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Analysis of the second order wave forces acted on a floating pontoon

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ABSTRACT

In order to analyse the second order wave force on a abnormity floating pontoon and the influences of dimensions of the appended pontoons and water depth on the second order wave force acted on the structure, the second order hydroelasticity theory of ships was used to calculate the second order wave force. The second order wave force include the difference-frequency and sum-frequency fluid forces. The first order wave potentials and responses are sure to make the major contributions to the second order hydrodynamic actions, so the first order motions of the structure were calculated and verified by comparison with the experimental results. The second order wave force of the abnormity floating pontoon was calculated and compared in the different water depth. The results showed that the water depth has significant influence on the second order wave force on the pontoon. And the results also showed that the mean drift force was only sensitive to the wave direction in the medium and high wave frequency domain.

Keywords: hydroelasticity theory, second order wave force, wave potentials, water depth

1 INTRODUCTION

When a pontoon is floating in the presence of an incident wave system, a pressure force and moment will be exerted on the floating body by the surrounding fluid. This force and moment will include not only the conventional unsteady exciting components which give rise to the oscillations of the ship in waves, but also higher order steady forces due to various nonlinear effects. The second order steady horizontal force and vertical moment are derived for a free floating ship in regular waves [1]. However, the solution of the second order problem results in mean forces, difference frequency and sum frequency forces [2]. The nonlinear hydrodynamics of a flexible body is initially presented by Wu et al.[3], and the codes of the theory are compiled by Chen et al [4] where the second order forces of a floating flexible box-type barge are firstly calculated by the procedure. The characteristics of the difference and sum frequency coordinates of a very large floating structure in multidirectional irregular waves are discussed in detail based on the method of the second order hydroelasticity [5]. A method for predicting the extreme value of bending moments of a very large floating plate considering the linear and nonlinear wave force is introduced by Chen et al, and the influence of nonlinear fluid force on the predicted extreme bending moment may be as large as 22% of the linear wave exciting forces[6]. So prediction of the second order wave force is important for the floating structure. The numerical methods and the calculated results of the non-linear hydroelastic responses of a ship traveling in rough seas were investigated based on the second order hydroelastic theory of ships [7]. The horizontal slow drift excitation forces increase significantly with the decreasing water depth and the second-order velocity potential gives dominant contribution in a frequency range of importance for moored ships in shallow water [8]. The mean drift forces on two connected semi-submersible platform modules in different water depth and different wave directions are studied in the reference [9]. Both numerical and physical model tests have been carried out to investigate the hydrodynamic performances of a moored pontoon in finite depth water [10]. The experimental and numerical calculation of the slow-drift motion of a rectangular barge moored at different positions along an inclined beach are presented [11]. Both numerical and experimental studies have been conducted to investigate the hydrodynamic performance of a barge in shallow water [12].

In this paper, the second order nonlinear hydroelastic analysis method of a floating body is presented firstly, and the second order nonlinear hydroelastic equations of a floating body are established; then the physical and numerical models are introduced, the first order motions of the numerical simulations are verified by the model tests; then the second order force and principal coordinates of the pontoon are calculated and discussed. Finally, several conclusions are presented.

2 NUMERICAL SIMULATION

2.1 Potential theory and motion formulations

The dynamic analysis of the pontoon subjected to the incident regular waves in the finite water is calculated in the time domain in this paper based on the theories of the potential flow and diffraction/radiation theory with linear wave assumption. The fluid is assumed to be inviscid, continuous and incompressible, the flow can be considered as irrotational flow, so the potential flow theory can be used to calculate the interaction of free surface waves with the pontoon. Besides, the pontoon is assumed as a rigid body which has six degrees of freedom, namely surge, sway, heave, roll, pitch and yaw. Under these assumptions, the velocity potential $\Phi(X,Y,Z;t)$ can be separated into three parts.

$$\Phi(X,Y,Z;t)=\left[(\varphi_{\mathrm{I}}+\varphi_{\mathrm{D}})+\sum_{j=1}^{6}\varphi_{\mathrm{r}j}x_j\right]\mathrm{e}^{-\mathrm{i}\omega t} \tag{1}$$

where φ_{I} is the incident wave potential, φ_{D} is the diffraction wave potential, $\varphi_{\mathrm{r}j}$ is the radiation wave potential due to j-th component of motion, (X,Y,Z) is the coordinate of the field points in the fixed reference system, x_j is the j-th motion mode and $j=1,2,\ldots,6$, ω is the wave angular frequency, i is the standard imaginary unit, t is time.

The velocity potential should satisfy the Laplace equation $\nabla^2\Phi=0$, as well as the boundary conditions, such as linear free-surface, the stationary bottom surface, the wetted surface of the floating body and the far-field condition. And the incident wave potential can be written as:

$$\phi_{\mathrm{I}}(X,Y,Z;t)=\varphi_{\mathrm{I}}(X,Y,Z)\mathrm{e}^{-\mathrm{i}\omega t}=-\frac{\mathrm{i}gA\cosh\left[k(Z+h)\right]}{\omega\cosh(kh)}\mathrm{e}^{\mathrm{i}\left[-\omega t+k_0(X\cos\beta+Y\sin\beta)+\alpha\right]} \tag{2}$$

it will automatically satisfy the free-surface condition and bottom condition, where A is the amplitude of the incident waves, d is the water depth, β is the incident wave direction, k is the wave number which is related to the angular frequency ω through the dispersion relation:

$$\omega^2=\mathrm{g}k\tanh(kd) \tag{3}$$

The diffraction wave potential $\phi_{\mathrm{D}}(X,Y,Z)$ and radiation wave potential $\phi_j(X,Y,Z)$ should satisfy the governing equation and the boundary conditions, and they will be expressed in terms of pulsating sources distributed over the mean wetted surface of the floating pontoons by means of Green's theorem. Then added mass, damping coefficients and wave exciting forces including Froude-Krylov force and diffraction force will be calculated through the wave potential. The catenary mooring systems were used in this research in the time domain. Therefore the response amplitude operators (RAOs) will be obtained through the linear motion equations.

$$\left[-\omega^2\left(\boldsymbol{M}_{\mathrm{S}}+\boldsymbol{A}(\omega)\right)-i\omega\boldsymbol{B}(\omega)+\boldsymbol{K}_{\mathrm{hys}}\right]\left[x_j\right]=\left[F_{\mathrm{W}j}\right] \tag{4}$$

where $\boldsymbol{M}_{\mathrm{S}}$ is the structural mass matrix, $\boldsymbol{A}(\omega)$ is the added mass matrix, $\boldsymbol{B}(\omega)$ is the radiation damping matrix, $\boldsymbol{K}_{\mathrm{hys}}$ is the hydrostatic stiffness, $F_{\mathrm{W}j}$ is the wave exciting forces matrix, and x_j is the motions of the body's center of gravity excited by an incident regular wave with unit amplitude. The response amplitude operators (RAOs) can be got through calculating the set of linear algebraic equations.

