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EXPERIMENTAL INVESTIGATION OF THE INERTIA AND DAMPING COEFFICIENTS OF A SPHEROID AND SURFACE SHIP IN FREE HEAVE

by

G. WEINBLUM, S. BROOKS and P. GOLOVATO

Abstract

Investigation is made at the David Taylor Model Basin of the damping and added mass coefficients of a 7 to 1 prolate spheroid subjected to small free-heaving oscillations near the surface. Tests were performed at various depths from surfaced to deep submergence, over a range of Froude numbers, and for a limited frequency range. Similar tests were performed for a model of a surfaced aircraft carrier of 0.6 prismatic at several displacements.

A detailed evaluation is made of previous work in the field of near-surface oscillations. Although of limited frequency range, the results of the present tests show reasonable agreement with available theoretical results. Tests for surface ship models using forced-oscillation techniques have been performed recently by the third author [20].

INTRODUCTION

Purpose

The purpose of the present report is to furnish data primarily needed in the theory of ship motions in a seaway; in addition, some results may be useful in treating problems of directional stability, steering, etc.

Historical background

The determination of the path of a body moving at or near the free calm surface under the influence of given extraneous forces leads to a complicated *boundary* problem. The problem becomes still more complicated when the body moves in waves [1, 2].

Krylov [3] made a basic contribution to the theory of oscillations of surface ships, but the sovereign omission of hydrodynamic considerations limits its immediate application. However, the practical value of his basic concept is continuously being increased by the introduction of hydrodynamic concepts and relations. These have led, among other things, to a determination of the coefficients in the linear differential equations of oscillations, especially the added masses and damping values.

Haskind has developed a theory of motions of surface ships in a regular seaway based on the source and sink concept [4, 5]. He claims that his method leads to a general solution of the hydrodynamic problems involved. However, this is not the case; essentially it yields information on the added mass and damping and on some coupling terms, and while extremely interesting, the paper is not more than a valuable hydrodynamic contribution to Krylov's basic concept.

The senior author has applied Krylov's reasoning to problems of completely submerged bodies and has shown that in this case hydrodynamic effects must be considered when determining exciting forces. Further, extensive work has been published on the influence of the free surface on added masses [2, 6, 24].

Havelock has computed the damping coefficients in heave and pitch for a submerged spheroid at zero speed and compared these results to the two-dimensional solution commonly in use [7]. Also, he has derived the dependence of the pitch damping on forward speed for a long, narrow, floating plank [8].

The flow around bodies (ships) moving in the vicinity of a free surface is commonly pictured by sources and sinks derived for a deeply submerged condition. However, this method fails to disclose a variability of the added masses with Froude numbers and frequency parameters, and therefore a second approximation becomes necessary. Conditions are simpler when dealing with damping. Here the first approximation yields useful results.

It is important to note that the calculated wave damping given by the aforementioned theory leads to a linear damping term. This is obviously due to the linear character of the hydrodynamic theory.

Mention should also be made of an interesting effect which Kochin, stimulated by the work of Bjerknes, calculated for the first time. Kochin [9] has shown that a translational oscillation below a free surface leads to a nonzero vertical average force whose magnitude and direction depend upon the frequency of oscillation. This solution is valid only for zero speed of advance. Attention is drawn to this little known effect since it may become necessary to consider it when more elaborate investigations of forced oscillations will be initiated.

Plan of the study

At the present state of the hydrodynamic theory, experimental evidence is valuable.

The theory of seaworthiness of surface ships and submerged bodies has to rely upon the study of free and forced oscillations. At present, there is a tendency in theoretical naval architecture to devote more attention to forced effects. This trend is enhanced by similar development in the hydrodynamics of the free surface.

In designing a facility, which is briefly described later, provisions were made for the investigation of free and forced oscillations. The present study was made before the "forced part" of the facility was completed. Our experiments have been initiated to some extent in an accidental way. We intended at first to establish a technique for determining the periods and the damping of free oscillations as an indispensable part of model investigations in a seaway. During preliminary tests however, it appeared that more general solutions could be reached by the free-oscillation method, notwithstanding its inherent limitations. Therefore the decision was made to extend the original program, especially because it seemed to be questionable at that time whether the "forced part" of the facility would be built at all.

This decision involved some difficulties. First, we had to use a temporary recorder which did not meet the requirements for systematic work. This caused some restrictions in planning and a slight loss in the accuracy of results obtained. Secondly, although free oscillations are the simpler problem from the viewpoint of experimentation as well as from that of general mechanics, the hydrodynamic aspect presents more complications than in the case of forced oscillations. Therefore we have to interpret our results by theoretical considerations which are not strictly applicable to our case.

We investigated experimentally the free-heaving oscillations of a spheroid and of a surface-ship model. The experiments yielded added mass and damping values for various speeds of advance, for some frequencies, and, in the case of the spheroid, for several depths of immersion, or, better, for some dimensionless parameters derived from these values.

For purposes of seaworthiness studies, no high accuracy is needed in the final results for the damping and added mass values.

Because of the simplifications admitted when defining these magnitudes in a seaway, the results have to some extent a qualitative meaning only. Conditions can be different when using these concepts for the computation of the path in calm water. Here, in addition, there is the question of how results obtained from the oscillator can be applied to path problems.

The internal damping of the facility turned out to be higher than was expected, and this fact impairs some results obtained for the deepest immersion of the spheroid.

THEORETICAL CONSIDERATIONS

Validity of the method of free oscillations

Inertia coefficients as well as damping values can be different when derived from forced and from free oscillations since the flow patterns are different. It was believed, however, that an approximate agreement between the values of inertia and damping coefficient for free and forced oscillations would be obtained by employing results obtained at corresponding values of certain characteristic parameters. We shall define these characteristic parameters in the following section.

It has been mentioned already that *free* oscillations of a body close to the free surface present a more difficult hydrodynamic problem than *forced* oscillations. In the latter case, the frequency has a known constant value. With free oscillations, conditions are much more complex. Few results are available so far.

However, there is a simple practical aspect of the problem. Although the theoretical solutions lead in principle to cumbersome expressions, we have tried to analyze our experimental results using the simple formulations presented by the theory of damped harmonic oscillations with viscous damping. This procedure is suggested by the inspection of the experimental extinction curves obtained; these agree well with the familiar pattern corresponding to damped linear harmonic oscillations, at least for small and moderate damping. The concept of constant periods and of well-known damping coefficients like the logarithmic decrement can be retained.

Further, since there is a close relation between the simple theory of free and forced harmonic oscillations, the following crude working hypothesis will be adopted: As a first orientation, basic parameters of the hydrodynamic problem will be borrowed from the theory of forced oscillations performed by a body in an ideal liquid close to the free surface; in addition, it will be assumed that some quantitative information on free oscillations can be obtained by considering them as forced.

Dimensionless parameters

We investigated immersions h beginning with $h = 0$, the floating condition at the surface, down to a depth where the surface effects become small or even negligible. The definition of h for a body of revolution follows from Fig. 1. The familiar geometric dimensionless parameter h/d is used throughout this report, although from theoretical considerations the ratio h/l is preferable where l is length of body.

In the theory of forced oscillations, the dependence of the inertial and damping coefficients upon the depth of immersion is analyzed using the dimensionless frequency parameter $\Omega_h = \omega\sqrt{h/g}$.

For h constant, one obtains appropriate frequency parameters: For elongated bodies of revolution $\Omega_d = \omega\sqrt{d/g}$, and for surface ships $\Omega_B = \omega\sqrt{B/g}$ where B is the beam. The Froude number, $F = V/\sqrt{gl}$ appears as a parameter when the body is

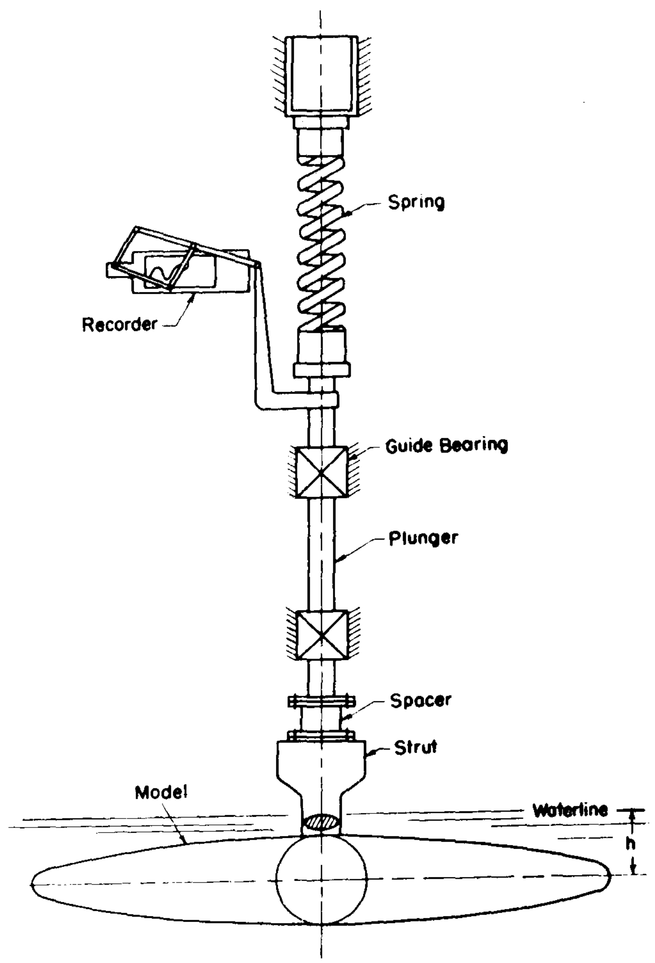

Fig. 1. *Experimental Setup*

advancing with a uniform speed V. Another parameter which is frequently more significant is $V/c = (2\pi V)/gT$, where $c = g/\omega = gT/2\pi$ is the celerity of the free plane wave corresponding to a frequency ω and period T. At a critical value, $V/c = 1/4$, the wave pattern generated by the oscillations changes markedly. To wit, when $V/c < 1/4$, the waves spread in all directions, whereas this becomes impossible for $V/c > 1/4$ [10].

