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Sustainable Public Transportation using Markov Chains: Case Study Hamburg Public Transportation

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Purpose: *Intelligent public transportation systems have been largely focused on improving the planning, and monitoring the transportation flows during recent years. Advancements in public transportation systems increase service levels and encourage more usage of public transportation. The forecast of buses' arrival time to stations and having a dynamic system to anticipate the real-time possible events for users, significantly increase passenger satisfaction. This paper has studied the literature considering dynamic public transportation systems and also matters of environmental emissions.*

Methodology: *The paper has developed a method to predict bus arrivals at stations by considering the buses' operation parameters and variables with stochastic characteristics by applying Markov Chains. The paper also applied the assignment problem technique and multi-objective planning to enable a framework for public transportation resource assignment considering the perspectives mentioned earlier.*

Findings: *The real data of Hamburg public transportation has been used to verify the capabilities of the platform. The findings show that the model validity of the platform and enabled effective strategic planning for public resource assignment.*

Originality: *This paper has studied the related literature and discussed the considerable gap for proposing a dynamic public transportation system that brings satisfaction from the side of the users and also mutually minimizing environmental emissions.*

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1 Introduction

Public network transportation systems are an indispensable part of each city's composition that the daily lives of many people depend on its structure. Efficient public transport enriches a society for having a dynamic community that could largely impact its inhabitant's quality of life. Many people commute with Public transportation systems and the satisfaction of these people is one of the factors that has been considered always in the planning. However, in addition to the service level of passengers, recently due to the large amount of pollution that produced in this sector, environmental issues have been regarded for implementing a suitable urban transportation planning.

For achieving less CO2 emissions and having a clean environment in the world, all countries in the world must have an integrated program and coordinate fully with each other to implement this program effectively. Germany as one of the pioneer countries in the protection of the environment, discussing and regarding these issues from various aspects, and also, they are working with other countries to fulfill programs for protecting the environment. There is a climate protection plan act 2050 (Klimaschutzplan 2050) that was made by German politicians in 2016 to reduce emissions and greenhouse gases according to a plan by 2050. This plan is also integrated with the Paris Climate Agreement that is agreed on at the international level. According to studies, the transportation sector is one of the main sectors that is not yet reduced its CO2 emissions significantly since the number of cars is getting more on roads, air travel and road freight are increasing.

According to the European Commission studies, 40% of CO2 emissions come directly from urban mobility while up to 70 % of other pollutants also have originality from transportation. This triggers a common challenge to most major cities in Europe, where to intensify movement and increase service level, the congestion, pollution, and environmental criteria should be controlled. Thus, for finding the right responses to this challenge, it is recommended to apply Sustainability Assessment (SA) as a critical tool to analyze the environmental aspects by common SA's criteria (Ribeiro et al., 2020). The importance of sustainability in public transport has got more attention since international laws and domestic regulations are impacting transportation for sustainability concerns. The focus of sustainability is to establish principles on reliably sufficing the requirements of public transportation according to the three pillars: economic, environmental, and social (also known informally as profits, planet, and people). So, SA as an appraisal method can be applied to support long-term and short-term decisions for transportation planning fulfilling the three aforementioned pillars.

In terms of environmental criteria, SA encourages governments to consider main prospects that influence the wellbeing of the environment, such as reducing greenhouse gases and emissions, preserve the ecosystem, and hinder the progress of global warming. In public transportation, the consumption of fuels that produce various air pollutants is the primary concern that effects environmental criteria (Tang et al., 2020). These pollutants mostly incorporate Carbon dioxide, Nitrogen oxides, Methane, and particulates. So, an efficient transport planning system with sustainable considerations must seek to diminish harmful gases. Simultaneously, from the social and economic perspectives, sustainable transportation must consider the preferences of its citizens with the proper transportation service level.

Public transport can be analyzed from two perspectives; in the view of users, the cost of transportation, service level and reliability are among the most favored factors (Mishalan et al., 2006). While from the other perspective, the target is to represent public transport as a viable alternative to self-driving, which both satisfies the logistics requirements of passengers and also will be more successful to fulfill the urban environmental traffic regulations. So, it is a significant challenge that needs a proper solution to fulfill the both citizens' service level and also SA considerations mutually.

The research roadmap is structured as follows. In section 2, the paper has conducted a literature review for studies of Markov Chains in public transportation. In section 3, the problem is formulated. Section 4 discusses the problem based on a case study that considers the Hamburg city center and then reviews the results.

2 Literature Review

The public transportation resource assignment is well-known in literature, see (Guihaire & Hao, 2008) for a comprehensive list of references. (Lin & Bertini, 2002) studied existing algorithm for predicting the bus arrival time to stations at free flow traffic without of congestion consideration. They formulate a Markov Chain to analyze the behavior of a bus in possible scenarios of delays.

(Xiao et al., 2018) obtained the errors state transition probability matrix by employing a Markov Chain model to analyze the error fluctuation of the neural network prediction results. Their results were shown in the rail transit line 6, 7, 13 in Beijing as input data of the model and the volume of passengers are output of the model that verified with rail transit data of line 1 and 2. They also studied its case in Beijing by formulating the spillover effect with Markov Chains. They applied Hidden Markov Chains to identify submarkets as hidden states. Then they obtained the transition probability Matrix and analyze a ranges of spillover type through regression analysis. (Huang et al., 2017) studied the service efficiency in public transportation of China and analyzed the process of delay of buses at bays where they divided delay into two kind of delays as entering delay and exiting delay. Then by forming the queuing model for delay they propose Markov Chains to obtain the steady-state of the equilibrium through calculation of entering delay at bays that are helpful to assess the dwell time distribution and evaluate efficiency of bus bays. (Li, 2014) also combined Markov Chains model with Grey Markov Chain and compare them together, where he also predicted the volume of passengers travel in the bus route.