2.2 Second order wave force

There are three methods that can calculate the second order wave forces of the floating structure, such as near field solution approach, middle field solution approach and far field solution approach. The near field solution approach is used to calculate the mean drift forces of the floating pontoon, the second order wave exciting force $F^{(2)}$ and moment $M^{(2)}$ can be written as:

$$\begin{aligned}\boldsymbol{F}^{(2)} &= -\frac{1}{2}\rho \mathrm{g}\oint_{WL} \varsigma_{\mathrm{r}}^{(1)} \cdot \varsigma_{\mathrm{r}}^{(1)} \boldsymbol{n} \mathrm{d}l + \frac{1}{2}\rho \iint_{S_0} \left[\nabla \boldsymbol{\Phi}^{(1)} \cdot \nabla \boldsymbol{\Phi}^{(1)}\right] \boldsymbol{n} \mathrm{d}S + \\ &\rho \iint_{S_0} \left[\boldsymbol{X}^{(1)} \cdot \nabla \frac{\mathrm{d}\boldsymbol{\Phi}^{(1)}}{\mathrm{d}t}\right] \boldsymbol{n} \mathrm{d}S + \boldsymbol{\alpha}^{(1)} \times \boldsymbol{F}^{(1)} + \rho \iint_{S_0} \frac{\mathrm{d}\boldsymbol{\Phi}^{(2)}}{\mathrm{d}t} \boldsymbol{n} \mathrm{d}S\end{aligned} \tag{5}$$

$$\begin{aligned}\boldsymbol{M}^{(2)} &= -\frac{1}{2}\rho \mathrm{g}\oint_{WL} \varsigma_{\mathrm{r}}^{(1)} \cdot \varsigma_{\mathrm{r}}^{(1)} (\boldsymbol{x} \times \boldsymbol{n}) \mathrm{d}l + \frac{1}{2}\rho \iint_{S_0} \left[\nabla \boldsymbol{\Phi}^{(1)} \cdot \nabla \boldsymbol{\Phi}^{(1)}\right] \cdot (\boldsymbol{x} \times \boldsymbol{n}) \mathrm{d}S + \\ &\rho \iint_{S_0} \left[\boldsymbol{X}^{(1)} \cdot \nabla \frac{\mathrm{d}\boldsymbol{\Phi}^{(1)}}{\mathrm{d}t}\right] \cdot (\boldsymbol{x} \times \boldsymbol{n}) \mathrm{d}S + \boldsymbol{\alpha}^{(1)} \times \boldsymbol{M}^{(1)} + \rho \iint_{S_0} \frac{\mathrm{d}\boldsymbol{\Phi}^{(2)}}{\mathrm{d}t} \cdot (\boldsymbol{x} \times \boldsymbol{n}) \mathrm{d}S\end{aligned} \tag{6}$$

Where $\boldsymbol{F}^{(1)}$ 、 $\boldsymbol{M}^{(1)}$ is the first order wave force and moment of the floating pontoon, $\varsigma_{\mathrm{r}}^{(1)}$ is the relative wave elevation along the mean undisturbed water line, $\boldsymbol{\alpha}^{(1)}$ is the first order rotational motion of the center of gravity in the fixed reference axes, $\boldsymbol{X}^{(1)}$ is the first order translational motion of the center of gravity in the fixed reference axes, $\boldsymbol{x}$ is the position of the point in the local structure axes, and $\boldsymbol{n}$ is the normal vecter.

The second order wave forces consist of the mean drift force and double frequency forces under the regular wave. In cases with irregular waves, there will be difference frequency force components and sum frequency force components in the second order forces. And the mean drift forces $\overline{F^{(2)}}$ can be written as:

$$\begin{aligned}\overline{F^{(2)}} &= \sum_{m=1}^{N_{\mathrm{d}}} \sum_{n=1}^{N_{\mathrm{d}}} \sum_{j=1}^{N_{\mathrm{m}}} A_{jm} A_{jn} \left\{P_{jjmn}^{-} \cos\left(\alpha_{jm} - \alpha_{jn}\right) - Q_{jjmn}^{-} \sin\left(\alpha_{jm} - \alpha_{jn}\right)\right\} \\ &= \sum_{m=1}^{N_{\mathrm{d}}} \sum_{n=1}^{N_{\mathrm{d}}} \sum_{j=1}^{N_{\mathrm{m}}} \overline{f^{(2)}}\left(\omega_j, \beta_m, \beta_n\right)\end{aligned} \tag{7}$$

Where m and n are m-th and n-th wave direction, j is j-th wave frequency, N_{d} is the number of wave directions, N_{m} and N_{n} are the numbers of wave components in the m-th and the n-th wave directions respectively, A_{jm} and A_{jn} are the amplitude, ω_j is the wave frequency, α_{jm} and α_{jn} is the phases, P_{jjmn}^{-} and Q_{jjmn}^{-} are the coefficients of the mean drift forces.

3 EXPERIMENTS AND NUMERICAL MODEL

3.1 Description of the barge and the numerical model

In this paper, a floating pontoon was examined for obtaining the motions and second order forces of it in the frequency domain, and the parameters of the pontoons in prototype and model scale are listed in Table 1. The numbers in the parentheses of length mean the length of added width of the bow and stern of the pontoon, and the numbers of the parentheses of width mean the total width of the bow and stern, respectively. In accordance with the dimensions of the wave tank and the pontoon and the test working conditions, the selected scale was 1:16. The numerical model of the pontoon under the waterline is shown in Figure 1.

