Proceedings of the Second (1992) International Offshore and Polar Engineering Conference
San Francisco, USA, 14-19 June 1992

ISBN 1-880653-00-1 (Set); ISBN 1-880653-02-8 (Vol II)

DYNAMIC CABLE STIFFNESS AND DYNAMIC INTERACTION BETWEEN CABLE AND BEAM

Mathias Kutterer and Uwe starossck
University of Stuttgart
Stuttgart, Germany

ABSTRACT: Closed-form expressions for the dynamic stiffness functions of a damped cable and a damped, axially prestressed beam element are given. A dynamic damping phenomenon of a single cable is described. For a multiply-guyed mast, dynamic response functions are evaluated. Influence of cable dynamics is investigated. In case of internal resonance, the dynamic stiffness function of a cable decreases and may become negative. Nevertheless, the response function for the composed system is favourably influenced by cable vibration. This effect is quantified by means of a stochastic analysis of the system response to random excitation. The study is restricted to linear theory; validity of linear theory is discussed.

KEY WORDS: Dynamics, Dynamic stiffness, Cable, Beam, Guyed mast, Damping, Random vibration.

Nomenclature

A	effective cross-sectional area of cable
c	damping force per unit length and velocity
d	cable sag perpendicular to cable chord
D	compression force
E	Young's modulus of elasticity of cable
EI	flexural rigidity
g	gravitational acceleration (decreased by buoyancy effect)
i	imaginary unit
K	horizontal dynamic stiffness of single cable according to eq. (1)
$\boldsymbol{k}$	local stiffness matrix
$\boldsymbol{K}$	global stiffness matrix
L_e	cable parameter according to eq. (4)
l	length of cable chord, or length of beam element respectively
$\bar{l}$	horizontal span of cable
m	cable mass per unit length (increased by virtual mass effect due to fluid), or beam element mass per unit length
P	nodal forces
$\boldsymbol{P}$	global load vector
q	uniform load applied to beam element
$\boldsymbol{r}$	global displacement vector
T_Θ	static cable tension at point where cable is parallel to chord
γ	cable weight per unit length and cross sectional area
ϵ	cable element parameter according to eq. (3), or axial force parameter of beam element according to eq. (10)
Θ	angle of inclination of cable chord (measured from a horizontal line)
κ	Ω_c-dependent auxiliary term defined by eq. (5)
λ^2	fundamental cable parameter according to eq. (2)
ξ	dimensionless damping parameter defined by eqs. (7)
σ	initial cable tension (pretension)
Ω_c	dimensionless frequency-damping parameter defined by eq. (6), or eq. (11) respectively
ω	circular frequency of harmonic motion
ω_c	frequency-damping parameter defined by eqs. (7)

0. Survey

Dynamic stiffness matrices provide a convenient way for carrying out the linear dynamic analysis of composed systems (dynamic direct-stiffness analysis). In this study, a closed-form expression for the dynamic stiffness function of an extensible sagging cable is given. Dynamic stiffness parts induced by motion in chord direction and perpendicular to chord (in plane and out of plane) as well as the effect of viscous damping are considered. Influence of cable parameters (static cable tension, angle of inclination, damping etc.) can be investigated easily. Dynamic effects due to motion in chord direction and perpendicular to chord may cancel each other out. By tuning the cable parameters appropriately, this mechanism can be used to limit cable vibrations.

The dynamic stiffness formulation of a beam element with axial force and viscous damping is also given. In the analysis of composed systems, dynamic matrices of beam and cable elements lead to a global dynamic stiffness matrix. Dynamic loads are considered by means of a dynamic load vector. The eigenvalue problem as well as the response problem can be solved. The system's response is evaluated for a single frequency. By repeating this procedure for a set of frequencies, a frequency-response function (admittance function) is obtained. This function is suitable for stochastic analysis of random vibrations.

For a multiply-guyed mast, response functions are evaluated using three different methods. The first method neglegts the influence of cable vibration. For the second method, cable dynamics is included into analysis. For the last method, system asymmetrie caused by external static loading is also considered.

The influence of cable dynamics is notable when there is internal resonance, a phenomenon which can be characterized by the occurence of strongly coupled modes of vibration, that is, of modes with both cable and beam displacements. In the range of internal resonance, the dynamic stiffness function of a cable decreases and may become negative. Nevertheless, the response function for the composed system is favourably influenced by cable vibration: The 'resonance cone' is splitted into two smaller cones. The limiting effect of damping increases.

The favourable influence of cable dynamics is quantified by means of a stochastic analysis of the system response to random excitation. The validity of linear dynamic theory is discussed.

1. Dynamic Stiffness of Sagging Cable

Static analysis of a mechanical system usually requires knowledge of the load-deformation behavior of the system elements. This behavior can be described in compact form by stiffness matrices. When limited to the steady-state response (harmonic oscillation), it is possible to transfer this concept to the investigation of dynamic mechanisms, which implies the development of dynamic impedance or stiffness matrices (Clough and Penzien, 1975). Elements of such matrices are functions of the frequency of motion but invariable in time.

