



X International Conference on Structural Dynamics, EURODYN 2017

# Nonlinear model parameter identification for ice-induced vibrations

Leo Dostal<sup>a,\*</sup>, Eliz-Mari Lourens<sup>b</sup>, Andrei Metrikine<sup>b</sup>

<sup>a</sup>*Institute of Mechanics and Ocean Engineering, Hamburg University of Technology, Hamburg 22179, Germany*

<sup>b</sup>*Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands*

---

## Abstract

The problem of level ice interacting with compliant structures is addressed, where the ice loads can depend on the dynamical behavior of the structures. We are interested in a special type of ice-induced vibration, known as frequency lock-in, and characterized by having the dominant frequency of the ice forces near a natural frequency of the structure. It is shown that accurate estimates of the model parameters for the well-known Määttänen's model for ice-induced vibrations can be obtained from measurements of the structural vibrations and the ice velocity. Määttänen's model uses a state-dependent piecewise nonlinear function for the ice crushing strength, which leads to nonlinear negative damping in the equations of motion of the considered structure. The identification is achieved by means of an Unscented Kalman Filter using simulated noisy measurements of the structural behavior.

© 2017 The Authors. Published by Elsevier Ltd.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

*Keywords:* Parameter identification; ice-induced vibrations; Unscented Kalman Filter

---

## 1. Introduction

Level ice crushing against bottom-founded compliant structures can result in ice-induced vibrations. Depending on the velocity of the moving ice, different regimes of interaction between the ice and the structure have been distinguished. These are defined as intermittent crushing, frequency lock-in, and continuous brittle crushing. During the last few decades, numerous models have been proposed in an attempt to simulate this complex interaction and to predict ice-induced vibrations [1–4]. Depending on the model, the interaction is described as a function of different sets of parameters related to the ice and the structure. While the relevant structural properties are often known quite well or identifiable through modal testing, the ice properties are in general more difficult to come by and in most cases also time-varying. Consider for instance parameters related to the strength of the ice or the contact area between the ice and the structure.

The aim of this work is the identification of these parameters from measurements of the structural vibrations and the velocity of the ice. The problem is cast in a coupled parameter and state estimation form, and solved in the time domain using the Unscented Kalman filter [5]. This framework allows for the identification of the time-varying model parameters in conjunction with the displacement and velocity profiles (referred to as the states) of the structure from

---

\* Corresponding author. Tel.: +49 40 42878 2209 ; fax: +49 40 42878 2028.

E-mail address: [dostal@tuhh.de](mailto:dostal@tuhh.de)

noise-contaminated response measurements. Research on ice-induced vibrations has been dominated by reduced-order models using only the first mode of the considered structure, such that its dynamics can be modeled as a single degree of freedom (DOF) oscillator. However, it was shown by Määttänen [6,7], that it is inadequate to consider only the first mode. Principally, all modes of a structure can be excited by ice forces, but the first two modes are usually the most dominant. In this contribution, the simulated response to the ice loads is obtained using a multi-DOF beam, modally reduced to a 2-DOF system. It is shown that the corresponding modal states as well as 4 model parameters can be successfully identified from two displacement and two velocity measurements.

## 2. State space model of the compliant structure

We consider a linear dynamic model of the compliant structure with viscous damping and  $n_{\text{DOF}}$  degrees of freedom, which can be described by  $n_{\text{DOF}}$  second-order differential equations

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{S}_f f(\mathbf{u}, t), \quad (1)$$

where  $\mathbf{u}(t) \in \mathbb{R}^{n_{\text{DOF}}}$  is the vector of degrees of freedom,  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K} \in \mathbb{R}^{n_{\text{DOF}} \times n_{\text{DOF}}}$  denote the mass, damping and stiffness matrices, which are obtained using a finite element method, and  $\mathbf{S}_f$  determines the position where the state dependent load  $f(\mathbf{u}, t)$  acts on the structure.