Thus five characteristic parameters h/d, Ω_h, F, V/c and Ω_d or Ω_B can be used in analyzing our experimental data with respect to free surface effects. Of these, only three are independent.

The range of variation of Ω_d and Ω_B investigated so far is not large. It is clearly easier to vary Ω_d and Ω_B by the method of forced oscillations.

Average lift due to free surface

The magnitude of hydrodynamic forces and moments experienced by a body moving uniformly in a horizontal plane at or near the surface has been investigated rather extensively. We treat it here briefly, because the effects involved influence the evaluation of the extinction curve.

By running the non-oscillating model at various Froude numbers, we measured linear or angular displacements which correspond to a new equilibrium position under the influence of the hydrodynamic force or moment. It was assumed that this position coincides with the mean position of the oscillating body. From what has been said before on the average oscillatory force, it follows that such an assumption is not strictly true. However, no corrections have been applied since the accuracy of our present recording is such that we cannot introduce corrections based on experiments, and theoretical solutions are not sufficiently developed.

Analysis of the linearized equations of free heave

The formalism used in analyzing our experiments is extremely elementary. We intentionally restricted ourselves to motions with one degree of freedom. We have to determine the coefficients in the differential equation of free heave:

$$m^* \ddot{z} + 2N \dot{z} + C' z = 0 \ldots\ldots \quad (1)$$

i.e. to establish the dependence of the inertia factor m^* and damping factor $2N$ on variables like the frequency of oscillation, speed of advance, and depth of immersion. These factors will be defined below in dimensionless form. It will further appear that an investigation of the restoring factor C' becomes indispensable. In Equation (1) $m^* = m + \tilde{m}_z$ is the *apparent* mass where m is the mass of the model and $\tilde{m}_z$ is the "added" hydrodynamic value. We denote by $m_0 = \varrho \cdot V$ the mass of the displaced water, and put $\tilde{k}_z = \tilde{m}_z/m_0$. The tilde indicates that the inertia factors are valid for free surface conditions [6].

Assuming an initial displacement z_1, zero initial velocity, and less than critical damping, we obtain the "extinction curve":

$$z = z_1\, e^{-nt} \left[\cos \omega t + \frac{n}{\omega} \sin \omega t\right] \ldots \quad (2)$$

where:

$$2n = \frac{2N}{m^*} \quad \ldots\ldots \quad (3)$$

and:

$$\omega^2 = \frac{C'}{m^*} - n^2 \quad \ldots\ldots \quad (4)$$

We introduce the definitions:

$$T = \frac{2\pi}{\omega} \text{ period of damped motion} \quad \ldots \quad (5)$$

$$2\delta = 2n\sqrt{m^*/C'}$$

dimensionless damping coefficient .. (6)

As long as the damping is not too strong, say $\delta \leqq 0.2$, we let:

$$2\delta = \frac{nT}{\pi} \quad \text{and} \quad 2N = \frac{C'}{2\pi^2}\, n\, T^2$$

Higher accuracy is needed in the evaluation of the masses. From Equations [4], [5] and [6]:

$$m^* = m + \tilde{m}_z = \frac{T^2\, C'\, (1 - \delta^2)}{4\pi^2} \ldots \quad (7)$$

when $\delta^2 \leqq 0.005$ or $\delta \leqq 0.07$:

$$\tilde{m}_z = m^* - m = \frac{T^2\, C'}{4\pi^2} - m \quad \ldots \quad (8)$$

2δ suffers from a serious deficiency since it is inherently mass dependent, $2\delta = (2N)/(m^*\omega)$. During the course of the experiments, the mass of the model may have varied from 5 to 10 per cent. and the added mass fluctuates greatly at depths shallower than $h/d = 1.5$. Some scatter in a 2δ plot might be due to this fact.

The damping coefficient $2N$, suitably non-dimensionalized, appears to be a logical choice as a measure of the damping since for fixed frequency, Froude number, and submergence, it should depend on the geometry alone. Also it allows a direct comparison with results of forced oscillation tests. In this paper, both 2δ and $2N$ will be reported.

Restoring force coefficient

At zero and low speeds of advance, hydrodynamic effects are negligible; thus one can put:

$$C' = C + C_0 = C + \varrho g A \ldots\ldots \quad (9)$$

Here C is the spring constant and $\varrho g A$ the buoyant force per unit immersion, with A representing a waterline area. In the case of a surface ship for example A is the area of the load waterline A_w, and when dealing with a submerged body, A is the cross section of the strut A_s. In the latter case, $\varrho g A$ is generally negligible compared with C except for soft springs and large struts. Large struts must be used for large depths of immersion.

As assumed in the linear theory of ship oscillations, A_w (and A_s) must be sensibly constant within the range of immersion, i.e., $(\partial A_w)/\partial z = 0$.

At higher speeds of advance, the influence of hydrodynamic effects on the restoring force and moment must be considered. This problem is interesting from a general point of view, quite apart from our present investigation.

In the case of the heaving motion, the surface ship experiences a downward suction force $\bar{Z}$ (positive lift occurs only at planing speeds beyond the range of Froude numbers tested) which for a given body is a function of the Froude number F and the draft H, $Z = Z(F, H)$. For the spheroid, the function $Z(F, H)$ can be estimated in the range of smaller values of F [11], but generally it must be found from experiments on the oscillator by towing the body at different speeds and drafts. Thus a set of curves $Z(F, H)$ can be obtained, and therefrom:

$$\frac{\partial Z\,(F, H)}{\partial\, H} = \frac{\partial Z\,(F, H_0 + z)}{\partial z}$$

where H_0 is the initial draft at rest.

$Z\ (F, h)$ is similarly obtained for the submerged body and:

$$\frac{\partial Z\ (F, h)}{\partial h} = \frac{\partial Z\ (F, h_0 + z)}{\partial z}$$

where, again, h_0 is the depth of immersion at rest. In the case of the submerged body, $[\partial \bar{Z}\ (F, h_0 + z)]/\partial z$ will generally be negligible, except when the body is very close to the surface; here, a breakdown of the flow may occur which complicates the reasoning.

Compared with the spheroid near the surface, the derivative $[\partial Z\ (F, H)]/\partial z$ for the surface ship model was relatively small in the cases tested. We have therefore not applied any hydrodynamic correction to the restoring coefficient except for the case of the spheroid near the surface. It is emphasized, however, that the problem should be reconsidered when a better recorder has been installed.

Damping effects

We are primarily interested in wave effects. In addition there clearly exists a "viscous" damping of oscillation. The designation "viscous" includes effects due to skin friction (shear stresses) and to the generation of vortex sheets. At high Froude numbers, the omission of viscous effects may lead to errors, as will follow from our experiments and can be demonstrated by the following estimate.

We assume that our body is deeply submerged. Skin friction is neglected. We assume further that towed with a uniform speed V under a small angle of attack a this body experiences a lift force $L = C_L'\ (\varrho/2)\ AV^2 a$ where $C_L' = \partial C_L/\partial a$ and A is an area of reference, say that of a maximum waterline (horizontal section). For the spheroid, $A = (\pi\ dl)/4$.

C_L' is known for some bodies of revolution and some ship models. Assuming now that the "steady state" value C_L' is applicable to a heaving motion, we obtain the well-known solution for the damping force:

$$F_D = C_L' \frac{\varrho}{2} AV \dot{z} \quad \text{and} \quad 2N = C_L' \frac{\varrho}{2} AV$$

For a body of neutral buoyancy:

$$2n = \frac{C_L'}{2} \frac{A}{(1 + k_z)\ \mathcal{V}} V \quad \ldots . \quad (10)$$

For the spheroid:

$$2n = \frac{3}{4\,(1 + k_z)}\, C_L{}' \,\frac{V}{d} \quad \ldots\ldots \; (11)$$

We non-dimensionalize $2n$ by dividing it by V/l:

$$\frac{2nl}{V} = \frac{C_L{}'}{2\,(1 + k_z)}\,\frac{Al}{V} \quad \ldots\ldots \; (12)$$

or for the spheroid:

$$\frac{2nl}{V} = \frac{3}{4\,(1 + k_z)}\, C_L{}' \,\frac{l}{} \quad \ldots\ldots \; (13)$$

It will be shown later in discussing damping in heave (see Fig. 10) that results obtained from this formula by inserting the appropriate values are significant and agree as to the order of magnitude with experimental findings, provided the Strouhal number $\omega l/V$ is small. This condition is necessary for the applicability of the analysis which holds only for small angles of incidence, since, at constant amplitude, $\dot{z}$ is proportional to ω. When the Strouhal number is large, some other expression for the damping may be introduced, for example $F_D = C_D\,(\varrho/2)\,A\dot{z}^2$. This destroys the linearity of the damping term. We shall not persue the idea further.