Tab. 1 The parameters of the pontoons

Parameters	Pontoon	
	Prototype	Model
Length(m)	30(6)	1.875(0.375)
Width(m)	6(10)	0.375(0.625)
Hight(m)	1.8	0.113
Draft(m)	0.35	0.022
Displacement(kg)	81800	19.971
Gravity center(m)	(15,0,0.40)	(0.938,0,0.025)
R_{xx}(m)	2.912	0.182
R_{yy}(m)	9.771	0.611
R_{zz}(m)	10.131	0.633

Figure 1 Numerical model of the pontoon under the waterline

3.2 Model tests of the pontoon

The 3D (three dimensional) physical model tests of pontoon were conducted in the wind-wave-current tank at the Jiangsu University of Science and Technology in Zhenjiang city (China). The wave tank is 38 m long, 15 m wide and 1.2 m deep, and the water depth is 0.8m. The whole experimental system and its arrangements are shown in Figure 2 where the “X” type of mooring system was used for avoiding the interaction between the mooring chains of two pontoons and adjusting the water region.

Waves are created by the piston-type wave maker, and dissipative material is set up at the side wall and the opposite end of the wave tank to prevent the reflection of transmitted wave. An optical non-contact type six degree of freedom (6-DOF) measuring instrument was used to capture the 6-DOF motions of the reference point of the pontoons. And the motions of the pontoon were obtained by the measuring instrument. The model of seabed was built based on the elevation information of one island above the tank bottom. Fig. 2 shows the pontoon model test in the wave tank when the incident wave angle was 0 degree. The motion of the pontoon were also calculated by using the ANSYS-AQWA in time domain[10].

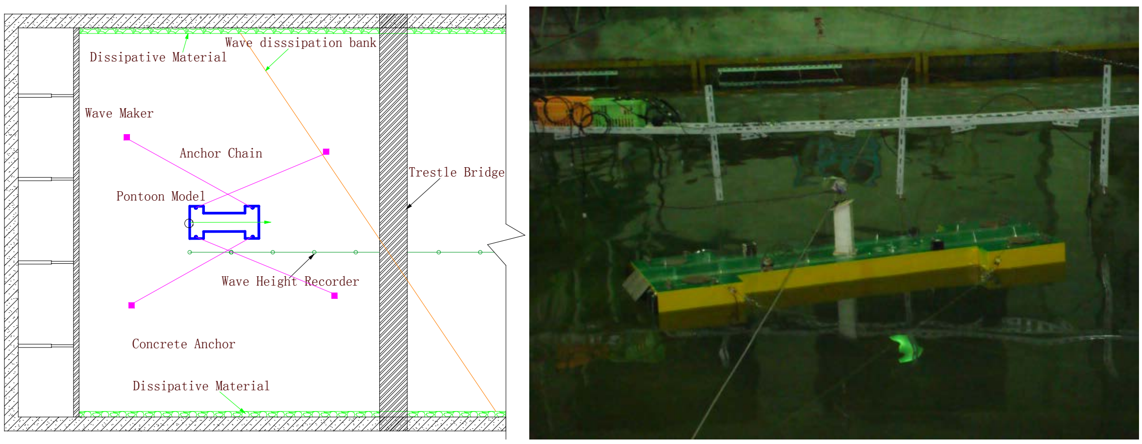


Figure 2 Layout of the tests arrangement (left) and Current picture of the wave tank (right).

3.3 First order motions of the pontoon

Figure 3 presents the comparison of the computed RAOs as a function of the wave frequency using the present numerical models (named as THAFTS) versus the results of AQWA and corresponding experimental results for the cases of the floating pontoon model in the wave 0^{o},and the Figure 4 shows the results of the

THAFTS and AQWA in the oblique wave 330°. In these figures, the “THAFTS-inf” means results of THAFTS in the infinite depth water, the “THAFTS-12.8m” means results of THAFTS in the 12.8 meters depth water, and “AQWA-inf” and “AQWA-12.8m” are results of AQWA in the infinite and 12.8m depth water, respectively, and “Test” means the model test results.

For the surge and pitch RAOs, the value of numerical calculation and model tests is same in the high frequency range, however, obvious value difference can be found in the low frequency. For the calculated RAOs in heave with the present numerical method is in a very good agreement with the model tests in the wave basin when the wave frequency is bigger than 0.8rad/s. In the curves, the main factor that the absolute values of the model test are bigger than the numerical value may be the diffraction waves caused by the wave dissipation bank. The figures also shown that the results of THAFTS and AQWA are same in the same water depth.

