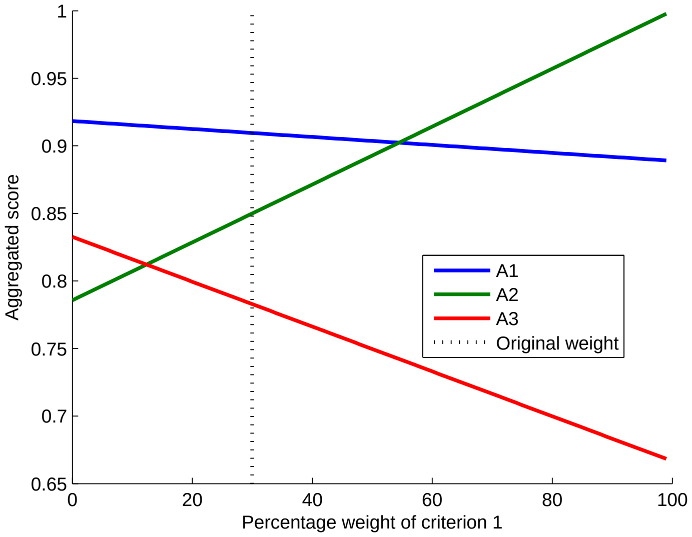


Figure 4.6: Interactive Sensitivity Analysis for the Weighting Factor of C_1 in an Aircraft Selection Example

from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting, the interactive sensitivity analysis for the weighting factor of C_1 is illustrated as an example in Figure 4.6, where an intersection of two lines indicates that there is a ranking change between two alternatives.

4.5 Global Sensitivity Analysis using Partial Rank Correlation Coefficients

In contrast to local sensitivity analysis, global sensitivity analysis allows the variations of all input variables over the full range simultaneously. Many techniques have been developed to perform global sensitivity analysis, among which Monte-Carlo sampling and correlation analysis [18], [79], [55] and variance decomposition analysis [110] are two most popular methods.

In this research, considering that inherent uncertainties in the decision analysis process, especially the subjectivities of weighting factors, have significant impacts on the final result of a decision making problem, statistical techniques are capable of effectively dealing with these uncertainties. Therefore, global sensitivity analysis based on Monte-Carlo sampling and correlation analysis is further investigated.

4.5.1 Correlation Coefficients and Statistical Significance Test

In the decision analysis process, decision criteria and preference information are main input variables. The output variables of the MCDA model are the overall performances of alternatives,

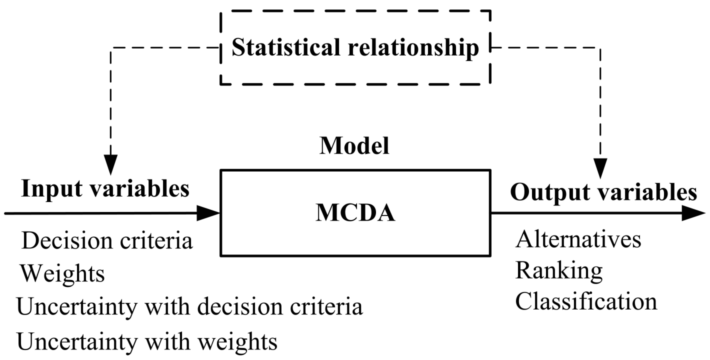


Figure 4.7: Input Variables and Output Variables in the Decision Analysis Process

in the form of alternatives' ranking or classification [14], [44]. The input variables and output variables in MCDA models for statistical analysis are illustrated in Figure 4.7.

The degree of association is one way to describe the statistical relationship between input variables and output variables in the decision analysis process. Association between two variables exists when knowing the value of one variable provides information about the likely value of the other variable, while correlation between the two variables exists when the association is linear [56]. There are several correlation coefficients measuring the degree of association: Pearson correlation coefficient, Spearman rank correlation coefficient, and partial rank correlation coefficient [113]. The following part of this subsection introduces these three correlation coefficients and statistical significance test.

Pearson Correlation Coefficient

Pearson correlation coefficient r is one common measure of linear relationship between two variables. Assume that two variables X and Y, with sample values $x_1, x_2, ..., x_n$ and $y_1, y_2, ..., y_n$, are well approximated by normal distributions, and their joint probability distribution is a bivariate normal distribution. Pearson correlation coefficient is calculated by Equation 4.11.

$$r = \frac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{var}(X)}\sqrt{\mathrm{var}(Y)}} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}} \tag{4.11}$$

where *cov* represents the covariance of two variables, *var* represents the variance of one variable, $\bar{x}$ is the mean of the sample values for X, and $\bar{y}$ is the mean of the sample values for Y.

Pearson correlation coefficient r ranges from -1 to +1. A value of -1 indicates a perfect negative linear relationship between variables X and Y, a value of +1 implies a perfect positive linear relationship, and a value of 0 shows that there is no linear correlation.

Spearman Rank Correlation Coefficient

Spearman rank correlation coefficient r_s is a non-parametric measure of association between two variables, which are measured in ordinal scale, without the assumption that the variables are normally distributed. When the association between X and Y is nonlinear, the relationship can be transferred into a linear one by using the ranking of the variables R_{x_i} and R_{y_i} rather than their actual values. The result of Equation 4.11 with rank transformed variables is called Spearman rank correlation coefficient. If there are no tied ranks, Spearman rank correlation coefficient can also be calculated by Equation 4.12 [69].

$$r_s = 1 - \frac{6 \sum\limits_{i=1}^{n} \left(R_{x_i} - R_{y_i}\right)^2}{n\left(n^2 - 1\right)} \tag{4.12}$$

If tied ranks occur, the same rank has to be assigned to the equal values, Equation 4.11 should be used to calculate Spearman rank correlation coefficient.

Spearman rank correlation coefficient ranges from -1 to +1. A value of -1 indicates a perfect negative correlation between the two ranked variables, a value of +1 implies a perfect positive correlation, and a value of 0 shows that there is no correlation.

Partial Rank Correlation Coefficient

Partial correlation coefficient measures the monotonic association between two variables, if they were not correlated with any other variables [76]. It is the association between any two of the variables, while eliminating indirect associations due to other variables [113]. Assume three variables X, Y, and Z, with sample values $x_1, x_2, ..., x_n$, $y_1, y_2, ..., y_n$, and $z_1, z_2, ..., z_n$. The partial correlation coefficient between X and Y, when eliminating indirect associations due to relationships that may exist between X and Z or Y and Z, equals to Pearson correlation coefficient between the two residuals $X - \hat{X}$ and $Y - \hat{Y}$. As shown in Equation 4.13, $\hat{X}$ and $\hat{Y}$ is the linear regression between X, Y and Z, respectively. The partial correlation coefficient between X and Y is given by Equation 4.14.

$$\begin{aligned} \hat{X} &= a_0 + a_1 Z \\ \hat{Y} &= b_0 + b_1 Z \end{aligned} \tag{4.13}$$

$$r_{XY.Z} = \frac{r_{XY} - r_{XZ} r_{YZ}}{\sqrt{\left(1 - r_{XZ}^2\right)\left(1 - r_{YZ}^2\right)}} \tag{4.14}$$

Partial rank correlation coefficient r_p calculates the partial correlation coefficient for the rank-transformed variables, which characterizes the monotonic relationship between the rankings of the two variables while eliminating indirect associations due to other variables.

Partial rank correlation coefficient varies between -1 and +1, where -1 represents strongest negative association between two variables, and +1 represents strongest positive association.

Statistical Significance Test

The degree of association itself cannot uncover the relationship between two variables without statistical significance test. A strong association is not necessarily statistically significant [107], the interpretation of the association could be misleading without statistical significance test. Therefore, it is crucial to conduct a measure of association and statistical significance test in order to avoid improper decisions [49].

Hypothesis testing can be performed to evaluate whether the measure of association between two variables is statistically significant, which involves the calculation of a test statistic based on a random sample from the population to determine whether to reject a given hypothesis [89].

In addition, p-value provides another way to assess the statistical significance of the test statistic [89]. The p-value is the probability value that the test statistic is at least as large as the observed one, given that the null hypothesis H_0 is true. A lower p-value provides stronger evidence to reject the null hypothesis H_0 in favor of the alternative hypothesis H_1.

4.5.2 Proposed Approach to Perform Global Sensitivity Analysis

Partial rank correlation coefficient is one popular sampling-based global sensitivity analysis index. It has been widely used to infer biochemical interactions in systems biology [18],[79]. In the decision analysis process, partial rank correlation coefficient can be utilized to determine the global sensitivity of the ranking or classification of alternatives to input variables. A higher magnitude of partial rank correlation coefficient indicates a larger impact on the ranking or classification of alternatives.

In this study, global sensitivity analysis using partial rank correlation coefficient in the decision analysis process is performed, according to a step by step approach emphasized on measure of association together with statistical significance test. The proposed step by step approach is presented as follows.

Step 1: Define Probability Distributions for Input Variables

In the decision analysis process, input variables are the values of decision criteria and weighting factors to reflect DM's preference information. When the amount of available data is not sufficient to construct probability distribution functions, uniform or normal distributions are two popular alternatives for probability distribution functions. In a given problem, physical constraints of decision criteria usually serve as the range of variable variation, while the weighting factors range from 0 to 1.

Step 2: Perform Latin Hypercube Sampling

Latin Hypercube Sampling (LHS) is a type of stratified Monte-Carlo sampling technique [81], where the distributions of input variables are divided into N equal probability intervals and the value of each input variable is then randomly sampled. The entire range for each variable is explored in a way that each value of each variable is used exactly once. LHS has the advantage that it requires fewer samples than simple random sampling to achieve the same accuracy [81]. The efficiency of LHS enables to vary all variables at the same time with low computational cost in global sensitivity analysis.

The minimum value of sample size N for LHS is $\frac{3}{4}k$, where k is the number of input variables that are varied [18]. However, it is not necessary that the result is better when a larger sample size is used. In addition to higher computational costs, larger sample size can make very weak relationship become significant. The significance of a weak relationship is not necessarily important in real-world applications [83].

Step 3: Rank Transformation for both Input Variables and MCDA Output

For each combination of the sampled values from decision criteria and weighting factors, MCDA methods are utilized to calculate the overall performances of alternatives. The input variables (decision criteria and weighting factors) and MCDA output (alternatives' performances) are transformed into ranks in ascending order. Although the ascending order seems contrary against the ranking of alternatives, it does not influence the calculation results of partial rank correlation coefficients, since both input variables and MCDA output are transformed into ranks in a consistent manner.

For the scoring MCDA methods, it is straightforward to transform the scores into ranks in ascending order. Regarding tied ranks, the average rank is used instead. For example, for a score vector $[0.01 \;\; 0.02 \;\; 0.03 \;\; 0.05 \;\; 0.02\,]$, counting from smallest to largest, 0.01 ranks first, the two 0.02 ranks second and third, thus, the average rank $(2+3)/2 = 2.5$ is used for both of them. The transformed ranks in ascending order are $[1 \;\; 2.5 \;\; 4 \;\; 5 \;\; 2.5\,]$.

For the classification MCDA methods, for instance, ELECTRE, the outrank set is assigned scores first: non-dominated alternatives are assigned score 1, while dominated alternatives are assigned score 0. Next, the outrank set with scores is transformed into ranks. For example, considering five alternatives (A_1, A_2, A_3, A_4, A_5), where A_1, A_3, and A_4 are non-dominated alternatives, while A_2 and A_5 are dominated alternatives. In the first step, A_1, A_3, and A_4 are assigned score 1, while A_2 and A_5 are assigned score 0. Thus, the assigned score vector for the five alternatives is $[1 \;\; 0 \;\; 1 \;\; 1 \;\; 0\,]$.

Next, the assigned score vector with tied values is transformed into ranks. Counting from smallest to largest, the two 0 rank first and second, then the average rank is $(1 + 2)/2 = 1.5$.

The three 1 rank third, fourth and fifth, their average rank is $(3+4+5)/3 = 4$. The transformed ranks of the outrank set in ELECTRE are [4 1.5 4 4 1.5].

Attention should be paid that too many tied ranks may reduce the statistical power of partial rank correlation coefficients. This will be shown in Chapter 6.

Step 4: Calculate Partial Rank Correlation Coefficients

With the rank-transformed data, partial rank correlation coefficients can be calculated. The partial rank correlation coefficients in global sensitivity analysis are used to characterize the monotonic statistical relationship between input variables and model output [18]. Besides, it is recommended that before initiating global sensitivity analysis, it is necessary to examine the scatter plots to detect the non-monotonicities between input variables and model output.

Step 5: Conduct Statistical Significance Test

The measure of association alone cannot uncover the statistical relationship between variables without statistical significance test. In the study, p-value is computed to assess the statistical significance of partial rank correlation coefficient. A lower p-value provides stronger evidence to reject the null hypothesis H_0 (there is no partial correlation between the rank transformed variables) in favor of the alternative hypothesis H_1 (there is nonzero partial correlation between the rank transformed variables).

Step 6: Results Interpretation

It is crucial to interpret partial rank correlation coefficients together with statistical significance test. Usually, p-values less than 0.05 indicate that the partial rank correlation coefficients are statistically significant. Partial rank correlation coefficients can offer the DM more insights into the relative contribution of input variables to the total performances of alternatives explicitly.

It is important to note that there are two components in a global sensitivity coefficient: the range of the input variable and the sensitivity coefficient of the output to this input variable [88]. An input variable is identified as important in global sensitivity analysis if it has a wider range and larger sensitivity coefficient. On the contrary, an input variable is not identified as important in global sensitivity analysis if it has a narrow range, or if has a small sensitivity coefficient.

Global Sensitivity Analysis for an Aircraft Selection Example

One example of global sensitivity analysis for an aircraft selection example, as described in Subsection 2.3.4, is conducted in this subsection. The decision matrix is repeated in Table 4.7 for the convenience of calculation.

Table 4.7: Decision Matrix of an Aircraft Selection Example for Global Sensitivity Analysis

	Criteria		
	C_1: Comfort	C_2: Cost	C_3: Environmental friendliness
Alternatives	w_1: 0.3	w_2: 0.4	w_3: 0.3
Aircraft A	8	7	10
Aircraft B	9	6	5
Aircraft C	6	7	8

When SAW is used to solve the aircraft selection problem, the ranking of the alternatives is $[A_1 \quad A_2 \quad A_3]$ ([Aircraft A Aircraft B Aircraft C]). The proposed approach for global sensitivity analysis is performed, with emphasis on measure of association together with statistical significance test. The partial rank correlation coefficients with p-values for A_1 (Aircraft A) is illustrated in Figure 4.8, where the horizontal axis represents the partial rank correlation coefficients, and the vertical axis stands for the six input variables for A_1 (Aircraft A), including three criteria values and their weighting factors.

P-values for partial rank correlation coefficients are next to the bars. Lower p-values provide stronger evidence of statistical significance. In this aircraft selection example, p-values less than 0.05 indicate that the partial rank correlation coefficients are statistically significant.

It can be observed from Figure 4.8 that C_3 (environmental friendliness) shows the strongest statistically significant correlations with the overall performance of Aircraft A among the six input variables, followed by C_1 (comfort) and C_2 (cost).

4.6 An Uncertainty Assessment Module

The proposed new approach is implemented and an uncertainty assessment module is developed and integrated into the multi-criteria decision support system. The user guide of the uncertainty assessment module can be found in Appendix A. As shown in Figure A.9 in Appendix A, the DM can simply go through the uncertainty assessment process according to the instructions. In addition, the mathematical calculation steps for four MCDA techniques: SAW, multiplicative weighting method, TOPSIS, and ELECTRE, are also built in the uncertainty assessment module, which highly facilitates the uncertainty assessment in the decision analysis process.

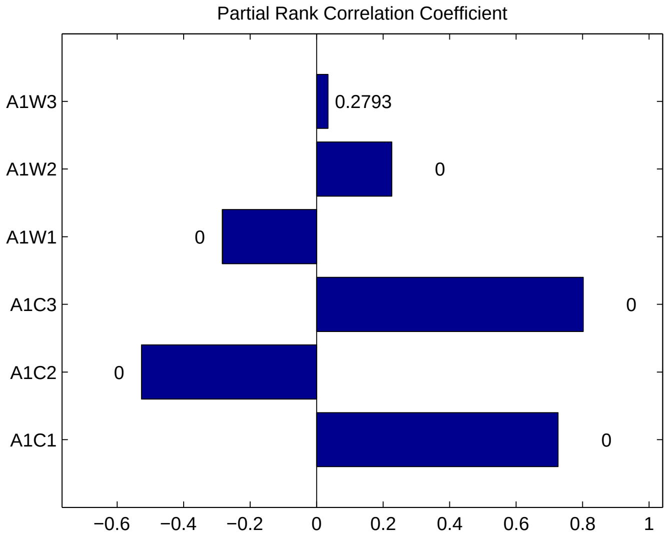


Figure 4.8: Partial Rank Correlation Coefficients for A_1 in an Aircraft Selection Example

4.7 Chapter Summary

A new approach for uncertainty assessment in the decision analysis process was proposed in this chapter. This approach consists of four steps: uncertainty characterization, uncertainty analysis, local and global sensitivity analysis. The proposed approach was implemented and an uncertainty assessment module was developed and integrated into the intelligent multi-criteria decision support system, as discussed in Chapter 3. This novel approach for uncertainty assessment can be used to aggregate input data from tools with different fidelity levels and is capable of propagating uncertainties in an assessment chain. Specifically, the different fidelity levels can be effectively captured by the confidence level in the uncertainty characterization step.

Furthermore, a step by step approach to perform global sensitivity analysis using partial rank correlation coefficients was proposed, with emphasis on measure of association and statistical significance test. The proposed approach can be extended to investigate statistical relationships between variables in complex analysis problems.

5

Proof of Concept 1: MCDA in Aircraft Design

The third objective of this research is to demonstrate the effectiveness of implementing the most appropriate MCDA techniques in aircraft design and evaluation processes. In this chapter, the feasibility and added values of applying MCDA techniques in aircraft design are explored. A new optimization framework incorporating MCDA techniques in aircraft conceptual design process is established, as illustrated in Figure 5.1. An improved MCDA method is utilized to aggregate multiple design criteria into one composite figure of merit, which serves as an objective function in the optimization process. The proposed optimization framework can support designers to quickly assess the compromised design alternatives, which is valuable especially in aircraft conceptual design stage.

The chapter is organized as follows. Section 5.1 defines the aircraft design problem. Section 5.2 presents the selection of the most appropriate MCDA method, through the intelligent multi-criteria decision support system, as described in Chapter 3. Section 5.3 presents the results of applying an improved MCDA method in the proposed multi-criteria optimization framework. In Section 5.4, surrogate models for design criteria in terms of weighting factors are developed. Section 5.5 presents uncertainty assessment based on the developed surrogate models, following the new approach proposed in Chapter 4. Section 5.6 discusses the implementation of MCDA techniques in aircraft design problems.

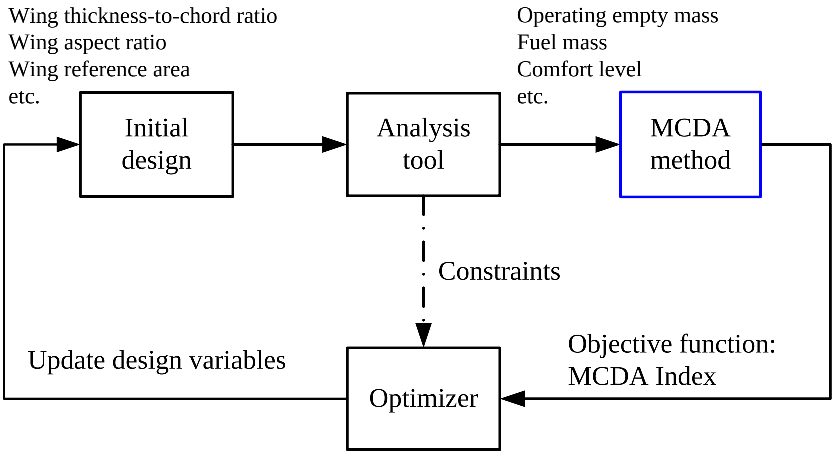


Figure 5.1: The Framework of Incorporating MCDA Techniques in Aircraft Design Process

5.1 Definition of the Decision Making Problem

The design of an A320-like commercial airliner is implemented as a proof of concept with the aircraft conceptual design tool VAMPzero (Virtual Aircraft Multidisciplinary Analysis and Design Processes) [19]. VAMPzero is developed at German Aerospace Center (DLR e.V.) and licensed under the Apache 2.0 license. The design has 150 passenger, twin engine with 3200 km range. The simplified mission profile is illustrated in Figure 5.2.

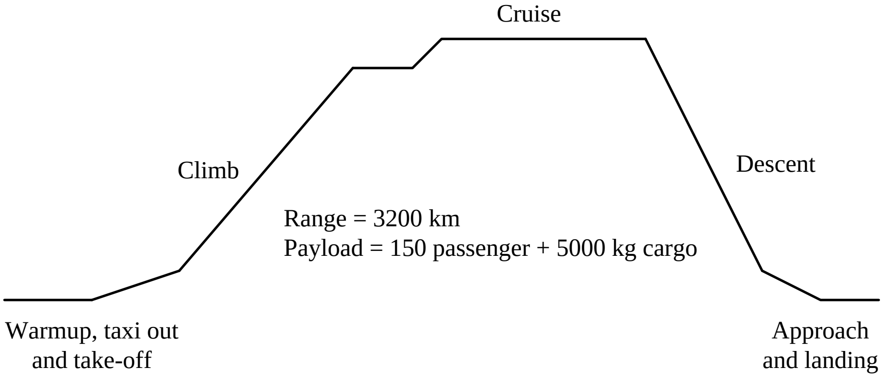


Figure 5.2: The Simplified Aircraft Mission Profile

The optimization framework shown in Figure 5.1 focuses on the assessment of added values of incorporating MCDA techniques in aircraft conceptual design process. Thus, in order to keep the design process transparent, the complexity of the design problem is limited. Five design variables are considered in this study: wing thickness-to-chord ratio, wing aspect ratio, wing reference area, cruise Mach number, and fuselage diameter. The baseline, minimum, and maximum values for the five design variables are listed in Table 5.1.

Table 5.1: The Baseline and Ranges of Design Variables

	Thickness-to-chord ratio	Aspect ratio	Reference area (m^2)	Cruise Mach number	Fuselage diameter (m)
Baseline	0.13	9.396	122.4	0.78	4
Minimum values	0.1	8	80	0.7	3.8
Maximum values	0.2	12	140	0.84	4.2

5.1.1 Identification of Design Criteria

The design criteria of interest are categorized into four groups: cost-based, weight-based, operation-based, and comfort-based. The four groups are described as follows.

Cost-based criteria

- **DOC**: DOC calculates all the direct operating costs per block hour, including fuel cost, maintenance cost, depreciation cost, crew cost, and miscellaneous cost.
- **Fuel cost**: Fuel cost calculates the mission fuel costs per block hour, as shown in Equation 5.1. Fuel price is set to 0.85 Dollars per kilogram.
- **Aircraft price**: An estimation of aircraft price based on OEM, is shown in Equation 5.2 [62]. The exchange rate from Dollar to Euro is set to 0.73.

Weight-based criteria

- **OEM**: Operating Empty Mass (OEM) calculates the operating empty mass from the components, including fuselage, wing, engine, landing gear, horizontal tail plane, vertical tail plane, and pylon, and operator's items mass.
- **Fuel mass**: Fuel mass calculates the fuel needed for the complete mission via the sum of all mission segment fuel masses, including take-off, climb, cruise, descent, and reserve.
- **TOM**: Take-off Mass (TOM) is the sum of OEM, fuel mass, and payload.

Operation-based criteria

- **Annual utilization**: Annual utilization defines the number of flight hours relative to the number of possible flight hours, with the assumption that the aircraft is grounded for a quarter of an hour. Its formula is shown in Equation 5.3 [54].
- **Block time**: Block time calculates the time from engines *on* to engines *off* for the design mission [62]. Utilization/(block time) ratio provides the number of flight, as shown in Equation 5.4.

Comfort-based criteria

- **Passenger density**: Passenger density is defined by the number of passenger seats divided by cabin base area, where cabin base area is calculated by the product of fuselage diameter and cabin length. Its mathematical formula is shown in Equation 5.5.

$$\text{Fuel Cost} = \left(\frac{\text{Fuel mass} \times \text{Fuel price}}{\text{Block time}}\right)(\text{Exchange rate}) \tag{5.1}$$

$$\text{Aircraft Price} = \left(0.8109\left(\frac{\text{OEM}}{1000}\right) + 6.3722\right)(\text{Exchange rate})(\text{Inflation rate})10^6 \tag{5.2}$$

$$\text{Annual Utilization} = \frac{4198}{1 + \frac{0.75}{\text{Block time}}} \tag{5.3}$$

$$\text{Utilization/(Block time)} = \frac{4198}{0.75 + \text{Block time}} \tag{5.4}$$

$$\text{Passenger Density} = \frac{\text{Number of passenger seats}}{\text{Fuselage diameter} \times \text{Cabin length}} \tag{5.5}$$

Selection of appropriate design criteria is critical to the determination of an optimal design. Some recommendations were provided in [101]: the design criterion should represent a non-trivial and calculable indication of the worth of the concept, it should be significantly affected by the design variables and constraints, it should have clear meaning to designers and customers, and it needs clear rationale for methods and factors used for blending if it is blended.

In our case, the question is: Which design criteria are more appropriate to be fed into the MCDA method? In order to better answer this question, parametric studies of design criteria are conducted first, followed by the determination of which design criteria would be further fed into the MCDA method.

5.1.2 Parametric Studies of Design Criteria

The parametric study for cruise Mach number is illustrated in Figure 5.3. The increase of cruise Mach number has a higher fuel consumption for a given mission range and more fuel needs to be carried with the aircraft. Due to the increased aircraft weight, the aircraft price is also increased. Besides, the wave drag of the aircraft increases dramatically with cruise Mach number. Furthermore, it can be seen from Figure 5.3 that there are optimal points for cruise Mach number concerning the minimization of OEM, fuel mass, aircraft price, and TOM, respectively. Utilization/(block time), DOC, and fuel cost increase with cruise Mach number. Cruise Mach number has no influence on passenger density. It is also important to point out that there does exist optimal cruise Mach number regarding the minimization of total DOC (Euro) instead of DOC per block hour.

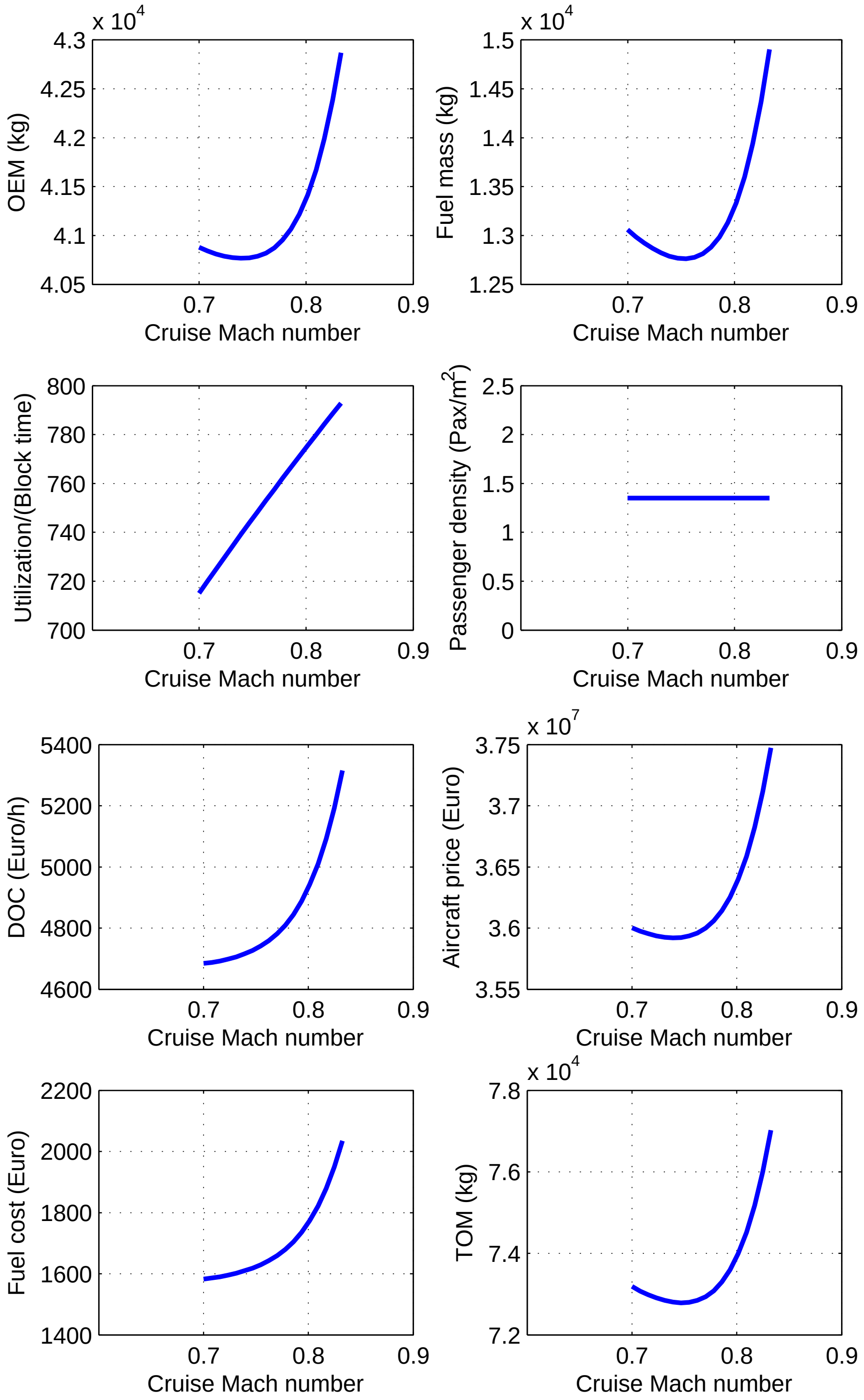


Figure 5.3: Parametric Study of Cruise Mach Number versus OEM, Fuel Mass, Utilization/(Block time), Passenger Density, DOC, Aircraft Price, Fuel Cost, and TOM

Parametric studies for wing thickness-to-chord ratio, aspect ratio, reference area, and fuselage diameter are presented in Figure B.1, Figure B.2, Figure B.3, and Figure B.4 in Appendix B, respectively. The increase of thickness-to-chord ratio reduces the wing weight and more fuel volume can be obtained. However, with the increase of thickness-to-chord ratio, the wave drag of the aircraft is also increased, especially at high speed. It can be observed that there are optimal settings of thickness-to-chord ratio with regard to the minimization of OEM, aircraft price, DOC, and TOM. With the increase of thickness-to-chord ratio, fuel mass and fuel cost increase significantly. Thickness-to-chord ratio has no influence on utilization/(block time) and passenger density.

The increase of wing aspect ratio can reduce the induced drag of the wing and thus the overall drag of the aircraft will be reduced. Thus, less fuel is required to fly a given mission range. However, the increase of wing aspect ratio also leads to a heavier wing weight. It can be seen from Figure B.2 that there is one optimum of aspect ratio regarding the minimization of DOC. Besides, OEM, aircraft price, and TOM increase with aspect ratio, while fuel mass and fuel cost decrease. Aspect ratio has no influence on utilization/(block time) and passenger density.

A larger wing reference area has a small drag coefficient, thus, less fuel is required to fly a given mission. However, the increase of reference area leads to a larger wing and hence a heavier aircraft. Figure B.3 shows that there are optimum points for reference area to minimize DOC and TOM. OEM and aircraft price increase with reference area, while fuel mass and fuel cost decrease. Reference area has no impact on utilization/(block time) and passenger density.

The increase of fuselage diameter can increase the cabin volume, but the fuselage weight is increased. The overall drag of fuselage is also increased when the wetted area of fuselage is increased. Moreover, Figure B.4 shows that OEM, fuel mass, DOC, aircraft price, fuel cost, and TOM all increase with fuselage diameter, while passenger density decreases. Fuselage diameter has no influence on utilization/(block time).

Another observation obtained from parametric studies is that all design variables under investigation are continuous, and design criteria with respect to the design variables in the conceptual aircraft design tool (VAMPzero) are rather smooth. This observation can help to choose the optimization routine for the proposed framework in Section 5.3.

Determination of Evaluation Criteria

The common practice of using DOC as objective function in the optimization is not appropriate in this study, considering that DOC has high correlation with all other design criteria. Besides, aircraft price is highly correlated to OEM, and fuel cost is calculated by fuel mass and block time. Payload is fixed in this case, and TOM is merely determined by OEM and fuel mass.

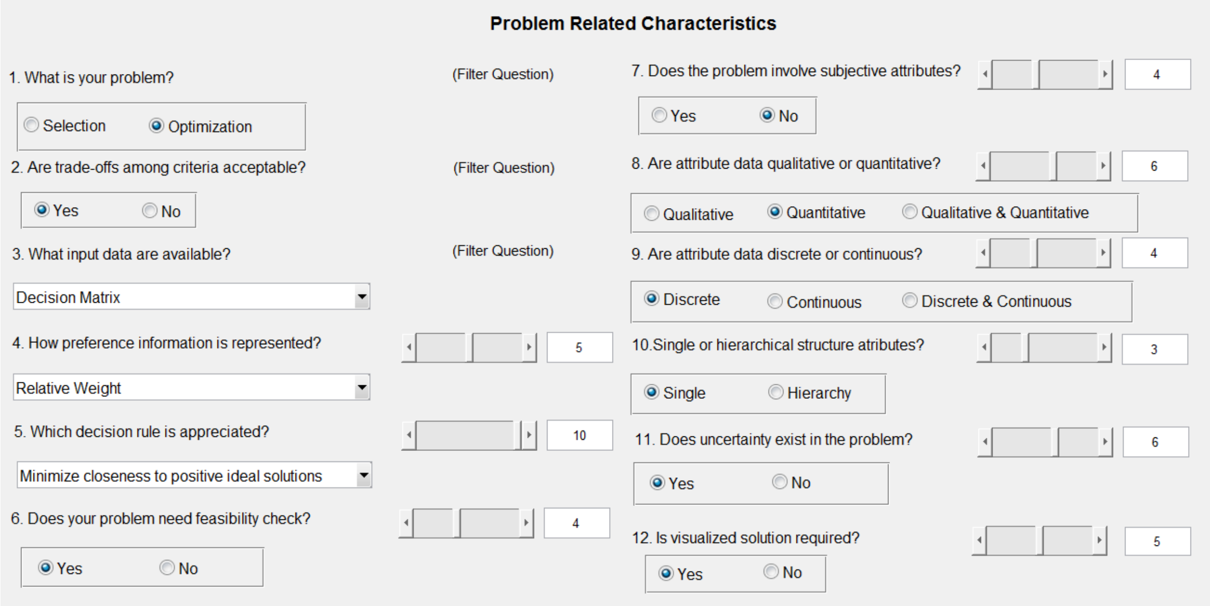
Therefore, in order to explore the interrelationships among the interests of manufacturers, fuel-based emissions, the concerns of airliners, and the consideration of passenger comfort explicitly, four design criteria: OEM, fuel mass, utilization/(block time), and passenger density, are selected to feed into the MCDA method. The other unselected design criteria of interest: DOC, aircraft price, fuel cost, and TOM, are traced as aircraft performance measures during the optimization process. The five design variables are listed in Table 5.1. The constraints imposed in the aircraft design process are wing span, fuel mass, take-off field length, landing field length, take-off wing loading, and cruise thrust. The design variables, constraints, and design criteria for this simplistic aircraft design model are summarized in Table 5.2.

Table 5.2: Summary of Design Variables, Constraints, and Design Criteria in Aircraft Optimization Process

	Units	Values
Design variables		
Wing thickness-to-chord ratio	–	$[0.1, 0.2]$
Wing aspect ratio	–	$[8, 12]$
Wing reference area	m^2	$[80, 140]$
Cruise Mach number	–	$[0.70, 0.84]$
Fuselage diameter	m	$[3.8, 4.2]$
Constraints		
Wing span	m	≤ 36
Fuel mass	kg	$\leq$ Fuel tank volume
Take-off field length	m	≤ 3000
Landing field length	m	≤ 2000
Take-off wing loading	kg/m^2	≤ 600
Cruise thrust	N	≤ 0.9 Take-off thrust
Design criteria		
OEM	kg	–
Fuel mass	kg	–
Utilization/(block time)	–	–
Passenger density	Pax/m^2	–

5.2 Selection of an Appropriate MCDA Method

In this section, the selection of the most appropriate MCDA method for the aircraft design problem is presented, through the developed intelligent multi-criteria decision support system, as described in Chapter 3. The user guide can be found in Appendix A. The step by step method selection process is discussed in the following subsections.



Problem Related Characteristics

1. What is your problem? (Filter Question)
○ Selection ● Optimization

2. Are trade-offs among criteria acceptable? (Filter Question)
● Yes ○ No

3. What input data are available? (Filter Question)
Decision Matrix

4. How preference information is represented? 5
Relative Weight

5. Which decision rule is appreciated? 10
Minimize closeness to positive ideal solutions

6. Does your problem need feasibility check? 4
● Yes ○ No

7. Does the problem involve subjective attributes? 4
○ Yes ● No

8. Are attribute data qualitative or quantitative? 6
○ Qualitative ● Quantitative ○ Qualitative & Quantitative

9. Are attribute data discrete or continuous? 4
● Discrete ○ Continuous ○ Discrete & Continuous

10.Single or hierarchical structure atributes? 3
● Single ○ Hierarchy

11. Does uncertainty exist in the problem? 6
● Yes ○ No

12. Is visualized solution required? 5
● Yes ○ No

Figure 5.4: Questions Related to Evaluation Criteria for Method Selection in Aircraft Design Process

Step 1: Define the Problem

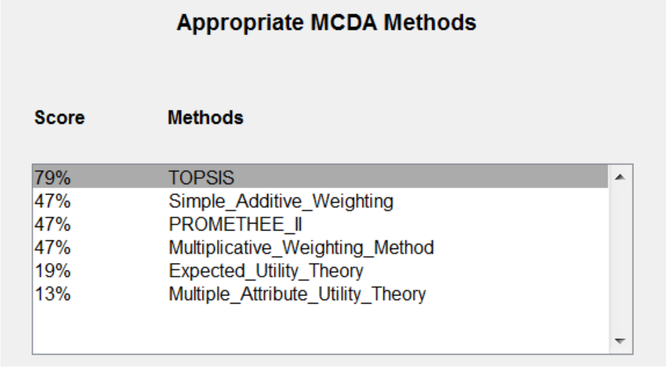
As discussed in Section 5.1, the decision making problem in this simplistic aircraft design is to aggregate the four design criteria into one compound figure of merit using one appropriate MCDA method. The proposed intelligent multi-criteria decision support tool is employed to facilitate this decision making process.

