Received: 30 June 2023 | Revised: 11 July 2023 | Accepted: 23 August 2023

DOI: 10.1002/pamm.202300141

PAMM Proceedings in Applied Mathematics and Mechanics

RESEARCH ARTICLE

Hydrodynamic forces acting on mechanical systems in linear and nonlinear ocean waves

Marten Hollm | **Robert Seifried**

Institute of Mechanics and Ocean Engineering, Hamburg University of Technology, Hamburg, Germany

Correspondence
Marten Hollm, Institute of Mechanics and Ocean Engineering, Hamburg University of Technology, 21073 Hamburg, Germany.
Email: marten.hollm@tuhh.de

Abstract
The hydrodynamic forces acting on prescribed free-floating mechanical systems are studied. Regular and irregular sea states are considered using Stokes wave theory of first and second order. It is analyzed how much the magnitude and temporal behavior of the forces can differ using nonlinear theories of different orders instead of the linear wave theory. The corresponding effects on the mechanical dynamics of the system are investigated. Therefore, the fluid–structure interaction and hydrodynamic forces are computed, solving the corresponding partial differential equations of the linear and nonlinear wave theory. In this way, the nonlinear interactions of the different wave components in irregular seas and the disturbances of the sea due to the presence of the mechanical system are captured.

1 | INTRODUCTION

In the real ocean, various ships and offshore structures differ in their geometric shape and the depth of water into which they reach. When they are constructed, their safety limits have to be determined. The hydrodynamic forces acting on the various mechanical structures have to be known for this. In order to compute the acting forces occurring in oceans as accurately as possible, the corresponding sea states and fluid–structure interactions (FSI) have to be modeled in the same way. This can be done by solving the governing equations of fluid mechanics. However, solving them is computationally very expensive since they are highly nonlinear. In order to simplify the problem, the corresponding equations can be approximated by linear and nonlinear wave theories [1]. While it is computationally cheap to deal with the linear wave theory, the related results are only accurate for small wave amplitudes. For higher wave amplitudes, nonlinear wave theories have to be used.

The use of nonlinear theories introduces some effects that influence the wave dynamics. For example, the phase velocity of waves modeled by the linear wave theory is independent of the wave amplitude. However, this changes when nonlinear theories are used. Here, waves of larger wave amplitude move faster than waves of smaller wave amplitude [2]. Therefore, it is impossible, for example, to precisely model with a linear theory the behavior of real random waves with high wave heights, which propagate several hundred wavelengths.

While in linear theory the sum of two solutions is also a solution, this is not the case in nonlinear theory. In nonlinear theory, interactions between the different solutions have to be calculated [3]. However, considering all these interactions needs much computation time [4]. This is a big drawback when it comes, for example, to the optimization of the system parameters of a mechanical system interacting with a nonlinear sea.

This work studies the effects on a given floating mechanical system when the corresponding nonlinear wave theory replaces the linear wave theory. Therefore, in the presence of given linear and nonlinear water waves, the hydrodynamic forces and the dynamics of the considered system are investigated and compared with each other. From this, it can be deduced whether a linear wave theory is sufficient to approximate the behavior of the given mechanical system in real ocean waves or whether nonlinear wave theories must be used for this purpose.

The work is organized as follows. In Section 2, all fundamental equations to model the FSI between ocean waves and floating mechanical systems are introduced. Afterward, the basic equations for generating regular and random nonlinear ocean waves are presented in Section 3. In Section 4, a mechanical structure is presented, and its dynamic motions in linear and nonlinear waves are studied in Section 5. Finally, this work ends with a conclusion in Section 6.

2 | MODELING OF WATER WAVES

Knowing the temporal behavior of the water waves as well as the dynamics of the water beyond the sea surface, the hydrodynamic pressure acting on prescribed mechanical structures can be computed. This allows the calculation of the corresponding hydrodynamic forces and the motion of mechanical structures floating in real sea. In this section, all fundamental equations to model the FSI between ocean waves and floating mechanical systems are introduced. Since solving the fully nonlinear equations is computationally expensive, the fundamental equations for an efficient calculation of nonlinear water waves and the corresponding FSI with a given mechanical body are developed using Stokes' expansions.