The dynamic stiffness matrix of an extensible, flexible and sagging cable was presented by Starossek (1991a,b). This matrix can be utilized for the dynamic analysis of compound systems such as guyed masts. The cable is considered as a continuum. Only small displacements are admissible (linear theory). Viscous damping due to external fluid forces is taken into consideration. Trigonometrical solution functions with complex arguments are utilized, which implies a substantial simplification in the analysis of damped vibrations.

For this study, the horizontal dynamic stiffness at the upper end of an inclined cable pinned at its lower end (see Fig. 1) is of special interest. By following paper Starossek (1991a), it is found that

$$K := \frac{F}{\Delta} = \frac{EA}{L_e}\cos^2\Theta \, \frac{\left[1 + \frac{1}{2}\epsilon \tan\Theta \, (\kappa - 1)\right]^2}{1 + \frac{\lambda^2}{\Omega_c^2}(\kappa - 1)} + \frac{T_\Theta}{l}\sin^2\Theta \, \Omega_c \cot \Omega_c \tag{1}$$

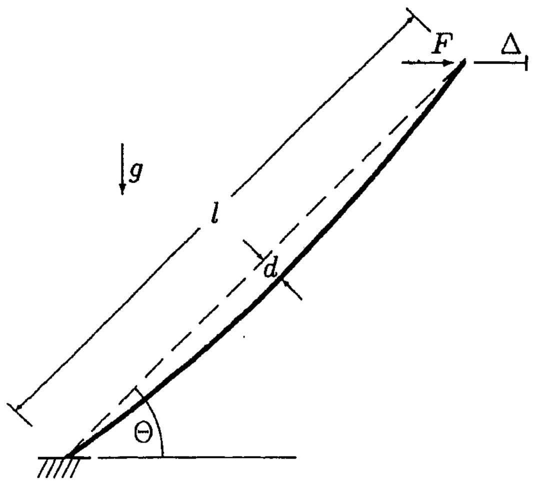


Figure 1: Single cable with horizontally moveable upper support.

where

$$\lambda^2 = \left(\frac{mgl}{T_\Theta}\right)^2 \frac{EA\, l}{T_\Theta L_e} \cos^2\Theta = \epsilon^2 \frac{EA}{T_\Theta} \frac{l}{L_e} \ , \tag{2}$$

$$\epsilon = \frac{mgl}{T_\Theta}\cos\Theta = \frac{8d}{l} \ , \tag{3}$$

$$L_e \simeq l\,(1 + \tfrac{1}{8}\epsilon^2) \tag{4}$$

are cable parameters, and

$$\kappa = \kappa(\Omega_c) = \frac{\tan(\Omega_c/2)}{\Omega_c/2} \tag{5}$$

is an auxiliary function which is dependent solely on the dimensionless frequency-damping parameter

$$\Omega_c = \omega_c l \sqrt{\frac{m}{T_\Theta}} \ , \tag{6}$$

where

$$\omega_c = \omega\sqrt{1 - 2\xi i} \qquad \text{and} \qquad \xi = \frac{c}{2m\omega} \ , \tag{7}$$

and ω is the circular frequency. Finally, T_Θ is static tension in the cable at the point where the cable is parallel to chord and corresponds approximately to the average cable tension.

Dynamic Damping Phenomenon of Single Cable

The internal detuning, or 'system damping' occurs in certain kinds of lightweight guyed structures. By examining a very specific phenomenon of cable dynamics, this effect can be investigated. Figure 2 shows the dimensionless stiffness-frequency diagrams of an inclined undamped cable, 30 m of horizontal span and prestressed by 100, 200, 300 and 400 MN/m^2 respectively.

The cable S30/300 apparently loses its first natural frequency peak in stiffness. The enlarged section shows that two phase changes occur very close together. It can be explained by the fact, that a horizontal excitation of the inclined cable implies an excitation both parallel and perpendicular to the chord, which cause different responses that appear to cancel each other out.

The equations of dynamic stiffness and deflections as given by Starossek (1991a) allow us to comprehend this process. As shown by Kutterer (1991) the cable parameters may be tuned with a view to control oscillations at the first natural frequency. The cable tension is then found thus

$$\sigma^2 = \frac{2\gamma \bar{l} E}{\Omega_1^2 \tan\Theta}\,, \tag{8}$$

with $\gamma = mg/A$ and $\bar{l} = l\,\cos\Theta$.