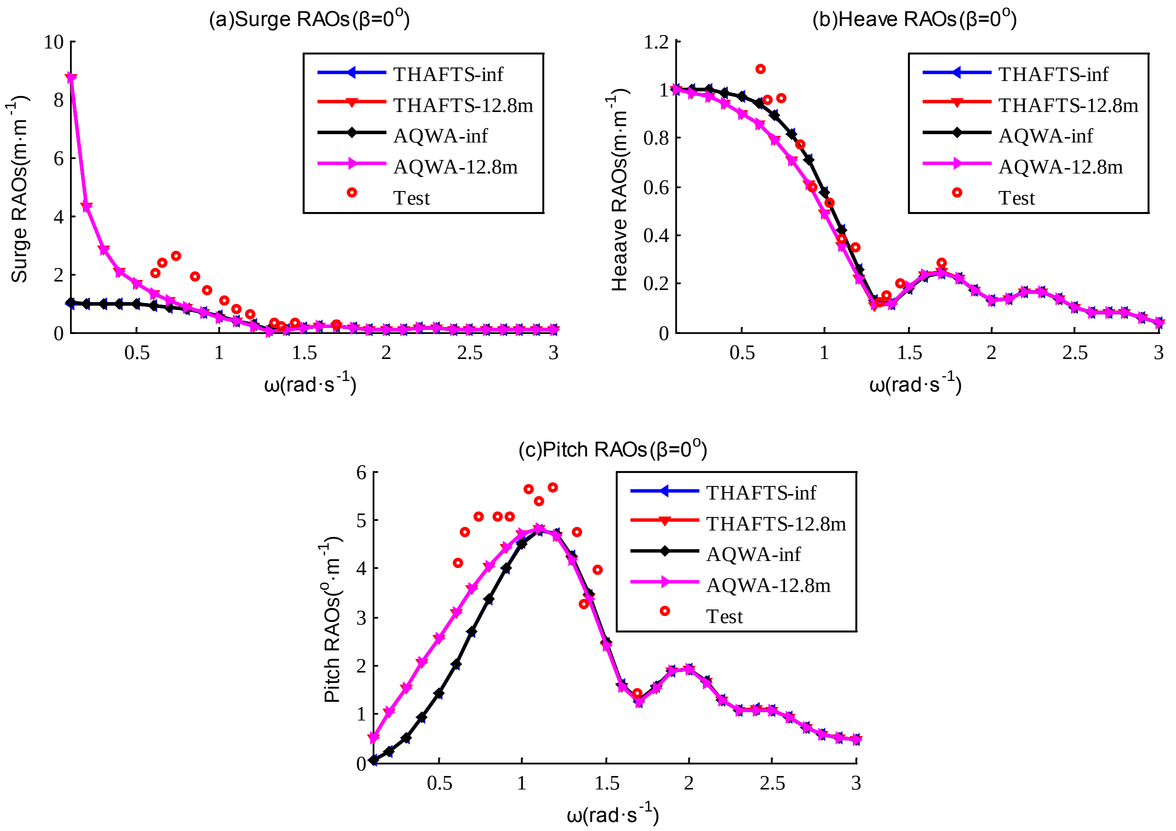
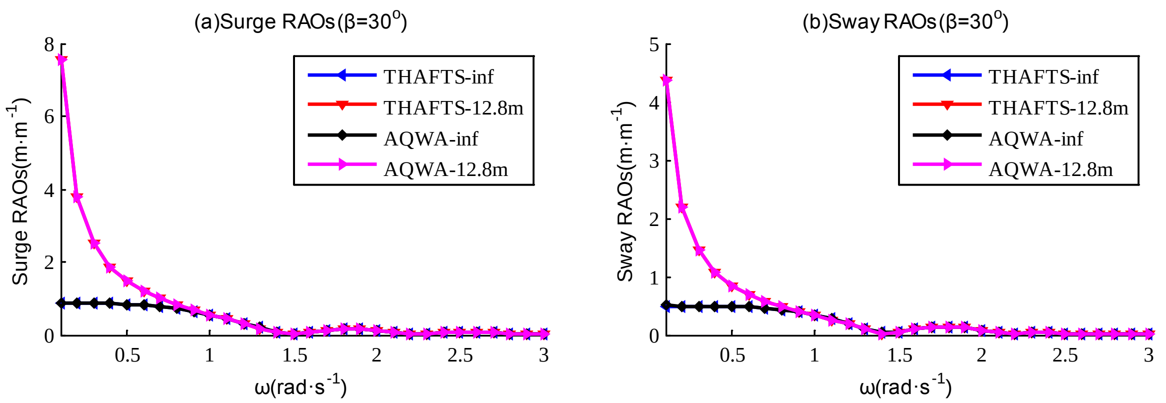


Figure 3 Comparison of the first order motion RAOs with numerical and model tests in wave 0°



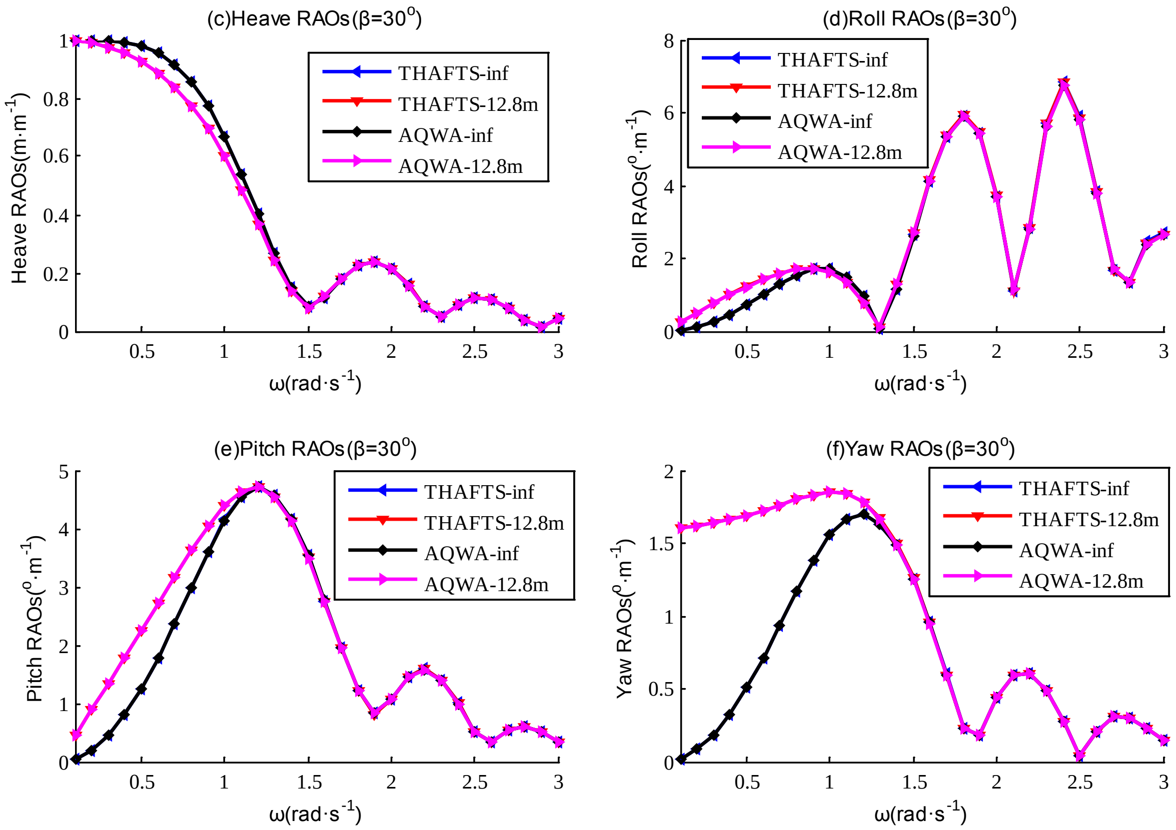
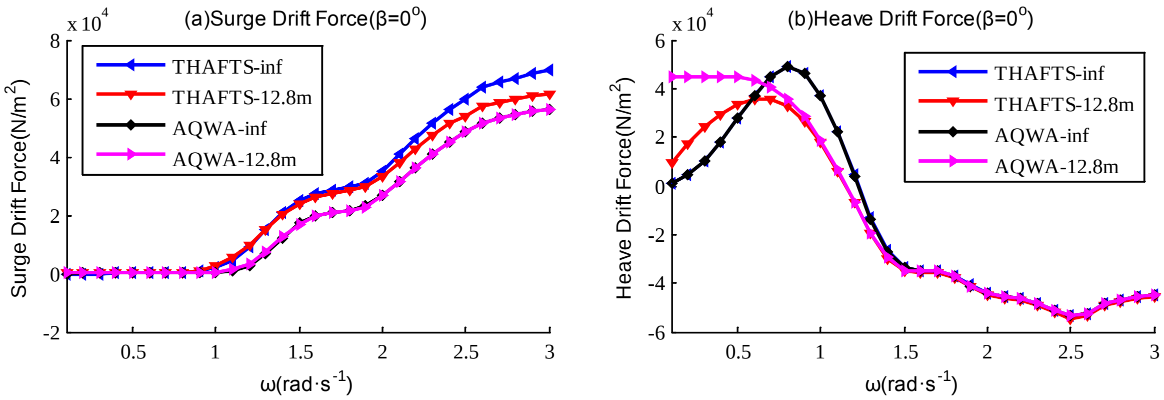


