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The Shallow Water Effects- Do Steady Disturbances Always Result in Steady Responses?

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The Sixth Georg Weinblum Memorial Lecture

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Do Steady Disturbances Always Result in Steady Responses?

by

Theodore Yaotsu Wu

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VI Georg-Weinblum-Gedachtnis-Vorlesung

The Shallow Water Effects -

Do Steady Disturbances

Always Result in Steady Responses?

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A recently identified phenomenon reveals that the wave-making resistance of a ship model as measured in towing tank experiments can indeed experience a periodic fluctuation as the model moves steadily within a certain range of the transcritical speeds. In this velocity range, two-dimensional solitary waves, spanning the tank, are generated, one after another, and move down the tank ahead of the model forming a sequence of free solitary waves; the longer the run, the more the solitons that appear, periodically and indefinitely. This process of generating 'runaway solitons' has also been predicted by numerical results based on a nonlinear, dispersive long-wave model for a moving surface pressure and a moving bottom bump. The underlying mechanism of this phenomenon can be ascribed to an interaction between the nonlinear and dispersive effects, with a weak resonance between the system and the external forcing excitation. It is with this well matched interplay that we encounter this interesting and beautiful example that steady disturbances do not necessarily always result in steady responses.

1. Introduction

Shallow water effects on the resistance and motion of a ship have long been recognized as problems of great importance in naval architecture. They are of theoretical interest since on linear theory, whether or not it be derived exclusively for long waves, the calculated wave-making resistance has certain well-known singular behaviors as the depth Froude number $F_h = U / \sqrt{gh} \rightarrow 1$, where U is the constant ship velocity and h is the original uniform water depth. They are also of practical value as actual service conditions more frequently, than before, encounter those of the shallow water limit and critical speed. With the modern advances in ocean engineering, this problem has also become quite acutely related to those of internal waves in stratified fluids and their dynamic loading on offshore structures.

Considerable insight into the complexities of the problem has been acquired experimentally by means of towing-tank studies, though they are no less challenging a task than theoretical ones. Generally speaking, it has been found invariably difficult to measure stable values of the flow field and wave-making resistance within a transcritical range of the depth Froude number $F_h \approx 1$. Some of these difficulties and the related perplexing nature of the problem have been made explicit in the literature, as can be found from the included list of bibliography on the subject. However, no attempt is implied to make the list exhaustive, for it affords a broad cross reference. No reviews of the literature are attempted here either, for they have been dealt with by several recent reviews listed. Nevertheless, for the present purpose of our discussion we cite in particular the contribution from Graff, Kracht and Weinblum (1964), which provides us with a clear delineation of the state of the art, possible sources of

experimental difficulties and the very nature of the problem. It describes how in shallow water, both viscous resistance and wave resistance are changed. The changes begin "as the depth Froude number $F_h \approx 0.7-0.8$ where local velocities in the surface pattern reach the critical value. The actual magnitude of the limit mentioned depends upon the slenderness of the ship and upon the depth of water.

"At this point the critical range of speed begins. It is marked by an essential transformation of the velocity field and of the wave pattern around the ship. In the model test this state is perceptible by periodical oscillations of the resistance, which reaches its final value after a long run only. A steep increase of resistance is connected with a similar increase of trim and sinkage. Finally, the system of diverging and transverse waves develops into one transverse-wave crest at the fore part of the ship. As the critical speed of this crest depends on its effective depth, the critical range exceeds somewhat the value $(gh)^{1/2}$. On the basis of the linearized wave-resistance theory, essential problems connected with the critical zone cannot be solved except when the (water depth to ship length) ratio h/L is no longer small."

It further pointed out that "the finite width of the model tank poses serious problems when converting test data to full size in the critical range"

This just mentioned deep insight of Weiblum's and coworkers' has turned out to be a far foresight, for the issue of there being 'a finite width of the model tank' is found to play a crucial role in the newly identified phenomenon which is the theme of this lecture.

The new phenomenon to be discussed presently is concerned with the generation of long waves and solitary waves by some forcing disturbances in shallow water, in particular by a ship moving in a canal, under

the circumstances when both the nonlinear and dispersive effects play a significant role. The external disturbances can be a surface pressure distribution, a bottom bump, a submerged obstacle – either two- or three-dimensional in shape – and a ship, moving through or on the layer of shallow water. The phenomenon becomes especially striking, even contrary to expectation, in the special case when the disturbance is maintained, after an impulsive start, at a steady state of forward motion.

More specifically, when a ship moves at a constant velocity atop a shallow water contained in a rectangular channel, it has been observed to generate, in addition to a train of waves following the ship, a sequence of two-dimensional solitary waves, one after another spanning the tank and surging ahead of the ship in succession to form an upstream moving train of solitons. This phenomenon manifests under a set of appropriate conditions when first, the depth Froude number lies within a certain transcritical range, $F_h \approx 1$, and second, the disturbance is sufficiently strong, but not too strong. Virtually the same phenomenon has been found to result from other forms of forcing excitations. For ship models in towing tank experiments, the phenomenon has been observed and investigated by Huang et al. (1982a, b) and successively by Ertekin (1984). For the case of two-dimensional surface pressures, the so-called 'runaway solitons' was first discovered numerically by Wu & Wu (1982) based on the theoretical model developed by Wu (1981, 1982). In addition to these recent developments, there was the communication from Sun (1982) regarding the towing tank experiments at the Zhongshan University in Guangzhou, China, on what is evidently the same phenomenon. As a note added in proof, the reports by Thews and Landweber (1935, 1936), which were brought to my attention

only shortly before the delivery of this lecture in the U. S. A. on March 20, 1984, appear to be the first laboratory report of such tank-spanning waves. We should further note that the solitary wave first described by John Scott Russell (1838, 1845) that formed "in a state of violent agitation", to roll forward from a canal boat, when the boat drawn by a pair of horses suddenly stopped, is a close relative of this family (of runaway solitons). In effect, it represents a particular case of the more general category covering the generation of solitary waves by unsteady disturbances, namely, in Russell's case, by a forcing excitation which, while underway, was suddenly turned off. In this regard, some searching questions may well be raised as to how the mass of water that was accumulated round the vessel reorganized itself to form a large, smooth solitary wave, and if there existed any relation between the stoppage of pulling the boat by the working horses and the resistance on the vessel.

A point of singular interest is the observed result that when upstream running solitons are generated in a towing tank by a forcing disturbance, they are always found to be two-dimensional, spanning uniformly across the tank, whether the disturbance is two-dimensional, a correspondance which would naturally be expected, or three-dimensional in distribution, which is less trivial to explain. This naturally leads one to question what happens as the tank becomes wider and wider, and thus bring the tank width issue mentioned by Weinblum and coworkers to an accented new light.

While some questions will remain to be clarified, this lecture is intended to present a recapitulation of the theoretical basis of the fomulation, some computed results for moving surface pressure and bottom bump as forcing disturbances, a related numerical experiment, and an expository investigation of the basic mechanism underlying the phenomenon in question.

