Increasing Tightness by Introduction of Intertemporal Constraints in MILP Unit Commitment

Tobias Zimmermann[1,*], Tobias Cors[2], Gerrit Erichsen[1], Alfons Kather[1]
[1]Institute for Energy Systems, Hamburg University of Technology, Hamburg, Germany
[2]Institute for Operations Management, University of Hamburg, Hamburg, Germany
[*]tobias.zimmermann@tuhh.de, Tel.: +49-40-42878-3280

Abstract

Flexibility requirements of controllable units in energy systems increase with high shares of volatile renewable energy (VRE) production. In this work, an existing unit commitment optimisation, formulated as mixed-integer linear problem (MILP) for energy systems is extended with intertemporal constraints to achieve a more realistic and detailed portrayal of the producer park against the backdrop of need for higher flexibilities. Those extensions mainly consider start-up times as well as minimum operating and standstill periods from a technical perspective, which are associated with rising complexity as well as tightness of the MILP. A small-scale energy system designed as a single copper plate with island restrictions, consisting of a conventional power plant, a short-term and a long-term energy storage and a VRE producer park is used to investigate the effects of the integrated intertemporal constraints.
The study shows, that unit commitment is widely influenced by these extensions and the scattering of units, observed in the reference system without these extensions, is eliminated. The number of starts and operating hours therefore results in a more realistic unit commitment. Even though the equation numbers of the MILP increases drastically, the computation time is lowered significantly.

Keywords – mixed integer linear programming, unit commitment, computational efficiency, intertemporal constraints

1 Introduction

The energy sector is facing a massive change, which is heavily focussing on the integration of more volatile renewable energy (VRE). The resulting residual load has to be balanced by controllable units with sufficient flexibility to match said volatility. In line with the flexibility requirements, complexity of system control increases and needs to be addressed by the underlying unit commitment (UC) model.
A popular approach for this is the formulation of the energy system as a mixed-integer linear problem (MILP). In a previous work [1], such an optimisation model has been formulated, which will be called reference model throughout this paper.
As described in [1], unrealistic system behaviour such as scattering of multiple units or in general inadequate UC on the operational level can be observed, attributed to missing constraints reflecting start-up procedures and operating time requirements.

1.1 Literature Review

In the literature, one can find two levels of extensions to advance a MILP UC in terms of a more realistic operational behaviour:
Level 1: Advancing the basic physical constraints, such as ramping limits, to reflect different limitations at start-up or shut-down, adding (economically motivated) minimum operating and standstill periods constraints and incorporating start-up cost approximations in the objective function. [2–4]
Level 2: Including an approximation of the start-up process from a technical perspective, which then is also considered in the objective function with the respective costs or emissions. [5, 6]

i) Tightness and Compactness of MILP formulations
In both levels, the extensions result in a higher degree of complexity to the MILP UC formulations and lead to larger models that have to be processed by the underlying solver. Although computational power and solver capabilities increased, the time to solve the MILP still is a critical limitation [2, 7–9]. For this reason, a stream of literature seeks to reduce the computational burden by improving the before mentioned formulations for both model levels.
In general, those solvers use the branch-and-cut algorithm to find the optimal solution. While the algorithm searches for solutions in an enumeration tree (branching), feasible integer solutions (upper bounds) and linear problem (LP) relaxations (lower bounds), where integrality constraints are relaxed, are computed. Mainly, the formulation tightness (distance between upper and lower bound) and compactness (amount of data to be processed) is influencing computational performance of a MILP. In the context of a branch-and-cut algorithm, a higher tightness allows pruning more nodes earlier as the lower bounds of the LP relaxation lies nearer to the optimal integer solution. Hence, tightening allows a faster search for the optimal integer solution by removing inefficient solutions from the

feasible region and thereby reducing the search space that needs to be explored by the solver. However, methods to tighten MILP formulations are typically worsening their compactness, since additional constraints and/or variables are added, leading to larger LPs that have to be solved repeatedly. [2, 3, 6, 9, 10]

