

# Nonlinear Flatness-Based Control of Driveline Oscillations for a Powertrain with Backlash Traversing

Truc Pham<sup>\*,\*\*</sup> Robert Seifried<sup>\*</sup> Andreas Hock<sup>\*\*</sup>  
Christian Scholz<sup>\*\*</sup>

<sup>\*</sup> *Institute of Mechanics and Ocean Engineering, Hamburg University of Technology, 21073 Hamburg, Germany (e-mail: robert.seifried@tuhh.de).*

<sup>\*\*</sup> *Porsche Research and Development Centre, 71287 Weissach, Germany (e-mail: {hong\_truc.pham, andreas.hock, christian.scholz}@porsche.de)*

**Abstract:** Load changes in automotive powertrains may cause uncomfortable vibrations during vehicle acceleration. These driveline oscillations occur due to flexible and underdamped shafts. Furthermore, backlashes in the drive train represent hard nonlinearities and lead to challenges in control. Reducing vibration only by feedback control may destabilize the closed loop due to time delays in the powertrain system. This paper proposes a flatness-based feedforward control law based on a two-mass powertrain model with backlash to compensate oscillations. In addition, a proportional feedback controller is used to damp remaining oscillations arising from model mismatches and disturbances after the set-point transition. Simulation results show improved tracking performance in comparison to a feedforward approach without considering backlash explicitly.

© 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

**Keywords:** Driveline Oscillation, Feedforward Control, Backlash, Differential Flatness, Load Change, Set-Point Transition.

## 1. INTRODUCTION

A challenging area in control of driveline oscillations is backlash traversing. When the backlash is traversed no torque is transmitted, but when the first contact is achieved, torque is abruptly introduced through the shaft, as discussed in Lagerberg (2001). The effect of this hard nonlinearity is known as "shudder". Especially, abrupt driving maneuvers, where the driver causes the traversing by tip-in and tip-out maneuver such that the vehicle changes from coast to drive condition and vice-versa, induce driveline oscillations. These are due to flexible shafts and backlashes. The requested torque change of the driver corresponds to a set-point transition of the powertrain system. Driveline oscillations are uncomfortable for passengers and stressing for mechanical parts of the powertrain.

There are several studies on driveline oscillation control without considering backlash explicitly in control design. For instance, in Bruce et al. (2005), Grotjahn et al. (2006) feedforward and feedback controller for powertrain models without backlashes are presented. A sliding mode observer and controller for a simplified powertrain model without backlash are investigated in Angeringer et al. (2012). The observer and controller are evaluated by a detailed powertrain model with backlash. These control systems have to be designed conservative enough such that the impact of backlashes is small, see Lagerberg (2001).

On the other hand, control systems, which take backlashes explicitly into account, can prevent driveline oscillations caused by backlashes and therefore can be designed more efficiently. A feedback controller with an optimization based backlash handling is designed in Templin and Egardt (2009). The backlash handling strategy introduces a torque hold level, such that the requested engine torque is limited, while the backlash is traversed. In Lagerberg and Egardt (2005) a switching controller is presented. There, a state feedback controller is used in contact mode. When the backlash gap has to be traversed, the control system switches to a model predictive controller.

In general, automotive control loops have to be robust to time delays, external disturbances and ensure good tracking performance. Time delays can occur due to signal delays (communication delay between different control units, filtering algorithm) and physical delays (delay of torque build-up of the engine). However, a feedback controller, which is robust to time delay, may lead to degradation of tracking performance. A feedforward controller can complement a feedback controller to improve the tracking performance. Few studies have addressed the problem of feedforward control design of powertrains to fulfill set-point transition. Especially, there remains a gap in feedforward control design which take backlash into account.

An important concept for analytical feedforward control design is differential flatness. It was introduced in Fliess