2.1 | Fully nonlinear equations

The behavior of homogeneous, non-viscous, incompressible fluids, where surface tensions are neglected, can be described by the continuity equation of motion and three boundary conditions (BCs). Let t denote the time and K: $\{O, x, y, z\}$ be a Cartesian coordinate system, whereby $z = 0$ describes the plane of the undisturbed free sea surface and the z-axis is positive upward. The vertical displacement of any point on the free sea surface may be defined by the function $z = \eta = \eta(x, y, t)$. Here, it is assumed that the water waves are not breaking such that the value of $\eta(x, y, t)$ is uniquely defined everywhere.

Furthermore, it is assumed that the fluid is irrotational. Then, there exists a velocity potential $\phi = \phi(x, y, z, t)$ such that the gradient of the potential $\nabla\phi$ is the velocity vector $\mathbf{v}$ of the fluid flow, that is, $\nabla\phi = \mathbf{v}$. Using this notation, the governing equations of fluid motion result in [2]

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0, \qquad \text{for} \quad -h \leq z \leq \eta(x, y, t), \tag{1a}$$

$$\eta_t + \phi_x\eta_x + \phi_y\eta_y = \phi_z, \qquad \text{for} \quad z = \eta(x, y, t), \tag{1b}$$

$$\phi_t + \frac{1}{2}\nabla\phi \cdot \nabla\phi + g\eta = 0, \qquad \text{for} \quad z = \eta(x, y, t), \tag{1c}$$

$$\phi_z = 0, \qquad \text{for} \quad z = -h. \tag{1d}$$

Here and in the rest of this work, f_s denotes the partial derivative of a function f with respect to the coordinate $s \in \{x, y, z, t\}$, that is, $f_s = \frac{\partial f}{\partial s}$. Furthermore, g is the gravitational acceleration and h is the constant water depth with a rigid, impermeable plane. It has to be noted that the external pressure exerted on the sea surface η is assumed to be zero.

The dynamics of water waves on the open sea can be computed solving Equation (1). However, if there are additional boundaries like the surface of walls or mechanical structures, additional equations have to be formulated to model the corresponding FSI. In this work, general floating bodies are considered, which are impermeable to water. Let S_B be the wetted surface of the mechanical structure and $\mathbf{u}$ the velocity of a point on S_B. Then, the BC at S_B is given by

$$\nabla\phi \cdot \mathbf{n} = \mathbf{u} \cdot \mathbf{n} \quad \text{at } (x, y, z) \in S_B, \tag{2}$$

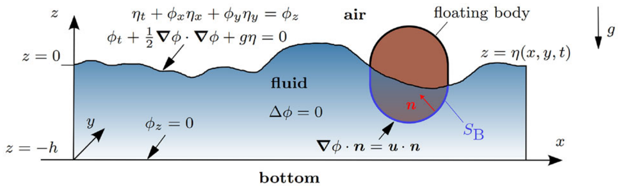

FIGURE 1 Governing equations of the nonlinear water wave problem and the FSI with a floating body of arbitrary shape.

whereby $\mathbf{n}$ denotes the normal vector at S_B pointing out of the fluid and hence into the body. The governing equations (1a)–(1d) and (2) as well as the wetted surface S_B of a floating body of arbitrary shape are shown in Figure 1. After calculating the velocity potential ϕ and the surface displacement η, the hydrodynamic pressure p can be calculated using Bernoulli's equation. Finally, integrating the pressure over S_B, the hydrodynamic force and torque with respect to O acting on the body are given by

$$\mathbf{F} = \iint_{S_B} p\mathbf{n}\,\mathrm{d}S, \quad \mathbf{M} = \iint_{S_B} p(\mathbf{r}\times\mathbf{n})\,\mathrm{d}S, \quad \text{with } p(x,y,z,t) = -\rho\left(\phi_t + \frac{1}{2}\nabla\phi\cdot\nabla\phi + gz\right). \tag{3}$$

2.2 | Approximation of governing equations

In this section, wave theories are introduced, which approximate the governing equations of fluid motion. Since the nonlinearity of Equation (1) comes from the BCs (1b) and (1c), which are stated at $z = \eta$, this section focuses on an appropriate approximation of these equations. The first step is to combine these two BCs into one single BC. This BC is exact and only depends on ϕ, except that it has to be applied at the unknown sea surface $z = \eta$. Therefore, a systematic procedure is introduced to expand this BC from the exact free sea surface $z = \eta(x, y, t)$ to $z = 0$. This is done by approximating all derivatives by Taylor series expansions. Depending on the approximation order of the Taylor series expansion, a sequence of BCs valid on $z = 0$ is obtained. Considering all terms up to order $\mathcal{O}(\phi^3)$, the resulting BC is given by [2]

$$\phi_{tt} + g\phi_z + 2\nabla\phi\cdot\nabla\phi_t - \frac{1}{g}\phi_t\frac{\partial}{\partial z}(\phi_{tt} + g\phi_z) = 0 + \mathcal{O}(\phi^3). \tag{4}$$

By deriving the BC (4), the calculation of the velocity potential ϕ has been decoupled from the free sea surface displacement η. However, the nonlinearity in Equation (4) makes the computation of solutions ϕ very inefficient. Therefore, the BC (4) is approximated by a sequence of linear equations in the following.

