



Strongly Different Adhesion Reduction for 1D or 2D Random Fractal Roughness, and an Extension of the BAM Model to Anisotropic Surfaces

M. Ciavarella^{1,2} · F. Pérez-Ràfols³

Received: 20 July 2024 / Accepted: 26 August 2024
© The Author(s) 2024

Abstract

The influence of roughness on adhesion has been studied since the time of Fuller and Tabor, but recently there has been debate about how roughness exactly seems to kill (but sometimes enhance!) adhesion, particularly with reference to the accepted model of fractal roughness. We show that the Persson–Tosatti criterion does not depend on anisotropy of the surface for a typical power law PSD if written in terms of rms roughness and magnification. Instead, a very simple extension of the Bearing Area Model (BAM) of Ciavarella to anisotropic fractal surface shows a weak but clear dependence on the anisotropy, with higher adhesion in the 1D case, showing better agreement than the Persson–Tosatti criterion to actual numerical results of Afferrante Violano and Dini. However, neither of the two models permit to capture the strong hysteresis found in experiments between loading and unloading, which is very likely to enhance adhesion more as we move from the isotropic to the full 1D case. This suggests the mechanism of load amplification along contact lines and the associated elastic instabilities, is not captured by either the Persson–Tosatti or the BAM model applied to anisotropic surfaces.

Keywords Adhesion · Pull-off · Persson–Tosatti · BAM model · Roughness · Elastic instabilities

1 Introduction

Adhesion in contact mechanics has important applications in bio and soft materials whose low elastic modulus makes them easily conform even to relatively rough surfaces permitting exploitation of van der Waals adhesive forces [10]. Early investigation about the effect of roughness on adhesion was given by [12], who addressed what is also called as the adhesion "paradox" namely why relatively strong van der Waals adhesive forces are not felt at the macroscopic scale. Fuller and Tabor used the asperity Greenwood and Williamson (GW) model of identical spherical asperities of

radius R whose summit heights followed a Gaussian distribution. In fact, their theory seemed to fit reasonably well experimental results on rubber spheres contacting a perspex plane with varying roughness amplitudes, which may be fortuitous. Indeed, as discussed by [8], given the very strong assumptions in the theory, of identical and independent JKR asperities with PSD measured only at a crude resolution, and finally that they had really made an equivalence between the pull-off of the smooth case and that with all the asperities in the model detaching at the same time, the results cannot be quantitative.

Classical approaches to adhesion contact are based on short range adhesion (JKR theory) which permits an energetic treatment which is equivalent to Griffith theory of fracture if materials are elastic, and the DMT approach, which is applicable for more long-range adhesion (see, e.g., [10]). Johnson [17] demonstrated that for a single sinusoidal rough contact, the solution in terms of mean pressure divided by the mean pressure in full contact depended on a single parameter: the ratio of the surface energy in the nominal area A_0 to the elastic strain energy in the state of full contact for that area (Johnson's parameter)

✉ M. Ciavarella
mciava@poliba.it

¹ Politecnico di BARI, DMMM Department, Viale Gentile 182, 70126 Bari, Italy

² Department of Mechanical Engineering, Hamburg University of Technology, Am Schwarzenberg-Campus 1, 21073 Hamburg, Germany

³ Serra Hunter Fellow at Universitat Politècnica de Catalunya, EPSEM, 08243 Manresa, Spain

$$\alpha = \frac{\Delta\gamma A_0}{U_{el}} \tag{1}$$

and they found that when α is greater than a value of the order of 1, jump to full contact is predicted, i.e., the surface is very ‘sticky.’

1.1 Persson and Tosatti Criterion

Persson and Tosatti [25] later extended the concept to study the effect of any nominal shape of a body for which a JKR solution is known, by reducing the surface energy available for adhesion of an amount related to the strain energy in the state of full contact U_{el}

$$\Delta\gamma_{eff} = \frac{A}{A_0} \Delta\gamma - \frac{U_{el}}{A_0} \tag{2}$$

where A is not the real contact area, but rather an area in full contact, increased with respect to the nominal one A_0 , because of an effect of roughness-induced increase of contact area, $\frac{A}{A_0} > 1$, which attempted to model the occasional finding of an roughness-increased adhesion. An alternative much more powerful explanation for roughness enhanced adhesion was suggested much later [14], see also [5] which can be understood by comparing the JKR pull-off of a smooth sphere vs the possibly orders of magnitude higher pull-off force for a sphere with a rather contrived rough surface as a set of concentric waves. We shall return to this point when we discuss one of the main issues of this note.

The term U_{el} for a halfspace having roughness with isotropic power spectrum $C(q)$ ¹ is given by

$$\frac{U_{el}(\zeta)}{A_0} = \frac{\pi E^*}{2} \int_{q_0}^{q_1} q^2 C(q) dq = E^* l(\zeta) \tag{3}$$

where we have integrated over wavevectors in the range q_0 , q_1 , and we have introduced in (3) a length scale $l(\zeta)$ where $\zeta = q_1/q_0$ is the so called “magnification.” For a nominally flat rough surface, we neglect the effect of $\frac{A}{A_0}$ since this was introduced to attempt to explain the apparent roughness-induced increase of adhesion in some experiments with spheres, (where it is questionable anyway as it cannot really explain more than a factor 1.4–1.5 increase). Also, we assume that the range of adhesive forces remains the same, then we could write from (2,3) then the tension at pull-off for the contact of nominally flat rough surfaces should decrease with respect to the theoretical one σ_0 as

$$\frac{\sigma_{pulloff}}{\sigma_0} = \frac{\Delta\gamma_{eff}}{\Delta\gamma} = 1 - \frac{U_{el}}{\Delta\gamma A_0} = 1 - \frac{1}{\alpha(\zeta)} = 1 - \frac{l(\zeta)}{l_a} \tag{4}$$

where $l_a = \Delta\gamma/E^*$ is a characteristic length of adhesion and $\alpha(\zeta)$ a “generalized Johnson parameter” as defined in [9]. Therefore, the PT criterion gives complete absence of pull-off when $\alpha(\zeta) = 1$, which defines a “stickiness” range.