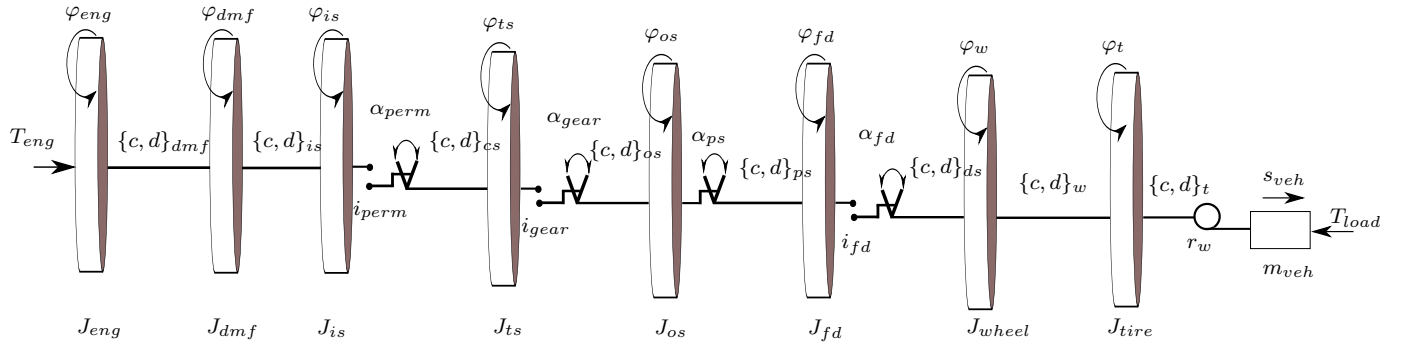


Fig. 1. Detailed powertrain model with lumped inertias  $J$ , lumped mass  $m$ , Torque  $T$ , rotational angle  $\varphi$ , distance  $s$ , stiffness  $c$ , damping  $d$ , gear ratio  $i$ , backlash angle  $\alpha$ , and wheel radius  $r_w$ . The indices denote engine (eng), dual mass flywheel (dmf), input shaft (is), permanent (perm), counter shaft (cs), transmission shaft (ts), gearbox (gear), output shaft (os), propeller shaft (ps), final drive (fd), drive shaft (ds), wheels (w), tire (t) and vehicle (veh).

et al. (1995); see also Sira-Ramírez and Agrawal (2004). Differential flatness was used in automotive control, for instance in Feldt et al. (2010), Gasper and Abel (2010) and Pham and Bushnell (2015). However, these works do not consider backlash explicitly.

This paper focuses on flatness-based feedforward controller design to fulfill set-point transition with damped driveline oscillations. Thereby backlash is considered in the feedforward control. The following specific contributions are made: A nonlinear two-mass powertrain model with backlash is identified. The equations of motion of the derived model are applied to design a flatness-based feedforward controller. A proportional feedback controller is activated after set-point transition to damp remaining driveline oscillations. The approach is evaluated through a simulation study and compared to a flatness-based feedforward controller without taking backlash into account. It is shown that the presented approach leads to significant less oscillations.

The paper is organized as follows. Section 2 provides a system model for vehicle powertrains with backlash. In Section 3 a nonlinear feedforward controller based on a two-mass powertrain model is presented. Section 4 contains simulation results for the presented control approach and Section 5 concludes the paper.

## 2. MODELING POWERTRAINS WITH BACKLASH

In this section a detailed powertrain model is presented. The model is used to derive a two-mass control model and for simulation study.

### 2.1 Detailed Powertrain Model

In order to represent the longitudinal dynamics of a vehicle powertrain, a powertrain is usually modeled as a chain of lumped inertias coupled by spring and damper elements, e.g. see Kiencke and Nielsen (2005). Fig. 1 illustrates the chain structure by a typical rear-driven powertrain model with eight lumped inertias and a lumped vehicle mass  $m_{veh}$ . The degrees of freedom of a lumped powertrain model are the rotation phases  $\varphi$  of each inertia and the

traveled distance of the vehicle  $s_{veh}$ . The system input is usually the engine torque  $T_{eng}$ , which acts on the first inertia. A disturbance  $T_{load}$  influences the vehicle mass  $m_{veh}$ . Flexibilities of the shafts are represented by stiffness and damping functions depending on the phase and speed difference, respectively. Especially shaft flexibility, tire characteristic, wind disturbances and backlash are responsible for the nonlinear behavior of the powertrain, see Gillespie (1992) and Kiencke and Nielsen (2005).

As depicted in Fig. 2, backlash is clearance between mechanical parts. In powertrains these parts are mainly mated gear teeth. A dead-zone model can describe the dynamic of backlashes, see Lagerberg (2001);

$$T_{flex}(\Delta\varphi, \Delta\dot{\varphi}) = \begin{cases} c(\Delta\varphi + \alpha) + d\Delta\dot{\varphi}, & \Delta\varphi \leq -\frac{\alpha}{2} \\ 0, & |\Delta\varphi| < \frac{\alpha}{2} \\ c(\Delta\varphi - \alpha) + d\Delta\dot{\varphi}, & \Delta\varphi \geq \frac{\alpha}{2} \end{cases} \quad (1)$$

Stiffness and damping coefficients are denoted by  $c$  and  $d$ ,  $\Delta\varphi$  is the phase difference and  $\Delta\omega$  is the rotational speed difference between two inertias. The backlash angle is denoted by  $\alpha$ . As long as the amount of the phase difference is smaller than the half backlash angle, no torque is transmitted.