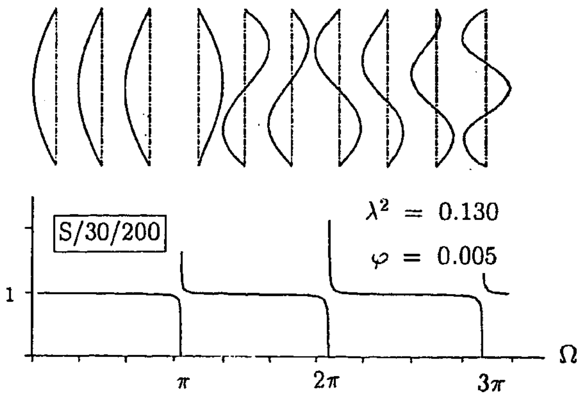
The following approximation for Ω_1 from Bauer [cited in (Starossek,1991a)] can be used

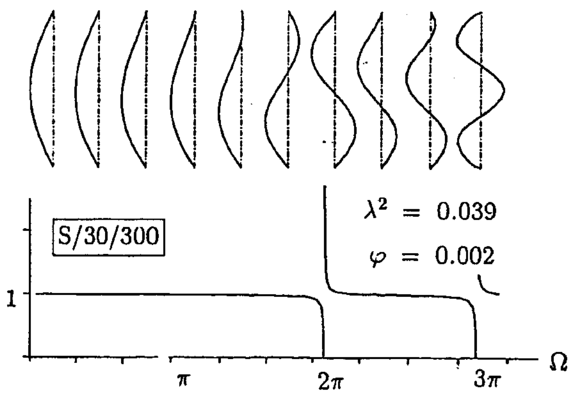
$$\Omega_1 \simeq \pi\,\sqrt{1+8\lambda^2/\pi^4}\,. \tag{9}$$

Note $\Omega_1 \simeq \pi$ for cables with small λ^2. The critical pretension for the cable considered above is then

$$\sigma = \sqrt{\frac{2\cdot 0.085\cdot 30\cdot 160\,000}{\pi^2\cdot 1}} = 288\ \mathrm{MN/m^2}\,.$$

As equation (8) shows, there will be a certain limit for this kind of tuning as the pretension cannot exceed a certain order. With a presumed pretension of 300 MN/m^2 the tuning is limited to a horizontal length for steel-cables of about $\bar{l} \simeq 30\,\mathrm{m}\cdot\tan\Theta$, and for a lightweight composite tensile member of about $\bar{l} \simeq 400\,\mathrm{m}\cdot\tan\Theta$.





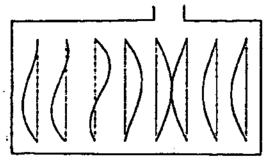
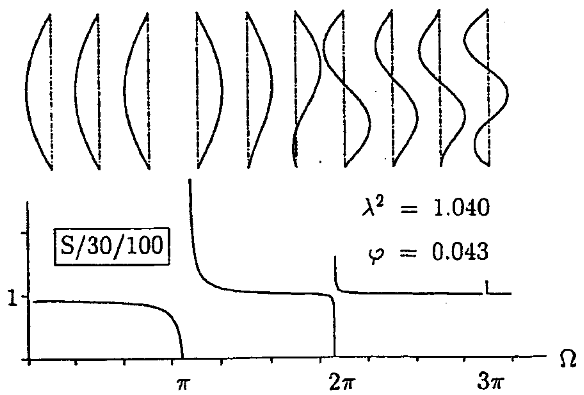
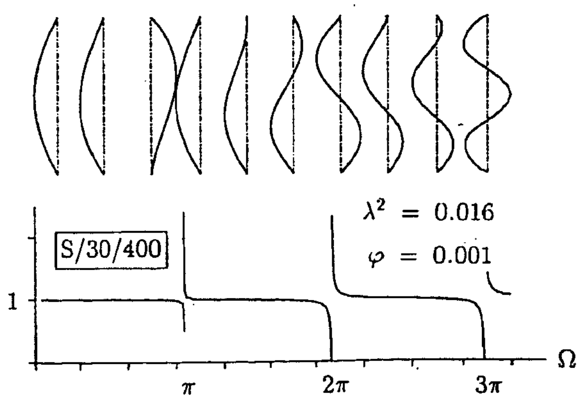


Figure 2:
Stiffness-frequency diagrams, horizontal length 30 m pretension 100-400 MN/m^2

The dynamic stiffness is related to the static stiffness of a cable without sag.





2. Dynamic Stiffness of Beam Element

The dynamic stiffness of the beam element with axial force, again with viscous damping and in complex notation, is given by Starossek (1991a) and Kutterer (1991).

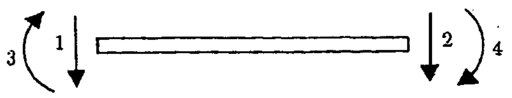


Figure 3: Degrees of freedom

The element parameters are: axial compression force D, length l, flexural-rigidity EI, mass per unit length m, and damping force per unit length c. (The notations l, m, c, ω_c correspond to those for cable-elements.)

