

Using Particle Swarm Optimization for Source Seeking in Multi-Agent Systems

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Abstract: This paper presents a novel approach to the source seeking problem, where a group of mobile agents tries to locate the maximum of a scalar field defined on the space in which they are moving. The agents know their position and the local value of the field, and by communicating with their neighbors estimate the gradient direction of the field. A distributed cooperative control scheme is then designed that drives the group towards the maximum while maintaining a specified formation. Previously proposed control schemes that are based on a combination of H_∞ -optimal formation control and local gradient estimation suffer from premature convergence to local maxima. To overcome this problem, here the use of particle swarm optimization for locating the global maximum is proposed. Agents take the role of particles and an information flow filter approach is employed to separate the consensus dynamics from the local feedback loops governing the agent dynamics. Stability of the overall scheme is established based on the small gain theorem, and by decomposing the synthesis problem for the distributed information flow filter the problem size is reduced to that of a single agent. Simulation results with multiple maxima and quadcopter models as agents illustrate the practicality of the approach.

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1. INTRODUCTION

Every source of a physical quantity such as toxic gas or atomic radiation causes a scalar field distribution in space with a global maximum at the source. A group of mobile agents can be used to locate the source by measuring the value of the scalar field distribution at their location, communicating the measurements and their position and calculating a direction pointing towards the source.

Previous solutions to this problem are presented, amongst others, in Rosero and Werner (2014a) and Rosero and Werner (2014b) using local gradient estimation together with a cooperative formation control scheme for a fixed communication topology. The drawback of this solution is that it has a strong tendency to converge to a local maximum and thus fails for scalar fields without a single isolated maximum, causing an unrealistic restriction for scenarios with multiple sources or disturbed scalar fields. Therefore a different source seeking scheme needs to be used.

A candidate can be found from the analogy to finding extrema of a nonlinear function, for which evolutionary schemes like particle swarm optimization (PSO) schemes (Zelinka et al. (2013)) are used. Regarding PSO the fully informed particle swarm (FIPS) method has been shown

by Mendes et al. (2004) to consistently find the optimum of a nonlinear function with acceptable effort. This method uses virtual particles with a fixed communication topology to find the optimum. Here we adopt this technique as part of a control scheme for multi-agent systems (MAS), in which the agents take the role of particles. This approach has already been used in robotic context in Doctor et al. (2004) and Hereford (2006).

In contrast to the particles defined as point masses, the agents are physical systems with more complicated dynamics. An elegant way to handle this is by separating the agent dynamics from the consensus. Such a decoupling of the agents has been originally proposed by Fax and Murray (2004) and further developed towards the information flow filter design by Pilz et al. (2011). This framework consists of a consensus loop to jointly determine the position reference of every agent and local feedback loops to control the agent dynamics. Within the consensus loop the agents can thus be handled similar to the particles of the PSO scheme. Stability criteria for the consensus-based formation control scheme are given in Pilz et al. (2011) and Bartels and Werner (2014), the latter based on Popov and Werner (2012) and Pilz and Werner (2012), where a synthesis approach is proposed. It has been shown by Bartels and Werner (2014) that the performance and

synthesis cost of the formation control can be improved by using a consensus-based approach instead of a cooperative approach.

The main contribution of this paper is that it jointly models the search algorithm, the agent dynamics and the communication topology to synthesize a information flow filter which is optimized under consideration of the model. This poses the challenge to adapt the PSO approach for application to the consensus-based formation control scheme which includes real systems with complex dynamics instead of virtual particles with simple dynamics. An additional challenge is to guarantee stability and establish a satisfactory performance in finding the global maximum of an unknown scalar field distribution.

The outline of this paper is to establish a general control scheme and to introduce the different parts in Section 4 after the problem statement in Section 2 and the needed fundamentals in Section 3. Afterwards the source seeking scheme based on the PSO scheme will be presented together with a stability criterion in the second part of Section 4. Additionally, a gradient-based source seeking scheme will be presented which serves as a comparison for the simulations presented in Section 5.

