

# **MODELING OF AERATED UPFLOWED FIXED BED REACTORS FOR THE NITRIFICATION**

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## **ABSTRACT**

A mathematical model for the nitrification in an aerated fixed bed reactor has been developed. This model is based on material balances in the bulk liquid, gas phase and in the biofilm area. The fixed bed is divided into a number of cells according to the reduced remixing behaviour. A fixed bed cell consists of 4 compartments: the support, the gas phase, the bulk liquid phase and the stagnant volume containing the biofilm. In the stagnant volume the biological transmutation of the ammonia is located. The transport phenomena are modelled with mass transfer formulations so that the balances could be formulated as an initial value problem. The results of the simulation and experiments are compared.

## **KEYWORDS**

Biofilm systems; waste water treatment; nitrification; fixed bed reactor; modelling

## **INTRODUCTION**

The increase of the demands on the waste water treatment with simultaneous reduction of costs leads to new concepts. It has been shown that it is not always favourable to treat different waste waters together in one treatment work. Therefore, reactors for the effective and economic treatment of split flows are required which are operated either as a pre-treatment stage, before discharge to the sewer, or as a treatment stage for process waste waters. To comply with future regulations post process units are needed. For this purpose fixed bed reactors appear to be suitable. Besides the elimination of dissolved pollutants the suspended particles will be reduced evidently.

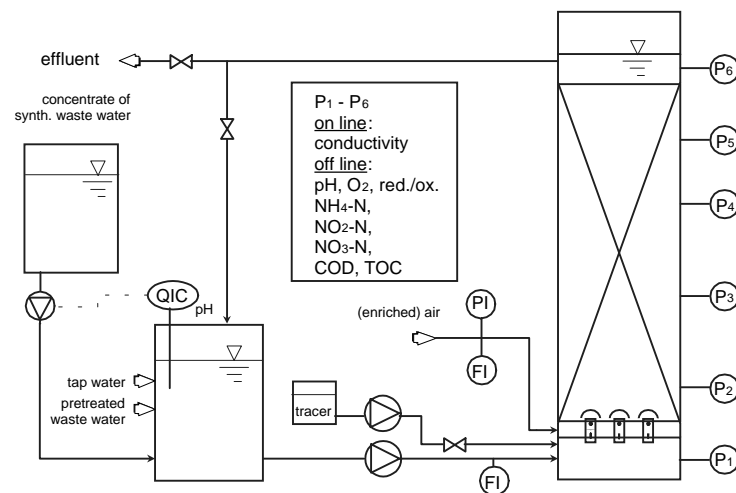
In practice fixed bed reactors are designed without considerations of the effects of residence time distribution and mass transfer on the reaction kinetics. The conventional design bases on mean volumetric loadings and empirical surface flow rates. This leads to large volumes, inefficient operation or unsatisfactory interceptions of variations of loads. For reasons of reaction technology a fixed bed reactor shall be pursued with reduced remixing. Generally, great temporal varieties of the load of pollutant have a detrimental effect. A reactor with reduced remixing reacts more sensitive on variations than a completely mixed reactor. Options to intercept the variations in the load have to be found. For this purpose the primary aim of the process modelation is to perform the design and operation and to enable an integration in a clarification system.

## **MATERIALS AND METHODS**

In the following a process model for an aerated fixed bed reactor with the example of the nitrification will be explained. For aerobic processes the nitrification is the most sensitive process caused by the low growth rates of the nitrifying bacteria and the high oxygen demand. The immobilised biomass and the good oxygen

transfer in fixed bed reactors promotes the nitrification. In nitrifying fixed bed reactors the biofilms are formed as grown, nearly irreversibly attached biomass.

Experimental results were obtained in a pilot scale fixed bed reactor that has been operated in cocurrent upflow by air and water as shown in Fig. 1. As support material beads of burned clay in a diameter range of  $d_p = 4$  to 8 mm were used. The reactor had a total height of about 6 m and an inner diameter of 0.5 m. A 4.5 m fixed bed was situated over a special construction at the bottom with nozzles for water and air distribution. The reactor was mainly operated with synthetic waste water (tap water with addition of  $(\text{NH}_4)_2\text{SO}_4$  and  $\text{NaHCO}_3$ ). For putting in operation pre-treated waste water from a nitrifying pilot scale activated sludge plant was added to the synthetic waste water. For tracer experiments NaCl was used and measured by conductivity. After at least 2 months of proper operation stable nitrifying conditions were obtained by washing intervals of once a week.