Figure 4 Comparison of the first order motion RAOs with AQWA and THAFTS in wave 30°

4 ANALYSIS OF THE SECOND ORDER FORCE OF THE PONTOON

4.1 The mean drift force

Figure 5 and Figure 6 present the comparison of the mean drift forces as a function of the wave frequency using the present numerical models versus the results of AQWA for the cases of the floating pontoon in the wave 0°and 30°, respectively. For the roll and pitch mean drift force, the values of THAFTS calculation are agreement with the results of AQWA in the infinite and 12.8m depth water in wave 0° and 30°.



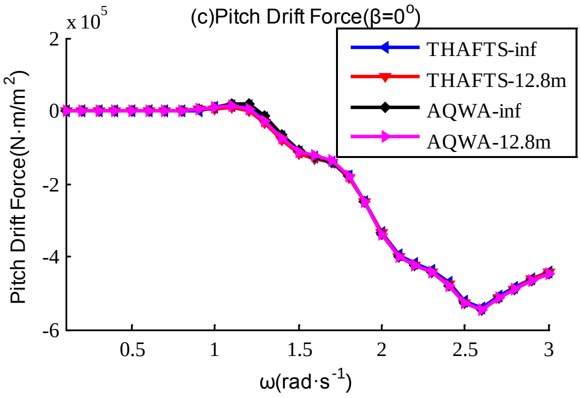


Figure 5 Comparison of the mean drift forces with AQWA and THAFTS in wave 0°

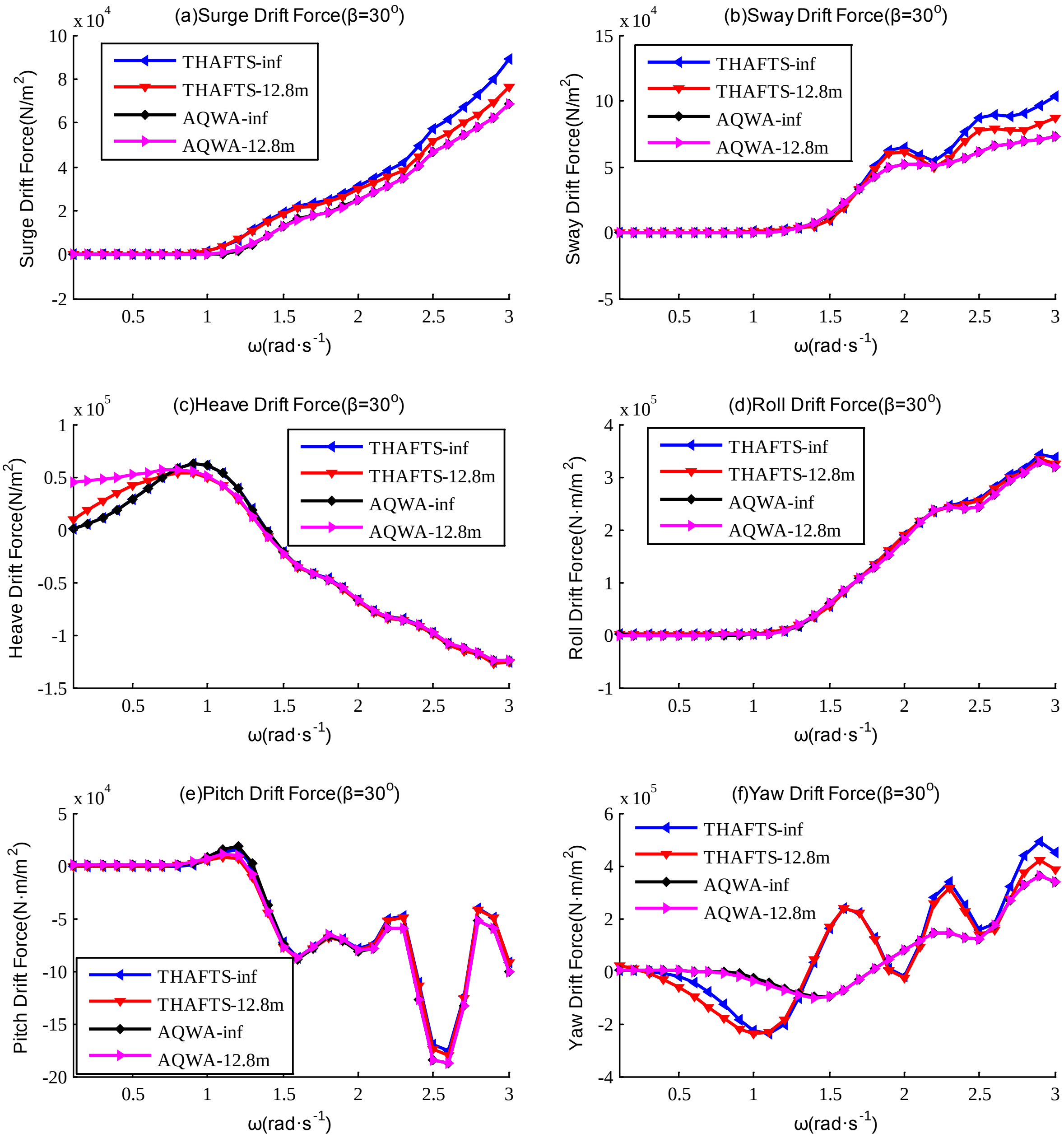


Figure 6 Comparison of the mean drift forces with AQWA and THAFTS in wave 30°

Figure 5(b) and Figure 6(c) also show that the heave mean drift forces calculated by THAFTS are also in a very good agreement with the results of AQWA in the infinite water depth, however, obvious value differences can be found in the low wave frequency when the water depth become smaller. The reason of the above difference may be that the instantaneous wet surface has been considered by the THAFTS, but not by AQWA.