It is assumed that ϕ, η, and the velocity vector $\mathbf{u}$ can be expressed by perturbation series of the form [1]

$$\phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + \varepsilon^3\phi^{(3)} + \mathcal{O}(\varepsilon^4), \tag{5a}$$

$$\eta = \varepsilon\eta^{(1)} + \varepsilon^2\eta^{(2)} + \varepsilon^3\eta^{(3)} + \mathcal{O}(\varepsilon^4), \tag{5b}$$

$$\mathbf{u} = \varepsilon\mathbf{u}^{(1)} + \varepsilon^2\mathbf{u}^{(2)} + \varepsilon^3\mathbf{u}^{(3)} + \mathcal{O}(\varepsilon^4). \tag{5c}$$

Here, ε is a perturbation parameter related to the wave steepness and is assumed to be small. Based on the pioneering work of Stokes [5], who used this ansatz to find periodic solutions of nonlinear water waves, these perturbation series are known as Stokes' expansions. Substituting Equation (5a) into Equation (4) and sorting for the resulting exponents in ε^i, it follows:

$$\varepsilon\left(\phi_{tt}^{(1)} + g\phi_z^{(1)}\right) + \varepsilon^2\left(\phi_{tt}^{(2)} + g\phi_z^{(2)} + 2\nabla\phi^{(1)}\cdot\nabla\phi_t^{(1)} - \frac{1}{g}\phi_t^{(1)}\frac{\partial}{\partial z}\left(\phi_{tt}^{(1)} + g\phi_z^{(1)}\right)\right) = 0 + \mathcal{O}(\varepsilon^3) \quad \text{for} \quad z = 0. \tag{6}$$

This BC (6) holds if the terms multiplied by ε^i vanish for each $i = 1, 2, \ldots$ As a result of this, the following equation for the computation of the first two velocity potentials $\phi^{(1)}$ and $\phi^{(2)}$ can be derived:

$$\phi^{(i)}_{xx} + \phi^{(i)}_{yy} + \phi^{(i)}_{zz} = 0,\ i = 1, 2, \qquad \text{for} \quad -h \le z \le 0, \tag{7a}$$

$$\phi^{(i)}_{z} = 0,\ i = 1, 2, \qquad \text{for} \quad z = -h, \tag{7b}$$

$$\nabla\phi^{(i)} \cdot \mathbf{n} = \mathbf{u}^{(i)} \cdot \mathbf{n},\ i = 1, 2, \qquad \text{for} \quad (x, y, z) \in S_0, \tag{7c}$$

$$\phi^{(1)}_{tt} + g\phi^{(1)}_{z} = 0, \qquad \text{for} \quad z = 0, \tag{7d}$$

$$\phi^{(2)}_{tt} + g\phi^{(2)}_{z} = -2\nabla\phi^{(1)} \cdot \nabla\phi^{(1)}_{t} + \frac{1}{g}\phi^{(1)}_{t}\frac{\partial}{\partial z}\left(\phi^{(1)}_{tt} + g\phi^{(1)}_{z}\right), \qquad \text{for} \quad z = 0. \tag{7e}$$

Here, S_0 is the mean wetted surface of the body, for which the free sea surface displacement η is zero. Instead of formulating the BC (7c) at the actual wetted surface of the body S_{B}, the BCs are applied at S_0. This is due to the fact that the velocity potentials are computed only at positions located below the rest position of the sea surface, that is, for $z \le 0$.

It has to be noted that BCs (7d) and (7e) have been found several times in the literature, see, for example, [6, 7]. An advantage of these new equations is that the lower-order potential $\phi^{(1)}$ is independent of the higher-order potential $\phi^{(2)}$. In practice, this means that the velocity potential $\phi^{(1)}$ can be computed independently from $\phi^{(2)}$. Afterward, $\phi^{(2)}$ can be calculated.