Step 2: Define the Evaluation Criteria

In order to identify the most appropriate method, sixteen widely used MCDA methods are studied and their characteristics are stored in the knowledge base. To compare the appropriateness of the methods with respect to the given problem, each method is evaluated based on the proposed twelve evaluation criteria. The twelve evaluation criteria can be captured by answering twelve questions relevant to the characteristics of the methods, as shown in Figure 5.4.

Step 3: Perform Initial Screening

In this step, infeasible MCDA methods are eliminated by three filtering questions. Considering that in this aircraft design problem, the compound figure of merit for the four design criteria aggregated by MCDA methods serves as an objective function in the optimization, scoring methods are more appropriate than classification methods. Meanwhile, all non-compensatory methods are excluded since compensation is allowed in the aircraft optimization process.



Appropriate MCDA Methods

Score	Methods
79%	TOPSIS
47%	Simple_Additive_Weighting
47%	PROMETHEE_II
47%	Multiplicative_Weighting_Method
19%	Expected_Utility_Theory
13%	Multiple_Attribute_Utility_Theory

Figure 5.5: MCDA Methods Ranking List with Scores in Aircraft Design Process

Step 4: Define the Preferences on Evaluation Criteria

Since DM may consider one criterion to be more important than another when selecting the most appropriate method, weighting factors are to be defined for each criterion to reflect the DM's preference information. The DM's preference information on the evaluation criteria can be defined using slide bars in our integrated user interface, with a subjective scale of 0 to 10, where 0 stands for extremely unimportant criterion and 10 represents extremely important.

Step 5: Calculate the Appropriateness Index

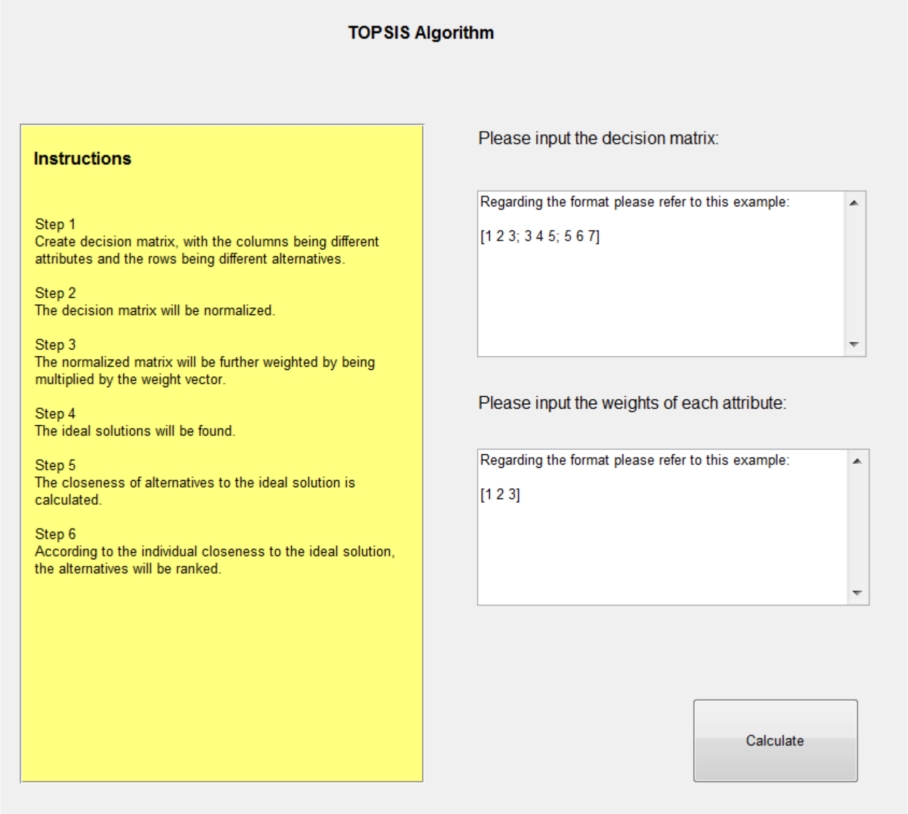
Essentially, AI is used to determine how the characteristics of a method match the characteristics of the given decision making problem. In this step, AI for each MCDA method is calculated by Equation 3.1, as described in Subsection 3.2.5.

Step 6: Evaluate the MCDA methods

Based on the calculation, AI of the MCDA methods are obtained and shown in Figure 5.5, where higher score represents more appropriateness of the method when solving the given problem.

Step 7: Choose the Most Suitable Method

In this example, as indicated in Figure 5.5, TOPSIS gets the highest score among the MCDA methods. In Subsection 3.2.5, it is shown that high value of AI indicates the method is more appropriate to solve a given decision problem. Therefore, TOPSIS is selected as the most appropriate method to solve the aircraft design problem. In the decision support system, the DM can simply click the name of the method and methodology instructions of TOPSIS will be displayed to guide the DM to solve the given problem, as illustrated in Figure 5.6.



Instructions

Step 1
Create decision matrix, with the columns being different attributes and the rows being different alternatives.

Step 2
The decision matrix will be normalized.

Step 3
The normalized matrix will be further weighted by being multiplied by the weight vector.

Step 4
The ideal solutions will be found.

Step 5
The closeness of alternatives to the ideal solution is calculated.

Step 6
According to the individual closeness to the ideal solution, the alternatives will be ranked.

Please input the decision matrix:

Regarding the format please refer to this example:

[1 2 3; 3 4 5; 5 6 7]

Please input the weights of each attribute:

Regarding the format please refer to this example:

[1 2 3]

Figure 5.6: Methodology Instructions for TOPSIS

Step 8: Conduct Sensitivity Analysis

Since different DMs often have different answers to the twelve questions, sensitivity analysis to the variation of input data should be performed on the MCDA method selection process. In our integrated user interface, the DM can adjust the weights of each criterion by moving the slide bars. In this example, with the current input data, it can be seen from Figure 5.5 that SAW, PROMETHEE, and multiplicative weighting method, are ranked second by the multi-criteria decision support system. According to the methodology description in Chapter 2, PROMETHEE needs three threshold values for each criterion: indifference threshold, strict preference threshold, and an intermediate value between indifference and strict preference threshold. These extra twelve thresholds for the four design criteria increase the complexity of the aircraft design problem significantly. Moreover, these extra twelve threshold values are rather subjective and different DMs often have different threshold values. Besides, the difference between SAW and multiplicative weighting method is the multiplicative property of the weighting factors. Therefore, considering that SAW is one widely used MCDA method, SAW is used in the aircraft design problem for the purpose of comparison.

5.2.1 An Improved TOPSIS (ITOPSIS) Technique

TOPSIS technique is recommended by the multi-criteria decision support system as the most appropriate one to solve the aircraft design decision problem. In TOPSIS method, two ideal solutions are hypothesized: positive ideal solution which has all the best criteria values, and negative ideal solution which has all the worst criteria values. TOPSIS selects the alternative that is closest to the positive ideal solution and farthest from the negative ideal solution.

For the purpose of illustration, it can be imagined that TOPSIS puts the alternatives into a coordinate system. For example, if there are three criteria, it is a three-dimension coordinate system, as shown in Figure 5.7, where the green dot represents the positive ideal solution, and the red dot represents the negative ideal solution. TOPSIS ranks the alternatives based on the Euclidean distance to these two ideal solutions.

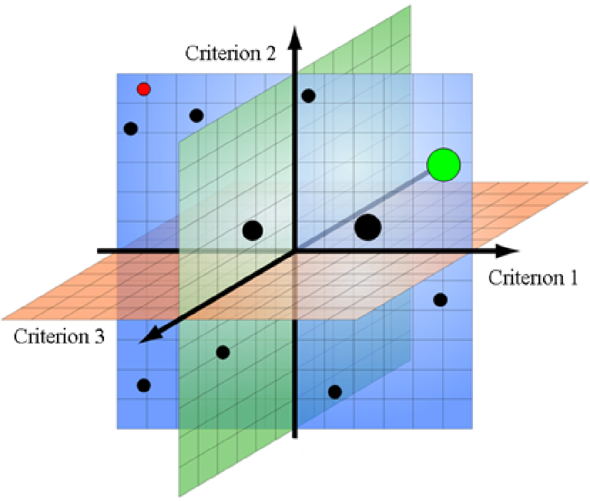


Figure 5.7: TOPSIS in Three Dimensions Coordinate System

However, in the original TOPSIS method, when an alternative is removed from or added to the candidate alternatives, the two hypothetical ideal solutions will probably change and the Euclidean distances to the two hypothetical ideal solutions will also change. Thus, the top-ranked alternative would possibly become inconsistent when the candidate alternatives are changed. It has been pointed out that the cause of rank inconsistency with TOPSIS lies in the calculation step of determining the two hypothetical ideal solutions [30].

In this study, an Improved TOPSIS (ITOPSIS) is utilized to aggregate the four design criteria into one compound figure of merit for optimization. The positive ideal solution and negative ideal solution are set beforehand in order to maintain the ranking consistency. In this aircraft design decision problem, two kinds of optimizations are conducted for each of the four design

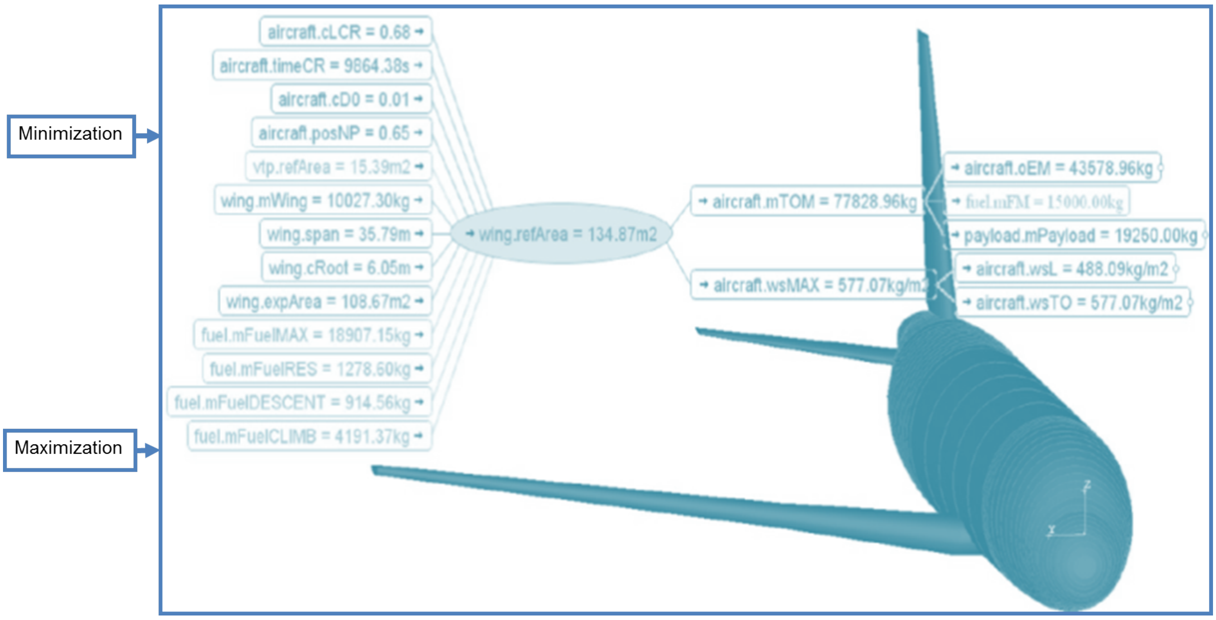


Figure 5.8: An Improved TOPSIS (ITOPSIS) in Aircraft Design Decision Problem

criteria: minimization and maximization, as illustrated in Figure 5.8.

For instance, in order to find the ideal solutions for fuel mass, two kinds of optimizations for fuel mass are conducted: minimization and maximization. The minimum value of fuel mass serves as the positive ideal solution, while the maximum value of fuel mass serves as the negative ideal solution. The ideal solutions for the other three design criteria are searched in a similar way. These ideal solutions for the four design criteria are summarized in Table 5.3. It should be noted that utilization/(block time) ratio is a benefit criterion, and the other three design criteria are cost criteria.

Table 5.3: The Positive Ideal Solution and Negative Ideal Solution in ITOPSIS

Ideal solutions	OEM (kg)	Fuel mass (kg)	Utilization/ (block time)	Passenger density (Pax/m^2)
Positive	36943.4992	11766.8787	796.8551	1.2875
Negative	50521.0972	20864.0399	715.0679	1.4063

5.3 Proposed Multi-Criteria Optimization Framework

Considerable research has been devoted to the development of optimization methods in order to deal with multiple, conflicting objectives (criteria), such as multi-objective Genetic Algorithms (GA) [36]. For instance, a three-objective GA was used to explore the trade-offs among noise, emissions, and operating costs in the aircraft conceptual design stage [10]. A two-objective GA was applied to balance fuel, NO_X emission, and DOC [71]. However, multi-objective GA suffer from expensive computation. Different runs of GA may generate different optimization results for the same problem. Furthermore, evolutionary multi-objective optimization techniques are not easily applicable for handling a large number of objectives [37].

A new multi-criteria optimization framework incorporating MCDA techniques in aircraft conceptual design process is established, as illustrated in Figure 5.1. ITOPSIS is utilized to aggregate the multiple design criteria into one composite figure of merit. The composite figure of merit serves as an objective function during the optimization. This framework supports designers to quickly assess the compromised design alternatives. Moreover, MCDA techniques have the ability to handle large number of objectives.

In this section, optimization algorithms are briefly reviewed first. Then, optimization results of typical weighting scenarios are presented. At last, optimizations using ITOPSIS index and SAW index as objective functions are compared.

5.3.1 Numerical Optimization Techniques

There are several optimization algorithms currently available, among which gradient-based methods and GA are most widely used in aircraft design.

Gradient-based methods compute the gradient of the objective function with respect to design variables, the gradient vector establishes a search direction of the deepest slope, the objective function changes most rapidly in this direction [67]. Gradient-based methods can provide efficient design solutions. However, gradient-based methods have problems with discontinuous functions and functions that have discrete variables. In addition, when the objective function varies in a non-smooth fashion, gradient-based methods may have the risk of ending up in a local optimum.

GA are stochastic evolutionary algorithms inspired by biological evolution, they operate on a population of candidate solutions and apply the principle of survival of the fittest to evolve the candidate solutions towards the desired optimal solutions [36]. Continuous and discrete variables can be included in GA simultaneously, where the continuous variables are discretized with a reasonable resolution. Additionally, GA consider the whole design space, thus, the risk of convergence to a local optimum can be avoided. However, GA suffer from expensive computation, and different optimization runs may result in different optimal solutions.

Which optimization method to use depends on the optimization problem under consideration. If all design variables are continuous and objective functions are smooth, gradient-based methods should be used in the optimization process. If there are discrete variables and objective functions are noisy, GA should be employed.

According to parametric studies performed in Subsection 5.1.2, it is observed that all design variables under investigation are continuous, and objective functions with respect to the design variables in the conceptual aircraft design tool (VAMPzero) are rather smooth. Therefore, gradient-based methods are used in the established optimization framework.

Evaluation of Gradient-based Optimization with Different Starting Points

It is important to note that gradient-based methods are prone to finding a local optimum, depending on the location of the starting point. In order to assess whether the gradient-based optimizer (sequential quadratic programming algorithm) can converge towards the same optimal design in the aircraft optimization process, optimization tests using ITOPSIS index as an objective function starting from different initial points are conducted in this subsection.

The baseline and ranges for the five design variables under consideration were summarized in Table 5.1 in Section 5.1. Random starting points are generated within their lower bounds and upper bounds, as shown in Equation 5.6.

$$(\text{upper bound} - \text{lower bound}) \times \text{random number} + \text{lower bound} \tag{5.6}$$

where $0 \leq \text{random number} \leq 1$. The lower bounds and upper bounds of design variables are the minimum values and maximum values scaled against baseline. Ten sets of random starting points are listed in Table 5.4. The optimized designs using these ten sets of different starting points are summarized in Table 5.5.

Table 5.4: Ten Sets of Random Starting Points in the Optimization Process

Set	Thickness-to-chord ratio	Aspect ratio	Reference area (m^2)	Cruise Mach number	Fuselage diameter (m)	Optimization time (s)
1	0.1058	10.0875	96.5858	0.7763	4.1628	1165
2	0.1995	8.6434	137.8836	0.8358	4.1288	606
3	0.1310	9.5031	81.0778	0.7455	4.0204	3666
4	0.1406	11.4378	115.9128	0.7674	4.1024	400
5	0.1151	9.1611	127.7489	0.7323	4.0620	390
6	0.1551	11.1465	132.9876	0.7114	3.8276	382
7	0.1266	10.4230	88.1035	0.8208	3.9396	442
8	0.1610	11.6172	105.6067	0.8032	3.8596	392
9	0.1763	8.1698	100.1110	0.7870	3.8872	483
10	0.1889	9.9757	119.2910	0.7268	3.9680	339

It is observed that the gradient-based optimizer is able to find the same optimal design starting from different initial points. Furthermore, computation times for the optimization

Table 5.5: Optimized Designs using Ten Sets of Random Starting Points

Set	Thickness-to-chord ratio	Aspect ratio	Reference area (m^2)	Cruise Mach number	Fuselage diameter (m)
1	0.1349	9.3783	116.9663	0.7603	3.8
2	0.1344	9.3697	116.9928	0.7611	3.8
3	0.1350	9.3923	116.9975	0.7613	3.8
4	0.1351	9.3999	116.9855	0.7600	3.8
5	0.1349	9.3929	116.9864	0.7601	3.8
6	0.1347	9.3733	116.9708	0.7606	3.8
7	0.1351	9.4015	116.9810	0.7596	3.8
8	0.1351	9.4014	116.9878	0.7599	3.8
9	0.1349	9.3948	116.9891	0.7600	3.8
10	0.1350	9.3954	116.9825	0.7600	3.8

starting from different initial points have also been recorded. It is noted that the Set 1 and Set 3 took unusual longer time than other sets, this can be attributed to that the starting points of reference area and thickness-to-chord ratio are far away from the optimal design, thus, the optimizer needs more iterations to converge towards the optimal design solution.

5.3.2 Optimization Results of Typical Weighting Scenarios

In this subsection, several typical weighting scenarios in the optimization process are investigated, ranging from one criterion preferred to evenly distributed. This is one approach to simulate DM's preference information. Optimization results for single criterion are summarized in Table 5.6, and optimization results with equal weighting factors among the four design criteria are summarized in Table 5.7, respectively.

It can be seen from Table 5.6, when optimizing OEM, fuselage diameter is reduced to the lower boundary, aspect ratio is reduced by 14%, reference area is decreased by 5%, and thickness-to-chord ratio is increased by 21%. The decrease of aspect ratio and reference area leads to a reduction in wing weight, which contributes to a reduction in OEM and TOM. As expected, aircraft price is also reduced by 8% because of the reduction in OEM. Fuel cost is reduced by 4% and DOC is decreased by 5%. However, the decrease of aspect ratio and reference area and the increase of thickness-to-chord ratio result in an increment of the overall drag of the aircraft and 9% reduction in cruise Mach number. The reduction in cruise Mach number leads to a 5% decrease in utilization/(block time). Besides, the decrease of fuselage diameter leads to a 5% increase of passenger density.

When optimizing the aircraft for fuel mass, aspect ratio is increased by 24%, reference area is increased by 8%, and thickness-to-chord ratio is decreased by 6%. The increase of aspect ratio and reference area leads to a larger span and an increase in wing weight, which further leads to the increase of OEM, TOM, and aircraft price. Flying slower (low cruise Mach number) can also reduce the consumption of fuel for certain mission range. However, lower cruise Mach number will prolong block time, thus, utilization/(block time) ratio is decreased. In addition, the overall drag of the aircraft can be reduced when the wetted area of fuselage is reduced, this is the reason why fuselage diameter is decreased to the lower boundary.

When optimizing the aircraft for utilization/(block time), cruise Mach number is increased to the upper boundary, fuselage diameter is reduced so that the wet area of fuselage is reduced, reference area is increased by 5%. The decrease of fuselage diameter and increase of reference area lead to the reduction of the overall drag of the aircraft. However, the increase of cruise Mach number will burn more fuel for specific mission range, thus, fuel mass and fuel cost are increased 19% and 25%, respectively. DOC is also increased by 12%, considering the dominant role of fuel cost. The increase of reference area leads to the increase of OEM, TOM, and aircraft price. Besides, the decrease of fuselage diameter results in 4% increase of passenger density.

When optimizing the aircraft for passenger density, fuselage diameter is increased to its upper limit. Reference area is increased slightly by 3%, thickness-to-chord ratio, aspect ratio, and

Table 5.6: Optimization Results for Single Criterion

	Baseline design	Min. OEM	Min. Fuel mass	Max. Utilization/ (block time)	Min. Passenger density
Design variables					
Thickness-to-chord ratio	0.13	0.1585	0.1220	0.1286	0.1301
Aspect ratio	9.4	8.0347	11.6740	9.3237	9.3608
Reference area (m^2)	122.40	116.18	132.05	128.53	125.77
Cruise Mach number	0.78	0.71	0.73	0.84	0.77
Fuselage diameter (m)	4	3.8	3.8	3.9	4.2
Design criteria					
OEM (kg)	40980	36949	43725	42974	42426
Fuel mass (kg)	12903	13280	11771	15319	13312
Utilization/(block time)	763	722	734	797	759
Passenger density (pax/m^2)	1.35	1.4211	1.4211	1.3863	1.2981
Traced performance measures					
DOC (Euro/h)	4818	4577	4672	5402	4925
Aircraft price (Euro)	36077718	33100305	38106043	37551218	37146224
Fuel cost (Euro/h)	1685	1626	1470	2104	1728
TOM (kg)	73133	69479	74746	77544	74988

Table 5.7: Optimization Results with Equal Weighting Factors

	Baseline design	Optimized design	Relative change (%)
Design variables			
Thickness-to-chord ratio	0.13	0.135	3.84
Aspect ratio	9.396	9.414	0.19
Reference area (m^2)	122.4	117.01	-4.40
Cruise Mach number	0.78	0.76	-2.55
Fuselage diameter (m)	4	3.8	-5
Design criteria			
OEM (kg)	40980	38705	-5.55
Fuel mass (kg)	12903	12242	-5.12
Utilization/(block time)	763	752	-1.53
Passenger density (pax/m^2)	1.35	1.4211	5.26
Traced performance measures			
DOC (Euro/h)	4818	4588	-4.76
Aircraft price (Euro)	36077718	34397326	-4.66
Fuel cost (Euro/h)	1686	1571	-6.79
TOM (kg)	73133	70197	-4.01

cruise Mach number almost do not change. Except utilization/(block time) ratio has decreased slightly, all other criteria have been increased by around 2.5%.

The conflicting design criteria are further explored when weighting factors are evenly distributed, as summarized in Table 5.7. Thickness-to-chord ratio is increased by 4%, aspect ratio almost does not change, reference area is decreased by 4%, cruise Mach number is decreased by 2.5%, and fuselage diameter is decreased to its lower boundary. The reduction of OEM and fuel mass is compromised by the decrease of utilization/(block time) ratio and the increase of passenger density.

Moreover, it can be observed from Table 5.7 that except for utilization/(block time) ratio is decreased by 1.5%, the other three design criteria have around 5% change. Therefore, utilization/(block time) ratio is less sensitive than other three design criteria in this simplistic aircraft design example.

The similar observation can be obtained when the relative changes of the four traced aircraft performances are compared. Fuel cost is decreased by around 6%, while the other three traced aircraft performances are all decreased by around 4%. Thus, fuel cost is more sensitive than other three traced aircraft performances in this simplistic aircraft design example.

5.3.3 Comparison Using Different MCDA Indices as Objective Functions

For the purpose of comparison, the proposed optimization framework is also performed when using SAW index as an objective function, optimization results are summarized in Table 5.8. The comparison of relative changes for the design criteria and traced performance measures, when using ITOPSIS index as an objective function (Table 5.7) and SAW index as an objective function (Table 5.8), are presented in Figure 5.9.

Table 5.8: Optimization Results using SAW Index as an Objective Function, when Weighting Factors are Evenly Distributed

	Baseline design	Optimized design	Relative change (%)
Design variables			
Thickness-to-chord ratio	0.13	0.1304	0.28
Aspect ratio	9.396	9.118	-2.95
Reference area (m^2)	122.4	116.9	-4.48
Cruise Mach number	0.78	0.77	-1.50
Fuselage diameter (m)	4	3.8	-5
Design criteria			
OEM (kg)	40980	38552	-5.92
Fuel mass (kg)	12903	12344	-4.33
Utilization/(Block time)	763.3	756.5	-0.89
Passenger density (pax/m^2)	1.35	1.4211	5.26
Traced performance measures			
DOC (Euro/h)	4818	4612	-4.27
Aircraft price (Euro)	36077718	34284714	-4.97
Fuel cost (Euro/h)	1686	1596	-5.32
TOM (kg)	73133	70147	-4.08

It is observed from Figure 5.9 that with equally assigned weighting factors, the optimized design using ITOPSIS index as an objective function is heavier but more fuel efficient than the design which is optimized using SAW index as an objective function. Furthermore, in the same running environment (Windows 7, 2.66 GHz Intel Core 2 Quad CPU, 4 GB RAM, and Matlab 2010a version), convergence rates when using ITOPSIS index and using SAW index as objective functions are summarized in Table 5.9. It is seen that the optimization using ITOPSIS index as an objective function needs less iterations and less computation time than using SAW index as an objective function.

However, only with one set of weighting factors, we cannot conclude which MCDA method is

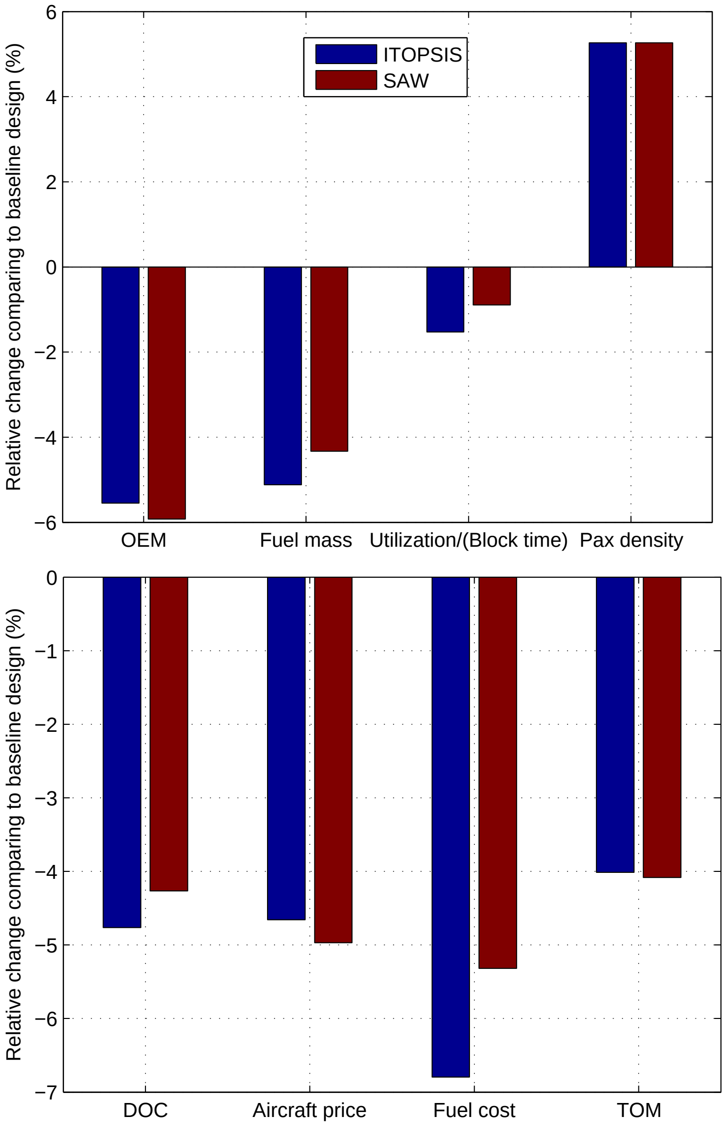


Figure 5.9: Comparison of Relative Changes for Design Criteria and Traced Performance Measures, using ITOPSIS Index and SAW Index as Objective Functions

Table 5.9: Comparison of Convergence Rates, using ITOPSIS Index and SAW Index as Objective Functions

Objective function	Iterations	Optimization time (seconds)
ITOPSIS index	5	304
SAW index	39	3005

more appropriate for the optimization, considering that the optimal design will change with the variation of weighting factors. Uncertainty assessment for exploring how the optimal design will change with the variation of weighting factors is further investigated in the following section.

5.4 Surrogate Model Development for Design Criteria in terms of Weighting Factors

Weighting factors create a compound figure of merit. The compound figure of merit serves as an objective function for optimization. Different weighting schemes result in different compound figure of merits. The selection of weighting factors is critical to the determination of an optimal design, since if a design is optimized for the wrong figure of merit, it will not be the best design in terms of the real important measure.

Especially, inherent uncertainties and subjectivities of weighting factors have significant impacts on the design solution. An uncertainty assessment that demonstrates this impact must consider different combinations of weighting factors. However, in the proposed multi-criteria optimization framework, the computation time for one set of weighting factors is at least 5 minutes. In this research, Monte Carlo is used to imitate decision makers preferences among the design criteria. A Monte Carlo based uncertainty analysis with 10,000 samples would take at least 35 days. The long computation time makes the uncertainty assessment an intractable computational task.

In this study, surrogate models for the four design criteria in terms of weighting factors are developed. Each point of this surrogate model represents an optimized aircraft design for a given set of weighting factors. The whole framework of incorporating MCDA techniques in aircraft design process is treated as a black box. An overview of surrogate modeling development for design criteria in terms of weighting factors is shown in Figure 5.10. The developed surrogate models provide efficient analysis tools for uncertainty assessment.

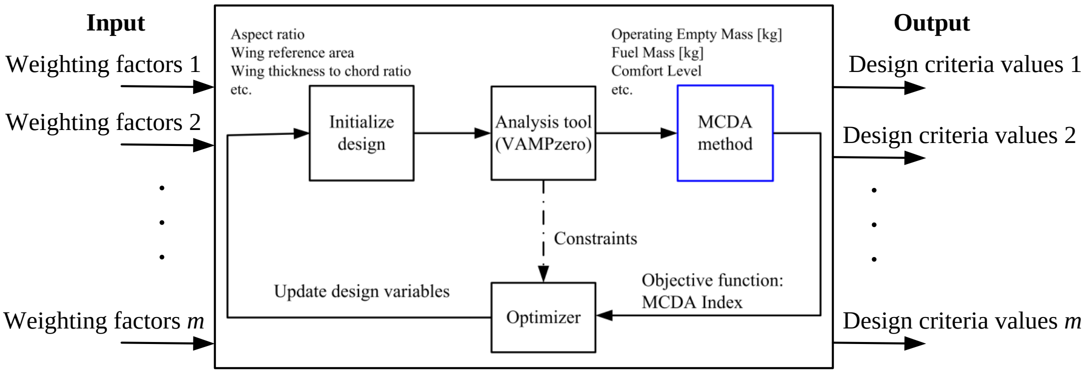


Figure 5.10: Overview of Surrogate Modeling Development for Design Criteria in terms of Weighting Factors

There are typically four steps in surrogate model building process: sample the design space using experimental design, choose a model to represent the input and output data, select a method to fit the model, and validate the constructed model [47]. Surrogate model development for design criteria in terms of weighting factors follow this process. Each step is discussed in detail in the following subsections.

5.4.1 Experimental Design

Experimental design is a sequence of experiments to be performed, expressed in terms of factor settings at specified levels [91]. Experimental designs were originally developed for effective physical experiments, they are being applied to computer experiments with the purpose of reducing the computation time and increasing the efficiency.

In order to explore the design space thoroughly, experimental design with spatially uniform distribution is one effective approach. There are several space filling strategies [75], among which Latin Hypercube Sampling (LHS) is one reliable method to generate random samples, with guarantee that these samples are relatively uniformly distributed in the design space [81].

In this study, weighting factors $[w_1, w_2, ..., w_n]$ generated by experimental design have to satisfy two conditions:

1. $0 \leq w_i \leq 1$
2. $\sum_{i=1}^{n} w_i = 1$

When standard LHS is utilized to generate m sets of weighting factors for n criteria ($w_{m \times n}$), for each experimental run, the factor setting w_{ij} ($i = 1, 2, ..., m, j = 1, 2, ..., n$) is randomly sampled from each interval $(0, 1/m), (1/m, 2/m), ..., (1 - 1/m, 1)$. The standard LHS meets the Condition 1 that all the factor settings range from 0 to 1. However, for each experimental run, the sum of the factor settings in each run does not equal to one. The normalization of the factor settings can fulfill the Condition 2, however, the hypercube is deformed and the Latin properties may not be guaranteed.

In this case, in order to generate experimental designs fulfilling the two conditions, standard LHS is conducted first, then the samples generated by LHS are rectified by Dirichlet distribution.

One Modified LHS with Dirichlet Distribution

Dirichlet distribution is a family of continuous multivariate probability distributions parameterized by a vector $\alpha = (\alpha_1, \alpha_2, ..., \alpha_k)$ of positive reals. Dirichlet distribution is one multivariate generalization of beta distribution and is defined as Equation 5.7.

$$Dir(X, \alpha) = \frac{\Gamma(\alpha_1 + \alpha_2 + ... + \alpha_k)}{\Gamma(\alpha_1)\Gamma(\alpha_2)...\Gamma(\alpha_k)} \prod ({x_1}^{\alpha_1 - 1} {x_2}^{\alpha_2 - 1} ... {x_k}^{\alpha_k - 1}) \tag{5.7}$$

where $X = (x_1, x_2, ..., x_{k-1})$, satisfying $x_i > 0$ and $\sum_{i=1}^{k-1} x_i < 1$. Besides, $x_k = 1 - x_1 - x_2 - ... - x_{k-1}$. A Dirichlet distribution is symmetric when the components of vector α are equal. If each component of α is 1, the symmetric Dirichlet distribution is equivalent to a uniform distribution; if each component of α is bigger than 1, it prefers dense, evenly distributed distributions; and if each component of α is smaller than 1, it prefers sparse distributions.

When using the modified LHS with Dirichlet distribution, although the modified sample values are not strictly uniformly distributed, Dirichlet distribution can keep the ranges of sample values larger once they are normalized, while maintaining the appealing Latin properties.