2. The generalized Boussinesq equations

To recapitulate the formulation adopted from Wu (1979, 1981) we consider the generation and propagation of three-dimensional long gravity waves of finite amplitude, having the degrees of freedom to propagate in two horizontal dimensions $\underline{r} = (x, y)$ in a layer of water, bounded below by a possibly deformable floor at a prescribed depth $z = -h(\underline{r}, t)$ and above by a free surface, $z = \zeta(\underline{r}, t)$, which initially (at time $t = 0$), if unperturbed, is at $z = 0$, but may be subjected to a free-surface pressure distribution $p_0(\underline{r}, t)$. The surface pressure can represent disturbances of meteorological and naval architectural nature such as in applications to air-cushion vehicle and ships, while the floor movement can simulate tsunamic-genic disturbances of the ocean floor or what is apparent to a vehicle moving in water of variable depth. In our exposition, we shall assume the fluid to be inviscid and incompressible, and its motion irrotational, possessing in the flow domain a velocity potential $\phi(x, y, z, t) = \phi(\underline{r}, a, t)$. Further, the forcing functions will be supposed to have such a scale and strength that the resulting waves will have a typical length λ , primarily large compared with the characteristic water depth, h_0 , and will have a typical amplitude, a , small relative to h_0 , i. e.,

$$\epsilon = h_0/\lambda \ll 1, \quad \alpha = a/h_0 \ll 1. \quad (1)$$

While the condition of $\epsilon \ll 1$ holds, by definition, for long waves, special attention will be given to the case when the Ursell number

$$Ur = \alpha/\epsilon^2 = a\lambda^2/h_0^3 = O(1). \quad (2)$$

Under the aforementioned assumptions a set of basic equations have been obtained by Wu (1979, 1981, there equations 37 and 38):

$$\zeta_t + \nabla \cdot [(h + \zeta) \bar{\mathbf{u}}] = -h_t, \quad (3)$$

$$\bar{\mathbf{u}}_t + \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} + g \nabla \zeta = -\frac{1}{\rho} \nabla p_o + \frac{h}{2} \frac{\partial}{\partial t} \nabla [h_t + \nabla \cdot (h \bar{\mathbf{u}})] - \frac{h^2}{6} \frac{\partial}{\partial t} \nabla^2 \bar{\mathbf{u}}, \quad (4)$$

where $\nabla \equiv \partial/\partial \mathbf{r} = (\partial/\partial x, \partial/\partial y)$ is the two-dimensional vector operator in the horizontal plane of $\mathbf{r} = (x, y)$, $\mathbf{u} = \nabla \phi(\mathbf{r}, z, t) = (\partial \phi/\partial x, \partial \phi/\partial y)$ is a vector composed of the two horizontal velocity components, and $\bar{\mathbf{u}}$ denotes the layer-mean value of \mathbf{u} as defined by

$$\bar{\mathbf{u}}(\mathbf{r}, t) = \frac{1}{\eta} \int_{-h}^{\zeta} \mathbf{u}(\mathbf{r}, z, t) dz, \quad (\eta = h + \zeta). \quad (5)$$

Here, equation (3) is exact, while the momentum equation (4) is an approximation with an error of $O(\alpha \epsilon^4, \alpha^2 \epsilon^2)$.

Some simplification is gained (in numerical computation) if use is made of the layer-mean velocity potential

$$\bar{\phi}(\mathbf{r}, t) = \frac{1}{\eta} \int_{-h}^{\zeta} \phi(\mathbf{r}, z, t) dz \quad (6)$$

rather than the layer-mean horizontal velocity $\bar{\mathbf{u}} = \overline{(\nabla \phi)}$. Along this line of development we have (see Wu 1979, 1981, eqs. 41 and 42):

$$\zeta_t + \nabla \cdot [(h + \zeta) \nabla \bar{\phi}] = -h_t + \nabla \cdot \left\{ \left[\frac{h}{2} (h_t + \nabla \cdot (h \nabla \bar{\phi})) - \frac{h^2}{3} \nabla^2 \bar{\phi} \right] \nabla h \right\}, \quad (7)$$

$$(\bar{\phi}) + \frac{1}{2} (\nabla \bar{\phi})^2 + g\zeta + \frac{1}{\rho} p_o = \frac{h}{2} \frac{\partial}{\partial t} [h_t + \nabla \cdot (h \nabla \bar{\phi})] - \frac{h^2}{6} \nabla^2 \bar{\phi}_t . \quad (8)$$

Equations (7) and (8) may be regarded as an alternative set of basic equations to (3) and (4), now both being approximate, each with an error of $O(\alpha \epsilon^4, \alpha^2 \epsilon^2)$. We further note that ϕ and $\bar{\phi}$ are related by

$$\phi - \bar{\phi} = - (z + \frac{1}{2} h) [h_t + \nabla \cdot (h \nabla \bar{\phi})] - \frac{1}{2} (z^2 - \frac{1}{3} h^2) \nabla^2 \bar{\phi} , \quad (9)$$

and that (8) is accordingly the first integral of (4) while (3) becomes (7). From this relation one can readily deduce the velocity distribution as (ϕ_x, ϕ_y, ϕ_z) and the pressure field from the Bernoulli equation.

$$\frac{1}{\rho} p = -gz - \phi_t - \frac{1}{2} (\phi_x^2 + \phi_y^2 + \phi_z^2) . \quad (10)$$

The original set of equations (3) and (4), or the alternative set (7) and (8) can be taken as to form a generalized Boussinesq class, possessing the new features that (i) the medium is now nonhomogeneous and unsteady (due to the spatial and temporal variation of h), (ii) the system is mechanically open (to exchanges of mechanical work and energy with external agencies), and (iii) there are two horizontal dimensions for wave propagation. In the special case of two-dimensional homogeneous motion, that is with $h = \text{const.}$ and $p_o = 0$, (3) and (4) reduce to the classic Boussinesq equations, which admit solitons (either right or left running) as exact solutions. For three-dimensional homogeneous and steady motions, (7) and (8) can be combined to give the equation of Mei (1976), which is a theoretical model for non-linear steady flow around a thin body moving in shallow water.

Finally, we note that our set of generalized Boussinesq equations contains the Korteweg-de Vries equation (or the KdV equation) as a subset for the case of unidirectional (in x , say) wave motion in shallow water of uniform depth, since in this case one can follow Whitham (1974, p. 466) to derive from (3) and (4) the KdV equation (with normalization) for right-running waves:

$$\zeta_t + \zeta_x + \frac{3}{2} \zeta \zeta_x + \frac{1}{6} \zeta_{xxx} = 0. \quad (11)$$

We further note that (11) has for solitons a one-parameter family of solutions

$$\zeta = a \operatorname{sech}^2 \left\{ \left(\frac{3a}{4h} \right)^{1/2} (x - ct - x_0) \right\}, \quad (12a)$$

$$c^2 = g(h + a). \quad (12b)$$

Here and in the sequel, the gravity constant g , the undisturbed water h and the fluid density ρ can be normalized to unity, as in (11), and can be restored for clarity of physical interpretation, as in (12a) and (12b).

It is of significance to point out that while the KdV equation (11), as well as the basic equations (3) and (4) for the more general case are originally meant to hold valid under conditions (1) and (2), that is, $\epsilon \ll 1$, $\alpha \ll 1$, and $\alpha/\epsilon^2 = O(1)$, the validity of (11) has been tested experimentally as a model for moderate-amplitude, unidirectional waves in shallow water of uniform depth. Hammack & Segur (1974) found the agreement between experiment and ^{the} KdV equation predictions to lie within about 20% over the entire range of experiments examined, including those with initial data for which the non-decayed amplitudes (i. e. with corrections for the viscous decay of the measured wave amplitudes) of the leading soliton exceed half the fluid depth, that is, up to $\alpha = a/h_0 = 0.5$ at least,

implying that $\epsilon < 0.7$. Also, Boussinesq's profile fits both observation and the more accurate theoretical calculations rather well for $\alpha < 0.5$ and appears to be superior to the other models (see Miles, 1980). Such a moderate value of α is only slightly less than the highest solitary wave that can be maintained without breaking (see Miles, 1980, p. 21)

$$\alpha_{\max} = 0.827 \quad (\text{with corresponding } F_h = 1.286). \quad (13)$$

Based on the extended range of validity established for the KdV and the Boussinesq models, it would be reasonable to presume the same to hold for the more general case of equations (3) and (4), though such a conjecture must still require experimental verification.