For the surge and sway mean drift forces of the pontoon, the results calculated by the THAFTS is in very good agreement with the results of AQWA when the wave frequency is lower than 1.0rad/s or 1.5rad/s, respectively. The difference between the two calculation methods become bigger with the wave frequency becoming bigger. And the surge mean drift force of the pontoon calculated by AQWA is same in the infinite water depth and in the 12.8m water depth, but the results of THAFTS have some difference in the difference water depth. As shown in the figures, the same performance appear in the curves of the sway mean drift force.

Some of the peaks are shown in the curves of the yaw mean drift force calculated by THAFTS, but not in the curves of the results of AQWA. The reason of the peaks of the yaw maybe expand on the explanation regarding the irregular frequency

4.2 The difference-frequency and sum-frequency force

Figure 7~Figure 10 show some difference frequency forces and sum frequency forces of the pontoon. Figure 7 and Figure 8 are the difference frequency forces calculated by THAFTS under following wave in the 12.8m and in the infinite water depth, respectively. Figure 9 and Figure 10 are the sum frequency forces calculated by THAFTS under following wave in the 12.8m and in the infinite water depth, respectively. There are some peaks when the first wave frequency or the second wave frequency is small, and these peaks are bigger than the value of diagonal element.

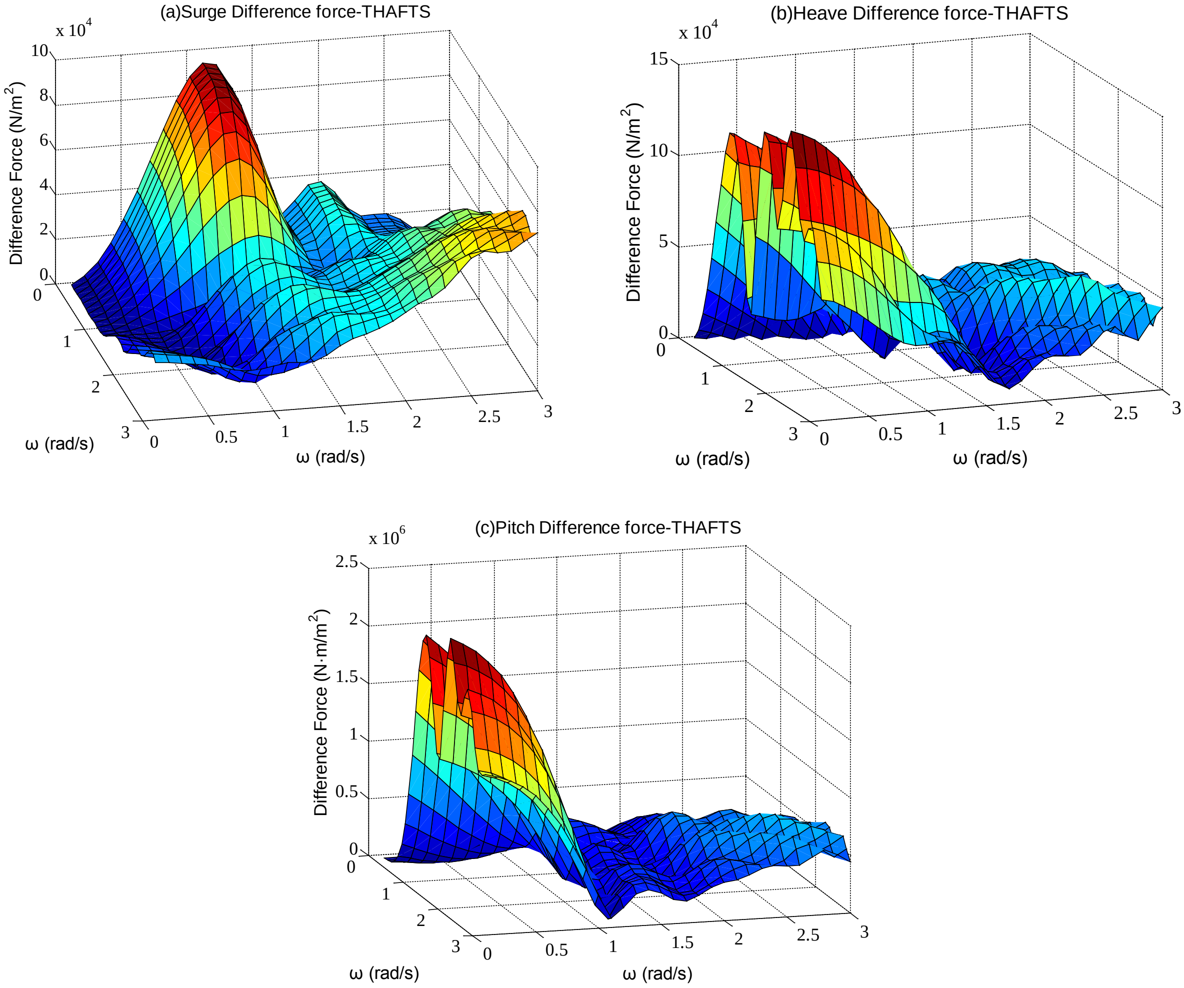


Figure 7 Difference frequency force under following wave in the 12.8m (THAFTS)