1.1 Notation

I_q denotes the $q \times q$ identity matrix. The set of $p \times q$ matrices is denoted by $\mathbb{R}^{p \times q}$. For the i -th row and the j -th column entry of a matrix A the notation A_{ij} is used. To describe the parallelism and communication, two Kronecker extensions are used: These are $M_{(q)} = M \otimes I_q$ and $\hat{M} = I_N \otimes M$.

2. PROBLEM STATEMENT

Consider a scalar field describing for example a scenario with some sort of contamination. It assigns a level of contamination $\psi(x)$ to every position $x \in \mathbb{R}^3$. In this paper it will be assumed that the scalar field is time-invariant and continuous with respect to x . It also has some upper bound and is allowed to have multiple maxima.

Additionally consider N agents with equal dynamics $P(s)$. Every agent i measures its position y_i as well as the scalar field value $\psi(y_i)$ and receives these measurements from its neighbor agents j , which form the set $\mathcal{N}_i \subseteq \mathcal{V}$. The communication is fixed and specified by the communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the set of nodes \mathcal{V} representing the agents and the set of edges \mathcal{E} representing the communication links. From the graph the normalized adjacency matrix $\mathcal{A} \in \mathbb{R}^{N \times N}$ can be derived element-wise as

$$\mathcal{A}_{ij} = \begin{cases} \frac{1}{|\mathcal{N}_i|} & \text{if } i \neq j \text{ and } (i, j) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}. \quad (1)$$

The normalized adjacency matrix can be used to define the normalized Laplacian \mathcal{L} as $\mathcal{L} = I_N - \mathcal{A}$.

The goal of this paper is to find a distributed control scheme which establishes the desired formation specified by the reference r and leads the agents towards the global maximum of the scalar field. This can be expressed in

$$\lim_{t \rightarrow \infty} \frac{1}{N} \sum_i y_i(t) = y_{max}$$

where y_{max} is the location of the global maximum of ψ . To achieve this goal only the information of the respective agent and its neighbors is to be used.

3. PRELIMINARIES

This section presents the fundamentals taken from literature. The first one is the idea of FIPS which will be required for Section 4 since it is adapted there for source seeking. The second one is the gradient-based source seeking which is required for comparison in Section 5.

3.1 Fully Informed Particle Swarm

The Fully Informed Particle Swarm is based on the idea that the search space is scanned by a large number of particles with simple dynamics to find the maximum or minimum of a nonlinear function $f(y)$:

$$\max_y f(y), f : \mathbb{R}^q \rightarrow \mathbb{R}$$

The equations for the dynamics of the particles are originated from Mendes et al. (2004) and shown in equation 2 for the velocity and equation 3 for the position.

$$v(t + t_s) = \chi \cdot (v(t) + \phi \cdot (m(t) - y(t))) \quad (2)$$

$$y(t + t_s) = y(t) + v(t) \quad (3)$$

In the equations $v(t) \in \mathbb{R}^q$ and $y(t) \in \mathbb{R}^q$ denote the velocity and the position at time t , respectively and t_s is the sampling time. The parameters ϕ and χ are randomly chosen in every time step and uniformly distributed in an interval from 0 to some upper value. The parameter $m(t)$ is calculated for every particle i individually as

$$m_i = \frac{\sum_{k \in \mathcal{N}_i} \varphi_{ik} y_k^* + \varphi_{ii} y_i^*}{\sum_{k \in \mathcal{N}_i} \varphi_{ik} + \varphi_{ii}}, \quad (4)$$

where $\varphi_{ik}, \varphi_{ii}$ are random factors which are uniformly distributed in the interval $[0, \varphi_{max}]$ where φ_{max} is a fixed upper bound and y_i^* is evaluated as

$$y_i^* = \arg \max_{y_i \in T_i} f(y_i), \quad (5)$$

denoting the position on the trajectory T_i of agent i where this agent has evaluated the highest value of the function $f(y)$ of all times from 0 to t . This value is also received from the neighbors denoted as y_k^* . The value m_i represents a randomly weighted average of the own and the neighbors' "best" positions.

The strategy of this scheme is that the particles communicate the values y_k^* to each other, calculate the $m_i(t)$ in each time step and then, according to (2), change their velocity randomly weighted towards the position $m_i(t)$. This means that the signal $m_i - y_i$ acts similar to a gradient pointing towards the global maximum and the particles move towards this position. The random parameters allow the agents to search in an area between their current positions for higher function values. The described process continues until a consensus has been reached.