In order to reduce the total number of unknowns to be estimated, a reduced-order state-space model is used for the identification, constructed using only the lower-order modes typically present in ice-induced vibrations of bottom-founded offshore structures. We assume proportional damping, in which case general complex modes can be replaced by normal modes. Then, the model can be transformed to the normal-mode state-space model

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}f(\mathbf{x}, t) \quad (2)$$

with the states  $\mathbf{x} \in \mathbb{R}^n$  given by

$$\mathbf{x} = \begin{pmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{pmatrix}, \quad (3)$$

and modal coordinates  $\mathbf{z} \in \mathbb{R}^{n_m}$ , where the used number of modes  $1 \leq n_m \leq 2n_{\text{DOF}}$ , and the number of states  $n = 2n_m$ . The vector of degrees of freedom is then obtained as  $\mathbf{u}(t) = \mathbf{\Phi}\mathbf{z}(t)$ , where the matrix  $\mathbf{\Phi} \in \mathbb{R}^{n_{\text{DOF}} \times n_m}$  is determined from an eigenvalue problem corresponding to equation (1):

$$\mathbf{K}\mathbf{\Phi} = \mathbf{M}\mathbf{\Phi}\mathbf{\Omega}^2, \quad (4)$$

with the diagonal matrix  $\mathbf{\Omega}$  containing the eigenfrequencies and the orthogonality conditions:

$$\mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi} = \mathbf{I}, \quad \mathbf{\Phi}^T \mathbf{K} \mathbf{\Phi} = \mathbf{\Omega}^2. \quad (5)$$

The matrix  $\mathbf{A} \in \mathbb{R}^{n \times n}$  and the vector  $\mathbf{B} \in \mathbb{R}^n$  are then given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega} & -\mathbf{\Gamma} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^T \mathbf{S}_f \end{bmatrix}, \quad (6)$$

where  $\mathbf{\Gamma} = \mathbf{\Phi}^T \mathbf{C} \mathbf{\Phi}$  due to the assumption of proportional damping.

## 3. Määttänen's model for ice crushing strength

A model for the synchronized resonant frequency lock-in of ice-induced vibrations was proposed by Määttänen in [8]. This frequency lock-in has been widely observed in full scale measurements. Määttänen's model for the ice crushing strength  $\sigma_c$  is given by

$$\dot{\sigma} = (V_{\text{ice}} - \dot{u}_s) \frac{8\sigma_0}{\pi D}$$

$$\sigma_c = \begin{cases} 0 & \dot{\sigma} < 0, \\ a\dot{\sigma} + b\dot{\sigma}^2 + c\dot{\sigma}^3 + d\dot{\sigma}^4 + 2 & 0 \leq \dot{\sigma} \leq 1.3 \text{ MPa/s}, \\ 1 & 1.3 \text{ MPa/s} < \dot{\sigma}, \end{cases} \quad (7)$$

where  $\dot{u}_s$  is the velocity of the structure at the ice level,  $\sigma_0$  is a reference ice strength,  $D$  is the diameter of the cylindrical structure,  $V_{ice}$  is the undisturbed velocity of the ice, and  $\dot{\sigma}$  is the stress rate. With this model the resulting ice load  $f(\dot{u}_s)$  is given by

$$f(\dot{u}_s) = A \sigma_c \sqrt{\frac{A_0}{A}}, \tag{8}$$

with the projected contact area  $A$ , and a reference area  $A_0$ . The ice crushing strength in Määtänen’s model is defined piecewise, including a discontinuity at  $\dot{\sigma} = 0$ , a polynomial region for  $0 \leq \dot{\sigma} \leq 1.3$  MPa/s, and a constant region for  $\dot{\sigma} > 1.3$  MPa/s. Using the specific parameter values from Table 1, the ice crushing strength is shown in Fig. 1. Since the discontinuity at  $\dot{\sigma} = 0$  leads to numerical problems during the parameter identification procedure, the ice crushing strength  $\sigma_c$  from equation (7) is multiplied by  $\frac{1}{2}(1 + \tanh(100\dot{\sigma}))$  in order to obtain a continuous version of the ice crushing strength.

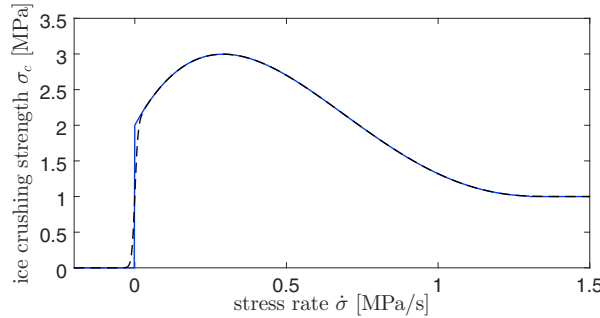


Fig. 1. Ice crushing strength  $\sigma_c$  (solid) and continuous approximation using tanh (dashed).