**Figure 1.** Schematic view of the fixed bed reactor

The fixed bed reactor can be operated as batch reactor or continuously. During operation under batch conditions, the addition of ammonia is controlled by the pH by a stoichiometric relation between ammonia and hydrogen carbonate (1:2). Samples could be taken in several heights and analysed off-line.

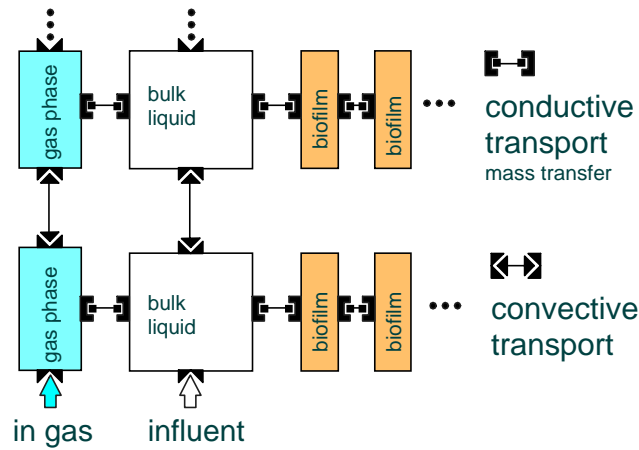
## MODELING

In the last year the modelling of biofilms has been given a lot of attention. Wanner O. and Gujer W. (1986) presented a biofilm mixed culture model that bases on transport and transformation processes. This model was improved by incorporate new properties, like heterogeneities and attachment/detachment of partikels (Wanner O. and Reichert P. (1996)). These one-dimensional modes bases still on a continuum approach and are suitable for describing the development of a biofilm (e.g. Horn H. (1992)). With the confocal scanning laser microscopy the internal structure of biofilms were investigated that resulted in the observation, biofilms are growing in cell clusters and channels are passing the matrix from the surface to the bottom, Bishop P.L. (1996). Therefore structured models were used recently to describe the biofilm structure properly (e.g. Picioreanu C. et al. (1998)). These models are very complex and inapplicable for the modeling of biofilm reactors in practice.

The modeling of biofilm reactors bases usually on one-dimensional homogeneous models (Henze M. et al. (1995), Picioreanu C. et al. (1996)). In fixed bed reactors a transport limitation occurs from the bulk liquid phase to the biofilm. In this case a reaction of first order concerning the concentration in the bulk liquid phase takes place (Henze M. et al. (1995)).

The development of the process model will be explained exemplarily for the nitrification process. It is known that the fixed bed reactor has a reduced remixing and stagnant volumes are not avoidable. For this purpose it appears wise to understand a fixed bed reactor as a serial circuit of elementary cells of a fixed bed reactor.

Fig. 2 shows a pictogram of the fixed bed reactor model. The liquid phase and the gas phase are passing the cell in uniflow. No reaction takes place in the gas phase that is coupled by mass transfer with the bulk liquid phase. Only in the stagnant volume, that includes the biofilm and will be called only biofilm later on, reactions take place. The biofilm is supplied with substrate by mass transfer from the bulk liquid phase, and reaction products are liberated into the bulk liquid phase vice versa.



**Figure 2.** Pictogram of a fixed bed reactor model

With these considerations a mathematical model for the nitrification in a fixed bed reactor can be formulated. The model bases on mass balances of the involved compounds in the 3 fluid compartments. Depending on question formulation, simplifications were carried out within the model. The mathematical solutions are received by using developed computer programs in the Department of Waste Water Management (TUHH) and the simulation tool AQUASIM (Swiss Federal Institute for Environmental Science and Technology, EAWAG).

### Mathematical formulation of the fixed bed reactor model

For the  $i^{\text{th}}$  elementary cell the following differential equation system has to be solved:

$$\text{Bulk liquid phase: } \frac{dc_{n,i}^l}{dt} = \frac{v_{FF}}{\varphi_l \Delta H} (c_{n,i-1}^l - c_{n,i}^l) + (k_L a_n)_{l/b} (c_{n,i}^b - c_{n,i}^l) + (k_L a_n)_{g/l} (c_{n,i}^s - c_{n,i}^l) \quad (1)$$

$$\text{Gas phase: } \frac{dc_{n,i}^g}{dt} = \frac{v_{FG}}{\varphi_g \Delta H} (c_{n,i-1}^g - c_{n,i}^g) - (k_L a_n)_{g/l} (c_{n,i}^s - c_{n,i}^l) \frac{\varphi_l}{\varphi_g} \quad (2)$$