Recently, [11] have shown detailed experimental data of adhesion of soft spheres with rough substrates with roughness measured down to almost the atomic scale, confirming that the Persson and Tosatti theory gave satisfactory predictions of the apparent work of adhesion only during loading. Papangelo and Ciavarella [19] tried to combine the Guduru idea and a generalized Johnson parameter to explain the non-monotonic effect of roughness amplitude on pull-off of such spheres. However, recently [18] in generalizing Persson–Tosatti’s criterion to layered solids, found again a good correlation during loading, but an impossible correlation during unloading at least within the range of data observed, which showed no clear drop of the pull-off up to 90 nm of roughness, contradicting also the [19] at least in the form which was tuned to the Dalvi data.

Considering a typical power law isotropic PSD $C(q) = Zq^{-2(1+H)}$ for $q > q_0 = \frac{2\pi}{\lambda_L}$, where H is the Hurst exponent (equal to $3 - D$ where D is the fractal dimension of the surface), where $Z = \frac{H}{2\pi} \left(\frac{h_0}{q_0}\right)^2 \left(\frac{1}{q_0}\right)^{-2(H+1)}$ and $h_0^2 = 2h_{rms}^2$, for $H \neq 0.5$

$$l(\zeta) = \frac{\pi}{2} \int_{q_0}^{q_1} q^2 C(q) dq = \frac{\pi Z}{2} \int_{q_0}^{q_1} q^{-2H} dq = q_0 \frac{h_{rms}^2}{2} H \frac{\zeta^{-2H+1} - 1}{-2H + 1} \tag{5}$$

In Appendix, we show that the final result is equally valid for 1D or indeed anisotropic roughness showing that, according to the Persson–Tosatti criterion, there should be absolutely no change in between 1D and 2D roughness.

For the usual case of $H > 0.5$ (low D) the integral converges quickly, is relatively insensitive to high wavevector truncation and indeed for practical purposes we can use the limit value

$$l(\infty)_{lowD} = \pi \frac{h_{rms}^2}{\lambda_L} \frac{H}{2H - 1} \tag{6}$$

Persson [22] includes an extension of the Persson–Tosatti’s theory to partial contact but it implies a much more complex calculation.

¹ Notice we use the original Persson’s convention and notation for $C(q)$. See [25] and al other numerous papers by Persson on the subject.

1.2 Ciavarella’s BAM Criterion

In [7], it was shown that the "Persson–Tosatti" stickiness criterion (PT) has exactly the same qualitative form as the one obtained from a DMT like approach introduced by Ciavarella called BAM (Bearing Area Model) in [6], which adds the repulsive load solution of Persson (which is a simple closed form result for not too large compressive loads) to a geometric estimate of the adhesive force (which is also a simple closed form result). The two, BAM and Persson–Tosatti, give stickiness for a pure power law PSD only for (for relatively large enough magnification)

$$h_{\text{rms}} < \sqrt{\beta l_a \frac{2\pi}{q_0}} \tag{7}$$

where $\beta_{PT} = 0.24$ and $\beta_{BAM} = 0.6$ for pure power law PSD and $H = 0.8$, so that the prefactor are even quantitatively close — even closer they will appear considering the factor A/A_0 as discussed in [7].

The fact that neither Persson–Tosatti nor the Ciavarella BAM models can predict the difference between loading and unloading is clearly a strong limitation of both.

1.3 About Persson–Tosatti Versus Ciavarella’s BAM Model at High Fractal Dimensions

Wang and Müser [27] have a very detailed numerical investigation for the Hurst exponent $H = 0.8$, showing that each surface has its own clearly threshold for stickiness (he shows for example one case for $\alpha(\zeta) \simeq 1/2$), whereas by averaging over various realization, the threshold in $\alpha(\zeta)$ is more "blurred" (not clear or distinct). The single realization drop corresponds to a particularly dramatic decrease from a close-to-percolating contact to a single-asperity contact and an equally drastic reduction of repulsive contact area. For $H = 0.8$, $\alpha(\zeta) = \frac{l_a}{l(\zeta)} \sim 1/h_{\text{rms}}^2$, the factor $\beta_{BAM}/\beta_{PT} = 0.6/0.24 = 2.5$ means that the threshold for stickiness expected by BAM, given that by Persson–Tosatti is $\alpha(\zeta)|_{PT} = 1$, is instead $\alpha(\zeta)|_{BAM} = \frac{1}{2.5} = 0.4$, in agreement with some of the surfaces in [27].

More in general, however, [26] show that BAM’s prefactor β converges in the fractal limit (i.e., infinite magnification, where we don’t specify the large wavevector cutoff), whereas Persson–Tosatti doesn’t, and hence for high fractal dimension the two criteria differ. Indeed, for very high magnifications, and considering the case of high fractal dimensions $H < 0.5$

$$\begin{aligned} \lim_{\zeta \rightarrow \infty} \beta_{BAM} &= (0.9H - 0.1)^2, \\ \lim_{\zeta \rightarrow \infty} \beta_{PT}(\zeta) &= \frac{1}{\pi H} \frac{1 - 2H}{\zeta^{1-2H} - 1} \rightarrow 0 \end{aligned} \tag{8}$$

However, the value reported in [26] is relative to $\zeta > 10^7$, i.e., for very large magnifications. Even in BAM, in other words, there is a significant dependence on magnification, before reaching this limit, as it is very clear from [10], in particular their Fig. 6b.