(Więcek et al., 2019) based on Markov Chain proposed an approach for prediction that were presented based on real-life data in order to optimize energy and cost in the public transport system. They considered the flow of passengers are stochastic to use Markov processes for determining occupancy level of buses and estimating the transition probability Matrix. The Matrix specified according to the historical data set for each bus stop in the heterogeneous Markov Chains. (Rajbhandari et al., 2003) applied Markov Chains in order to determine the propagation of bus delay and propose a model for using these delays of buses in the estimation of bus arrival times. He obtained the transition probability matrix based on Homogeneous and Heterogeneous propagation of delay between time-points for one-way trip bus transport system in New Jersey from Newark Penn's station to Woodbridge center Mall that is last about 1 hour 40 minutes travel time. (Saadi et al., 2016) studied a survey data of Belgian Household daily travel to estimate transport related variable for forecasting travel behavior by Markov Chain. (Şahin, 2017) reviewed the sequence of train departure and arrival times in stochastic process by using Markov Chains, where all the delays at stations were considered as possible states and the fluctuations among them were predict possible scenarios to predict steady state delay probabilities.

(Huang et al., 2017) focused on optimizing the bus standard driving stile based on route conditions and efficient fuel consumption, where predicted driving cycles of specific bus routes. They proposed an estimation model between 27 inter-stations, which four of them were represented the driving cycles features among all stations based on the multiple linear regression model. Iterative Markov Chains was applied for the bus lines assignment. (Delaram & Valilai, 2016) the mutual service level satisfaction of passengers, and environmental emissions in public transportation are considered as a motivational gap for this research. (Sodachi et al., 2020) optimized transportation planning in Hamburg city center regarding efficient of service level and minimizing emissions. They propose the routes that buses can travel while all stations are visited and passengers' satisfaction are adapted. This paper contributed to the (Sodachi et al., 2020) and studied its result from the other point of view to propose new solution to the model and also by employing the Markov Chains properties, analyze and forecast the delay at the assumed stations of the proposed model accordingly.

From the related literature, it is clear that the goal of the most of papers was to predict possible time delay at stations through obtaining the transitions probabilities of the Markov Chains model in various perspectives. They pursued this through the side of the passengers and they main goal was to predict possible delays in order to enhancing customer satisfaction. In this paper as well, we seek to maintain high service level, but also from the model we considering SA criteria as well. Thus, the Markov Chains is applied on the model that both optimizing service level from the side of passengers and environmental emissions from the side of SA.

3 Problem Formulation

In this section, the main goal is to implement a dynamic transportation model on (Sodachi et al., 2020) model by considering Markov Process properties. By this, we can mutually observe sustainability assessment issues in transportation from one side and also the passengers' satisfaction from the other side through predicting delays. According to literature, sustainability in transportation is an important trend that most developed countries are planning and seeking to adapt fully its criteria. So, it is not possible only to optimize the model from the side of the passengers. For this reason, achieving the sustainable transportation model is very crucial and the paper target here is to propose a dynamic transportation system that is mutually suitable for customer satisfaction, and friendly working peace to the environment. Due to this, the paper aims to study the results of the previous paper by (Sodachi et al., 2020). According to upcoming subsections, the paper first analyzes the sustainability assessment criteria in public transportation and review the results of references. Then for having a dynamic model in transportation, the Markov Process is applied to determine bus delay's propagation that is a basis for employing bus delay in the prediction of bus arrival time.

3.1 Sustainable Public Transportation with Service Level Efficiency

Sustainability assessment in public transportation depends on various criteria. The study of sustainability is not limited only to environmental metrics such as CO2 emissions. However, according to the literature, the impact of CO2 emissions is much more significant. For observing the most effective criteria in public transportation, the minimization of CO2 emissions and waiting time of passengers at stations were studied (Sodachi et al., 2020).

They minimized both the emissions of buses and the waiting time of passengers at stations by a Multi-Objective Optimization Problem (MOOP). The objective function of fuel consumption of buses is obtained based on three factors: the vehicle type, the distance traveled, and the load carrier as follows (Molina et al., 2014).

$$\min(F) = min \sum_i \sum_j \sum_k \sum_t ef^{CO_2r}.d_{ij}.x_{ijt}^K\left(fe^K + feu^K.L_{ij}^K\right) \tag{1}$$

Where the parameter of (1) are as following

ef^{CO_2r}: The amount of CO_2 emitted per unit of fuel consumed as an emission factor

fe^K: The amount of fuel consumed while the vehicle is empty

feu^K: The amount of fuel consumed based on the additional load in the vehicle

L_{ij}^K: The load carried by the vehicle between the considered stations

The mathematical model for minimizing the waiting time of passengers at stations is proposed according to the following equation

$$\min(\overline{W_j}) = \min \sum_{j=1}^{n} (h_j - \frac{\lambda_j(\lambda_j - 1)}{2.h_j})$$

$$= \min \sum_{j=1}^{n} (\sum_K y_{jt}^k - \sum_K y_{j(t-1)}^k - \frac{\lambda_j(\lambda_j - 1)}{2\left(\sum_K y_{jt}^k - \sum_K y_{j(t-1)}^k\right)}) \tag{2}$$

while (2) constraints are

$$\sum_{j=1}^{n} x_{0jt}^k = 1 \qquad \forall t, k \tag{3}$$

$$\sum_{j=1, j\neq i}^{n} x_{ijt}^k - \sum_{j=1, j\neq i}^{n} x_{jit}^k = 0 \qquad \forall i, t, k \tag{4}$$

$$\sum_{k=1}^{m} \sum_{i=1}^{n} x_{ijt}^k = 1 \qquad \forall j, t \tag{5}$$

$$\sum_{i=1, i\neq j}^{n} \sum_{k=1}^{m} x_{ijt}^k\left(y_{it}^k - y_{jt}^k\right) = 0 \tag{6}$$

$$\begin{cases} y_{jt}^{k} \geq x_{0jt}^{k}.t_{0j} & \forall k,t \\ y_{jt}^{k} = \sum_{i=1}^{n} xy_{ijt}^{k} + x_{ijt}^{k} * s_{i}^{k} + x_{ijt}^{k} * t_{ij} & \forall j,t,k \\ xy_{ijt}^{k} \leq M.x_{ijt}^{k} & \forall i,j,t,k \\ M\left(1 - x_{ijt}^{k}\right) + xy_{ijt}^{k} \geq y_{it}^{k} & \forall i,j,t,k \\ xy_{ijt}^{k} \leq y_{it}^{k} & \forall i,j,t,k \end{cases} \tag{7}$$