$$\text{Biofilm: } \frac{dc_{n,i}^b}{dt} = (k_L a_n)_{l/b} \frac{\varphi_l}{\varphi_b} (c_{n,i}^b - c_{n,i}^l) - r_n \quad (3)$$

where  $c$  = concentration [mmol/m<sup>3</sup>],  $v_{FF}$  = filter velocity [m<sup>3</sup>/(m<sup>2</sup> h)],  $v_{FG}$  = superficial gas velocity [m<sup>3</sup>/(m<sup>2</sup> h)],  $(k_L a)_{l/b}$  = mass transfer coefficient (liquid/biofilm) [h<sup>-1</sup>],  $(k_L a)_{g/l}$  = mass transfer coefficient (gas/liquid) [h<sup>-1</sup>],  $\Delta H$  = height of an elementary cell

[m];  $\phi$  = fraction [ $\text{m}^3/\text{m}^3$ ],  $t$  = time [h],  $r$  = volumetric reaction rate [ $\text{mol}/(\text{m}^3 \text{ h})$ ]; indices:  $n$  = compound,  $i$  = number of the elementary cell,  $l$  = liquid,  $g$  = gas,  $b$  = biofilm,  $s$  = saturation.

The influence of the substrate and oxygen concentration can be described by a formal kinetic approach. With the use of the concentration of the undissociated substrate molecules the pH influence will be taken into consideration. In the following equations (4) to (6) the formal kinetic approaches for the ammonia-, nitrite-, oxygen- and alkalinity uptake will be represented:

$$\text{Ammonium:} \quad r_{\text{NH}_4} = r_{\text{max,NH}_4} \frac{\phi_b}{\phi_b^*} \frac{c_{\text{NH}_3}}{K_{\text{S,NH}_3} \left( 1 + \frac{c_{\text{HNO}_2}}{K_{\text{I,HNO}_2}} \right) + c_{\text{NH}_3}} \frac{c_{\text{O}_2}}{K_{\text{O}_2,\text{NH}_3} + c_{\text{O}_2}} \quad (4)$$

$$\text{Nitrite:} \quad r_{\text{NO}_2} = r_{\text{max,NO}_2} \frac{\phi_b}{\phi_b^*} \frac{c_{\text{HNO}_2}}{K_{\text{S,HNO}_2} \left( 1 + \frac{c_{\text{NH}_3}}{K_{\text{I,NH}_3}} \right) + c_{\text{HNO}_2}} \frac{c_{\text{O}_2}}{K_{\text{O}_2,\text{HNO}_2} + c_{\text{O}_2}} \quad (5)$$

$$\text{Oxygen:} \quad r_{\text{O}_2} = Y_{\text{O}_2/\text{NH}_3} r_{\text{NH}_4} + Y_{\text{O}_2/\text{HNO}_2} r_{\text{NO}_2} \quad (6)$$

$$\text{Alkalinity:} \quad r_{\text{al}} = Y_{\text{H}^+/\text{NH}_3} r_{\text{NH}_4} \quad (7)$$

where  $r_{\text{max}}$  = maximum reaction rate [ $\text{mol}/(\text{m}^3 \text{ h})$ ],  $\phi_b^*$  = related volume fraction [ $\text{m}^3/\text{m}^3$ ],  $K_{\text{S,NH}_3}$  =  $\text{NH}_3$  saturation constant [ $\text{mol}/\text{m}^3$ ],  $K_{\text{O}_2,\text{NH}_3}$  =  $\text{O}_2$  saturation constant for the  $\text{NH}_3$  degradation [ $\text{mol}/\text{m}^3$ ],  $K_{\text{I,HNO}_2}$  =  $\text{HNO}_2$  competitive inhibition constant for the  $\text{NH}_3$  degradation [ $\text{mol}/\text{m}^3$ ],  $K_{\text{S,HNO}_2}$  =  $\text{HNO}_2$  saturation constant [ $\text{mol}/\text{m}^3$ ],  $K_{\text{I,NH}_3}$  =  $\text{NH}_3$  competitive inhibition constant for the  $\text{HNO}_2$  degradation [ $\text{mol}/\text{m}^3$ ],  $K_{\text{O}_2,\text{HNO}_2}$  =  $\text{O}_2$  saturation constant for the  $\text{HNO}_2$  degradation [ $\text{mol}/\text{m}^3$ ],  $Y_{\text{O}_2/\text{NH}_3}$  = yield coefficient  $\text{O}_2/\text{NH}_3$  [ $\text{mol O}_2/\text{mol NH}_3$ ],  $Y_{\text{O}_2/\text{HNO}_2}$  = yield coefficient  $\text{O}_2/\text{HNO}_2$  [ $\text{mol O}_2/\text{mol HNO}_2$ ],  $Y_{\text{H}^+/\text{NH}_3}$  = yield coefficient  $\text{H}_3\text{O}^+/\text{NH}_3$  [ $\text{mol H}_3\text{O}^+/\text{mol NH}_3$ ].