3. Numerical results

For the general case, it is possible to obtain solutions of (3) and (4), or (7) and (8) by numerical computation. In our calculations, we primarily followed the original method of Wu & Wu (1982), while adopting further improvements (including replacement of the iterative step by a direct calculation after suitable rearrangements of the equations as well as new modifications for increased accuracy) by Lee (1983-1984) for the case of two-dimensional initial-boundary problems. Thus we integrate (3) and (4) using a modified Euler's predictor-corrector method in advancing the time and the central difference approximation for the spatial derivatives. A mixed implicit-explicit scheme is adopted for the forward difference computation of ζ and \bar{u} in the fluid frame of reference, the implicit part being incorporated in order to achieve the desired numerical stability and accuracy with relatively large time step Δt . The open boundary condition of Wu & Wu (1982) has proven successful, so that the region of computation

can be taken relatively small. We present below some typical numerical results so obtained.

For the surface pressure we consider the distribution

$$p_o = p_{om} \frac{1}{2} \left[1 - \cos \left(2\pi \frac{x+Ut}{L} \right) \right] \quad (0 < x+Ut < L, t > 0) \quad (14)$$

$$\text{with } p_{om} = 0.15, h = 1, L = 2, F_h = U / \sqrt{gh} = U = 1.0, \quad (15)$$

and $p_{om} = 0$ elsewhere for $t > 0$ as well as for all $x, t < 0$. And at $t = 0$, the water was at rest. The transient motion resulting from the application of this p_o moving at the critical speed was computed by using the present numerical method over the region $-30 < x+Ut < 40$, with $\Delta x = 0.2$ and $\Delta t = 0.2$. The numerical result for the free surface elevation ζ is shown in figure 1, here plotted with reference to the moving pressure disturbance, marked by two vertical lines for its extent, over the time period (normalized vs. L / \sqrt{gh}) in $0 < t < 80$.

From figure 1 it is clear to see that the first solitary wave emerges at about $t = 20$, then surges ahead to become a runaway soliton while a new solitary wave starts being generated and subsequently 'born', at about $t = 40$, only to be followed by another, and still another, in succession. There are nearly three solitons having run away by the end of this computation period of $t = 80$. The slowly increasing phase velocity of the runaway solitons and of the trailing cnoidal-like waves reflects the nonlinear and dispersive effects; there is, however, an ever prolonging region of depression in water depth just behind the disturbance. This numerical result bears the improved accuracy of the present numerical code over its previous version of Wu & Wu (1982); in one respect, the rate of growth of

the solitons after separation is in this case negligible, much weaker than the earlier result of Wu & Wu (1982).

The wave resistance, D_w , experienced by the surface pressure (per unit width) has the coefficient

$$C_{Dw} = D_w / \rho ghL = - \frac{1}{\rho ghL} \int_0^L p_o(x, t) \frac{\partial \zeta}{\partial x} dx . \quad (16)$$

And the coefficient of rate of working by p_o is

$$C_{(\dot{W}/U)} = \dot{W} / \rho ghLU = \bar{\lambda} \frac{1}{\rho ghLU} \int_0^L p_o(x, t) \frac{\partial \zeta}{\partial t} dx . \quad (17)$$

The result of C_{Dw} and $C_{(\dot{W}/U)}$ corresponding to (14) and (15) is given in figure 2, in which also shown is the coefficient $C_{(\dot{W}'/U)} = C_{(\dot{W}/U)} - C_{Dw}$. The wave resistance is thus seen to vary considerably during the course when each soliton is generated and runs away. The period of C_{Dw} variations provides an excellent indicator of the 'generation period' of runaway solitons, for it gives very uniform readings of the generation period in succession, which is $T = 20$ for $p_{om} = 0.15$.

The phenomenon of generation of runaway solitons is found to manifest over a range of the depth Froude number from as low as $F_h = 0.2$ to about $F_h = 1.2$, depending on the form and extent of p_o distribution. At low subcritical speeds ($0.3 < F_h < 0.7$), the solitons generated are relatively weak, but definitely discernible both numerically and experimentally, and the period of generation is relatively short. The periodically generated solitons gain in magnitude with increasing F_h , whereas the trailing wave train becomes gradually weaker and the period of generation becomes somewhat more prolonged, especially when F_h increases beyond 1.

As F_h reaches a certain value of about 1.2, the leading soliton was found experimentally to break into a turbulent bore and numerically to evanesce for still higher values of F_h .

Generation of solitons has also been investigated for a bottom bump, having a curved top surface and flat base and moving along the bottom of a water layer. Figure 3 presents the numerical result of the free surface elevation ζ produced by the motion of the cosine bump:

$$h = h_0 - h_1, \quad h_1 = \frac{1}{2} H_0 \left[1 - \cos \left(2\pi \frac{x + Ut}{L} \right) \right], \quad (18)$$

$$\text{with} \quad H_0 = 0.2, \quad h = 1, \quad L = 2, \quad F_h = U = 1.0, \quad (19)$$

the qualifying conditions for (18) and the initial conditions for ζ and \bar{u} are the same as that for (14). To calculate the wave resistance and the rate of working required for maintaining the bump motion we simply replace, in (16) and (17), p_0 by the pressure at the bump surface, and ζ by $h_1(x, t)$; the result for the drag coefficient is given in figure 4. From these figures we find that the free solitons generated ahead and the cnoidal-like waves trailing the moving bump closely resemble those produced by a surface pressure having the same strength distribution and moving at the same transcritical speed. Comparisons between the numerical solutions and the preliminary experimental results obtained by Lee (1983, 1984) have shown that the present theoretical model appears to be very satisfactory in predicting the overall results such as the generation period and amplitudes of the runaway solitons and the wave pattern over a wide range of the Froude number F_h and forcing strength H_0 .

Having thus acquired a general estimate of the validity of the present theoretical model, we can make use of numerical experimentation as a

powerful tool to ascertain the role played by various physical effects. For instance, by removing from (3) and (4), or from (7) and (8) separately the nonlinear terms and in turn all the highest order derivatives, the latter being responsible for causing waves to disperse, the phenomenon of soliton generation was found by Wu & Wu (1982) to cease to occur. Hence the assertion follows that the phenomenon arises only from the interaction between the nonlinear and dispersive effects.

Another numerical experiment of interest is to remove the surface pressure disturbance (14), (15) shortly before the natural separation of the first soliton. On physical ground, we expect that the mass of water accumulated around the pressure disturbance will be left on its own to continue to evolve, after the removal of p_0 , eventually appearing as a single soliton, with in general an undular tail. In fact, this large-time picture readily follows from the asymptotic solution of Hammack & Segur (1974) reached after long evolution from known initial data, since the wave profile at the instant of removal of p_0 can be used as the initial data. Only when the initial data satisfy certain characteristic equations can we expect to see finally a single smooth solitary wave (i. e. without an undular tail) as the one John Scott Russell/first encountered. There could not be more than one leading soliton to appear in the final result because the initial Ursell number was not even large enough to induce a natural soliton separation prior to the removal of p_0 . See figure 5 for the details.

4. Basic underlying principles

The conservation equations and some basic principles can be used to develop an alternative method for evaluating an approximate solution to the water wave problem just described and now illustrated in figure 6.