$$\begin{cases} y_{it}^{k} + s_{j}^{k} + t_{ij} \leq y_{jt}^{k} + T^{k}\left(1 - x_{ijt}^{k}\right) \\ t_{0j} \leq y_{jt}^{k} + T^{k}\left(1 - x_{0jt}^{k}\right) \end{cases} \tag{8}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_{ijt}^{k}(t_{ij} + s_{j}^{k}) \leq T^{k} \quad \forall t,k \tag{9}$$

$$\begin{cases} xy_{ijt}^{k} \geq 0 & \forall i,j,t,k \\ y_{ijt}^{k} \geq 0 & \forall i,j,t,k \\ x_{ijt}^{k} = (0,1) & \forall i,j,t,k \end{cases} \tag{10}$$

The parameters were defined according to the Vehicle Routing Problem as following

$$G(V,A): \begin{cases} V = (v_0, v_1, \dots, v_n),\ v_0: depot,\ V' = V\backslash\{v_0\}: Stations \\ A = \left\{(v_i, v_j) | v_i, v_j \in V, i \neq j\right\} \end{cases}$$

$K = (k_1, k_2, \dots, k_m)$: Vehicles which are assigned individually to each route,

d_{ij}: Distance between station i and$j$,

t_{ij}: Travel time between different stations,

T^k: Maximum allowing travel time for vehicle k,

S_i^k: Service time at station i.

And also, Decision Variables are:

$x_{ijt}^{k}: \begin{cases} 1, & If\ vehicle\ k\ travel\ from\ station\ i\ to\ j\ at\ period\ t \\ 0, & Otherwise \end{cases}$,

y_{jt}^{k}: Arrival time of vehicle k at station j in period t,

xy_{ijt}^{k}: Dummy variable for linearization, By y_{jt}^{k} definition, the headway at station j could be obtained as

$$h_j = \sum_{K} y_{jt}^{k} - \sum_{K} y_{j(t-1)}^{k} \tag{11}$$

3.2 Transportation Delays with Markov Process

In this section, the dynamic transportation system is considered in order to obtain the propagation of bus delays. For this reason, a Markov process has been applied to predict bus arrival times. Traffic condition situations can be categorized into homogeneous and heterogeneous, which could be predicted with transition probabilities. For the construction of transition probabilities of a Markov Chain, it is required all possible states are defined in transportation based on delays between time points. Hence, the states for the delay in bus transportation could be categorized at each time-points regards to three states as the arrival of bus exactly according to the determined scheduled, early arrival, and late arrival. These states are debriefed with “e” for early arrival, “l” for late arrival, and “o” for on-time arrival.

By knowing the possible states, then the transition probabilities between these states are calculated as follows:

$$P_{ij}^{r} = \frac{n_i}{\sum_{j=1}^{m} n_{ij}}, m \in \{1,2,3\}, r \in \{r_1, r_2, \dots, r_r, \dots, r_R\} \tag{12}$$

Where,

m: Total number of delay states that are classified into three mentioned states,

n_{ij}: number of events that states jump from i to j,

r: number of time-points

So, the matrix of transition probabilities is obtained as follows:

$$P_{ij} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} ee & el & eo \\ le & ll & lo \\ oe & ol & oo \end{bmatrix} \tag{13}$$

As shown in (4) each p_{ij} determines the probability of being in a state delay j at downstream time-point, while the upstream time-point has a delay state of i. For example, p_{11} represents the probability where the delay state is "early" at the downstream time-point, while the delay state at the origin time-point is also "early". So, for obtaining the transition probabilities, it is required to combinate the upstream time-point delay state and downstream time point delay state as shown in the next section.

3.3 Prediction of Transition Probabilities

In transportation, transition probabilities for Markov Chains are calculated to determine the possibility of fluctuation based on all available states with the combinations of time points. In transportation planning, these transition probabilities are computed based on homogeneous and heterogeneous propagation of time delay between time-points, where these transition probabilities of delay states are predicted at time-points regarding time-point originality.

Assume downstream time-point r_R and r_1 based on homogeneous delay propagation are determined as follows:

$$P_{r_1-r_R} = P_{r_1-r_2} \times P_{r_2-r_3} \times \ldots \times P_{r_{R-1}-r_R} \tag{14}$$

Because of the homogeneous delay propagation, all these probabilities are equal as follows:

$$P_{r_1-r_2} = P_{r_2-r_3} = P_{r_3-r_2} = \cdots = P_{r_{R-1}-r_R} \tag{15}$$

$$P_{r_1-r_R} = (P_{r_1-r_2})^R = \begin{bmatrix} ee & el & eo \\ le & ll & lo \\ oe & ol & oo \end{bmatrix}_{r_1-r_2}^R \tag{16}$$

Similarly transition probabilities between downstream time-points $r_{R-1}$and r_1, and also finally $r_{R-2}$and r_1 based on homogeneous delay propagation are determined as follows:

$$P_{r_1-r_{R-1}} = (P_{r_1-r_2})^{R-1} = \begin{bmatrix} ee & el & eo \\ le & ll & lo \\ oe & ol & oo \end{bmatrix}_{r_1-r_2}^{R-1} \tag{17}$$

$$P_{r_1-r_2} = \begin{bmatrix} ee & el & eo \\ le & ll & lo \\ oe & ol & oo \end{bmatrix}_{r_1-r_2} \tag{18}$$