BAM was found to predict well the [20] data for adhesive rough surfaces both at high and low fractal dimensions, see more discussion in [9, 10]. BAM was found in reasonable agreement also comparing with a more extensive data set of JTB theory [16]. The JTB theory is a very accurate model which builds on an elegant recursive and multiscale numerical procedure to determine the effect of adding a small packet of waves on the probability distribution function (PDF) of the local interfacial gap. By this procedure, it is seen that the PDF converges at large wavenumber, and hence they have definitively demonstrated that the problem of truncation does not occur, as was already suggested by the BAM model. In [10] we already gave detailed comparison between BAM results and the JTB theory [16] finding generally (at least qualitative) excellent agreement. In particular, while at low fractal dimensions, the curves of force-separation, pull-off, and effective adhesion energy are all almost completely independent on high truncation wavenumber, some small sensitivity appears at high fractal dimension namely a slow convergence toward a “fractal limit.” Nevertheless, a fractal limit clearly exists, and is found by both the JTB theory and BAM, whereas it doesn’t exist in the Persson and Tosatti theory.

2 An Extension of BAM’s Model to Anisotropic Rough Surfaces

We assume that the surfaces have a Gaussian height distribution. However, we consider anisotropic, self-affine surfaces, with a power spectrum given by [1, 3] with $q = \sqrt{q_x^2 + q_y^2}$

$$C(q) = \begin{cases} C(q_x^2 + \beta^2 q_y^2)^{-1-H} & q_0 \leq q \leq q_1 \\ 0 & \text{else} \end{cases} \tag{9}$$

where H is the Hurst exponent, q_x and q_y are the wave-numbers (related to wavelength as $q = 2\pi/\lambda$) and β controls anisotropy. The surfaces considered have $H = 0.8$, which is a common value in real surfaces [23], and cut-off wave-numbers given by $q_0 = 8(2\pi/L)$ and $q_1 = 64(2\pi/L)$, where L is the size of the domain. The appropriateness of these values for isotropic surfaces is discussed in [21]. The approach by [15] is used to generate the Gaussian surface. Figure 1 show, as an example, the shape of the various surfaces considered.

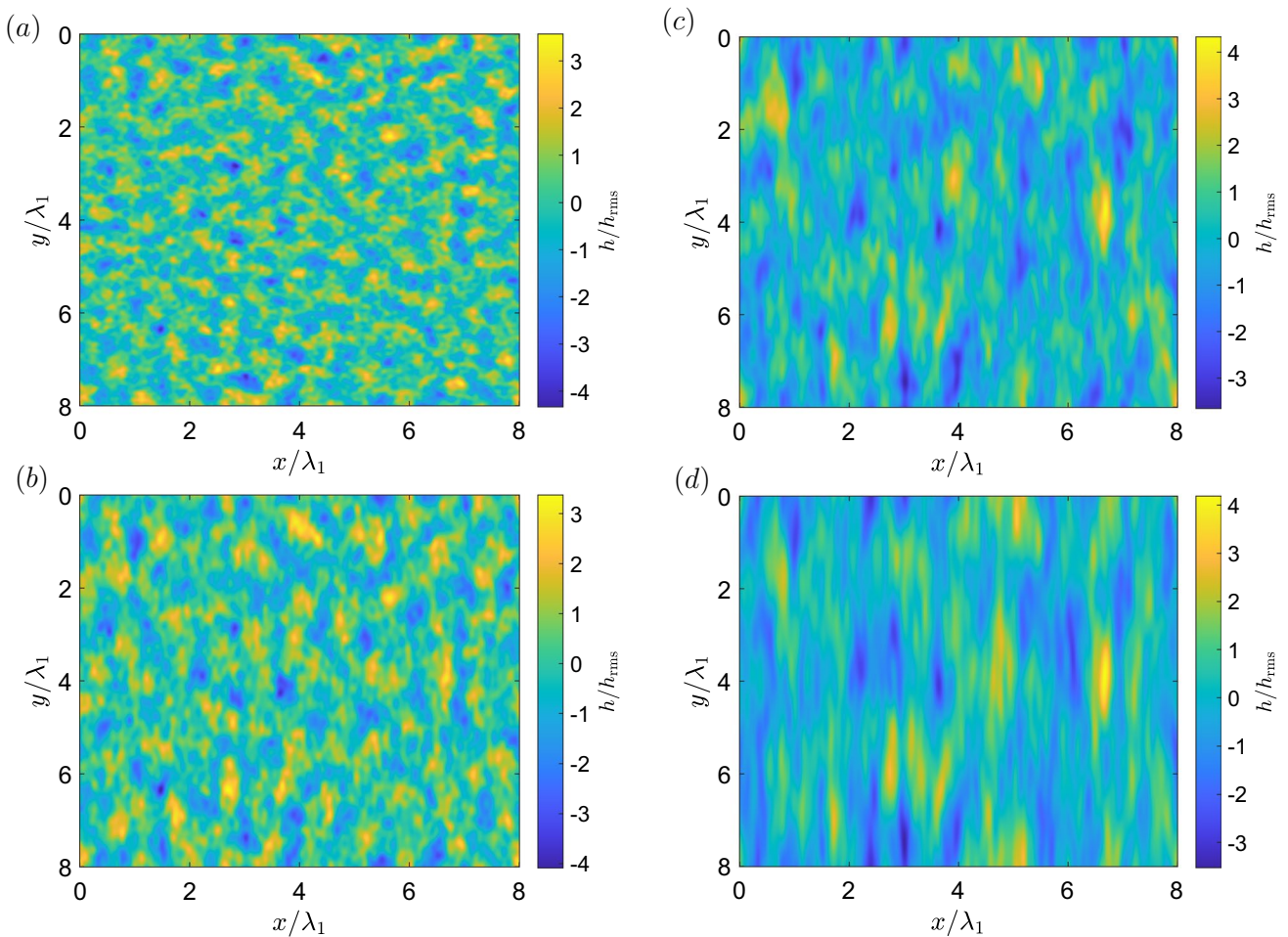


Fig. 1 Examples of the surfaces considered with increasing anisotropy from **a** $\beta = 1$ (isotropic), **b** $\beta = 2$ **c** $\beta = 4$ and **d** $\beta = 8$