ii) Level 1 UC models

Coming back to the above-introduced level 1 models, a tight and compact model formulation is presented in [2] and compared to those in [3] and [4]. The model in [2] is in principle a three binary variable (BiV) formulation including one BiV each for operation, start-up and shut-down status. With the addition of another BiV, defining the start-up type and the corresponding constraints, it is shown that, although the model is less compact, the tightness is increased in comparison to those in [3] and [4] showing an overall better computational performance in their test cases. The model in [4] is an earlier attempt to increase the model compactness by using only one operation BiV, which was, after solver capabilities increased, shown to be less efficient than the advantage that could be gained by cuts on the additional but strongly dependent start-up and shut-down BiV [2]. However, the comparison in [2] only considers the formulation with just one start-up type. In terms of a system analysis due to flexibility and against the backdrop of the rise of VRE, the level 1 models lack the respective operational detail, which is mainly due to a weak approximation of the start-up process of thermal power plants.

iii) Level 2 UC models

The level 2 models overcome the shortcomings of level 1 models on the operational level with better approximations but are still less represented in literature. In [5] a level 2 model is presented, that considers three different start-up process types (hot, warm, cold) dependent on the prior standstill period. Those directly correspond to the timestep quantity and the power output during the respective start-up period. The formulation is a 3 BiV setup for operation, start-up and shut-down decision, but has been extended by further sets of binary variables. Following [5], an improved model is introduced in [6] considering multiple start-up types as well as start-up and shut-down ramps with respective power trajectories.

1.2 Contribution

This work aims to take into consideration the aforementioned increasing flexibility requirements of the energy system by incorporating intertemporal constraints. The focus herby lies on a tight problem formulation, which leads to a more realistic operation plan while keeping the computational effort on a reasonable level. The introduced MILP model is a 1 BiV formulation which considers a three-step (hot, warm, cold) start-up formulation, while maintaining the same amount of (binary) variables as in the reference model. In general, this work aims to make an addition to the level 2 model formulations and applications by providing a technical formulation of start-up procedures at a reasonable computational effort.

2 Model Description and Scenario Setup

The reference model presented in [1] acts as a basis for this paper. There, the UC for the energy system is described as a MILP using the approach of a rolling horizon with a non-linear control model between periods. It distinguishes between conventional (power plants) and renewable energy storage devices, while VRE production and demand are given as input data. The model allows for multi-objective optimisations, considering (operational) costs and CO_2 emissions. For this paper, the optimisation goal is set to minimise CO_2 emissions only to achieve comparability with [1]. For the complete description of the reference model, please refer to [1].

2.1 Intertemporal Constraints

The model extensions of this work are designed to consider start-up procedures, minimum standstill and operating periods. Start-up is divided between hot start-up (<8h), warm start-up (8-48h) and cold start-up (>48h) in accordance with [11].

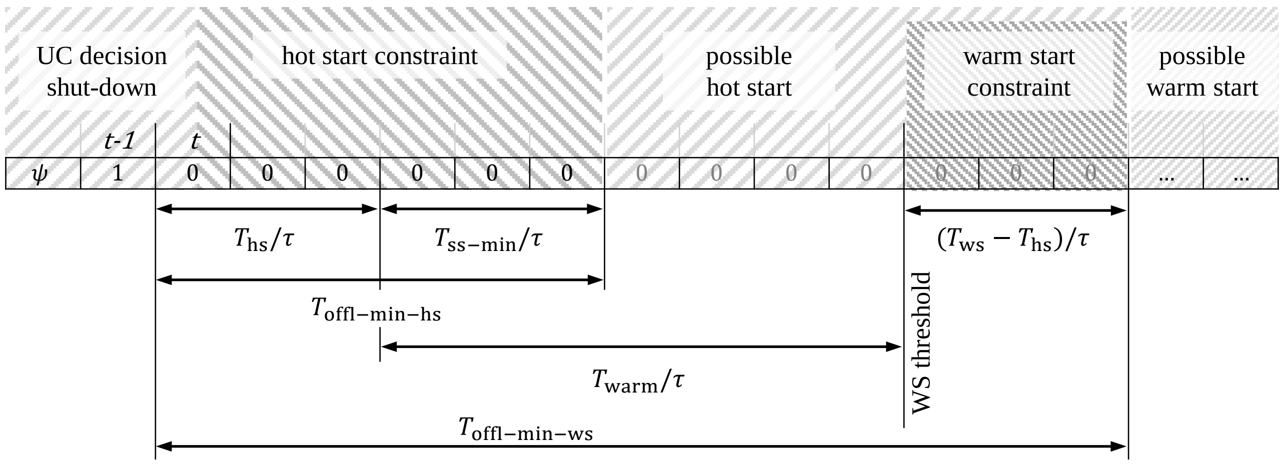

Figure 1 Visualisation of temporal parameters for hot start-up and warm start-up constraints in the extended model