One Example of Standard LHS, Normalized LHS, and Modified LHS with Dirichlet Distribution

One example of standard LHS, normalized LHS, and the modified LHS with Dirichlet distribution is demonstrated as follows. In order to generate ten sets of weighting factors for three criteria, standard LHS is conducted first, as shown in Figure 5.11, where S_1, S_2, and S_3 represent the sample values for the three criteria. It is noted that there is exactly one point in each row and each column in the two dimensional projections, and the sample values range from 0 to 1 (which meets the Condition 1), however, the sum of one set of the sample values is not equal to

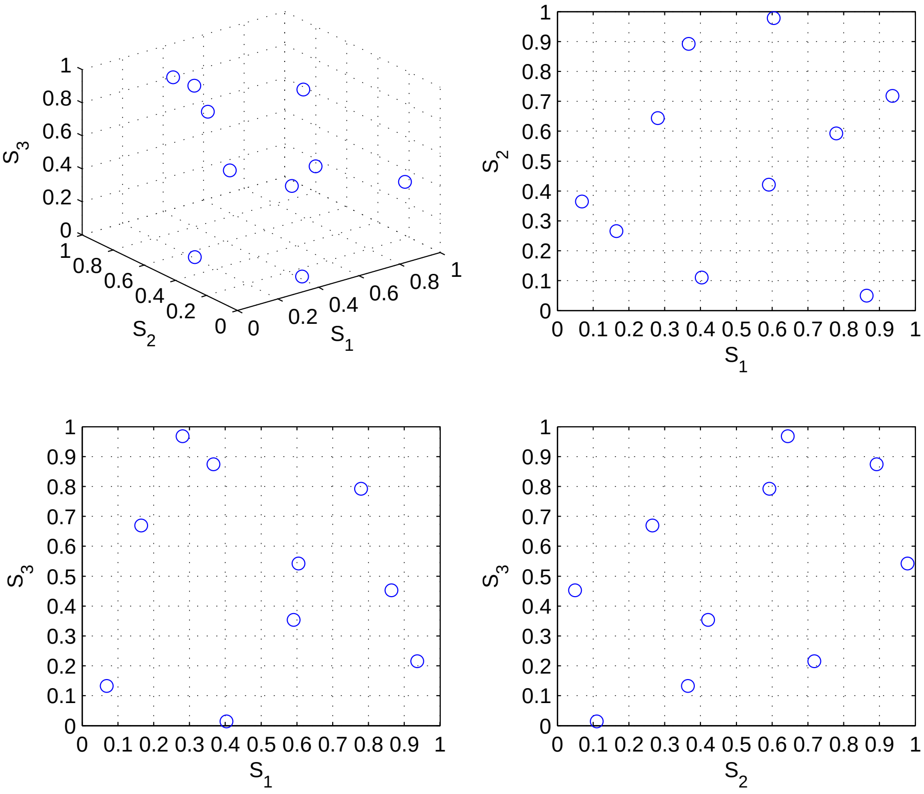


Figure 5.11: Standard Latin Hypercube Sampling in Three Dimensions and with Two Dimensional Projections

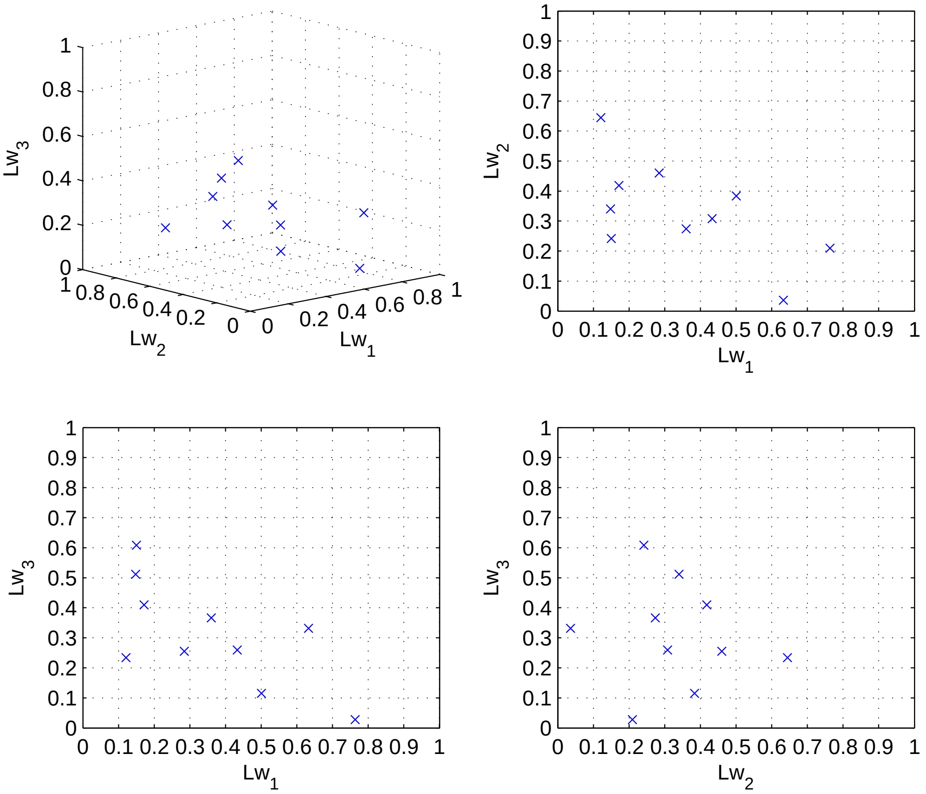


Figure 5.12: Normalized Latin Hypercube Sampling by Row Sum in Three Dimensions and with Two Dimensional Projections

one (which does not meet the Condition 2).

Thus, in order to fulfill the Condition 2, standard LHS can be normalized by its row sum, as shown in Figure 5.12, where Lw_1, Lw_2, and Lw_3 represent the normalized sample values for the three criteria. It is observed that the range of the normalized sample values shrinks into 0 to 0.8. Moreover, there is no point in the bins which are bigger than 0.8, thus, the hypercube is deformed and the Latin properties is not maintained.

The modified LHS with Dirichlet distribution are shown in Figure 5.13, where LDw_1, LDw_2, and LDw_3 represent the sample values rectified by Dirichlet distribution for the three criteria. It is observed that the range of the sample values are recovered from 0 to 1, although there is not exactly one point in each row and each column in the two dimensional projections.

In this study, one hundred sets of weighting factors are generated by the modified LHS with Dirichlet distribution. The data is attached in Table C.1 in Appendix C.1. The weighing factors reflect the relative importance of the design criteria. For instance, the first row in Table C.1 is $W_1 = [0.4333\ 0.0176\ 0.3719\ 0.1772]$. This set of weighting factors indicates that the first design criterion (OEM) is most important, followed by the third design criterion (utilization/block

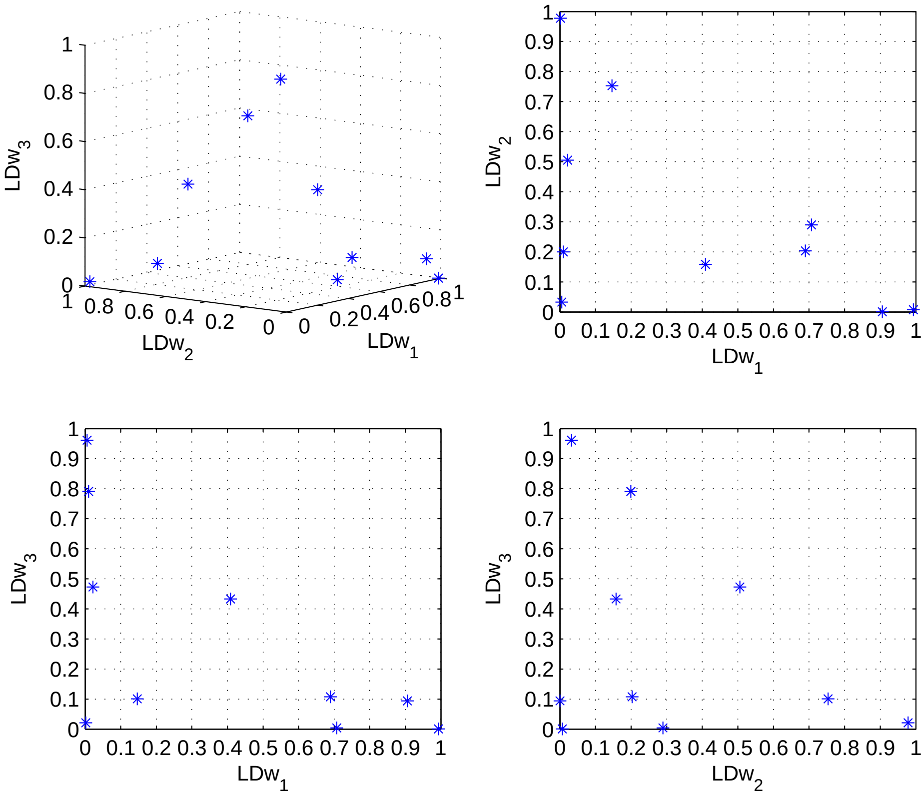


Figure 5.13: Modified Latin Hypercube Sampling with Dirichlet Distribution in Three Dimensions and with Two Dimensional Projections

time) and the fourth design criterion (passenger density), while the second design criterion (fuel mass) is least important. The other 99 sets of weighting factors have similar explanations.

5.4.2 Model Choice

Response surface is one popular approach to build surrogate models [91]. Response surface typically involves least square regression to fit a polynomial model of the observed response values. The most common response surface models are low-order polynomials. For an unknown function of interest $y(x)$, as defined in Equation 5.8.

$$y(x) = f(x) + \epsilon \tag{5.8}$$

where $f(x)$ is a polynomial function, ϵ is random error, which is normally distributed with mean zero and variance σ^2. A second-order polynomial model is shown in Equation 5.9. The parameters of the polynomial in Equation 5.9 are determined through least square regression, which minimizes the sum of the squares of the predicted values $\hat{y}(x)$ from the actual values $y(x)$.

$$\hat{y}(x) = a_0 + \sum_{i=1}^{k} a_i x_i + \sum_{i=1}^{k} a_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} a_{ij} x_i x_j + \epsilon \tag{5.9}$$

Response surface models have been widely used in the surrogate model development in engineering design. There are several advantages using response surface models, such as ease of implementation, minimal efforts required to train models, and suitability for uncertainty analysis. In this research, a fourth-order response surface is utilized to construct the surrogate models.

5.4.3 Model Fitting

A widely used statistics software package JMP *(http://www.jmp.com)* is employed to fit response surface models. Before the construction of response surface models, the correlations among the four design criteria and the traced aircraft performances are assessed. The pairwise correlation coefficients are summarized in Table 5.10.

Table 5.10: Pairwise Correlation Coefficients for Design Criteria of Interest

Correlations	OEM	Fuel mass	Utilization/ (block time)	Passenger density	DOC	Aircraft price	Fuel cost	TOM
OEM	1.0000	-0.1879	0.4779	-0.4840	0.6573	1.0000	0.0781	0.9613
Fuel mass		1.0000	0.1535	-0.5872	0.5480	-0.1879	0.8811	0.0899
Utilization/ (block time)			1.0000	-0.1202	0.7498	0.4779	0.6013	0.5277
Passenger density				1.0000	-0.6845	-0.4840	-0.5352	-0.6554
DOC					1.0000	0.6573	0.8026	0.8202
Aircraft price						1.0000	0.0781	0.9613
Fuel cost							1.0000	0.3263
TOM								1.0000

It is observed from Table 5.10 that DOC shows high correlation with all other criteria, the correlation coefficient between aircraft price and OEM is 1, fuel cost is highly correlated with fuel mass, and TOM have strong correlation with OEM. These observations are consistent with the analytical explanation of the determination of design criteria, as described in Section 5.1. Thus, Table 5.10 serves as one evidence that the selected four design criteria are more appropriate to be fed into the MCDA method for aggregation.

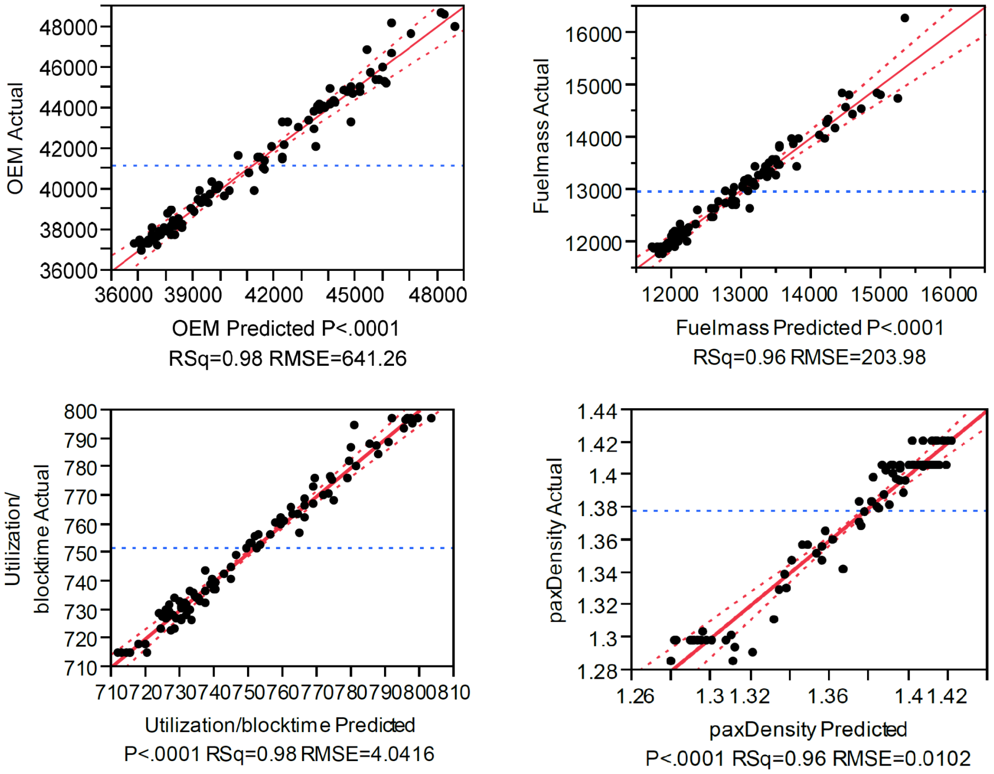


Figure 5.14: The Actual by Predicted Plots for OEM, Fuel Mass, Utilization/(Block time), and Passenger Density, when using ITOPSIS Index as an Objective Function

5.4.4 Model Validation

In this subsection, the accuracy of response surface models is assessed by the actual versus predicted plots first, and is further evaluated by running additional untried data points.

Model Accuracy Evaluation by the Actual Versus Predicted Plots

The actual values versus the predicted values for the four design criteria aggregated by ITOPSIS and by SAW are shown in Figure 5.14 and Figure 5.15, respectively. In the actual by predicted plot, the horizontal dotted blue line represents the mean of actual values, the red line shows 45 degree diagonal line, and the two red dotted lines show 95% confidence intervals.

The actual by predicted plots illustrate how well the predicted responses match the actual data. A quick assessment of the model is to eyeball a 45 degree pattern in these plots. In our case, the scatter plots when ITOPSIS is used for the multiple criteria aggregation and when SAW is used for the multiple criteria aggregation all follow a 45 degree pattern. Specifically, the scatter plots for ITOPSIS are less divergent along the diagonal line than the scatter plots

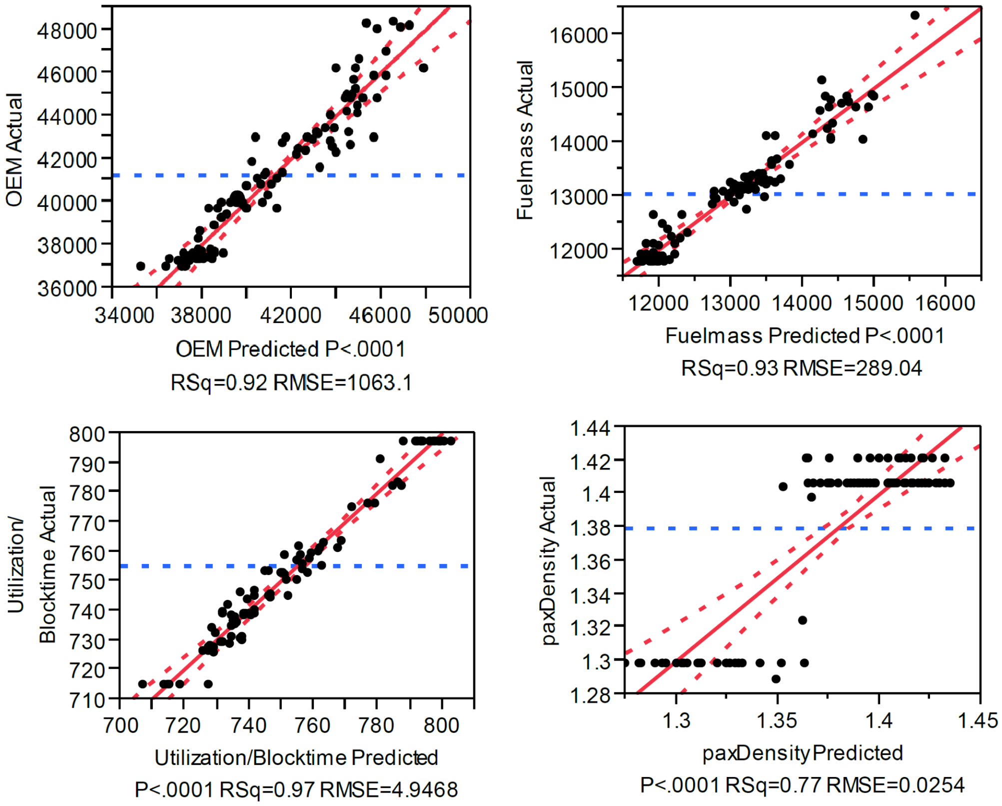


Figure 5.15: The Actual by Predicted Plots for OEM, Fuel Mass, Utilization/(Block time), and Passenger Density, when using SAW Index as an Objective Function

for SAW. This is one indicator of better goodness of fit when ITOPSIS is used for the multiple criteria aggregation than SAW.

The diagnostics of each response surface model, including R^2, R^2_{Adj}, and Root Mean Square Error (RMSE) in percentage, are listed in Table 5.11. R^2 measures the proportion of the variation explained by the regressed polynomial model, R^2_{Adj} adjusts the R^2 value to make it more comparable over models with different numbers of parameters, and RSME estimates the standard deviation of the random error. The percent RMSE shown in Table 5.11 is normalized by its mean of response.

Higher values of R^2 and R^2_{Adj} and lower values of percent RSME are strong evidences of goodness of fit. It is observed from Table 5.11 that the values of R^2 and R^2_{Adj}, when ITOPSIS is used for the aggregation of the four design criteria, are all higher than when SAW is used. The percent RSME, when ITOPSIS is used for the aggregation of the four design criteria, are all lower than when SAW is used. Especially, R^2 of passenger density when ITOPSIS is used is 0.957, while it is only 0.774 when SAW is used. Therefore, it is obtained that the developed response surface models using ITOPSIS for multiple criteria aggregation are better fitted than

Table 5.11: Diagnostics of Response Surface Models for Design Criteria, using ITOPSIS Index and SAW Index as Objective Functions

Diagnostics	OEM	Fuel mass	Utilization/(block time)	Passenger density
ITOPSIS				
R^2	0.975	0.964	0.983	0.957
R^2_{Adj}	0.963	0.951	0.976	0.945
Percent RMSE	1.56%	1.57%	0.54%	0.74%
SAW				
R^2	0.916	0.934	0.973	0.774
R^2_{Adj}	0.9	0.92	0.965	0.743
Percent RMSE	2.58%	2.22%	0.66%	1.84%

Table 5.12: Relative Errors between Actual and Predicted Values for Design Criteria

	OEM	Fuel mass	Utilization/(block time)	Passenger density
Percent μ	0.42%	-0.68%	0.28%	-0.06%
Percent σ	1.80%	1.44%	0.71%	1.00%

using SAW for multiple criteria aggregation. In summary, ITOPSIS index is a more appropriate objective function for the optimization framework of incorporating MCDA techniques in aircraft design process than the traditional SAW index.

Model Accuracy Evaluation by Running Additional Data Points

The accuracy of response surface models when ITOPSIS is used for aggregation are further evaluated by running additional untried data points. The additional untried data points are attached in Appendix C.2. The error analysis between the actual values produced by the original analysis tool (VAMPzero) and the predicted values generated by the response surface models are performed. The means and standard deviations of these errors are summarized in Table 5.12. It is found that the means of relative errors for these four design criteria are all less than 0.7% and the standard deviations are less than 2%. The minor errors support that the response surface models predict sufficiently.

In conclusion, the response surface models can provide adequate approximations to the analysis tool (VAMPzero). In the following sections, the response surface models when ITOPSIS is used for aggregation are further investigated. The quartic response surface models for OEM, Fuel mass, Utilization/(Block time), and Passenger density are shown in Equation 5.10, Equation 5.11, Equation 5.12, and Equation 5.13, respectively. The developed response surface models are further utilized to conduct uncertainty assessment in Section 5.5.

$$\begin{aligned}
\text{OEM} \quad = \quad & 1633729.28 - 5559654.9w_1 - 10111949.62w_2 + 5871087.01w_3 - 1583434.53w_4 \\
& +3966831.61w_1^2 - 37080.75w_1^3 + 33608.3w_1^4 \\
& +8344035.19w_2^2 + 291467.03w_2^3 - 113577.3w_2^4 \\
& -7325819.2w_3^2 - 338809.53w_3^3 + 203414.72w_3^4 \\
& -7658.01w_4^2 - 25705.92w_4^3 + 27207.1w_4^4 \\
& +12183210.34w_1w_2 - 3549670.4w_1w_3 + 3970538.3w_1w_4 \\
& +847842.19w_1w_2^2 - 505382w_1w_2^3 + 65502.92w_1w_3^2 - 17714.77w_1w_4^2 \\
& +1124737.44w_2w_3 + 8609064.34w_2w_4 + 453253.31w_2w_1^2 - 130141.67w_2w_1^3 \\
& -90457.4w_2w_3^2 + 99352.58w_2w_3^3 - 50086.28w_2w_4^2 - 7494192.78w_3w_4 \\
& +62899.5w_3w_1^2 - 2352.33w_3w_2^2 + 109246.01w_3w_4^2 \\
& -47802.27w_4w_2^2 - 300525.86w_4w_3^2 + 459079.26w_4w_3^3 - 511084.68w_1^2w_2^2 \\
& -124351.06w_1^2w_3^2 + 30456.18w_2^2w_3^2 + 101187.68w_2^2w_4^2 - 12394.79w_1w_3w_4 \qquad (5.10)
\end{aligned}$$

$$\begin{aligned}
\text{Fuelmass} \quad = \quad & -805894.11 + 819277.67w_1 + 822014.87w_2 + 816225.54w_3 - 2507037.13w_4 \\
& -39.73w_1^2 - 21303.96w_2^2 + 29216.79w_2^3 - 12140.04w_2^4 \\
& +8362.3w_3^2 - 5499.64w_3^3 + 10377960.3w_4^2 - 7051799.87w_4^3 \\
& +1610.42w_1w_2 - 650.27w_1w_3 + 3324358.46w_1w_4 \\
& -7036989.87w_1w_4^2 - 11836.2w_2w_3 + 3304017.13w_2w_4 \\
& -3057.23w_2w_1^2 - 7042375.09w_2w_4^2 + 3331260.35w_3w_4 \\
& +3148.82w_3w_1^2 + 32999.73w_3w_2^2 - 20516.74w_3w_2^3 \\
& -7035605.77w_3w_4^2 - 13847.45w_3w_4^3 + 53126.58w_4w_2^2 \\
& -37155.22w_4w_2^3 - 15647.52w_4w_3^2 + 23283.24w_4w_3^3 \\
& -8538.25w_1w_3w_4 + 18073.74w_1w_3w_4^2 \\
& -14473.02w_2w_3w_4 + 29239.37w_3w_4w_2^2 \qquad (5.11)
\end{aligned}$$

$$\begin{aligned}\frac{\text{Utilization}}{\text{Blocktime}} = {} & -7812.78 + 8676.09w_1 + 8503.01w_2 + 8632.79w_3 + 8667.41w_4 \\ & -462.14w_1^2 + 321.22w_1^3 + 6w_1^4 \\ & +46.86w_2^2 + 254.13w_2^3 - 256.71w_2^4 \\ & -40.17w_3^2 + 16.63w_3^3 - 433.64w_4^2 + 302.05w_4^3 \\ & -375.99w_1w_2 + 86.08w_1w_3 + 56.8w_1w_4 \\ & -81.42w_1w_3^2 - 143.58w_1w_4^2 - 92.08w_2w_3 + 295.9w_2w_4 \\ & +713.78w_2w_1^2 + 207.52w_2w_3^2 - 67.18w_2w_3^3 + 198.5w_3w_4 \\ & +203.25w_3w_1^2 - 2345.64w_3w_4^2 + 3127.85w_3w_4^3 - 647.75w_4w_1^2 \\ & +554.78w_4w_1^3 - 597.79w_4w_2^2 - 122.21w_4w_3^2 \\ & -401.82w_1^2w_3^2 + 808.56w_1^2w_4^2 + 1749w_3^2w_4^2 \end{aligned} \tag{5.12}$$

$$\begin{aligned}\text{paxDensity} = {} & 12.62 - 11.39w_1 - 11.33w_2 - 10.87w_3 - 10.58w_4 \\ & +0.18w_1^2 + 0.12w_2^2 - 0.23w_3^2 - 0.14w_3^3 - 2.14w_4^2 + 1.41w_4^3 \\ & +0.24w_1w_2 + 0.01w_1w_3 + 0.41w_1w_4 \\ & -1.05w_1w_3^2 + 1.17w_1w_3^3 + 0.12w_1w_4^2 \\ & -0.27w_2w_3 - 0.55w_2w_4 + 0.02w_2w_3^2 + 1.16w_2w_4^2 \\ & -1.5w_3w_4 + 1.38w_3w_4^2 - 0.69w_4w_1^2 - 0.59w_4w_3^2 \\ & -0.23w_1^2w_4^2 + 1.52w_3^2w_4^2 - 0.36w_1w_3w_4 \\ & +0.71w_1w_4w_3^2 + 2.21w_2w_3w_4 \\ & -4.6w_2w_3w_4^2 - 0.08w_2w_4w_3^2 \end{aligned} \tag{5.13}$$

5.5 Uncertainty Assessment for Weighting Factors via Surrogate Models

As noted in Section 5.4, inherent uncertainties and subjectivities of weighting factors have significant impacts on the design solution in the proposed multi-criteria optimization framework. The intractable computation task in uncertainty assessment process is alleviated by the development of surrogate models. This section presents uncertainty assessment via surrogate models, following the new approach proposed in Chapter 4.

5.5.1 Uncertainty Characterization

As described previously in Section 4.2, uncertainties in weighting factors are described by percentage uncertainties with confidence levels first. In our case, when the weighting factors are evenly distributed among the four design criteria, the mean of the weighting factors is

$$\mu_W = [0.25\ 0.25\ 0.25\ 0.25]^{\mathrm{T}}$$

Assume that there exists 20% uncertainty in the weighting factor of OEM with 90% confidence level. In other words, it is 90% confident that the weighting factor of OEM would fall within the interval $[w_1(1-20\%), w_1(1+20\%)]$. The percentage untertainties and confidence levels of other design criteria in the weighting factors have similar explanations. The weighting factors with percentage uncertainties and confidence levels are summarized in Table 5.13.

Table 5.13: Uncertainty Characterization for Weighting Factors

	OEM w_1	Fuel mass w_2	Utilization/(block time) w_3	Passenger density w_4
Percentage uncertainty	20%	30%	20%	10%
Confidence level	90%	80%	70%	80%

Secondly, percentage uncertainties with confidence levels are transferred into standard deviations through Equation 4.4 and Equation 4.6, as described in Subsection 4.2.2. For example, the number of standard deviation for w_1 with 20% uncertainty at 90% confidence level, is calclulated by Equation 5.14. The standard deviation for w_1 is calculated by Equation 5.15.

$$n_{w_1} = \sqrt{2}erf^{-1}(\text{Confidence level}) = \sqrt{2}erf^{-1}(90\%) = 1.6449 \tag{5.14}$$

$$\sigma_{w_1} = \frac{\text{Relative error}(\%)\mu_{w_1}}{n_{w_1}} = \frac{(20\%)(0.25)}{1.6449} = 0.0304 \tag{5.15}$$

The same calculation is done for all design criteria. The standard deviation for the weighting factors is

$$\sigma_W = [0.0304\ 0.0585\ 0.0482\ 0.0195]^{\mathrm{T}}$$

In this step, uncertainties in the weighting factors, characterized by percentage uncertainties and confidence levels, are transferred into means and standard deviations. Moreover, μ_W and σ_W are the input for the error propagation calculation step.

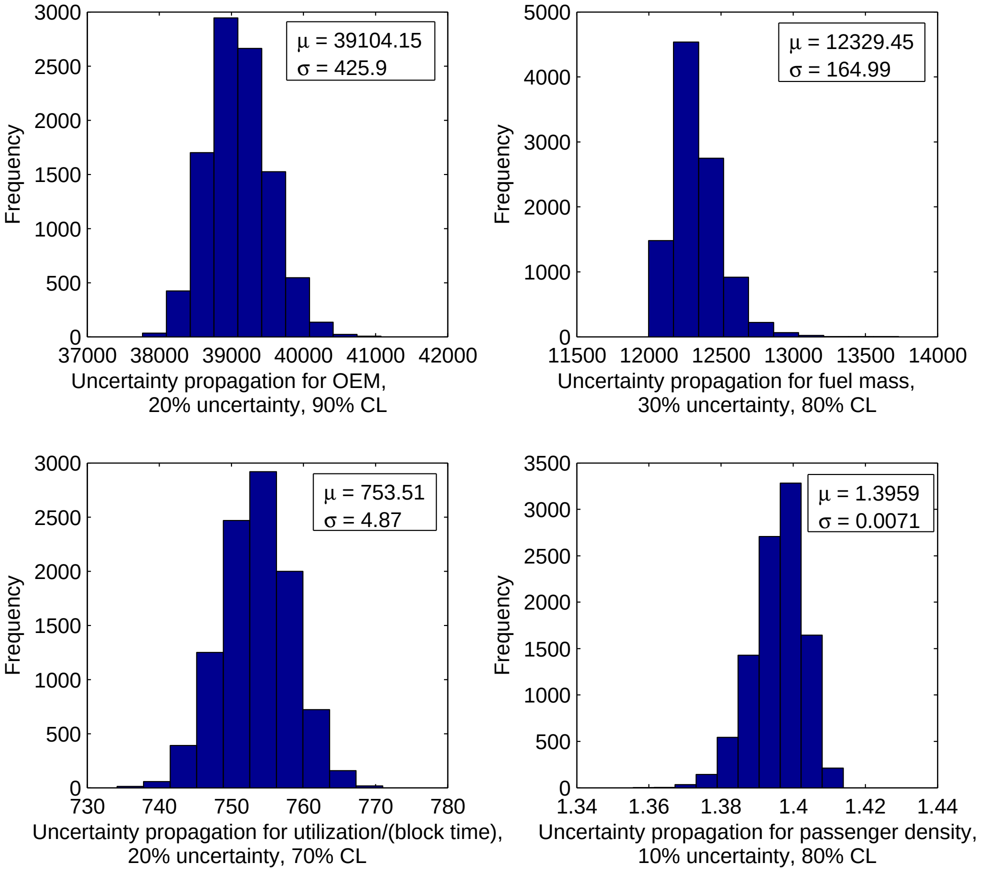


Figure 5.16: Histograms of Uncertainty Propagation for OEM, Fuel Mass, Utilization/(Block time), and Passenger Density

5.5.2 Uncertainty Analysis

As discussed in Section 4.3, Monte Carlo-based numerical error propagation technique is applied to propagate uncertainty through surrogate models. 10,000 iterations are performed from normal distribution with parameters μ_W and σ_W. The histograms of the design criteria with uncertainty propagated from the weighting factors via surrogate models are presented in Figure 5.16, where CL stands for confidence level. The x-axis represents the values of the design criteria, the y-axis stands for the number of times the value occurred when uncertainty exists. The mean values and standard deviations for the design criteria are also calculated and integrated in the figure. It can be seen that except for fuel mass, the distribution of the propagated uncertainties from the weighting factors for the other three design criteria can be approximately represented by normal distributions.

Robustness Measurement using Signal-to-Noise Ratio

The design criteria with deterministic weighting factors were shown in Table 5.7 in Subsection 5.3.2. The comparison of design criteria with propagated uncertainty from weighting factors and with deterministic weighting factors is summarized in Table 5.14, including mean, standard deviation, and SNR (Signal-to-Noise Ratio).

Table 5.14: Comparison of Design Criteria with Deterministic and Uncertain Weighting Factors

Design criteria	Deterministic design	Uncertain design		
		Mean	Standard deviation	SNR (dB)
OEM	38705.03	$\mu = 39104.15$	$\sigma = 425.9$	39.26
Fuel mass	12242.18	$\mu = 12329.45$	$\sigma = 164.99$	37.47
Utilization/(block time)	751.64	$\mu = 753.31$	$\sigma = 4.87$	43.79
Passenger density	1.4211	$\mu = 1.3959$	$\sigma = 0.0071$	45.87

Larger SNR value indicates more robustness against uncertainty. For instance, in Table 5.14, 39.26 (dB) means that the magnitude of mean for OEM is $10^{\frac{39.26}{20}} \approx 92$ times the magnitude of its standard deviation. The other SNR values for the other design criteria have similar explanations.

The largest value of SNR for passenger density in Table 5.14 indicates that passenger density is relatively robust to the uncertainty in the weighting factors, while fuel mass is relatively sensitive among the four design design criteria. On one side, the largest value of SNR for passenger density may be due to the smallest percentage uncertainty assigned in Table 5.13; on the other side, the linearity of passenger density regarding the five design variables, as shown in parametric studies of design criteria in Section 5.1.2, can also leads to highest SNR value of passenger density.

Likewise, the second-higher SNR value of utilization/(block time) ratio among the four design criteria can also be attributed to its linearity with regards to the five design variables. Furthermore, one reason of the smallest SNR for fuel mass probably is also the biggest percentage uncertainty assigned in Table 5.13, another reason can also be attributed to its non-linearity with regards to the five design variables.

Uncertainty Variation in Percentage Uncertainty and Confidence Level

Since uncertainty characterization has substantial impact on the distribution shape and robustness of the design criteria, uncertainty variation in the percentage uncertainty and confidence level are investigated. Especially, the impact behavior of percentage uncertainty is compared with confidence level on the distribution shapes of design criteria.

The percentage uncertainty under investigation ranges from 10%, 30%, and 50%, with con-

Table 5.15: Uncertainty Variation for Weighting Factors, Regarding Percentage Uncertainty and Confidence Level

	Confidence level				
Percentage uncertainty	50%	60%	70%	80%	90%
10%	0.0371	0.0297	0.0241	0.0195	0.0152
30%	0.1112	0.0891	0.0724	0.0585	0.0456
50%	0.1853	0.1485	0.1206	0.0975	0.0760

fidence level ranges from 50%, 60%, 70%, 80%, and 90%, as presented in Table 5.15. These percentage uncertainties with confidence levels are transferred into standard deviations using Equation 4.4 and Equation 4.6, as described in Subsection 4.2.2. It is observed from Table 5.15 that with the same percentage uncertainty, the growth of confidence level reduces the standard deviation of the weighting factors. Likewise, at equal confidence level, the increase of percentage uncertainty leads to higher standard deviation of the weighting factors.

Robustness Comparison

10,000 Monte Carlo simulations are conducted through the developed surrogate modesl for the four design criteria with equal weighting factor μ_W and standard deviation presented in Table 5.15. In order to measure the robustness of the design criteria against uncertainty in the weighting factors, SNR is also calculated using Equation 4.10. As an example, the SNR for OEM is presented in Figure 5.17. The SNR analysis indicates consistent conclusions previously drawn from the histograms of uncertainty variation. For the same percentage uncertainty, the growth of confidence level leads to the increase of SNR. Since a larger SNR indicates more robustness against uncertainty, the robustness of design criteria can be strengthened by the growth of confidence level.

5.5.3 Sensitivity Analysis

As noted in Section 4.4, sensitivity analysis can identify the relative contribution of input variables to the variability of model output. Local sensitivity analysis via iterative binary search algorithm and global sensitivity analysis using partial rank correlation coefficients are not followed, since they are established for evaluation decision making problems.

In this study, when MCDA techniques are implemented in design decision making problems, sensitivity analysis can be performed via surrogate models. The prediction profiler in JMP provides one effective approach to perform this task. Thus, it is utilized to perform sensitivity analysis for the weighting factors in the aircraft design problem.

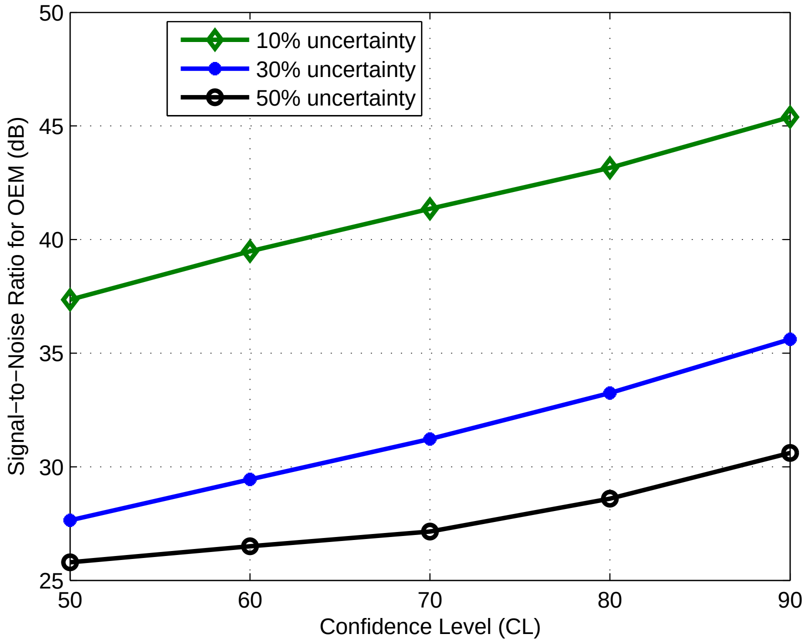


Figure 5.17: Robustness Comparison for OEM

In this example, equal weighting factors are assigned to the four design criteria, and one linear constraint ($w_1 + w_2 + w_3 + w_4 = 1$) is imposed on the weighting factors. The prediction profiles for the four design criteria are illustrated in Figure 5.18, where the vertical dotted red line for each variable shows its current value, the horizontal dotted red line shows the predicted value of each design criterion for the current values of weighting set. The black lines within the plots show how the predicted value changes when the current value of a variable is changed. The role of the weighting factors in the prediction of the four design criteria can be visualized, by moving the vertical dotted line or by directly entering a variable value.

The steepness of the prediction trace can reflect the sensitivity of variables. It can be observed from Figure 5.18 that the prediction traces on the blue diagonal line have the steepest slopes. In other words, they are the most sensitive variables for the predicted criteria on each row using the developed response surface model. This is consistent with physical explanation. For instance, w_1 has the steepest negative gradient in the first row when predicting OEM, considering that w_1 is the weighting factor for OEM during optimization, thus, OEM will decrease with the increase of w_1. For the same reason, w_2 is the most sensitive variable in predicting fuel mass, w_3 is the most sensitive variable in predicting utilization/(block time) ratio, and w_4 is the most sensitive variable in predicting passenger density.

The non-linearity of the prediction profilers can be explained by the four-order response surface models, as shown in Equation 5.10 - 5.13 in Section 5.4.4.