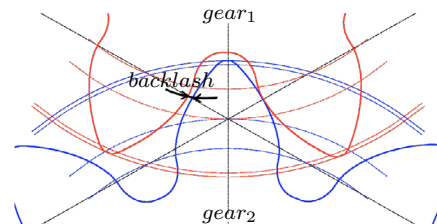


Fig. 2. Backlash due to clearance between mated gear teeth.

### 2.2 Nonlinear Two-Mass Control Model

Due to different inertias and flexibilities in the detailed model, the powertrain can oscillate in various modes. But one dominant mode for set-point transition, which can

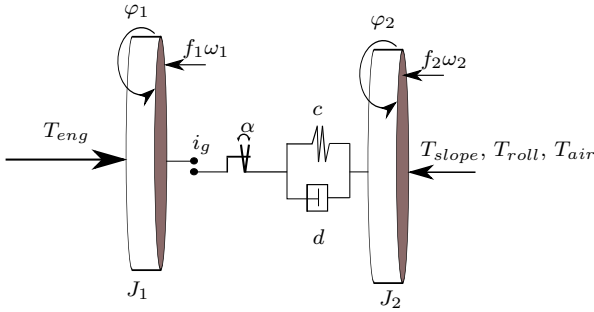


Fig. 3. Two-mass model with backlash

be experienced by the passenger, is the low-frequency shuffle mode, which is typically in the range of 1-7 Hz. To represent the shuffle mode, a two-mass model, as shown in Fig. 3 and used in Grotjahn et al. (2006) and Bruce et al. (2005), is sufficient. The equations of motion of the two-mass model with backlash are;

$$\begin{aligned}\Delta\dot{\varphi} &= \frac{\omega_1}{i_g} - \omega_2, \\ \dot{\omega}_1 &= \frac{1}{J_1 i_g} [-T_{flex}(\Delta\varphi, \Delta\dot{\varphi})] - \frac{f_1}{J_1} \omega_1 + \frac{1}{J_1} T_{eng}, \\ \dot{\omega}_2 &= \frac{1}{J_2} [T_{flex}(\Delta\varphi, \Delta\dot{\varphi}) - f_2 \omega_2 - T_{slope} - T_{roll} - T_{air}],\end{aligned}\quad (2)$$

with the piecewise torque flexibility function given in (1) and total gear ratio  $i_g$ . The system states are the phase difference  $\Delta\varphi$ , rotational speed of the first inertia  $\omega_1$  and rotational speed of the second inertia  $\omega_2$ . Viscous friction coefficients are denoted by  $f_1$  and  $f_2$ . The three disturbances are defined by

$$\begin{aligned}T_{slope} &= m_{veh} r_w g \sin \theta, \\ T_{roll} &= m_{veh} r_w (f_{r1} + f_{r2} \omega_2 r_w), \\ T_{air} &= \frac{1}{2} \rho_{air} c_{air} A_{veh} \omega_2^2 r_w^3,\end{aligned}\quad (3)$$

representing slope, rolling and air resistance. Thereby  $m_{veh}$  denotes vehicle mass,  $g$  gravitational constant,  $\theta$  road inclination,  $r_w$  wheel radius,  $f_{r1}$  rolling offset,  $f_{r2}$  rolling resistance,  $\rho_{air}$  air density,  $c_{air}$  drag coefficient and  $A_{veh}$  reference area of the vehicle, see Kiencke and Nielsen (2005) for details.

Using equations (3) and the abbreviations

$$\begin{aligned}b_1 &= f_1, \\ b_2 &= f_{r2} r_{wheel}^2 m_{veh} + f_2,\end{aligned}\quad (4)$$

the equations of motion (2) becomes

$$\begin{aligned}\Delta\dot{\varphi} &= \frac{\omega_1}{i_g} - \omega_2, \\ \dot{\omega}_1 &= \frac{1}{J_1 i_g} [-T_{flex}(\Delta\varphi, \Delta\dot{\varphi})] - \frac{b_1}{J_1} \omega_1 + \frac{1}{J_1} T_{eng}, \\ \dot{\omega}_2 &= \frac{1}{J_2} [T_{flex}(\Delta\varphi, \Delta\dot{\varphi}) - b_2 \omega_2 - T_{air}(\omega_2) - \delta],\end{aligned}\quad (5)$$

with disturbance function  $\delta$  without system states;