$$\big(\psi(u,t-1)-\psi(u,t)\big)\cdot T_{\mathrm{offl-hs}}(u,t) \leq \sum_{i=t}^{t+T_{\mathrm{offl-hs}}(u)-1} \big(1-\psi(u,i)\big) \qquad \forall u \in U, \quad \forall t \in 2 \ldots T \tag{1}$$

$$with \qquad T_{\mathrm{offl-hs}}(u,t) = \begin{cases} T_{\mathrm{offl-min-hs}}(u)-1, & if\ T-t \geq T_{\mathrm{offl-min-hs}}(u)-1 \\ T-t+1, & else \end{cases}$$

$$\big(T_{\mathrm{offl-ws}}(u,t)+1\big)\cdot\left(\big(\psi(u,t-1)-\psi(u,t)\big)-\sum_{i=t}^{t+T_{\mathrm{offl-th-ws}}(u)-1}\psi(u,i)\right) \leq \sum_{i=t+T_{\mathrm{offl-th-ws}}(u)}^{t+T_{\mathrm{offl-th-ws}}(u)+T_{\mathrm{offl-ws}}(u,t)} \big(1-\psi(u,i)\big) \qquad \forall u \in U,\ \forall t \in 2 \ldots T-T_{\mathrm{offl-th-ws}}(u)-1 \tag{2}$$

$$with \qquad T_{\mathrm{offl-ws}}(u,t) = \begin{cases} T_{\mathrm{offl-min-ws}}(u)-T_{\mathrm{offl-th-ws}}(u)-1, & if\ T-t \geq T_{\mathrm{offl-min-ws}}(u)-1 \\ T-t-T_{\mathrm{offl-th-ws}}(u), & else \end{cases}$$

The functionality of start-up constraints is visualised exemplarily for hot and warm start-up in **Figure 1**, which are represented in the MILP by the following constraints in equation (1) and (2). A cold start-up is modelled accordingly. Note, that the arrangement of temporal parameters as presented in **Figure 1** represents the mathematical formulation of real plant behaviour. Ordering the temporal parameters according to operational logic would lead to equations that are even more complex. An initial shut-down is represented as a switch of the binary status variable ψ from 1 to 0 in timestep t. When the UC "decides" for this unit shut-down, the plant has to stay offline for a number of timesteps $T_{\mathrm{offl-min-hs}}$, which consists of the hot start-up time T_{hs} and the minimum standstill period $T_{\mathrm{ss-min}}$. Afterwards, a possible start-up is considered a hot start. The timestep length τ translates time to discrete timesteps. Equation (1) is the mathematical representation of this hot start-up procedure, which is applied and valid for all units u in the set of modelled units U. Note, that the temporal validity is only given for timesteps 2 to T of the current rolling horizon interval, because for the first timestep, $t = 1$, the initial values resulting of the last interval of the rolling horizon have to be accounted for. In the mathematical representation, the scope of applicable timesteps in the equations has to be adjusted according to the time frame of the rolling horizon. Therefore, in case of the hot start-up, $T_{\mathrm{offl-hs}}$ has to be calculated accordingly for each timestep.

The same principle is applied for a warm start-up, where the minimum offline time $T_{\mathrm{offl-min-ws}}$ consists of the threshold time for a start-up to be considered warm T_{warm}, the hot start-up time T_{hs} and the difference between the hot and warm start-up time $T_{\mathrm{ws}} - T_{\mathrm{hs}}$. A cold start-up is modelled accordingly to the warm start-up formulation in (2). A similar formulation approach is shown in [12].

In addition to the technically limiting constraints, start-up CO_2 emissions are considered in the objective function, which aims to minimize the CO_2 emissions of the energy system.