On the other hand, if the propagation of time delays is considered heterogeneous, then the predicted time delay at downstream time point r_R and r_1 are as following:

$$P_{r_1-r_R} = P_{r_1-r_2} * P_{r_2-r_3} * \dots * P_{r_{r-1}-r_r} * P_{r_{r-1}-r_r} * P_{r_r-r_{r+1}} * \dots * P_{r_{R-1}-r_R} \quad (19)$$

$$P_{r_1-r_2} \neq P_{r_2-r_3} \neq P_{r_3-r_2} \neq \cdots \neq P_{r_{R-1}-r_R} \quad (20)$$

$$P_{r_1-r_R} = \begin{bmatrix} ee & el & eo \\ le & ll & lo \\ oe & ol & oo \end{bmatrix}_{r_1-r_2} \times \begin{bmatrix} ee & el & eo \\ le & ll & lo \\ oe & ol & oo \end{bmatrix}_{r_2-r_3} \times \dots \times \begin{bmatrix} ee & el & eo \\ le & ll & lo \\ oe & ol & oo \end{bmatrix}_{r_{R-1}-r_R} \quad (21)$$

Therefore, by using above equations, the $P_{ij}^{r}$for all downstream time-points were determined concerning time-point r_1.

4 Case Study

Efficient public transportation in Hamburg is a very important topic from various aspects. Hamburg is known as a free and Hanseatic city that is the second-largest city in Germany after Berlin and also the 7th largest city in the European Union with a population of over 1.84 million (Wikipedia). Hamburg has a strategic position among German cities and because of its key role in the German economy, it is required to have optimized public transportation since, in large cities as Hamburg, public transportation has an enormous effect on the everyday life of its inhabitants, and it is a significant role for the dynamic face of the city. Accessibility to transportation or easy access, level of service, comfortability, effectiveness, the value of cost, and other aspects are major factors that encourage people to use public transportation in their everyday travels. But from the other side, the change in the structures of the cities and the human manipulation cause a gigantic effect on the environment. Sustainability criteria in general and protection of the environment are issues that are indispensable in future public transportation planning. So, optimization of public transportation without consideration of sustainability is not enough and complete. Certainly, sustainability in general and environmental protection, in particular, is one of the main issues that is a trend for policymaking in most governments.

According to Hamburg municipality divisions, Hamburg is divided into five rings that most of the city covered with rings A and B around the city center. Other rings are covering other parts up to 60 km far from the city. The most strategic and populous part of Hamburg is covered with rings A and B based on Figure 1.

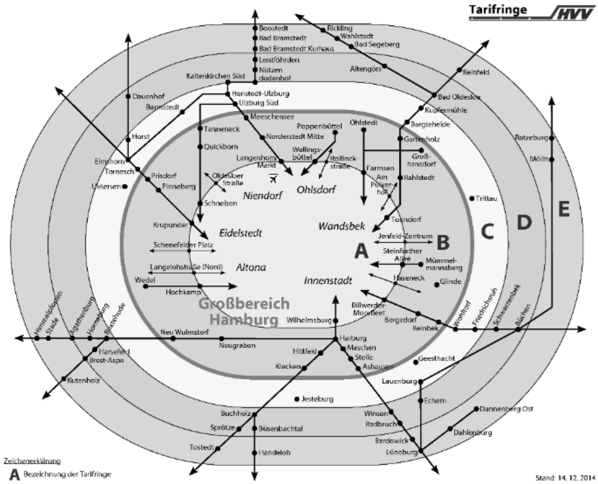

Figure 1: Divisions of Hamburg and its suburbs into Rings, (adapted from www.nimmbus.de)

Based on Figure 2, this paper focused on city center that is part of the city in Ring A since it is crowded with a high demand of a large number of passengers that they use everyday travels around the city. In this region, there are some fixed stations as nodes as shown in Figure 3. All possible direct travel between these stations is also determined in this figure. Table 1 also indicates the available stations by their names and determined numbers.

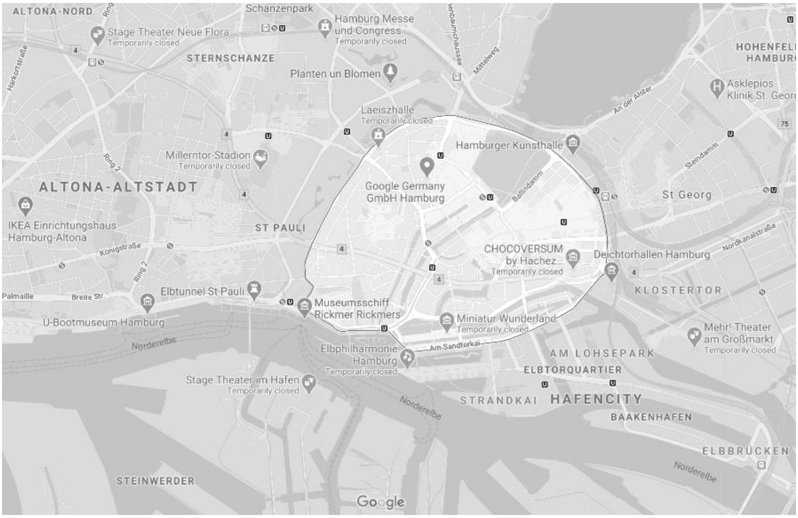

Figure 28: City center of Hamburg that is focused area in this paper obtained from Google© (Date 03/2021)

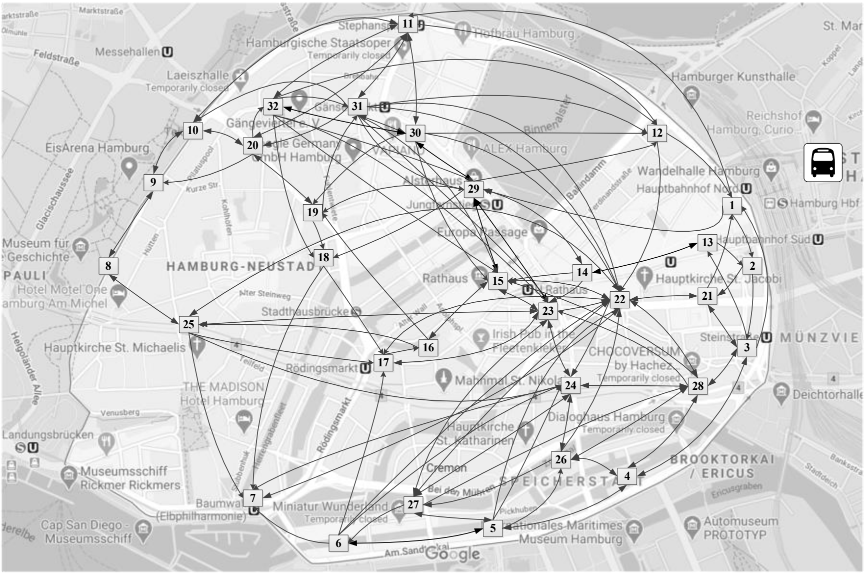

Figure 3: Bus stations in Hamburg city center and related connecting graph obtained from Google© (Date 03/2021)