DESCRIPTION OF EXPERIMENTS

The heave oscillator

The tests employed a heave oscillator which has been developed for investigating free and forced oscillations of surface ship models and submerged bodies. In its present form it is a free vibrating system of one degree of freedom.

The instrument consists of a vertical plunger and a recorder. The plunger is suspended from a coil spring clamped at each end by inner and outer cylinders in which the helix of the spring has been cut. Clamping is secured by screwing the outer cylinder down onto a projecting flange of the inner cylinder; see Fig. 1. The recorder is a simple paper drive upon which a ball point pen traces the vertical movement of the plunger. The pen is actuated by means of a pantograph linkage connected to the plunger.

The motion of the plunger is limited to the true vertical and held in lateral alignment by splines riding on ball rollers. Motion is induced by manual compression or extension of the spring and a quick release mechanism. Control of the frequency of oscillation of the plunger is limited by the natural frequencies of the four springs available for use with this instrument. Input amplitude is controlled by varying the initial displacement of the springs when compressed or extended.

This instrument can be adapted to test surface or subsurface models by the use of various combinations of struts and spacers.

A heaving test requires one strut to be rigidly connected to the model and to the plunger. This strut is usually centered over the center of buoyancy. Surface and subsurface models and all component parts of the oscillating system are carefully weighed to determine the true mass of the system.

The test work performed in conjunction with this report has necessitated frequent changes of the springs. The method of locking the springs in the spring caps allows a slight variation in active coil length whenever the springs are changed. To insure that the proper spring factor is used for each test, a calibration was conducted at each spring change by placing calibrated weights on a weight pan built in the plunger and then recording the deflections. These deflections were measured from the tape and the spring factor calculated. To prove the accuracy of this method and the repeatability of the instrument, concurrent calibrations were conducted by measuring the plunger deflections with a machinist's dial indicator graduated in 0.001-inch intervals. The indicator and tape readings have agreed to within 1 per cent. and repeatability is positive.

The models

The experimental data were obtained from tests on a surface and on a subsurface model. The experiments with the subsurface model are a continuing study on a 7 to 1 prolate spheroid 9 feet long.

Surface data were obtained from experiments on an available model whose dimensions were suitable for use in the heave oscillator. It represented a proposed aircraft carrier (see Fig. 2) and had a length of 10.73 feet, a beam of 1.354 feet, and a displacement of 198 pounds at a draft of 0.363 foot. The block coefficient was 0.589 and the prismatic coefficient 0.605. This model had an extremely flat stern bounded by two skegs which protruded quite far aft. The center of flotation was approximately 0.56 of the length aft of the forward perpendicular.

General remarks on the experiments

In the deeply immersed condition, the spring is clearly an indispensable part of the experimental setup.

Models floating on the surface, however, experience restoring forces due to changes of buoyancy. Therefore, in principle, such models can be tested without a spring so that the experiments reflect the natural conditions. However, the damping is generally so strong that the extinction curves die out very quickly and it is rather difficult to evaluate properly the period and the effect of the added mass. By introducing an additional spring force, one is enabled to find the period and the added mass

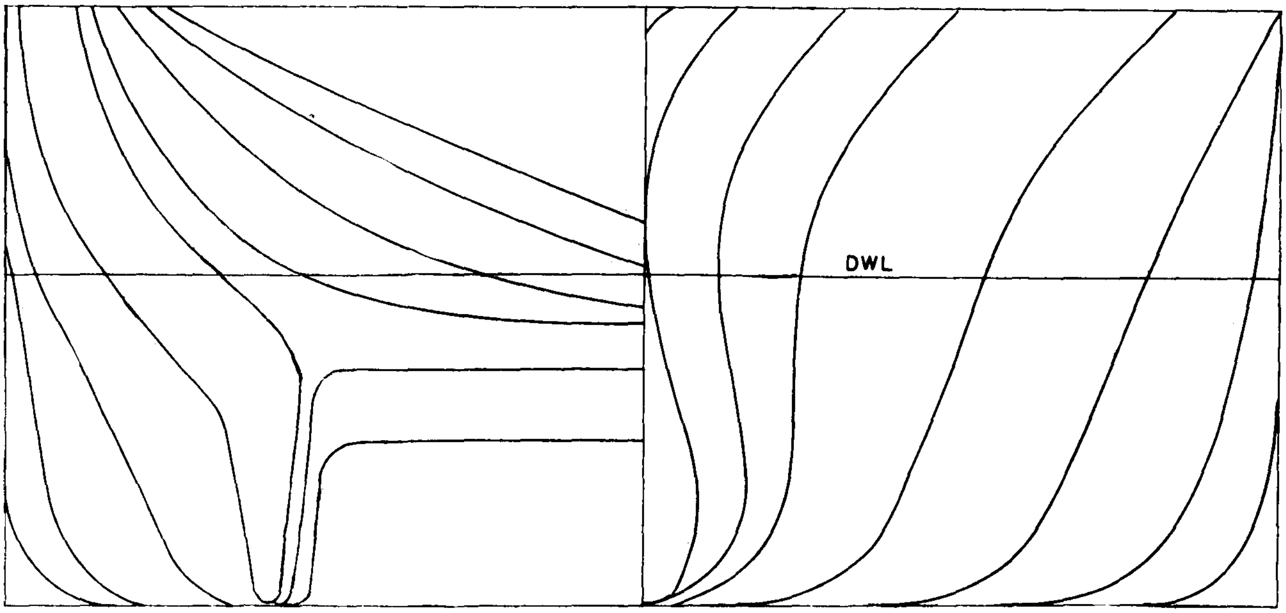

Fig. 2. *Body Plan of Surface Model*

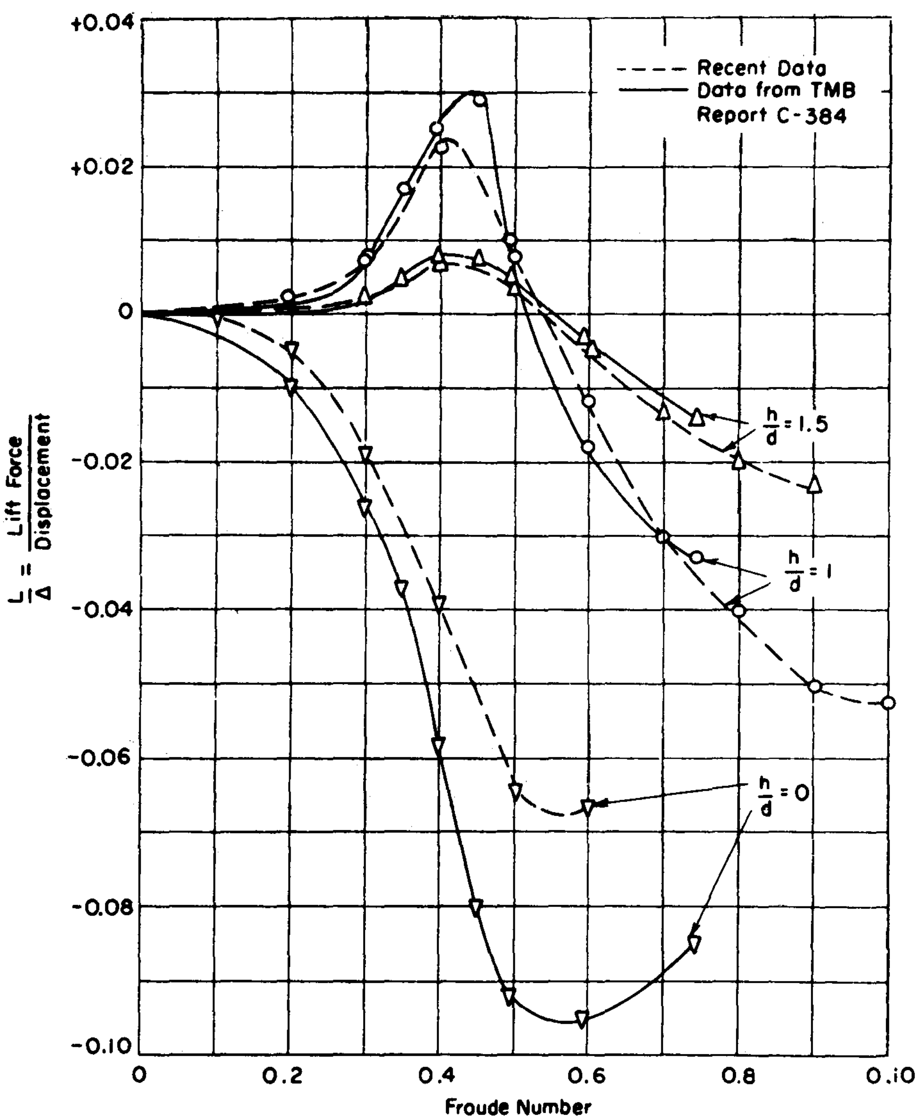

Fig. 3. *Lift Force Versus Froude Number for Spheroid*

with reasonable accuracy. Obviously, the values so obtained do not correspond to the natural frequency of the full-size ship. By changing the stiffness of the spring and the masses, variations in frequencies $\omega = 2\pi/T$ can be obtained. This was done in several cases, but as has been pointed out before, no comprehensive investigations on the dependence of free-surface effects upon Ω_d or Ω_B have been made since such variations can be performed much easier by forced oscillations. Therefore the results presented are meagre and can serve only as a first orientation.