The concentrations of the undissociated substrate molecules can be calculated for ammonia

$$c_{\text{NH}_3} = \frac{K_{\text{D,NH}_3} c_{\text{NH}_4}}{K_{\text{D,NH}_3} + c_{\text{H}_3\text{O}^+}} \quad (8)$$

and for nitrous acid respectively

$$c_{\text{HNO}_2} = \frac{K_{\text{D,HNO}_2} c_{\text{HNO}_2} + c_{\text{H}_3\text{O}^+}}{K_{\text{D,HNO}_2} + c_{\text{H}_3\text{O}^+}} \quad (9)$$

where  $c_{\text{H}_3\text{O}^+}$  = concentration of  $\text{H}_3\text{O}^+$ -ions [ $\text{mol}/\text{m}^3$ ],  $K_{\text{D,NH}_3}$  = dissociation constant for the system  $\text{NH}_4^+/\text{NH}_3$  [ $\text{mol}/\text{m}^3$ ],  $K_{\text{D,HNO}_2}$  = dissociation constant for the system  $\text{HNO}_2/\text{NO}_2^-$  [ $\text{mol}/\text{m}^3$ ].

In this paper the ammonia concentration  $c_{\text{NH}_4}$  means the sum of ammonia and ammoniac ( $c_{\text{NH}_4} = c_{\text{NH}_3} + c_{\text{NH}_4^+}$ ) and the nitrite concentration means the sum of nitrous acid and nitrite ( $c_{\text{NO}_2} = c_{\text{HNO}_2} + c_{\text{NO}_2^-}$ ) respectively with respect to the control parameters. The pH has great influence on the process and has to be considered. The carbon dioxide can be stripped from the water phase and has to be calculated from the equilibrium conditions of the inorganic carbon.

$$c_{\text{CO}_2} = c_{\text{C}} \left( 1 + \frac{K_{\text{D,CO}_2}}{c_{\text{H}_3\text{O}^+}} + \frac{K_{\text{D,CO}_2} K_{\text{D,HCO}_3^-}}{c_{\text{H}_3\text{O}^+}^2} \right) \quad (10)$$

$$c_{\text{HCO}_3^-} = c_{\text{C}} \left( \frac{c_{\text{H}_3\text{O}^+}}{K_{\text{D,CO}_2}} + 1 + \frac{K_{\text{D,HCO}_3^-}}{c_{\text{H}_3\text{O}^+}} \right) \quad (11)$$

$$c_{\text{CO}_3^{2-}} = c_{\text{C}} \left( \frac{c_{\text{H}_3\text{O}^+}^2}{K_{\text{D,CO}_2} K_{\text{D,HCO}_3^-}} + \frac{c_{\text{H}_3\text{O}^+}}{K_{\text{D,HCO}_3^-}} + 1 \right) \quad (12)$$

where  $c_{\text{C}} = (c_{\text{CO}_2} + c_{\text{HCO}_3^-} + c_{\text{CO}_3^{2-}})$  = concentration of inorganic carbon [ $\text{mol}/\text{m}^3$ ],  $K_{\text{D,CO}_2}$  = dissociation constant for the system  $\text{CO}_2/\text{HCO}_3^-$  [ $\text{mol}/\text{m}^3$ ],  $K_{\text{D,HCO}_3^-}$  = dissociation constant for the system  $\text{HCO}_3^-/\text{CO}_3^{2-}$  [ $\text{mol}/\text{m}^3$ ].