Table 1: Bus stations in the Hamburg city center

1	HBF/ Spitalerstraße	9	Handwerkskammer	17	U Rödingsmarkt 2	25	Michaeliskirche
2	Hauptbhf./ Steintorwall	10	Johannes-Brahms-Platz 1	18	Axel Springer Square 1	26	Bei St Annen

1	HBF/ Spitalerstraße	9	Handwerkskammer	17	U Rödingsmarkt 2	25	Michaeliskirche
3	U Steinstraße (Deichtorplatz)	11	U Stephansplatz	19	Axel Springer Square 2	27	Auf dem Sande (Speicherstadt)
4	Singapurstraße	12	Kunsthalle	20	Johannes-Brahms-Platz 2	28	Meßberg
5	Am Sandtorkai	13	HBF/Mönckebergstraße	21	U Steinstraße	29	US Jungfernstieg
6	Am Kaiserkai	14	Gerhard Hauptmann Square	22	Jakobikirchof	30	U Gänsemarkt
7	Baumwall	15	Rathausemarkt	23	Rathausemarkt (Petrikirche)	31	Valentinskamp
8	Museum für Hamburgische	16	U Rödingsmarkt 1	24	Brandstwiete	32	Dragonerstall

Sodachi et al. (2020) found the optimum routes which incorporate all stations effectively for minimizing the waiting time of the passengers at these stations and simultaneously minimizing effectively the produced environmental emissions such as CO2 by buses that are traveling around this region. In Figure 4 the optimum assigned routes of buses based on the two objective functions are determined.

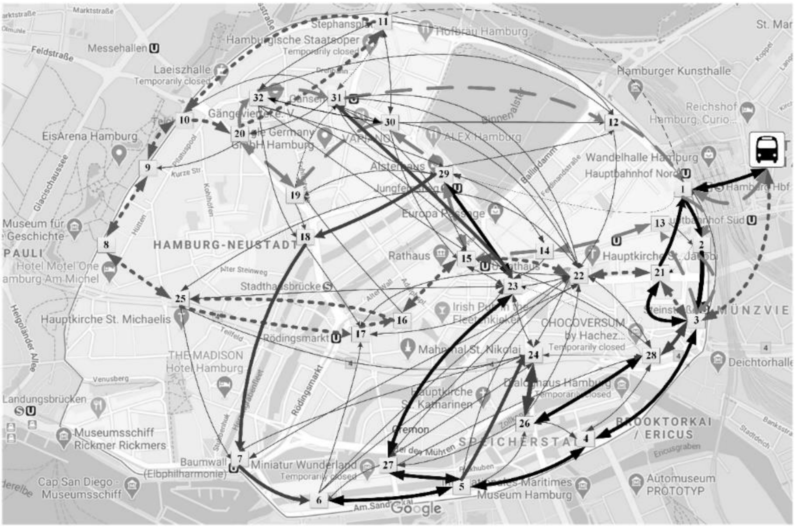

Figure 4: Graph of optimum assigned routes for three buses obtained from Sodachi et al. (2020)

They find three lines of travel for the transportation of buses in the city center. According to Figure 4, by assigning buses to these lines, the service level of passengers would be enhanced. According to the calculations, the waiting time of passengers decreased 20% and also environmental emissions was decreased largely accordingly. So, this model is worked properly both for the protection of the environment and also satisfying passengers. However, this model was not considered the propagation of bus delays in the downstream time points. The model just found the optimum routes for traveling in order to protect the environment and enhance service level in general. But time-delays in traveling specifically in some part-time of the day could differ situation for traveling. So, the proposed model could be remodeled based on dynamic conditions.

Arrival time at stops is developed by using a stochastic approach to predict bus travel time and propagation of bus delays in the determined lines in Figure 4. In each line, buses are traveling in processes that include travel time data correspond to periods. Generally, a bus starts its travel in a specific time-points and then finishes the travel in another specific time-points. The interval between successive time points is standard traveling time. Standard traveling time obtains from the difference between the bus door open time at the following stop and this door close time at the preceding stop. Table 2 demonstrate the standard time-points between stations and also the length of travel between these three lines continuously. In this Table, the depot is considered a place around the central rail station that from where buses are started their travels and then after the service in each line they will back again to this assumed location as the depot.

Table 2: The standard time-points between stations of each line (Blue, Black, and Red lines)

stations numbers (Blue line)	starting time-point	ending time-point	Length
Depot - 1	0	0.5	110
1-2	0.5	1.5	220
2-3	1.5	2.3	180
3-13	2.3	8.5	1600
13-14	8.5	10	400
14-15	10	11	300
15-29	11	12	260

stations numbers (Blue line)	starting time-point	ending time-point	Length
29-30	12	13	450
30-19	13	15	600
19-20	15	16	350
20-32	16	17	300
32-31	17	17.5	150
31-12	17.5	22	1300
12-1	22	24.5	500

stations numbers (Black Line)	starting time-point	ending time-point	Length
Depot - 1	0	0.5	110
1-2	0.5	1.5	220
2-3	1.5	2.3	180
3-4	2.3	5	1000