#### 4. Parameter estimation using the Unscented Kalman Filter

In order to handle the higher-order nonlinearities characterizing the ice-structure interaction, the Unscented Kalman filter (UKF) is employed. The UKF is intended for the estimation of the states  $\mathbf{x} \in \mathbb{R}^n$  of nonlinear systems, which are disturbed by a vector of zero-mean normal distributed white noise  $\zeta$  using measurements, cf. [5]. The measurements are in general also disturbed by a vector  $\xi$  of zero-mean normal distributed white noise and stated as a function of the unknown states. This function is denoted as the sensor function  $\mathbf{h}$ . If unknown parameters of the considered model are to be estimated, a common approach is to introduce them as additional state space variables. The state vector is then augmented by a parameter vector  $\mathbf{p} \in \mathbb{R}^{n_p}$ , where  $\dot{\mathbf{p}} = \mathbf{0}$ , since the parameters are constants. Due to this constraint, the expected value of these parameters changes only in the correction step of the UKF. Considering the state space model from equation (2), the resulting equations for the filter problem are given by

$$\begin{aligned} \dot{\mathbf{x}}_p &= \mathbf{A}_p \mathbf{x}_p + \mathbf{B}_p f(\mathbf{x}_p, t) + \zeta, \\ \mathbf{h} &= \mathbf{g}(\mathbf{x}_p, t) + \xi, \end{aligned} \tag{9}$$

with the augmented state vector  $\mathbf{x}_p \in \mathbb{R}^{n+n_p}$  relating the measurements to the state vector, a suitable function  $\mathbf{g}(\mathbf{x}_p, t)$ , and with corresponding  $\mathbf{A}_p \in \mathbb{R}^{(n+n_p) \times (n+n_p)}$  and  $\mathbf{B}_p \in \mathbb{R}^{n+n_p}$  such that

$$\mathbf{x}_p(t) = \begin{bmatrix} \mathbf{x} \\ \mathbf{p} \end{bmatrix}, \quad \mathbf{A}_p = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{B}_p = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix}. \tag{10}$$

If the sensor function consists of measurements of acceleration, velocity, and/or displacement, and if the state vector is given by the modal coordinates according to equation (3), where  $n_d$  is the number of measurements, then the sensor function becomes

$$\mathbf{h} = \mathbf{G}\mathbf{x} + \mathbf{J}f(\mathbf{x}_p, t) + \xi. \tag{11}$$

Herein, the output influence matrix  $\mathbf{G} \in \mathbb{R}^{n_d \times n}$  and the direct transmission matrix  $\mathbf{J} \in \mathbb{R}^{n_d}$  are defined as

$$\mathbf{G} = \begin{bmatrix} \mathbf{S}_d \Phi - \mathbf{S}_a \Phi \Omega^2 & \mathbf{S}_v \Phi - \mathbf{S}_a \Phi \Gamma \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} \mathbf{S}_a \Phi \Phi^T \mathbf{S}_f \end{bmatrix}, \tag{12}$$

where the selection matrices  $\mathbf{S}_a$ ,  $\mathbf{S}_v$ , and  $\mathbf{S}_d \in \mathbb{R}^{n_q \times n_{\text{DOF}}}$  are populated according to the spatial location at which acceleration, velocity, and/or displacement is measured. From an initial state estimate  $\hat{\mathbf{x}}_0$  with an initial variance, the UKF algorithm propagates the expected value and sigma-points according to the variance through the system dynamics equations. The measurements of the response of the considered structure are subtracted from the predicted measurements of the UKF system, by which means updates are calculated and fed back to the estimated states  $\hat{\mathbf{x}}$  and their estimated variance. This iterative procedure determines state estimates of the unknown states of the nonlinear system. Of course, one can not expect convergence if the number of measurements is low and the number of unknown parameters is high, or if the noise disturbance is too severe or not Gaussian.