2.3 | Computation of the disturbance of the body

In general, the presence of the floating body will disturb the dynamics of the incoming water waves. In order to compute this disturbance separately, the total velocity potential ϕ is divided into two potentials: the potential ϕ_0 of the incoming water waves, which are not disturbed by the presence of the body, and the potential ϕ_{B} corresponding to the body disturbance. Both potentials ϕ_0 and ϕ_{B} are expanded following the Stokes expansion procedure presented in Equation (5). This results into

$$\phi = \phi_0 + \phi_{\mathrm{B}} = \varepsilon(\phi^{(1)}_0 + \phi^{(1)}_{\mathrm{B}}) + \varepsilon^2(\phi^{(2)}_0 + \phi^{(2)}_{\mathrm{B}}) + \mathcal{O}(\varepsilon^3). \tag{8}$$

Here, $\phi^{(i)}_0$ and $\phi^{(i)}_{\mathrm{B}}$ are the respected potentials coming from the Stokes expansion of ϕ_0 and ϕ_{B}. It has to be noted that in the absence of a body, the velocity potential corresponding to the body disturbance, ϕ_{B}, vanishes. This means that the velocity potentials $\phi^{(1)}_0$ and $\phi^{(2)}_0$ have to satisfy Equations (7a), (7b), (7d), and (7e). For regular and irregular water waves, some analytical solutions for $\phi^{(1)}_0$ and $\phi^{(2)}_0$ are given in Section 3.

Knowing the incident potentials $\phi^{(i)}_0$, $i = 1, 2, 3, \ldots$, the corresponding equations for velocity potentials corresponding to the body disturbance, $\phi^{(i)}_{\mathrm{B}}$, can be formulated by substituting $\phi^{(i)} = \phi^{(i)}_0 + \phi^{(i)}_{\mathrm{B}}$ into Equation (7). For example, the BC for the computation of the velocity potential $\phi^{(2)}_{\mathrm{B}}$ at $z = 0$ is given by

$$\phi^{(2)}_{\mathrm{B},tt} + g\phi^{(2)}_{\mathrm{B},z} = F(\phi^{(1)}_0 + \phi^{(1)}_{\mathrm{B}}) - F(\phi^{(1)}_0), \qquad \text{with} \quad F(\phi) = -2\nabla\phi \cdot \nabla\phi_t + \frac{1}{g}\phi_t\frac{\partial}{\partial z}\left(\phi_{tt} + g\phi_z\right). \tag{9}$$

It has to be noted that a radiation condition has to be formulated which determines the behavior of $\phi^{(i)}_{\mathrm{B}}$ far away from the floating body. For details, it is referred to [1]. However, in this work the velocity potentials $\phi^{(i)}_{\mathrm{B}}$ are computed numerically. Therefore, suitable absorbing BCs are formulated at the end of the considered spatial domains instead of radiation conditions.

3 | REGULAR HIGHER ORDER WAVES

In Section 2, the fundamental equations for modeling the nonlinear FSI have been derived. The basic idea is that the total velocity potential ϕ of the sea disturbed by the presence of the body is decomposed into the velocity potential ϕ_0 of the incoming undisturbed water waves and the potential ϕ_B corresponding to the body disturbance. For the case of regular and random waves, analytical solutions for $\phi_0^{(1)}$ and $\phi_0^{(2)}$ are known, which will be presented here.

For incoming regular waves with wave amplitude A and frequency ω, the sea surface displacement $\eta^{(1)}$ is given by

$$\varepsilon\eta^{(1)}(x,y,t) = A\cos(\theta), \quad \text{with } \theta = k(\cos(\chi)x + \sin(\chi)y) - \omega t + \beta. \tag{10}$$

Here, k is the wave number, χ is the angle between the wave direction and the x-axis, and β is a phase shift. The corresponding velocity potentials $\phi_0^{(1)}$ and $\phi_0^{(2)}$ are given by

$$\varepsilon\phi_0^{(1)}(x,y,z,t) = \frac{gA}{\omega}\frac{\cosh(k(z+h))}{\cosh(kh)}\sin(\theta), \quad \varepsilon^2\phi_0^{(2)} = \frac{3\omega A^2}{8}\frac{\cosh(2k(z+h))}{\sinh^4(kh)}\sin(2\theta). \tag{11}$$

In order to fulfill the BCs (7d) and (7e) at $z = 0$, k and ω are related by the dispersion relation $\omega^2 = kg\tanh(kh)$.