Since our analysis is based on linear differential equations, it is essential to keep the linear displacements small. However, comprehensive treatment of free-surface effects requires the investigation of several initial displacements to establish deviations from the linear law. Such experiments were frequently performed. It is thought that this amplitude dependency should be investigated later when the facility is completed. In the present report we confined ourselves to the average of the results obtained.

The proper way to perform heaving tests clearly requires correcting before each run for the comparatively small linear displacements which a body in uniform motion near a free surface would incur. The desired mean position of the body when advancing and oscillating is thus approximately obtained. Within the limits of the accuracy attainable at present, such a refinement did not appear to be necessary.

The hydrodynamic force can be calculated (Fig. 3) from the deflection of the spring. A comparison with former results [12] showed a reasonable general agreement in the case of submerged bodies; this is all that can be expected from the present records.

Some difficulties arise in the interpretation of results for surface conditions. Assume first that the vertical force is measured when the model is kept in a fixed vertical position. This gives a true value of the hydrodynamic suction force as differentiated from the buoyancy force only as long as the average level of the water around the model coincides with the original calm water surface, i.e., at lower Froude numbers up to slightly below $F = 0.3$, depending upon the shape of the body. At higher Froude numbers, the vertical force measured in such a way presents a purely qualitative value. Using a spring, a fixed condition can be approximated by lifting the model before each run by such an amount that the initial draft is maintained underway.

When the model is free to perform a vertical motion, obviously no force measurement is possible. One can only register the bodily sinkage of the model.

Using a spring, some intermediate condition between the fixed and free position is reached. Again, the force calculated from the spring elongation can be interpreted properly only at low Froude numbers. The fixed condition can be approximated by lifting the model before the run by an appropriate amount.

Experimental procedures

Heaving tests on a submerged body were usually conducted according to the following procedure. The model was carefully aligned to assure that the angles of yaw and pitch were zero; it was then submerged to a definite depth. A static (non-oscillating) run was made to establish the mean position of the oscillating body due to the surface influence for the various speeds of advance at which the model was to be tested. This change in vertical displacement indicated the lift force at the vertical position assumed. The procedure was followed for tests at the several basic depths of submergence.

Surface models were similarly tested. Towards the end of the experimental work for this project, however, a more accurate measure of the lift force was obtained by removing the spring from the system and restraining the model to its initial position by weights. As the lift force varied, the model was returned to its original position by adding or removing known weights. The model was tested at several displacements and various speeds of advance.

Data for determining damping and added mass in heave were obtained from the records of the models oscillating vertically at the various depths and speeds of advance. In addition the input amplitude was varied to study the resulting effects, and different springs were employed to study the effects of the change in frequency.

Analysis of the data

For damped free vibrations, the amplitudes of any two consecutive maxima decreases from $z_k = z_0 e^{-nt}$ to $z_{k+1} = z_0 e^{-n(t+T)}$; the ratio of any two consecutive maxima is a constant e^{-nT}. Thus $\log (z_{k+1}/z_k) = -nT$.

The maximum amplitudes for all heaving tests were plotted on semilog paper and a straight line faired through the points. When multiplied by T/π, the slope of this line, n, yields the damping coefficients 2δ.

The damping coefficient $2N$ can be computed from a knowledge of the slope n and the period T since $2N = (C'/2\pi^2)\, nT^2$.

EXPERIMENTAL RESULTS

Spheroid

Lift Force Due to the Free Surface

Non-oscillating runs were made at various depths of submergence and speeds primarily to determine the mean position of the oscillating body in order to allow proper analysis of the extinction curve in heaving motion. However, the resulting spring extension allows the computation of the vertical force due to uniform motion of the spheroid below the free surface.

This force, non-dimensionalized with respect to the total displacement, is plotted in Fig. 3 against Froude number for various submergences. The general trend of the curves agrees with the results obtained earlier on a lift balance (Reference 12, Fig. 10). Several curves from this reference are plotted in Fig. 3 for purposes of comparison.

Damping in Heave

Heaving tests were run for a range of submergences and speeds using 80, 55, 28 and 14 pounds per inch springs. Initial displacements 3/4 to 3 inches in either direction from the zero speed equilibrium position of the model were used. Initial amplitude effects were not significant so that the periods and damping values were averaged from tests of varying initial displacements but otherwise similar conditions. These averaged data points and appropriate faired curves for all heaving tests of the spheroid appear in Figs. 4 and 5 where 2δ and $2N' = [2N/(\Delta/\sqrt{gl})]$ are respectively plotted against Ω_2 for constant h/d and Froude number.

Also plotted in Figs. 4 and 5 are curves of 2δ and $2N'$ respectively as a function of frequency from heaving tests in air at frequencies and amplitudes corresponding to those used in the heaving tests in water. In these figures the curve marked "instrument damping" must represent the frictional