## 5. Results

We illustrate the application of the proposed scheme for the identification of multiple model parameters governing the ice-structure interaction. In order to verify whether the model parameters can be successfully identified using the proposed scheme, measurement data are numerically generated for a specific compliant structure excited by a moving ice sheet. As the compliant structure we consider a vertical cylindrical beam, which is founded on the seabed and excited by a load due to ice, as shown in Fig. 2. Any forces due to wind or water are neglected. The cylindrical beam is modeled using 21 nodes (20 elements), such that the ice load acts at node 14. Each node has one translational and one rotational degree of freedom, resulting in a total number of 40 degrees of freedom. The first two natural frequencies of the beam are 0.5 Hz and 3.15 Hz. All selected parameter values of the beam, as well as of the model for the ice crushing strength from section 3 are specified in Table 1. Our goal is to identify the four unknown parameters  $a$ ,  $b$ ,  $c$ ,

Table 1. Parameter specifications.

$V_{\text{ice}}$	$\sigma_0$	$D$	$A$	$A_0$	$a$	$b$	$c$	$d$
0.1 m/s	2 MPa/s	1 m	0.5 m <sup>2</sup>	1 m <sup>2</sup>	7.8	18.57	13	-2.91

and  $d$  from Määttänen's model (7) using measurements of the dynamical behavior of the beam and the ice velocity. The true parameters were set according to the values in table 1. Retaining a reduced-order state space model for the beam using two modes leads to

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -9.967 & 0 & -0.063 & 0 \\ 0 & -390.889 & 0 & -0.395 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ 0.337 \\ -0.302 \end{bmatrix} \quad (13)$$

in equation (6). The UKF algorithm is now used to estimate the unknown states  $\mathbf{x}_p$  from the noisy measurements  $\mathbf{h}$ , as presented in section 4. In the following, we show the identification results of the unknown parameters  $a$ ,  $b$ ,  $c$ , and  $d$  from Määttänen's model, cf. Eq. (7), where we assume that the ice velocity  $V_{\text{ice}}$  is known. All unknown parameters are initialized by zero. The system noise is set to  $\boldsymbol{\zeta} = \mathbf{0}$  for the considered problem. The difficulty in the presented setup lies in the estimation of the modal states of the second mode. If this estimation is not sufficient, then the identification of the parameters  $a$ ,  $b$ ,  $c$ , and  $d$  does not lead to acceptable results. Without measurement noise, the true parameters could be estimated using measurements of the displacement and velocity at node 14, only. The convergence of the parameter estimates to the true values is shown in figure 5. Adding very small measurement noise  $\boldsymbol{\xi}$  of 0.1% or more of the standard deviation of the respective displacement or velocity, the parameter identification procedure does not lead to sufficient results, if only displacement and velocity at node 14 are measured. The reason lies in the more difficult estimation of the modal states  $z_2$  and  $\dot{z}_2$  of the second mode. This problem can be overcome by using additional measurements of displacement and velocity at a node where the output influence is still low for the first mode and high for the second mode. Then the modal coordinates  $z_2$  and  $\dot{z}_2$  are better identifiable by the UKF. This can be achieved by taking measurements at node 7, where the corresponding absolute value of the entry in the matrix  $\mathbf{G}$  for the modal coordinates of the first mode is 0.088 and 0.338 for the second mode. For the case of displacement and velocity measurements at node 7 and node 14, the matrices in the sensor function (11) are explicitly given by  $\mathbf{J} = \mathbf{0}$  and

$$\mathbf{G} = \begin{bmatrix} 0.088 & -0.338 & 0 & 0 \\ 0.337 & -0.302 & 0 & 0 \\ 0 & 0 & 0.088 & -0.338 \\ 0 & 0 & 0.337 & -0.302 \end{bmatrix}. \quad (14)$$

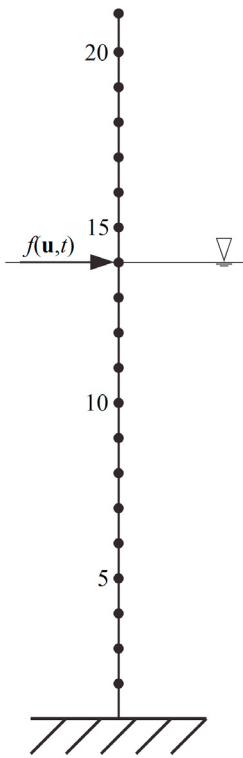


Fig. 2. Depiction of the discretized beam and node numbers.