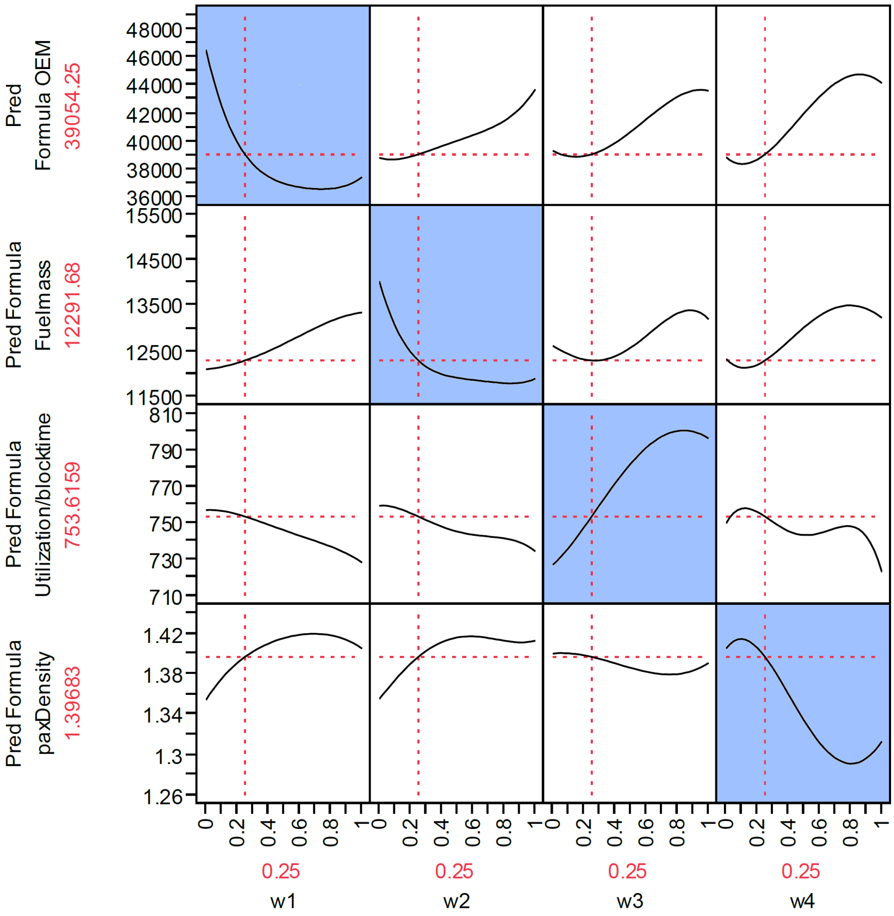


Figure 5.18: Prediction Profiles for Four Design Criteria

5.6 Discussion

This chapter explored the feasibility and assessed the added values of implementing MCDA techniques in aircraft design process. A new optimization framework incorporating MCDA techniques for aircraft conceptual design was established. The developed intelligent multi-criteria decision support system was used to select an appropriate MCDA method. It was demonstrated that the chosen MCDA method with improvement (ITOPSIS) provided a better objective function for the optimization than the traditional weighted sum (SAW) method.

Furthermore, considering that inherent uncertainties and subjectivities of weighting factors have crucial impacts on the design solution, surrogate models for design criteria in terms of weighting factors were developed to efficiently assess the uncertainties related to the subjective preference information in aircraft design process.

In this section, optimization algorithms used in aircraft design are discussed, followed by surrogate model development for design criteria in terms of weighting factors.

Optimization Algorithms in Aircraft Design

As noted in Subsection 5.3.1, there are several optimization algorithms available, among which gradient-based methods and genetic algorithms are most widely used in aircraft design. Which optimization method to use depends on the optimization problem under consideration.

The choice of gradient-based methods for the proposed optimization framework was based on the parametric studies performed in Subsection 5.1.2, where all design variables under investigation were continuous, and the objective functions with respect to the design variables in the conceptual aircraft design tool were rather smooth.

Furthermore, the focus of this research has been on developing the framework of incorporating MCDA techniques in aircraft design process, particularly on exploring the feasibility and assessing the added values, not on the optimization itself. A hybrid optimizer combining genetic algorithms and gradient-based methods could be also used in order to provide a more global optimization and include discrete design variables. However, this is beyond the scope of this study and can be regarded as future research.

Surrogate Model Development for Design Criteria in terms of Weighting Factors

As noted in Section 5.4, there are typically four steps in constructing the surrogate models: experimental design, model choice, model fitting, and model validation [47].

The choice of experimental design has a critical impact on the accuracy of the surrogate models. In this study, experimental designs for weighting factors have to satisfy that for each experimental run, the sum of the factor settings equals to 1. One modified LHS with Dirichlet distribution was employed, as presented in Subsection 5.4.1. Other sampling strategies with space filling properties could be also investigated.

Response surface model was utilized to construct the surrogate models in the model choice step. Furthermore, Kriging models are alternative techniques to construct surrogate models with more sound statistical meaning [109]. Kriging models interpolate the observed data and fit the model using maximum likelihood estimate.

A comparison of response surface model and Kriging model for multidisciplinary design optimization was presented in [114], with the application to the design of an aerospike nozzle. The authors concluded that the second-order response surface models and Kriging models using a constant underlying global model and a Gaussian correlation function yielded comparable results. Besides, it was stated that the choice of the modeling technique depends on the expectations of what the underlying response might look like [47]. Future research can be conducted on using Kriging model to construct the surrogate models.

6

Proof of Concept 2: MCDA in Aircraft Evaluation

In this chapter, the effectiveness of implementing the most appropriate MCDA techniques in aircraft evaluation process is demonstrated, following a three-step framework: definition of the decision making problem, selection of the most appropriate MCDA method, and uncertainty assessment in the decision analysis process.

The chapter is organized as follows. Section 6.1 defines the business aircraft evaluation problem. Section 6.2 presents the selection of the most appropriate MCDA method, through the developed intelligent multi-criteria decision support system, as described in Chapter 3. Section 6.3 presents the results of applying the appropriate MCDA method in the business aircraft evaluation problem. Section 6.4 presents uncertainty assessment in decision analysis process, following the new approach proposed in Chapter 4. Section 6.5 discusses the implementation of MCDA techniques in aircraft evaluation problems.

6.1 Definition of the Decision Making Problem

Assume that one business aviation customer needs to purchase a business jet. At present, there are six major business jet manufacturers: Canadian Bombardier, American Cessna, French Dassault, Brazilian Embraer, American Gulfstream, and American Hawker. There are five different segments for different types of the product models: very light jets, light jets, medium jets, large jets, and large corporate airliners. The segmentation is primarily determined by a combination of price, range, and cabin volume, as summarized in Table 6.1.

The six major business jet manufacturers are briefly introduced as follows. Bombardier offers three families of business jets: Learjet, Challenger, and Global. Cessna mainly offers light to medium size business aircrafts. Dassault produces medium to large size business jets. Embraer

Table 6.1: Segmentation Criteria for Business Jets [20]

Business aircraft segmentation	Price ($ Millions)	Range (km)	Cabin volume (m^3)
Very light jets	< 7	< 3148	< 8.5
Light jets	7 - 18	3148 - 5741	8.5 - 19.8
Medium jets	18 - 42	5741 - 9260	19.8 - 42.5
Large jets	46 - 68	> 9260	42.5 - 85.0
Large corporate airliners	> 68	> 9260	> 85

offers five product models of business jets, ranging from light to large size aircrafts. Gulfstream offers light, medium, and large business aircrafts. Hawker produces mainly light and medium business jets. In addition, Airbus and Boeing also offer Airbus Corporate Jet (ACJ) and Boeing Business Jet (BBJ), based on their A319 and B737 series, respectively. These large size aircrafts are most expensive in the business jet market.

There are more than forty different types of business aircraft available in the current market, costing from $ 1 million to almost $ 100 millions. How to choose an appropriate aircraft to meet the needs of the business aviation customer is a complicated decision making process. In addition to costs, there are several other criteria to be evaluated at the same time. For instance, aircraft configuration, aircraft performances, environmental impacts, and level of comfort. Therefore, considering these multiple conflicting criteria simultaneously, the evaluation and selection of a business jet is a typical MCDA problem and needs to be prudently conducted.

In the following subsections, the identification of evaluation criteria for business aircraft is discussed first, followed by the quantification of additional soft criteria.

6.1.1 Identification of Evaluation Criteria

The specifications of business aircraft are presented in Figure 6.1. Based on the specifications, the evaluation criteria for business aircraft can be categorized into four groups:

- **Economic criteria**: purchase price and operating costs.
- **Performance criteria**: maximum payload, maximum range, cruise speed, fuel consumption, and take-off field length.
- **Environmental criteria**: noise and CO_2 emissions.
- **Additional soft criteria**: passenger comfort level, product support level, and manufacturers reputation.

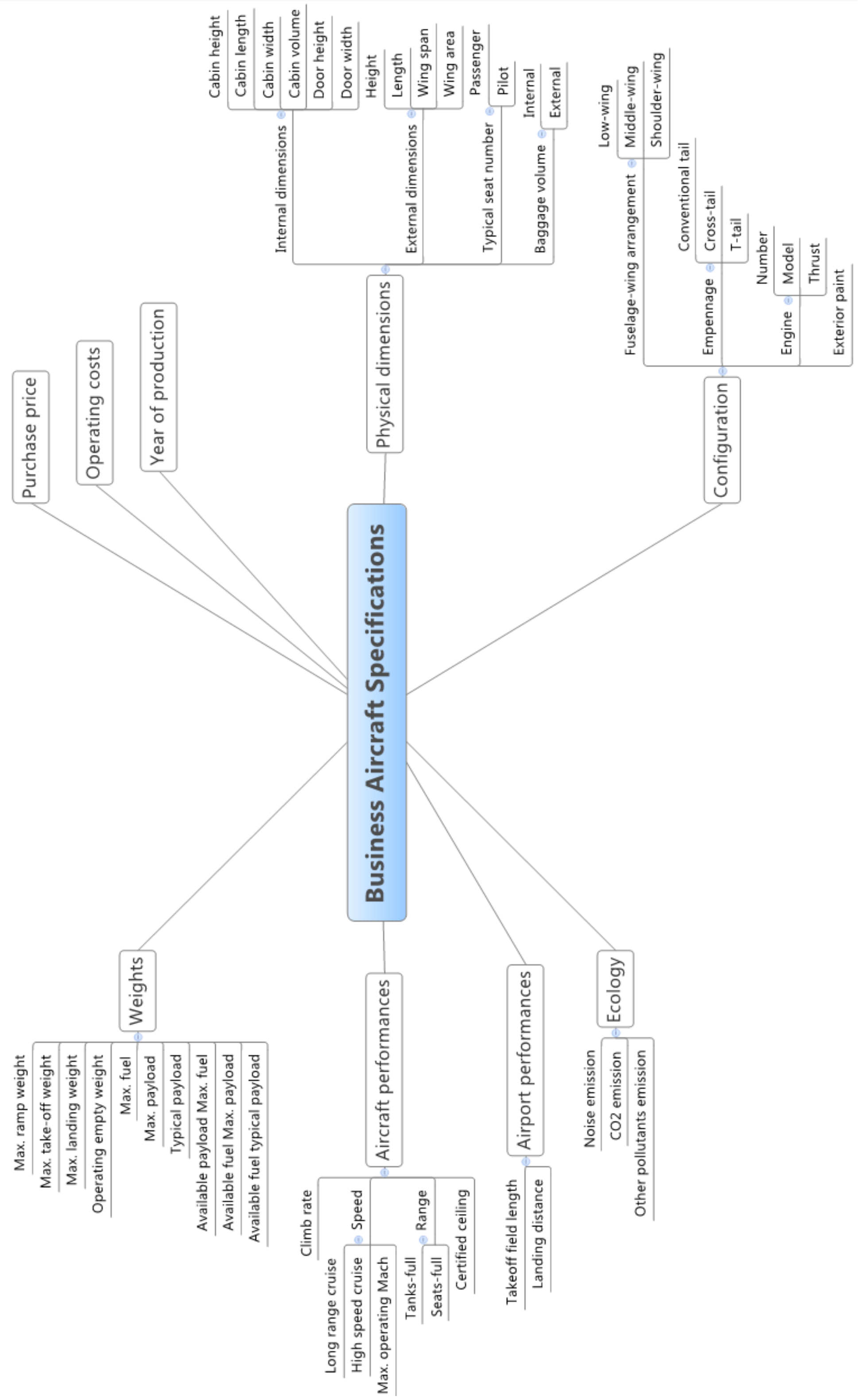


Figure 6.1: The Specifications of Business Aircraft [94]

We are confronted with the similar question as in Subsection 5.1.1: Which evaluation criteria are most appropriate to be fed into the MCDA method for the business aircraft evaluation problem? In order to better answer this question, the quantification of additional soft criteria is presented first, followed by the determination of which evaluation criteria would be further feed into the MCDA method.

6.1.2 Quantification of Additional Soft Criteria

Among these four groups, additional soft criteria are decisive in the business aircraft evaluation problem. However, these soft criteria cannot be fed into the MCDA method directly without quantification. In this subsection, the quantification of passenger comfort level, product support level, and manufacturer's reputation are presented, respectively.

Quantification of Passenger Comfort Level

Passenger comfort level can be influenced by several factors, for instance, space utilization, cabin noise, and vibration. Among these factors, space utilization is known as predominant for passenger comfort, thus, we focus on space utilization in this research. The passenger seating configuration, cabin height, cabin width, cabin length, and cabin volume determine the space utilization. Passenger comfort level can be quantified by cabin volume per passenger (m^3/pax), as calculated in Equation 6.1.

$$\text{Cabin volume per passenger} = \frac{\text{Cabin volume}}{\text{Typical passenger seat number}} \tag{6.1}$$

Quantification of Product Support Level

Product support level is quantified based on the aviation international news 2010 product support survey [124]. The product support survey is conducted entirely on the Internet, qualified readers are asked to rate their business aircraft, engines, and avionics in ten categories. The ten categories are summarized in Table 6.2, where the explanations of key points that the survey participants were asked to consider are also included. The rating scale ranges from 1 (inadequate) to 10 (excellent), as illustrated in Figure 6.2.

Table 6.2: Ten Categories of the Aviation International News 2010 Product Survey [124]

Categories	Explanations of key points
1. Authorized service center	Estimated cost versus actual cost, on-time performance, scheduling ease, and service experience.
2. Factory service center	The same as with the authorized service center.
3. Parts availability	In stock versus back order and shipping time.
4. Costs of parts	Value for price paid.
5. Aircraft on ground response	The speed, accuracy, and cost to get a grounded aircraft back in the air as soon as possible.
6. Warranty fulfillment	Ease of paperwork and extent of coverage.
7. Technical manuals	Ease of use, formats available, timeliness of updating.
8. Technical representatives	Response time, knowledge, and effectiveness.
9. Maintenance tracking programs	Cost, ease of use, accuracy, and reliability.
10. Overall aircraft reliability	Product's overall reliability and quality against the competition's.

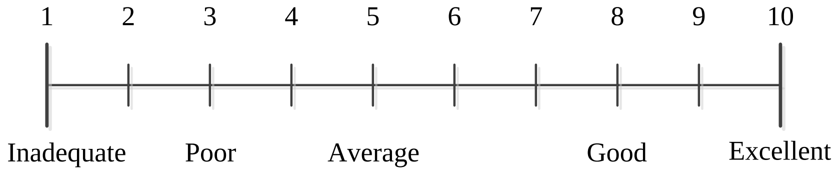


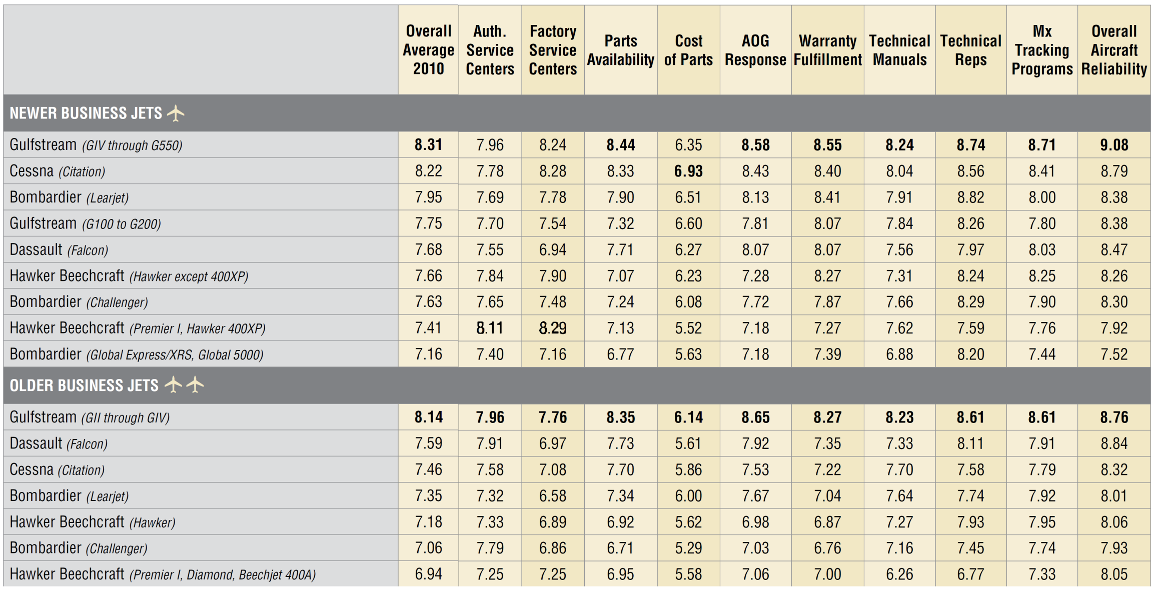
Figure 6.2: Rating Scale of the Aviation International News 2010 Product Survey [124]

The 2010 product survey invited 17,284 readers to participate and 921 completed the survey, with a return rate of 5.3%. The results of the 2010 product survey are presented in Figure 6.3, where the aircraft are listed in the order of their overall average scores. The newer business jets are less than ten years old, and the older business jets are more than ten years old. The bold number indicates the highest number in each category.

According to the survey results shown in Figure 6.3, product support level of Gulfstream ranked first for both newer and older business jets in 2010.

Quantification of Manufacturer's Reputation

Manufacturer's reputation is quantified according to aviation week's 16th annual top-performing companies study [9]. The top-performing companies study was launched in 1996 by *Aviation Week & Space Technology*, with the purpose of assessing the operational performance of publicly traded companies in the aerospace and defense industries. The company ranking is based on a composite scoring of four equally weighted performance categories. The scores range from 1 (worst performance) to a maximum value 99 (best performance). The four categories are



	Overall Average 2010	Auth. Service Centers	Factory Service Centers	Parts Availability	Cost of Parts	AOG Response	Warranty Fulfillment	Technical Manuals	Technical Reps	Mx Tracking Programs	Overall Aircraft Reliability
NEWER BUSINESS JETS											
Gulfstream *(GIV through G550)*	**8.31**	7.96	8.24	**8.44**	6.35	**8.58**	**8.55**	**8.24**	**8.74**	**8.71**	**9.08**
Cessna *(Citation)*	8.22	7.78	8.28	8.33	**6.93**	8.43	8.40	8.04	8.56	8.41	8.79
Bombardier *(Learjet)*	7.95	7.69	7.78	7.90	6.51	8.13	8.41	7.91	8.82	8.00	8.38
Gulfstream *(G100 to G200)*	7.75	7.70	7.54	7.32	6.60	7.81	8.07	7.84	8.26	7.80	8.38
Dassault *(Falcon)*	7.68	7.55	6.94	7.71	6.27	8.07	8.07	7.56	7.97	8.03	8.47
Hawker Beechcraft *(Hawker except 400XP)*	7.66	7.84	7.90	7.07	6.23	7.28	8.27	7.31	8.24	8.25	8.26
Bombardier *(Challenger)*	7.63	7.65	7.48	7.24	6.08	7.72	7.87	7.66	8.29	7.90	8.30
Hawker Beechcraft *(Premier I, Hawker 400XP)*	7.41	**8.11**	**8.29**	7.13	5.52	7.18	7.27	7.62	7.59	7.76	7.92
Bombardier *(Global Express/XRS, Global 5000)*	7.16	7.40	7.16	6.77	5.63	7.18	7.39	6.88	8.20	7.44	7.52
OLDER BUSINESS JETS											
Gulfstream *(GII through GIV)*	**8.14**	**7.96**	**7.76**	**8.35**	**6.14**	**8.65**	**8.27**	**8.23**	**8.61**	**8.61**	**8.76**
Dassault *(Falcon)*	7.59	7.91	6.97	7.73	5.61	7.92	7.35	7.33	8.11	7.91	8.84
Cessna *(Citation)*	7.46	7.58	7.08	7.70	5.86	7.53	7.22	7.70	7.58	7.79	8.32
Bombardier *(Learjet)*	7.35	7.32	6.58	7.34	6.00	7.67	7.04	7.64	7.74	7.92	8.01
Hawker Beechcraft *(Hawker)*	7.18	7.33	6.89	6.92	5.62	6.98	6.87	7.27	7.93	7.95	8.06
Bombardier *(Challenger)*	7.06	7.79	6.86	6.71	5.29	7.03	6.76	7.16	7.45	7.74	7.93
Hawker Beechcraft *(Premier I, Diamond, Beechjet 400A)*	6.94	7.25	7.25	6.95	5.58	7.06	7.00	6.26	6.77	7.33	8.05

Figure 6.3: Results of the Aviation International News 2010 Product Survey [124]

Table 6.3: Four Categories of the Aviation Week's 16th Annual Top-Performing Companies Study [9]

Categories	Measurement
1. Return on invested capital	Investment decisions, companies with superior operating profit are rewarded.
2. Earning momentum	Earning quality and revenue expansion.
3. Asset management	Efficiency in employing the resources.
4. Financial health	Overall solvency and available liquidity.

summarized in Table 6.3.

For the purpose of this study, scores for the six major business jet manufacturers are presented in Table 6.4. It should be noted that Cessna, Gulfstream, and Hawker are not explicitly on the list of the top-performing companies study. Thus, the scores of their parent companies are used instead. A higher score in the top-performing companies study represents better reputation. According to the scores shown in Table 6.4, Gulfstream has the highest reputation, while Cessna has the lowest reputation.

In summary, in the additional soft criteria group, passenger comfort level is quantified by cabin volume per passenger (m^3/pax), product support level is quantified according to the overall average scores obtained via the aviation international news 2010 product survey, as shown in Figure 6.3, and manufacturer's reputation is quantified based on the aviation week's 16th annual top-performing companies study, as summarized in Table 6.4.

Table 6.4: Scores of the Six Major Business Jet Manufacturers [9]

Manufacturers	Scores
Bombardier	55
Cessna (Textron)	39
Dassault	74
Embraer	60
Gulfstream (General Dynamics)	82
Hawker (Raytheon)	78

Determination of Evaluation Criteria

Empirical studies in consumer behavior and industrial market context have shown that the quality of a decision has an inverted U-shaped relationship with the number of alternatives, and the number of intensively discussed alternatives is less than five [48]. In practice, a small number of alternatives can be obtained by a simple check-list of desirable features [136].

In this business aircraft evaluation problem, typical passenger seat number, maximum range, and purchase price are utilized as filter criteria for initial screening in the first phase of the decision making process. The filter criteria can highly facilitate evaluating the business aircraft by reducing the number of alternatives under consideration.

Furthermore, the operating costs will not be fed into the MCDA method, the reasons are listed as follows. The operating costs are composed of fixed costs and variable costs. Fixed costs are irrespective of aircraft utilization, and thus include insurance, training costs, and other miscellaneous costs. Variable costs vary with aircraft utilization, consisting of fuel costs, maintenance costs, and miscellaneous trip expenses. Fixed costs are directly proportional to the purchase price, while variable costs are directly related to fuel consumption. Additionally, CO_2 emission is also largely fuel-based. Thus, instead of using operating costs as an independent evaluation criterion, aircraft purchase price is used to approximate the fixed operating costs, and fuel consumption is utilized as a proxy for the variable operating costs and CO_2 emission.

In summary, three filter criteria and seven decision criteria are summarized in Table 6.5, where EPNdB represents the decibels of Effective Perceived Noise.

Table 6.5: Ten Evaluation Criteria for Business Aircraft

	Name	Units
Filter criteria	Typical passenger seat number	pax
	Maximum range	km
	Purchase price	$ Millions
Decision criteria	Fuel consumption per seat kilometer	kg/pax/km
	High-speed cruise speed	km/h
	Take-off field length	m
	Noise	EPNdB
	Cabin volume per passenger	m^3/pax
	Product support level	-
	Manufacturer's reputation	-

One Scenario for Business Aviation Customer

Assume that one business aviation customer considers to purchase a business jet with 8 to 10 typical passengers on board. The aircraft range with maximum fuel and available payload should be around 5500 km to 6500 km, and the purchase price is between $ 20 millions and $ 25 millions.

In the available business jet market, four business jet alternatives satisfy the needs of the customer. The values of the three filter criteria and seven decision criteria for the four business jet alternatives are summarized in Table 6.6.

In Table 6.6, maximum range is when the aircraft is with full fuel and maximum available payload, and with the National Business Aviation Association (NBAA) Instrument Flight Rules (IFR) fuel reserves (370.4 km or 200 nm alternate). Purchase price is *Business & Commercial Aviation (BCA)* equipped price published in May 2011 issue [94]. Fuel consumption is calculated based on the fuel used for the mission of flying 1852 km (1000 nm) with four passengers on board. Noise is calculated by the average of take-off, sideline, and approach noise. It should be noted that the seven decision criteria (from C_1 to C_7) are further fed into the MCDA method.

Table 6.6: The Values of Evaluation Criteria for the Four Business Jet Alternatives

	Alternatives			
	A_1 Bombardier Challenger 300	A_2 Cessna Citation X	A_3 Gulfstream G200	A_4 Hawker H4000
				
Filter criteria				
F_1: Typical passenger seat number	8	9	10	8
F_2: Maximum range (km)	5975	5656	6378	5808
F_3: Purchase price ($ Millions)	24.7500	21.6330	23.3250	22.9089
Decision criteria				
C_1: Fuel consumption per seat kilometer (kg/pax/km)	0.2396	0.2720	0.2264	0.2624
C_2: High-speed cruise speed (km/h)	870	952	870	870
C_3: Take-off field length (m)	1466	1567	1854	1545
C_4: Noise (EPNdB)	84.2333	82.4333	86.7333	86.1000
C_5: Cabin volume per passenger (m^3/pax)	4.0500	2.3556	3.1000	3.4375
C_6: Product support level	7.63	8.22	7.75	7.66
C_7: Manufacturer's reputation	55	39	82	78

6.2 Selection of an Appropriate MCDA Method

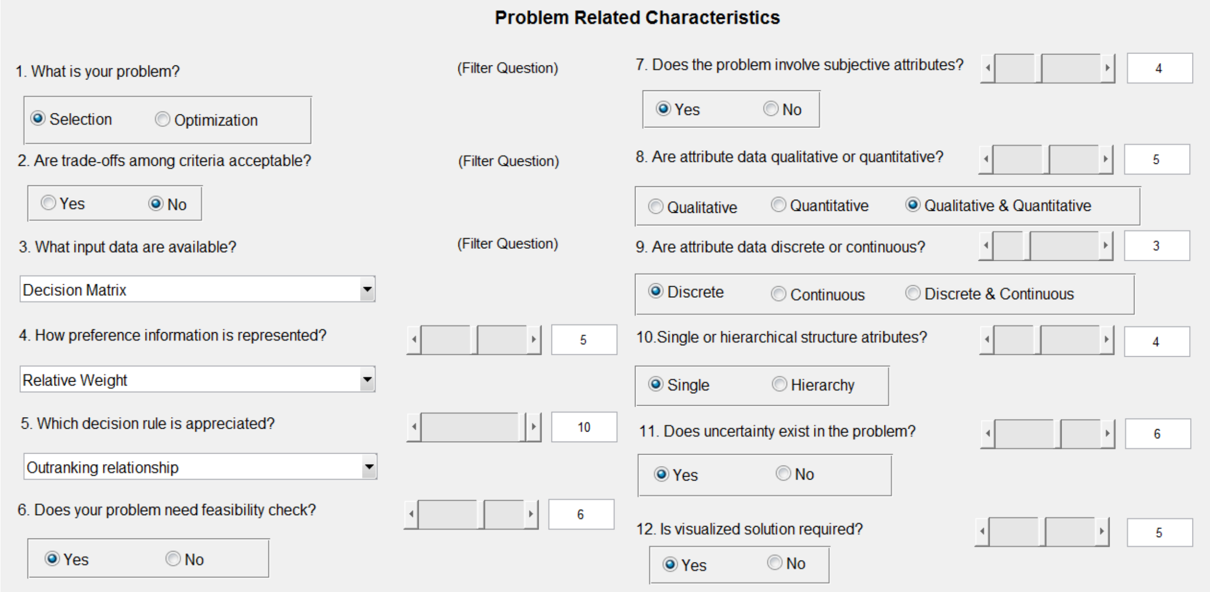
The selection of the most appropriate MCDA method for the business aircraft evaluation problem is presented in this section, through the developed intelligent multi-criteria decision support system, as discussed in Chapter 3. The user guide can be found in Appendix A. The step by step problem solving process is explained in the following subsections.

Step 1: Define the Problem

As discussed in Section 6.1, the objective of this decision making problem is to evaluate the performance of the business jets and identify which one has the best compromised performance using one appropriate MCDA method. The developed intelligent multi-criteria decision support tool is employed to facilitate this decision analysis process.

Step 2: Define the Evaluation Criteria

With the purpose of identifying the most appropriate method, sixteen widely used MCDA methods are studied and their characteristics are stored in the knowledge base. To compare



Problem Related Characteristics

1. What is your problem? (Filter Question)
● Selection ○ Optimization

2. Are trade-offs among criteria acceptable? (Filter Question)
○ Yes ● No

3. What input data are available? (Filter Question)
Decision Matrix

4. How preference information is represented? 5
Relative Weight

5. Which decision rule is appreciated? 10
Outranking relationship

6. Does your problem need feasibility check? 6
● Yes ○ No

7. Does the problem involve subjective attributes? 4
● Yes ○ No

8. Are attribute data qualitative or quantitative? 5
○ Qualitative ○ Quantitative ● Qualitative & Quantitative

9. Are attribute data discrete or continuous? 3
● Discrete ○ Continuous ○ Discrete & Continuous

10.Single or hierarchical structure atributes? 4
● Single ○ Hierarchy

11. Does uncertainty exist in the problem? 6
● Yes ○ No

12. Is visualized solution required? 5
● Yes ○ No

Figure 6.4: Questions Related to Evaluation Criteria for Method Selection in Business Aircraft Evaluation Process

the appropriateness of the methods with respect to the given problem, each method is assessed based on the proposed twelve evaluation criteria. The twelve evaluation criteria are captured by answering twelve questions, as shown in Figure 6.4.

Step 3: Perform Initial Screening

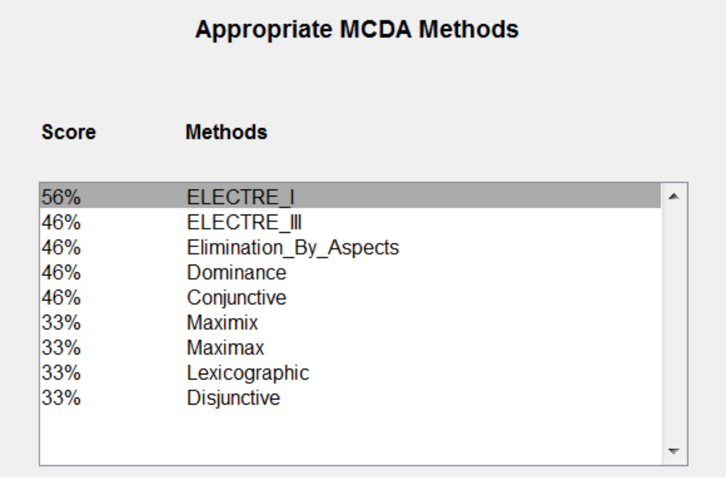
The infeasible MCDA methods are eliminated by the three filtering questions. For the business aircraft evaluation problem, with the assumption that trade-offs among criteria are not permitted, all compensatory methods are excluded and only non-compensatory methods remain as candidate methods for further selection.

Step 4: Define the Preferences on Evaluation Criteria

When selecting the most appropriate method, the DM's preference information on the evaluation criteria can be defined using slide bars in the integrated user interface, where 0 stands for extremely unimportant criterion and 10 represents extremely important.

Step 5: Calculate the Appropriateness Index

The match of a particular method and the given problem is quantified by AI. In this step, AI for each MCDA method is calculated by Equation 3.1, as described in Subsection 3.2.5.



Appropriate MCDA Methods

Score	Methods
56%	ELECTRE_I
46%	ELECTRE_III
46%	Elimination_By_Aspects
46%	Dominance
46%	Conjunctive
33%	Maximix
33%	Maximax
33%	Lexicographic
33%	Disjunctive

Figure 6.5: MCDA Methods Ranking List in Business Aircraft Evaluation Process

Step 6: Evaluate the MCDA methods

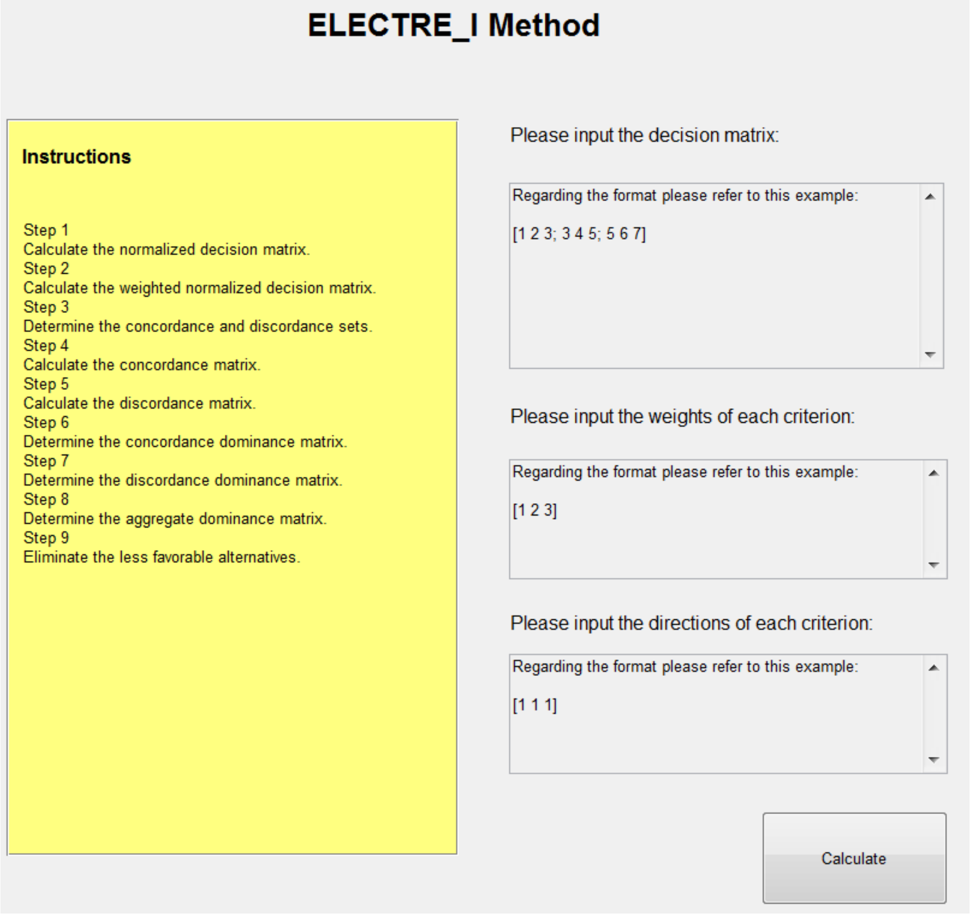
According to Step 5, AI of the MCDA methods are obtained and presented in Figure 6.5, where higher score represents more appropriateness of the method for the given problem.

Step 7: Choose the Most Suitable Method

As shown in Figure 6.5, ELECTRE I gets the highest score among the MCDA methods, therefore, it is selected as the most appropriate method to solve the business aircraft evaluation problem. Its mathematical calculation steps are built in the decision support system, thus, the DM can simply click the name of the method and methodology instructions of ELECTRE I will be displayed to guide the DM to solve the given problem and get the final solution, as illustrated in Figure 6.6. The evaluation results using ELECTRE I are presented in Section 6.3.

Step 8: Conduct Sensitivity Analysis

The answers to the twelve questions can be varied in the method selection process. In our integrated user interface, the DM can adjust the weights of each criterion by moving the slide bars. In this example, with the current input data, it is observed from Figure 6.5 that ELECTRE I is ranked first by the multi-criteria decision support system. Therefore, ELECTRE I is further used to solve the business aircraft evaluation problem.



Instructions

Step 1
Calculate the normalized decision matrix.
Step 2
Calculate the weighted normalized decision matrix.
Step 3
Determine the concordance and discordance sets.
Step 4
Calculate the concordance matrix.
Step 5
Calculate the discordance matrix.
Step 6
Determine the concordance dominance matrix.
Step 7
Determine the discordance dominance matrix.
Step 8
Determine the aggregate dominance matrix.
Step 9
Eliminate the less favorable alternatives.

Please input the decision matrix:

Regarding the format please refer to this example:
[1 2 3; 3 4 5; 5 6 7]

Please input the weights of each criterion:

Regarding the format please refer to this example:
[1 2 3]

Please input the directions of each criterion:

Regarding the format please refer to this example:
[1 1 1]

Figure 6.6: Methodology Instructions for ELECTRE I

6.3 Evaluation Results using ELECTRE I

When ELECTRE I is utilized to solve the business aircraft evaluation problem, it requires a decision matrix as input data and weighting factors as the presentation of DM's preference information. For this example, the decision matrix is shown in matrix D, where each row corresponds to one business jet alternative, and each column corresponds to one decision criterion. In the first round of evaluation, equal weighting factors are considered, as shown in vector W.

$$D = \begin{bmatrix} 0.2396 & 870 & 1466 & 84.2333 & 4.0500 & 7.63 & 55 \\ 0.2720 & 952 & 1567 & 82.4333 & 2.3556 & 8.22 & 39 \\ 0.2264 & 870 & 1854 & 86.7333 & 3.1000 & 7.75 & 82 \\ 0.2624 & 870 & 1545 & 86.1000 & 3.4375 & 7.66 & 78 \end{bmatrix}$$

$$W = [0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.14290.1429\ ]^{\mathrm{T}}$$

The stepwise calculations of ELECTRE I are presented in detail in the following subsection, based on the methodology description in Subsection 2.3.4 .