Next, the incident velocity potentials of random waves are discussed, which satisfies the equations of Stokes' theory of second-order. An irregular short-crested sea surface displacement $\varepsilon\eta^{(1)}$ can be written in the absence of any structure as [3]

$$\varepsilon\eta^{(1)}(x,y,t) = \sum_{m=1}^{M} a_m\cos(\theta_m), \quad \text{with } \theta_m = k_m(\cos(\chi_m)x + \sin(\chi_m)y) - \omega_m t + \beta(\omega_m,\chi_m). \tag{12}$$

Here, β is a random phase shift, which is uniformly distributed in $[0, 2\pi)$. Furthermore, the amplitudes a_m depend on the underlying sea state given by the corresponding one-sided spectral density $S(\omega)$ and the spread function $D(\chi)$, cf. [8, 9]. The corresponding incident velocity potentials $\varepsilon\phi_0^{(1)}$ and $\varepsilon\phi_0^{(2)}$ are given by [3]

$$\begin{aligned}\varepsilon\phi_0^{(1)} &= \sum_{m=1}^{M}\frac{a_m g}{\omega_m}\frac{\cosh(k_m(h+z))}{\cosh(k_m h)}\sin(\theta_m),\\ \varepsilon^2\phi_0^{(2)} &= \sum_{m=1}^{M}\sum_{n=1}^{M} c_{mn}^{-}\frac{\cosh(|\mathbf{k}_m-\mathbf{k}_n|(h+z))}{\cosh(|\mathbf{k}_m-\mathbf{k}_n|h)}\sin(\theta_m-\theta_n) + c_{mn}^{+}\frac{\cosh(|\mathbf{k}_m+\mathbf{k}_n|(h+z))}{\cosh(|\mathbf{k}_m+\mathbf{k}_n|h)}\sin(\theta_m+\theta_n).\end{aligned} \tag{13}$$

Here, $\mathbf{k}_m = k_m[\cos(\chi_m), \sin(\chi_m)]^{\mathrm{T}}$ is the vector consisting of the wave numbers in the x- and y- direction, respectively. The constants c_{mn}^{-} and c_{mn}^{+} can be found in [3].

In the formulation of $\phi_0^{(2)}$, interactions between the components of the first-order velocity potential $\phi_0^{(1)}$ are considered. The number of interactions is given by M^2. It follows that the calculation of all nonlinear interactions becomes increasingly inefficient as the number of regular components in the first-order irregular sea state increases. As a result, in the presence of random waves, a Stokes theory of higher order needs much computation time [4]. This is a big drawback when, for example, the system parameters of the considered mechanical system are optimized. Comparing the resulting dynamics of a mechanical system in linear and nonlinear water waves, this work investigates whether the consideration of a nonlinear wave theory is necessary or whether using a linear wave theory is enough to compute the main dynamics of the system.

4 | DYNAMICS OF MECHANICAL STRUCTURES IN NONLINEAR WATER WAVES

After showing how the velocity potential of a sea disturbed by a given structure can be calculated, it is shown how this potential can be used to calculate the motion of the structure efficiently. This section introduces the mechanical structure whose dynamics in linear and nonlinear waves are studied in this work. The hydrodynamic loads acting on the given body and resulting from the disturbed velocity potential are calculated, and the equations of motion are formulated.

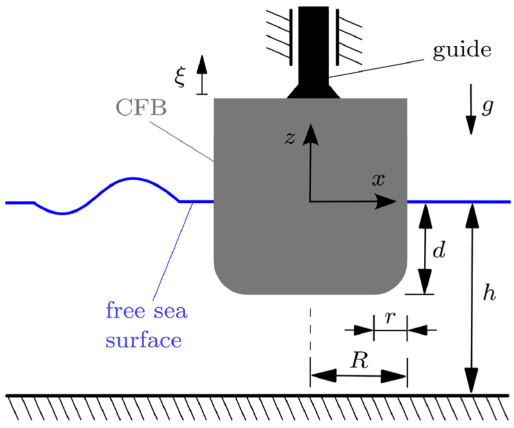

FIGURE 2 Sketch of the considered mechanical system.

4.1 | Mechanical structure

In this work, a cylindrical floating body (CFB) is considered, which moves along a guidance in the vertical direction. Figure 2 shows a sketch of the mechanical system in the x-z-plane. The displacement of the CFB against its resting position is denoted by ξ. This work investigates how the motion of the system changes if the incoming water waves and the FSI are modeled using a nonlinear instead of a linear wave theory.