For simplicity, we shall confine ourselves to the case of two-dimensional motion depending on x and z , but not on the horizontal axis of y , as the basic principles involved will remain essentially the same. To begin with, we have the layer-mean equations expressing conservation of mass, momentum and energy (see Wu, 1979, 1981) as

$$\partial N / \partial t + \partial Q / \partial x = 0 \quad (N = \rho \eta = \rho(h + \zeta)), \quad (20)$$

$$\partial Q / \partial t + \partial S / \partial x = X, \quad (21)$$

$$\partial \mathcal{E} / \partial t + \partial F / \partial x = \dot{W}, \quad (22)$$

where

$$Q = \int_{-h}^{\zeta} \rho u(x, z, t) dz, \quad (23)$$

$$S = \int_{-h}^{\zeta} (\rho u^2 + p) dz, \quad (24)$$

$$X = p_o \partial \zeta / \partial x + p_h \partial h / \partial x, \quad (25)$$

$$\mathcal{E} = \int_{-h}^{\zeta} \rho \left[\frac{1}{2} (u^2 + w^2) + gz \right] dz \quad (w = \frac{\partial \phi}{\partial x}), \quad (26)$$

$$F = \int_{-h}^{\zeta} \left[p + \frac{1}{2} \rho (u^2 + w^2) + \rho gz \right] u dz, \quad (27)$$

$$\dot{W} = - (p_o \partial \zeta / \partial t + p_h \partial h / \partial t). \quad (28)$$

Here, Q signifies the mass flux across a vertical section, which is also the x -component of momentum density. S may be called the 'total momentum flux' across a vertical section, and X is the x -component of

the differential force arising from the boundary pressure acting on the fluid, p_0 at the top, and p_h at the bottom surface. \mathcal{E} is the layer energy density, F is the total energy flux through a vertical section and \dot{W} represents the rate of working on the fluid by the boundary pressure.

These equations are exact and applicable to open systems that can have exchanges of momentum, X , and energy, \dot{W} , with external agencies. If $h = h_0(x) + h_1(x, t)$ and if $h_1 + \zeta$ is absolutely integrable at infinity, bounded for all x and t , (20) has the first integral stating that the 'excess mass' is conserved,

$$m = \int_{-\infty}^{\infty} \rho(\zeta + h_1) dx = \text{const.} \quad (29)$$

With similar conditions on the integrability of S and F , we obtain the conservation of horizontal momentum

$$\dot{I} \equiv \frac{d}{dt} \int_{-\infty}^{\infty} Q dx = \int_{-\infty}^{\infty} X(x, t) dx, \quad (30)$$

where I is the x component of momentum impulse. We also have the conservation of the total energy as

$$\dot{E} = \frac{d}{dt} \int_{-\infty}^{\infty} \mathcal{E} dx = \int_{-\infty}^{\infty} \dot{W} dx. \quad (31)$$

If the surface pressure is uniform, $p_0 = 0$, and if everywhere $h = \text{const.}$, then $X = \dot{W} = 0$ and the system becomes closed, with the corresponding momentum impulse and total energy both conserved,

$$I = \int_{-\infty}^{\infty} dx \int_{-h}^{\zeta} \rho u(x, z, t) dz = \text{const.}, \quad (32)$$

$$E = \int_{-\infty}^{\infty} \mathcal{E} \, dx = \text{const.} \quad (33)$$

In particular, the soliton solution (12) is a closed system.

Substituting (12) in (29), (32) and (33) readily yields

$$m_s = \rho 4h^2 \left(\frac{\alpha}{3}\right)^{1/2}, \quad (\alpha = \frac{a}{h}) \quad (34)$$

$$I_s = m_s c \left(1 + \frac{2}{3} \alpha\right), \quad (35)$$

$$E_s = \rho c^2 h^2 \left(\frac{4\alpha}{3}\right)^{3/2}, \quad (36)$$

where c is given by (12b) and the subscript S signifies the quantity of a soliton. All three quantities m_s , I_s , and E_s depend on one parameter, namely, α .

For the problem at hand concerning soliton generation from rest by a steadily moving bottom bump with a transcritical velocity U , we explore the motion illustrated in figure 6a. In the fluid frame of reference, we have (i) the forward surging solitons moving with velocity C_a given by (12b) for solitons with height a , (ii) a stretch of depressed layer of water of constant depth h_1 , say, immediately behind the moving bump, followed by (iii) a train of cnoidal-like waves with amplitude very gradually decreasing with increasing distance (equal to $(U - Cg_e)t$ from the bump), but slowly increasing with time, whose back end advances at the velocity equal to the group velocity Cg_e at the end. For smoothly shaped bumps, any initial disturbance that would propagate outwards in both directions with velocity \sqrt{gh} must be very weak and can be neglected. Relative to the bump, the motion apparent to the body frame of reference

is shown in figure 6b. On physical ground, we shall assume that the variations in wave properties are sufficiently gradual, and the stretch of depressed water and the trailing wave train are great enough in length for the local motion to be considered as quasi-steady over the period large with respect to that of the trailing wave but small compared with the generation period of the runaway solitons. The method has been developed by Whitham (1962) for water waves to second order; similar idea will be employed for the present purpose.

Based on the assumptions just stipulated, the conservation equations (20)-(22) can be integrated with respect to x from a section upstream of the leading solitary wave to a section of the uniform flow within the stretch of depressed water, of depth h_1 , behind the bump. We note that the flow is locally uniform at both sections. Integrals of the time-derivative terms in (20)-(22) may be approximated by the respective quantities of a single soliton, of amplitude a , divided by T , the period of generating a runaway soliton. Thus we obtain

$$\rho U_1 h_1 + m_s/T = \rho U h \quad , \quad (37)$$

$$\rho(U_1^2 + \frac{1}{2} g h_1) h_1 + I_s/T = \rho(U^2 + \frac{1}{2} g h) h - D \quad , \quad (38)$$

$$(\frac{1}{2} \rho U^2 + \rho g h)(U_1 h_1 - U h) + E_s/T + \mathcal{E}_o(U - C_g) = D U \quad , \quad (39)$$

$$D = - \int p_h \frac{\partial h}{\partial x} dx \quad (\text{over the bump}) \quad , \quad (40)$$

where the subscript 1 refers to the section of depressed water, m_s , I_s and E_s are given by (34)-(36), D is the drag on the bump, and \mathcal{E}_o denotes the energy density per unit area of water surface for the trailing wave train. In addition, since the pressure is constant at the water surface,

$$\frac{1}{2} U_1^2 + gh_1 = \frac{1}{2} U^2 + gh \equiv B. \quad (41)$$

Equations (37)-(39) and (41) are four equations for five unknowns, U_1 , h_1 , T , D , and $\alpha = a/h$, noting that \mathcal{E}_0 is known for given amplitude, and we are left with the problem of finding an additional equation.

For a smooth bump with a base chord, L , large compared with the water depth h , the required equation can be acquired by taking an approximate account of the drag D . As a first approximation, we assume that the free-surface flow above the bump, with thickness profile $z = h_b(x)$, is quasi-one-dimensional and steady, taking place in the flow region

$$h_b(x) < z < \tilde{h}(x), \quad 0 < x < L. \quad (42)$$

The continuity condition along the channel requires the velocity \tilde{U} to be such that

$$\tilde{U}(x) [\tilde{h}(x) - h_b(x)] = Uh. \quad (0 < x < L) \quad (43)$$

Since the pressure is constant at the water surface,

$$\frac{1}{2} \tilde{U}^2(x) + g\tilde{h}(x) = \frac{1}{2} U^2 + gh \equiv B. \quad (44)$$

The pressure at the bump surface is then given by the Bernoulli equation

$$p_h = \rho[B - \frac{1}{2} \tilde{U}^2(x) - gh_b(x)] = \rho g[\tilde{h}(x) - h_b(x)], \quad (45)$$

after making use of (44). Substituting (45) in (40) yields

$$\begin{aligned}
D &= \rho g \int_0^L [\tilde{h}(x) - h_b(x)] (\partial h_b / \partial x) dx \\
&= \rho g \int_0^L \tilde{h}(x) \frac{\partial h_b}{\partial x} dx, \quad (46)
\end{aligned}$$

since $h_b(0) = h_b(L) = 0$ as assumed. With this solution for D , calculation of the other quantities can now be carried out.