However, for later comparisons we also use a roll-off region where the PSD is flat, which improves the Gaussianity of the surfaces, and corresponds to the PSD of [2].

The extension of BAM is not in closed form, because the load-separation repulsive solution from Persson does not exist, so we use the same BEM code already discussed in [21], while the adhesive contribution comes from geometric estimates which depend only on the height distribution, and hence this term is identical to the original BAM theory and is the difference between two error functions [6]. In particular, the BAM model is *exact* for a single sphere, and assumes a Maugis-Dugdale force separation law, so that the adhesive traction can be computed as $\sigma_0 A_{ad}$, where σ_0 is cohesive strength (the surface energy is therefore $\Delta\gamma = \sigma_0\epsilon$ where ϵ is range of attractive forces). Deformations are neglected to estimate the adhesive area as

$$A_{ad}(\bar{u}) \approx B(\bar{u} + \epsilon) - B(\bar{u}), \tag{10}$$

where $B(\bar{u})$ is the classical *bearing area*, that is the area over which the bodies, if taken as rigid, would interpenetrate each other when moved together up to a mean separation \bar{u} . For a Gaussian nominally flat surface, either isotropic or anisotropic, the results does not change, and is

$$\frac{A_{ad}}{A_0} = \frac{1}{2} \left[\text{Erfc} \left(\frac{\bar{u} - \epsilon}{\sqrt{2}h_{rms}} \right) - \text{Erfc} \left(\frac{\bar{u}}{\sqrt{2}h_{rms}} \right) \right] \tag{11}$$

where h_{rms} is rms amplitude of roughness.

The repulsive pressure at mean separation \bar{u} which is easily obtained with Persson’s theory only for isotropic surfaces. This gives, for a surface with the pure power law spectrum and $H = 0.8$ for sufficiently small loads,

$$\frac{p_{rep}(\bar{u})}{E^*} \approx q_0 h_{rms} \exp \left(\frac{-\bar{u}}{\gamma h_{rms}} \right), \tag{12}$$

where $\gamma \approx 0.5$ is a corrective factor. For anisotropic surfaces, we estimate $p_{rep}(\bar{u})$ by the BEM numerical code.

3 Results and Comparisons

In a recent paper, Afferrante et al. (AVD in some figure captions in the following [2]), the authors conduct a Finite Element investigation on random rough 1D profile, adding some experimental investigation (which are however on 2D roughness so cannot really be compared with the numerical results), but their 1D results are interesting and deserve some further comment. They seem to conclude that adhesion is very different for small or large fractal dimensions, and that their results support Persson–Tosatti stickiness criterion rather than the Ciavarella’s BAM.² There is to date no quantitative criterion for adhesion or stickiness in 1D profiles, and no comparison has been attempted with the Persson–Tosatti criterion, but, as we show in Appendix, the Persson–Tosatti criterion predicts no difference at all in the two types of roughness.

In [2] case, the Young modulus is $E = 380\text{MPa}$, Poisson’s ratio $\nu = 0.49$, surface energy $\Delta\gamma = 0.05\text{J/m}^2$ and hence $l_a = \Delta\gamma/E^* = \frac{0.05}{380 \times 10^6} (1 - 0.49^2) = 0.1\text{ nm}$. Also, there is a small region of roll-off in the PSD, but there is no great effect if we assume a pure power law by assuming an average value for small wavevector cutoff between the long cutoff and the roll-off cutoff, at $q_0 \simeq 2 \times 10^6\text{m}^{-1}$. We gather from the figures at very low roughness that $\sigma_0 \simeq 0.05E^* = 0.05 \times \frac{380 \times 10^6}{(1 - 0.49^2)} = 25\text{MPa}$ which is in any case an appropriate value for a Lennard Jones force-separation law, leading to $\varepsilon = \frac{0.05}{25 \times 10^6} = 2\text{ nm}$.

First of all, we reproduce in Fig. 2 the results of Fig. 3 in [2] with starred marks on retraction and with a power law trend curve fitting the data as solid black line, where $h_{\text{rms}} = 20\text{ nm}$, and $\zeta = 64$, together with the prediction from the Persson–Tosatti criterion (blue solid line), and those of the new extended BAM criterion with $\beta = 1, 2, 4, 8$ (dashed lines with red, green, purple, and magenta lines), where we have used the same PSD of the AVD paper. Figure 2a in particular contains the data of $H = 0.8$, while Fig. 2b the case of $H = 0.4$. As it is clear from the figure, adhesion persists in 1D roughness for one or two orders of magnitude larger rms roughness. Precise details depend on how we decide to define stickiness, since here pull-off seems to decay as an exponential function so there is not a real threshold as there is instead in the Persson–Tosatti, and in the BAM criteria.