$$\delta = m_{veh} r_w (f_{r1} + g \sin(\theta)). \quad (6)$$

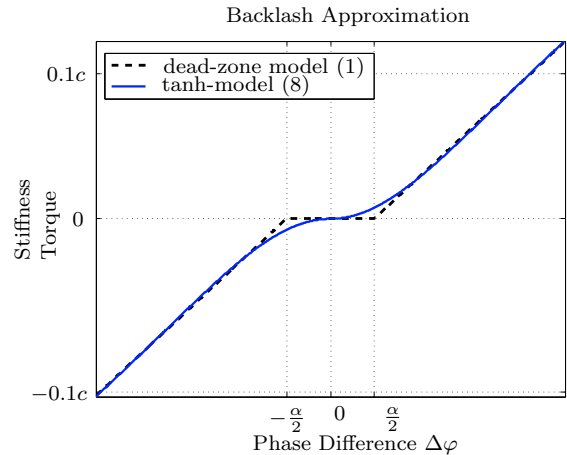
As discussed in Kiencke and Nielsen (2005), the influence of air resistance is small in low gears and since oscillations caused by set-point transition mainly occur in low gears, (5) can be simplified to

$$\begin{aligned}\Delta\dot{\varphi} &= \frac{\omega_1}{i_g} - \omega_2, \\ \dot{\omega}_1 &= \frac{1}{J_1 i_g} [-T_{flex}(\Delta\varphi, \Delta\dot{\varphi})] - \frac{b_1}{J_1} \omega_1 + \frac{1}{J_1} T_{eng}, \\ \dot{\omega}_2 &= \frac{1}{J_2} [T_{flex}(\Delta\varphi, \Delta\dot{\varphi}) - b_2 \omega_2 - \delta].\end{aligned}\quad (7)$$

The equations of motion (7) are nonlinear in the piecewise defined function  $T_{flex}(\Delta\varphi, \Delta\dot{\varphi})$ . This function is not differentiable for  $|\Delta\varphi| = \frac{\alpha}{2}$ . In order to design a flatness-based feedforward controller in Section 3 the dead-zone model (1) is approximated with the help of a tanh-function;

$$T_{flex}(\Delta\varphi, \Delta\dot{\varphi}) = (c\Delta\varphi + d\Delta\dot{\varphi}) q \tanh(p|\Delta\varphi|). \quad (8)$$

The tuning parameters  $p$  and  $q$  can be used to fit the curve to the dead-zone model. Fig. 4 shows a good approximation of a piecewise defined stiffness torque function (1) by a tanh-function (8). It can be seen that the approximation model transmits small stiffness torque, even in the gap area. This behavior does not indicate a problem, since in a general powertrain there are several backlashes, which transmit torque at different times. Additionally, oil between mated gear teeth can transmit torque, too. Therefore, zero torque transmission in the gap area is not necessary, as discussed in Haschka and Krebs (2007).

Fig. 4. Approximation of a dead-zone model (1) by a tanh-model (8) with damping parameter  $d = 0$ .

Using (8), the equations of motion (7) become

$$\begin{aligned}\Delta\dot{\varphi} &= \frac{\omega_1}{i_g} - \omega_2, \\ \dot{\omega}_1 &= \frac{1}{J_1 i_g} [-(c\Delta\varphi + d\Delta\dot{\varphi}) q \tanh(p|\Delta\varphi|)] - \frac{b_1}{J_1} \omega_1 + \frac{1}{J_1} T_{eng}, \\ \dot{\omega}_2 &= \frac{1}{J_2} [(c\Delta\varphi + d\Delta\dot{\varphi}) q \tanh(p|\Delta\varphi|) - b_2 \omega_2 - \delta].\end{aligned}\quad (9)$$

### 2.3 Parameter Identification

The system parameters  $J_1, J_2, c, d, b_1, b_2$ , backlash parameters  $p, q$  and disturbance parameter  $f_{r1}$  have to be identified for the equations of motion (9). Vehicle mass  $m_{veh}$  and wheel radius  $r_w$  are usually known and road inclination  $\theta$  is

measurable. For simplification, the parameter  $q$  in (9) can be considered in  $c, d$  and therefore only eight parameters have to be determined.

As a reference to find the parameters of the two-mass model, a detailed powertrain model, which is validated by measurement data, or the measurement data itself, can be used. The following error function can be minimized to determine the system parameters

$$e = \sum_k \left[ (\omega_{eng,ref}(k) - \omega_1(k))^2 + (a_{veh,ref}(k) - \dot{\omega}_2(k)r_w)^2 \right] \quad (10)$$

over all time steps  $k$  in the considered time interval. Engine speed  $\omega_{eng}$  and vehicle acceleration  $a_{veh}$  are common measurable values in cars and hence, are used to determine the deviation from the two-mass model to these references.