6.3.1 Stepwise Calculations of ELECTRE I

There are two kinds of criteria: benefit criteria and cost criteria. Bigger values of benefit criteria and smaller values of cost criteria are preferred. In the business aircraft evaluation problem, benefit criteria are high-speed cruise speed (C_2), cabin volume per passenger (C_5), product support level (C_6), and manufacturer's reputation (C_7), while fuel consumption per seat kilometer (C_1), take-off field length (C_3), and noise (C_4) are cost criteria. Before conducting the normalization, cost criteria are transformed into benefit criteria by taking the reciprocal values.

1. Normalize the decision matrix D.

$$D_n = \begin{bmatrix} 0.5178 & 0.4881 & 0.5423 & 0.5035 & 0.6149 & 0.4879 & 0.4175 \\ 0.4561 & 0.5341 & 0.5073 & 0.5145 & 0.3577 & 0.5257 & 0.2960 \\ 0.5480 & 0.4881 & 0.4288 & 0.4890 & 0.4707 & 0.4956 & 0.6225 \\ 0.4728 & 0.4881 & 0.5145 & 0.4926 & 0.5219 & 0.4899 & 0.5921 \end{bmatrix}$$

2. Calculate the weighted normalized decision matrix D_{nw}.

$$D_{nw} = \begin{bmatrix} 0.0740 & 0.0697 & 0.0775 & 0.0720 & 0.0879 & 0.0697 & 0.0597 \\ 0.0652 & 0.0763 & 0.0725 & 0.0735 & 0.0511 & 0.0751 & 0.0423 \\ 0.0783 & 0.0697 & 0.0613 & 0.0699 & 0.0673 & 0.0708 & 0.0890 \\ 0.0676 & 0.0697 & 0.0735 & 0.0704 & 0.0746 & 0.0700 & 0.0846 \end{bmatrix}$$

3. Determine the concordance and discordance sets.

   For instance, for the pair of alternatives A_1 and A_2, the set of decision criteria is divided into two disjoint subsets. The concordance set C_{12} is composed of all criteria which support that A_1 is preferred to A_2. The discordance set D_{12} is the complementary set of the concordance set C_{12}, with respect to the decision criteria set $\{1, 2, 3, 4, 5, 6, 7\}$.

$$C_{12} = \{1, 3, 5, 7\} \qquad D_{12} = \{2, 4, 6\}$$

$$C_{13} = \{2, 3, 4, 5\} \qquad D_{13} = \{1, 6, 7\}$$

$$C_{14} = \{1, 2, 3, 4, 5\} \qquad D_{14} = \{6, 7\}$$

$$C_{21} = \{2, 4, 6\} \qquad D_{21} = \{1, 3, 5, 7\}$$

$$C_{23} = \{2, 3, 4, 6\} \qquad D_{23} = \{1, 5, 7\}$$

$$C_{24} = \{2, 4, 6\} \qquad D_{24} = \{1, 3, 5, 7\}$$

$$C_{31} = \{1, 2, 6, 7\} \qquad D_{31} = \{3, 4, 5\}$$

$$C_{32} = \{1, 5, 7\} \qquad D_{32} = \{2, 3, 4, 6\}$$

$$C_{34} = \{1, 2, 6, 7\} \qquad D_{34} = \{3, 4, 5\}$$

$$C_{41} = \{2, 6, 7\} \qquad D_{41} = \{1, 3, 4, 5\}$$

$$C_{42} = \{1, 3, 5, 7\} \qquad D_{42} = \{2, 4, 6\}$$

$$C_{43} = \{2, 3, 4, 5\} \qquad D_{43} = \{1, 6, 7\}$$

4. Calculate the concordance matrix $M_{concordance}$.

   Each element of the concordance matrix is calculated by the sum of criteria weights which are contained in the concordance set. For example, the element $M_{concordance_{12}}$ between A_1 and A_2 is calculated by Equation 6.2.

$$M_{concordance} = \begin{bmatrix} - & 0.5716 & 0.5716 & 0.7145 \\ 0.4287 & - & 0.5716 & 0.4287 \\ 0.5716 & 0.4287 & - & 0.5716 \\ 0.4287 & 0.5716 & 0.5716 & - \end{bmatrix}$$

$$M_{concordance_{12}} = \sum_{j \in C_{12}} w_j = w_1 + w_3 + w_5 + w_7 = 0.5716 \tag{6.2}$$

5. Calculate the discordance matrix $M_{discordance}$.

   Each element of the discordance matrix reflects the degree to which one alternative is worse than the other. For instance, the element $M_{discordance_{12}}$ between A_1 and A_2 is calculated by Equation 6.3.

$$M_{discordance} = \begin{bmatrix} - & 0.1793 & 1.0000 & 1.0000 \\ 1.0000 & - & 1.0000 & 1.0000 \\ 0.7038 & 0.2406 & - & 1.0000 \\ 0.5327 & 0.1554 & 0.8767 & - \end{bmatrix}$$

$$\begin{aligned} M_{discordance_{12}} &= \frac{\max\limits_{j \in D_{12}} \left| D_{nw_{1j}} - D_{nw_{2j}} \right|}{\max\limits_{j \in (1,2,\ldots,7)} \left| D_{nw_{1j}} - D_{nw_{2j}} \right|} \\ &= \frac{\max\{0.0066, 0.0016, 0.0054\}}{\max\{0.0088, 0.0066, 0.0050, 0.0016, 0.0368, 0.0054, 0.0174\}} \\ &= \frac{0.0066}{0.0368} = 0.1793 \end{aligned} \tag{6.3}$$

6. Determine the concordance dominance matrix $M_{concordance\ dominance}$.

   A concordance threshold c needs to be chosen to perform the concordance test. In this study, the average value of the elements in the concordance matrix $M_{concordance}$ is used, $c = 0.5359$. For instance, A_1 possibly dominates alternative A_2, if $M_{concordance_{12}} \geq c$. In this example, $M_{concordance_{12}} \geq c$ $(0.5716 \geq 0.5359)$, thus, the concordance test is passed and the element of the concordance dominance matrix is 1. Otherwise, the element is 0.

$$M_{concordance\ \ dominance} = \begin{bmatrix} - & 1 & 1 & 1 \\ 0 & - & 1 & 0 \\ 1 & 0 & - & 1 \\ 0 & 1 & 1 & - \end{bmatrix}$$

7. Determine the discordance dominance matrix $M_{discordance\ \ dominance}$.

   A discordance threshold d needs to be chosen to perform the discordance test. In this study, the average value of the elements in the discordance matrix $M_{discordance}$ is used, $d = 0.7240$. For instance, A_1 possibly dominates A_2, if $M_{discordance_{12}} \leq d$. In this example, $M_{discordance_{12}} \leq d$ $(0.1793 \leq 0.7240)$, thus, the discordance test is passed and the element of the discordance dominance matrix is 1. Otherwise, the element is 0.

$$M_{discordance\ \ dominance} = \begin{bmatrix} - & 1 & 0 & 0 \\ 0 & - & 0 & 0 \\ 1 & 1 & - & 0 \\ 1 & 1 & 0 & - \end{bmatrix}$$

8. Aggregate the dominance matrix $M_{aggregated\ \ dominance}$.

   The aggregated dominance matrix is calculated by an element-to-element product of the concordance dominance matrix and the discordance dominance matrix.

$$M_{aggregated\ \ dominance} = \begin{array}{cc} & \text{is dominated by} \\ \text{dominates} & \begin{bmatrix} - & 1 & 0 & 0 \\ 0 & - & 0 & 0 \\ 1 & 0 & - & 0 \\ 0 & 1 & 0 & - \end{bmatrix} \end{array}$$

9. Eliminate the dominated alternatives.

   In the aggregated dominance matrix, the element 1 in the column indicates that this alternative is dominated by other alternatives. In this example, it can be identified that A_1 is dominated by A_3, A_2 is dominated by A_1 and A_4. Thus, A_1 and A_2 are dominated alternatives and can be excluded by ELECTRE I.

It can be obtained that when weighting factors are evenly distributed among the seven criteria, A_1 and A_2 are dominated by A_3 and A_4. In other words, A_1 (Bombardier Challenger 300) and A_2 (Cessna Citation X) should be excluded from the candidates of business jets. But the outranking relationship between A_3 (Gulfstream G200) and A_4 (Hawker H4000) cannot be identified in the current set of weighting factors.

6.3.2 Typical Weighting Scenarios for ELECTRE I

Weighting factors play an important role in the decision analysis process. In this study, in order to better simulate DM's preference information, typical weighting scenarios for the seven criteria are generated from eleven levels of experimental design. The weighting factors for the seven criteria are the combination of seven numbers from the set [0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1], with the constraint that the sum is one. Since the seven decision criteria need to be considered simultaneously in the decision analysis process, all the seven numbers are required to be bigger than zero. Thus, 84 sets of weighting factors are generated and attached in Table C.4 in Appendix C.3.

The weighting factors reflect the relative importance of the decision criteria. For instance, the first row in Table C.4 is [0.4 0.1 0.1 0.1 0.1 0.1 0.1]. This set of weighting factors indicates that C_1 (fuel consumption per seat kilometer) is the most important decision criterion, and the other six decision criteria have the same level of importance. The other 83 sets of weighting factors have similar explanations.

The evaluation results using ELECTRE I for the 84 sets of weighting factors are summarized in Table 6.7. It is observed that when the DM takes into account all the seven criteria, A_4 has the highest frequency to be a non-dominated alternative, and A_2 has the highest frequency to be a dominated alternative. Therefore, it can be concluded that for the scenario considered in this study, A_2 (Cessna Citation X) should be excluded from the candidates of business jets and A_4 (Hawker H4000) should be recommended for the business aviation customer to purchase.

Table 6.7: Evaluation Results for 84 Sets of Weighting Factors using ELECTRE I

	A_1	A_2	A_3	A_4
Non-dominated times	50	34	51	59
Dominated times	34	50	33	25
Non-dominated frequency	59.52%	40.48%	60.71%	70.24%
Dominated frequency	40.48%	59.52%	39.29%	29.76%

6.4 Uncertainty Assessment

In the business aircraft evaluation problem, weighting factors and criteria values are the main input data utilized to solve the decision problem. It is observed that weighting factors are often highly subjective considering that they are elicited based on the DM's experience or estimation, while there are always uncertainties existing in the criteria values due to incomplete information. The inherent uncertainties and subjectivities associated with the input data have significant impacts on the final result of a decision making problem. Thus, it is critical to effectively address these uncertainties in the decision making process in order to get more accurate results. In this section, uncertainty assessment for weighting factors and criteria values is performed, following the new uncertainty assessment approach proposed in Chapter 4.

6.4.1 Uncertainty Characterization

As discussed in Section 4.2, uncertainties for weighting factors and criteria values are represented by percentage uncertainties with confidence levels. For example, if a DM assigns 15% uncertainty to the weight of the first decision criterion (w_1) with 90% confidence level, it implies that the DM is 90% confident that w_1 would fall within the interval $[w_1(1-15\%), w_1(1+15\%)]$. For this example, the uncertainty characterization for weighting factors and criteria values are summarized in Table 6.8.

Table 6.8: Uncertainty Characterization for Weighting Factors and Criteria Values

	Weighting factors						
	w_1	w_2	w_3	w_4	w_5	w_6	w_7
Percentage uncertainty	15%	10%	15%	10%	25%	30%	30%
Confidence level	90%	95%	85%	90%	70%	80%	90%
	Criteria values						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Percentage uncertainty	10%	5%	15%	10%	20%	20%	20%
Confidence level	90%	90%	85%	95%	80%	90%	95%

Secondly, percentage uncertainties with confidence levels are transferred into standard deviations using Equation 4.4 and Equation 4.6 in Subsection 4.2.2.

When the weighting factors are evenly distributed among the seven decision criteria, the mean of weighting factors μ_W equals to normalized weighting factors. The standard deviation of weighting factors σ_W is shown as follows.

$$\mu_W = [0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.14290.1429\ ]^{\mathrm{T}}$$

$$\sigma_W = [0.0130\ 0.0073\ 0.0149\ 0087\ 0.0345\ 0.0335\ 0.0261]^{\mathrm{T}}$$

For instance, the standard deviation of w_1 with 15% uncertainty at 90% confidence level, is calculated by Equation 6.4 and Equation 6.5, respectively.

$$n_{w_1} = \sqrt{2}erf^{-1}(\text{Confidence level}) = \sqrt{2}erf^{-1}(90\%) = 1.6449 \tag{6.4}$$

$$\sigma_{w_1} = \frac{\text{Relative error}(\%)\mu_{w_1}}{n_{w_1}} = \frac{(15\%)(0.1429)}{1.6449} = 0.0130 \tag{6.5}$$

The similar calculation is carried out for other weighting factors and criteria values. The normalized decision matrix D can be taken as μ_D, and the standard deviation of the decision matrix is shown in σ_D.

$$\mu_D = \begin{bmatrix} 0.5178 & 0.4881 & 0.5423 & 0.5035 & 0.6149 & 0.4879 & 0.4175 \\ 0.4561 & 0.5341 & 0.5073 & 0.5145 & 0.3577 & 0.5257 & 0.2960 \\ 0.5480 & 0.4881 & 0.4288 & 0.4890 & 0.4707 & 0.4956 & 0.6225 \\ 0.4728 & 0.4881 & 0.5145 & 0.4926 & 0.5219 & 0.4899 & 0.5921 \end{bmatrix}$$

$$\sigma_D = \begin{bmatrix} 0.0191 & 0.0090 & 0.0393 & 0.0178 & 0.0490 & 0.0412 & 0.0296 \\ 0.0169 & 0.0099 & 0.0367 & 0.0182 & 0.0285 & 0.0444 & 0.0210 \\ 0.0203 & 0.0090 & 0.0310 & 0.0173 & 0.0375 & 0.0419 & 0.0441 \\ 0.0175 & 0.0090 & 0.0372 & 0.0175 & 0.0416 & 0.0414 & 0.0420 \end{bmatrix}$$

In this step, uncertainties in the weighting factors and criteria values are transferred into means and standard deviations. μ_D, μ_W, σ_D, and σ_W are the input for the error propagation calculation in the uncertainty analysis step.

6.4.2 Uncertainty Analysis

As noted in Section 4.3, Monte Carlo-based numerical error propagation technique is applied to perform uncertainty analysis for ELECTRE I. 10,000 runs are performed from normal distribution with parameters μ_D, μ_W, σ_D, and σ_W. In this study, three scenarios are considered: uncertainty propagated from weighting factors, criteria values, and both from weighting factors and criteria values, as summarized in Table 6.9.

Table 6.9: Three Scenarios for Uncertainty Analysis

	Uncertainty incorporation	
Scenario	Weighting factors	Criteria values
1	✓	
2		✓
3	✓	✓

The probabilistic outranking relationships for each alternative in the three scenarios are presented in Table 6.10. It can be observed that with evenly distributed weighting factors among the seven decision criteria, A_4 (Hawker H4000) has the highest probability to be non-dominated, while A_2 (Cessna Citation X) has the highest probability to be dominated. The results of

Table 6.10: The Probabilistic Outranking Relationships in Three Scenarios

	Alternatives			
	A_1	A_2	A_3	A_4
Scenario 1				
Non-dominated	48.84%	11.50%	89.22%	99.71%
Dominated	51.16%	88.50%	10.78%	0.29%
Scenario 2				
Non-dominated	67.79%	9.16%	64.93%	72.37%
Dominated	32.21%	90.84%	35.07%	27.63%
Scenario 3				
Non-dominated	67.20%	10.04%	63.98%	70.34%
Dominated	32.80%	89.96%	36.02%	29.66%

uncertainty analysis are consistent with the evaluation results for the 84 sets of weighting factors presented in Table 6.7 in Subsection 6.3.2.

Besides, it also should be noted that in the three scenarios, the non-dominance or dominance status of A_2, A_3, and A_4 are preserved, while the dominance status of A_1 is not preserved in Scenario 2 and Scenario 3. The unstable status of A_1 can be attributed to its sensitivity to weighting factors and criteria values. The sensitivity of the alternatives to weighting factors and criteria values is investigated in Subsection 6.4.3.

Confidence Quantification of Sampling-based Error Propagation Technique

Since the numerical error propagation technique is sampling-based, a large number of samples are required in order to recreate the probability distributions for the input parameters. However, with the same input parameters, the results of uncertainty analysis will not be the same because of the randomness of the sampling method. In this study, the degree of confidence for the uncertainty analysis results is quantified through confidence intervals. The nested simulation loop for the confidence quantification is shown in Figure 6.7.

In our case, 10,000 Monte-Carlo simulation runs are performed in uncertainty analysis pro-

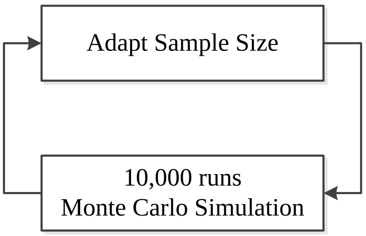


Figure 6.7: Nested Monte Carlo Simulation Loop for Confidence Quantification

cess. Considering that mean and standard deviation for 10,000 Monte-Carlo simulation runs are unknown, we can suppose that the sample mean $\overline{x}$ follows t distribution with mean μ and standard deviation $s/\sqrt{n}$, where s is the estimated standard deviation, n is the sample size [89]. The t distribution with sample size n has $n-1$ degree of freedom. The confidence interval is calculated by Equation 6.6.

$$[\overline{x} - t^* s/\sqrt{n}, \overline{x} + t^* s/\sqrt{n}] \tag{6.6}$$

where t^* is the upper $(1-\mathrm{CL})/2$ critical value for the t distribution with $n-1$ degree of freedom, CL is confidence level.

In this example, we take the sample size $n = 100$, CL $= 95\%$, the 0.025 critical value for 99 degree of freedom is $t^* = 1.984$. The 95% confidence intervals for the probabilistic outranking relationship in the three scenarios are summarized in Table 6.11. For instance, for the non-dominance probability of A_1 in Scenario 1, the sample mean is 48.35%, the sample standard deviation is 0.5535%. The 95% confidence interval is calculated by Equation 6.7.

$$\begin{aligned}
[\overline{x} - t^* s/\sqrt{n}, \overline{x} + t^* s/\sqrt{n}] &= [0.4835 - 1.984 \times\ 0.005535/\sqrt{100}, \\
&\quad\ 0.4835 + 1.984 \times\ 0.005535/\sqrt{100}] \\
&= [48.24\%, 48.46\%]
\end{aligned} \tag{6.7}$$

The tight confidence intervals in Table 6.11 verify that sampling-based error propagation technique can generate accurate results in the uncertainty analysis process for the business aircraft evaluation problem.

Table 6.11: The 95% Confidence Intervals for the Probabilistic Outranking Relationship in Three Scenarios

	Alternatives			
	A_1	A_2	A_3	A_4
Scenario 1				
Non-dominated	[48.24%,48.46%]	[11.53%,11.68%]	[89.81%,89.93%]	[99.74%,99.76%]
Dominated	[51.54%,51.76%]	[88.32%,88.47%]	[10.07%,10.19%]	[0.24%,0.26%]
Scenario 2				
Non-dominated	[67.46%,67.65%]	[9.00%,9.13%]	[64.69%,64.88%]	[72.02%,72.21%]
Dominated	[32.35%,32.54%]	[90.87%,91.00%]	[35.12 %,35.31%]	[27.79%,27.98%]
Scenario 3				
Non-dominated	[67.07%,67.24%]	[9.68%,9.80%]	[63.65%,63.86%]	[70.47%,70.65%]
Dominated	[32.76%,32.93%]	[90.20%,90.32%]	[36.14%,36.35%]	[29.35%,29.53%]

6.4.3 Sensitivity Analysis

Local sensitivity analysis based on iterative binary search algorithm and global sensitivity analysis using partial rank correlation coefficients are conducted for the business aircraft evaluation problem in the following subsections, respectively.

Local Sensitivity Analysis Based on Iterative Binary Search Algorithm

As discussed in Section 4.4, local sensitivity analysis varies input variables one at a time to determine which variables have the greatest effect on the model output, while holding the others fixed at nominal values. In the business aircraft evaluation problem using ELECTRE I, with equally distributed weighting factors among the seven criteria, A_3 and A_4 are non-dominated alternatives, while A_1 and A_2 are dominated alternatives. The developed iterative binary search algorithm can answer the question: What is the minimum change in the weighting factors or criteria values so that the non-dominance or dominance status of an alternative can be altered?

Local Sensitivity Analysis for Weighting Factors

The absolute minimum changes in weighting factors which can alter the non-dominance or dominance status of alternatives are summarized in Table 6.12. For the convenience of comparison, the relative minimum changes are also presented in Table 6.13. The relative minimum changes are the absolute minimum changes scaled against the original values of weighting factors. In the two tables, N/F (Non-Feasible) means that it is not mathematically feasible to alter the non-dominance or dominance status of alternatives through the change of the current parameter.

It can be seen from the first row in Table 6.13 that for dominated alternative A_1, it is not feasible to change the weighting factor of C_2 to switch A_1 into non-dominated alternative, while only around 3% change in C_5 or in C_7 can make A_1 become non-dominated alternative. Therefore, it can be concluded that A_1 is most robust against the weighting factor of C_2 and most sensitive to the weighting factors of C_5 and C_7.

Table 6.12: Absolute Minimum Changes in Weighting Factors to Alter the Non-dominance or Dominance Status of Alternatives

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1 to Non-dominance	-0.0396	**N/F**	0.0416	0.0716	**0.0048**	-0.0715	**-0.0049**
A_2 to Non-dominance	-0.0715	0.0478	-0.0715	0.0716	-0.0715	0.0716	-0.0715
A_3 to Dominance	-0.0272	0.5814	0.0324	1.4440	0.1281	1.6962	-0.0632
A_4 to Dominance	0.0868	0.8808	-0.0550	1.9280	0.2535	1.1968	-0.0841

Table 6.13: Relative Minimum Changes in Weighting Factors to Alter the Non-dominance or Dominance Status of Alternatives

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1 to Non-dominance	-27.67%	**N/F**	29.05%	50.06%	**3.33%**	-49.99%	**-3.41%**
A_2 to Non-dominance	-49.99%	33.39%	-49.99%	50.06%	-49.99%	50.06%	-49.99%
A_3 to Dominance	-18.98%	406.80%	22.65%	1010.50%	89.59%	1186.95%	-44.20%
A_4 to Dominance	60.69%	616.37%	-38.47%	1349.15%	177.39%	837.45%	-58.81%

Interactive Sensitivity Analysis for Weighting Factors

In this study, interactive sensitivity analysis for the weighting factors is developed with the purpose of providing the DM more vivid decision aiding, as shown in Figure 6.8, where the green bar represents that the alternative is non-dominated. The DM can simply move the slide bar of the weighting factor, and the change of the non-dominance or dominance status of the four alternatives is displayed simultaneously. The main idea of the interactive sensitivity analysis of weighting factors is to vary the weighting of one criterion from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting.

The interactive weighting plot for C_1 is presented in Figure 6.9, where *Non.* represents non-dominated and *Dom.* represents dominated. The interactive weighting plots for other six criteria

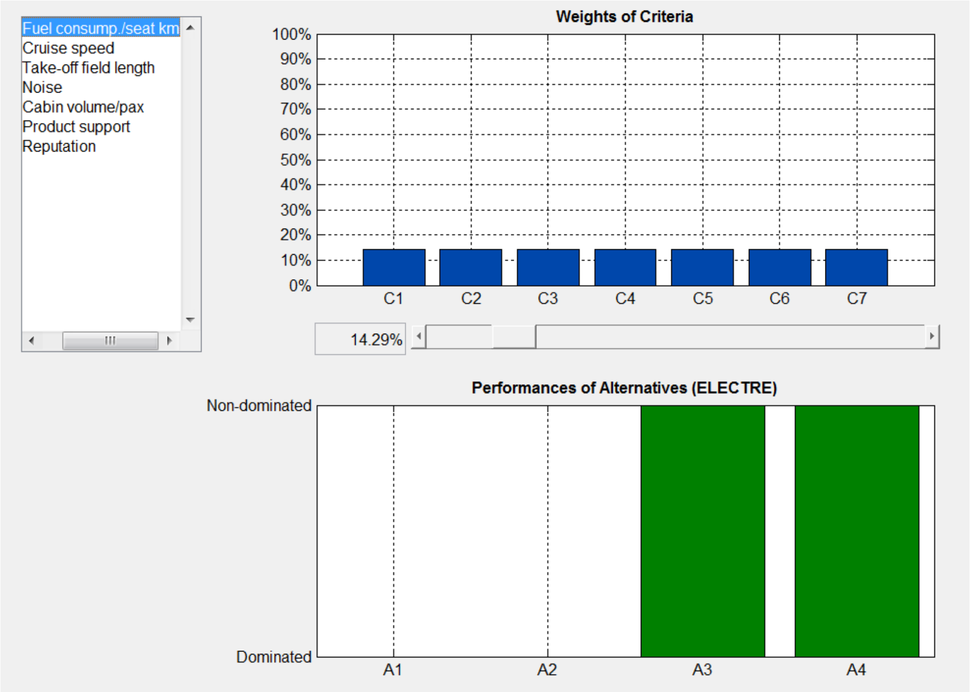


Figure 6.8: Interactive Sensitivity Analysis for Weighting Factors

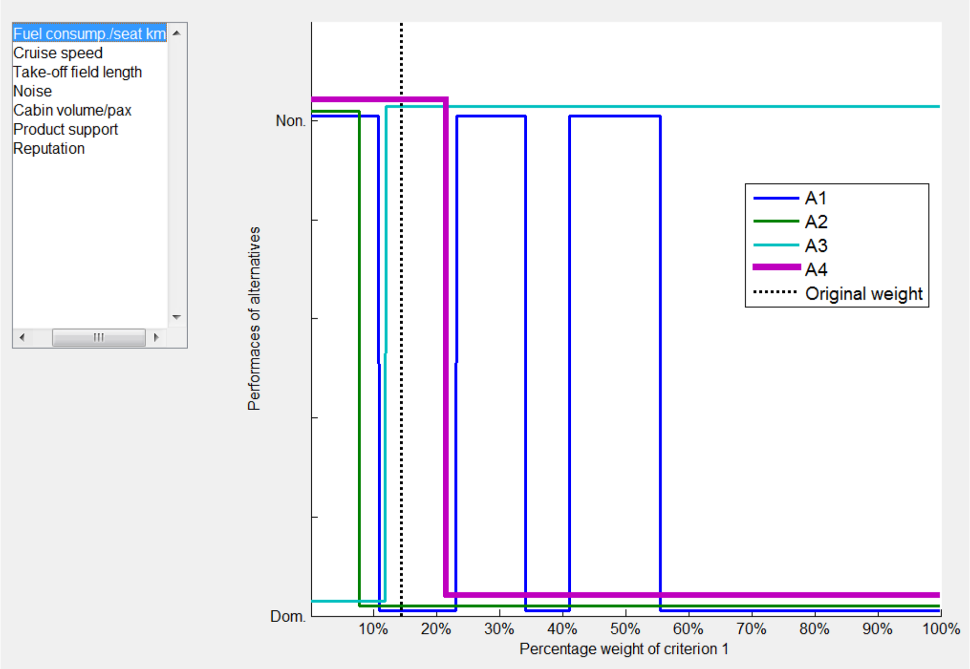


Figure 6.9: Interactive Weighting Plot for Criterion 1

(C_2 to C_7) are attached in Appendix B.2. The four alternatives are marked with different colors. The count of the vertical line stands for the change frequency of non-dominance or dominance status for one alternative.

For instance, in Figure 6.9, the purple line represents A_4, one purple vertical line tell us that when varying the weighting of C_1 from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting, A_4 changes one time from non-dominated to dominated alternative. Similarly, it can be observed that A_1 changes five times, A_2 and A_3 change one time, respectively.

The frequency of status changes for the four alternatives, when varying the weighting factors of the seven decision criteria from 0 to 100% individually, is summarized in Table 6.14. The row sum represents that for one alternative, how many times the status of this alternative has been changed, when varying the weighting factors of the seven decision criteria from 0 to 100% individually. The column sum represents that for one criterion, how many times the non-dominance or dominance status of the four alternatives have been changed, when varying the weighting of this criterion from 0 to 100%.

In Table 6.14, the biggest column sum of C_1 shows that C_1 has the highest frequency to change the non-dominance or dominance status of the four alternatives, when varying the weighting of this criterion from 0 to 100%. The biggest row sum of A_1 shows that A_1 has the highest

Table 6.14: Frequency of Status Changes for Alternatives in Interactive Weighting Plots

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7	Row sum
A_1	5	0	2	2	1	1	1	**12**
A_2	1	1	1	1	1	1	1	7
A_3	1	3	1	1	1	1	1	9
A_4	1	1	2	1	1	1	2	9
Column sum	**8**	5	6	5	4	4	5	

frequency of changing the non-dominance or dominance status, when varying the weighting factors of the seven decision criteria from 0 to 100%, individually. In summary, among the four alternatives, A_1 is most sensitive to the weighting factors. The sensitivity of A_1 to the weighting factors is consistent with the results shown in Table 6.12 and Table 6.13.

Furthermore, it is important to note that Table 6.12, Table 6.13, and Table 6.14 address different aspects of local sensitivity analysis for the weighting factors. Table 6.12 and Table 6.13 show the minimum changes in the weighting factors when the non-dominance or dominance status of alternatives is altered around the region of the nominal values of the weighting factors, which are located in the vicinity of the dot-dashed line in the interactive weighting plots. Table 6.14 summarizes the total frequency for the non-dominance or dominance status change of alternatives when varying the weighting of one criterion from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting.

Local Sensitivity Analysis for Criteria Values

Local sensitivity analysis for criteria values investigates how to change the criteria values so that the non-dominance or dominance status of alternatives can be altered. The developed iterative binary search algorithm can provide the mathematically feasible change of the criteria values to alter the non-dominance or dominance status of alternatives. However, for the business aircraft evaluation problem, mathematical feasibility does not necessarily guarantee physical feasibility. For instance, when the value of C_2 (high-speed cruise speed) is changed, it should be less than its maximum operating speed. The physical constraints of the decision criteria in the business aircraft evaluation problem are summarized in Table 6.15. Any change which violates these constraints is physically non-feasible.

In Table 6.15, M_{MO} represents maximum operating Mach number. According to BCA [94], the M_{MO} for the four business jets are 1016 km/h (0.83 Mach), 1126 km/h (0.92 Mach), 1040 km/h (0.85 Mach), and 1028 km/h (0.84 Mach), respectively. The constraint for C_5 is calculated by $42.5/8 = 5.3125$, which is based on the maximum cabin volume per passenger for the medium

Table 6.15: Physical Constraints of Decision Criteria for Business Aircraft

Decision criteria	Constraints
C_1: Fuel consumption per seat kilometer (kg/pax/km)	-
C_2: High-speed cruise speed (km/h)	$\leq M_{MO}$
C_3: Take-off field length (m)	[1300, 1900]
C_4: Noise (EPNdB)	[80, 90]
C_5: Cabin volume per passenger (m^3/pax)	$\leq$ 5.3125
C_6: Product support level	[1,10]
C_7: Manufacturer's reputation	[1,99]

jets, as shown in Table 6.1. The constraint for the product support level is based on the overall average scores obtained via the aviation international news 2010 product survey, as shown in Figure 6.3. The constraint for manufacturer's reputation is based on the aviation week's 16th annual top-performing companies study, as summarized in Table 6.4.

The absolute minimum changes in the criteria values which can alter the non-dominance or dominance status of alternatives are summarized in Table 6.16, and the relative minimum changes are summarized in Table 6.17. The relative minimum changes are the absolute minimum changes scaled against the original criteria values of the alternatives. In the two tables, N/F (Non-Feasible) represents that it is not mathematically feasible to alter the non-dominance or dominance status of alternatives through the change of the current parameter, and PN/F (Physically Non-Feasible) represents that the changed parameter violates its physical constraint.

The first four rows in Table 6.17 show the minimum changes in the criteria values of A_1 so that the non-dominance or dominance status of the four alternatives can be altered. It can be seen that it is not feasible to change any criteria value of A_1 in order to alter the dominance status of A_2.

Similarly, it can be observed from the second four rows in Table 6.17 that it is not feasible to change any criteria value of A_2 so that the non-dominance or dominance status of A_1, A_2, and A_4 can be altered. The third four rows show that it is not feasible to change any criteria value of A_3 to alter the dominance status of A_2. The fourth four rows show that it is not feasible to change any criteria value of A_4 so that the dominance status of A_2 can be modified.

The whole Table 6.17 shows that the criterion value C_2 of A_1 is most sensitive to the dominance status of A_1, while the criterion value C_4 is most robust against the change of the non-dominance or dominance status of the four alternatives.

Table 6.16: Absolute Minimum Changes in Criteria Values to Alter the Non-dominance or Dominance Status of Alternatives

Criteria values changed	C_1	C_2	C_3	C_4	C_5	C_6	C_7	Alternative status changed
A_1	N/F	**0.01**	N/F	**N/F**	N/F	0.13	N/F	A_1
A_1	N/F	N/F	N/F	**N/F**	N/F	N/F	N/F	A_2
A_1	-0.11	PN/F	N/F	**PN/F**	N/F	PN/F	N/F	A_3
A_1	-0.08	N/F	PN/F	**N/F**	1.13	N/F	13.5	A_4
A_2	N/F	N/F	N/F	**N/F**	N/F	N/F	N/F	A_1
A_2	N/F	N/F	N/F	**N/F**	N/F	N/F	N/F	A_2
A_2	N/F	N/F	PN/F	**PN/F**	N/F	N/F	53.26	A_3
A_2	N/F	N/F	N/F	**N/F**	PN/F	N/F	PN/F	A_4
A_3	N/F	PN/F	-122.19	**PN/F**	N/F	2.19	N/F	A_1
A_3	N/F	N/F	N/F	**N/F**	N/F	N/F	N/F	A_2
A_3	0.01	-175.08	PN/F	**PN/F**	-0.33	-1.63	-13.99	A_3
A_3	-0.02	PN/F	N/F	**N/F**	N/F	1.93	N/F	A_4
A_4	-0.02	PN/F	N/F	**N/F**	N/F	N/F	N/F	A_1
A_4	N/F	N/F	N/F	**N/F**	N/F	N/F	N/F	A_2
A_4	-0.01	PN/F	-61.51	**PN/F**	0.42	1.85	19.61	A_3
A_4	0.03	-192.24	106.68	**PN/F**	-0.84	-1.62	-10.45	A_4

Summary of Local Sensitivity Analysis for Weighting Factors and Criteria Values

According to the results of local sensitivity analysis for the weighting factors and criteria values shown in Table 6.13 and Table 6.17, we can summarize that in the business aircraft evaluation problem, A_1 is most sensitive to the weighting factor of C_5, the weighting factor of C_7, and the criterion value C_2, while the criterion value C_4 is most robust against the change of the non-dominance or dominance status of the four alternatives. The sensitivity of A_1 explains its unstable status shown in Table 6.10.

Attention should be paid that these minimum changes in the weighting factors and criteria values, shown in Table 6.13 and Table 6.17, are obtained using local sensitivity analysis. In other words, only one variable is varied at a time around its nominal value and the interactions among the input variables may not be captured. The simultaneous variations of all variables and the effects of the interactions among the input variables are investigated in global sensitivity analysis in the next subsection.

Global Sensitivity Analysis using Partial Rank Correlation Coefficients

In contrast to local sensitivity analysis, global sensitivity analysis allows the variations of all variables over the full range at the same time. In this subsection, global sensitivity analysis using

Table 6.17: Relative Minimum Changes in Criteria Values to Alter the Non-dominance or Dominance Status of Alternatives

Criteria values changed	C_1	C_2	C_3	C_4	C_5	C_6	C_7	Alternative status changed
A_1	N/F	**0.01%**	N/F	**N/F**	N/F	1.58%	N/F	A_1
A_1	N/F	N/F	N/F	**N/F**	N/F	N/F	N/F	A_2
A_1	-42.86%	PN/F	N/F	**PN/F**	N/F	PN/F	N/F	A_3
A_1	-29.88%	N/F	-30.81%	**N/F**	27.70%	N/F	24.55%	A_4
A_2	N/F	N/F	N/F	**N/F**	N/F	N/F	N/F	A_1
A_2	N/F	N/F	N/F	**N/F**	N/F	N/F	N/F	A_2
A_2	N/F	N/F	-41.10%	**PN/F**	N/F	N/F	136.56%	A_3
A_2	N/F	N/F	N/F	**N/F**	PN/F	N/F	PN/F	A_4
A_3	N/F	PN/F	-6.60%	**PN/F**	N/F	28.22%	N/F	A_1
A_3	N/F	N/F	N/F	**N/F**	N/F	N/F	N/F	A_2
A_3	2.78%	-20.13%	PN/F	**PN/F**	-10.39%	-21.03%	-17.06%	A_3
A_3	-8.20%	PN/F	N/F	**N/F**	N/F	24.83%	N/F	A_4
A_4	-4.12%	PN/F	N/F	**N/F**	N/F	N/F	N/F	A_1
A_4	N/F	N/F	N/F	**N/F**	N/F	N/F	N/F	A_2
A_4	-2.86%	PN/F	-3.99%	**PN/F**	11.98%	24.15%	25.15%	A_3
A_4	10.06%	-22.10%	6.91%	**PN/F**	-24.17%	-21.15%	-13.40%	A_4

partial rank correlation coefficients for the business aircraft evaluation problem is presented, following the proposed approach in Section 4.5.