Assuming the pH mainly being controlled by the inorganic carbon, a balance of the charges leads to an equilibrium equation for the determination the concentration of  $\text{H}_3\text{O}^+$ -ions  $c_{\text{H}_3\text{O}^+}$ :

$$c_{\text{al}} = c_{\text{C}} \left( \frac{K_{\text{D,CO}_2} c_{\text{H}_3\text{O}^+} + 2 K_{\text{D,CO}_2} K_{\text{D,HCO}_3^-}}{c_{\text{H}_3\text{O}^+}^2 + K_{\text{D,CO}_2} c_{\text{H}_3\text{O}^+} + K_{\text{D,CO}_2} K_{\text{D,HCO}_3^-}} \right) \quad (13)$$

that leads to the pH that is defined:

$$\text{pH} := -\lg_{10} \frac{a_{\text{H}_3\text{O}^+}}{c_{\text{H}_3\text{O}^+}^0} \approx -\lg_{10} \frac{c_{\text{H}_3\text{O}^+}}{c_{\text{H}_3\text{O}^+}^0} \quad (14)$$

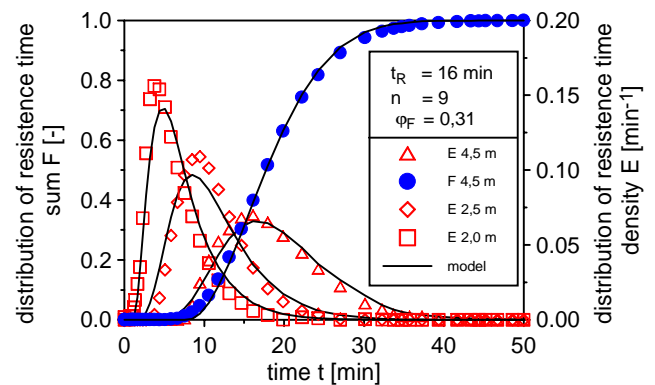
where  $a_{\text{H}_3\text{O}^+}$  = activity of  $\text{H}_3\text{O}^+$ -ions [ $\text{mol}/\text{m}^3$ ],  $c_{\text{H}_3\text{O}^+}^0$  = reference unit ( $c_{\text{H}_3\text{O}^+}^0 = 1000 \text{ mol}/\text{m}^3$ )

## RESULTS

### Hydraulics

Fixed-bed reactors without recirculation show a strong restriction of remixing affecting reaction kinetics beneficially, if small effluent concentrations should be achieved. The result of a tracer (NaCl) measurement is represented in Fig. 3 exemplarily. It clearly shows, that the hydraulic behaviour of a plug flow fixed-bed reactor can be described by a cascade model.

The tracer experiments yielded a great influence of the filter velocity  $v_{\text{FF}}$  and of the superficial gas velocity  $v_{\text{FG}}$  on the gas hold up and the stagnant volume. The stagnant volume can be reduced considerably by an increase of the filter velocity.



**Figure 3.** Distribution of the residence time of an aerated fixed bed reactor

### Mass transfer

Certainly, the oxygen transfer is very decisive for the transformation rates of aerobic processes in the fixed-bed reactor, as e.g. nitrification. Oxygen transfer can only occur by diffusion into the stagnant volumes. Therefore, a relatively high oxygen concentration must appear in the bulk liquid. This is modelled by a mass

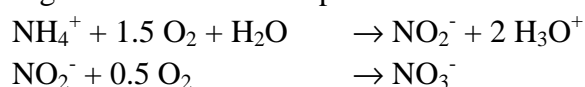
transfer approach. In this case, it is assumed that the mass transfer liquid/liquid is proportional to the mass transport gas/liquid. In Tab. 1 the mass transfer coefficients are represented exemplarily referring to total volume depending on the operating parameters. Oxygen transport can be influenced by the operating parameters superficial gas velocity and oxygen concentration in the gas phase (enrichment with technical oxygen) and the fixed bed height too.

**Table 1.** Mass transfer coefficients, referring to total volume ( $k_{La} \phi_F$ ), in aerated fixed bed reactors depending on the operating parameters filter velocity  $v_{FF}$  and gas superficial velocity  $v_{FG}$

$k_{La} \phi_F$ [ $h^{-1}$ ] for $T=10^\circ C$	$v_{FG}= 5$ $m^3/(m^2 h)$	$v_{FG}= 10$ $m^3/(m^2 h)$	$v_{FG}= 15$ $m^3/(m^2 h)$	$v_{FG}= 20$ $m^3/(m^2 h)$	$v_{FG}= 25$ $m^3/(m^2 h)$	$v_{FG}= 30$ $m^3/(m^2 h)$
$v_{FF} = 5 m^3/(m^2 h)$	11	13	14	15	16	18
$v_{FF} = 10 m^3/(m^2 h)$	17	22	25	28	32	36
$v_{FF} = 15 m^3/(m^2 h)$	21	28	33	39	44	50
$v_{FF} = 20 m^3/(m^2 h)$	26	35	42	49	57	64
$v_{FF} = 25 m^3/(m^2 h)$	31	43	51	61	71	80
$v_{FF} = 30 m^3/(m^2 h)$	36	51	60	72	84	95