Optimization techniques can be used to minimize the error function (10) over the parameters  $J_1, J_2, c, d, p, q, b_1, b_2$  and  $f_{r1}$ . For instance a direct search method is applied in this paper. The advantages are that it is easy to implement, applicable to a broad class of nonlinear optimization problems and work well in practice, see Lewis et al. (2000). The disadvantages of direct search method in terms of robustness, guarantee of convergence and global minimum are for this problem less significant, since good initial values are available based on physical estimations. The same applies to convergence time, due to the small number of optimization variables.

### 3. NONLINEAR CONTROL OF A POWERTRAIN WITH BACKLASH

In this section, a flatness based feedforward controller for a nonlinear two-mass powertrain model with backlash is presented to ensure good tracking performance during set-point transition. The control design is based on a two-mass model with a continuous backlash function (8). Trajectory planning is discussed, since the derived feedforward controller needs planned reference trajectories. Then, a proportional feedback controller is described, which is activated after the set-point transition to damp remaining driveline oscillations.

#### 3.1 Differential Flat Feedforward Control

The concept of differential flatness allows to describe system states, input, and output in terms of the *flat output* and its derivatives. Meaning the system is inverted and has no zero dynamics. The whole system dynamics are given by the flat output and its derivatives.

Flatness for a general nonlinear system is defined as follows:

*Definition 1.* (Fliess et al. (1995)). The nonlinear SISO system

$$\begin{aligned} \dot{x} &= f(x, u), & x(0) &= x_0 \\ y &= h(x) \end{aligned} \quad (11)$$

with  $x \in \mathbb{R}^n$  and  $u, y \in \mathbb{R}$  is said to be *differentially flat*, if and only if there exists a flat output  $z \in \mathbb{R}$ , such that

- the flat output  $z$  is a function of the state variables  $x$ :  $z = \lambda(x)$ ,

- the system state, input and output can be parametrized with  $z$  and a final number of its derivatives:

$$\begin{aligned} x &= \Phi_x(z, \dot{z}, \dots, z^{(n-1)}), & u &= \Phi_u(z, \dot{z}, \dots, z^{(n)}), \\ y &= \Phi_y(z, \dot{z}, \dots, z^{(n-r)}), \end{aligned}$$

where  $r$  is the relative degree of (11).

The main challenge in nonlinear feedforward control design based on differentially flatness theory is to find a flat output  $z$ . To design a feedforward controller for a two-mass powertrain model with backlash, the third order system (9) is reduced to a system with system order  $n = 2$ . The damping terms  $\frac{b_1}{J_1}\omega_1$  and  $\frac{b_2}{J_2}\omega_2$  are neglected in system (9), since the effect on the system behavior is small. Defining

$$x(t) = [\Delta\varphi, \Delta\omega]^T$$

as state vector with the initial condition  $x(0) = x_0 \in \mathbb{R}^2$ , the engine torque  $T_{eng}$  as system input  $u$  and the rotational speed difference  $\Delta\omega$  as system output  $y$  yields

$$\begin{aligned} \dot{x} &= \begin{bmatrix} x_2 \\ -\xi \tanh(p|x_1|)(cx_1 + dx_2) + \frac{1}{J_1 i_g}u + \frac{1}{J_2}\delta \end{bmatrix}, \\ y &= x_2, \end{aligned} \quad (12)$$

with  $\xi = \frac{J_1 i_g^2 + J_2}{J_1 J_2 i_g^2}$ . The parameter  $q$  in (9) is incorporated in  $c$  and  $d$ . The disturbance  $\delta$  is independent of system states and is defined in (6).

As shown in Pham and Bushnell (2015) for linear powertrain systems without backlash, the output  $z = \Delta\varphi$  is also a flat output for the nonlinear backlash system (12). This is shown next.

The  $n$  derivatives of  $z$  are

$$\begin{aligned} z &= x_1, \\ \dot{z} &= x_2, \\ \ddot{z} &= -\xi \tanh(p|x_1|)(cx_1 + dx_2) + \frac{1}{J_1 i_g}u + \frac{1}{J_2}\delta. \end{aligned} \quad (13)$$

Using the equations in (13), the state parametrization  $x = \Phi_x(z, \dot{z})$  is

$$\begin{aligned} x_1 &= z, \\ x_2 &= \dot{z}. \end{aligned}$$

The input parametrization  $u = \Phi_u(z, \dot{z}, \ddot{z})$  is

$$u = J_1 i_g \left[ \ddot{z} + \xi \tanh(p|z|)(cz + d\dot{z}) - \frac{1}{J_2}\delta \right] \quad (14)$$

and the output parametrization  $y = \Phi_y(z, \dot{z})$  is

$$y = \dot{z}.$$

When (14) is supplied as the feedforward input

$$u_{FW} = J_1 i_g \left[ \ddot{z}_{ref} + \xi \tanh(p|z_{ref}|)(cz_{ref} + d\dot{z}_{ref}) - \frac{1}{J_2}\delta \right], \quad (15)$$

the system trajectory of (12) satisfies

$$x = [z_{ref}, \dot{z}_{ref}]^T.$$

#### 3.2 Trajectory Planning

To fulfill a set-point transition using feedforward control law (15) reference trajectories  $z_{ref}$ ,  $\dot{z}_{ref}$  and  $\ddot{z}_{ref}$  have to

be planned. The trajectory  $z_{ref}$  has to be at least twice differentiable. For instance, a polynomial function can be used.