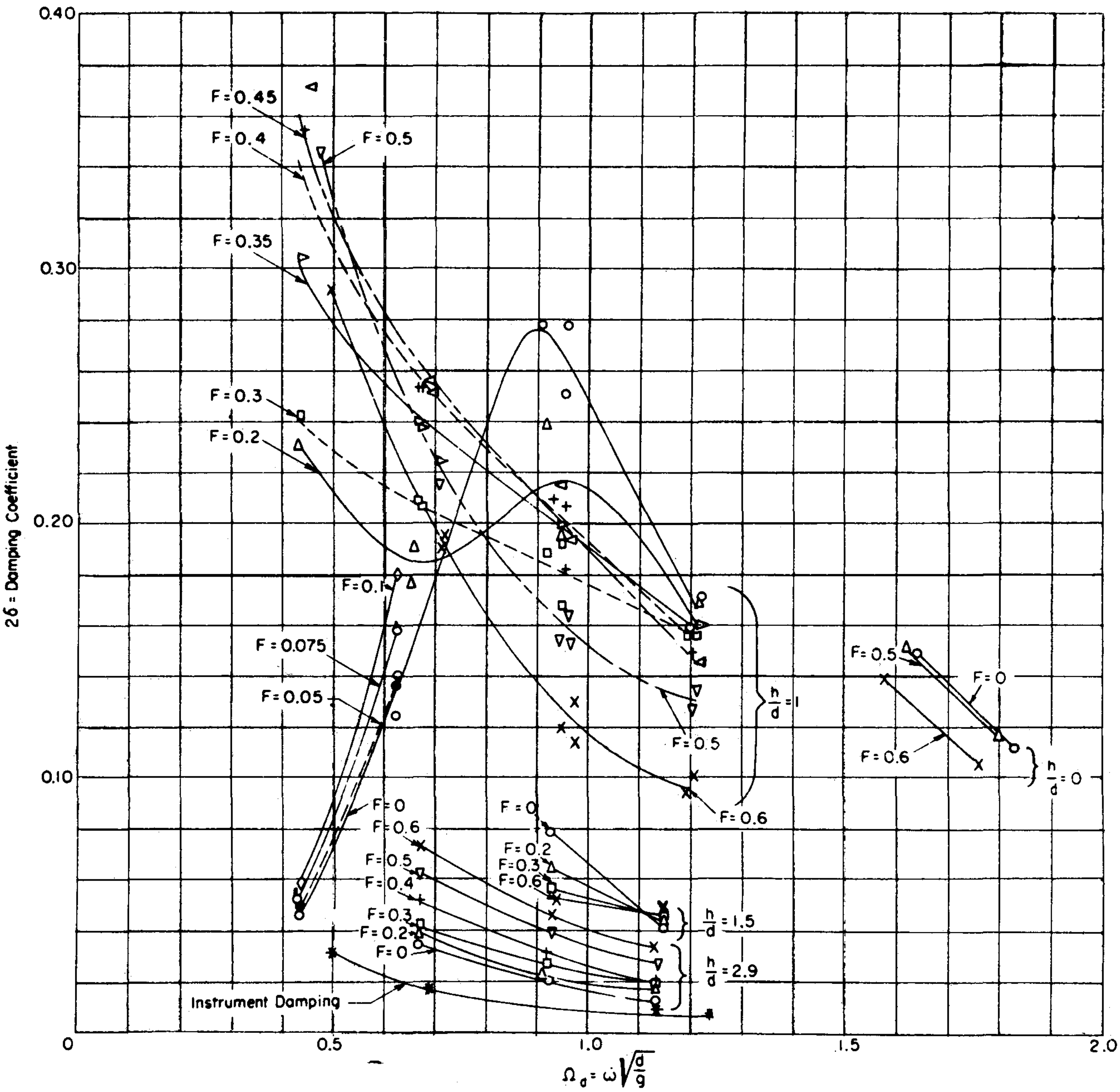

Fig. 4. *Damping Coefficient 2δ Versus Ω_d for Constant h/d and F for Spheroid*

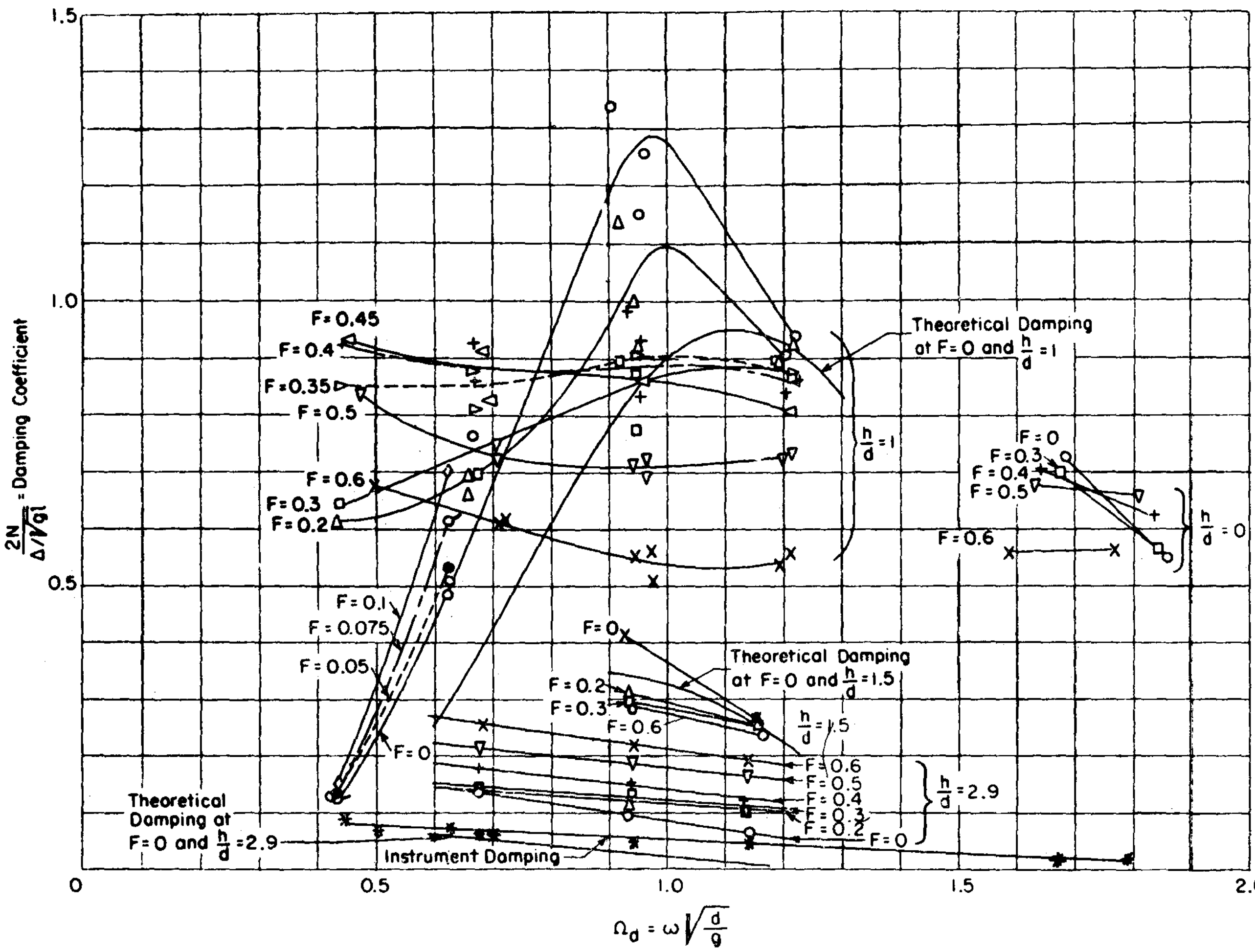

Fig. 5. *Damping Coefficient 2N′ Versus Ω_d for Constant h/d and F for Spheroid*

damping of the instrument, and it can be seen that it is a considerable portion of the total damping measured at comparatively deep submergence $h/d = 2.9$.

If the assumption is made that the damping coefficient $2N$ is a function of frequency, forward speed, and depth alone, then the value of $2N$ in air may be subtracted from $2N$ at lesser depths for corresponding frequencies and speeds in order to obtain the hydrodynamic damping. The logarithmic decrements cannot be subtracted in this manner since it is mass dependent and the added mass varies considerably from shallow to deep submergence.

A strip method based on Havelock's work has been used to determine theoretically the wave damping for a body performing small oscillations below a free surface at zero forward speed.

Assume a cylinder of radius r with its axis horizontal and submerged a distance h below the free surface. The cylinder is performing small heaving oscillations $z_0 \sin \omega t$. It will be assumed that the cylinder may be replaced by a doublet with vertical axis whose strength is oscillating, the instantaneous magnitude corresponding to the strength necessary to generate a cylinder of radius r in an infinite fluid with instantaneous velocity $z_0\omega \cos \omega t$.

The amplitude of the regular waves generated by a submerged oscillating doublet is given [13] as:

$$a = 2\pi\, z_0\, r^2\, k_0^2\, e^{-k_0 h} \quad \ldots\ldots \quad (14)$$

where $k_0 = \omega^2/g$.

This amplitude is precisely the same as that for a swaying cylinder (Equation (12), Reference 13).

Under assumptions of forced simple harmonic vibration and small radiated energy, Havelock [14] suggests finding the damping coefficient by equating the mean rate of propagation of energy outwards in the wave motion to the mean value of work done by the damping force. The mean rate of propagation of energy outwards per unit length is found to be:

$$E' = \frac{g^2\, \varrho a^2}{2\omega} = 2\pi^2\, \frac{\varrho\omega^7\, r^4\, z_0^2}{g^2} e^{-2k_0 h} \quad (15)$$

Using a strip method, the rate of total energy propagation E can be computed by integrating E' over the length of the spheroid $2a$:

$$E = \frac{32\pi^2\, b^4\, a\varrho\omega^7\, z_0^2}{15\, g^2}\, e^{-2k_0 h} \quad .... \; (16)$$

with $2b$ equal to the maximum beam.

The total wave energy propagation Equation (16) can be equated to the mean rate of the work done by the damping force $\frac{1}{2}(2N)\, z_0^2\, \omega^2$. Solving for $2N'$, the non-dimensional damping.

$$2N' = \frac{2N}{\Delta/\sqrt{gl}} = \frac{4\pi}{5}\, \Omega_d^5 \sqrt{\frac{l}{d}}\, e^{-2\Omega_h^2} \quad .. \; (17)$$

The damping coefficient 2δ is:

$$2\delta = \frac{2N}{m^*\, \omega} = \frac{4\pi}{5}\, \frac{\varrho V}{m^*}\, \Omega_h^4\, \frac{d^2}{h^2}\, e^{-2\Omega_h^2} \quad .. \; (18)$$

Comparison can now be made between these theoretical results and the experimentally determined values at zero Froude number. The coefficient 2δ of Equation (18) does not lend itself well to such comparison since m^* is an unknown function of frequency which must be determined experimentally. Thus it cannot be said to be a "theoretical" result.

The coefficient $2N'$ of Equation (17) has been plotted in Fig. 5 for $h/d = 1.0$, 1.5 and 2.9. Differentiation of Equation (17) indicates that a maximum exists at $\Omega_d = \sqrt{5d/4h}$. Thus for $h/d = 1$, the theoretical peak is at $\Omega_d = 1.12$. The experimental data indicate a peak in damping in this vicinity, but insufficient data were obtained to define it accurately. The measured damping curve is higher than the theoretical curve at $h/d = 1$. This may be due to the fact that the approximation used for the doublet distribution loses accuracy with diminishing submergence. For greater depths of submergence, $h/d = 1.5$ and 2.9, the comparison is good. If the instrument damping curve is subtracted from the experimental curve at $h/d = 1.5$ and 2.9, the resulting curve closely follows the theoretical wave damping.

Fig. 6 and 7 (obtained from Fig. 4) and Figs. 8 and 9 (obtained from Fig. 5) are cross plots showing the damping versus Froude number relation for constant frequency. Fig. 8 offers evidence that the choice of the damping coefficient $2N'$ to describe damping forces in heave affords advantages since the data collapse above Froude numbers of 0.35. The damping coefficient is substantially independent of frequency above this Froude number. This region of frequency independence corresponds to relatively small angles of attack. For Froude numbers above 0.35 and with the initial displacement no greater than 3 inches, the maximum instanta-