With the characteristic parameter

$$\epsilon := l\sqrt{\frac{D}{EI}} \; . \tag{10}$$

and the dimensionless frequency

$$\Omega_c = l^2\,\omega_c\,\sqrt{\frac{m}{EI}} \tag{11}$$

and further expressions

$$\Phi := \sqrt{-\epsilon^2/2 + \sqrt{(\epsilon^2/2)^2 + \Omega_c^2}} \tag{12}$$

$$\Psi := \sqrt{+\epsilon^2/2 + \sqrt{(\epsilon^2/2)^2 + \Omega_c^2}}\,, \tag{13}$$

$$\mathrm{s} := \sin\Phi,\ \mathrm{c} := \cos\Phi,\ \mathrm{S} := \sinh\Psi,\ \mathrm{C} := \cosh\Psi, \tag{14}$$

$$\Pi := \frac{\epsilon^2/2}{\Omega_c}\,,\quad \Lambda := \sqrt{\Pi^2+1} \tag{15}$$

the dynamic stiffness matrix is

$$\boldsymbol{k} = \frac{EI}{l^2}\cdot\frac{1}{1-\mathrm{Cc}-\Pi\mathrm{Ss}} \; . \tag{16}$$

$$\begin{bmatrix} (\mathrm{Cs}\Phi+\mathrm{Sc}\Psi)\Omega_c\Lambda/l & [(1-\mathrm{Cc})\Pi+\mathrm{Ss}]\Omega_c & -(\mathrm{s}\Phi+\mathrm{S}\Psi)\Omega_c\Lambda/l & (\mathrm{C}-\mathrm{c})\Omega_c\Lambda \\ [(1-\mathrm{Cc})\Pi+\mathrm{Ss}]\Omega_c & (\mathrm{Cs}\Psi-\mathrm{Sc}\Phi)\Lambda\,l & (\mathrm{C}-\mathrm{c})\Omega_c\Lambda & (\mathrm{S}\Phi-\mathrm{s}\Psi)\Lambda\,l \\ -(\mathrm{s}\Phi+\mathrm{S}\Psi)\Omega_c\Lambda/l & (\mathrm{C}-\mathrm{c})\Omega_c\Lambda & (\mathrm{Cs}\Phi+\mathrm{Sc}\Psi)\Omega_c\Lambda/l & -[(1-\mathrm{Cc})\Pi+\mathrm{Ss}]\Omega_c \\ (\mathrm{C}-\mathrm{c})\Omega_c\Lambda & (\mathrm{S}\Phi-\mathrm{s}\Psi)\Lambda\,l & -[(1-\mathrm{Cc})\Pi+\mathrm{Ss}]\Omega_c & (\mathrm{Cs}\Psi-\mathrm{Sc}\Phi)\Lambda\,l \end{bmatrix}$$

Nodal Forces

The dissection of a uniform dynamic element loading into nodal forces $P_1 - P_4$, again with consideration of viscous damping and in complex notation, has been derived by Kutterer (1991). With the same abbreviations as before, this leads to

$$P_1 \;=\; \frac{\Lambda\,[\Psi\mathrm{S}(\mathrm{c}-1)+\Phi\mathrm{s}(\mathrm{C}-1)]}{\Omega_c(\mathrm{Cc}-\Pi\mathrm{Ss}-1)}\,q\,l \;=\; P_3 \tag{17}$$

$$P_2 \;=\; \frac{\Pi(\mathrm{Cc}-1)-\Lambda(\mathrm{C}-\mathrm{c})+\mathrm{Ss}}{\Omega_c(\mathrm{Cc}-\Pi\mathrm{Ss}-1)}\,q\,l^2 \;=\; -P_4 \tag{18}$$

The transition to the element with no axial loading leads to $\Phi = \Psi = \sqrt{\Omega}$, $\Pi = 0$ and $\Lambda = 1$.

As $\Omega \to 0$ the well-known static values result:

$$\lim_{\Omega\to 0} P_1 = \frac{ql}{2}\;,\quad \lim_{\Omega\to 0} P_2 = \frac{ql^2}{12}\,.$$

3. Global Dynamic Stiffness Matrix

As usual in the static direct stiffness method, the local element stiffness functions are transformed to global degrees of freedom r_j and added into the system matrix $\boldsymbol{K}$. Thus the dynamic stiffness of a cable element or a cable-cluster can be inserted exactly in the same way as dynamic beam-elements or any other contribution to the stiffness system.

The viscous damping, which can be chosen individually for each element, as well as the phasing of the system quantities, are taken into account by complex calculation.

The nodal forces due to element loading or discrete nodal forces constitute the load vector $\boldsymbol{P}$.