## Nitrification.

The nitrification i.e. the biological transformation of ammonia via nitrite to nitrate, is performed by slowly growing chemo-litho-autotrophical bacteria:



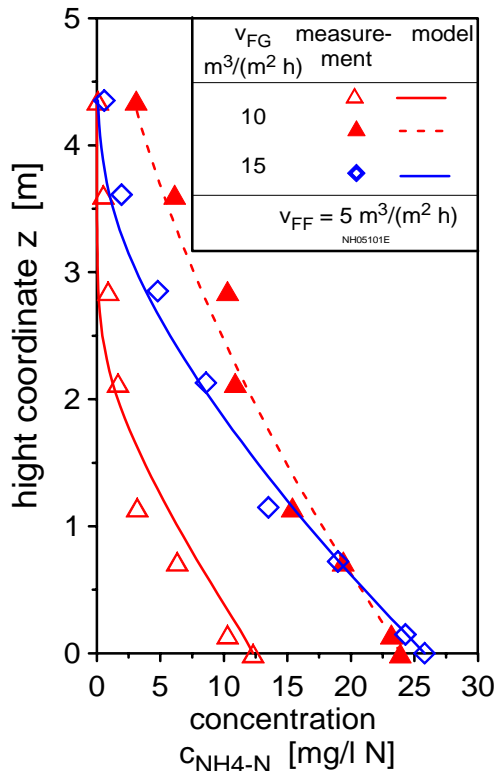
For this a lot of oxygen is necessary and alkalinity is needed. Therefore, a process is useful which

- holds slowly growing bacteria in the system,
- allows an effective oxygen transfer and
- reacts tolerantly compared to variations of influent loads.

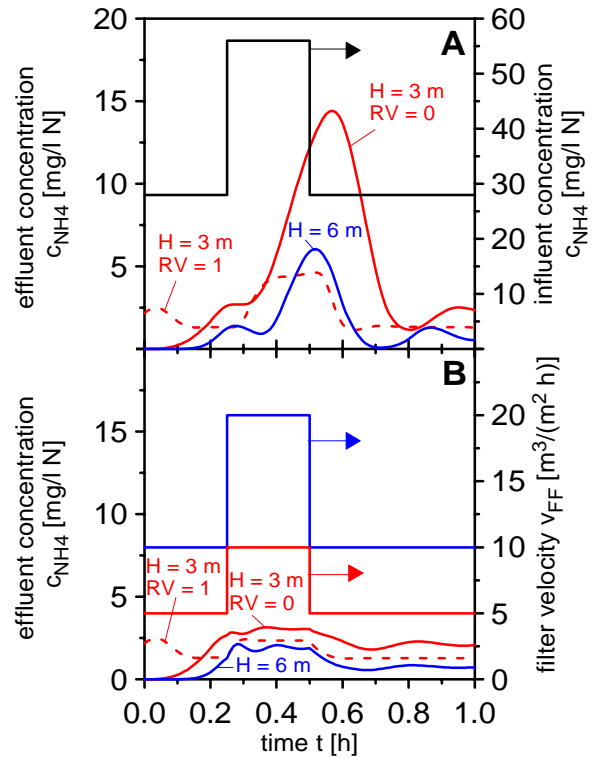
If low effluent ammonia concentrations are required additionally, fixed bed reactors fulfil the first two conditions. Variations must then be managed by an adapted control technology or pre-connected units (in the case of clarifying systems). To investigate the actuating variables for nitrification in the fixed bed reactor ammonia concentration profiles are determined in the fixed bed in the case of different operating conditions. Three  $NH_4-N$  profiles are represented in Fig. 4. A complete degradation within the total reactor height was intended (N-limited operation). It can be recognised that the profiles are represented well by the simulation model. A superficial gas velocity adapted to peak concentrations can adjust the ammonia effluent concentration to less than 1 mg/l N.