In the AVD paper the calculation involves obviously an approach phase and a retraction phase. Both Persson–Tosatti and Ciavarella’s theory do not distinguish between approach and retraction so it may be argued that they may work better

² In the AVD paper the calculation involves a long range LJ potential so both the Persson–Tosatti or BAM simple models are not expected to give the same results as they involve hard contact calculations.

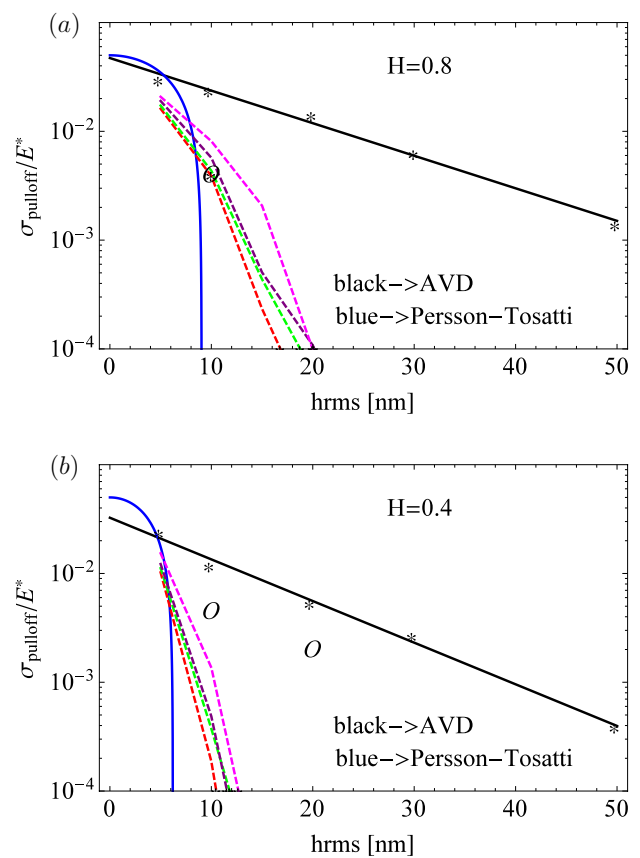


Fig. 2 Pull-off should decay $\frac{\sigma_{\text{pull-off}}}{E^*}$ as a function of rms roughness h_{rms} for the case discussed in Fig. 3 in [2] where magnification $\zeta = 64$. The numerical results of [2] are reported as starred marks (“*”) on retraction and “O” on approach, and with a power law trend curve fitting the data on retraction as solid black line, while the prediction from the Persson–Tosatti criterion predictions for isotropic or 1D roughness (4) (blue solid line), and those of the new extended BAM criterion with $\beta = 1, 2, 4, 8$ (dashed lines with red, green, purple, and magenta lines), where we have used the same PSD of the AVD paper. **a** $H = 0.8$, **b** $H = 0.4$ (Color figure online)

during approach. We have therefore plotted in Fig. 2 data both on approach (open “O” symbols) and on retraction (“*” stars symbol, which follow a power law fit). Although the data are clearly in much better agreement on approach, and particularly for the BAM method, the agreement is not perfect. The disagreement is particularly evident for the large fractal dimension case ($H=0.4$), and hence it is not due to the possible effect of long-range adhesion. In these respects, a theory like [24] one is likely to be extremely similar in results to BAM (in fact, they are based on the very similar DMT assumptions), except that Persson–Scaraggi makes a more complex convolution of gap distribution with possible long-range adhesion pressure-gap relationships, and hence not likely to explain the difference we observe with the FEM results of AVD.

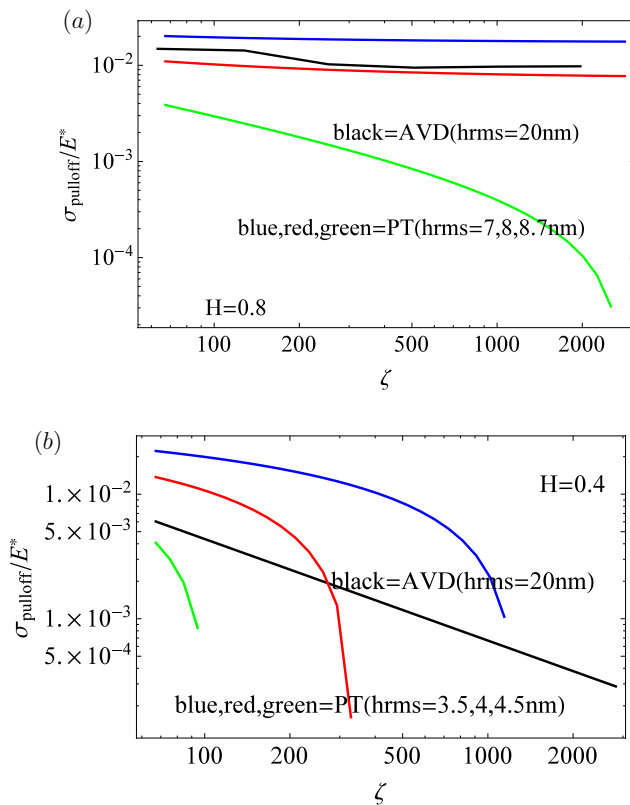


Fig. 3 Pull-off should decay $\frac{\sigma_{\text{pulloff}}}{E^*}$ with magnification ζ for $h_{\text{rms}} = 20\text{ nm}$ in the Fig. 5 case of [2] (black solid line for $H = 0.8$ (a) $H = 0.4$ (b)). Color lines are the Persson–Tosatti predictions for isotropic PSD roughness (4) for $h_{\text{rms}} = 7, 8, 8.7\text{ nm}$ (a) and $3.5, 4, 4.5\text{ nm}$ (b) (Color figure online)

The difference between approach and retraction clearly shows that there is a different mechanism at play, which is likely that of elastic instabilities of Guduru [14], see also [5] because the contact proceed on infinitely long contact lines where the load is amplified by the small scale roughness and produces instabilities and dissipation when the solution jumps from one crest to another crest of roughness. The new extended BAM criterion gives a better prediction than Persson–Tosatti in both cases of fractal dimensions, and gives also a correct trend of increasing adhesion toward the 1D case, but the dependence is anyway weak.