The CFB has a radius of R and a draft of d. In order to decrease the viscous damping of the water and, therefore, to increase the dynamics of the system, a semicircular bottom is used. Here, the sharp corners of the bottom of the cylinder are replaced by a spherical segment of radius r. Quantitatively, it has been observed by Tom et al. [10] that a floater with a semicircular bottom can experience a decrease in damping by as much as 50% when compared to a flat-bottom geometry.

4.2 | Hydrodynamic forces and nonlinear equations of motion

The wave loads acting on a mechanical structure are calculated by integrating the hydrodynamic pressure over the time-varying wetted surface S_B of the body, cf. Equation (3). One difficulty in efficiently calculating hydrodynamic forces is that the wetted surface S_B changes with time. The hydrodynamic forces depend not only on the velocity potential ϕ but also on the sea surface displacement η. Therefore, the integral over the wetted surface of the mechanical structure is split into the mean wetted surface S_0 and the time-dependent part $\Delta S(t)$.

If the displacement ξ of the mechanical structure sketched in Figure 2 is smaller than $|d-r|$, the cross-section of the mean wetted surface S_0 and the mean water surface does not change in time. This cross-section is denoted by C_0. Furthermore, it is assumed that the water comes only in contact with the lateral surface of the cylinder. Then, the hydrodynamic forces acting on a cylindrical body result into [6]

$$\mathbf{F} = \iint_{S_\mathrm{B}} p\mathbf{n}\,\mathrm{d}S = \mathbf{F}^{(0)} + \varepsilon^1\mathbf{F}^{(1)} + \varepsilon^2\mathbf{F}^{(2)} + \mathcal{O}(\varepsilon^3), \text{ with } \mathbf{F}^{(0)} = -\iint_{S_0} \rho g z\mathbf{n}\,\mathrm{d}S,$$
$$\mathbf{F}^{(1)} = -\iint_{S_0} \rho\phi_t^{(1)}\mathbf{n}\,\mathrm{d}S, \quad \mathbf{F}^{(2)} = -\iint_{S_0} \left(\rho\phi_t^{(2)} + \frac{\rho}{2}\nabla\phi^{(1)}\cdot\nabla\phi^{(1)}\right)\mathbf{n}\,\mathrm{d}S + \int_{C_0} \frac{\rho}{2g}\left(\phi_t^{(1)}\right)^2\mathbf{n}\,\mathrm{d}C. \tag{14}$$

After computing the nonlinear hydrodynamic forces acting on the CFB presented in Section 4.1, the equation of motion can be formulated. In the following, the component of the force $\mathbf{F}$ and $\mathbf{F}^{(i)}$ in the z-direction is denoted by F_3 and $F_3^{(i)}$, respectively. Then, the corresponding equation of motion is given by

$$m\ddot{\xi} = F_3 - mg - \lambda\dot{\xi} = F_3^{(0)} + \varepsilon F_3^{(1)} + \varepsilon^2 F_3^{(2)} - mg - \lambda\dot{\xi} + \mathcal{O}(\varepsilon^3). \tag{15}$$

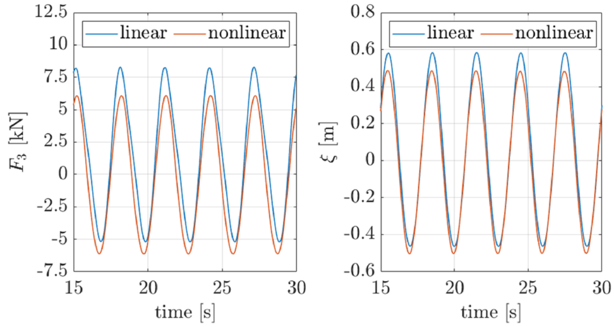

FIGURE 3 The CFB is excited by regular waves with amplitude $A = 0.35\,\mathrm{m}$, wave period $T_\mathrm{p} = 3\,\mathrm{s}$, direction $\chi = 0°$, and phase shift $\beta = 0°$. Left: acting hydrodynamic forces F_3. Right: displacement ξ of the CFB.

Here, m is the total mass of the CFB and the guiding rods. Furthermore, λ is a constant that accounts for the total velocity-dependent damping force from mechanical friction effects and viscous damping in the z-direction due to the fluid [11].