Prior to the planning of the trajectories, actual and requested set-points  $z_{sp}$  have to be calculated. The set-points of system (12) can be derived from the equilibrium;

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \Delta\omega_{sp} \\ -\xi \tanh(p|\Delta\varphi_{sp}|) (c\Delta\varphi_{sp} + d\Delta\omega_{sp}) + T_{sp} \end{bmatrix},$$

with  $T_{sp} = \frac{1}{J_1 i_g} u_{sp} + \frac{1}{J_2} \delta_{sp}$ . It is  $\Delta\omega_{sp} = 0$  and thus the following nonlinear equation has to be solved to find actual and requested set-points  $z_{sp}$ ;

$$-\xi \tanh(p|z_{sp}|) c z_{sp} + \frac{1}{J_1 i_g} u_{sp} + \frac{1}{J_2} \delta_{sp} = 0. \quad (16)$$

The nonlinear equation (16) depends on the input  $u_{sp}$ , which is the actual torque value  $T_{av}$  or, respectively, the requested torque  $T_{req}$ . The initial torque is measurable and the requested torque can be calculated by a pedal characteristic map using pedal and engine speed information, as shown schematically in Fig. 5.

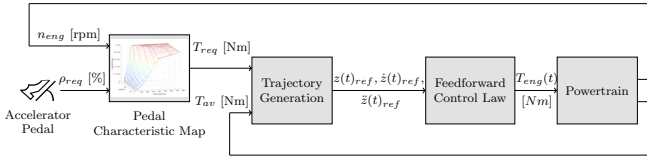


Fig. 5. Feedforward control loop with determination of the actual value torque  $T_{av}$  and requested torque  $T_{req}$ .

*Remark 1.* If disturbance  $\delta$ , which mainly depends on the road inclination, is constant during the set-point transition, it can be neglected in feedforward control design.

*Remark 2.* For implementation reason the nonlinear equation (16) can be solved offline for various set-point transitions by using various actual torque values  $T_{av}$  and requested torques  $T_{req}$ . Then, reference trajectories can be pre-calculated and saved in a look-up table.

### 3.3 Feedback Control After Set-Point Transition

A proportional feedback controller is activated after the set-point transition. The feedback control law

$$u_{FB} = -\psi(t)K\Delta\omega = -\psi(t)K \left( \frac{\omega_{eng}}{i_g} - \frac{v_{veh}}{r_w} \right), \quad (17)$$

with proportional gain  $K$  and function

$$\psi(t) = \begin{cases} 0, & t \leq t_T \\ \kappa(t - t_T), & t_T < t < t_T + \frac{1}{\kappa} \\ 1, & t \geq t_T + \frac{1}{\kappa} \end{cases}$$

should damp remaining torsional oscillation, which arise due to model mismatches and unknown disturbances during the set-point transition. To avoid discontinuities when the feedback controller is activated, a transition function  $\psi(t)$  is used. The factor  $\kappa$  defines the transition time from inactive to fully activated feedback controller. Large  $\kappa$  defines shorter transition time. If the rotational speed of the engine divided by gear ratio is greater than the

rotational speed of the wheels, engine torque is reduced by the feedback controller, such that the speed of the wheels can catch up and synchronize to the speed of the engine divided by gear ratio.

The proportional gain  $K$  has to be chosen such that the settling time is short and the closed loop system is stable even in the presence of time delay. In powertrains time delay can occur due to signal and physical actuator delays.

## 4. SIMULATION RESULTS

The proposed nonlinear flatness-based feedforward controller is evaluated on a detailed powertrain simulation model with backlashes via Matlab. The simulation model includes nine degrees of freedom and represents a powertrain with eight inertias, a vehicle mass and four backlashes as shown in Fig. 1. Backlash is described in the simulation model by the dead-zone function (1), while for the control design the continuous function (8) is used. In the following the simulation results show that this approximation is a valid choice in the control design as well as the reduction to a two degree of freedom system.