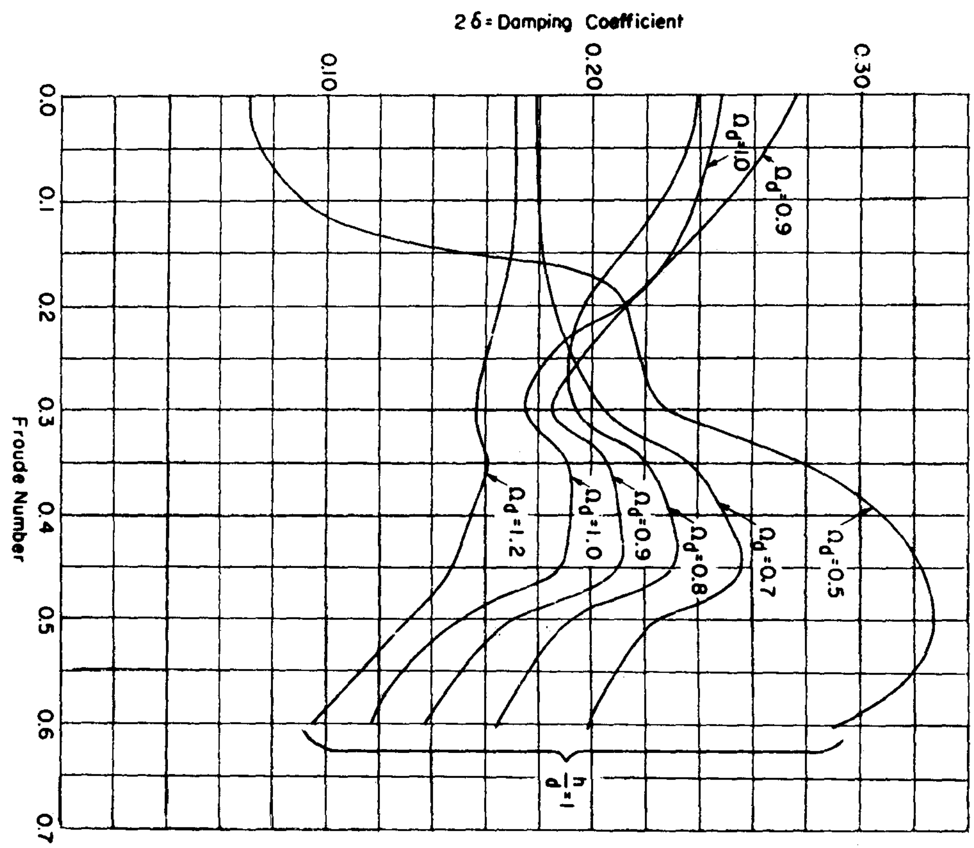

Fig. 6. *Damping Coefficient 2δ Versus Froude Number for $h/d = 1$ and Constant Ω_d for Spheroid*

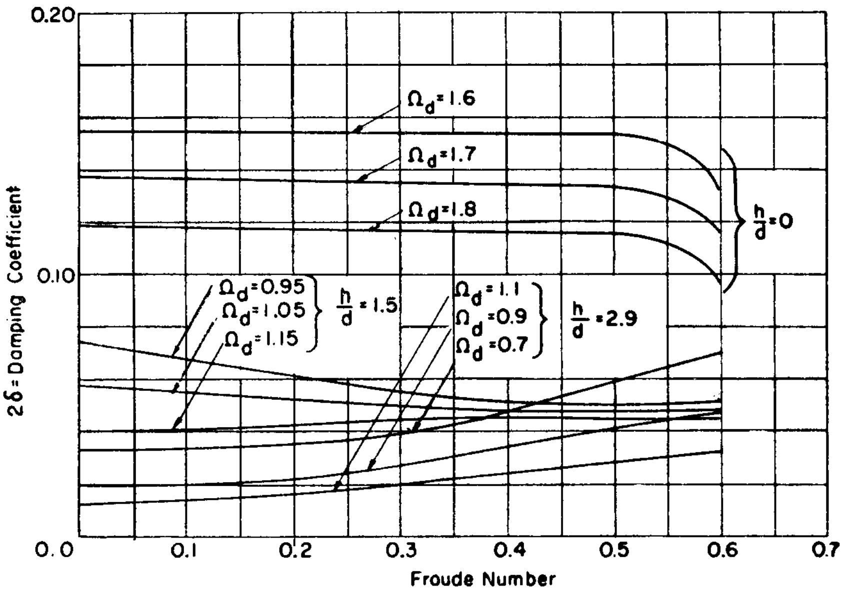

Fig. 7. *Damping Coefficient 2δ Versus Froude Number for h/d = 0, 1.5, 2.9 and Constant Ω_d for Spheroid*

neous angle of attack for the range of frequencies tested never exceeded 10 degrees.

The celerity c of a wave having the period of oscillation of the body is $c = gT/2\pi$. Hence a graph of the damping coefficient versus Froude number for constant frequency (Figs. 6 through 9) can be interpreted as a plot of damping coefficient versus V/c. For higher values of Ω_d, the total damping coefficient is decreasing with V/c and this applies a fortiori to the wave part of the damping. This result supports Haskind's findings for surface ships [4] (see also Equation (36) Reference 15).

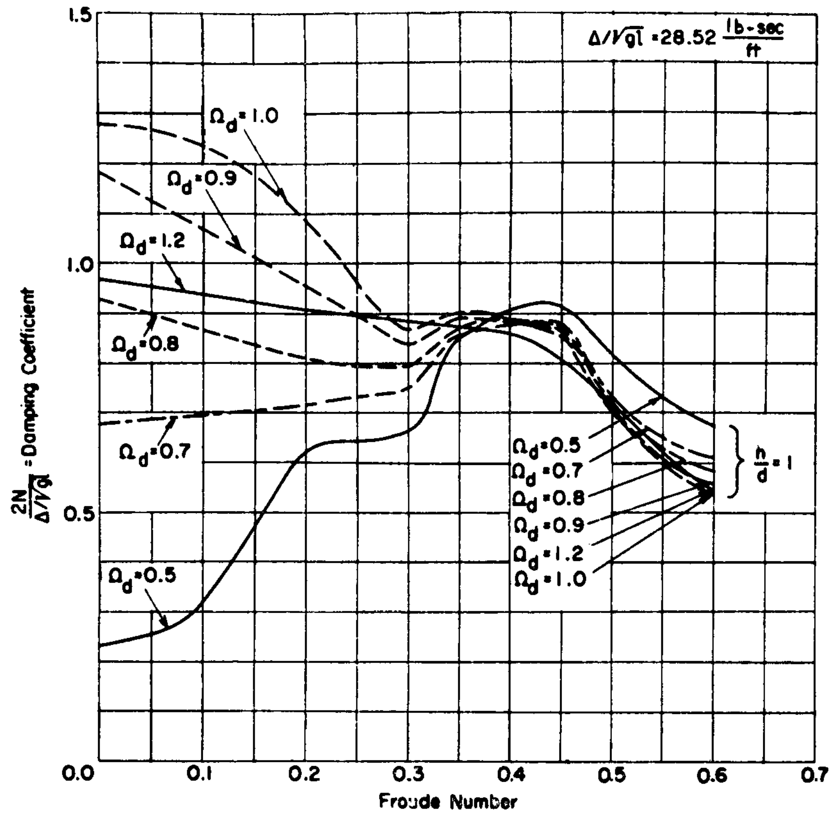

Fig. 8. *Damping Coefficient 2N′ Versus Froude Number for h/d = 1 and Constant Ω_d for Spheroid*

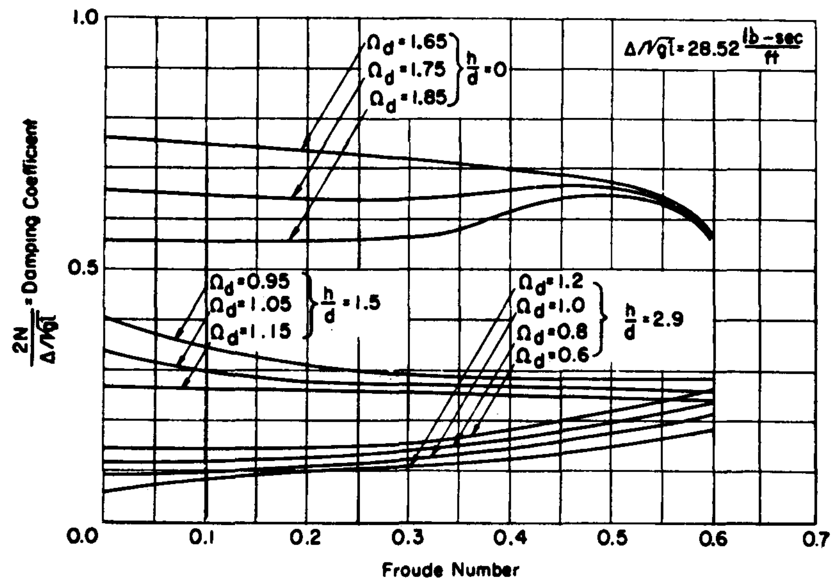

Fig. 9. *Damping Coefficient 2N′ Versus Froude Number for h/d = 0, 1.5, 2.9 and Constant Ω_d for Spheroid*

On the other hand, our measurements for $h/d = 0$ do not show the strong decline with V/c predicted by Haskind. The trend of the damping curves at $h/d = 1$ is rather unexpected for lower frequencies.