2.2 Scenario Setup

The analysed scenarios are based on [1]. A small energy system designed as single copper plate with island restrictions is the focus of this research. It consists of a combined cycle gas turbine (CCGT), a short-term and a long-term renewable energy storage device as well as VRE producers. While the short-term energy storage device is parameterised to represent a battery storage, the long-term storage device consists of an electrolyser for converting power to hydrogen and a CCGT for reconversion of hydrogen to power.

The rated power of these VRE producers is increased throughout the scenarios and is always distributed evenly between wind and solar power. It is given in MW_{each}, meaning the rated power each VRE type (wind and solar) has in a given configuration. For the sake of simplicity, a constant demand of 1000 MW is defined.

The time frame of the simulation is one year in a 15 minute resolution, using scaled grid operator data from 2015 for wind and solar production as used and shown in [1].

The technical specifications of the CCGT relevant for the intertemporal constraints are provided in **Table 1** in the Appendix. The CO_2 emissions for the different start-up procedures stem from [13].

As the model is able to consider minimum operating times, additional parameters have to be set. In [14] values are suggested, but only motivated economically without technical reasons given. Therefore, these parameters were set to zero as to leave it to the optimisation, whether short operating times are beneficial. All remaining input data and parameters can be found in [1].

3 Results

The optimisation aims to find a UC, matching the residual load in the energy system with minimum CO_2 emissions. While the overall results, hence the yearly CO_2 emissions, do not change significantly due to the model extensions, the detailed operation plans differ widely. Without

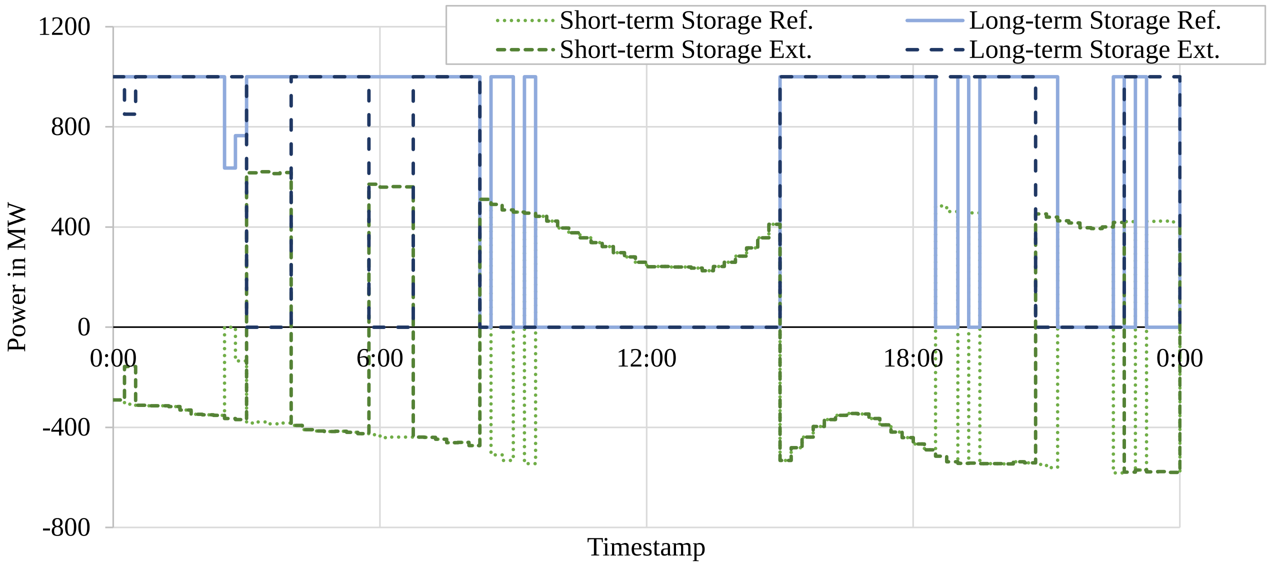

Figure 2 Time series excerpt of power of storages according to reference model (Ref.) and extended model (Ext.) on 17 January with 3000 MW_{each} VRE power installed. CCGT power for both models is not shown, since it is zero.

incorporation of intertemporal constraints, the results tend to include scattering [1], which is deemed an unrealistic behaviour. **Figure 2** displays a time series excerpt of the storage devices' powers for both models in a scenario with 3000 MW_{each} VRE rated power.