Step 1: Define Probability Distributions for Input Variables

In the business aircraft evaluation problem using ELECTRE I, input variables are seven decision criteria and their weighting factors. The outputs are the outranking relationships for the four alternatives. Since there is no sufficient data to construct the probability distribution functions for the fourteen input variables, uniform distribution is chosen. For the seven decision criteria, the physical constraints shown in Table 6.15 serve as the minimum and maximum values, where the range of C_1 and the minimum value of C_5 are given by an expert. The weighting factors range from 0.05 to 0.85 in order to take all seven criteria into consideration. The probability distributions for the fourteen input variables are summarized in Table 6.18.

Step 2: Perform Latin Hypercube Sampling

The efficient LHS enables to vary all variables at the same time with low computational cost in global sensitivity analysis. In the business aircraft evaluation problem using ELECTRE I, 1000

Table 6.18: Probability Distributions for Input Variables

Input variables	Min	Max	Distribution
C_1	0.2	0.4	Uniform
C_2	850	1016	Uniform
C_3	1300	1900	Uniform
C_4	80	90	Uniform
C_5	2	5.3125	Uniform
C_6	1	10	Uniform
C_7	1	99	Uniform
$W_i, i = 1, .., 7$	0.05	0.85	Uniform

LHS runs are carried out with the probability functions defined for the fourteen input variables in Step 1. The minimum value of sample size for LHS is $\frac{3}{4}k$, where k is the number of input parameters that are varied [18]. In this example, $k = 14$, thus, 1000 runs of LHS is adequate for the calculation of partial rank correlation coefficients.

For each combination of the sampled values for the decision criteria and weighting factors, ELECTRE I is utilized to calculate the overall performances of the alternatives.

Step 3: Rank Transformation for both Input Variables and MCDA Output

In this step, the fourteen input variables and ELECTRE I output are transformed into ranks. Since ELECTRE I output is the outranking relationship of alternatives instead of scoring, the rank transformation is performed as described in Section 4.5. At first, the outrank set is assigned scores as follows: the non-dominated alternatives are assigned score 1, while the dominated alternatives are assigned score 0. Next, the outrank set with scores is transformed into ranks.

For example, in the business aircraft evaluation problem with equal weighting factors, A_3 and A_4 are non-dominated alternatives, while A_1 and A_2 are dominated alternatives. Thus, in the first step, A_3 and A_4 are assigned score 1, while A_1 and A_2 are assigned score 0. Next, the assigned score vector [0 0 1 1] is transformed into ranks. Counting from smallest to largest, the two 0 rank first and second, the average rank is $(1 + 2)/2 = 1.5$. The two 1 rank third and fourth, their average rank is $(3 + 4)/2 = 3.5$. Thus, the transformed ranks of the outrank set in ELECTRE I are [1.5 1.5 3.5 3.5].

Step 4: Calculate Partial Rank Correlation Coefficients

With the rank-transformed data, partial rank correlation coefficients can be calculated. Tornado plots of partial rank correlation coefficients for the four alternatives are presented in Figure 6.10, where p-values for partial rank correlation coefficients are next to the bars.

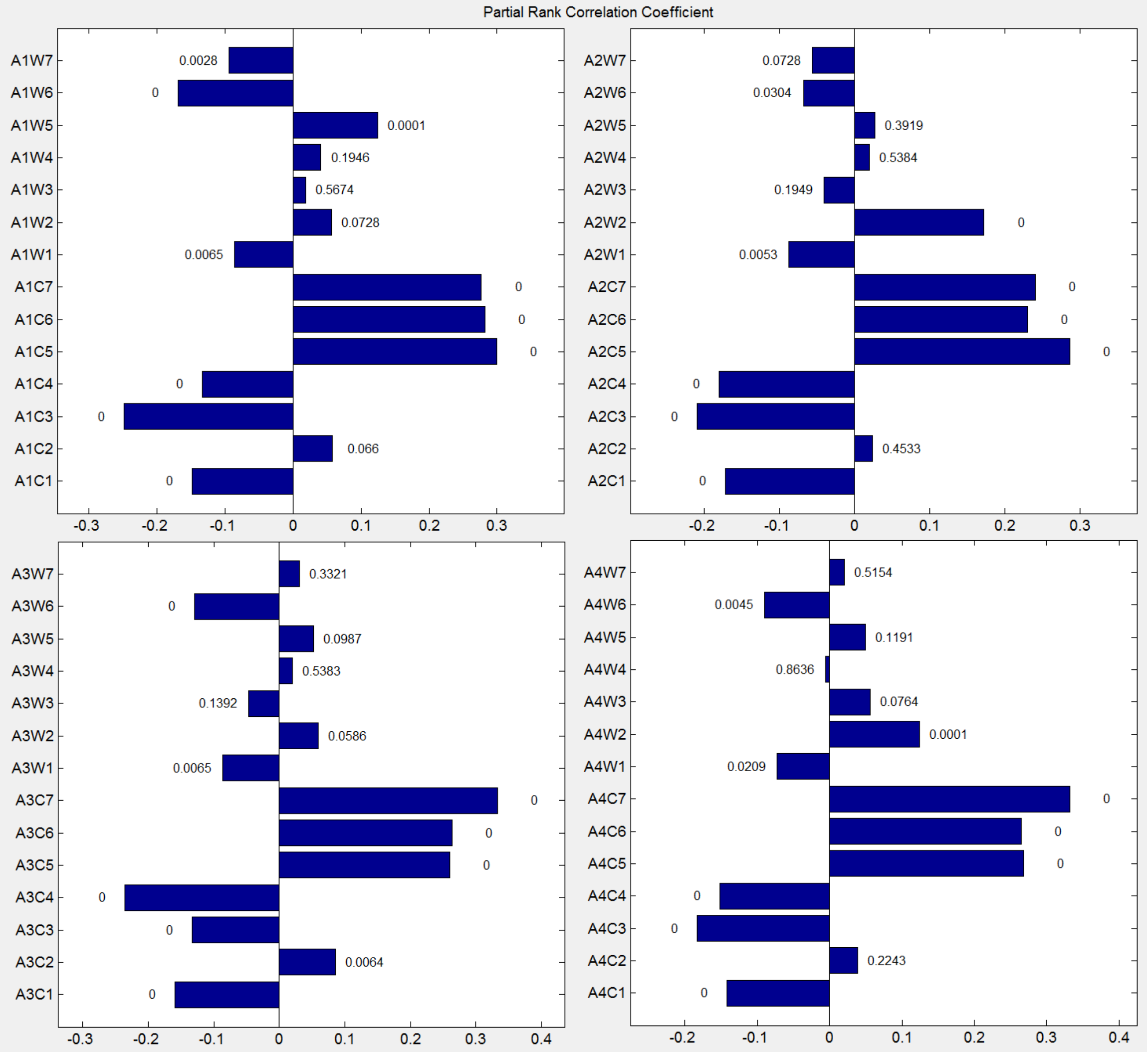


Figure 6.10: Tornado Plots of Partial Rank Correlation Coefficients for Four Alternatives using ELECTRE I, with p-values

Step 5: Conduct Statistical Significance Test

A lower p-value provides stronger evidence to reject the null hypothesis H_0 that there is no partial correlation between the rank transformed input variables and ELECTRE I output, in favor of the alternative hypothesis H_1 that there is nonzero partial correlation.

Step 6: Results Interpretation

Partial rank correlation coefficients should be interpreted together with statistical significance test. In this example, p-values less than 0.05 indicate that partial rank correlation coefficients are statistically significant.

It is observed from Figure 6.10 that in the business aircraft evaluation problem using ELEC-

TRE I, for the two non-dominated alternatives A_3 and A_4, input variable C_7 shows the strongest statistically significant correlations with the overall performances of the four alternatives, while for the two dominated alternatives A_1 and A_2, input variable C_5 shows the strongest statistically significant correlations with the overall performances of the four alternatives. Moreover, three input variables: C_5, C_6, and C_7, have the top three statistically significant correlations with the overall performances of the four alternatives.

The magnitude of partial rank correlation coefficients in global sensitivity analysis represents the relative importance of the influence of input variables on model output. Therefore, it is concluded that C_7 is most important for the performance of the non-dominated alternatives, C_5 is most important for the performance of the dominated alternatives, and C_5, C_6, and C_7, are most important in contributions to the overall performances of the four alternatives.

It is noted that there are two components in global sensitivity analysis for each input variable: range and sensitivity coefficient of the output to this input variable [88]. An input variable is identified as important in global sensitivity analysis if it has a wide range and large sensitivity coefficient. In our case, the reason why C_7 is detected as the most important input variable may be contributed by its wide range (1-99).

It is interesting to note that the three most important variables: C_5, C_6, and C_7, based on partial rank correlation coefficients in global sensitivity analysis, are the three additional soft decision criteria in the business aircraft evaluation problem. This shows that when evaluating the business aircraft, in addition to the technical *hard* criteria, it is also crucial to assess the additional soft criteria. The aggregation of the technical *hard* criteria and the additional soft criteria is the unique advantage of the MCDA methods.

Evaluation of Statistical Power of Partial Rank Correlation Coefficients

It is noted that when performing global sensitivity analysis for ELECTRE I, the magnitudes of partial rank correlation coefficients are relative small. This may be attributed to the rank transformation approach performed in Step 3, and too many tied ranks reduce the statistical power of partial rank correlation coefficients. Thus, in order to assess the statistical power of partial rank correlation coefficients in the decision analysis process, one popular scoring method, TOPSIS, is also utilized to solve the business aircraft evaluation problem.

With the same input variables, the seven decision criteria shown in decision matrix D and the weighting factors shown in W are repeated here for the convenience of calculation.

$$D = \begin{bmatrix} 0.2396 & 870 & 1466 & 84.2333 & 4.0500 & 7.63 & 55 \\ 0.2720 & 952 & 1567 & 82.4333 & 2.3556 & 8.22 & 39 \\ 0.2264 & 870 & 1854 & 86.7333 & 3.1000 & 7.75 & 82 \\ 0.2624 & 870 & 1545 & 86.1000 & 3.4375 & 7.66 & 78 \end{bmatrix}$$

$$W = [0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.14290.1429\ ]^{\mathrm{T}}$$

The ranking of the four alternatives using TOPSIS are $[A_4\ A_3\ A_1\ A_2]$. The results are consistent with the evaluation results using ELECTRE I that A_3 and A_4 are non-dominated alternatives, while A_1 and A_2 are dominated alternatives.

Partial rank correlation coefficients for the four alternatives, when TOPSIS is utilized to solve the business aircraft evaluation problem, are presented in Figure 6.11, where p-values for partial rank correlation coefficients are next to the bars.

It is observed from Figure 6.11 that the three input variables: C_5, C_6, and C_7, have the top three statistically significant correlations with the overall performances of the four alternatives. This observation is consistent with when ELECTRE I is utilized to solve the business aircraft evaluation problem. Furthermore, the magnitudes of partial rank correlation coefficients between the input variables and TOPSIS scores are bigger, which shows the statistical power of partial rank correlation coefficients in the decision analysis process.

6.5 Discussion

The effectiveness of implementing the most appropriate MCDA techniques in aircraft evaluation process was demonstrated in this chapter. A three-step framework was followed: definition of the decision making problem, selection of the most appropriate MCDA method, and uncertainty assessment in the decision analysis process. For the scenario considered in this study, A_2 (Cessna Citation X) should be excluded from the candidates of business jets, and A_4 (Hawker H4000) could be recommended for the business aviation customer to purchase.

In this section, the quantification of soft criteria is discussed first, followed by the advantages and disadvantages of local sensitivity analysis and global sensitivity analysis. Furthermore, the potential limitations in applicability of the proposed method is also further discussed and the direction needed for improving the proposed approach in the future is suggested.

Soft Criteria Quantification

Soft criteria become decisive in aircraft evaluation process. The quantification of additional soft criteria was presented in Subsection 6.1.2. Passenger comfort level was quantified by cabin volume per passenger. However, there are several other factors influencing passenger comfort, for instance, available seats and tables, passenger cabin electronics, in-flight access to baggage, and in-flight food service. However, currently there is no available reliable data to quantify these factors. Thus, they are not included in the quantification of passenger comfort level. Further research is needed to quantify those factors for passenger comfort level.

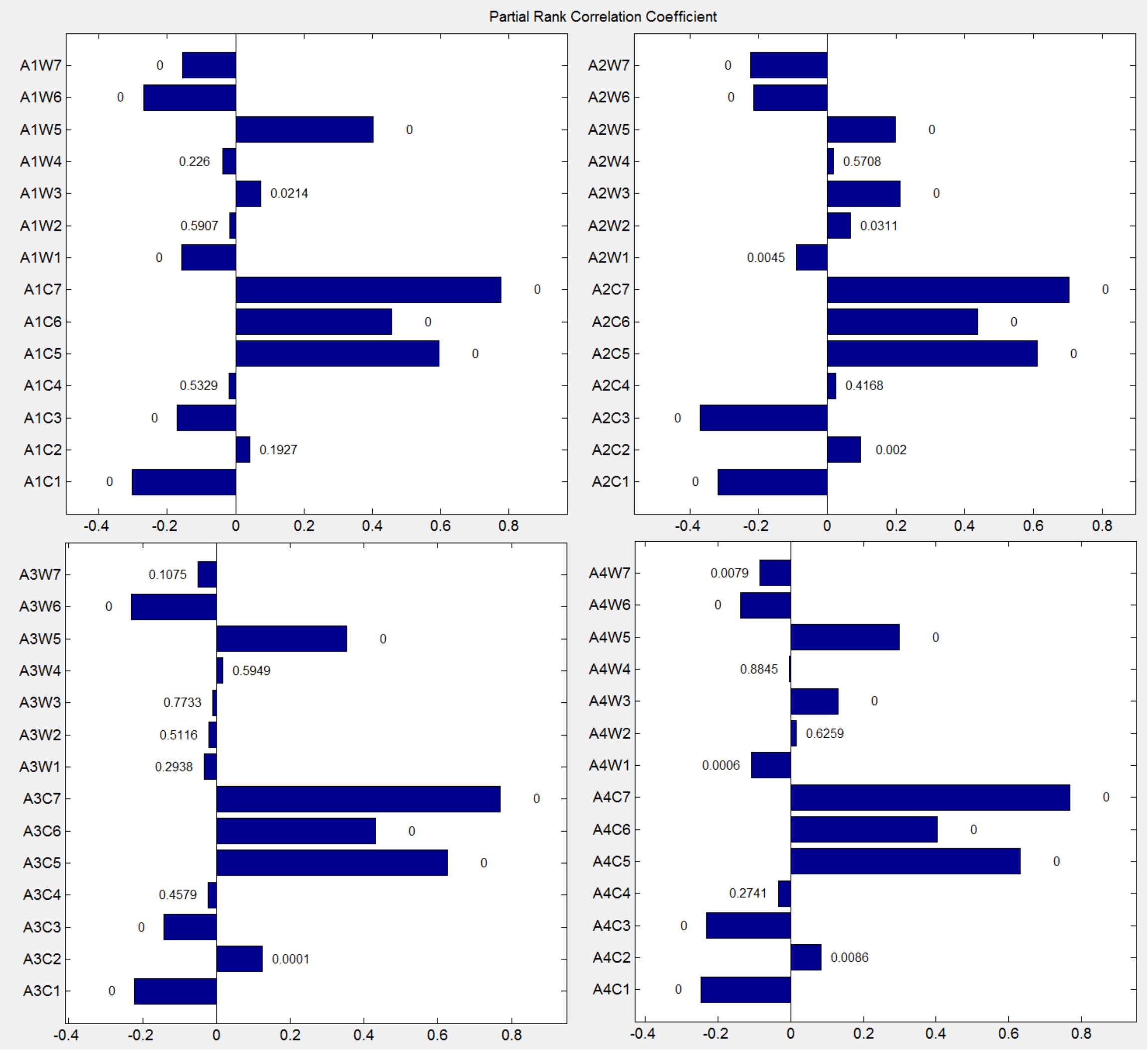


Figure 6.11: Tornado Plots of Partial Rank Correlation Coefficients for Four Alternatives using TOPSIS, with p-values

Local Versus Global Sensitivity Analysis

As discussed in Section 4.4 and Section 4.5, local sensitivity analysis varies one input variable at a time. It provides the sensitivity of an individual variable on model output over a small region around nominal values of input variables with efficient computation. However, when the model is nonlinear, or when several input variables are varied at the same time, local sensitivity analysis may not provide meaningful results. Global sensitivity analysis allows the variations of all input variables over their full range simultaneously and can capture the effects of interactions among input variables on model output, but with higher computational costs.

In the business aircraft evaluation problem, in order to obtain an initial understanding of the sensitivity of one individual variable on the MCDA outputs, local sensitivity analysis based on iterative binary search algorithm was conducted first. The results of local sensitivity analysis were summarized in Table 6.13 and Table 6.17, respectively.

In order to capture the effects of interactions among weighting factors and criteria values on the MCDA outputs, global sensitivity analysis using partial rank correlation coefficients were also performed. The results of global sensitivity analysis were presented in Figure 6.10.

According to Table 6.13 and Table 6.17, relative minimum changes of weighting factors and criteria values to alter the non-dominance or dominance status of the alternatives are ranked in ascending order. The top eight sensitive input variables identified by local sensitivity analysis are shown in the second column in Table 6.19.

For the purpose of comparison, partial rank correlation coefficients are ranked in descending order, according to Figure 6.10. The top eight important input variables with statistical significance identified by global sensitivity analysis are shown in the third column in Table 6.19.

As shown in Table 6.19, sensitivity rankings of input variables identified by local sensitivity analysis and global sensitivity analysis are different. One reason is that in local sensitivity analysis, input variables are varied one at a time and the interactions among input variables may not be captured.

However, this does not mean that the results of local sensitivity analysis are erroneous, because there are two distinct ways that the models are sensitive to input variables [53]: (1) small changes in input variables result in significant changes in the model output, and (2) the variation of input variables contributes substantially to the variation of model output. The former input variables are called *sensitive*, and the latter input variables are called *important*. An *important* variable is always *sensitive* because the variation of the variable will not appear in the model output unless the model is sensitive to this variable. However, a *sensitive* variable may not be *important* because the variable will have no influence on the variation of the model

Table 6.19: Comparison of Sensitivity Rankings for Input Variables Identified by Local and Global Sensitivity Analysis

Sensitivity rankings of input variables	Local sensitivity analysis	Global sensitivity analysis
1st	C_2	C_7
2nd	C_6	C_5
3rd	C_1	C_6
4th	W_5	C_3
5th	W_7	C_4
6th	C_3	C_1
7th	C_5	W_2
8th	C_7	W_6

output if the value of the variable is known precisely [53].

The top four *important* input variables (C_7, C_5, C_6, and C_3) and the sixth *important* input variable (C_1), are recognized as *sensitive* by local sensitivity analysis, although the ranking orders are different. The fifth, seventh, and eighth *important* input variables (C_4, W_2, and W_6) are not recognized as *sensitive* by local sensitivity analysis, which can be attributed to the reason that they are insensitive by themselves, however, when interacted with other input variables, their variations contribute substantially to the variation of the MCDA output.

For the same reason, the first, fourth, and fifth *sensitive* input variables (C_2, W_5, and W_7), are not identified as *important* by global sensitivity analysis, because they are sensitive by themselves, however, when interacting with other input variables, their variations do not contribute greatly to the variation of the MCDA output.

In summary, we take the perspective that local sensitivity analysis and global sensitivity analysis investigate model behaviors in different domains of input variables [138], and global sensitivity analysis should not precede local sensitivity analysis [50]. A complete understanding of the sensitivity of input variables on model output can be provided by performing both types of sensitivity analysis.

Potential Limitations and Further Improvement of the Proposed Approach

In this chapter, the most appropriate MCDA technique in aircraft evaluation process was implemented, following a three-step framework: definition of a decision making problem, selection of the most appropriate MCDA method for the given problem, and uncertainty assessment in the decision analysis process. This three-step framework provides a general guideline on how to structure and solve any given decision making problems.

In order to enable effective multi-criteria analysis and appropriately formulate the multi-criteria decision problem in the decision making problem definition step, the identification of alternatives and evaluation criteria can be facilitated with the help of problem structuring methods, such as value focused thinking and reasoning maps, as presented in Subsection 2.5.1 in Chapter 2.

In the uncertainty assessment step, global sensitivity analysis was based on partial rank correlation coefficients, with the assumption that the relationships between input variables and model output are monotonic. If non-monotonicities exist, variance decomposition analysis should be used to perform global sensitivity analysis.

7

Conclusions

The goal of this research is to investigate how MCDA techniques can be applied to provide better decision aiding for stakeholders in air transportation systems, by investigating how existing MCDA techniques could be improved to better solve complex decision problems, and how the improved MCDA techniques could be implemented in aircraft design and evaluation processes.

An advanced approach to effectively select the most appropriate MCDA method for a given decision making problem was presented, and a new approach for assessing the uncertainties propagated in the decision analysis process was proposed, respectively. The first proof of concept was the implementation of an improved MCDA method with uncertainty assessment in aircraft conceptual design process. The second proof of concept was the application of an appropriate MCDA technique with uncertainty assessment in business aircraft evaluation process.

7.1 Research Questions Answered

Question 1: *How to select the most appropriate MCDA method for the decision making problem under consideration?*

There are several MCDA techniques available to solve decision making problems, where different methods have different underlying assumptions, analysis models, and decision rules that are designed for solving a certain class of decision making problems. Thus, it is important to select the most appropriate MCDA method for a given problem.

An advanced approach to effectively select the most appropriate MCDA method for a given decision making problem was presented and an intelligent multi-criteria decision support system was developed. The match between the MCDA methods and a given problem was quantified by an Appropriateness Index, as proposed by **Hypothesis 1**. The MCDA method which has the highest score would be recommended as the most appropriate method for the DM to solve the given problem.

Question 2: *How to capture and assess the uncertainties propagated in the decision analysis process when solving decision making problems?*

When using the MCDA techniques to solve decision making problems, weighting factors and decision criteria are the main input data. Weighting factors are often highly subjective, considering that they are elicited based on the DM's experience or intuition, while there are always uncertainties existing in decision criteria due to incomplete information or limited knowledge. The inherent uncertainties of the input data have crucial impacts on the final solution for a decision making problem.

Hypothesis 2 proposed that statistical techniques are capable of effectively dealing with uncertainties. A new approach for uncertainty assessment was proposed. This approach consists of four steps: uncertainty characterization by percentage uncertainty with confidence level, uncertainty analysis using error propagation techniques, local sensitivity analysis based on iterative binary search algorithm, and global sensitivity analysis using partial rank correlation coefficients. This novel approach for uncertainty assessment can be used to aggregate input data from tools with different fidelity levels and is capable of propagating uncertainties in an assessment chain.

Question 3: *How to effectively implement the improved MCDA techniques in aircraft design and aircraft evaluation processes?*

As supported by **Hypothesis 3**, a new optimization framework incorporating MCDA techniques in aircraft conceptual design process was established. The intelligent multi-criteria decision support system was used to select an appropriate MCDA method. It was demonstrated that the chosen MCDA method with improvement (ITOPSIS) provided a better objective function for the optimization than the traditional weighted sum (SAW) method. Moreover, in order to efficiently assess the uncertainties related to the subjective preference information in aircraft design process, surrogate models for design criteria in terms of weighting factors were developed.

In the implementation of the MCDA techniques in business aircraft evaluation process, the selection of the most appropriate MCDA method was conducted through the intelligent multi-criteria decision support system. Moreover, three soft criteria were quantified: passenger comfort level, product support level, and manufacturer's reputation. The synergy of *hard* technical criteria and additional soft criteria is the unique advantage of the MCDA techniques.

7.2 Summary of Scientific Contributions

Four main scientific contributions of this research are summarized as follows.

1. An advanced approach to effectively select the most appropriate MCDA method for a given decision making problem is presented. This method selection approach is implemented and an intelligent multi-criteria decision support system is developed. Sixteen widely used MCDA methods are stored in the knowledge base as candidate methods for selection. Twelve criteria are proposed to evaluate the appropriateness of the method for a given decision making problem. The MCDA method which has the highest score is recommended as the most suitable method to solve the given problem.

2. A new uncertainty assessment approach in the decision analysis process is proposed, consisting of uncertainty characterization, uncertainty analysis, local sensitivity analysis, and global sensitivity analysis. This novel approach for uncertainty assessment can be used to aggregate input data from tools with different fidelity levels and is capable of propagating uncertainties in an assessment chain. Specifically, the different fidelity levels can be effectively captured by the confidence level in the uncertainty characterization step. Moreover, the step by step approach to perform global sensitivity analysis using partial rank correlation coefficients can be extended to investigate statistical relationships between variables in complex analysis problems.

3. A three-step framework for solving decision making problems is implemented: definition of a decision making problem, selection of the most appropriate MCDA method for the given problem, and uncertainty assessment in the decision analysis process. This three-step framework provides a general guideline on how to structure and solve any given decision making problems.

4. Two proofs of concept are conducted to demonstrate the effectiveness of utilizing the most appropriate MCDA techniques in aircraft design and evaluation processes. Surrogate models for design criteria in terms of weighting factors are developed to efficiently assess the uncertainties related to the subjective preference information in aircraft design process. Furthermore, the quantification of soft criteria in aircraft evaluation process permits the synergy of *hard* technical criteria and additional *soft* criteria for the MCDA techniques.

7.3 Recommendations

This section discusses some recommendations for future work. Regarding the proposed approach for uncertainty assessment, global sensitivity analysis was based on partial rank correlation coefficients, with the assumption that the relationships between input variables and model output are monotonic. If non-monotonicities exist, variance decomposition analysis should be used to perform global sensitivity analysis.

In the established optimization framework, incorporating MCDA techniques in aircraft conceptual design process, gradient-based methods were used. The focus was on exploring the feasibility and assessing the added values, not on the optimization itself. Genetic algorithms or hybrid optimizers combining genetic algorithms and gradient-based methods could also be investigated in the future.

Soft criteria become more decisive in the decision analysis process. In the business aircraft evaluation process, three soft criteria were quantified: passenger comfort level, product support level, and manufacturer's reputation. Further research could be conducted on the quantification of other soft criteria, such as aircraft safety and mission dispatch ability.

The MCDA techniques with uncertainty assessment were implemented in aircraft design and evaluation processes. The application of the MCDA techniques with uncertainty assessment could be extended into the assessment of air transportation systems, for balancing social, economic, ecological, and technical constraints.

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Appendix A

User Guide of an Intelligent Multi-Criteria Decision Support System

An intelligent multi-criteria decision support system is developed in MATLAB Graphical User Interface (GUI). The main interface is illustrated in Figure A.1. The decision support system has the capabilities to select the most appropriate method, use a specific method to solve a given problem, and perform uncertainty assessment in the decision analysis process. The user guide for each desired task is described in detail as follows.

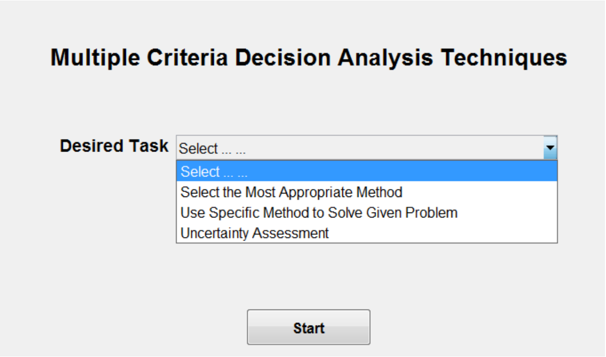
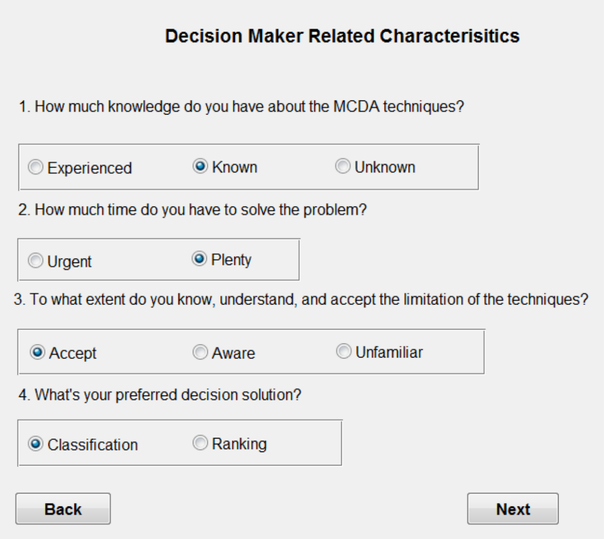


Figure A.1: Main Interface of an Intelligent Multi-Criteria Decision Support System

A.1 Select the Most Appropriate Method

When a DM wants to select the most appropriate method, DM-related requirements and problem-related requirements need to be defined in the first step. The interface of DM-related characteristics is illustrated in Figure A.2 and is summarized in Figure A.3. If the summary is not



Decision Maker Related Characterisitics

1. How much knowledge do you have about the MCDA techniques?

○ Experienced ● Known ○ Unknown

2. How much time do you have to solve the problem?

○ Urgent ● Plenty

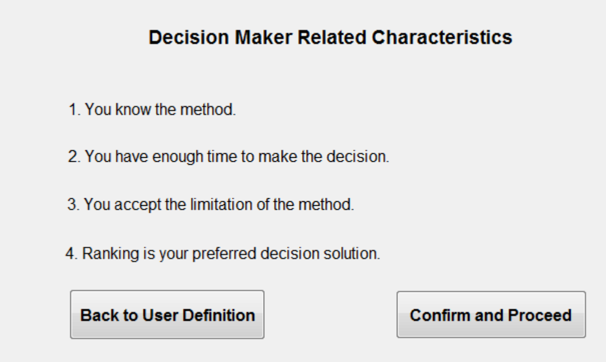
3. To what extent do you know, understand, and accept the limitation of the techniques?

● Accept ○ Aware ○ Unfamiliar

4. What's your preferred decision solution?

● Classification ○ Ranking

Figure A.2: Interface of Decision Maker Related Characteristics



Decision Maker Related Characteristics

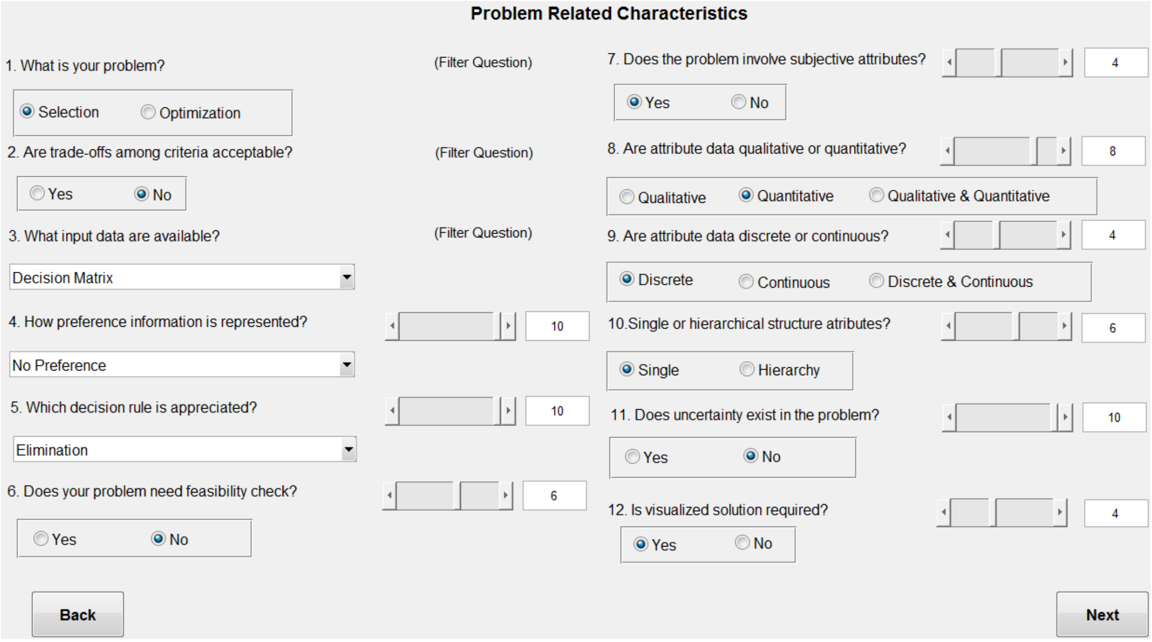
1. You know the method.
2. You have enough time to make the decision.
3. You accept the limitation of the method.
4. Ranking is your preferred decision solution.

Figure A.3: Summary of Decision Maker Related Characteristics

satisfying, then the DM can simply click *Back to User Definition* and redefine the requirements; otherwise, the DM can click *Confirm and Proceed* to move on to the next step.

When the DM is experienced about the MCDA techniques, the interface of all sixteen widely used MCDA methods is presented. This interface is discussed in Section A.2.

The interface of problem-related characteristics is most important, where the appropriateness score for each MCDA method is obtained. It is illustrated in Figure A.4 and is summarized in Figure A.5. If the summary is not satisfying, the DM can simply click *Back to User Definition* and redefine the requirements; otherwise, the DM can click *Confirm and Proceed*. The ranking of the MCDA methods with appropriateness scores is shown in Figure A.6.



Problem Related Characteristics

1. What is your problem? (Filter Question)
● Selection ○ Optimization

2. Are trade-offs among criteria acceptable? (Filter Question)
○ Yes ● No

3. What input data are available? (Filter Question)
Decision Matrix

4. How preference information is represented? 10
No Preference

5. Which decision rule is appreciated? 10
Elimination

6. Does your problem need feasibility check? 6
○ Yes ● No

7. Does the problem involve subjective attributes? 4
● Yes ○ No

8. Are attribute data qualitative or quantitative? 8
○ Qualitative ● Quantitative ○ Qualitative & Quantitative

9. Are attribute data discrete or continuous? 4
● Discrete ○ Continuous ○ Discrete & Continuous

10.Single or hierarchical structure atributes? 6
● Single ○ Hierarchy

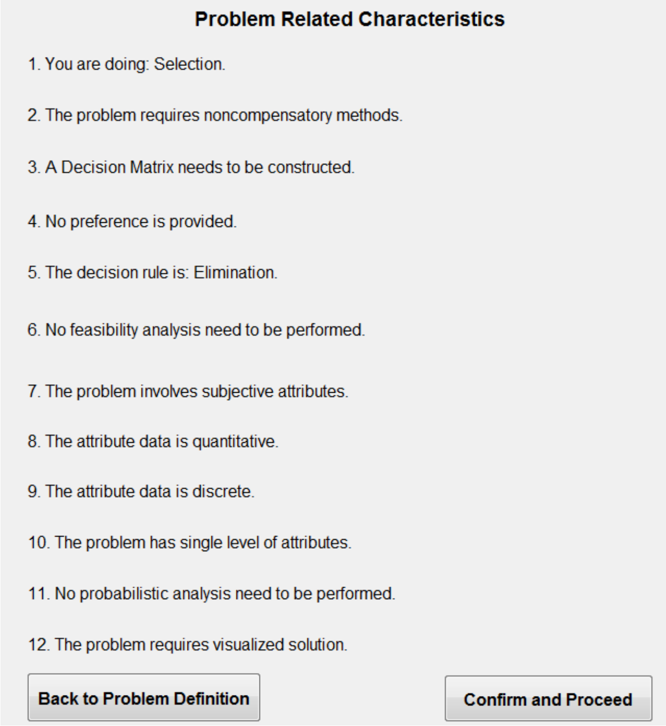
11. Does uncertainty exist in the problem? 10
○ Yes ● No

12. Is visualized solution required? 4
● Yes ○ No

Figure A.4: Interface of Problem Related Characteristics

The DM can simply click the name of the most appropriate method, and methodology instructions will be shown to guide the DM to get the final solution. In addition, the mathematical calculation steps are also built in the decision support system. Thus, for evaluation decision making problems, the DM can input the data according to the instruction, and get the final results by clicking one corresponding button. For instance, methodology instructions of the Dominance method are illustrated in Figure A.7.

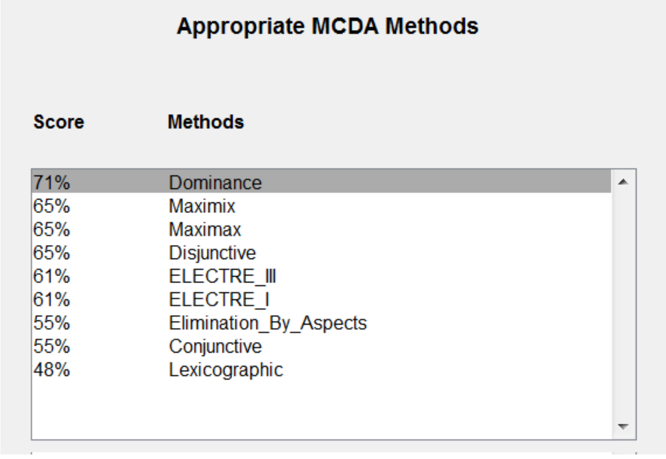
Attention should be paid that inconsistent input for the three filter questions will be rectified by the intelligent multi-criteria decision support system automatically. For instance, since compensation is always allowed in the optimization process, if the DM selects the MCDA methods for optimization, all non-compensatory MCDA methods which cannot offer scores will be excluded. Even if the DM selects optimization for the first filter question and non-compensatory for the second filter question, the system will rectify the conflicting input by offering compensatory MCDA methods for solving optimization problem.



Problem Related Characteristics

1. You are doing: Selection.
2. The problem requires noncompensatory methods.
3. A Decision Matrix needs to be constructed.
4. No preference is provided.
5. The decision rule is: Elimination.
6. No feasibility analysis need to be performed.
7. The problem involves subjective attributes.
8. The attribute data is quantitative.
9. The attribute data is discrete.
10. The problem has single level of attributes.
11. No probabilistic analysis need to be performed.
12. The problem requires visualized solution.