This strategy can be applied to a multi-agent system as well. The difference is that the particles are replaced by physical systems with complex dynamics, which are generally fewer in number and therefore cause a challenge to handle. Additionally, the nonlinear function is replaced by an unknown scalar field measured by the agents. The adaptation to these factors is described in Section 4.

3.2 Gradient-Based Source Seeking

Despite the goal of this paper to use an evolutionary scheme for source seeking, a gradient-based method will be presented for comparison and included as such in the simulation study.

The method is taken from Rosero and Werner (2014a) and Rosero and Werner (2014b) and uses the information of an agent and its neighbors to compute a gradient estimate. This is done by performing a least squares error method after acquiring a system equation from the first order Taylor approximation of the scalar field between neighbored agents.

The gradient estimate g_i for agent i is computed as

$$g_i = (R_i^T R_i)^{-1} R_i^T b_i, \quad (6)$$

where R_i is a matrix containing the distances between the agent and its neighbors and b_i a vector containing the differences of the scalar field values measured by the agent and its neighbors, see Rosero and Werner (2014a) for details. This gradient estimate is normalized to avoid an impact on the stability and the controller is tuned to minimize the gradient since it becomes zero if a maximum is reached.

In this paper the gradient estimation will be used as a source seeking scheme. The cooperative control scheme from Rosero and Werner (2014a) and Rosero and Werner (2014b) will be replaced by a consensus-based one since the focus of this paper is to compare the different source seeking schemes.

4. CONTROL SCHEME

This section will present the control scheme by first introducing a general setup and subsequently the individual parts of the general setup.

4.1 General Setup

In order to tackle the problem presented in the problem description, the system is divided in 3 subblocks with individual tasks and signals connecting them.

The setup is shown in Figure 1 and contains the following blocks:

- The source seeking scheme computing the pseudo-gradient signal g from the position vector y of the agents and the scalar field values measured at y
- The consensus loop containing the information flow filter $\hat{F}(s)$, which estimates a position reference p from communicated estimation errors and pseudo-gradient g

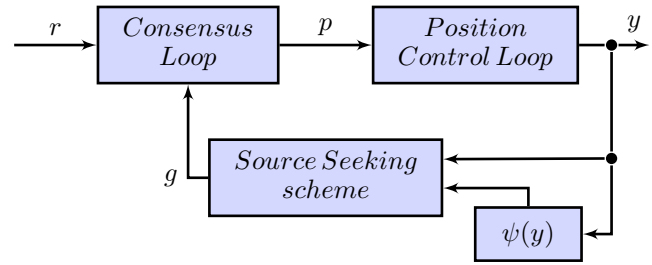


Fig. 1. General Control Loop

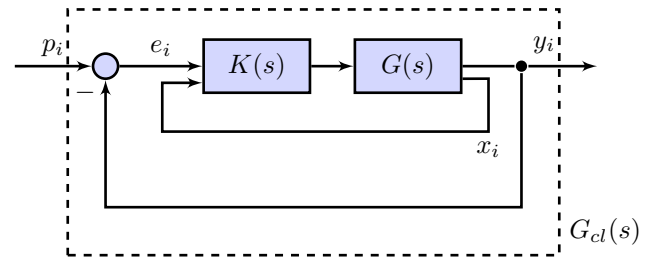


Fig. 2. Single Instance of Position Control Loop

- The position control loop containing the LTI agent model $G(s)$ with nu control inputs and $q+nx$ outputs containing the position and the nx states. A local LTI controller $K(s)$ with $q+nx$ inputs and nu outputs tracks the estimated reference p

It is important to note that all signals without index contain the information for all agents. Therefore the dimension of all signals is \mathbb{R}^{Nq} , where N is the number of agents and q the number of dimensions for a single agent.

The tasks for the individual blocks can be derived from the general control strategy. The strategy requires the source seeking scheme to calculate a signal g which points towards the maximum. The information flow filter combines the input information to calculate a position reference which establishes a formation and at the same time leads the agents towards the maximum. The position reference p is then tracked by the agents in the position control loop.

