61 | August 1959

SCHRIFTENREIHE SCHIFFBAU

H. Baumann

On a Instability of Surface Waves

TUHH
Technische Universität Hamburg-Harburg

On a Instability of Surface Waves

H. Baumann,, Hamburg, Technische Universität Hamburg-Harburg, 1959

Schriftenreihe Schiffbau
Schwarzenbergstraße 95c
D-21073 Hamburg

http://www.tuhh.de/vss

On an Instability of Surface Waves.

M. Schuler[1] has described a sudden change of the wave pattern occuring in the neighbourhood of a wave generator. Under certain conditions, suddenly transverse standing waves appear the period of which is double the period of the progressive waves. If a plunging sphere is used as generator, under certain conditions waves with radial (instead of circular) crests appear.

As until now a theoretical foundation of this phenomenon has not yet been given; we have tried to find out at least some of the conditions necessary for a sudden change of that kind.

The apparent gravity of any surface particle in a progressive wave of amplitude a and frequency $\omega/2\pi$ is the sum of gravity $g \approx 981\ cm/s^2$ and kinematic acceleration $b(t) = -a\omega^2 \sin \omega t$

$$g'(t) = g + b(t) = g\cdot[1+\beta(t)]. \tag{1}$$

The free surface condition for a superimposed flow with potential ϕ is thus

$$\frac{\partial^2}{\partial t^2}\phi + [1+\beta(t)]\cdot g\cdot\frac{\partial}{\partial z}\phi = 0, \tag{2}$$

z being the vertical coordinate. Now, if

$$\phi(z,y,t) = e^{-kz}\cdot\cos ky\cdot A(t) \tag{3}$$

is the potential of a standing transverse wave in a cross section $0 \leq y \leq B$ of the tank of breadth B,

1) M. Schuler, "Der Umschlag von Oberflächenwellen". ZAMM 13 (1933) 443 - 446.

the free surface condition can be written as a Mathieu equation

$$\frac{d^2}{dt^2}A(t) + [1+\beta(t)]\cdot g\cdot k\cdot A(t) = 0, \tag{4}$$

where from (3)

$$k\cdot B = n\cdot\pi, \qquad n \text{ integer}. \tag{5}$$

It is known that instability occurs when the natural period of the "oscillator" T_A is about twice the period of excitation T_β; more exactly: when

$$1-\tfrac{1}{4}\bar{\beta} < \frac{2T_\beta}{T_A} < 1+\tfrac{1}{4}\bar{\beta}, \tag{6}$$

$\bar{\beta} = \frac{a\omega^2}{g} = \frac{2\pi a}{\lambda}$ being the maximum acceleration or, which is the same, the maximum wave slope of the progressive wave. With regard to the wave lengths we have

$$\lambda_A = \frac{2B}{n} \qquad \text{from (5)}, \tag{7}$$

$$\lambda_{progr} \approx \tfrac{1}{4}\lambda_A = \frac{B}{2n} \qquad \text{from (6)}. \tag{8}$$

In consequence the first condition of instability may practically be written as

$$T_\beta \approx \sqrt{\frac{2\pi\lambda_{progr}}{g}} \approx \sqrt{\frac{\pi B}{g\cdot n}} \tag{9}$$

A second condition results from the damping of the "oscillator" which in the plane wave-formula (3) has been neglected and which cannot easily be found. In other connection[2] we have shown that in the case of a damped oscillator the acceleration $\overline{\beta}$ must exceed a limit

$$\overline{\beta} > \frac{2}{\pi} \cdot \ln \frac{A_n}{A_{n+1}} \qquad (10)$$

where A_n and A_{n+1} are subsequent amplitudes of the free damped oscillations (with a time difference of one period).

Under these conditions a small disturbance will suffice to cause a change in the wave pattern, characterized by feeding up transverse waves with energy taken from the progressive wave. This process probably will begin at the wave generator where the vertical acceleration is greater because of the additional flow necessary to satisfy the continuity and surface conditions at the plunger. A steady state will be attained finally either by breaking of the transverse waves, which means energy dissipation and consequently a reduced height of the progressive wave down the tank or by detuning the natural period of the transverse wave at finite amplitude without reduction of the amplitude of the progressive wave.

[2] "Roll Resonance in a Transverse Swell" by Dr. H. Baumann prepared for publication in the "Journal of Ship Research"

Hamburg, August 4, 1959

Baumann