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Bibliography on shallow water effects

Reviews

- Proc. 13th ITTC (Intern. Towing Tank Conf.) - Resistance Committee
Report - 1972. Eds., S. Schuster & M. Schmiechen. Berlin/Hamburg.
- Proc. 14th ITTC - Resistance Committee Report - 1975. Ed.,
S. T. Mathews. Ottawa.
- Proc. 15th ITTC - Resistance Committee Report - 1978. ed.,
M. W. C. Oosterveld. The Hague.
- Proc. 16th ITTC - Resistance Committee Report - 1981. Leningrad.
- Inui, T. 1957 Study on wave-making resistance of ships. 60th
Anniversary Series 2, 173-355. SNAJ.
- Millward, A. 1982 Fast ship in shallow water. Occasional Publ. of
RINA.
- Basin, A., et al. 1976 Shallow water ship hydro. (Russ.) Sudostr.
Publ., Leningrad.
- Tuck, E. O. 1978 Hydrodynamic problems of ships in restricted
waters. Ann. Rev. Fluid Mech. 10, 33-46.
- Wehausen, J. V. 1973 The wave resistance of ships. Adv. Appl.
Mech. 13, (ed. C. S. Yih) 93-245. Acad. Press.

References

- Anderson, B. J. 1976 On ship waves in shallow water. Proc. Intern.
Seminar on Wave Resistance, SNAJ, 341-348.
- Beck, R. F. 1981 Hydrodynamic forces caused by ship in confined
waters. ASCE JEMD 107 EM3, 523-546.
- Doctors, L. J. & Sharma, S. D. 1972 The wave resistance of an air-
cushion vehicle in steady and accelerated motion. J. Ship Res. 16,
248-260.

- Eggers, K. 1955 Über Widerstands verhältnisse von Zweikörperschiffen. Jahrbuch der Schiffbautech. Ges. 49, 516-539.
- Graff, W. & Binek, H. 1969 Untersuchung des Modell-tankeinflusses an einem Flachwasserschiff. Forsch. d. Landes Nordrh. - Nr. 1986. Westdeu. Verlag.
- Graff, W., Kracht, A. & Weinblum, G. 1964 Some extensions of D. W. Taylor's standard series. Trans. SNAME 72, 374-403.
- Inui, T. & Nishiyama, T. 1952 Initial wave resistance of a submerged sphere in shallow water. JSNA 84, 17-26.
- Kinoshita, M. 1954 On restricted water effect on ship resistance. JSNA 76, 173-213.
- Kinoshita, M. & Inui, T. 1953 Wave-making resistance of a submerged spheroid, ellipsoid and a ship in a shallow sea. JSNA 75, 119-155.
- Kirsch, M. 1966 Shallow water and channel effects on wave resistance. J. Ship Res. 10, 164-181.
- Lea, G. K. & Feldman, J. P. 1972 Transcritical flow past slender ships. Prof. 9th Symp. Naval Hydrodynamics. eds. R. Brard & A. Castera. 2, 1527-1541. ACR-203.
- Maruo, H. 1952 On the shallow water effect. JSNA 84.
- Mei, C. C. 1976 Flow around a thin body moving in shallow water. J. Fluid Mech. 77, 737-751.
- Newman, J. N. & Poole, F. A. P. 1962 The wave resistance of a moving pressure distribution in a canal. Schiffstechnik 9, 21-26.
- Sretenskii, L. N. 1937 A theoretical investigation on wave resistance (Russian) Tr. Tsent. Aero-Gidrodinam. Inst., vgp. 319, 56 pp.
- Tuck, E. O. 1966 Shallow-water flows past slender bodies. J. Fluid Mech. 26, 81-95.

General References

- Ertekin, R. C. 1984 Soliton generation by moving disturbances in shallow water: Theory, computation and experiment. Ph. D. Dissertation, University of California, Berkeley.
- Hammack, J. L. & Segur, H. 1974 The Korteweg-de Vries equation and water waves. Part 2. Comparison with experiments. J. Fluid Mech. 65, 289-314.
- Huang, De-Bo, Sibul, O. J., & Wehausen, J. V. 1982a Ships in very shallow water. Festkolloquium-Dedication to Professor Karl Wiegardt. March 1982, Institut für Schiffbau der Universität Hamburg.
- Huang, De-Bo, Sibul, O. J., Webster, W. C., Wehausen, J. V., Wu, De-Ming, & Wu, T. Y. 1982b Ship moving in the transcritical range. Proc. Conf. on Behavior of Ships in Restricted Waters. Vol. II, 26.1-26.12, Nov. 11-13, 1982, Varna, Bulgaria.
- Lee, S. J. 1983-84 Graduate research report. California Institute of Technology, Pasadena, California (unpublished).
- Miles, J. W. 1980 Solitary waves. Ann. Rev. Fluid Mech. 12:11-43, Annual Reviews Inc., Palo Alto, California.
- Russell, J. S. 1838 Report of the Committee on Waves. Rep. Meet. Brit. Assoc. Adv. Sci. 7th, Liverpool, 1837, 417-496. John Murray, London.
- Russell, J. S. 1845 Report on waves. Rep. Meeting Brit. Assoc. Adv. Sci. 14th, York, 1844, 311-390. John Murray, London.
- Sun, Ming-Guan 1982 Private communication.
- Thews, J. G. & Landweber, L. 1935 The influence of shallow water on the resistance of a cruiser model. U. S. Experimental Model Basin, Navy Yard, Washington, D. C., Rep. No. 408.

- Thews, J. G. & Landweber, L. 1936 A thirty-inch model of the S. S. Clairton in shallow water. U. S. Experimental Model Basin, Navy Yard, Washington, D. C., Rep. No. 414.
- Whitham, G. B. 1962 Mass, momentum and energy flux in water waves. J. Fluid Mech. 12, 135-147
- Whitham, G. B. 1974 Linear and Nonlinear Waves. John Wiley & Sons, New York.
- Wu, T. Y. 1979 On tsunami propagation - evaluation of existing models. in TSUNAMIS - Prof. National Science Foundation Workshop (eds. L. S. Hwang & Y. K. Lee) 110-149. Tetra Tech, Inc., Pasadena, Ca.
- Wu, T. Y. 1981 Long waves in ocean and coastal waters. Proc. ASCE, J. Eng. Mech. Div. 107, EM3, 501-522.
- Wu, De-Ming & Wu, T. Y. 1982 Three-dimensional nonlinear long waves due to moving surface pressure. Proc. 14th Symposium on Naval Hydrodynamics, 103-125. University of Michigan, August 23-27, 1982. National Academy Press, Washington, D. C.