This setup separates the control of local agents from their cooperation since it is adopted from the consensus-based control scheme presented in Popov and Werner (2012) and Bartels and Werner (2014). The block diagram of a single instance of the decoupled position control loop is shown in Figure 2. A parallel connection of N instances composes the position control loop shown in Figure 1. The controller $K(s)$ was synthesized as a H_∞ -optimal controller in a mixed sensitivity design according to Pilz and Werner (2012) and Bartels and Werner (2014), details are omitted here.

4.2 Information Flow Filter

The purpose of the information flow filter is to combine the pseudo-gradient g and the communicated formation error \tilde{e} into a position reference p for the agents as shown in Figure 3.

Here we use a 2-degree-of-freedom design in order to distinguish between the two inputs and thereby enable the designer to weight between source seeking and formation control.

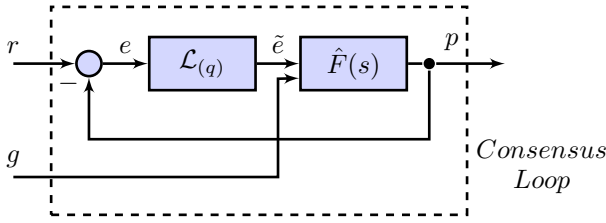


Fig. 3. Consensus Loop using a 2-Degree-of-Freedom Information Flow Filter $\hat{F}(s)$

The synthesis of the information flow filter was done using a robust H_∞/l_1 design. It was performed on the level of a single agent since the system can be brought into a linear fractional transformation (LFT) representation where the only non-block-diagonal parts are in the external loop as shown in the proof of Theorem 1. The synthesis problem contains an l_1 condition for stability and an H_∞ condition for performance as presented in Bartels and Werner (2014) and Pilz and Werner (2012). It is given in Section 4.4.

4.3 Source Seeking

For the source seeking block a suitable scheme has to be chosen. This will be a modified version of the FIPS scheme presented in Section 3. First, the modifications to the FIPS scheme will be described, followed by the derivation of a stability criterion. The strengths and weaknesses of the source seeking scheme will later be evaluated and compared in simulation in Section 5.

Adjustments of FIPS for Source Seeking in MAS For the general scheme a source seeking scheme is required which computes a signal pointing towards the global maximum. This signal can be found in the Fully Informed Particle Swarm as the term $m(t) - y(t)$, where $y(t)$ represents the position. The position-controlled agents $G_{cl}(s)$ as shown in Figure 2 replace the particles. Therefore, only this pseudo-gradient term has to be considered for the source seeking scheme block shown in the general control scheme.

Applying the original idea to the source seeking scheme leads to the pseudo gradient

$$g_i = m_i - y_i \quad (7)$$

where m_i is calculated like in equation 4 using the measurement value $\psi_i(y)$ as function value $f_i(y)$. It is evident that there are two contributions to the pseudo-gradient g . The first one is the signal $-y$, which can be included in the generalized plant by inserting a direct feedback of the position as shown in Figure 5.

The impact of the second contribution m is depended on the signal y^* . Each agent stores the position and the value of the highest measurement of ψ along its previous trajectory as value pair y^* and ψ^* . At each time instance y^* and ψ^* are updated if $\psi(y(t)) > \psi^*$. Thus, two cases can occur: "active" in case that the agent measures a higher scalar field value or "inactive" if there is no update for the value y^* . In the inactive case the signal y_k^* does not depend on y and can be treated as an unknown exogenous disturbance d as shown in Figure 5 and therefore doesn't affect the stability.

In the active case the agent finds a higher value of the scalar function. Hence, the value y_k^* changes to the current position of the agent. The update of the value happens every time the agents finds a higher value and thus keeps happening as long as the agent moves up a slope of the scalar field. This results in a feedback of the position y to the signal m via the communication network and random weighting. This feedback can be represented as $m_i = \Upsilon y$ corresponding to the structure shown in Figure 5 with the interconnection matrix Υ . Υ can be set up element-wise as

$$\Upsilon_{ij} = \begin{cases} 0 & \text{if } \mathcal{L}_{ij} = 0 \\ v_{ij} & \text{if } \mathcal{L}_{ij} \neq 0 \text{ and } y_j^* \text{ active} \\ 0 & \text{if } \mathcal{L}_{ij} \neq 0 \text{ and } y_j^* \text{ inactive} \end{cases}, \quad (8)$$

$$v_{ij} = \varphi_{ij} / \sum_{j \in \mathcal{N}_i} \varphi_{ij} \quad (9)$$

Therefore Υ has the same structure and dimensions as \mathcal{L} .