It has to be noted that $F_3^{(0)}$ represents the buoyancy force of the CFB. If the mechanical system is in its resting position at $\xi = 0$, the balance of forces in the z-direction results in $F_3^{(0)} = mg$. For $\xi \leq |d - r|$, it follows

$$F_3^{(0)} - mg = \rho g \pi R^2 \xi. \tag{16}$$

In the next step, a similar series expansion for the displacement ξ is assumed as for the velocity potential ϕ:

$$\xi = \varepsilon\, \xi^{(1)} + \varepsilon^2 \xi^{(2)} + \mathcal{O}(\varepsilon^3) \tag{17}$$

Finally, Equations (16) and (17) are substituted into Equation (15). Rearranging the resulting equation by the different orders of ε, the corresponding equations for $\xi^{(1)}$ and $\xi^{(2)}$ are given by

$$m\ddot{\xi}^{(1)} + \lambda\dot{\xi}^{(1)} + \rho g \pi R^2 \xi^{(1)} = F_3^{(1)}, \quad m\ddot{\xi}^{(2)} + \lambda\dot{\xi}^{(2)} + \rho g \pi R^2 \xi^{(2)} = F_3^{(2)}. \tag{18}$$

5 | NUMERICAL RESULTS

In this section, the motion of the mechanical system depicted in Figure 2 is investigated using linear and nonlinear ocean waves. Here, a CFB with $R = 1\,\mathrm{m}$, $r = 0.2\,\mathrm{m}$, and $d = 1.5\,\mathrm{m}$ is considered in a sea with water depth $h = 5\,\mathrm{m}$. Damping due to mechanical friction effects and viscous damping is accounted for using $\lambda = 3000\,\mathrm{Ns/m}$. The results are computed using Stokes' expansions of first and second order. The corresponding incident velocity potentials of the incoming water waves are given in Section 3. The velocity potentials $\phi_\mathrm{B}^{(1)}$ and $\phi_\mathrm{B}^{(2)}$ resulting from the disturbance of the body are computed by solving the corresponding partial differential equations numerically. This is done by applying a finite-difference scheme.

In the following, the total corresponding hydrodynamic forces in the vertical direction F_3 and the total displacement of the CFB ξ are investigated using the linear and nonlinear wave theory. These are given by $F_{3,\mathrm{lin}} = \varepsilon F_3^{(1)}$, $\xi_\mathrm{lin} = \varepsilon\xi^{(1)}$ in the linear case and $F_{3,\,\mathrm{nonlin}} = \varepsilon F_3^{(1)} + \varepsilon^2 F_3^{(2)}$, $\xi_\mathrm{nonlin} = \varepsilon\xi^{(1)} + \varepsilon^2\xi^{(2)}$ in the nonlinear case.

First of all, the dynamics of the considered CFB are investigated using regular water waves with $A = 0.35\,\mathrm{m}$, wave period $T_\mathrm{p} = 3\,\mathrm{s}$, direction $\chi = 0°$ and phase shift $\beta = 0°$. The CFB is released from the rest position. After 15 s, the CFB is moving in a steady state. In the following, only the results in the steady state are considered. Figure 3 shows on the left side the hydrodynamic forces acting on the CFB. The corresponding displacements of the CFB are presented in Figure 3 on the right side. It can be seen that considering the nonlinear wave theory results in a negative shift in the force and, therefore, in the displacement of the CFB. This comes due to the fact that the nonlinear wave force $F_3^{(2)}$ contains a negative mean component, which is constant in time. Furthermore, it can be seen that the nonlinearities significantly influence the amplitude of the hydrodynamic force. Thus, a nonlinear wave theory leads to a 9% lower force amplitude. In this example, the amplitude of the displacement changes also by 6%. This shows that a nonlinear wave theory can lead to significant changes in the dynamics of the CFB compared to the linear theory.

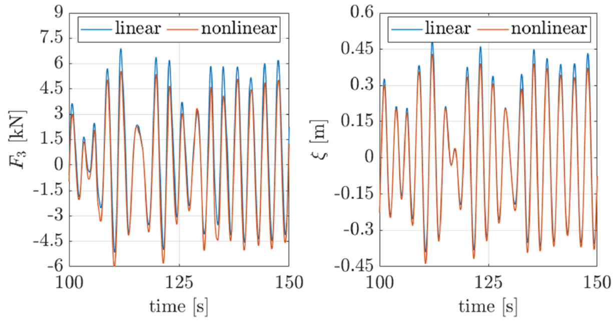

FIGURE 4 The CFB is excited by random long-crested linear and nonlinear water waves, whereby the amplitudes are computed using the JONSWAP-spectrum with a corresponding significant wave height of $H_s = 0.84$ m and peak period of $T_p = 3$ s. Left: acting hydrodynamic forces F_3. Right: displacement ξ of the CFB.