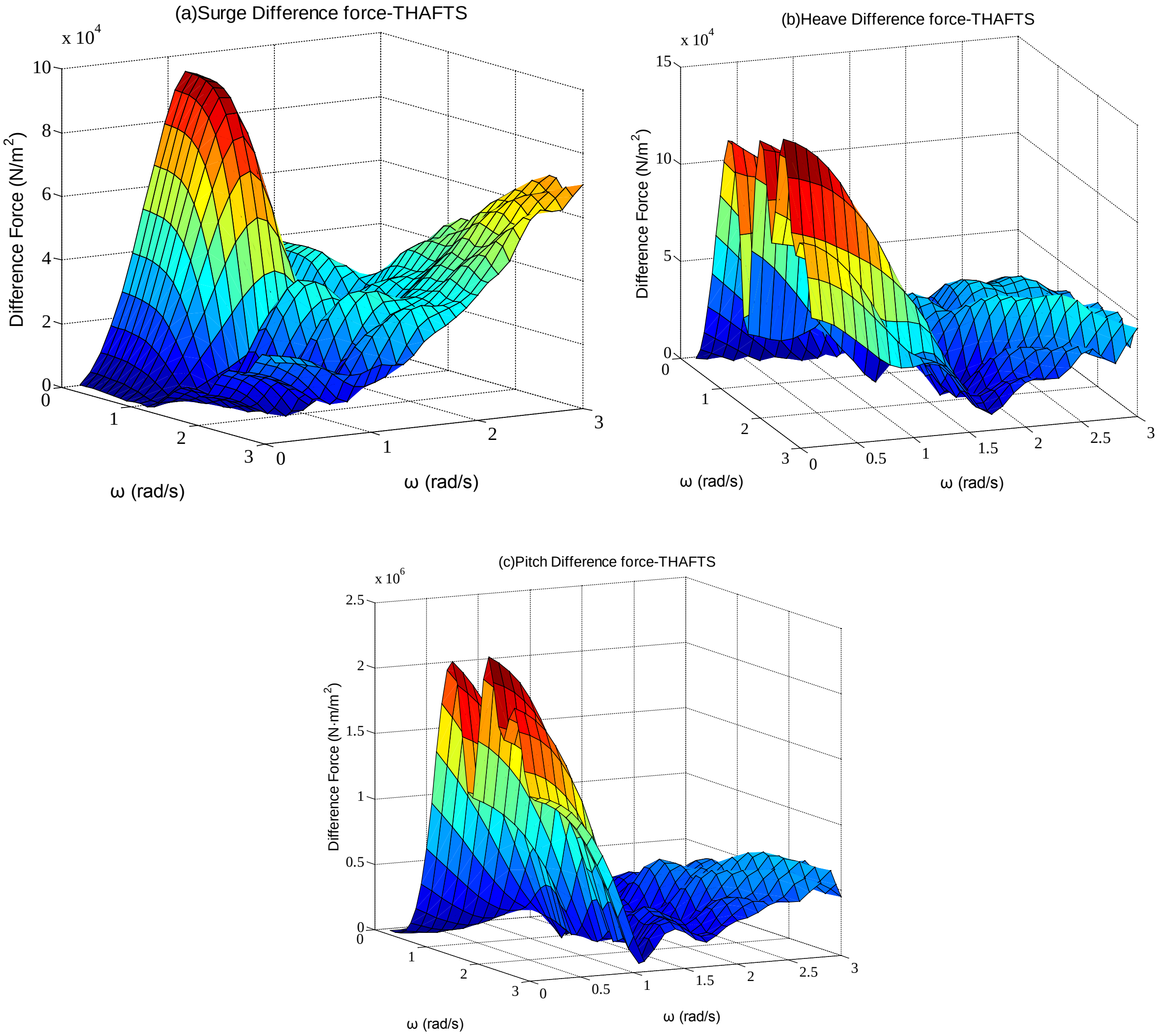
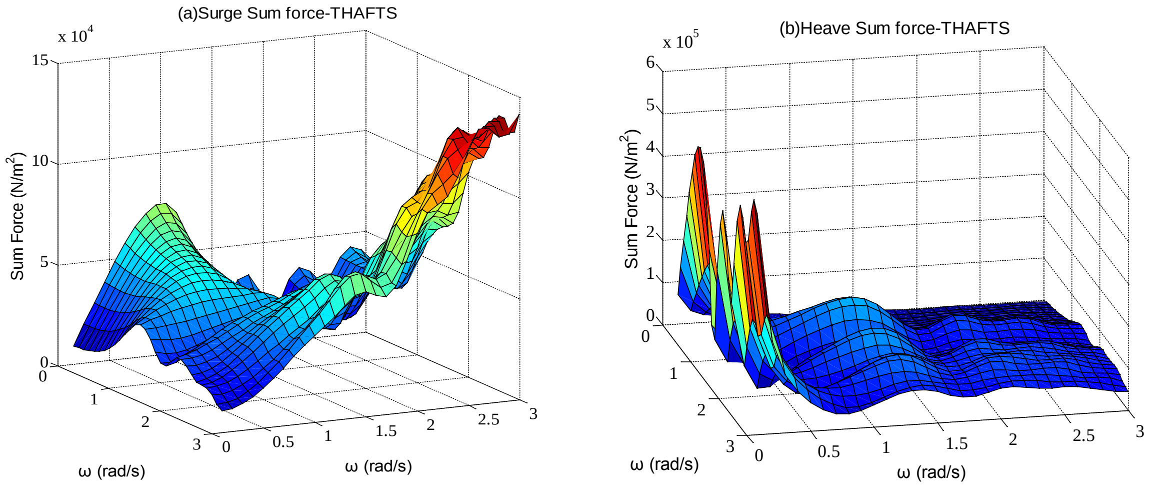