An important fact to note is that the number of agents is usually smaller than the number of particles used in nonlinear optimization since the particles do only exist virtually, whereas the agents consist of expensive hardware. This leads to the necessity for the agents to be spread wider in the search space to cover the same area.

Another important fact to mention is that there is a possibility that the agents might not converge at a maximum but instead close to it if the values of y^* converge and none of the agents' trajectories included the position of the maximum. This phenomenon can be reduced by using the output noise or adding an artificial noise to the signal y before passing it into the source seeking scheme and measuring the scalar value. This will cause the agents to move around their position and potentially measure higher values of the scalar field around their position.

4.4 Stability of Source Seeking Loop using FIPS

The source seeking scheme introduces a loop in the general control scheme (see Figure 1) through all three blocks and therefore has an impact on the stability of the whole system. In order to examine this, for the source seeking scheme derived from the FIPS scheme one can formulate a generalized plant considering both information flow filter and source seeking scheme and formulate the following stability criterion:

Theorem 1. The system shown in Figure 5 using the modified FIPS scheme characterized by Υ as defined in (8) is stable for an arbitrary communication topology, if there exists an invertible matrix $D \in \mathbb{R}^{q \times q}$ such that $\|DH_{11}(s)D^{-1}\|_1 < 1$ where $H_{11}(s)$ denotes the transfer function from w_Δ to z_Δ according to Figure 4.

Proof: The proof will have the following outline: Step 1 contains some observations about the properties and structure of the system. The system is then rearranged in step 2 to get an LFT representation. In step 3 the stability criterion will be derived using the properties of the system.

Step 1: A closer look at Figure 5 reveals that the FIPS update law can be related to the framework of the communication of the formation error since it is also written in matrix form containing the interaction. Another

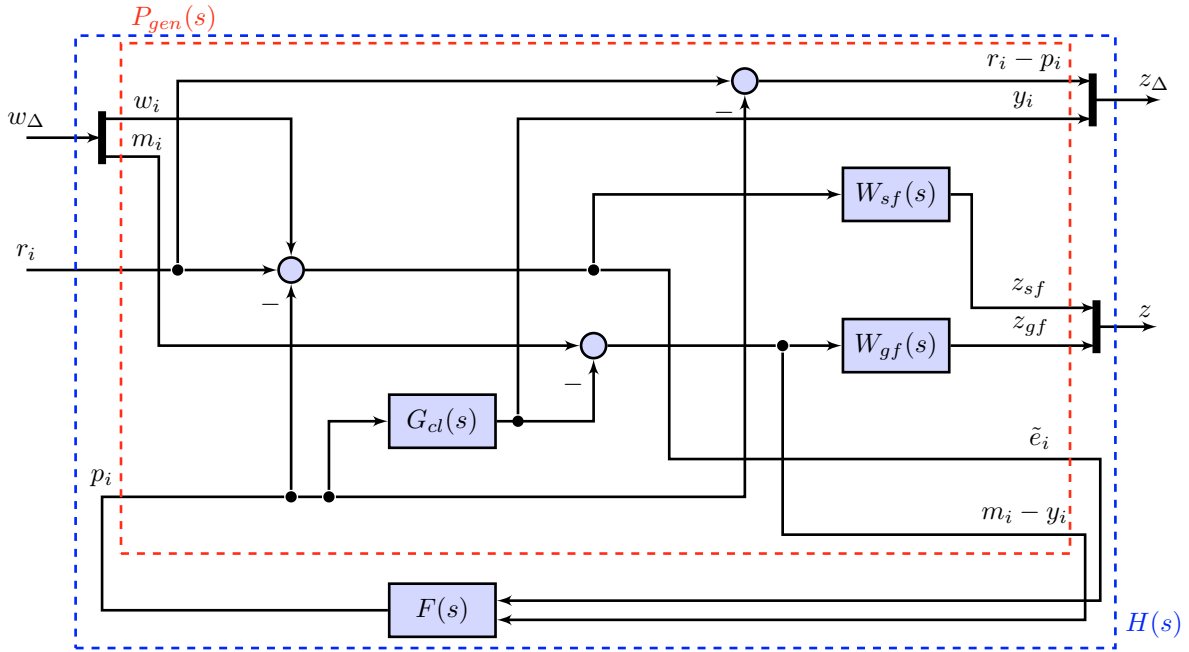


Fig. 4. Generalized Plant of the 2-Degree-of-Freedom Information Flow Filter for FIPS