stations numbers (Blue line)	starting time-point	ending time-point	Length
4-5	5	6	220
5-27	6	7	230
27-23	7	11	1100
23-29	11	14	850
29-18	14	16	700
18-7	16	19	1100
7-6	19	21	550
6-5	21	22	450
5-24	22	24	700
24-26	24	25	180
26-28	25	26	450
28-3	26	27	450
3-21	27	28.5	400

stations numbers (Blue line)	starting time-point	ending time-point	Length
21-1	28.5	30	450
1 - Depot	30	30.5	110

stations numbers (Red line)	starting time-point	ending time-point	Length
Depot - 3	0	1.5	510
3-21	1.5	3	400
21-22	3	4	350
22-15	4	5.5	450
15-16	5.5	7	500
16-25	7	9.5	850
25-8	9.5	11	450
8-9	11	12	300
9-10	12	12.7	150

stations numbers (Red line)	starting time-point	ending time-point	Length
10-20	12.7	13.25	100
20-11	13.25	15	710
11-10	15	17	800
10-9	17	17.7	150
9-8	17.7	18.7	300
8-25	18.7	20	450
25-17	20	23	650
17-16	23	23.5	100
16-15	23.5	25	500
15-22	25	27	500
22-21	27	28	350
21-3	28	29.5	400
3 – Depot	29.5	31	510

For estimation of bus delay propagation in each station with the formulation of the Markov process, it is required to obtain transition probabilities of delay states between time points. As in the previous section mentioned, the number of observed delays is classified as early arrival or “E”, late arrival or “L”, and on-time arrival or “O”. By using the (19) and (20) relationships, the transition probabilities in each line are determined as shown in Table 3, Table 4, and Table 5, respectively. These transition probabilities are obtained based on the Maximum Likelihood Estimation between time points.

Table 3: The standard time-points between stations of line 2 (Black line)

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
E	0.5	0.5	0	0	0.5	Depot – 1
L	0	0.9003	0.0997			
O	0.007	0.112	0.881			
E	0.795	0.004	0.201	0.5	1.5	1-2
L	0.033	0.713	0.254			
O	0.02	0.11	0.87			
E	0.732	0.004	0.264	1.5	2.3	2-3
L	0.008	0.782	0.21			
O	0.025	0.11	0.865			
E	0.48	0.23	0.29	2.3	8.5	3-13
L	0	0.921	0.079			

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
O	0.048	0.42	0.532			
E	0.89	0.053	0.057	8.5	10	13-14
L	0.03	0.66	0.31			
O	0.06	0.21	0.73			
E	0.55	0.03	0.42	10	11	14-15
L	0.036	0.584	0.38			
O	0	0.189	0.811			
E	0.41	0.12	0.47	11	12	15-29
L	0	0.89	0.11			
O	0.04	0.35	0.61			
E	0.84	0.04	0.12	12	13	29-30
L	0.0231	0.6339	0.343			
O	0	0.22	0.78			
E	0.529	0.006	0.465	13	15	30-19
L	0	0.728	0.272			
O	0.11	0.3	0.59			

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
E	0.71	0.002	0.288	15	16	19-20
L	0.011	0.559	0.43			
O	0.026	0.28	0.694			
E	0.81	0	0.19	16	17	20-32
L	0.008	0.502	0.49			
O	0.013	0.267	0.72			
E	0.82	0.0303	0.1497	17	17.5	32-31
L	0.02	0.66	0.32			
O	0.05	0.22	0.73			
E	0.322	0.54	0.138	17.5	22	31-12
L	0	0.89	0.11			
O	0	0.743	0.257			
E	0.586	0.12	0.294	22	24.5	12-1
L	0.07	0.59	0.34			
O	0.13	0.26	0.61			
E	0.48	0.52	0	24.5	25	1 – Depot

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
L	0.05	0.81	0.14			
O	0.002	0.15	0.848			

According to Table 3, transition probabilities between stations of Line 1 show that when a bus starts its route on time, then it would with great possibility arrive on time on next downstream stations 1, 2, and 3, respectively. However, results show that this on-time arrival contradicts travel between stations 3 and 13. It means here the bus may encounter some traffic or route problems that the late arrival has more possibility. This situation repeats between stations 31 and 12. However, among other stations, there are not any critical issues and when a bus starts its travel according to the time plan then it would arrive at the next station with satisfaction planning.

Table 4: The standard time-points between stations of line 3 (Red line color)

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
E	0.5	0.5	0	0	0.5	Depot – 1
L	0	0.9003	0.0997			
O	0.007	0.112	0.881			
E	0.795	0.004	0.201	0.5	1.5	1-2
L	0.033	0.713	0.254			
O	0.02	0.11	0.87			

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
E	0.732	0.004	0.264	1.5	2.3	2-3
L	0.008	0.782	0.21			
O	0.025	0.11	0.865			
E	0.48	0.23	0.29	2.3	8.5	3-13
L	0	0.921	0.079			
O	0.048	0.42	0.532			
E	0.89	0.053	0.057	8.5	10	13-14
L	0.03	0.66	0.31			
O	0.06	0.21	0.73			
E	0.55	0.03	0.42	10	11	14-15
L	0.036	0.584	0.38			
O	0	0.189	0.811			
E	0.41	0.12	0.47	11	12	15-29
L	0	0.89	0.11			
O	0.04	0.35	0.61			
E	0.84	0.04	0.12	12	13	29-30

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
L	0.0231	0.6339	0.343			
O	0	0.22	0.78			
E	0.529	0.006	0.465	13	15	30-19
L	0	0.728	0.272			
O	0.11	0.3	0.59			
E	0.71	0.002	0.288	15	16	19-20
L	0.011	0.559	0.43			
O	0.026	0.28	0.694			
E	0.81	0	0.19	16	17	20-32
L	0.008	0.502	0.49			
O	0.013	0.267	0.72			
E	0.82	0.0303	0.1497	17	17.5	32-31
L	0.02	0.66	0.32			
O	0.05	0.22	0.73			
E	0.322	0.54	0.138	17.5	22	31-12
L	0	0.89	0.11			