At first, the parameters of the reduced two-mass powertrain model as described in (12) are identified using the detailed powertrain simulation model as a reference. Both models apply the same input. The parameters  $J_1, J_2, c, d, p$  are found by minimizing the error function (10) for a set-point transition. As seen in Fig. 6, vehicle acceleration of the reduced second order powertrain model and the detailed model show a good correlation.

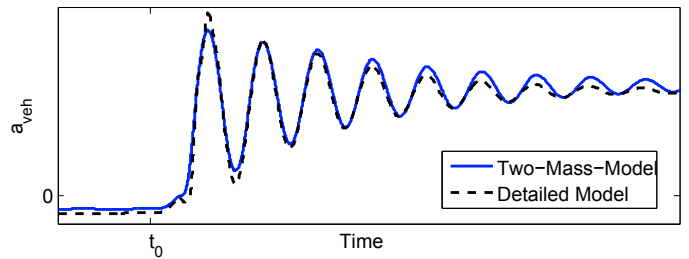


Fig. 6. Vehicle acceleration signal of the parameter identified two-mass model and detailed powertrain model for a set-point transition.

The demand in Fig. 7 is to change the initial torque  $T_{min}$  to a new requested torque  $T_{max}$  by a feedforward controller. Therefore the presented flatness-based feedforward controller with backlash is compared to a flatness-based feedforward control design without considering backlash and a torque ramp. The acceleration signal of the flatness-based feedforward controller with backlash shows least oscillation and best performance. The acceleration signals of the strategies without backlash consideration oscillate more strongly.

In Fig. 8, parameter sensitivity of our feedforward control approach is investigated. For the nominal case, feedforward control law (15) is supplied to the two-mass model (12). The acceleration signal shows perfect tracking. Then, feedforward control law (15) is supplied to the detailed powertrain model and the parameters of the model are varied. The upper figure shows that an increase of the



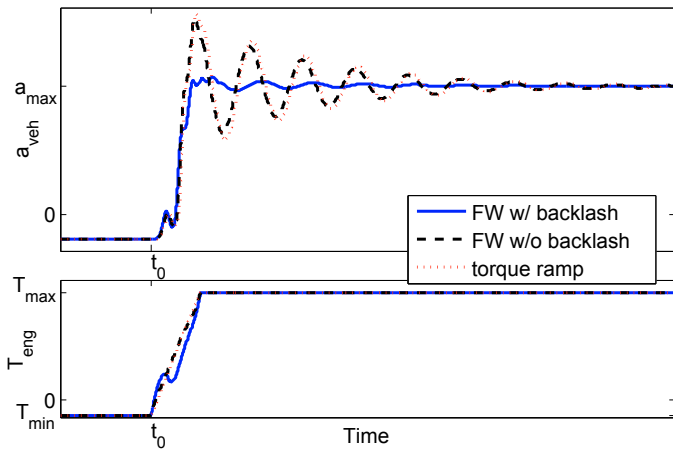


Fig. 7. Comparison of vehicle acceleration and engine torque signals derived by the presented controller and strategies without backlash consideration.

vehicle mass causes an offset in the acceleration signal and has little influence on the oscillation behavior. Air drag, as defined in (3), has little impact on the acceleration signal. The figure below shows that a more flexible drive shaft has the worst influence on the oscillation behavior of the system. However, even a large variation of the drive shaft stiffness ( $\pm 20\%$ ) does not have a large influence on the system vibration.

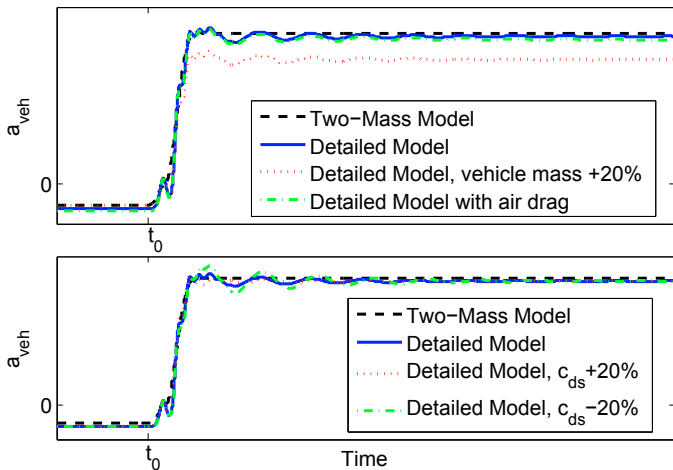


Fig. 8. Parameter sensitivity of the presented feedforward controller with backlash.