The reactor model should be used for checking the variability towards peak loadings and for the development of control strategies concerning different reactor designs. An important design parameter is the reactor height. In Fig. 5 the behaviour on peak loading ( $B_{NH_4}^R$  from 1.12 kg/( $m^3 d$ ) N to 2.24 kg/( $m^3 d$ ) N for 15 minutes) by raising the influent concentration (Fig. 5 A) and by raising the filter velocity (Fig. 5 B) is shown. A reactor height of 3 m (without and with recirculation) is compared to a reactor height of 6 m (without recirculation). The gas superficial velocity  $v_{FG}$  is controlled to be within a range of 5 to 30  $m_N^3/(m^2 h)$  to achieve an ammonia effluent concentration of 1 mg/l N. A high influent concentration leads to increased effluent concentrations (Fig. 5 A). It seems to be favourable to operation with a high filter velocity. Operating of the 3 m high fixed bed reactor with recirculation (i.e. high filter velocity  $v_{FF}$ ) shows results of

the same quality as a reactor of 6 m height, because the stagnant volume fraction is minimised leading to an increased ammonia transformation rate. Therefore, the hydraulic peak loading leads only to a moderate increase of the effluent concentrations (Fig. 5 B). The recirculation ratio should be considered also as an important control parameter.



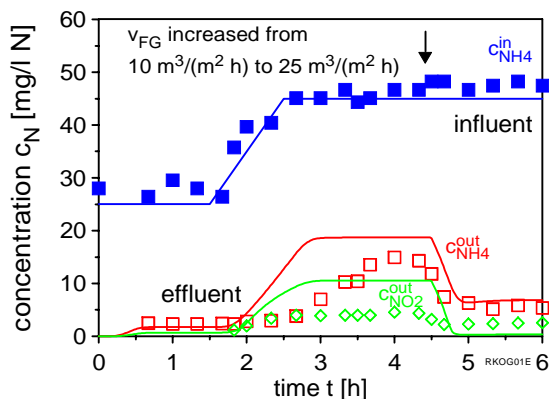
**Figure 4.** Profiles of the ammonia concentration in a fixed bed reactor by variation of the superficial air velocity (measurement and simulation)



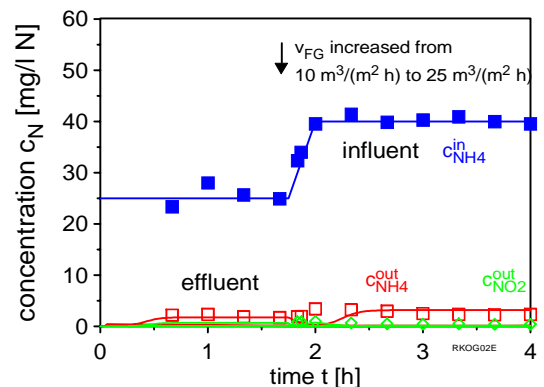
**Figure 5.** Response of a fixed bed reactor on peak ammonia loading for reactors with different heights

### Nitrification control in the fixed bed reactors

The basic inspection of the control strategy and of the fixed bed simulation occurred by means of pilot investigations. The influent and effluent concentration of ammonia is represented in Fig. 6 for a fixed-bed reactor. An increase of ammonia concentration in the influent can be found in the effluent with a temporal delay. After an increase of the gas superficial velocity the effluent concentration drops off again. By an adaptation of the gas superficial velocity to an increase of the influent concentration a rise of the effluent concentration may be prevented (see Fig. 7).



**Figure 6.** Simulation and measurement of a nitrifying pilot plant fixed bed reactor



**Figure 7.** Simulation and measurement of a nitrifying pilot plant fixed bed reactor

## CONCLUSION

For the successful application of modelling the nitrification in a fixed-bed reactor experiments for hydrodynamics and for the mass transfer in aerated fixed beds had to be carried out. The model was successful to describe the results obtained with a pilot system. Control strategies developed with the model could be transformed into practice successfully. For the design of fixed bed reactors the results allow an optimal dimensioning on account of basic load and the determination of the performance reserves could be achieved.

## REFERENCES

- Bishop P.L. (1996). Biofilm structure and kinetics. Third International IAQW Special Conference on Biofilm Systems. Copenhagen, Denmark.
- Henze M., Harremoës P., LaCour Jansen J. and Arvin E. (1995). *Wastewater Treatment: Biological and Chemical Processes*. Springer, Heidelberg.
- Horn H. (1992). Simultane Nitrifikation und Denitrifikation in einem heter-/autotrophen Biofilm unter Berücksichtigung der Sauerstoffkonzentration. *Gwf Wasser/Abwasser*, **134**(6), 287-292
- Picioreanu C., van Loosdrecht M.C.M. and Heijnen, J.J. (1998). Mathematical modeling of biofilm structure with a hybrid differential-discrete cellular automation approach. *Biotech. & Bioeng.*, **58**(1), 101-116
- Picioreanu C., van Loosdrecht M.C.M. and Heijnen, J.J. (1996). Modeling the effects of oxygen concentration on nitrite accumulation in a biofilm airlift suspension reactor. Third International IAQW Special Conference on Biofilm Systems, Copenhagen Denmark.
- Vayenas D.V. and Lybertatos G. (1994). A novel model for nitrifying trickling filters. *Wat. Res.*, **28**, 1275-1284
- Wanner O. and Gujer W. (1986). A multispecies biofilm model. *Biotech. & Bioen.*, **28**, 314-328
- Wanner O. and Reichert P. (1996). Mathematical modeling of mixed-culture biofilms. *Biotech. & Bioen.*, **49**, 172-184