Only simple harmonic vibrations are considered. In this steady state the time dependencies cancel out and a homogenous linear equation system

$$\boldsymbol{K}\cdot\boldsymbol{r} = \boldsymbol{P} \tag{19}$$

is obtained. This represents a classical response problem which differs from the static case only in the frequency-dependence and the complex nature of the coefficients.

With the homogenous equation system

$$\boldsymbol{K}\cdot\boldsymbol{r} = 0 \tag{20}$$

the eigenvalue problem is established.

Calculation of Mechanical Admittance Functions

Carrying out the analyses in close succession and plotting any system-response y (force or displacement quantity) against the frequency, a mechanical admittance function $y(f)$ is obtained, valid only for this system-quantity and the underlying dynamic load case.

4. Examination of a Simple System

The mechanical admittance has been established for a simply guyed mast (figure 3) with a triple-cluster at the top and examined with respect to its inner resonance.

The following data was assumed:

Cable diameter	0.044 m ,
Cable pretension	$\sigma = 400$ MN/m^2 ,
Cable modulus	$E_s = 160\,000$ MN/m^2 ,
Damping parameter	$\xi = 0.05$,
Flexural stiffnesses of mast	$EI = 12207$ MN/m^2 (stiff mast),
	$EI = 3612$ MN/m^2 (weak mast),
Mass per unit length of mast	$m = 190$ kg/m ,
Compressive force	$D = 3\sigma A \sin\Theta + G = 1.67$ MN .

With these values and an assumed uniform dynamic load the mechanical admittance of the displacement of the top was determined. Figure 5 shows the real and the imaginary parts of the displacement and the dynamic spring stiffness.
The left-hand side of the figure shows the system with the stiff mast and the right-hand side the more flexible one, plotted against circular frequency.

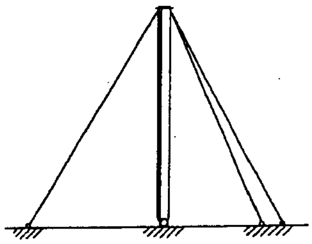


Figure 4:
Simply guyed mast

The upper diagrams do not consider cable-dynamics, whilst the lower ones do. This is clearly shown by the frequency-dependence of the cable stiffness.

In both examples there is a natural frequency of the cables meeting that of the total system (analysed without cable dynamics) so that the dynamic interaction can be clearly observed.

The assumption of a dangerous internal vibration excitation is not justified. On the contrary, the changing of the cable stiffness moves the resonance spectrum, so that no resonance can occur at the supposed critical point.

Even if the cable stiffness approaches zero, the beam has no tendency to start oscillation, because the natural frequency has now moved elsewhere.

Moving with the exciting frequency towards the critical point, the natural frequency of the system goes down as the cable stiffness decreases, and 'comes to meet' the excitation frequency. Descending from higher frequencies the process is reversed, so that eventually the resonance-hill is cut in two.

The dynamic interaction of the structural elements produces a reciprocal disturbance, which seems to be associated with the phenomenon of dynamic damping (as used, for example, in tuned mass dampers). A methodical exploitation of this effect is difficult, because the natural frequencies of the cables change significantly under load. Nevertheless one can suppose that a complex system with a dense spectrum of resonances might profit from this favourable effect.

5. Examination of a Complex System

To get a realistic idea of the dynamics of guyed systems a mast with three cable-clusters is investigated. The same cables are used as before, but with a pretension of $\sigma = 300$ MN/m^2. The flexural rigidity of the mast is increased to $EI = 37548$ MN/m^2 and the mass per unit length is $m = 190$ kg/m. The compression in the mast and the wind loading are assumed to be uniform for each section.

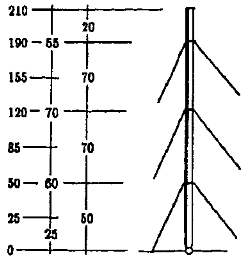


Figure 6:
Multiply guyed mast

The diagrams of figure 7 are built up as before, with the circular-frequency ω on the abscissa. The upper diagrams show the deflected shapes, with broken lines drawn at the levels of each anchorage. The central graph plots the deflection of the highest anchoring-point. The path of the determinante of the dynamic stiffness matrix (its real part) is drawn dimensionlessly to show the natural frequencies as zero-transits of the stiffness and also to give a qualitative idea of the systems characteristic. The lower diagram shows the equivalent spring-stiffness of the cable clusters. The topmost anchoring has the lowest stiffness.

The initial system (F0/A) was calculated as a beam on flexible supports, i.e. without consideration of cable-dynamics and thus with constant spring-stiffness. The first two resonance-modes are close (see zero-transits of determinante) but distinct, the first mode is dominated by the rigid-body behaviour and the second mode is that of an elastic supported beam.

The influence of cable dynamics causes disturbances in the path of the stiffness-determinant (F0/B), which is even more clear with the pretension being reduced (F0/C). The cable vibrations now produce further zero-transits of the determinant and various hybrid modes are excited.