The difference in the results between the two fractal dimensions is significant, but not large enough to make definitive statements, neither in the simulations nor in the Persson–Tosatti criterion, also because the magnification is low, $\zeta = 64$. In any case BAM gives a better prediction than the Persson–Tosatti criterion.

Moving to the results in Fig. 5 in [2] which report the pull-off should decay $\frac{\sigma_{\text{pulloff}}}{E^*}$ with magnification ζ for $h_{\text{rms}} = 20\text{ nm}$, Fig. 2 shows their results as black solid line for $H = 0.8$ (a) $H = 0.4$ (b) as "AVD." To avoid confusion,

we report separately the case of $H = 0.8$ (Fig. 3a), and $H = 0.4$ (Fig. 3b). Once again, we find the 1D rough profiles are much more sticky than the isotropic counterpart, so we don't show in the figure the Persson–Tosatti prediction for the correct roughness amplitude, which would have no pull-off value at all. Instead, we report much smaller rms amplitudes for comparative purposes, namely to evaluate if the expected trend with magnification is correctly predicted, since this is one of the main points raised by [2], and by the debate about low vs high fractal dimension.

For the case of $H = 0.8$ (a) we find that PT criterion would lead to expect similar pull-off for a surface with roughness of about $h_{\text{rms}} = 8\text{ nm}$, and that there is indeed almost no dependence on magnification for rms roughness smaller than the threshold, but notice that at just larger roughness $h_{\text{rms}} = 8.7\text{ nm}$ (which is near the threshold for stickiness) a very large dependence of pull-off on magnification appears. Moving to the case of high fractal dimension $H = 0.4$, Fig. 3b shows that there is no rms amplitude of roughness which seems to imitate reasonably well the behavior of simulations, although close to correct prediction occurs for $h_{\text{rms}} = 4\text{ nm}$, i.e., a much larger distance appears between the Persson–Tosatti prediction and the effective 1D simulation. This is a sign that the larger dependence on magnification for high fractal dimensions of the Persson–Tosatti criterion may not be correct at all, which is somehow in contrast to the conclusions in [2].

Regarding BAM, using the BAM criterion for isotropic surfaces, we can say that at very high magnifications ($\zeta > 10^7$) the limit for stickiness is

$$h_{\text{rms}} < (0.9H - 0.1) \times 17 \text{ [nm]} \quad (13)$$

so $H = 0.8$ gives $h_{\text{rms}} < 10.5\text{ nm}$ and $H = 0.4$ gives 4.4 nm : all the profiles in the simulations again are predicted not sticky. So in this sense also BAM does not seem to work for these 1D profiles, and on the other hand, the 1D profiles look much more adhesive than their expected isotropic counterparts. However, considering results already obtained in [10], convergence of pull-off with magnification can require ζ as large as $\zeta = 10^4$ or 10^6 and hence the qualitative trend in Fig. 5 in [2] are not incompatible with BAM results, although at a different rms amplitude of roughness.

4 Discussion

As recollected in the introduction, the BAM criterion for stickiness and the entire prediction of pull-off was found reasonably good to predict well the [20] data for adhesive rough surfaces both at high and low fractal dimensions, and also in comparison with the extensive data set of JTB theory [16], again at both low and high fractal dimensions. The JTB

theory is a strong reason to believe the fractal limit is well defined and there must be convergence of the gap between surfaces, as well as pull-off and other quantities with magnification. After all, when the surfaces introduce amplitude of roughness below the range of attraction of the adhesive forces, no difference can be felt by the problem.

There are many papers now which show that there is a strong difference between loading and unloading (hysteresis) and this is also geometry-dependent. For the simulations of ideally nominally flat surfaces, there is no possibility of roughness-enhancement since the theoretical limit with no roughness cannot be overpassed. The situation changes when we explore pull-off of spheres, where the JKR result for a smooth sphere can be overpassed, and indeed has been found experimentally to be overpassed by a factor of more than one order of magnitude with very particular roughness (1D or axisymmetric) by [14].

In the paper [2], the authors use a 1D profile, but argue about the isotropic Persson–Tosatti criterion. This has motivated the present authors to check if there is any dependence in the Persson–Tosatti criterion on the anisotropy of the surface, and we demonstrated here that there is none. Next, we generalized the BAM model of [6] to anisotropic surfaces, and we show here that it leads to a distinct, although weak, dependence on anisotropy, namely adhesion increases with anisotropy and is highest for 1D surface, in qualitative agreement with theoretical experimental or numerical observations [4, 14], but not in quantitative one, because BAM is essentially a DMT model, and cannot capture the mechanism of adhesion increase by elastic instabilities.

It is certainly true that small scale wavelengths affect adhesion in a more pronounced form for high fractal dimension, as it was found already in the JTB theory, but also in BAM, see Fig. 6b in [10], but the trend predicted by the Persson–Tosatti criterion doesn’t seem to agree well with the 1D simulations of [2], as we have discussed.

5 Conclusion

We have shown that for power law PSD, the Persson–Tosatti criterion would give the same stickiness for 1D or 2D rough surfaces, in terms of rms amplitude and magnification of the profile/surface. However, even considering the possible order of error for 2D surfaces in the approximate Persson–Tosatti criterion, clearly the results of the 1D simulations in [2] show a much enhanced adhesion which is likely due to the nature of the 1D roughness and the elastic amplifications of the load along infinitely long contact lines, an effect already noticed in single sinusoidal roughness [14], see also [5], but also in 1D rough profiles previously [4].