While both models match the residual load, major differences in unit commitment can be seen. As described above, the reference model results in a scattering behaviour, where especially the long-term storage device (solid line) repeatedly switches between discharging and no operation in between timesteps. Especially in terms of the discharging unit, consisting of a CCGT, this result is not technically feasible. With the introduction of intertemporal constraints in the extended model, the long-term storage device (long dashed line) follows a steadier schedule, resulting in longer blocks of operation. The short-term storage mirrors the behaviour of the long-term storage to match the residual load, as the conventional CCGT is not running.

It is worth noting, that at times one storage is charging while the other is discharging. This behaviour could be prohibited by constraints, but is optimal in terms of energy use and hence minimum CO_2 emissions of the respective system. This is due to the relative efficiency loss of the CCGT and the high efficiency of the battery as explained in [1]. Therefore, this behaviour was allowed.

Displayed in **Figure 3** are the boxplots showing the distribution of the CCGT's operating periods. The reference model without intertemporal constraints (solid) shows a clear tendency towards short operating periods well below a sensible value as e.g. 4 h, which is often regarded as economically feasible minimum operating period [15]. The presented model extensions eliminate this problem almost entirely, showing longer operating periods in the majority of cases (striped boxplots).

With higher shares of renewable energy, volatility of residual load increases, which in general causes a trend towards shorter operating periods. Therefore, also the extended model results in short operating periods for situations where it is optimal according to the optimisation objective. This also leads to the same minimum values for operating times in both models in all VRE configurations.

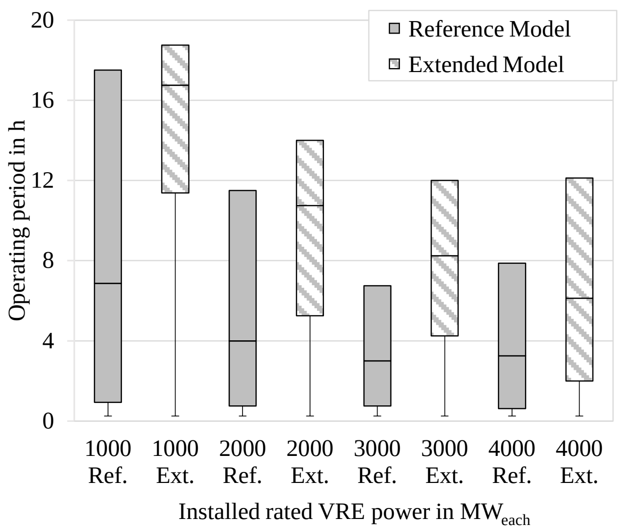

Figure 3 Comparison of CCGT operating periods in reference (Ref., solid) and extended (Ext., striped) model for different VRE configurations

In [6] it was shown, that the introduction of intertemporal constraints to MILP models of energy systems reduce the computation time despite the greater complexity, given the increased tightness of the problem formulation.

Different MILP solvers use heuristics and cuts that may dramatically influence the performance, but in general are strongly influenced by the formulation tightness [10]. Further, MILP solvers use enumeration tree and branching strategies, which change completely, when formulations with different integer variables are compared [2]. Because the reference model and the extended model use the exact same amount of BiV for the problem formulation and all calculations are carried out using CPLEX 12.7.1 [16] as a solver on a machine with an Intel i7-3770 with 16 GB RAM, the computational performance is also a measure for the formulation tightness. For this reason and because the time to solve is an important measure, we only discuss the CPU times offering a broader view of the model performance.

From **Figure 4**, showing the computation times of various configurations for both the reference model (grey, solid) and the extended model (orange, dashed), we confirm what was shown in [2] and [6] for our 1 BiV MILP formulation in the given scenario and configurations. It can be concluded, that the tightness of the problem formulation is increased in comparison to the reference model due to the addition of intertemporal constraints. This results in cutting of infeasible solution space early and therefore allowing the solver to achieve a faster search for an optimal and valid solution.