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
O	0	0.743	0.257			
E	0.586	0.12	0.294	22	24.5	12-1
L	0.07	0.59	0.34			
O	0.13	0.26	0.61			
E	0.48	0.52	0	24.5	25	1 – Depot
L	0.05	0.81	0.14			
O	0.002	0.15	0.848			

Results of transition probabilities between stations in Table 4 shows that when a bus starts its route on time, then it would with great possibility arrive on time to next downstream stations 1, 2, 3, 4, 5, and also with about 0.6 probability arrive on time from station 5 to station 27. However, results show that this on-time arrival contradicts between stations 27 and 23 that just with 0.22 probability it arrives on time while it was departed on time. Similarly, it means here the bus may encounter some traffic or route problems that the late arrival has more possibility. This situation repeats between stations 18 and 7 and stations 3 and 21. Table 4 also shows late arrival somehow between stations 5 and 24 and stations 26 and 28 has a considerable possibility that could not be ignored. However, among other stations, there are not any critical issues and when a bus starts its travel according to the time plan then it would arrive at the next station with an expected suitable time scheduling.

Table 5:Transition probabilities between time-points of line 1 (Blue line)

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
E	0.562	0.104	0.334	0	1.5	Depot – 3
L	0.005	0.63	0.365			
O	0.025	0.44	0.535			
E	0.74	0.03	0.23	1.5	3	3—21
L	0.04	0.44	0.52			
O	0.08	0.28	0.64			
E	0.75	0.016	0.234	3	4	21—22
L	0.13	0.43	0.44			
O	0.016	0.34	0.644			
E	0.474	0.12	0.406	4	5.5	22—15
L	0	0.872	0.128			
O	0	0.38	0.62			
E	0.495	0.15	0.355	5.5	7	15—16
L	0	0.834	0.166			
O	0.04	0.24	0.72			

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
E	0.354	0.245	0.401	7	9.5	16—25
L	0	0.932	0.068			
O	0	0.634	0.366			
E	0.685	0.13	0.185	9.5	11	25—8
L	0.018	0.422	0.56			
O	0.08	0.3	0.675			
E	0.46	0.07	0.47	11	12	8—9
L	0.04	0.63	0.33			
O	0.013	0.27	0.717			
E	0.53	0.03	0.44	12	12.7	9—10
L	0.008	0.632	0.36			
O	0.007	0.3	0.693			
E	0.43	0.021	0.549	12.7	13.25	10—20
L	0.075	0.68	0.245			
O	0.072	0.2	0.728			
E	0.289	0.26	0.451	13.25	15	20—11

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
L	0	0.81	0.19			
O	0	0.53	0.47			
E	0.485	0.1	0.415	15	17	11—10
L	0.003	0.71	0.287			
O	0.01	0.27	0.72			
E	0.51	0.05	0.44	17	17.7	10—9
L	0.01	0.76	0.23			
O	0.088	0.211	0.701			
E	0.449	0.12	0.431	17.7	18.7	9—8
L	0.008	0.71	0.282			
O	0.029	0.301	0.67			
E	0.73	0.073	0.197	18.7	20	8—25
L	0.1	0.49	0.41			
O	0.025	0.245	0.73			
E	0.32	0.28	0.4	20	23	25—17
L	0	0.86	0.14			

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
O	0	0.66	0.34			
E	0.56	0.03	0.41	23	23.5	17—16
L	0.028	0.77	0.202			
O	0.01	0.3	0.69			
E	0.43	0.089	0.481	23.5	25	16—15
L	0	0.92	0.08			
O	0.03	0.29	0.68			
E	0.44	0.08	0.48	25	27	15—22
L	0.005	0.839	0.156			
O	0.03	0.35	0.62			
E	0.772	0.018	0.21	27	28	22—21
L	0.13	0.41	0.46			
O	0.041	0.319	0.64			
E	0.781	0.02	0.199	28	29.5	21—3
L	0.038	0.47	0.492			
O	0.026	0.374	0.6			

	E	L	O	Upstream Time-Point	Downstream Time-Point	Station
E	0.493	0.127	0.38	29.5	31	3-Depot
L	0.02	0.66	0.32			
O	0.007	0.49	0.503			

Finally, for the third line, the results of Table 5 show that when a bus starts its route planning according to the time plan, due to the route and also the possible traffic around the central rail stations the bus may arrive with substantial delay to the next station, station number 3. But the bus can continue its travel without any significant delay from station 3 to other stations till station 16, where between station 16 and station 25 there is much more possible considerable delay. In addition, between stations 25 and 17, and between stations 20 and 11 are possible delays that are encountering delays that are greater than on-time arrival.

For prediction of transition probabilities for each line of transportation for all mentioned time-points in transportation routes, Markov Chains based on both homogeneous and heterogeneous propagation of time delay between time-points is applied. First assuming homogeneous time delay propagation between time-points of Table 2 stations. Based on (20) the transition probabilities are as following:

$$P^{T2}_{r_{Depot}-r_{Depot}} = P_{r_{Depot}-r_1} \times P_{r_1-r_2} \times \ldots \times P_{r_{12}-r_1} \times P_{r_1-r_{Depot}}$$

Due to the homogeneous assumption all transition probabilities between two successive time-point are equal as follows:

$$P_{r_{Depot}-r_1} = P_{r_1-r_2} = P_{r_2-r_3} = P_{r_3-r_{13}} = \cdots = P_{r_{12}-r_1} = P_{r_1-r_{Depot}}$$

Therefore, the transition probability for this line is as following:

$$P^{T2}_{r_{Depot}-r_{Depot}} = (P_{r_{Depot}-r_1})^{15} = \begin{bmatrix} 0.5 & 0 & 0,5 \\ 0 & 0.9003 & 0.0997 \\ 0.007 & 0.112 & 0.881 \end{bmatrix}^{15}_{r_{Depot}-r_1}$$
$$= \begin{bmatrix} 0.006482 & 0.529496 & 0.464022 \\ 0.006221 & 0.539603 & 0.454176 \\ 0.006907 & 0.510373 & 0.48272 \end{bmatrix}$$