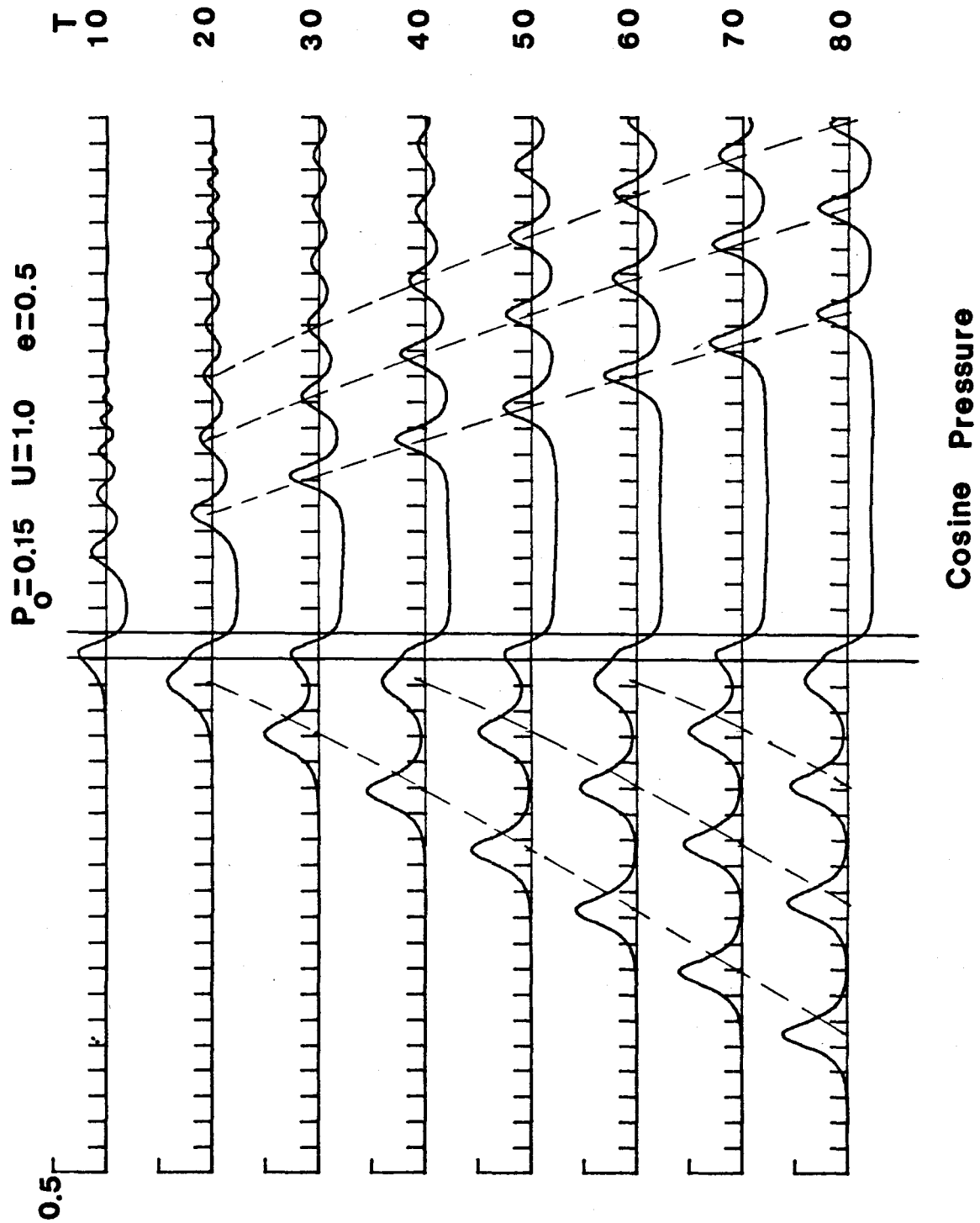


Figure 1. The free solitary waves generated ahead and cnoidal-like waves produced behind the free-surface pressure distribution (14) and (15).

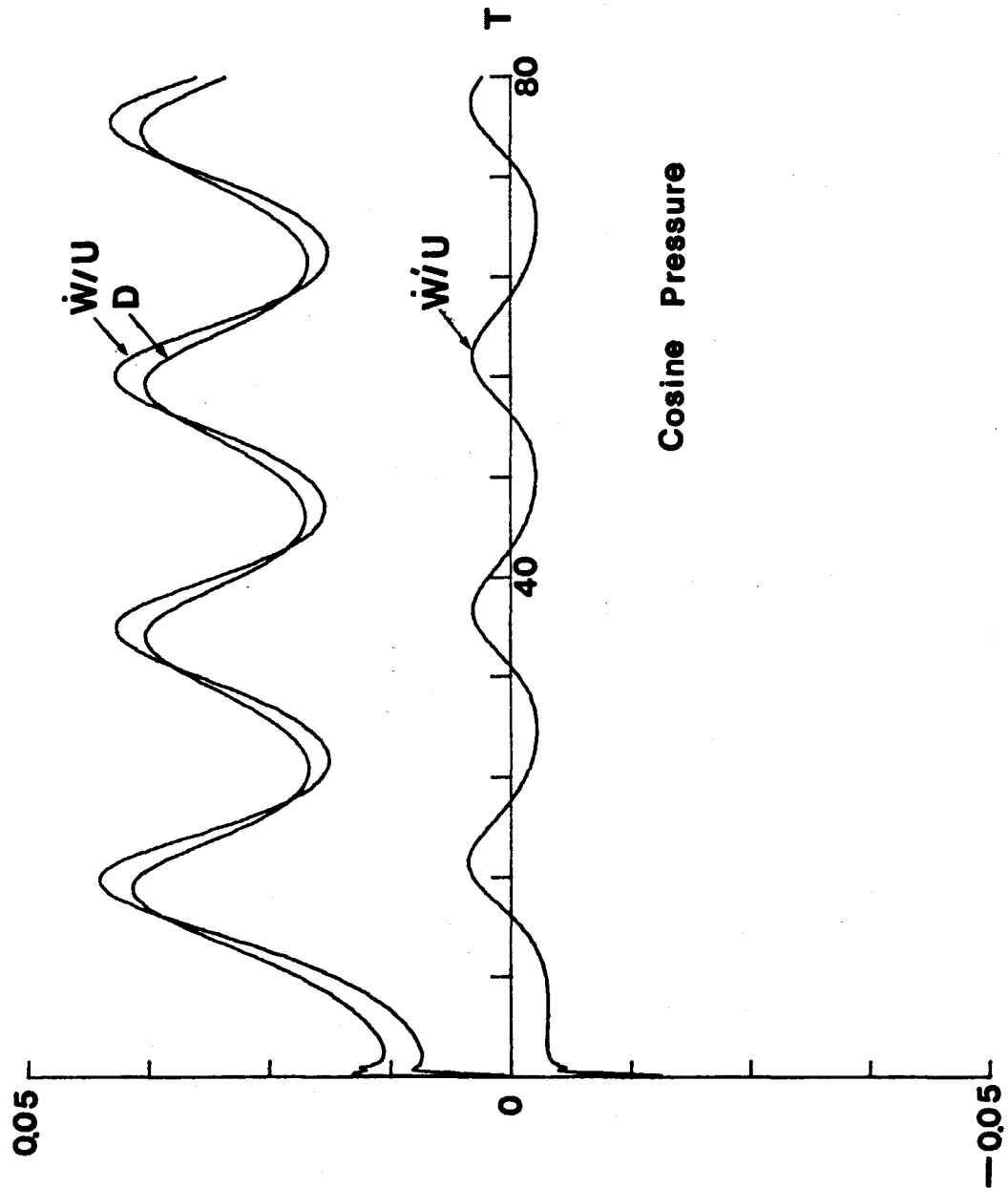


Figure 2. The wave resistance coefficient C_{DW} corresponding to the motion given in figure 1.