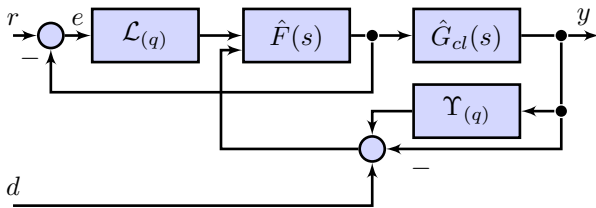


Fig. 5. Stability Loop for all agents

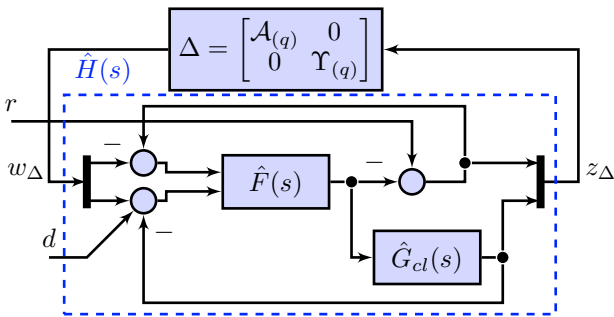


Fig. 6. LFT representation of the system

observation can be made about the properties of the matrix Υ . Due to the normalization of the uncertain parameters, the sums of the rows of Υ are less or equal to 1. This leads to a l_1 -norm less or equal 1.

Step 2: In this step the system shown in Figure 5 is rearranged into an LFT representation consisting of a block-diagonal system $\hat{H}(s)$ and a diagonal concatenation Δ of $\Upsilon(q)$ and $\mathcal{A}(q)$ as shown in Figure 6. The reasoning to pull out $\Upsilon(q)$ lies in the relation to the interconnection and the structure of this matrix.

Step 3: This step is done by means of Theorem 2 in Popov and Werner (2012). Due to the construction of \mathcal{A} and Υ it is clear that the rows of those matrices sum up to 1 and therefore the l_1 -norm is less or equal 1. If now the small

gain theorem is applied to the feedback loop of $\hat{H}(s)$ and Δ , from Popov and Werner (2012) we obtain the sufficient stability condition

$$\|\hat{D}\hat{H}_{11}(s)\hat{D}^{-1}\|_1 = \|DH_{11}(s)D^{-1}\|_1 < 1$$

The equation holds since the l_1 -norm of a block-diagonal matrix is equal to the maximum of the l_1 -norms of the blocks, which are equal here. ■

If the block Δ is symmetric, the l_1 -norm can be replaced by the H_∞ -norm.(Popov and Werner (2012))

Corollary 1. The system shown in Figure 5 is stable for any symmetric matrix $\Delta = \text{diag}(\mathcal{A}(q), \Upsilon(q))$ with \mathcal{A} according to Equation 1 and Υ according to Equation 8 if there exists an invertible matrix $D \in \mathbb{R}^{q \times q}$ such that $\|DH_{11}(s)D^{-1}\|_\infty < 1$.

Proof: The proof is almost identical to Popov and Werner (2012) except that $\mathcal{A}(p)$ is replaced by Δ also fulfilling $\|\Delta\|_1 \leq 1$ and therefore will be omitted here. ■

Information Flow Filter Synthesis For performance optimization, the shaping filters $W_{sf}(s)$ and $W_{gf}(s)$ are added to the generalized plant as shown in Figure 4 to generate the artificial signal z . The filter $W_{sf}(s)$ penalizes the formation error and thereby influences the formation control. The source seeking is influenced by $W_{gf}(s)$ through penalizing the pseudo gradient signal.