## NOMENCLATURE

### Symbols

$B_{NH_4}^R$	Ammonia space loading [kg/(m <sup>3</sup> d) N]	$OC_A$	oxygen transfer capacity [kg O <sub>2</sub> /(m <sup>2</sup> h)]
$c_{al}$	Alkalinity [mol/m <sup>3</sup> ]	$OC_N^{th}$	oxygen transfer efficiency [kg O <sub>2</sub> /kWh]
$c_C$	Concentration of inorganic carbon [mol/m <sup>3</sup> ]	$\Delta H$	height of an elementary cell [m]
$c_{NH_3}$	Ammonia concentration [mol/m <sup>3</sup> ]	$v_{FF}$	filter velocity [m <sup>3</sup> /(m <sup>2</sup> h)]
$c_{HNO_2}$	nitrous acid concentration [mol/m <sup>3</sup> ]	$v_{FG}$	gas superficial velocity [m <sup>3</sup> /(m <sup>2</sup> h)]
$c_{H_3O^+}$	concentration of protons [mol/m <sup>3</sup> ]	$r_{max,NH_3}$	maximum degradation rate of NH <sub>4</sub> = NH <sub>4</sub> <sup>+</sup> +NH <sub>3</sub> -N [mol/(m <sup>3</sup> h)]
$c_{O_2}$	concentration of dissolved oxygen [mol/m <sup>3</sup> ]	$r_{max,HNO_2}$	maximum degradation rate of NO <sub>2</sub> = NO <sub>2</sub> <sup>-</sup> +HNO <sub>2</sub> -N [mol/(m <sup>3</sup> h)]
$K_{D,CO_2}$	dissociation constant H <sub>2</sub> CO <sub>3</sub> /HCO <sub>3</sub> <sup>-</sup> [mol/m <sup>3</sup> ]	$RV$	recirculation ration [-]
$K_{D,HCO_3}$	dissociation constant HCO <sub>3</sub> <sup>-</sup> /CO <sub>3</sub> <sup>2-</sup> [mol/m <sup>3</sup> ]	$x_{O_2}$	oxygen content in the gas [m <sup>3</sup> /m <sup>3</sup> ]
$K_I$	inhibition coefficient for competitive inhibition [mol/m <sup>3</sup> ]	$Y_{O_2/NH_3}$	stoichiometric coefficient oxygen to ammonia (Y <sub>O<sub>2</sub>/NH<sub>3</sub></sub> = 1.5 mol O <sub>2</sub> / mol NH <sub>4</sub> -N)
$K_{O_2}$	half saturation coefficient for oxygen for degradation of NH <sub>3</sub> resp. HNO <sub>2</sub> [mol/m <sup>3</sup> ]	$Y_{O_2/HNO_2}$	stoichiometric coefficient oxygen to nitrite (Y <sub>O<sub>2</sub>/HNO<sub>2</sub></sub> = 0.5 mol O <sub>2</sub> / mol NO <sub>2</sub> -N)
$K_S$	half saturation coefficient for the substrate (NH <sub>3</sub> resp. HNO <sub>2</sub> ) [mol/m <sup>3</sup> ]	$\phi_l$	fraction of the free liquid phase [-]
$(k_{La})_{g/l}$	volumetric mass transfer coefficient (gas/liq.) [h <sup>-1</sup> ]	$\phi_g$	fraction of the gas phase [-]
$(k_{La})_{l/b}$	volumetric mass transfer coefficient into the biofilm (liquid/biofilm) [h <sup>-1</sup> ]	$\phi_b$	fraction of the biofilm phase [-]
<b>Indices</b>			
b	biofilm phase	l	liquid phase
g	gas phase	n	index for substance
i	number of elementary cell (i = 0: influent; i = N: effluent)	S	saturation
		*	reference quantity