In discussing the theory of damping effects, attention was called to the fact that the omission of viscous effects may become misleading in the determination of total damping. Equation (10) and (11) were developed for the damping force coefficient in the deeply submerged condition using a "steady state" value for the lift-curve slope. The lift-curve slope can be taken from Reference 12 where, for h/d equal to 3, $\partial C_L/\partial \alpha$ equals 0.10. If $2N$ is non-dimensionalized by the factor $(\varrho/2)\ AV$, where A is the maximum waterline area, the non-dimensional damping coefficient is $\partial C_L/\partial \alpha$, a constant for small angles of attack. The value of 0.10 for this body is not small and cannot be neglected. In Fig. 10, $2N/(\varrho/2)\ AV$ is plotted against Strou-

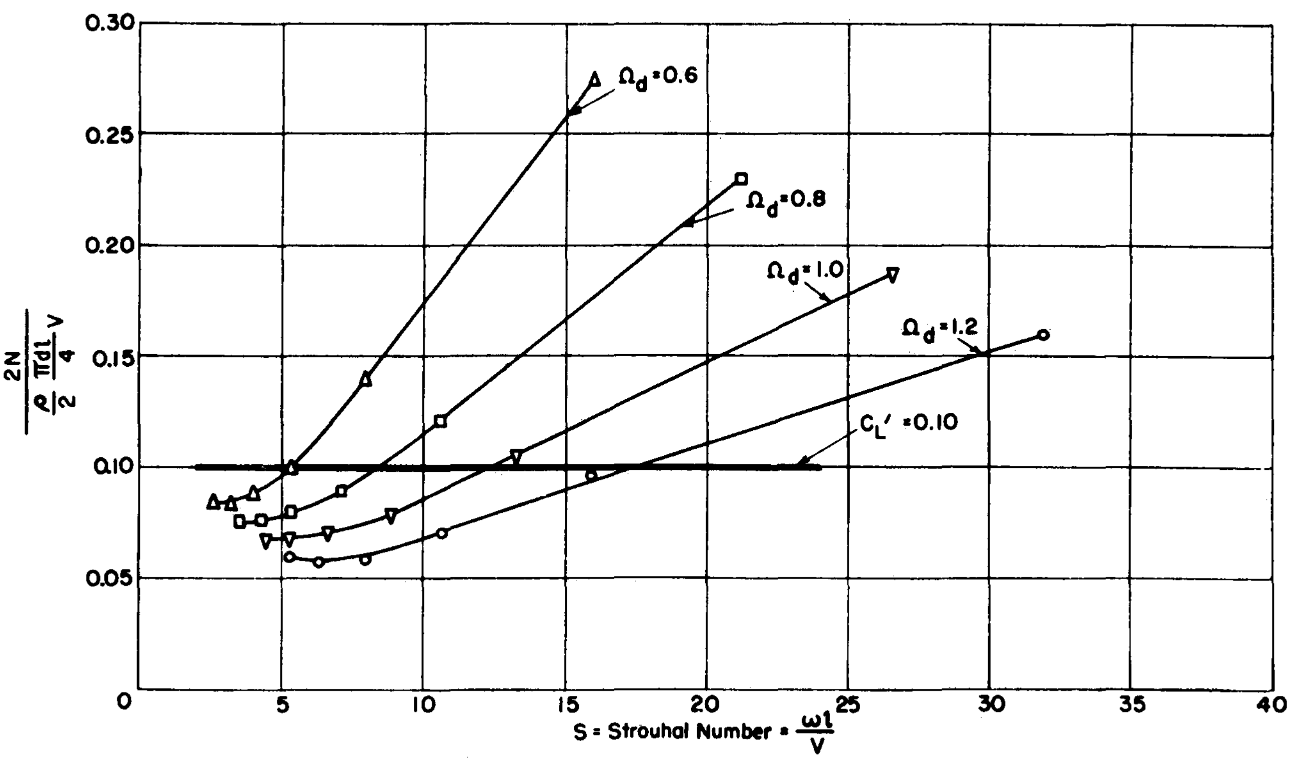

Fig. 10. *Damping Coefficient Versus Strouhal Number at h/d = 2.9 for Spheroid*

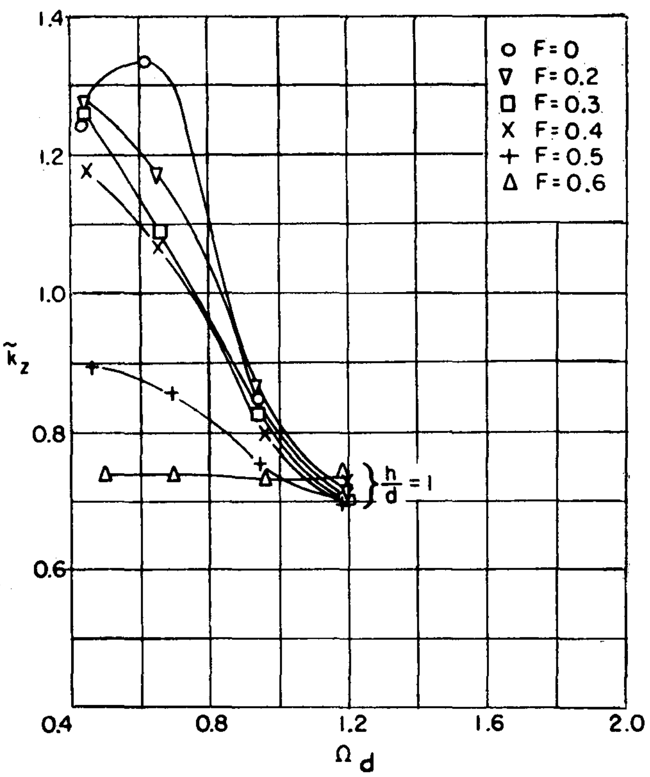

Fig. 11. *Added Mass Coefficient Versus Ω_d for $h/d = 1$ and Constant Froude Number for Spheroid*

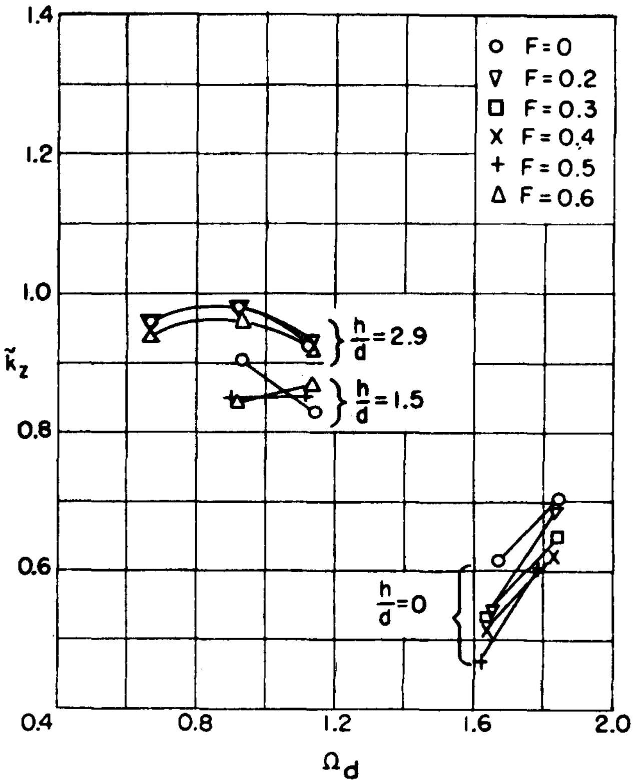

Fig. 12. *Added Mass Coefficient Versus Ω_d for h/d = 0, 1.5, 2.9 and Constant Froude Number for Spheroid*

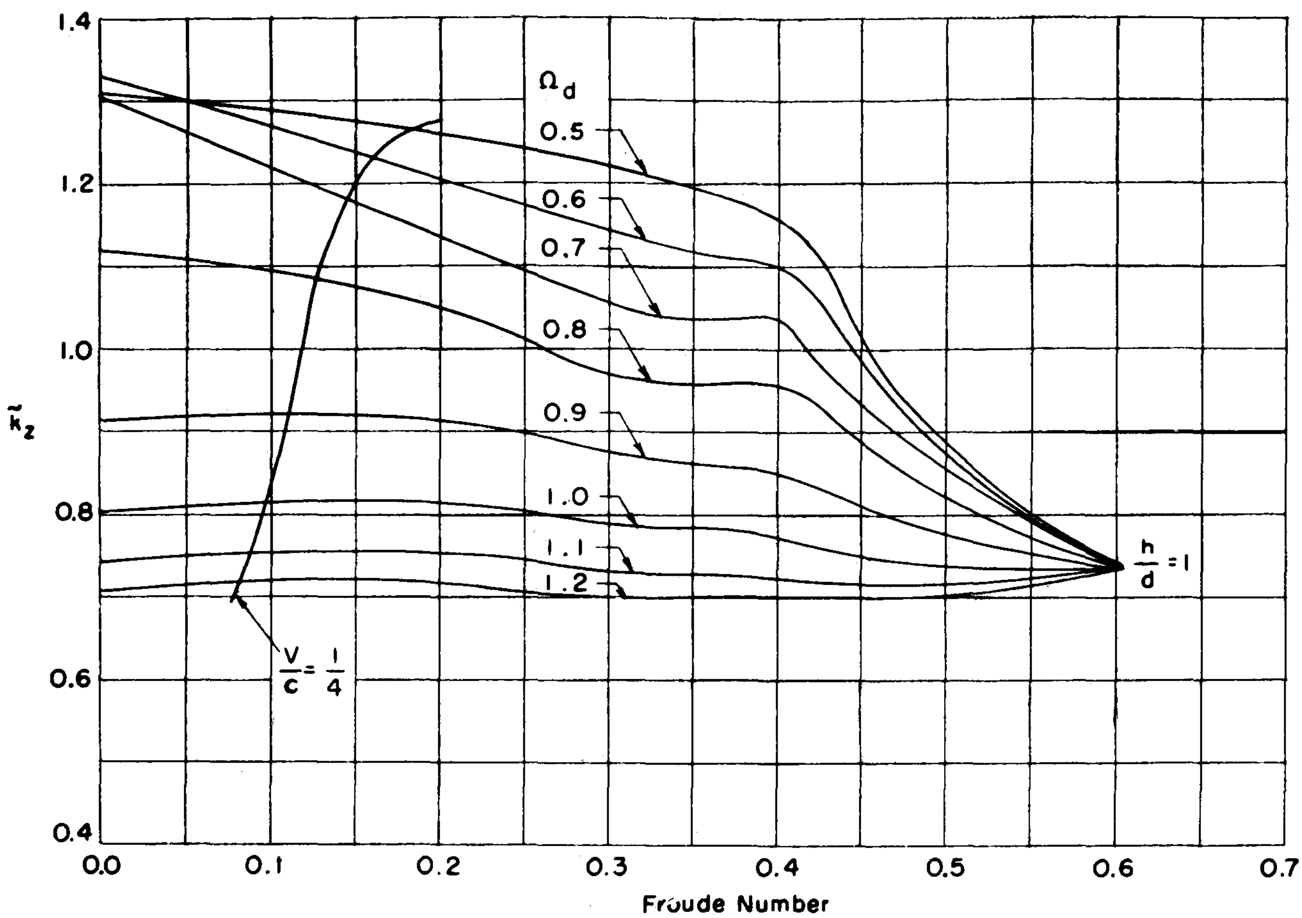

Fig. 13. *Added Mass Coefficient Versus Froude Number for $h/d = 1$ and Constant Ω_d for Spheroid*