The proportional feedback controller (17) is added to the control loop after set-point transition to overcome model mismatches and external disturbances. The acceleration and torque signals, shown in Fig. 9, demonstrate that a model mismatch of the drive shaft stiffness can be compensated by a feedback controller. The presence of a road gradient (slope), as an external disturbance, does not strongly affect the system vibration.

## 5. CONCLUSION

In order to damp driveline oscillation caused by set-point transition with backlash traversing, a flatness-based feedforward controller in combination with a proportional

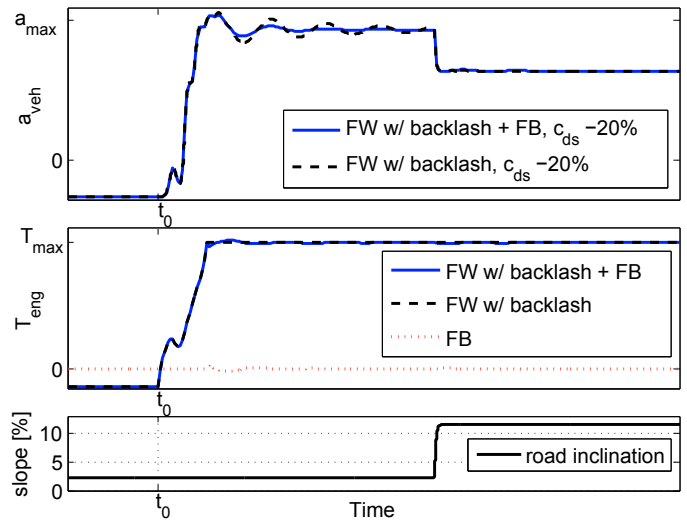


Fig. 9. Feedforward controller with and without proportional feedback controller under slope disturbance.

controller has been developed and evaluated in simulation studies. The focus of this paper was on the feedforward controller, which explicitly considers backlash in a nonlinear two-mass powertrain control model by using a tanh-function. To apply the controller, trajectory planning was discussed in the context of realization. Simulation results showed that the approach enables a set-point transition with backlash traversing with significantly few driveline oscillations. Furthermore, the simulation study demonstrated low sensitivity to parameter variations.

## REFERENCES

- Angeringer, U., Horn, M., and Reichhartinger, M. (2012). Drive line control for electrically driven vehicles using generalized second order sliding modes. *2012 IFAC Workshop on Engine and Powertrain Control, Simulation and Modeling*.
- Bruce, M., Egardt, B., and Pettersson, S. (2005). On powertrain oscillation damping using feedforward and LQ feedback control. *IEEE International Conference on Control Application (CCA)*.
- Feldt, M., Kopf, S., Eichhorn, M., and Konigorski, U. (2010). Flatness-based control and online trajectory generation for electrical actuators in combustion engines. *6th IFAC Symposium Advances in Automotive Control*.
- Fliess, M., Levine, J., and Rouchon, P. (1995). Flatness and defect of nonlinear systems: Introductory theory and examples. *International Journal of Control*, 61.
- Gasper, R. and Abel, D. (2010). Flatness based control of a parallel hybrid drivetrain. *6th IFAC Symposium Advances in Automotive Control*.
- Gillespie, T.D. (1992). *Fundamentals of Vehicle Dynamics*. Society of Automotive Engineers.
- Grotjahn, M., Quernheim, L., and Zemke, S. (2006). Modelling and identification of car driveline dynamics for anti-jerk controller design. *IEEE International Conference on Mechatronics*.
- Haschka, M.S. and Krebs, V. (2007). Observing the torque of a powertrain with backlash. *at Automatisierungstechnik*, 55.

- Kiencke, U. and Nielsen, L. (2005). *Automotive Control Systems: For Engine, Driveline, and Vehicle*. Springer.
- Lagerberg, A. (2001). *A Literature Survey on Control of Automotive Powertrains with Backlash*. Chalmers tekniska högsk.
- Lagerberg, A. and Egardt, B. (2005). Model predictive control of automotive powertrains with backlash. *16th IFAC World Congress*.
- Lewis, R.M., Torczon, V., and Trosset, M.W. (2000). Direct search methods: then and now. *Journal of Computational and Applied Mathematics*, 124.
- Pham, T. and Bushnell, L. (2015). Two-degree-of-freedom damping control of driveline oscillations caused by pedal tip-in maneuver. *American Control Conference (ACC)*.
- Sira-Ramírez, H. and Agrawal, S.K. (2004). *Differentially Flat Systems*. Control Engineering Series, Marcel Dekker Inc.
- Templin, P. and Egardt, B. (2009). Experimental results for a powertrain LQR-torque compensator with backlash handling. *IFAC Workshop on Engine and Powertrain Control, Simulation and Modeling*.