We have therefore extended the BAM model of [6] to anisotropic surfaces, and found that it has a weak dependence

on anisotropy, and in the correct direction of increasing adhesion toward the 1D limit case.

Concerning the effect of fractal dimensions, it is certainly true that the necessarily limited simulations in [2] seem to show a reduced dependence on magnification, but the Persson–Tosatti and BAM criteria also show a possibly large dependence on magnification both at large and small fractal dimensions, so in these respects, the debate is not conclusive, except that they certainly do not apply precisely for 1D roughness.

Appendix: The Energy to Flatten a 1D Profile

The term U_{el} for a halfspace having roughness of height $h(x, y)$ with isotropic power spectrum $C(q)$ is given by (3). Here, 2D rough surfaces PSD is defined as³

$$C(\mathbf{q}) = \frac{1}{(2\pi)^2} \int d^2x \left\langle h(\mathbf{0})h(\mathbf{x}) \right\rangle e^{-i\mathbf{q}\cdot\mathbf{x}} \tag{14}$$

and 1D rough surfaces PSD is

$$C_{1D}(q) = \frac{1}{2\pi} \int dx \left\langle h(0)h(x) \right\rangle e^{-iqx} \tag{15}$$

where the connection between 1D and 2D PSD is

$$C_{2D}(\mathbf{q}) = \delta(q_y)C_{1D}(q_x) \tag{16}$$

It is known that 1D rough surfaces with PSD and 2D isotropic rough surfaces with PSD $C_{2D}(|\mathbf{q}|)$ behave exactly in the same way if they have the same angular average (see Geike and Popov [13]) and hence if

$$C_{1D}(q) = \pi q C_{2D}(q) \tag{17}$$

Therefore, this means that for 1D random rough profiles,

$$\frac{U_{el}(\zeta)}{A_0} = \frac{E^*}{2} \int_{q_0}^{q_1} q C_{1D}(q) dq. \tag{18}$$

Now having a power law PSD $C_{1D}(q) = Zq^{-1-2H}$ for $q_1 > q > q_0$,

$$h_{rms}^2 = 2 \int_{q_0}^{q_1} C_{1D}(q) dq = Z \frac{q_0^{-2H}}{H} \tag{19}$$

we get $Z = Hh_{rms}^2 q_0^{2H}$ and hence

³ Notice we use the original Persson’s convention and notation for $C(q)$, whereas other authors use $C_{2D}(q) = 4\pi^2 C(q)$.

$$l(\zeta) = \frac{1}{2} \int_{q_0}^{q_1} q C_{1D}(q) dq = \frac{Z}{2} \int_{q_0}^{q_1} q^{-2H} dq \quad (20)$$

$$= \frac{H h_{\text{rms}}^2 q_0}{2(1-2H)} (\zeta^{-2H+1} - 1)$$

which is exactly the same as the 2D counterpart (5). Therefore, the Persson–Tosatti criterion for 1D roughness is identical to the isotropic roughness counterpart, when written in terms of rms roughness and magnification for a power law PSD. The same result can also be obtained of course starting directly from a single sinusoid $h(x) = h_0 \cos(qx)$ in a plane contact [17] and integrating the strain energy of a single wave which is

$$U_0 = \frac{1}{2} h(x) p(x) = E^* \frac{q h_0^2}{8}. \quad (21)$$

More in general, for anisotropic, self-affine surfaces, with a power spectrum as used in this paper, using cylindrical coordinates, $q_x = q \cos \theta$ and $q_y = q \sin \theta$ so that $d^2q = q dq d\theta$

$$U_{\text{el}} = \frac{A_0 E^* Z}{4} \int_{q_0}^{q_1} \int_0^{2\pi} \frac{q^{-2H} dq d\theta}{(\cos^2 \theta + \beta^2 \sin^2 \theta)^{1+H}} \quad (22)$$

$$= \frac{A_0 E^* Z}{4} I(\beta, H) \int_{q_0}^{q_1} q^{-2H} dq$$

with $I(\beta, H) = \int_0^{2\pi} \frac{d\theta}{(\cos^2 \theta + \beta^2 \sin^2 \theta)^{1+H}}$. Now writing

$$\int_{q_0}^{q_1} q^{-2H} dq = q_0^{1-2H} \frac{1 - \zeta^{-2H+1}}{2H-1}, \text{ we get} \quad (23)$$

$$U_{\text{el}} = \frac{A_0 E^* Z}{4} I(\beta, H) q_0^{1-2H} \frac{1 - \zeta^{-2H+1}}{2H-1}$$

On the other hand,

$$h_{\text{rms}}^2 = \int d^2q C(q) = Z I(\beta, H) \int_{q_0}^{q_1} q^{-1-2H} dq \simeq Z I(\beta, H) \frac{q_0^{-2H}}{2H} \quad (24)$$

Hence,

$$U_{\text{el}} = \frac{A_0 E^*}{2} H h_{\text{rms}}^2 q_0 \frac{1 - \zeta^{-2H+1}}{2H-1} \quad (25)$$

which is the same result obtained for isotropic roughness, or for 1D roughness.

Acknowledgements MC acknowledges support from the Italian Ministry of Education, University and Research (MIUR) under the program "Departments of Excellence" (L.232/2016). The authors thank prof. J.R.Barber of University of Michigan for discussion about the Appendix.

Author Contributions The author being a sole author, has made contribution to all the parts of the process of the paper.