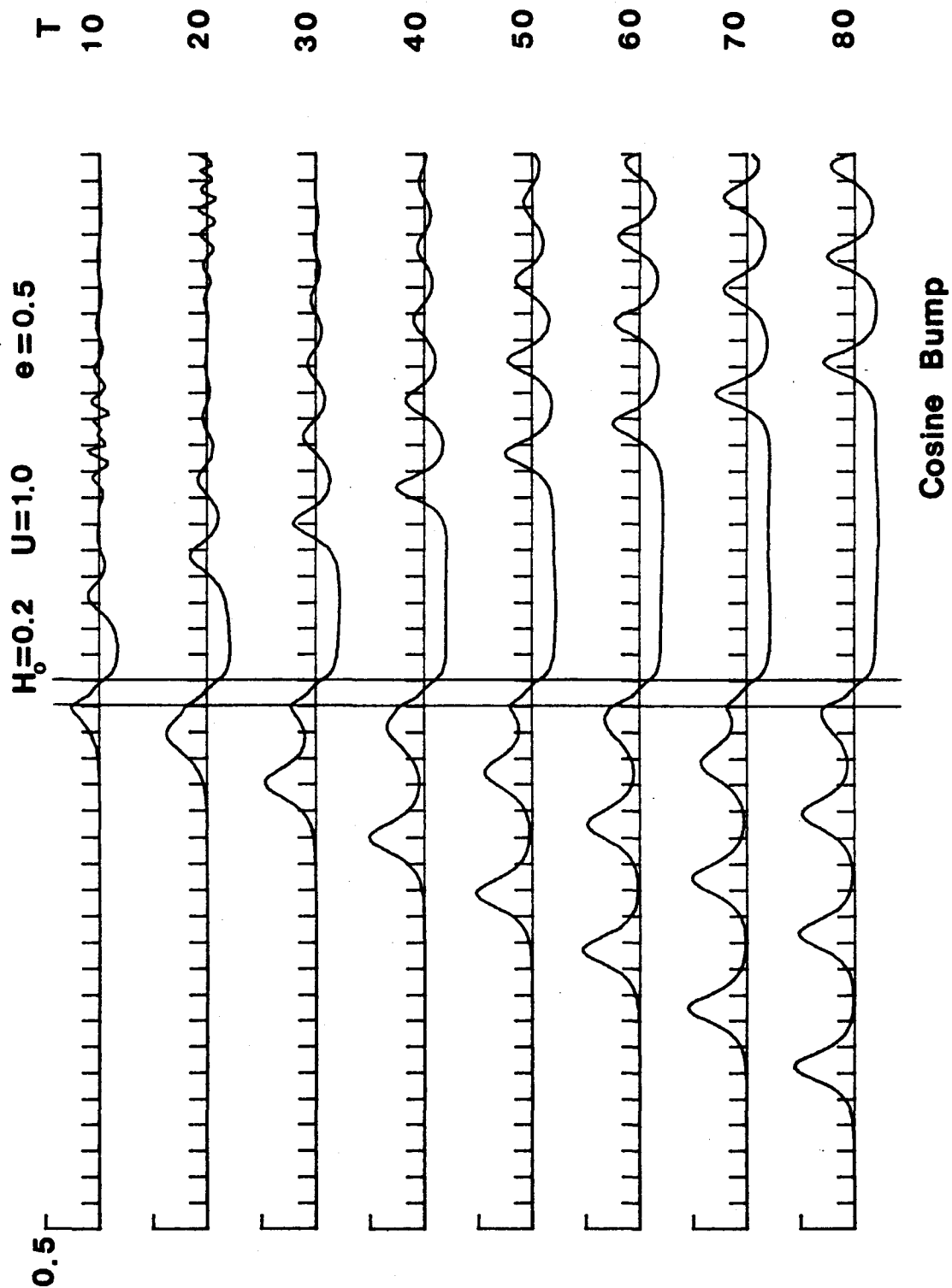


Figure 3. The free solitary waves generated ahead and cnoidal-like waves produced behind the free-surface pressure distribution (18) and (19).

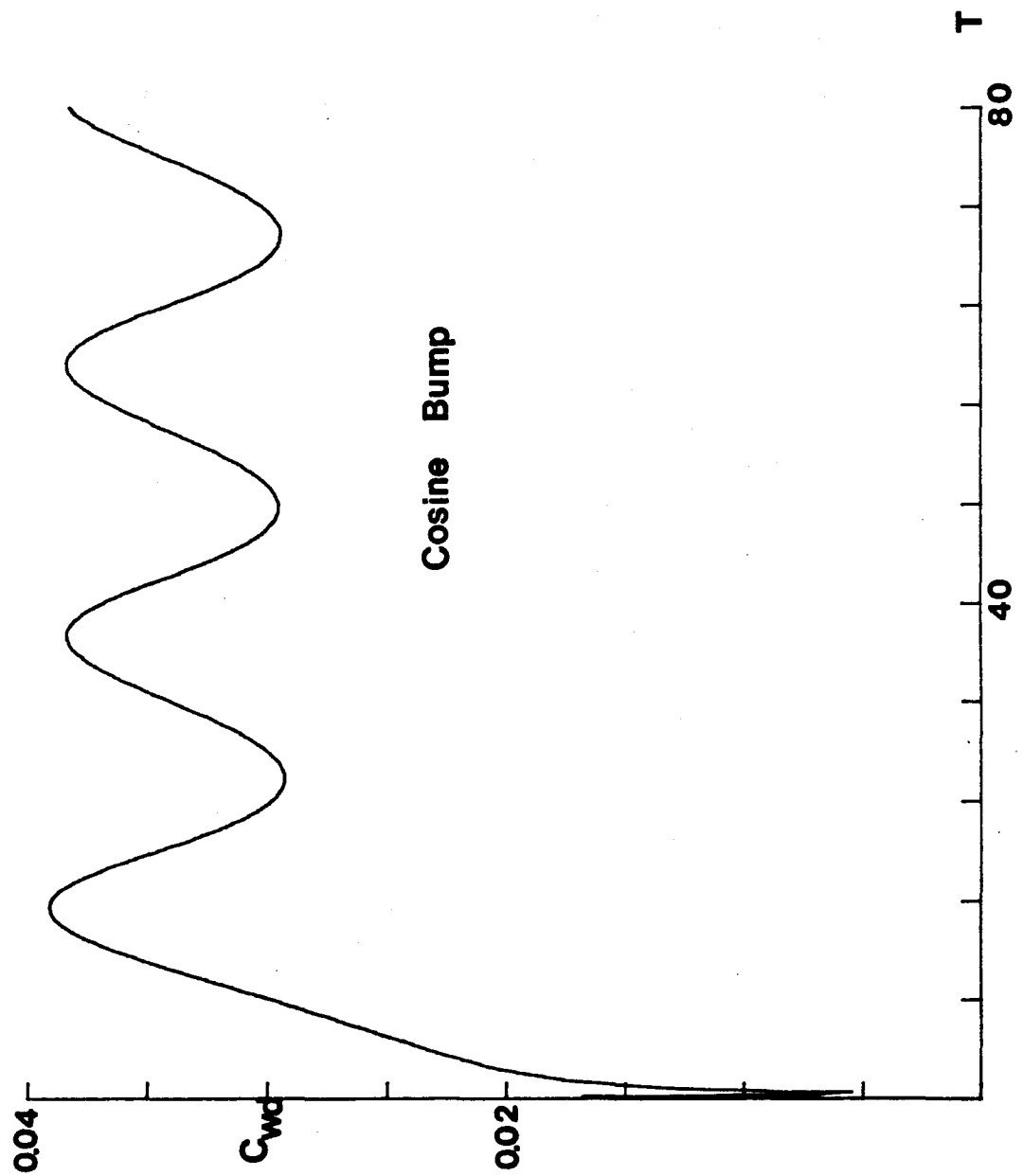


Figure 4. The wave resistance coefficient C_{Dw} corresponding to the motion given in figure 3.