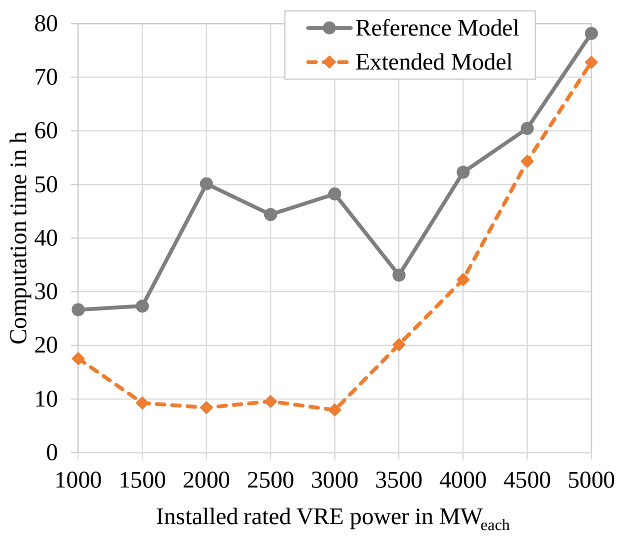

Figure 4 Comparison of computation time in reference (solid) and extended (dashed) model for different VRE configurations

For scenarios with higher installed rated VRE power, the computation time increases significantly for the given optimisation problem. Due to the low number of necessary CCGT operating periods, the solution space for minimisation of CO_2 emissions becomes flat, meaning that many close-to-optimal solutions can be found, hence leading to slower convergence.

4 Conclusion

In this work, the existing model for unit commitment from [1] has been extended with intertemporal constraints. These extensions mainly include start-up procedures and operating and standstill periods of the units in the energy system.

The goal of a more realistic operation plan is met, as exemplified by longer and less scattered operating periods of the power units. For flexibility analysis, it can be stated that operational detail matters and should be included via intertemporal constraints into MILP UC, especially for energy systems with highly volatile residual loads.

Furthermore, the computation times were reduced throughout all regarded scenario configurations, even though the size of the optimisation problem was heavily extended, showing the increased tightness of the MILP formulation.

5 Literature

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6 Nomenclature

Symbol	Unit	Description
i	-	Running index
t	-	Timestep
T_{hs}	h	Time period for hot start
$T_{\mathrm{offl-min-hs}}$	-	Minimum offline time for hot start
$T_{\mathrm{offl-min-ws}}$	-	Minimum offline time for warm start
$T_{\mathrm{offl-th-ws}}$	-	Threshold offline time for warm start
$T_{\mathrm{ss-min}}$	h	Minimum standstill period
T_{warm}	h	Threshold time for warm start
T_{ws}	h	Time period for warm start
U	-	Set of units
u	-	Index of unit in set U
τ	h	Length of timestep
ψ	-	Status variable for unit

Abbreviation	Description
BiV	Binary variable in MILP
CCGT	Combined Cycle Gas Turbine
MILP	Mixed integer linear problem
LP	Linear problem
UC	Unit Commitment
VRE	Volatile Renewable Energy

7 Appendix

Table 1 Intertemporal parameters used in the simulations. Values for parameters without source are own assumptions

Parameter	Unit	Value	Source
Cold start time CCGT	h	2	[11, 14]
Warm start time CCGT	h	1	[11, 14]
Hot start time CCGT	h	0.5	[11, 14]
Cold start time short-term storage charge/discharge	h	0	-
Warm start time short-term storage charge/discharge	h	0	-
Hot start time short-term storage charge/discharge	h	0	-
Cold start time long-term storage charge	h	0.25	[17]
Warm start time long-term storage charge	h	0	[17]
Hot start time long-term storage charge	h	0	[17]
Cold start time long-term storage discharge	h	2	[11, 14]
Warm start time long-term storage discharge	h	1	[11, 14]
Hot start time long-term storage discharge	h	0.5	[11, 14]
Minimum standstill time CCGT	h	0.5	[14]
Minimum standstill time short-term storage charge/discharge	h	0	-
Minimum standstill time long-term storage charge	h	0	[17]
Minimum standstill time long-term storage discharge	h	0.5	[14]