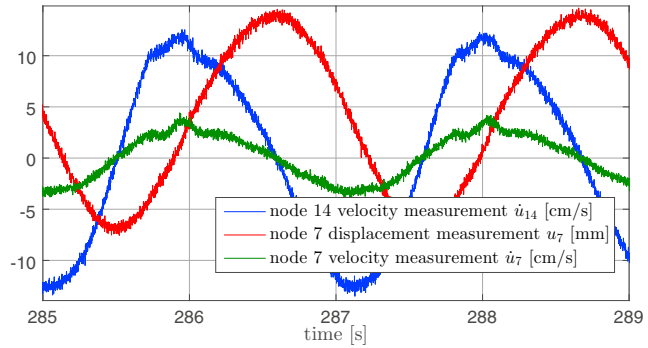


Fig. 3. Simulated displacement at node 7 and velocities at nodes 7 and 14. Introduced noise is 2% of the standard deviation of the respective displacement or velocity.

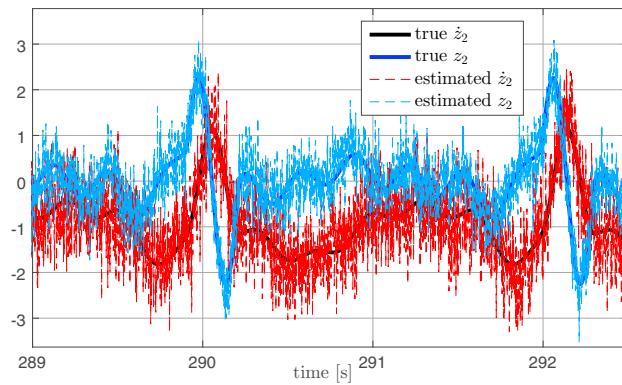


Fig. 4. Mode 2 estimates.

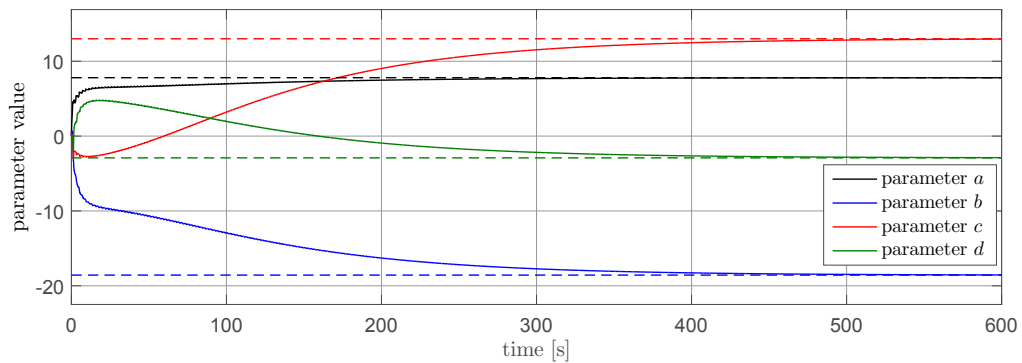


Fig. 5. Estimated parameters without measurement noise, using only measurements of displacement and velocity at node 14. The true values are shown as dashed lines.

Simulated displacement at node 7 and velocities at nodes 7 and 14 where the introduced noise  $\xi$  is 2% of the standard deviation of the respective displacement or velocity, are shown in Fig. 3. In this case a sufficiently accurate estimation of  $z_2$  and  $\dot{z}_2$  could be achieved, such that the parameter identification converged. A part of the corresponding estimated time series of  $z_2$  and  $\dot{z}_2$  is shown in Fig. 4. The response of this beam at node 14 and node 7 starting at rest can be seen in Fig. 6 and Fig. 7, respectively. The resulting time series of the parameter estimations are shown in Fig. 8, which is the main result. The true values of the parameters are shown as dashed lines. The parameters are sufficiently estimated after 100 seconds. In comparison to the results from Fig. 5, the estimations converged much faster, due to the additional measurements.

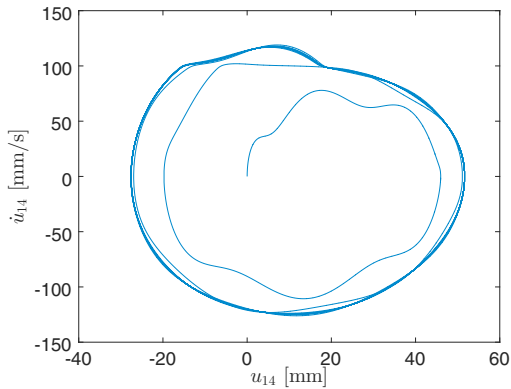


Fig. 6. Response of the beam at node 14 starting at rest.