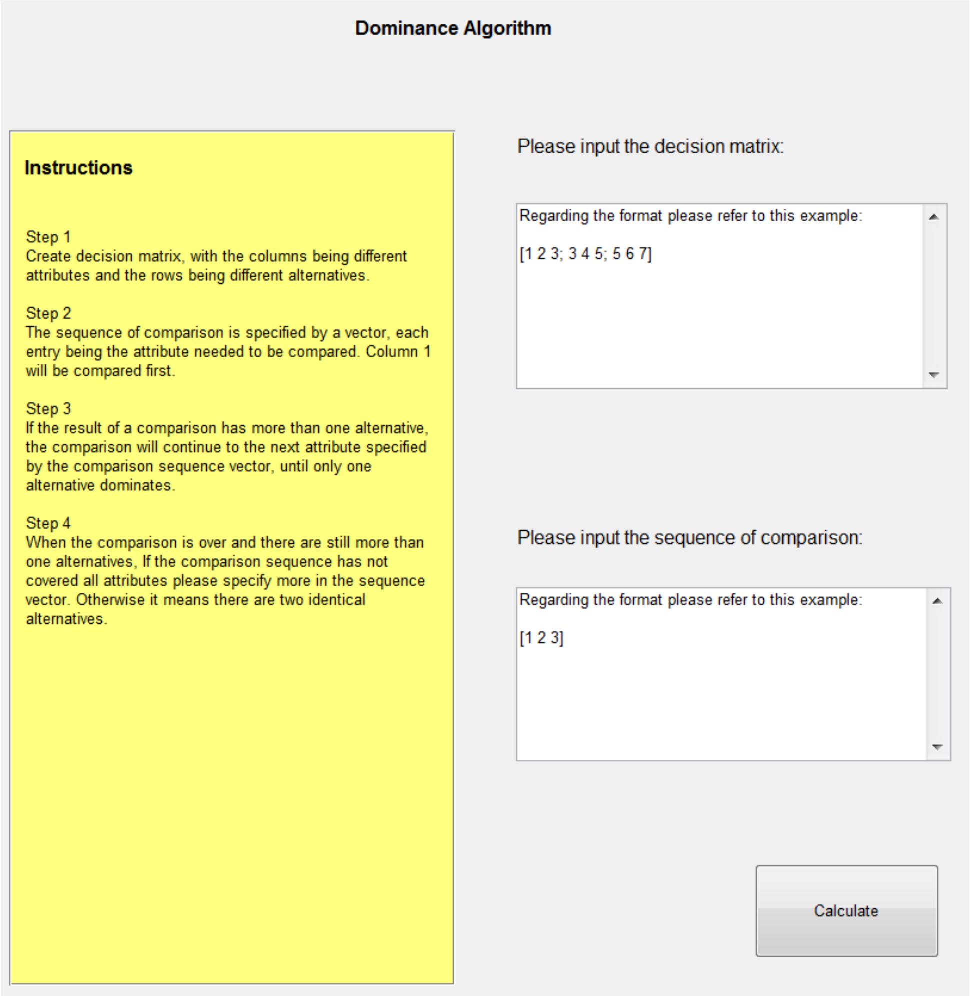
Figure A.5: Summary of Problem Related Characteristics



Appropriate MCDA Methods

Score	Methods
71%	Dominance
65%	Maximix
65%	Maximax
65%	Disjunctive
61%	ELECTRE_III
61%	ELECTRE_I
55%	Elimination_By_Aspects
55%	Conjunctive
48%	Lexicographic

Figure A.6: Ranking of MCDA Methods with Appropriateness Scores



Dominance Algorithm

Instructions

Step 1
Create decision matrix, with the columns being different attributes and the rows being different alternatives.

Step 2
The sequence of comparison is specified by a vector, each entry being the attribute needed to be compared. Column 1 will be compared first.

Step 3
If the result of a comparison has more than one alternative, the comparison will continue to the next attribute specified by the comparison sequence vector, until only one alternative dominates.

Step 4
When the comparison is over and there are still more than one alternatives, If the comparison sequence has not covered all attributes please specify more in the sequence vector. Otherwise it means there are two identical alternatives.

Please input the decision matrix:

Regarding the format please refer to this example:

[1 2 3; 3 4 5; 5 6 7]

Please input the sequence of comparison:

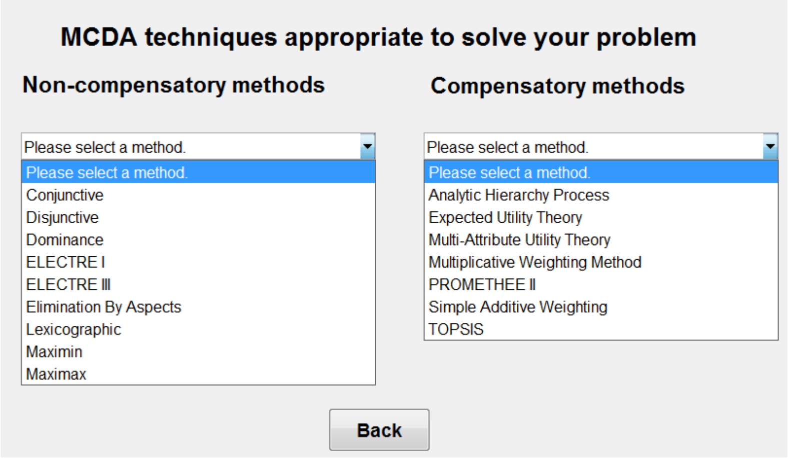
Regarding the format please refer to this example:

[1 2 3]

Figure A.7: Methodology Instructions for Dominance Method

A.2 Use Specific Method to Solve a Given Problem

When the DM wants to use specific method to solve a given problem, all sixteen widely used MCDA methods are listed in Figure A.8. The DM can simply click the name of the most appropriate MCDA method, and methodology instructions will be shown to guide the DM to get the final solution by using the selected method.



MCDA techniques appropriate to solve your problem

Non-compensatory methods

Please select a method.

- Please select a method.
- Conjunctive
- Disjunctive
- Dominance
- ELECTRE I
- ELECTRE III
- Elimination By Aspects
- Lexicographic
- Maximin
- Maximax

Compensatory methods

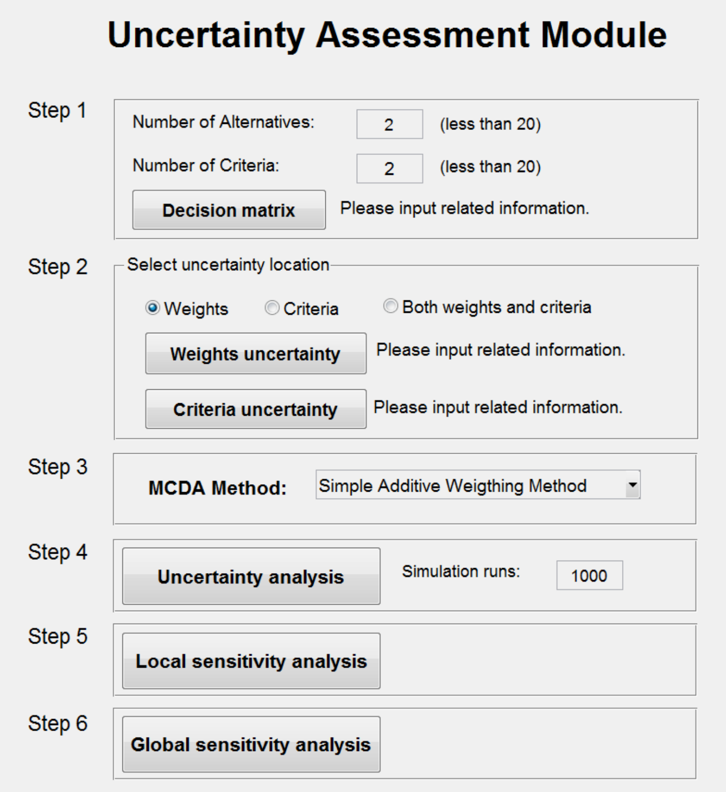
Please select a method.

- Please select a method.
- Analytic Hierarchy Process
- Expected Utility Theory
- Multi-Attribute Utility Theory
- Multiplicative Weighting Method
- PROMETHEE II
- Simple Additive Weighting
- TOPSIS

Figure A.8: List of Sixteen MCDA Methods

A.3 Uncertainty Assessment

When the DM wants to perform uncertainty assessment, the interface of the uncertainty assessment module is illustrated in Figure A.9. In the uncertainty assessment module, the DM can simply go through the uncertainty assessment process according to the instructions. In addition, the detailed mathematical calculation steps for four MCDA techniques: SAW, multiplicative weighting method, TOPSIS, and ELECTRE I, are also built in the uncertainty assessment module, which highly facilitates the uncertainty assessment in the decision analysis process.



Number of Alternatives: 2 (less than 20)

Number of Criteria: 2 (less than 20)

Please input related information.

● Weights ○ Criteria ○ Both weights and criteria

Please input related information.

Please input related information.

MCDA Method: Simple Additive Weigthing Method

Simulation runs: 1000

Figure A.9: Interface of Uncertainty Assessment Module

Appendix B

Additional Figures

B.1 Parametric Studies of Design Criteria

Parametric studies for wing thickness-to-chord ratio, aspect ratio, reference area, and fuselage diameter in the aircraft conceptual design tool (VAMPzero) are presented in Figure B.1, Figure B.2, Figure B.3, and Figure B.4, respectively.

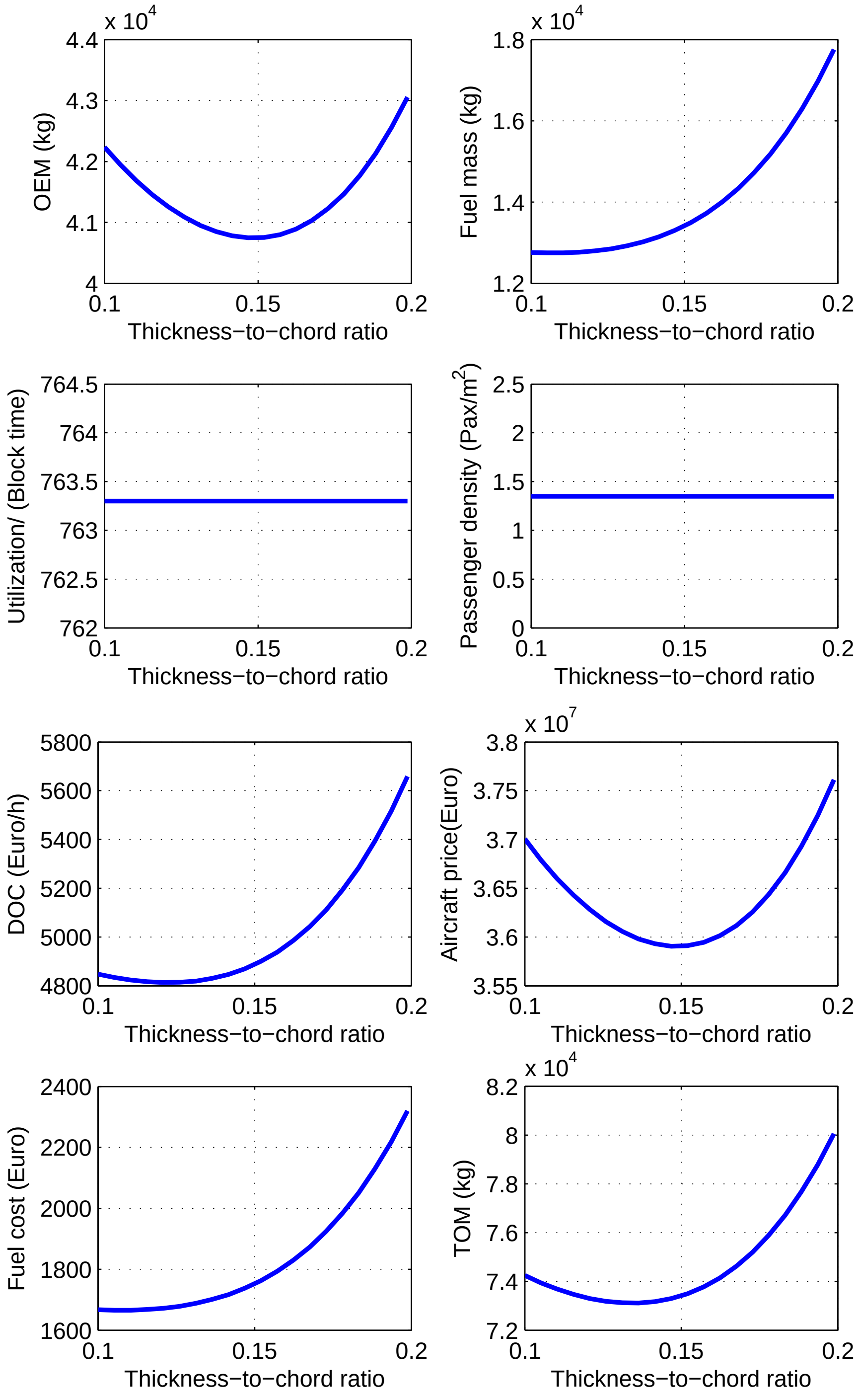


Figure B.1: Parametric Study of Thickness-to-chord Ratio versus OEM, Fuel Mass, Utilization/(Block time), Passenger Density, DOC, Aircraft Price, Fuel Cost, and TOM

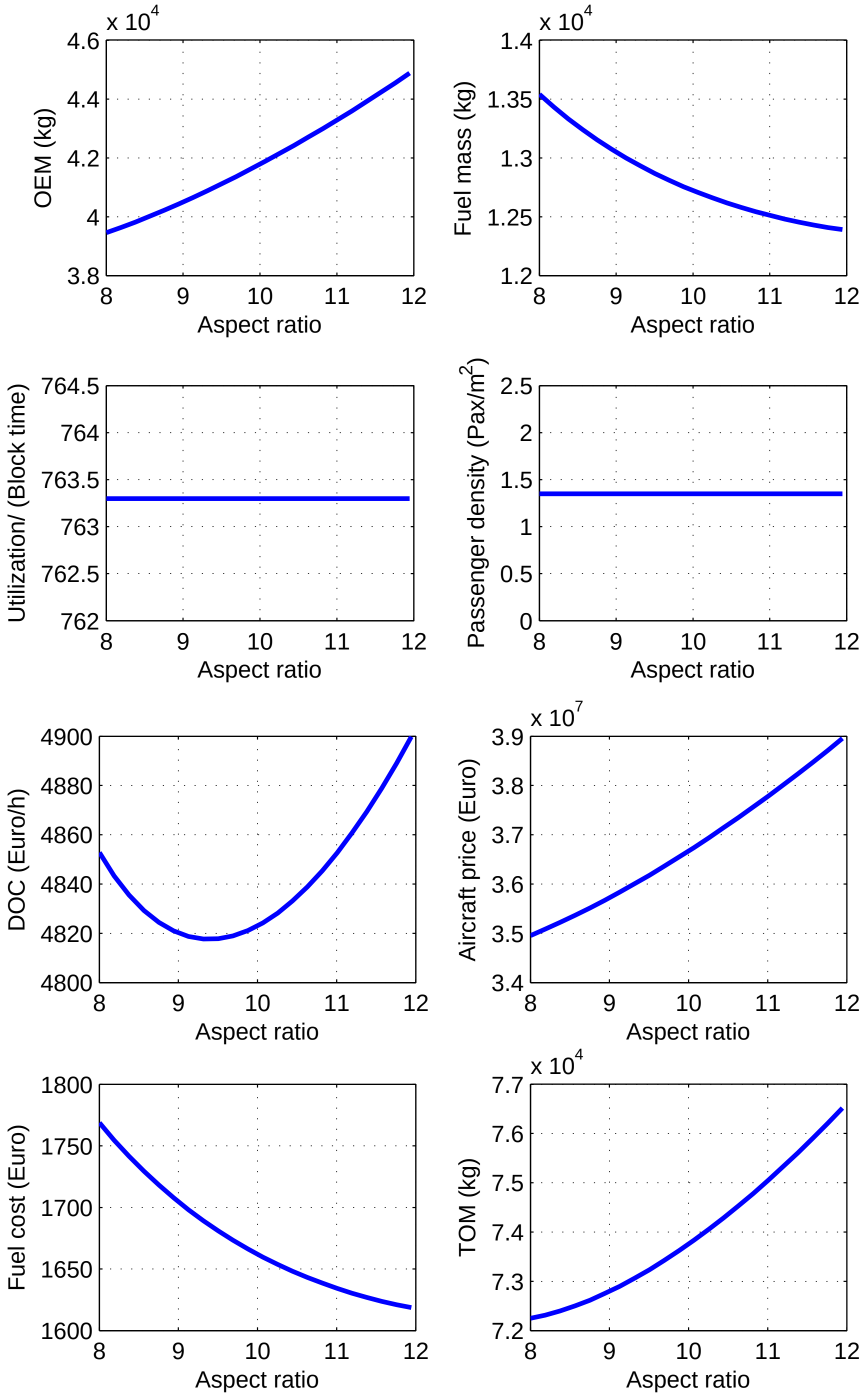


Figure B.2: Parametric Study of Aspect Ratio versus OEM, Fuel Mass, Utilization/(Block time), Passenger Density, DOC, Aircraft Price, Fuel Cost, and TOM

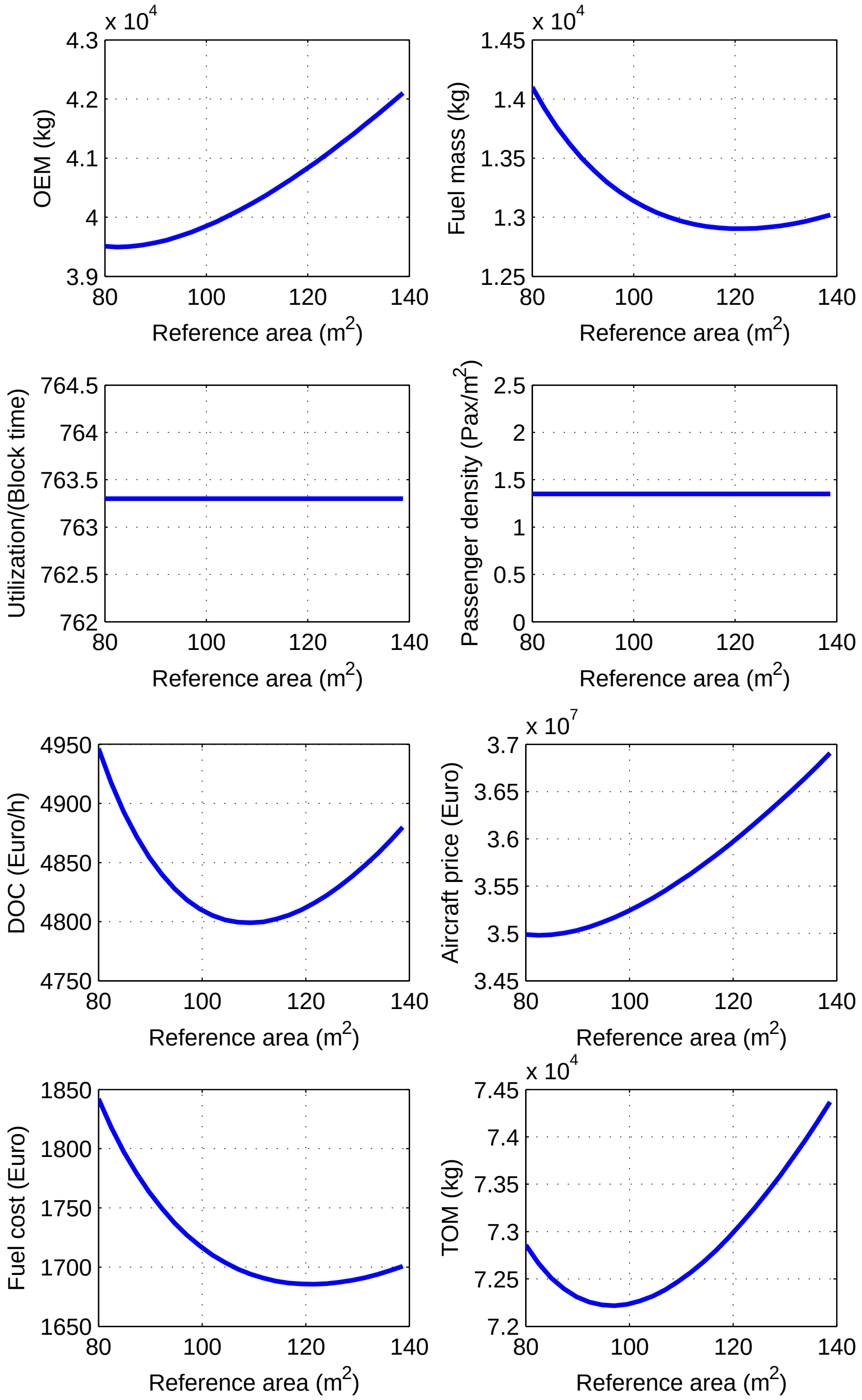


Figure B.3: Parametric Study of Reference Area versus OEM, Fuel Mass, Utilization/(Block time), Passenger Density, DOC, Aircraft Price, Fuel Cost, and TOM

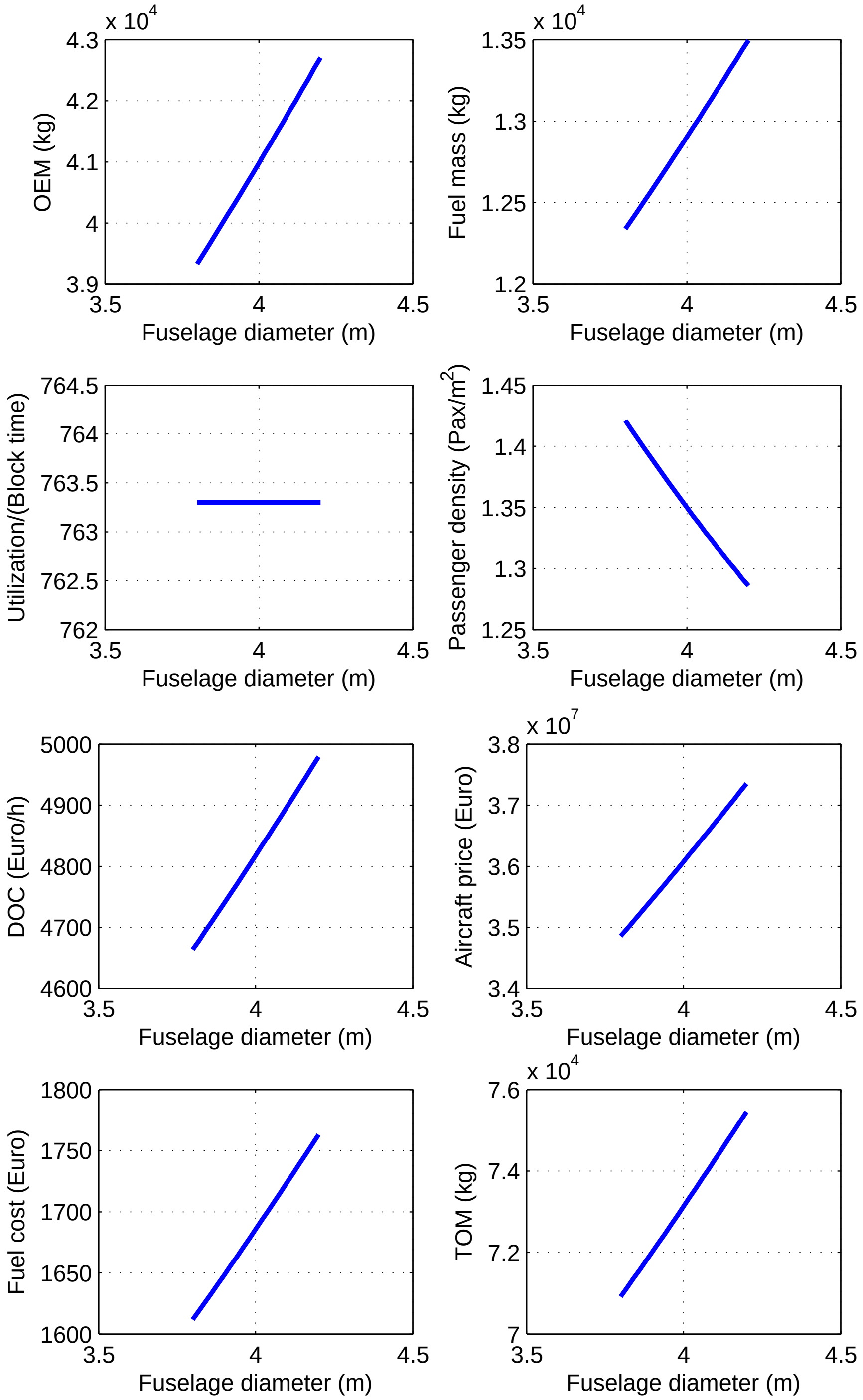


Figure B.4: Parametric Study of Fuselage Diameter versus OEM, Fuel Mass, Utilization/(Block time), Passenger Density, DOC, Aircraft Price, Fuel Cost, and TOM

B.2 Interactive Weighting Plots for Business Aircraft Evaluation

The main idea of the interactive sensitivity analysis for weighting factors is to vary the weight of one criterion from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting. In the business aircraft evaluation problem using ELECTRE I, the interactive weighting plots for C_2 to C_7 are presented in Figure B.5, Figure B.6, Figure B.7, Figure B.8, Figure B.9, and Figure B.10, respectively, where *Non.* represents non-dominated alternative, and *Dom.* represents dominated alternative.

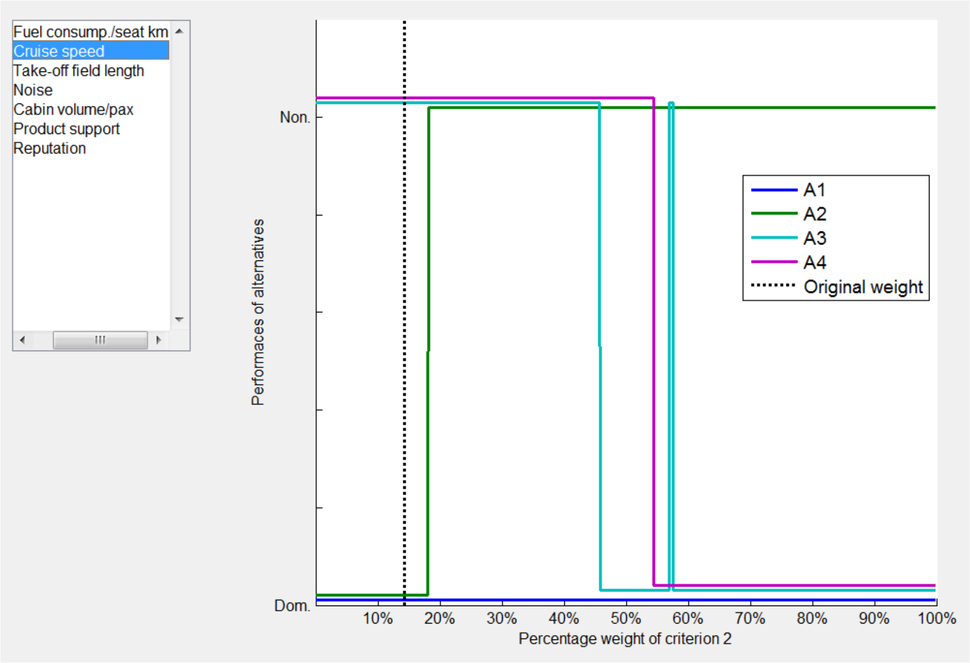


Figure B.5: Interactive Weighting Plot for Criterion 2

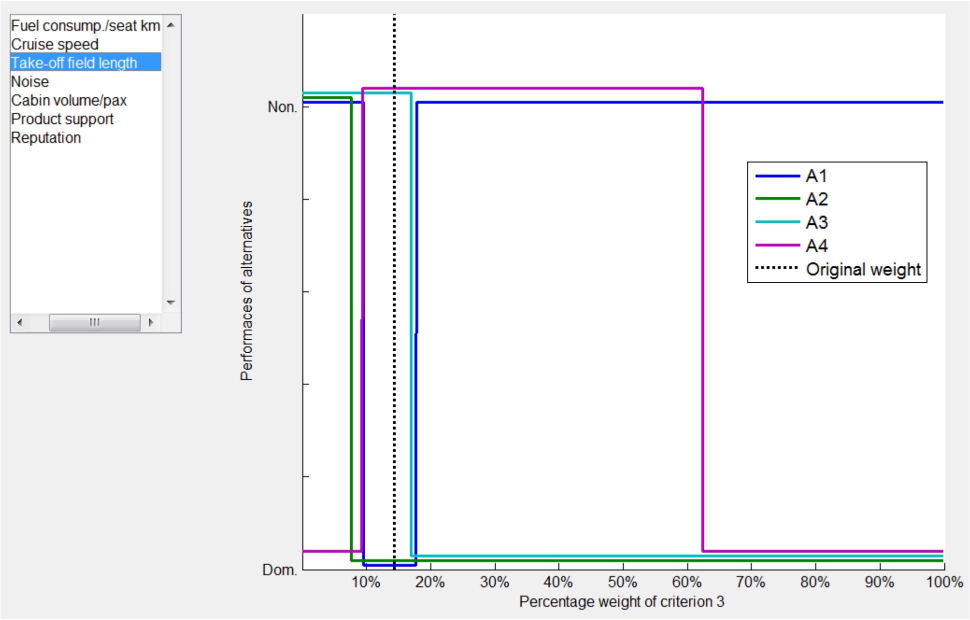


Figure B.6: Interactive Weighting Plot for Criterion 3

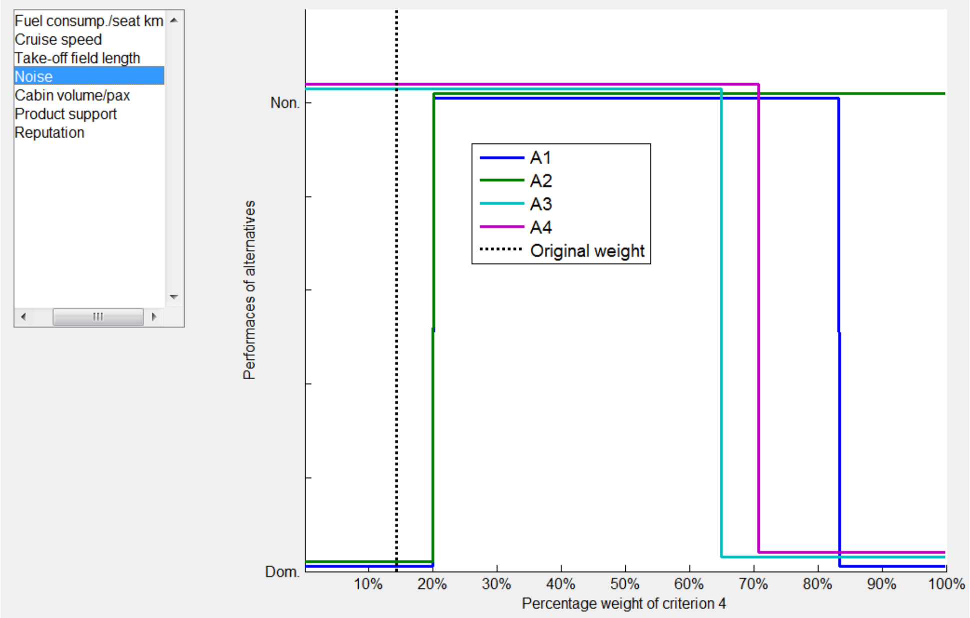


Figure B.7: Interactive Weighting Plot for Criterion 4

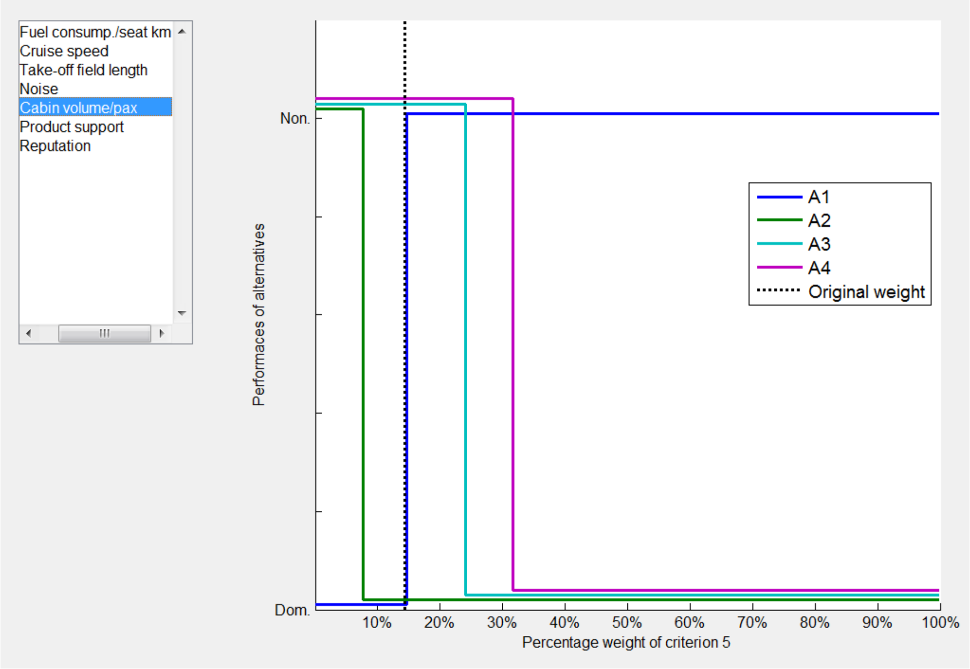


Figure B.8: Interactive Weighting Plot for Criterion 5

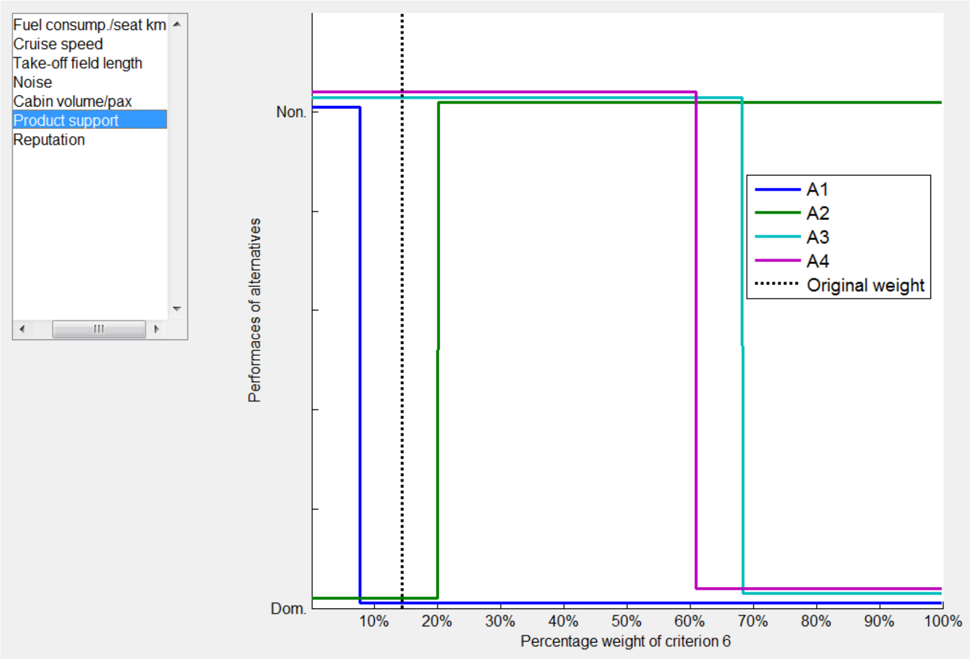


Figure B.9: Interactive Weighting Plot for Criterion 6

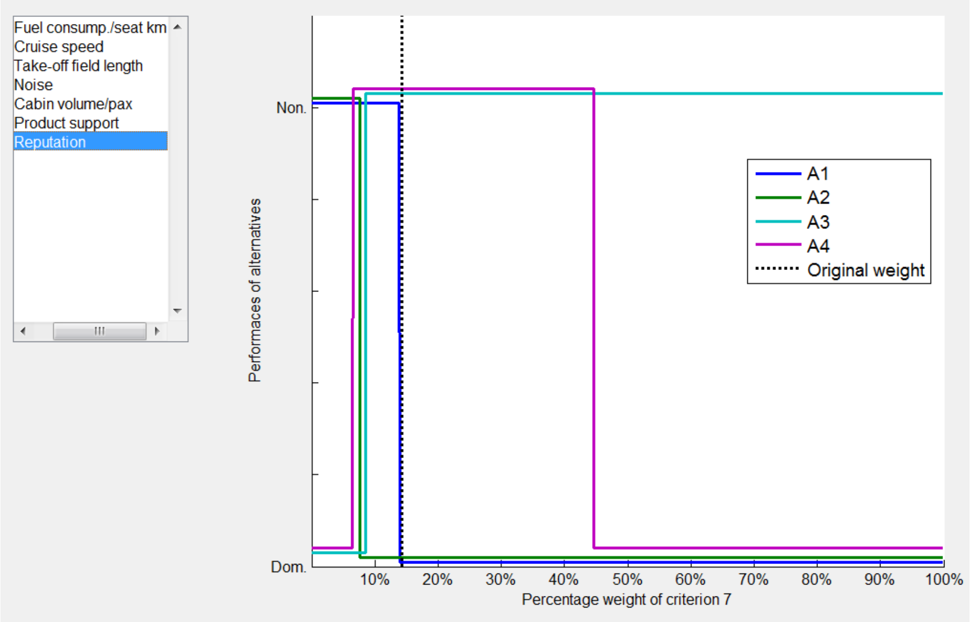


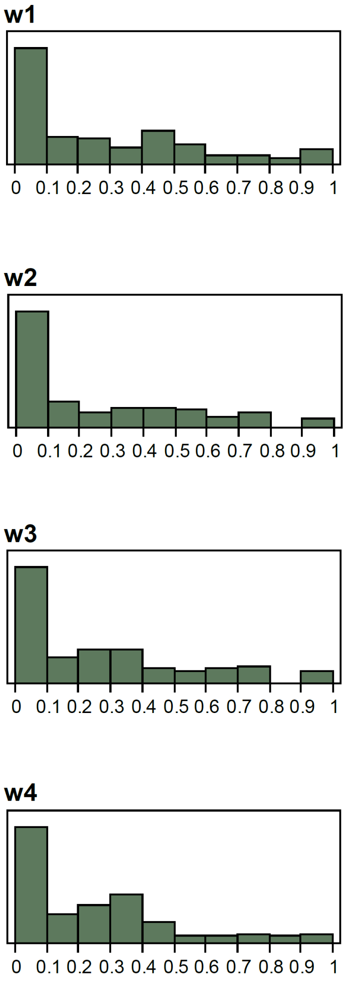
Figure B.10: Interactive Weighting Plot for Criterion 7

Appendix C

Data Sources

C.1 Data Points for Surrogate Model Development in terms of Weighting Factors

One hundred sets of weighting factors are generated by the modified Latin Hypercube Sampling (LHS) with Dirichlet distribution. Histograms of the one hundred sets of weighting factors are depicted in Figure C.1. The values of four design criteria (OEM, fuel mass, utilization/(block time), and passenger density) are listed in Table C.1.