According to the lines of Pilz and Werner (2012), the following synthesis problem can now be formulated:

Problem 1: Find an LTI information flow filter $F(s)$ with $2q$ inputs and q outputs that fulfills

$$\min \|H_{22}(s)\|_\infty \tag{10}$$

subject to

$$\|DH_{11}(s)D^{-1}\|_1 < 1 \tag{11}$$

with a suitable scaling matrix D , $H_{22}(s)$ denoting the closed loop transfer function from the reference signal r_i to $z = [z_{sf} \ z_{gf}]^T$ according to Figure 4.

5. SIMULATION RESULTS

In order to test the performance of the presented control scheme, some simulations have been performed using the two presented source seeking schemes. First the simulation scenario will be introduced and afterwards the simulation results for the different source seeking schemes will be shown in order to compare them at the end.

5.1 Simulation Setup

The simulation scenario includes 4 agents with the LTI model of a quadcopter taken from Lara et al. (2006) as plant model moving in the 3-dimensional space. Additionally, there is a scalar field, which consists of 3 superimposed Gaussian distributions and is defined as:

$$\begin{aligned} \psi(y) &= \sum_{i=1}^3 \psi_i(y) \\ \psi_1(y) &= 2e^{\|1/6(y-y_1)\|_2} \\ \psi_2(y) &= 6e^{\|1/5(y-y_2)\|_2} \\ \psi_3(y) &= 2e^{\|1/6(y-y_3)\|_2} \end{aligned}$$

$$y_1 = [1 \ 1 \ 1]^T, \quad y_2 = [5 \ 5 \ 5]^T, \quad y_3 = [8 \ 8 \ 8]^T$$

This scalar field has a global maximum at the position $[5 \ 5 \ 5]^T$ and local maxima at $[1 \ 1 \ 1]^T$ and $[8 \ 8 \ 8]^T$. The starting positions of the agents are

$$y_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad y_{02} = \begin{bmatrix} 4 \\ 4 \\ 5 \end{bmatrix}, \quad y_{03} = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} \quad \text{and} \quad y_{04} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}.$$

The formation reference is chosen as

$$r_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad r_2 = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}, \quad r_3 = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} \quad \text{and} \quad r_4 = \begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix}.$$

The communication topology is set to be full and constant over time. For 4 agents this is necessary since each agent needs at least 3 neighbors to calculate a gradient estimate. Therefore the Laplacian is obtained as

$$\mathcal{L} = \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & 1 & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 1 \end{bmatrix}. \quad (12)$$

The plots of the simulation results only show the x-position of the agents for simplicity. For the simulations using the FIPS a white Gaussian noise with the power 0.0001 and sampling time 0.01 is added to the output.

5.2 Gradient-Based Source Seeking

The result of the simulation using the gradient-based source seeking together with the 2-degree-of-freedom in-

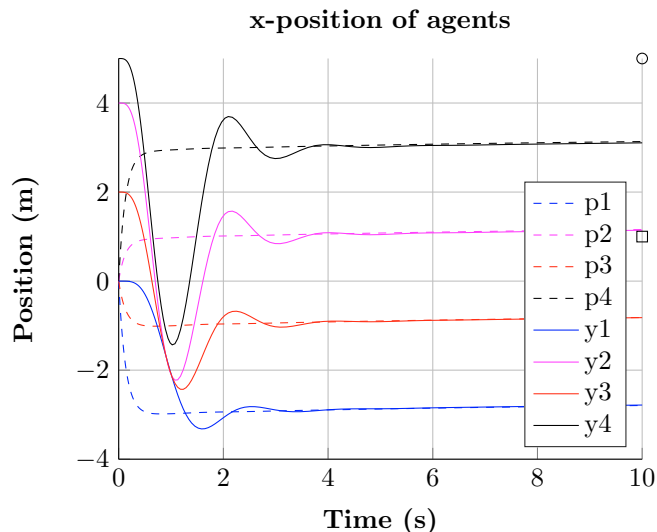


Fig. 7. Simulation Result in x-Direction using the Gradient-Based Source Seeking Scheme

formation flow filter and the setup defined in Section 5.1 is displayed in Figure 7.