On the other side, if considering the propagation of delay between time-points is heterogenous, then the predicted delay for the line is computed as following:

$$P_{r_{Depot}-r_1} \neq P_{r_1-r_2} \neq P_{r_2-r_3} \neq P_{r_3-r_{13}} \neq \cdots \neq P_{r_{12}-r_1} \neq P_{r_1-r_{Depot}}$$

$$P^{T2}_{r_{Depot}-r_{Depot}} = \begin{bmatrix} 0.5 & 0 & 0,5 \\ 0 & 0.9003 & 0.0997 \\ 0.007 & 0.112 & 0.881 \end{bmatrix}_{r_1-r_2} \times \begin{bmatrix} 0.795 & 0.004 & 0.201 \\ 0.033 & 0.713 & 0.254 \\ 0.02 & 0.11 & 0.87 \end{bmatrix}_{r_2-r_3} \times \ldots$$
$$\times \begin{bmatrix} 0.48 & 0.52 & 0 \\ 0.05 & 0.81 & 0.14 \\ 0.002 & 0.15 & 0.848 \end{bmatrix}_{r_1-r_{Depot}}$$
$$= \begin{bmatrix} 0.073457 & 0.523654 & 0.402889 \\ 0.073121 & 0.523804 & 0.403075 \\ 0.073195 & 0.52377 & 0.403035 \end{bmatrix}$$

Similarly, for the two other lines as in Table 3 and Table 4, the computation of transition probabilities for the homogeneous assumption of propagation of delay between time-points are the same as (18) according to the following matrices.

$$P^{T3}_{r_{Depot}-r_{Depot}} = (P_{r_{Depot}-r_1})^{19} = \begin{bmatrix} 0.5 & 0 & 0,5 \\ 0 & 0.9003 & 0.0997 \\ 0.007 & 0.112 & 0.881 \end{bmatrix}^{19}_{r_{Depot}-r_1}$$
$$= \begin{bmatrix} 0.006516 & 0.527113 & 0.466372 \\ 0.006421 & 0.531071 & 0.462508 \\ 0.00669 & 0.519632 & 0,473678 \end{bmatrix}$$

$$P^{T4}_{r_{Depot}-r_{Depot}} = (P_{r_{Depot}-r_3})^{22} = \begin{bmatrix} 0.562 & 0.104 & 0,334 \\ 0,005 & 0.63 & 0.365 \\ 0,025 & 0.44 & 0.535 \end{bmatrix}^{22}_{r_{Depot}-r_3}$$
$$= \begin{bmatrix} 0.03109 & 0.530312 & 0.438598 \\ 0.031088 & 0.530314 & 0.438598 \\ 0.031088 & 0.530314 & 0.438598 \end{bmatrix}$$

And accordingly, while the propagation of time delay between time-points is heterogeneous, then based on (7) the transition probabilities for the line are as follows, respectively:

$$P^{T3}_{r_{Depot}-r_{Depot}} = \begin{bmatrix} 0.5 & 0 & 0,5 \\ 0 & 0.9003 & 0.0997 \\ 0.007 & 0.112 & 0.881 \end{bmatrix}_{r_{Depot}-r_1} \times \begin{bmatrix} 0.795 & 0.004 & 0.201 \\ 0.033 & 0.713 & 0.254 \\ 0.02 & 0.11 & 0.87 \end{bmatrix}_{r_1-r_2} \times \ldots \times \begin{bmatrix} 0.48 & 0.52 & 0 \\ 0.05 & 0.81 & 0.14 \\ 0.002 & 0.15 & 0.848 \end{bmatrix}_{r_1-r_{Depot}} = \begin{bmatrix} 0,083427 & 0,551018 & 0,365554 \\ 083427 & 0,551018 & 0,365554 \\ 083427 & 0,551018 & 0,365554 \end{bmatrix}$$

$$P^{T4}_{r_{Depot}-r_{Depot}} = \begin{bmatrix} 0.562 & 0.104 & 0,334 \\ 0,005 & 0.63 & 0.365 \\ 0.025 & 0.44 & 0.535 \end{bmatrix}_{r_{Depot}-r_3} \times \begin{bmatrix} 0.74 & 0.03 & 0.23 \\ 0.04 & 0.44 & 0.52 \\ 0.08 & 0.28 & 0.64 \end{bmatrix}_{r_3-r_{21}} \times \ldots \times \begin{bmatrix} 0.493 & 0.127 & 0,38 \\ 0.02 & 0.66 & 0.32 \\ 0.007 & 0.49 & 0.503 \end{bmatrix}_{r_3-r_{Depot}} = \begin{bmatrix} 0,070961 & 0,515553 & 0,426346 \\ 0,070845 & 0,514709 & 0,425648 \\ 0,070851 & 0,514755 & 0,425686 \end{bmatrix}$$

5 Conclusions

The efficient public transportation system in large cities is significantly contributed to the inhabitants' quality of life. Having a public transportation system that works properly based on dynamic planning, and also have a cost-effective program encourages more and more people to use them for their daily traveling. By having a dynamic public transportation system, not only does the service level of passengers increase, but also most criteria of sustainability assessment as environment, society, and economic perspectives would be observed. From the previous paper, (Sodachi et al., 2020), it is concluded that proper management of public transportation creates paradoxical challenges for fulfilling the mutual service level satisfaction for the passengers and also the environmental perspective considerations. According to the Vehicle Routing Problem technique and also proposing a multi-objective optimization planning model, the best possible routes are obtained that simultaneously regarding service level of users and reduced emissions. This paper has also studied the obtained results and discussed them for predicting possible delays among stations. As the delay in a station propagates itself to other downstream stations, it could bring greater delays to other stations that cause the bus transport system encounter more disorders. So, by considering the Markov Chains, the paper defined three possible states for the arrival of buses in stations. Then the transition probabilities for every two successive stations are achieved through the Maximum Likelihood Estimation. Therefore, with Markov Chains, all possible delays in the available stations of each line of the bus transport system in Hamburg city center can be predicted and a dynamic bus transport system scheduling program can be proposed to satisfy users accordingly.

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