A complete complexity is produced by an asymmetrical pretension as caused by the wind-load (F0/D). This picture of confusion gives a realistic idea of the dynamic behaviour of a guyed lightweight mast. Such systems oscillate within an extensive resonance-band, which moreover is changing continually.

The path of the system-response shows the basic modes are still dominant and are clearly disturbed in a favourable way by the cable dynamics. Even a spring stiffness becoming negative does not cause a dominating vibration mode of large amplitude.

6. Evaluation by Spectral Method

The determined mechanical admittance functions have been evaluated with the spectrum of atmospheric turbulence of Davenport (1962) as an entrance function and with a constant aerodynamic admittance function. Assuming that the windspeed is fully correlated over the height there is only one load-process to be considered.

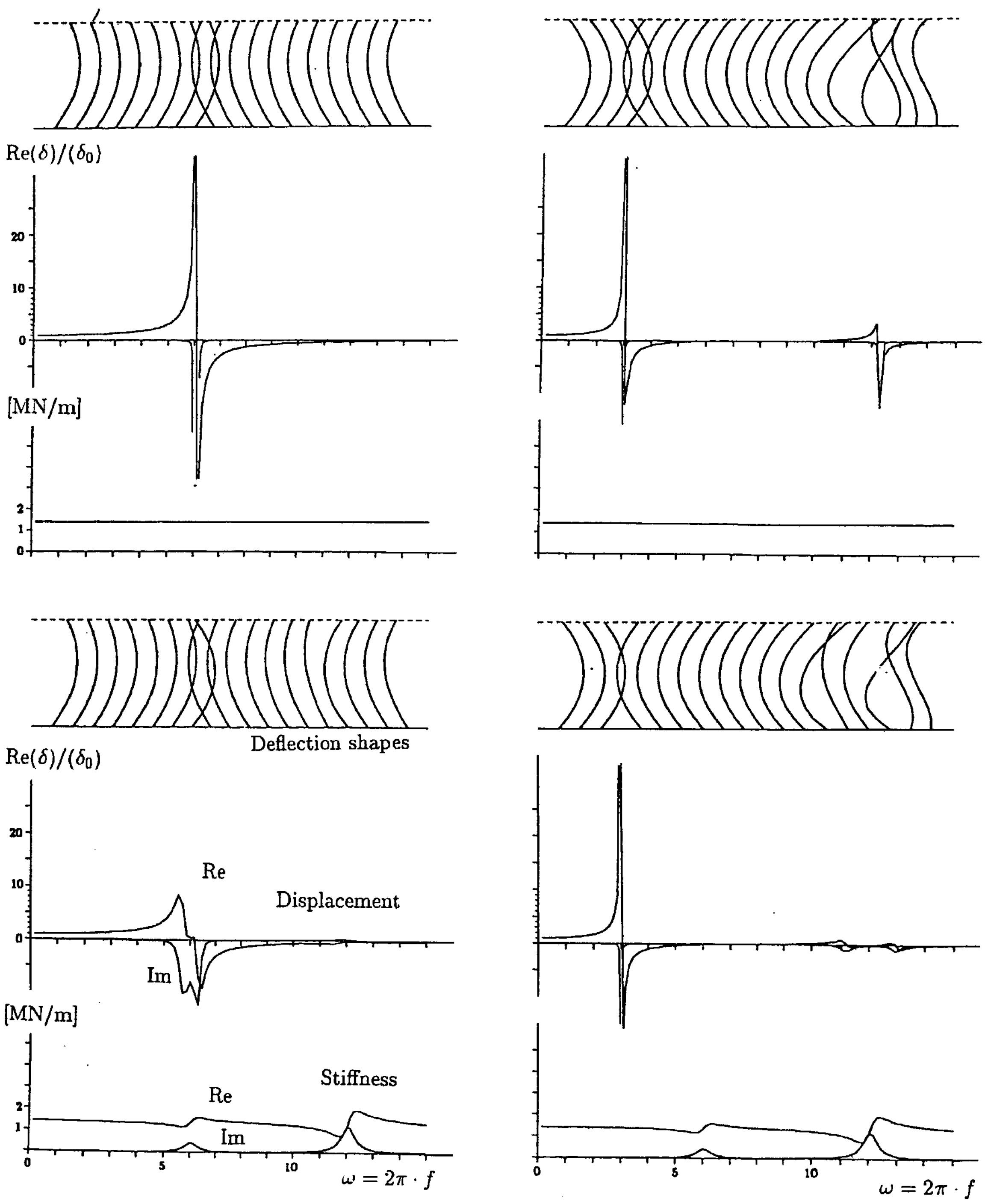


Figure 5: Admittance function of the stiff mast (left) and the weak mast (right)