Figure 8 Difference frequency force under following wave in the infinite depth (THAFTS)



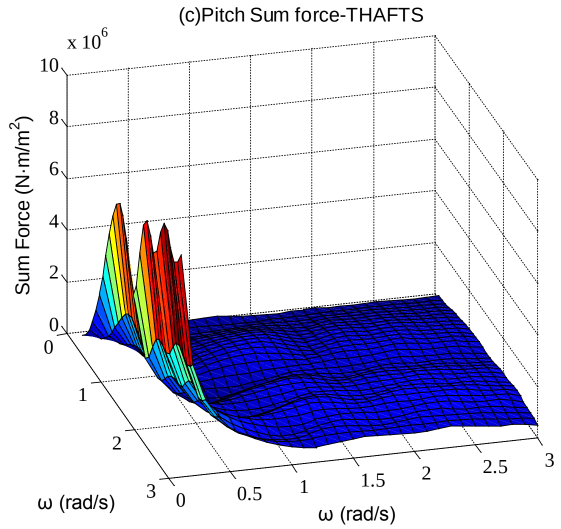


Figure 9 Sum frequency force under following wave in the 12.8m (THAFTS)

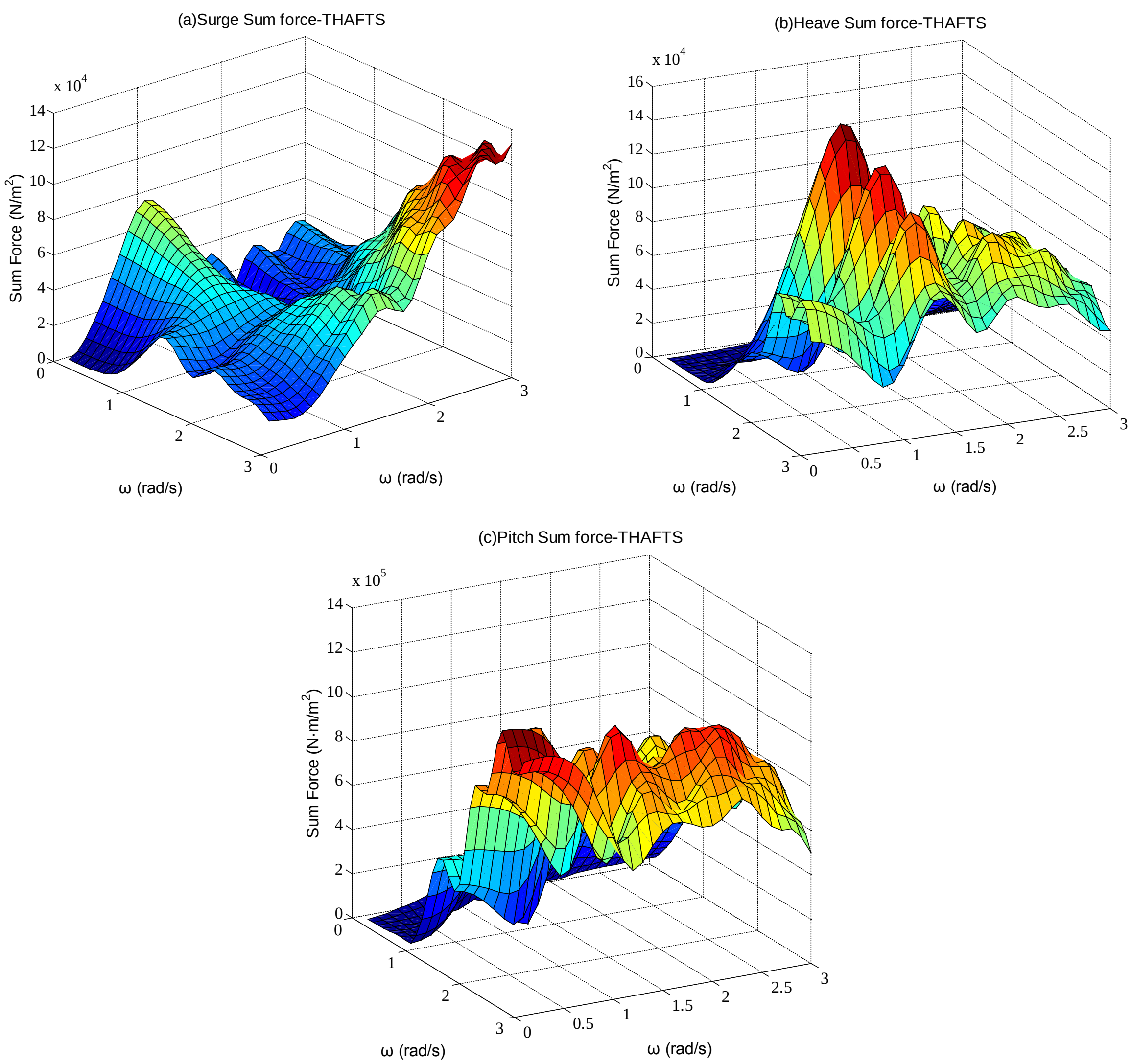


Figure 10 Sum frequency force under following wave in the infinite depth (THAFTS)

5 CONCLUSION

The hydrodynamic analyses of a floating pontoon interacted with the regular wave are studied by numerical simulations in the infinite and finite depth water. Based on the analyses, the main conclusions of this work are as follows: the results of the current numerical method are agreement with the model tests in majority cases, particularly, the change tendency of curves are essentially same in two ways. So the THAFTS can be used to forecast the pontoon hydrodynamic properties during the design stage. And the mean drift forces calculated by the THAFTS is same with the results of the AQWA except the yaw mean drift forces.

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