Next, the dynamics of the considered CFB are investigated using long-crested random water waves. Here, the amplitudes are generated using the JONSWAP-spectrum with a corresponding significant wave height of $H_s = 0.84$ m and peak period of $T_p = 3$ s. The CFB is released from its rest position. The acting hydrodynamic forces and the resulting displacement of the mechanical system are presented in Figure 4. A negative shift in the hydrodynamic force can again be seen. A direct comparison of the amplitude of the displacements, as in the case of a regular excitation, cannot be performed. However, it can be seen that the dynamical behavior of the CFB can significantly change going from linear to nonlinear theory.

6 | CONCLUSION

The difference between a linear and nonlinear wave theory is studied by investigating the motion of a given mechanical system and the corresponding acting hydrodynamic forces, respectively. The used approach is based on approximating the governing equation of fluid motion using Taylor series expansion and Stokes expansion. This leads to a sequence of different wave theories, which consider nonlinear wave effects. Considering a linear and nonlinear wave theory, it is presented how the FSI with regular and random waves can be computed.

It is shown that a change from a linear to a nonlinear wave theory can lead to significant changes in the dynamics of the system. For the considered mechanical structure, a decrease in the amplitude of the acting hydrodynamic forces by about 9% is observed for regular waves, which decreases the amplitude of the motion of the system by about 6%. Also in random waves, a significant change in the dynamic behavior of the CFB can be observed going from linear to nonlinear theory. Therefore, when modeling the motion of mechanical systems or optimizing system parameters such as those of the considered CFB, irregular nonlinear ocean waves should be considered to model the system behavior in water waves as realistically as possible.

ACKNOWLEDGMENTS

Open access funding enabled and organized by Projekt DEAL.

ORCID

Marten Hollm https://orcid.org/0000-0001-5139-8918

REFERENCES

1. Malenica, Š., & Molin, B. (1995). Third-harmonic wave diffraction by a vertical cylinder. *Journal of Fluid Mechanics*, *302*, 203–229.
2. Newman, J. N. (2018). *Marine hydrodynamics*. The MIT Press.
3. Sharma, J. N., & Dean, R. G. (1979). Development and evaluation of a procedure for simulating a random directional second-order sea surface and associated wave forces. [PhD thesis]. University of Delaware.
4. Slunyaev, A., Pelinovsky, E., & Guedes Soares, C. (2014). Reconstruction of extreme events through numerical simulations. *Journal of Offshore Mechanics and Arctic Engineering*, *136*(1), 011302.
5. Stokes, G. G. (1847). On the theory of oscillatory waves. *Transactions of the Cambridge Philosophical Society*, *8*, 441–455.
6. Taylor, R. E., & Hung, S. (1987). Second-order diffraction forces on a vertical cylinder in regular waves. *Applied Ocean Research*, *9*(1), 19–30.

7. Rahman, M., Bora, S., & Satish, M. (1999). A note on second-order wave forces on a circular cylinder in finite water depth. *Applied Mathematics Letters*, *12*(1), 63–70.
8. Clauss, G., Lehmann, E., & Östergaard, C. (1988). *Meerestechnische Konstruktionen*. Springer.
9. Mitsuyasu, H., Tasai, F., Suhara, T., Mizuno, S., Okhuso, M., Honda, T., & Rikiishi, K. (1975). Observation of the directional spectrum of ocean waves using a cloverleaf buoy. *Journal of Physical Oceanography*, *5*(4), 750–760.
10. Tom, N., & Yeung, R. W. (2013). Performance enhancements and validations of a generic ocean-wave energy extractor. *Journal of Offshore Mechanics and Arctic Engineering*, *135*(4), 041101.
11. Hollm, M., Dostal, L., Yurchenko, D., & Seifried, R. (2022). Performance increase of wave energy harvesting of a guided point absorber. *The European Physical Journal Special Topics*, *231*(8), 1465–1473.

How to cite this article: Hollm, M., & Seifried, R. (2023). Hydrodynamic forces acting on mechanical systems in linear and nonlinear ocean waves. *Proceedings in Applied Mathematics and Mechanics*, e202300141. https://doi.org/10.1002/pamm.202300141