Moments (w1)

Moments	
Mean	0.263379
Std Dev	0.2834573
Std Err Mean	0.0283457
Upper 95% Mean	0.3196231
Lower 95% Mean	0.2071349
N	100

Moments	
Mean	0.247492
Std Dev	0.2739894
Std Err Mean	0.0273989
Upper 95% Mean	0.3018574
Lower 95% Mean	0.1931266
N	100

Moments	
Mean	0.252911
Std Dev	0.275488
Std Err Mean	0.0275488
Upper 95% Mean	0.3075738
Lower 95% Mean	0.1982482
N	100

Moments	
Mean	0.236206
Std Dev	0.249093
Std Err Mean	0.0249093
Upper 95% Mean	0.2856314
Lower 95% Mean	0.1867806
N	100

Figure C.1: Histograms of One Hundred Sets of Weighting Factors Generated by Modified Latin Hypercube Sampling with Dirichlet Distribution

Table C.1: One Hundred Sets of Weighting Factors Generated by Modified Latin Hypercube Sampling with Dirichlet Distribution and Design Criteria Values

Set	w_1	w_2	w_3	w_4	OEM	Fuel mass	Utilization/ (block time)	Passenger density
1	0.4333	0.0176	0.3719	0.1772	37895.82	13291.65	766.7125	1.4062
2	0.0269	0.9322	0.0407	0.0001	41428.86	11883.51	740.8892	1.4062
3	0.1942	0.798	0.0077	0	40342.85	11976.27	727.2155	1.4062
4	0.0231	0.0886	0.7454	0.1429	45184.84	13279.81	794.9339	1.3564
5	0.0017	0.008	0.088	0.9023	47650.82	13842.98	794.4662	1.2981
6	0	0.0498	0.0302	0.9199	46678.28	12649.03	740.8611	1.2981
7	0.0033	0.5557	0.0032	0.4379	44164.19	12152.47	732.6296	1.3553
8	0.7703	0.0002	0.2081	0.0215	37406.54	13364.7	751.3264	1.4062
9	0.0012	0.998	0.0002	0.0006	43910.36	11868.22	734.5968	1.4062
10	0.1196	0.0007	0.0129	0.8668	39874.9	14846.64	715.0679	1.2981
11	0.292	0.3525	0.3555	0.0001	39293.32	12329.55	755.8107	1.4062
12	0.268	0.4633	0.2516	0.0171	39574.7	12161.37	744.7317	1.4062
13	0.9818	0.001	0.0007	0.0165	37279.76	13292.7	731.715	1.4062
14	0.3792	0	0.6033	0.0174	38533.39	13335.2	780.4182	1.4062
15	0.0059	0.1591	0.216	0.619	47934.89	12751.37	756.5135	1.2981
16	0.0135	0.7722	0.2143	0	44079.11	11889.21	760.8023	1.4062
17	0.7002	0.2848	0.0142	0.0007	37908.16	12719.78	728.1471	1.4062
18	0	0.3933	0.3362	0.2705	45670.25	12112.59	762.0093	1.3776
19	0.3687	0.1794	0.0627	0.3892	38369.88	12955.2	731.1664	1.3837
20	0.421	0.0319	0	0.547	38938.25	14563.35	715.0679	1.3291
21	0.0119	0.4165	0.5715	0	45020.94	12109.69	776.6504	1.4062
22	0.0466	0.1694	0.7773	0.0067	42997.04	12475.15	787.5743	1.4062
23	0	0.3998	0.2686	0.3316	45367.03	12173.38	753.0088	1.3595
24	0.8032	0.0098	0.0005	0.1865	37236.32	13560.23	717.7176	1.4049
25	0.0329	0	0.6159	0.3513	42928.7	14725.35	796.8851	1.2981
26	0.5056	0.0003	0	0.494	38441.14	14419.48	715.0679	1.3472
27	0.0352	0.766	0.0418	0.157	41376.68	11879.14	738.6567	1.4062
28	0.0407	0.3944	0.1865	0.3785	44847.48	12221.38	742.5613	1.3474
29	0.5923	0.066	0	0.3417	37636.91	13958.4	717.9521	1.3792
30	0.0992	0.6287	0.0034	0.2687	41504.01	11928.74	726.0321	1.3968
31	0.1354	0.0126	0.6272	0.2248	41553.97	14255.61	796.8851	1.3382
32	0.7565	0.0003	0.2322	0.011	37444.99	13333.38	752.7839	1.4062
33	0.0067	0.0147	0.0053	0.9734	44316.32	12753.82	726.8117	1.2981
34	0.2406	0.0015	0.38	0.3778	40017.59	13979.94	768.9911	1.3306
35	0.6052	0.2998	0.004	0.091	38063.29	12632.53	729.4745	1.4062

Set	w_1	w_2	w_3	w_4	OEM	Fuel mass	Utilization/ (block time)	Passenger density
36	0.2621	0.7258	0.0121	0	39967.69	12039.81	727.7391	1.4062
37	0.9446	0.0535	0.0018	0.0001	37287.83	13221.59	736.5761	1.4062
38	0.9157	0	0.0667	0.0176	37275.52	13276.58	743.8319	1.4062
39	0.1839	0.3162	0.3597	0.1403	40042.67	12242.96	762.1718	1.4062
40	0.9923	0.0047	0	0.003	37276.06	13418.7	722.9659	1.4062
41	0.5623	0.3139	0.1176	0.0062	38037.84	12646.94	732.2106	1.4062
42	0.002	0.2278	0.0097	0.7604	45992.66	12608.39	731.5576	1.3016
43	0.2451	0.011	0.2933	0.4506	40118.47	14173.88	756.1211	1.3108
44	0.4803	0.2422	0.0302	0.2472	38049.27	12641.76	728.0511	1.4062
45	0.4546	0.2123	0.0078	0.3252	37937.91	12703.52	727.4846	1.4062
46	0.3963	0.0912	0.2477	0.2648	37698.98	13163.99	756.1622	1.4024
47	0.112	0.1415	0.4848	0.2617	41546.1	12918.15	782.1182	1.365
48	0.2459	0	0.0145	0.7396	39728.33	14790.34	715.0679	1.303
49	0.6045	0.0059	0.3867	0.0029	37765.89	13298.83	763.2272	1.4062
50	0.4188	0.1814	0.067	0.3327	37868.71	12785.68	730.2064	1.4042
51	0.4102	0.0687	0.1709	0.3502	37721.02	13559.16	736.8785	1.3838
52	0.5535	0.0122	0.1728	0.2615	37188.6	13456.12	739.1604	1.4059
53	0.8347	0.133	0.0001	0.0322	37436.49	13056.45	730.646	1.4062
54	0.5616	0.0789	0.0309	0.3286	37614.04	13795.63	723.0046	1.3817
55	0.0004	0.0186	0.9475	0.0335	46824.92	13544.52	796.8851	1.3418
56	0.0422	0.5609	0.3625	0.0344	43397.63	11862.87	766.3542	1.4211
57	0.3347	0.0001	0.6348	0.0304	38774.75	13482.08	784.6366	1.4062
58	0.4379	0.0001	0.5606	0.0013	38276.79	13334.99	775.4621	1.4062
59	0.0654	0.5942	0.2612	0.0792	42023.53	11841.26	760.0369	1.4211
60	0.4001	0.2158	0.0149	0.3691	38159.24	12696.21	729.1825	1.4007
61	0.1627	0.0297	0.355	0.4527	41623.78	14327.67	776.0346	1.291
62	0.0001	0.4	0.2372	0.3627	45285.95	12210.96	748.9781	1.3518
63	0.4422	0.4512	0.0064	0.1001	38831.82	12318.31	728.4445	1.4062
64	0.6081	0.1174	0.0758	0.1987	37436.74	13058.77	735.54	1.4062
65	0.0302	0.7622	0.1212	0.0865	42135.08	11780.44	753.4676	1.4211
66	0	0.5144	0.0031	0.4825	44201.8	12139.96	734.222	1.357
67	0.5574	0.1117	0.0302	0.3007	37501.71	13317.47	729.5176	1.396
68	0.5882	0.0914	0.3203	0.0001	37720.14	13145.69	760.5986	1.4062
69	0.0006	0.0052	0.9879	0.0062	44975.87	13130.95	796.8851	1.3877
70	0.2094	0.0095	0.7634	0.0176	39058	13491.32	793.4018	1.4211

Set	w_1	w_2	w_3	w_4	OEM	Fuel mass	Utilization/ (block time)	Passenger density
71	0.0581	0.4849	0.2125	0.2445	43262.31	11964.33	751.1284	1.3888
72	0.1297	0.575	0.0001	0.2953	40919.2	11987.3	726.6097	1.3976
73	0.3782	0.4645	0.1531	0.0043	39049.01	12253.68	734.0083	1.4062
74	0	0.5184	0.4477	0.0338	44682.3	11908.9	770.7272	1.4211
75	0.0001	0.9878	0	0.0121	43817.67	11768.92	734.1318	1.4211
76	0.003	0.409	0.543	0.045	44795.68	11999.31	776.1666	1.4211
77	0.2354	0.0022	0	0.7624	39874.87	14847.17	715.0679	1.2981
78	0.0085	0.0079	0.4356	0.5479	43304.69	14519.47	796.8851	1.2981
79	0.0006	0.0511	0.7425	0.2057	48529.98	14046.5	796.8851	1.294
80	0.1498	0.0178	0.0004	0.8319	39878.4	14800.82	715.0679	1.2981
81	0.032	0.6551	0.0096	0.3033	43319.39	11963.59	732.1487	1.3799
82	0.9906	0	0.0032	0.0061	36947.09	13279.07	722.8448	1.4211
83	0.3111	0.655	0.0151	0.0189	39678.05	12098.1	726.9415	1.4062
84	0.429	0.0698	0.5011	0	38191.41	13201.07	773.1662	1.4062
85	0.0602	0.0714	0.4478	0.4206	43950.06	13881.94	787.057	1.2857
86	0.0002	0.18	0.7782	0.0416	45402.92	12464.13	788.5213	1.4062
87	0.0232	0.3681	0.3557	0.253	44783.21	12006.7	765.5023	1.3986
88	0.0109	0.114	0.2348	0.6403	48694.94	13019.07	768.3219	1.2857
89	0.2501	0.388	0.1352	0.2267	39450.37	12155.04	736.9588	1.4062
90	0.0249	0.4659	0.3909	0.1184	44115.42	11902.98	770.0601	1.4211
91	0.0011	0.0019	0.997	0	42087.9	13094.97	796.8851	1.4062
92	0.0092	0.0696	0.4829	0.4382	48113.29	13439.78	787.8327	1.2981
93	0.0129	0.5555	0.1046	0.327	44036	12029.99	736.3019	1.3708
94	0.0543	0.7987	0.0003	0.1467	41044.31	11887.42	729.5942	1.4062
95	0	0	0.641	0.359	45729.69	16276.61	796.8851	1.2981
96	0.489	0.026	0.135	0.35	37935.02	13970.83	726.1814	1.3687
97	0.1027	0.631	0.0548	0.2115	40748.32	11920.6	732.8819	1.4062
98	0.458	0.0788	0.3237	0.1395	37476.49	13025.54	763.1866	1.4211
99	0.1384	0.0501	0.7969	0.0146	39310.86	13431.58	796.5039	1.4211
100	0.0008	0.083	0.9151	0.0011	44887.95	12965.04	796.8428	1.4062

C.2 Additional Untried Data Points for Evaluation of Surrogate Model Accuracy

The 84 sets of additional untried data points for weighting factors and the actual values of four design criteria obtained by the analysis tool (VAMPzero), are listed in Table C.2.

The predicted values of four design criteria for the 84 additional untried data points of weighting factors, generated by the developed surrogated models, are listed in Table C.3. The relative error is the difference between the predicted values and the actual values.

C.2 Additional Untried Data Points for Evaluation of Surrogate Model Accuracy

Table C.2: The 84 Sets of Weighting Factors and Predicted Design Criteria Values, Obtained by the Analysis Tool

Set	w_1	w_2	w_3	w_4	OEM	Fuel mass	Utilization/ (block time)	Passenger density
1	0.1	0.1	0.1	0.7	42084.41	13197.05	739.95	1.2981
2	0.1	0.1	0.2	0.6	42341.09	13318.56	758.24	1.2981
3	0.1	0.1	0.3	0.5	42713.95	13500.98	772.32	1.2981
4	0.1	0.1	0.4	0.4	43005.37	13719.57	782.12	1.2953
5	0.1	0.1	0.5	0.3	42366.91	13573.82	786.28	1.3211
6	0.1	0.1	0.6	0.2	40645.46	13125.85	788.74	1.3828
7	0.1	0.1	0.7	0.1	40033.5	12977.04	790.11	1.4063
8	0.1	0.2	0.1	0.6	43927.88	12671.43	730.95	1.3084
9	0.1	0.2	0.2	0.5	43598.42	12656.58	746.36	1.3177
10	0.1	0.2	0.3	0.4	43509.83	12689.26	758.81	1.3253
11	0.1	0.2	0.4	0.3	42588.78	12561.7	771.13	1.3608
12	0.1	0.2	0.5	0.2	40981.43	12278.35	780.33	1.4211
13	0.1	0.2	0.6	0.1	40925.32	12344.49	782.6	1.4211
14	0.1	0.3	0.1	0.5	43928.19	12369.03	730.05	1.334
15	0.1	0.3	0.2	0.4	43221.94	12245.3	742.12	1.3546
16	0.1	0.3	0.3	0.3	42486.52	12216.92	757.76	1.3748
17	0.1	0.3	0.4	0.2	41583.82	12152.49	770.24	1.4063
18	0.1	0.3	0.5	0.1	41441.19	12126.11	775.59	1.4211
19	0.1	0.4	0.1	0.4	43174.65	12094.29	728.62	1.3661
20	0.1	0.4	0.2	0.3	41990.3	11968.47	744.85	1.3933
21	0.1	0.4	0.3	0.2	41147.74	11925.88	761.51	1.4211
22	0.1	0.4	0.4	0.1	41798.45	12088.73	767.56	1.4063
23	0.1	0.5	0.1	0.3	42256.88	11999.73	731.87	1.3826
24	0.1	0.5	0.2	0.2	39024.56	12896.65	715.07	1.4063
25	0.1	0.5	0.3	0.1	41559.88	11999.51	759.42	1.4063
26	0.1	0.6	0.1	0.2	40476.15	11819.08	738.77	1.4211
27	0.1	0.6	0.2	0.1	40839.1	11844.64	750.5	1.4211
28	0.1	0.7	0.1	0.1	40851.2	11921.26	738.8	1.4063
29	0.2	0.1	0.1	0.6	40426.28	13838.65	734.25	1.302
30	0.2	0.1	0.2	0.5	40243.59	13875.71	751.87	1.3103
31	0.2	0.1	0.3	0.4	39820.91	13608.73	764.65	1.338
32	0.2	0.1	0.4	0.3	39255.84	13260.78	776.65	1.3791
33	0.2	0.1	0.5	0.2	38983.05	13082.04	784.59	1.4063
34	0.2	0.1	0.6	0.1	39106.72	13104.04	786.58	1.4063
35	0.2	0.2	0.1	0.5	40827.52	12772.78	729.9	1.3406
36	0.2	0.2	0.2	0.4	40109.26	12582.87	743.24	1.3676
37	0.2	0.2	0.3	0.3	39549.13	12492	759.68	1.394
38	0.2	0.2	0.4	0.2	39421.6	12542.33	770.99	1.4063
39	0.2	0.2	0.5	0.1	39560.15	12632.38	776.52	1.4063
40	0.2	0.3	0.1	0.4	40881.05	12372.48	733.15	1.3663
41	0.2	0.3	0.2	0.3	39707.49	12183.51	745.69	1.4028
42	0.2	0.3	0.3	0.2	39816.73	12221.34	757.16	1.4063

Set	w_1	w_2	w_3	w_4	OEM	Fuel mass	Utilization/ (block time)	Passenger density
43	0.2	0.3	0.4	0.1	39889.22	12319.01	765.05	1.4063
44	0.2	0.4	0.1	0.3	39801.33	12071.75	732.91	1.4063
45	0.2	0.4	0.2	0.2	39836.48	12105.08	744.44	1.4063
46	0.2	0.4	0.3	0.1	40000.09	12153.63	754.26	1.4063
47	0.2	0.5	0.1	0.2	39929.03	12047.41	733.31	1.4063
48	0.2	0.5	0.2	0.1	40009.31	12064.93	743.08	1.4063
49	0.2	0.6	0.1	0.1	40057.1	12022.29	730.83	1.4063
50	0.3	0.1	0.1	0.5	38916.96	13947.73	732.46	1.338
51	0.3	0.1	0.2	0.4	38448.22	13562.88	745.3	1.3634
52	0.3	0.1	0.3	0.3	38281.35	13243.09	761.36	1.3868
53	0.3	0.1	0.4	0.2	38260.71	13079.76	772.99	1.4063
54	0.3	0.1	0.5	0.1	38515.93	13101.09	778.1	1.4063
55	0.3	0.2	0.1	0.4	38850.98	12669.81	732.21	1.3856
56	0.3	0.2	0.2	0.3	38061.69	12431.43	747.69	1.4211
57	0.3	0.2	0.3	0.2	38512.93	12631.18	758.71	1.4063
58	0.3	0.2	0.4	0.1	38336.97	12605.89	767.34	1.4211
59	0.3	0.3	0.1	0.3	38880.9	12304.95	735.19	1.4063
60	0.3	0.3	0.2	0.2	38564.49	12217.94	743.17	1.4211
61	0.3	0.3	0.3	0.1	38979.96	12392.75	753.44	1.4063
62	0.3	0.4	0.1	0.2	39181.04	12214.6	731.38	1.4063
63	0.3	0.4	0.2	0.1	39203.72	12231.56	740.89	1.4063
64	0.3	0.5	0.1	0.1	39413.73	12155.35	730.77	1.4063
65	0.4	0.1	0.1	0.4	38316.4	13849	731.16	1.3576
66	0.4	0.1	0.2	0.3	37788.12	13306.97	748.09	1.3907
67	0.4	0.1	0.3	0.2	37520.62	12970.04	763.34	1.4211
68	0.4	0.1	0.4	0.1	38073.13	13124.42	769.75	1.4063
69	0.4	0.2	0.1	0.3	37995.52	12667.69	733.27	1.4063
70	0.4	0.2	0.2	0.2	38056.93	12691.44	745.71	1.4063
71	0.4	0.2	0.3	0.1	38170.97	12766.63	757.09	1.4063
72	0.4	0.3	0.1	0.2	38488.01	12441.14	732.61	1.4063
73	0.4	0.3	0.2	0.1	38178.45	12344.36	741.95	1.4211
74	0.4	0.4	0.1	0.1	38806.09	12326.12	731.02	1.4063
75	0.5	0.1	0.1	0.3	37559.28	13460.97	735.36	1.3904
76	0.5	0.1	0.2	0.2	37530.17	13128.12	752.63	1.4063
77	0.5	0.1	0.3	0.1	37460.73	13002.29	762.31	1.4211
78	0.5	0.2	0.1	0.2	37855.33	12753.19	735.58	1.4063
79	0.5	0.2	0.2	0.1	37894.05	12783.5	745.67	1.4063
80	0.5	0.3	0.1	0.1	38237.9	12546.39	731.61	1.4063
81	0.6	0.1	0.1	0.2	37381.97	13143.11	741.88	1.4063
82	0.6	0.1	0.2	0.1	37522.52	13131.66	752.42	1.4063
83	0.6	0.2	0.1	0.1	37754.16	12815.92	727.53	1.4063
84	0.7	0.1	0.1	0.1	37067.43	13008.91	742.96	1.4211

C.2 Additional Untried Data Points for Evaluation of Surrogate Model Accuracy

Table C.3: Predicted Design Criteria Values for the 84 Data Points and Relative Error(%), Generated by Surrogated Models

Set	OEM	Error	Fuel mass	Error	Utilization/ (block time)	Error	Passenger density	Error
1	44121.58	4.84	13446.44	1.89	747.15	0.97	1.2969	-0.09
2	44280.13	4.58	13428.06	0.82	757.79	-0.06	1.3013	0.25
3	43544.43	1.94	13384.24	-0.86	764.34	-1.03	1.3131	1.16
4	42864.55	-0.33	13339.60	-2.77	771.67	-1.34	1.3267	2.42
5	42642.29	0.65	13296.15	-2.05	781.36	-0.63	1.3405	1.47
6	42731.12	5.13	13233.22	0.82	791.68	0.37	1.3563	-1.91
7	42436.23	6.00	13107.52	1.01	797.58	0.95	1.3798	-1.88
8	43956.08	0.06	12838.06	1.32	739.86	1.22	1.3154	0.54
9	43988.50	0.89	12709.90	0.42	747.93	0.21	1.3246	0.52
10	43326.04	-0.42	12587.12	-0.80	757.14	-0.22	1.3416	1.23
11	42768.23	0.42	12505.32	-0.45	768.78	-0.30	1.3603	-0.04
12	42566.32	3.87	12477.45	1.62	780.87	0.07	1.3780	-3.03
13	42423.25	3.66	12493.81	1.21	788.09	0.70	1.3958	-1.78
14	43362.23	-1.29	12487.26	0.96	735.96	0.81	1.3401	0.46
15	43130.25	-0.21	12298.43	0.43	744.71	0.35	1.3549	0.03
16	42520.69	0.08	12150.75	-0.54	756.77	-0.13	1.3753	0.03
17	42182.57	1.44	12090.76	-0.51	769.86	-0.05	1.3940	-0.87
18	42216.58	1.87	12142.36	0.13	778.43	0.37	1.4075	-0.95
19	42521.28	-1.51	12257.33	1.35	735.33	0.92	1.3667	0.05
20	42073.31	0.20	12057.71	0.75	746.14	0.17	1.3854	-0.57
21	41683.05	1.30	11939.92	0.12	759.36	-0.28	1.4043	-1.18
22	41848.95	0.12	11961.45	-1.05	769.17	0.21	1.4152	0.64
23	41650.02	-1.44	12071.60	0.60	737.24	0.73	1.3909	0.60
24	41221.22	5.63	11911.81	-7.64	749.59	4.83	1.4091	0.20
25	41403.36	-0.38	11879.45	-1.00	760.43	0.13	1.4189	0.90
26	41000.79	1.30	11913.42	0.80	740.33	0.21	1.4086	-0.88
27	41013.05	0.43	11844.84	0.00	751.85	0.18	1.4190	-0.14
28	40861.53	0.03	11826.21	-0.80	742.65	0.52	1.4155	0.66
29	41324.67	2.22	13708.68	-0.94	736.82	0.35	1.3192	1.33
30	41237.89	2.47	13501.86	-2.69	747.17	-0.62	1.3307	1.55
31	40584.85	1.92	13287.95	-2.36	758.11	-0.86	1.3450	0.52
32	40105.49	2.16	13113.39	-1.11	770.99	-0.73	1.3589	-1.46
33	39991.43	2.59	13001.98	-0.61	783.84	-0.10	1.3728	-2.38
34	39886.02	1.99	12954.85	-1.14	791.42	0.62	1.3907	-1.11
35	40966.72	0.34	12975.72	1.59	735.64	0.79	1.3461	0.41
36	40704.91	1.49	12711.93	1.03	745.68	0.33	1.3601	-0.54
37	40141.52	1.50	12495.74	0.03	758.47	-0.16	1.3763	-1.27
38	39865.91	1.13	12384.54	-1.26	771.80	0.10	1.3905	-1.12
39	39919.20	0.91	12413.08	-1.74	780.14	0.47	1.4022	-0.29
40	40557.23	-0.79	12512.06	1.13	735.56	0.33	1.3736	0.53
41	40087.88	0.96	12246.50	0.52	746.64	0.13	1.3894	-0.96
42	39699.55	-0.29	12088.16	-1.09	759.61	0.32	1.4036	-0.19

Set	OEM	Error	Fuel mass	Error	Utilization/ (block time)	Error	Passenger density	Error
43	39831.10	-0.15	12105.36	-1.73	768.65	0.47	1.4111	0.35
44	40160.92	0.90	12203.28	1.09	736.44	0.48	1.3975	-0.63
45	39638.23	-0.50	11991.92	-0.93	748.08	0.49	1.4114	0.37
46	39697.14	-0.76	11952.29	-1.66	757.66	0.45	1.4170	0.76
47	39878.07	-0.13	11995.02	-0.43	737.57	0.58	1.4135	0.51
48	39642.97	-0.92	11894.54	-1.41	747.38	0.58	1.4192	0.92
49	39844.54	-0.53	11892.93	-1.08	737.59	0.93	1.4174	0.79
50	39545.60	1.62	13692.47	-1.83	732.58	0.02	1.3450	0.53
51	39232.20	2.04	13357.27	-1.52	744.43	-0.12	1.3600	-0.25
52	38696.10	1.08	13064.44	-1.35	758.65	-0.36	1.3738	-0.94
53	38467.08	0.54	12882.19	-1.51	773.07	0.01	1.3851	-1.51
54	38526.61	0.03	12856.13	-1.87	782.21	0.53	1.3964	-0.70
55	39123.68	0.70	12922.42	1.99	734.82	0.36	1.3752	-0.75
56	38651.65	1.55	12598.63	1.35	747.04	-0.09	1.3897	-2.21
57	38286.88	-0.59	12395.73	-1.86	760.76	0.27	1.4008	-0.38
58	38408.61	0.19	12392.89	-1.69	770.23	0.38	1.4066	-1.02
59	38928.34	0.12	12433.42	1.04	736.18	0.13	1.4004	-0.42
60	38370.94	-0.50	12182.30	-0.29	748.27	0.69	1.4117	-0.66
61	38368.71	-1.57	12135.52	-2.08	757.93	0.60	1.4145	0.59
62	38907.81	-0.70	12133.32	-0.67	736.54	0.71	1.4163	0.72
63	38525.01	-1.73	12016.91	-1.75	746.13	0.71	1.4190	0.91
64	39045.87	-0.93	11990.04	-1.36	735.17	0.60	1.4188	0.89
65	38374.27	0.15	13505.81	-2.48	732.14	0.13	1.3716	1.03
66	37918.26	0.34	13121.48	-1.39	746.03	-0.28	1.3864	-0.31
67	37598.66	0.21	12860.02	-0.85	761.20	-0.28	1.3962	-1.75
68	37735.10	-0.89	12811.47	-2.38	771.94	0.28	1.4016	-0.33
69	38004.05	0.02	12786.20	0.94	735.74	0.34	1.3998	-0.46
70	37471.11	-1.54	12497.18	-1.53	749.15	0.46	1.4100	0.27
71	37469.83	-1.84	12433.40	-2.61	759.91	0.37	1.4116	0.38
72	38039.89	-1.16	12359.36	-0.66	736.81	0.57	1.4172	0.78
73	37609.04	-1.49	12232.99	-0.90	747.37	0.73	1.4183	-0.19
74	38313.51	-1.27	12155.45	-1.38	735.24	0.58	1.4197	0.96
75	37546.53	-0.03	13256.72	-1.52	733.85	-0.21	1.3955	0.37
76	37097.22	-1.15	12921.64	-1.57	749.09	-0.47	1.4063	0.00
77	37158.98	-0.81	12821.03	-1.39	761.62	-0.09	1.4083	-0.90
78	37330.91	-1.39	12675.06	-0.61	737.40	0.25	1.4161	0.70
79	36951.65	-2.49	12534.73	-1.95	749.75	0.55	1.4172	0.78
80	37602.16	-1.66	12397.90	-1.18	737.09	0.75	1.4202	0.99
81	36944.18	-1.17	13053.21	-0.68	736.68	-0.70	1.4130	0.48
82	36716.17	-2.15	12884.92	-1.88	751.34	-0.14	1.4155	0.66
83	36973.26	-2.07	12697.02	-0.93	739.39	1.63	1.4202	0.99
84	36594.99	-1.27	13003.29	-0.04	740.22	-0.37	1.4197	-0.10

C.3 Typical Weighting Scenarios for Business Aircraft Evaluation

In the business aircraft evaluation problem, 84 sets of weighting factors generated from eleven levels of experimental design and the evaluation results using ELECTRE I are summarized in Table C.4, where D represents the alternative is dominated, and N represents the alternative is non-dominated.

Table C.4: The 84 Sets of Weighting Factors for Business Aircraft Evaluation, D: Dominated, N: Non-dominated

Set	w_1	w_2	w_3	w_4	w_5	w_6	w_7	A_1	A_2	A_3	A_4
1	0.4	0.1	0.1	0.1	0.1	0.1	0.1	D	D	N	D
2	0.3	0.1	0.1	0.1	0.1	0.1	0.2	D	D	N	D
3	0.3	0.1	0.1	0.1	0.1	0.2	0.1	D	D	N	D
4	0.3	0.1	0.1	0.1	0.2	0.1	0.1	N	D	N	D
5	0.3	0.1	0.1	0.2	0.1	0.1	0.1	N	D	N	D
6	0.3	0.1	0.2	0.1	0.1	0.1	0.1	N	D	N	D
7	0.3	0.2	0.1	0.1	0.1	0.1	0.1	D	D	N	D
8	0.2	0.1	0.1	0.1	0.1	0.1	0.3	D	D	N	D
9	0.2	0.1	0.1	0.1	0.1	0.2	0.2	D	D	N	D
10	0.2	0.1	0.1	0.1	0.1	0.3	0.1	D	N	N	D
11	0.2	0.1	0.1	0.1	0.2	0.1	0.2	N	D	N	D
12	0.2	0.1	0.1	0.1	0.2	0.2	0.1	N	D	N	D
13	0.2	0.1	0.1	0.1	0.3	0.1	0.1	N	D	D	D
14	0.2	0.1	0.1	0.2	0.1	0.1	0.2	D	D	N	D
15	0.2	0.1	0.1	0.2	0.1	0.2	0.1	D	N	N	D
16	0.2	0.1	0.1	0.2	0.2	0.1	0.1	N	D	D	N
17	0.2	0.1	0.1	0.3	0.1	0.1	0.1	N	N	N	N
18	0.2	0.1	0.2	0.1	0.1	0.1	0.2	D	D	N	N
19	0.2	0.1	0.2	0.1	0.1	0.2	0.1	N	D	N	N
20	0.2	0.1	0.2	0.1	0.2	0.1	0.1	N	D	D	N
21	0.2	0.1	0.2	0.2	0.1	0.1	0.1	N	D	N	N
22	0.2	0.1	0.3	0.1	0.1	0.1	0.1	N	D	D	N
23	0.2	0.2	0.1	0.1	0.1	0.1	0.2	D	D	N	D
24	0.2	0.2	0.1	0.1	0.1	0.2	0.1	D	N	N	D
25	0.2	0.2	0.1	0.1	0.2	0.1	0.1	N	D	D	D
26	0.2	0.2	0.1	0.2	0.1	0.1	0.1	D	N	N	D
27	0.2	0.2	0.2	0.1	0.1	0.1	0.1	N	D	N	N
28	0.2	0.3	0.1	0.1	0.1	0.1	0.1	D	N	N	D
29	0.1	0.1	0.1	0.1	0.1	0.1	0.4	D	D	N	N
30	0.1	0.1	0.1	0.1	0.1	0.2	0.3	D	D	N	N
31	0.1	0.1	0.1	0.1	0.1	0.3	0.2	D	N	N	N
32	0.1	0.1	0.1	0.1	0.1	0.4	0.1	D	N	N	N
33	0.1	0.1	0.1	0.1	0.2	0.1	0.3	D	D	N	N
34	0.1	0.1	0.1	0.1	0.2	0.2	0.2	N	D	N	N
35	0.1	0.1	0.1	0.1	0.2	0.3	0.1	N	N	N	N
36	0.1	0.1	0.1	0.1	0.3	0.1	0.2	N	D	D	N
37	0.1	0.1	0.1	0.1	0.3	0.2	0.1	N	D	D	D
38	0.1	0.1	0.1	0.1	0.4	0.1	0.1	N	D	D	D
39	0.1	0.1	0.1	0.2	0.1	0.1	0.3	D	D	N	N
40	0.1	0.1	0.1	0.2	0.1	0.2	0.2	D	N	N	N

Set	w_1	w_2	w_3	w_4	w_5	w_6	w_7	A_1	A_2	A_3	A_4
41	0.1	0.1	0.1	0.2	0.1	0.3	0.1	D	N	N	N
42	0.1	0.1	0.1	0.2	0.2	0.1	0.2	N	D	N	N
43	0.1	0.1	0.1	0.2	0.2	0.2	0.1	N	N	D	N
44	0.1	0.1	0.1	0.2	0.3	0.1	0.1	N	D	D	D
45	0.1	0.1	0.1	0.3	0.1	0.1	0.2	N	N	N	N
46	0.1	0.1	0.1	0.3	0.1	0.2	0.1	N	N	N	N
47	0.1	0.1	0.1	0.3	0.2	0.1	0.1	N	N	D	N
48	0.1	0.1	0.1	0.4	0.1	0.1	0.1	N	N	N	N
49	0.1	0.1	0.2	0.1	0.1	0.1	0.3	D	D	N	N
50	0.1	0.1	0.2	0.1	0.1	0.2	0.2	D	D	N	N
51	0.1	0.1	0.2	0.1	0.1	0.3	0.1	N	N	N	N
52	0.1	0.1	0.2	0.1	0.2	0.1	0.2	N	D	D	N
53	0.1	0.1	0.2	0.1	0.2	0.2	0.1	N	D	D	N
54	0.1	0.1	0.2	0.1	0.3	0.1	0.1	N	D	D	D
55	0.1	0.1	0.2	0.2	0.1	0.1	0.2	N	D	D	N
56	0.1	0.1	0.2	0.2	0.1	0.2	0.1	N	N	D	N
57	0.1	0.1	0.2	0.2	0.2	0.1	0.1	N	D	D	N
58	0.1	0.1	0.2	0.3	0.1	0.1	0.1	N	N	D	N
59	0.1	0.1	0.3	0.1	0.1	0.1	0.2	N	D	D	N
60	0.1	0.1	0.3	0.1	0.1	0.2	0.1	N	D	D	N
61	0.1	0.1	0.3	0.1	0.2	0.1	0.1	N	D	D	N
62	0.1	0.1	0.3	0.2	0.1	0.1	0.1	N	D	D	N
63	0.1	0.1	0.4	0.1	0.1	0.1	0.1	N	D	D	N
64	0.1	0.2	0.1	0.1	0.1	0.1	0.3	D	D	N	N
65	0.1	0.2	0.1	0.1	0.1	0.2	0.2	D	N	N	N
66	0.1	0.2	0.1	0.1	0.1	0.3	0.1	D	N	N	N
67	0.1	0.2	0.1	0.1	0.2	0.1	0.2	N	D	N	N
68	0.1	0.2	0.1	0.1	0.2	0.2	0.1	N	N	D	N
69	0.1	0.2	0.1	0.1	0.3	0.1	0.1	N	D	D	D
70	0.1	0.2	0.1	0.2	0.1	0.1	0.2	D	N	N	N
71	0.1	0.2	0.1	0.2	0.1	0.2	0.1	D	N	N	N
72	0.1	0.2	0.1	0.2	0.2	0.1	0.1	N	N	D	N
73	0.1	0.2	0.1	0.3	0.1	0.1	0.1	N	N	N	N
74	0.1	0.2	0.2	0.1	0.1	0.1	0.2	D	D	D	N
75	0.1	0.2	0.2	0.1	0.1	0.2	0.1	N	N	D	N
76	0.1	0.2	0.2	0.1	0.2	0.1	0.1	N	D	D	N
77	0.1	0.2	0.2	0.2	0.1	0.1	0.1	N	N	D	N
78	0.1	0.2	0.3	0.1	0.1	0.1	0.1	N	D	D	N
79	0.1	0.3	0.1	0.1	0.1	0.1	0.2	D	N	N	N
80	0.1	0.3	0.1	0.1	0.1	0.2	0.1	D	N	N	N
81	0.1	0.3	0.1	0.1	0.2	0.1	0.1	N	N	D	N
82	0.1	0.3	0.1	0.2	0.1	0.1	0.1	D	N	N	N
83	0.1	0.3	0.2	0.1	0.1	0.1	0.1	N	N	D	N
84	0.1	0.4	0.1	0.1	0.1	0.1	0.1	D	N	N	N