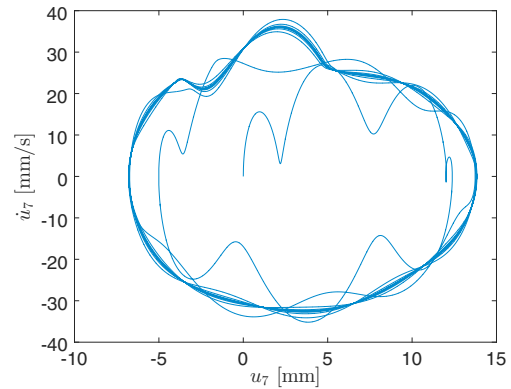


Fig. 7. Response of the beam at node 7 starting at rest.

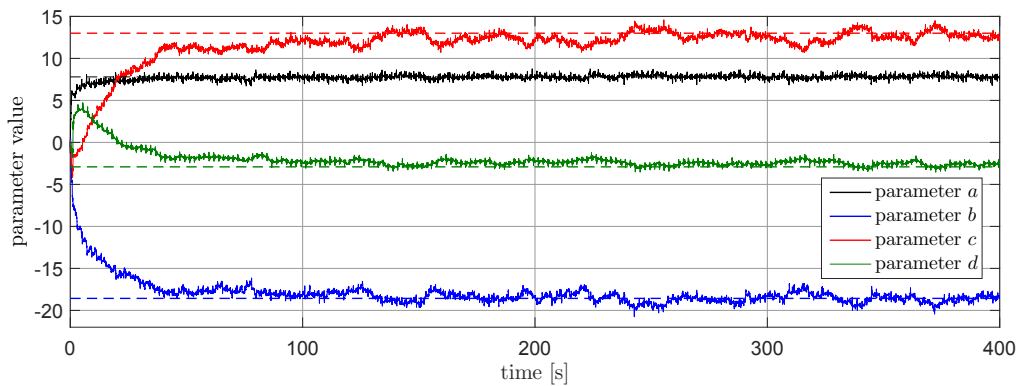


Fig. 8. Estimated parameters using measurements of displacement and velocity at nodes 7 and 14, for the case of 2% measurement noise, cf. Fig. 3. The true values are shown as dashed lines.

## 6. Conclusions

The results show that the proposed scheme can successfully be used for the identification of multiple model parameters governing the ice-structure interaction. The identification of four model parameters and two modal states from four noise-contaminated vibration measurements was shown to be feasible. Future work will include the application of the presented methodology to more advanced ice-induced vibration models.

## References

- [1] H. Hendrikse, A. Metrikine, Interpretation and prediction of ice induced vibrations based on contact area variation, *International Journal of Solids and Structures* 75 (2015) 336–348.
- [2] R. Gagnon, An explanation for the molikpaq may 12, 1986 event, *Cold Regions Science and Technology* 82 (2012) 75–93.
- [3] K. Blenkarn, et al., Measurement and analysis of ice forces on cook inlet structures, in: *Offshore Technology Conference, Offshore Technology Conference*, 1970.
- [4] M. Määttänen, Numerical model for ice-induced vibration load lock-in and synchronization, in: *Proceedings of the 14th International Symposium on Ice, Potsdam/New York/USA, volume 2, 1998*, pp. 923–930.
- [5] S. J. Julier, J. K. Uhlmann, Unscented filtering and nonlinear estimation, *Proceedings of the IEEE* 92 (2004) 401–422.
- [6] M. Määttänen, Experiences of ice forces against a steel lighthouse mounted on the seabed, and proposed constructional refinements, *Port and Ocean Engineering Under Arctic conditions (POAC)* (1975) 857–867.
- [7] M. Määttänen, Dynamic ice-structure interaction during continuous crushing, *Technical Report, DTIC Document*, 1983.
- [8] M. Määttänen, Numerical model for ice-induced vibration load lock-in and synchronization, *Ice in Surface Waters: Proceedings of the 14th International Symposium on Ice 2 (1998)* 923–930.