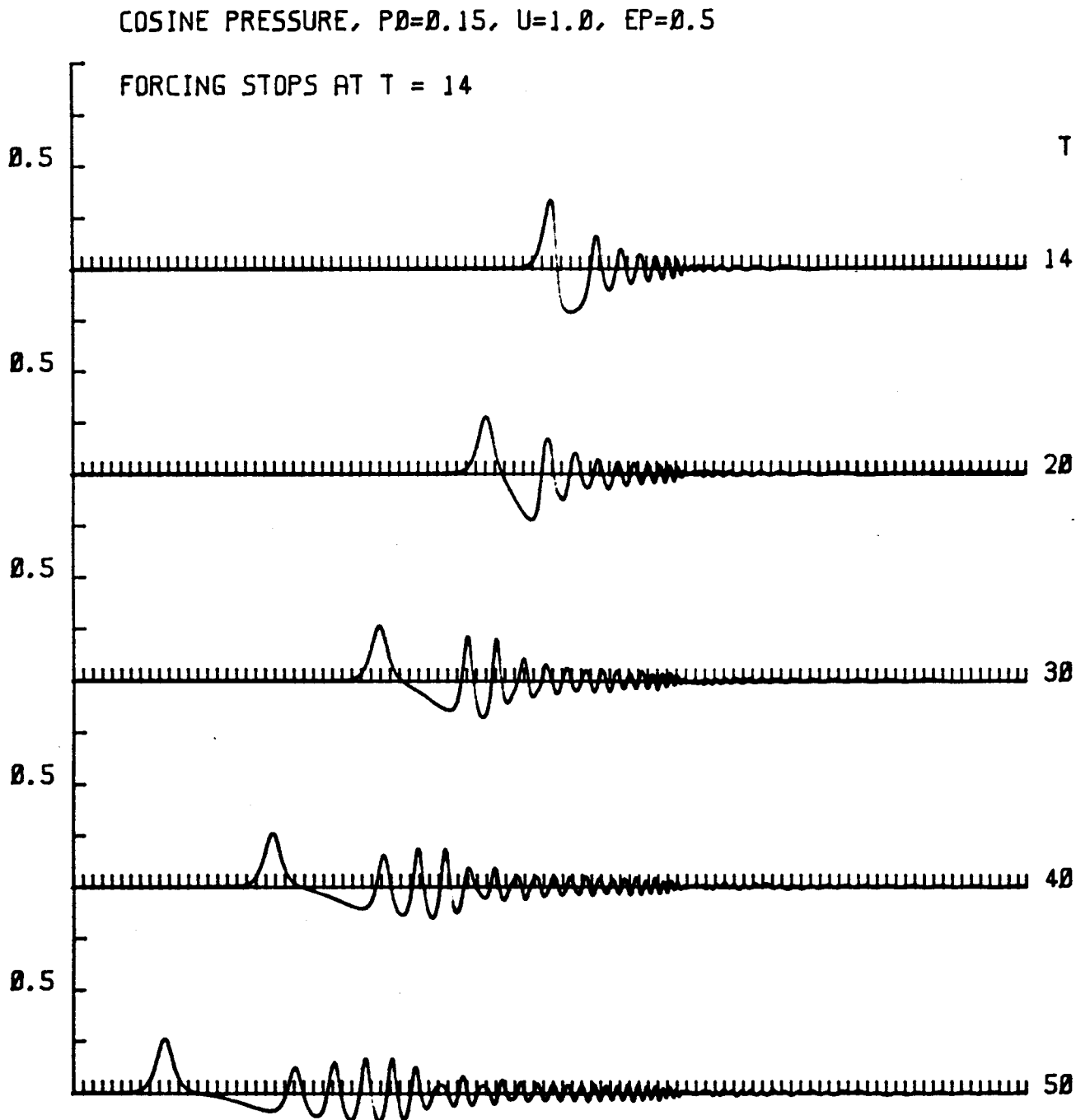


Figure 5. Evolution of the free wave after the forcing pressure given by (14), (15) was removed at $t = 14$.

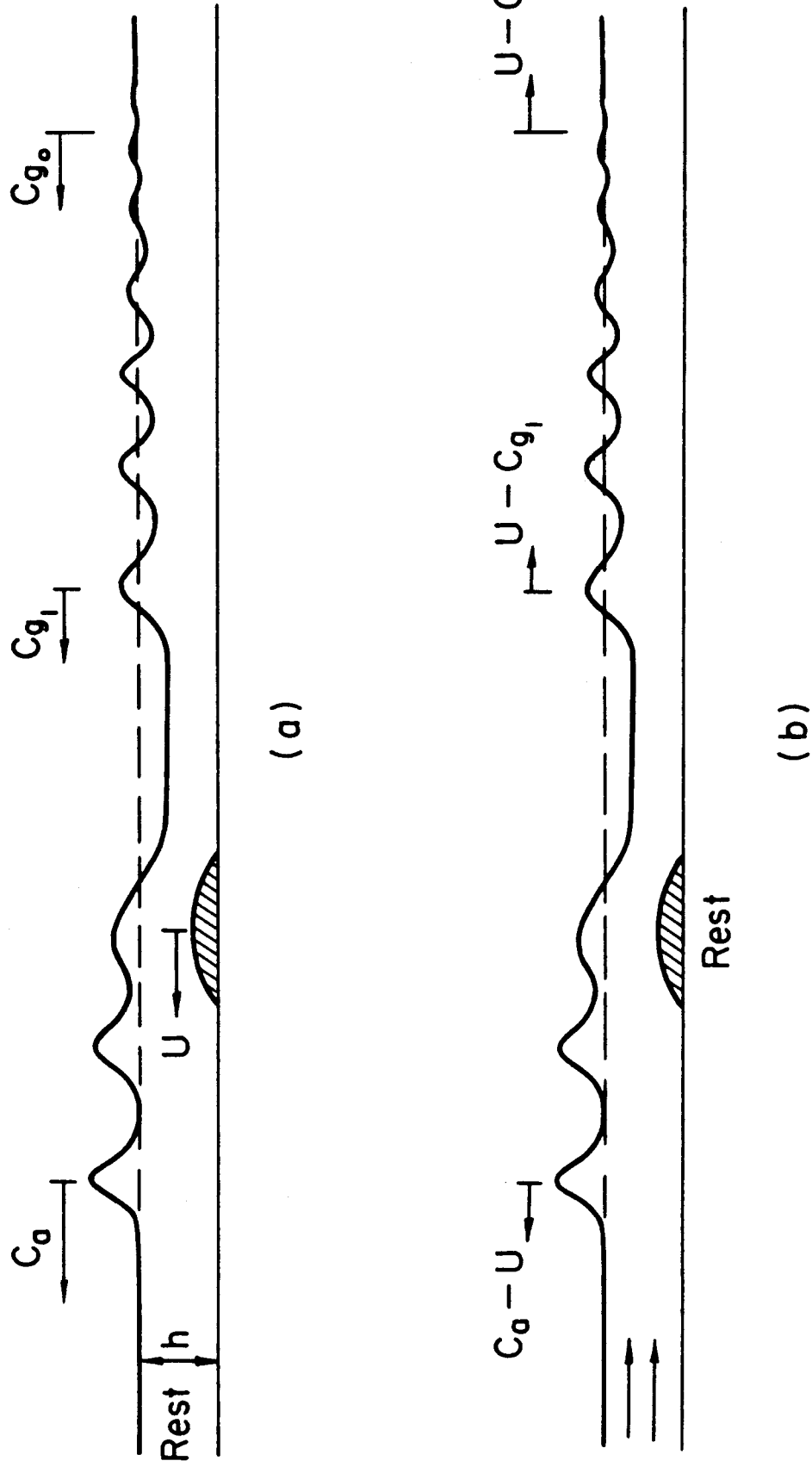


Figure 6. Generation of long waves as viewed in (a) the fluid frame of reference and in (b) the body frame of reference.