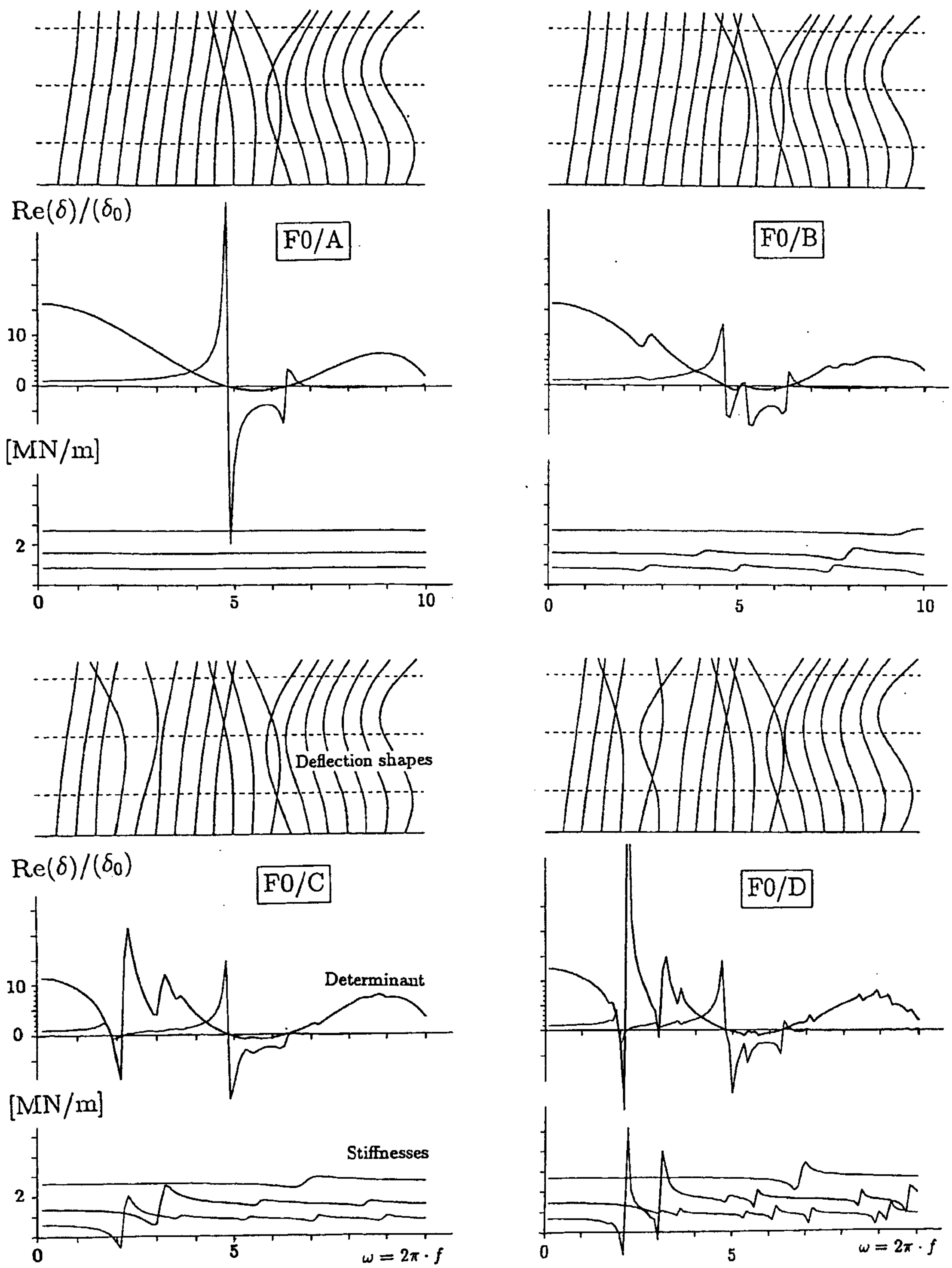


Figure 7: Admittance funcion, determinant and stiffness functions based on assumptions F0/A-D

The hourly mean wind speed $\bar{v}_{10}$ is 27.8 m/s. With the roughness-parameter k = 0.01, the exponent α = 0.22 and the reference height z = 200 m yield a turbulence intensity coming up to I_z = 0.016 (Ruscheweyh, 1982).

Stress in the Mast

To allow comparison between different systems, some absolute values of bending moments were calculated. Figure 8 shows the graphical evaluation with stress-profiles.

On the left the static moment diagram of the continuous beam on rigid supports is shown with an arbitrary series of dynamic moment diagrams. On the right hand side, besides the stress-profile of the basic system (F0), two other systems are represented, with varying cable sections (F1, F2).

All three of these systems display the favourable influence of cable dynamics (compare the fat line B with the thin one A). The calculation, needed for the frequency dependent formulation, can be worthwhile. However the calculation with static cable stiffness, at least in this study, is conservative.