On the one hand it can be observed that the agents establish the desired formation with a spacing of 2 between them. On the other hand the formation doesn't surround the global maximum at the position 5 which is marked with a circle, but rather converges at the local maximum at 1 which is marked with a square. The 5% settling time is 3.42.

This behavior is caused by the source seeking scheme which causes the agents to move along the gradient and therefore the formation converges at the local maximum. Additionally, as the reference position p is initialized with zero, the agents move towards the origin at first. Those two factors combined cause a convergence at the lower local maximum.

5.3 Evolutionary Scheme Source Seeking

Next up are the results for the 2-degree-of-freedom information flow filter design using the FIPS-based source seeking scheme. For this simulation study the information flow filter was obtained by solving Problem 1 as H_∞/H_∞ problem replacing the l_1 -norm by the H_∞ -norm according to corollary 1. This is valid as a symmetric interaction matrix (12) is considered. The shaping filters are chosen as:

$$\begin{aligned} W_{sf}(s) &= I_3 \cdot 1/(s + 0.02) \\ W_{gf}(s) &= I_3 \cdot 0.5/(s + 0.01) \end{aligned}$$

For the simulation the same setup was used as for the gradient-based version and the resulting plot of the signals p and y in x-direction is shown in Figure 8.

From the simulation it can be observed that the formation is established in the steady state but the spacing is less than 2 between the agents. The agents surround the global maximum even though the average position of the agents doesn't coincide with the global maximum. Similar behavior can be observed in the other directions. To show the reliability of this method in spite of its stochastic parts,

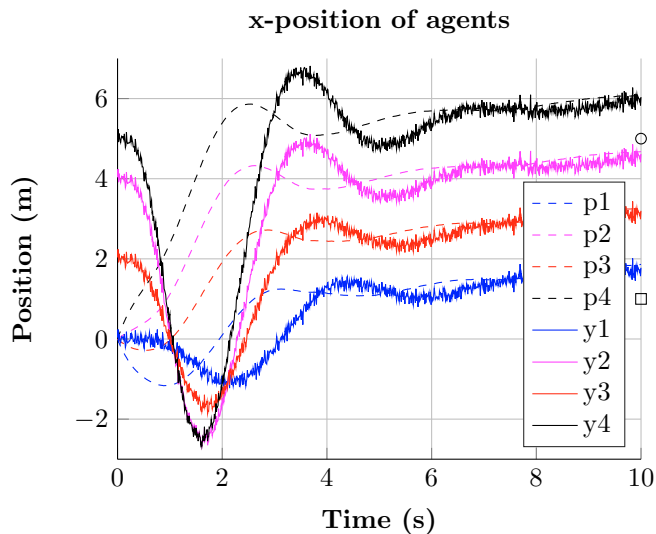


Fig. 8. Simulation Result in x-Direction using the FIPS-Based Source Seeking Scheme

Table 1. Performance Values FIPS-scheme

	mean	standard deviation
10% settling time (s)	7.41	0.43
average position (m)	3.87	0.01

the simulation was performed 50 times and a statistic evaluation was done regarding the properties average position and settling time. Results are shown in Table 1

The explanation of this behavior lies in the working principle of the source seeking scheme. The scheme causes the agents to move towards the position m which lies in the space spanned by the positions y^* . Additionally, there is the influence of the initial position reference pulling the agents towards the origin at first. The result is that the average position of the agents in steady state is below the global maximum. This result can't be fully prevented by the output noise which causes the agents to find higher values of the scalar field around their position.

A comparison of the dynamic behavior exhibits a domination of the gradient-based source seeking in terms of settling time and overshoot.

6. CONCLUSIONS

Due to the unknown shape of the scalar fields which occur in practical application, it is desired to use source seeking schemes capable of finding global maxima instead of converging at the local ones. Therefore it has been tried in this paper to adopt a PSO as source seeking scheme combined with a formation control scheme. This integration was done successfully and additionally a stability criterion for the source seeking loop was established.

The simulation results verified that the proposed scheme is able to find the global maximum of a scalar field even though a small position error remains. The dynamic behavior of the system using the PSO-based source seeking scheme is not as good compared to the gradient-based one, though the last mentioned one is unable to find the global maximum.

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