Funding Open access funding provided by Politecnico di Bari within the CRUI-CARE Agreement. The authors declare that no funds,

grants, or other support were received during the preparation of this manuscript.

Data Availability The data that support the findings of this study are available from the corresponding author, Michele Ciavarella, upon reasonable request.

Declarations

Conflict of interest The authors have no conflict of interest to declare that are relevant to the content of this article.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

- Afferrante, L., Putignano, C., Menga, N., Carbone, G.: Friction in rough contacts of linear viscoelastic surfaces with anisotropic statistical properties. *Eur. Phys. J. E* **42**, 1–8 (2019)
- Afferrante, L., Violano, G., Dini, D.: How does roughness kill adhesion? *J. Mech. Phys. Solids* (2023). <https://doi.org/10.1016/j.jmps.2023.105465>
- Carbone, G., Lorenz, B., Persson, B.N., Wohlers, A.: Contact mechanics and rubber friction for randomly rough surfaces with anisotropic statistical properties. *Eur. Phys. J. E* **29**, 275–284 (2009)
- Carbone, G., Pierro, E., Recchia, G.: Loading-unloading hysteresis loop of randomly rough adhesive contacts. *Phys. Rev. E* **92**, 062404 (2015)
- Ciavarella, M.: On roughness-induced adhesion enhancement. *J. Strain Anal. Eng. Des.* **51**, 473–481 (2016). <https://doi.org/10.1177/0309324716653003>
- Ciavarella, M.: A very simple estimate of adhesion of hard solids with rough surfaces based on a bearing area model. *Meccanica* **53**, 241–250 (2018)
- Ciavarella, M.: Universal features in “stickiness” criteria for soft adhesion with rough surfaces. *Tribol. Int.* **146**, 106031 (2020)
- Ciavarella, M.: Comments on old and recent theories and experiments of adhesion of a soft solid to a rough hard surface. *Tribol. Int.* **155**, 106779 (2021)
- Ciavarella, M., Papangelo, A.: A generalized Johnson parameter for pull-off decay in the adhesion of rough surfaces. *Phys. Meso-mech.* **21**, 67–75 (2018). <https://doi.org/10.1134/S102995991801009>
- Ciavarella, M., Papangelo, A.: Extensions and comparisons of BAM (bearing area model) for stickiness of hard multiscale randomly rough surfaces. *Tribol. Int.* **133**, 263–270 (2019)
- Dalvi, S., Gujrati, A., Khanal, S.R., Pastewka, L., Dhinojwala, A., Jacobs, T.D.: Linking energy loss in soft adhesion to surface roughness. *Proc. Natl. Acad. Sci.* **116**(51), 25484–25490 (2019)

12. Fuller, K.N.G., Tabor, D.: The effect of surface roughness on the adhesion of elastic solids. *Proc. R. Soc. Lond. A* **345**(1642), 327–342 (1975)
13. Geike, T., Popov, V.L.: Mapping of three-dimensional contact problems into one dimension. *Phys. Rev. E* **76**(3), 036710 (2007)
14. Guduru, P.R.: Detachment of a rigid solid from an elastic wavy surface: theory. *J. Mech. Phys. Solids* **55**, 445–472 (2007). <https://doi.org/10.1016/j.jmps.2006.09.004>
15. Hu, Y.Z., Tonder, K.: Simulation of 3-D random rough surface by 2-D digital filter and Fourier analysis. *Int. J. Mach. Tools Manuf.* **32**, 83–90 (1992)
16. Joe, J., Thouless, M.D., Barber, J.R.: Effect of roughness on the adhesive tractions between contacting bodies. *J. Mech. Phys. Solids* **118**, 365–373 (2018)
17. Johnson, K.L.: The adhesion of two elastic bodies with slightly wavy surfaces. *Int. J. Solids Struct.* **32**(3–4), 423–430 (1995)
18. Kumar, S., Gaire, B., Persson, B., Dhinojwala, A.: Impact of nanometer-thin stiff layer on adhesion to rough surfaces (2024). [arXiv: 2402.15014](https://arxiv.org/abs/2402.15014)
19. Papangelo, A., Ciavarella, M.: A criterion for the effective work of adhesion in loading and unloading of adhesive soft solids from rough surfaces. *Tribol. Lett.* **69**(1), 9 (2021)
20. Pastewka, L., Robbins, M.O.: Contact between rough surfaces and a criterion for macroscopic adhesion. *Proc. Natl. Acad. Sci.* **111**(9), 3298–3303 (2014)
21. Perez-Rafols, F., Ciavarella, M.: On the effect of the height distribution function in adhesion of non gaussian random rough surfaces, and a simple analytical model (bam). *Tribol. Int.* **193**, 5 (2024)
22. Persson, B.N.J.: Adhesion between an elastic body and a randomly rough hard surface. *Eur. Phys. J. E* **8**, 385–401 (2002)
23. Persson, B.N.J.: On the fractal dimension of rough surfaces. *NanoSci. Technol.* **31**, 235–248 (2015)
24. Persson, B.N., Scaraggi, M.: Theory of adhesion: role of surface roughness. *J. Chem. Phys.* **141**, 124701 (2014)
25. Persson, B.N.J., Tosatti, E.: The effect of surface roughness on the adhesion of elastic solids. *J. Chem. Phys.* **115**(12), 5597–5610 (2001)
26. Violano, G., Papangelo, A., Ciavarella, M.: Stickiness of randomly rough surfaces with high fractal dimension: is there a fractal limit? *Tribol. Int.* **159**, 106971 (2021)
27. Wang, A., Müser, M.H.: Is there more than one stickiness criterion? *Friction* **11**(6), 1027–1039 (2023)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.