An increase of stiffness in the upper part of the mast (F2) is favourable with respect to the bending stress. This is clearly better than decreasing stiffness with the height, as when using cable sections simply adapted to the load.

Stress in Cable Elements

Obviously the mast cannot be damaged by oscillating cables. The question remains, whether the cable elements themselves might be influenced in a negative way by the dynamic interaction with the mast.

The stochastic estimation of dynamic cable forces is carried out in a way analogous to that for the beam element.

The numerical investigations with various systems showed that the interaction between cable and beam remains benign, the dynamic magnification factor did not exceed φ_d = 1.30.

Limits of Linear Analysis

It must be checked that the amplitudes which occur, correspond to the assumptions of linear theory.

The amplitude of oscillation of the upper cable at midspan has been investigated with the stochastic method, assuming linear theory is valid and consequently the formulae as presented in Starossek (1991a).

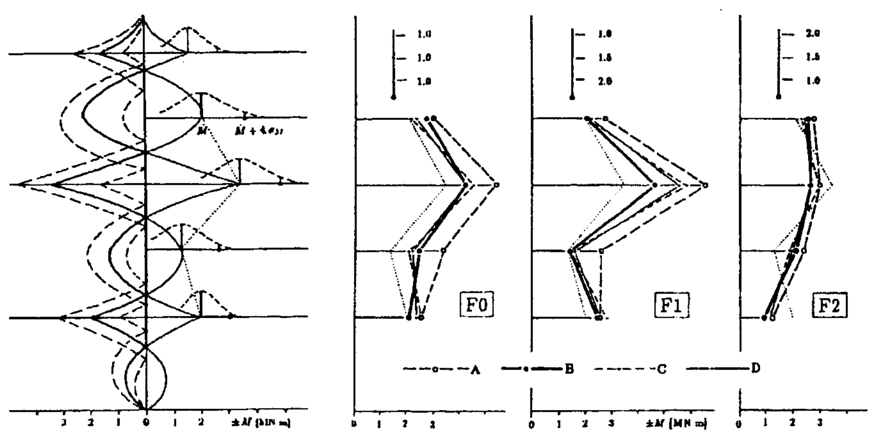


Figure 8: Stress profiles

The basic system F0 produces dynamic amplifications of 1.43 (F0/B) and 1.40 when the pretension is halved (F0/C). The dynamic deflections found were 66.2 cm and 63.8 cm respectively around the static position, where the cable sag is f = 1.09 m and f = 2.19 m respectively.

The cables of the simple guyed mast (EI = 12207 MN m^2), as described above, whose natural frequencies were close to those of the system, showed a dynamic amplification of 4.35 and a corresponding amplitude of 2.39 m.

These statistically determined amplitudes certainly cannot be regarded as small in sense of assumptions of linear theory. Whether or not nonlinear mechanisms of amplitude limitation can be taken into account by raising the damping factor can be clarified only with advanced methods. At this point the linear theory makes apparent its own limits.

Nevertheless, the presented kind of analysis represents a helpful tool for the designing engineer to examine the influence of different parameters. It quickly gives definite and comparable results. Structures with a certain sensibility to dynamics can be developed and optimised in this way.

Bibliography

Clough, RW, and Penzien J (1975). *Dynamics of Struct.*, McGraw-Hill, N. Y.

Irvine, HM (1981). *Cable Structures*, MIT Press, Cambridge, Massachusetts.

Kutterer, M (1991). *Abgespannte Maste — Statische und dynamische Besonderheiten* (Guyed masts — static and dynamic peculiarities), Diplomarbeit, Institut für Tragwerksentwurf und -konstruktion, Universität Stuttgart.

Starossek, U (1991a). *Brückendynamik — Winderregte Schwingungen von Seilbrücken* (Bridge Dynamics — Wind-Induced Vibration of Cable Supported Bridges), Doctorial thesis, Universität Stuttgart, published by Friedr. Vieweg & Sohn, Braunschweig.

Starossek, U (1991b). "Dynamic stiffness matrix of sagging cable," *ASCE, Journal of Engineering Mechanics*, Vol. 117, No. 12, pp. 2815–2829.

Veletsos, AS, and Darbre, GR (1983). "Dynamic stiffness of parabolic cables", *Earthquake Eng. and Structural Dyn.*, Vol. 11, No. 3, pp. 367–401.

Ruscheweyh, H (1982). *Dynamische Windwirkung an Bauwerken*, Bauverlag GmbH, Wiesbaden Berlin.

Petersen, C (1971). *Aerodynamische und seismische Einflüsse auf die Schwingungen insbesondere schlanker Bauwerke*, VDI-Verlag, Düsseldorf.

Davenport, AG, and Steels, GN (1965). "Dynamic Behaviour of Massive Guy Cables," *ASCE, Journal of the Struct. Div.*, Vol. 91, No. St 2, pp. 43–70.