hal number $\omega l/V$ for constant frequency. They agree in order of magnitude with the "theoretical" constant value of 0.10, provided the Strouhal number is small. This condition is necessary for the applicability of Equation (11). The lift in the oscillatory motion seems to be less than that derived from static conditions. It must be borne in mind that the large instrument damping values make it difficult to interpret the curves properly so that we do not wish to stress this point.

Added Masses

As emphasized before, it was not intended to cover a broad range of values of frequencies because this can be done much more easily by forced oscillations. The original purpose was to obtain a few data points using free oscillations and later to compare these with pertinent results derived from forced oscillations. However, some of the results are so interesting that a more general discussion appears to be justified.

Figs. 11 and 12 show the dependency of the added mass coefficient upon Ω_d for various Froude numbers and depths of immersion. Tests with several initial displacements but otherwise similar conditions were again averaged.

Fig. 11 reveals a maximum in the k_z versus Ω_d curve at $h/d = 1$, $F = 0$. Comparable graphs have been published by Ursell [2] and Haskind [21] for $h/d = 0$. Ursell's investigation leads to $\tilde{k}_z \to \to \infty$ as $\Omega_d \to 0$ while Haskind's curve shows a hump similar to that obtained in the present study.

The hump in the curve does not appear for nonzero Froude numbers due to the limited experimental range of frequencies. By using forced oscillations, it should be possible to clarify the whole problem satisfactorily.

Figs. 13 and 14 are cross plots taken from Figs. 11 and 12 to show the dependency of added mass coefficient upon Froude number for constant Ω_d and h/d. The dependence of $\tilde{k}_z$ upon Froude number is not too pronounced for $h/d = 0$, 1.5 and 2.9. This is also true for the higher frequencies at $h/d = 1$. However, a steep drop in $\tilde{k}_z$ with increasing F (or V/c) for lowest frequencies at $h/d = 1$ is most conspicuous. A line of constant $V/c = 1/4$ has been drawn in Fig. 13 and no abrupt change in $\tilde{k}_z$ at $V/c = 1/4$ can be found.

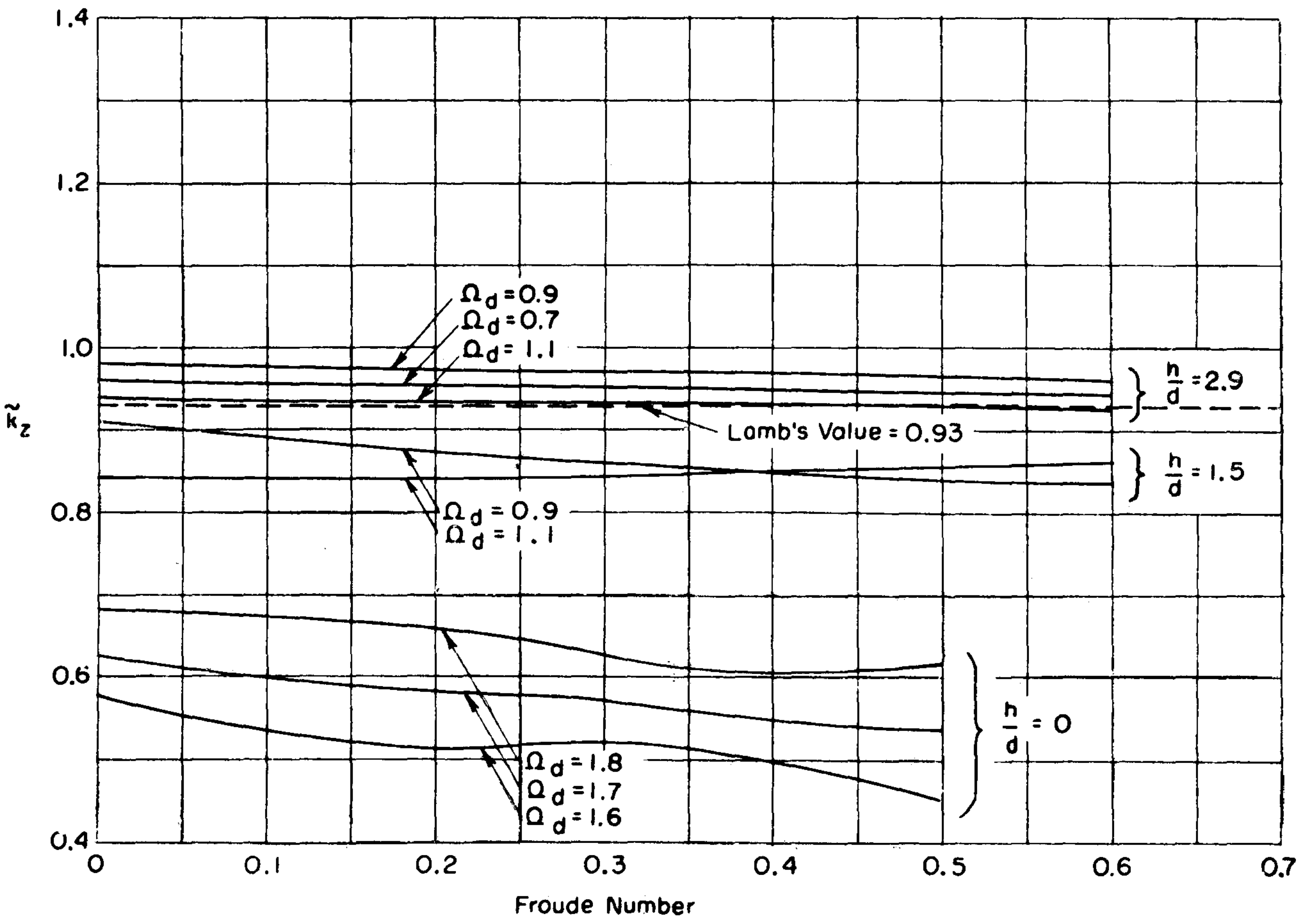

Fig. 14. *Added Mass Coefficient Versus Froude Number for $h/d = 0$, 1.5, 2.9 and Constant Ω_d for Spheroid*

At $h/d = 2.9$, we might expect close agreement of the experimental values with Lamb's values for infinite immersion of $\overset{\infty}{k}_z = 0.93$. This can be inferred from the well-known formula $\overset{\infty}{k}_z = 1 \pm \frac{1}{2} (d/2h)^2$ which applies to a circular cylinder moving below a rigid wall or free surface (the sign depending on the case). For the cylinder at $h/d = 2.9$, the value of k_z is within $1\frac{1}{2}$ per cent. of the infinite depth value. The experimental data for the spheroid shown in Fig. 14 are in the neighbourhood of Lamb's value.

Influence of Viscosity on the Added Mass and Damping

Although some experimental results obtained earlier agree satisfactorily with theoretical values, doubts have been frequently expressed whether the added mass coefficients determined for motion in an ideal medium can be applied to actual conditions with reasonable accuracy. Two effects must be considered: The thickening of the body by the boundary layer and possible separation. Since the displacement thickness of the boundary layer is generally small, it can be assumed that the first effect generally has no great influence. If we exclude bodies with abrupt changes in section, then no considerable separation should be expected.

Generally speaking, the flow pattern around a body under starting conditions closely resembles potential flow, provided the viscosity of the liquid is low (water). Thus, a priori, one would not expect a large influence of viscosity on transverse added masses.

Since it was not possible to eliminate the boundary layer, we went the opposite way. We roughened the smooth surface of the spheroid (30 mesh sand on wet paint). Under otherwise similar conditions of fluid motions, this may lead to a doubling of the boundary layer thickness as compared with the smooth condition.

As had been anticipated, oscillation tests made at $h/d = 2.9$ showed only a minute increase in periods as compared with the results for the smooth body (Fig. 15). Since these exploratory tests agree with our simple reasoning, there is apparently no need to bother about the influence of viscosity effects on such added mass factors as $\overset{\infty}{k}_z$, $\overset{\infty}{k}_{yy}$, $\overset{\infty}{k}_y$, $\overset{\infty}{k}_{zz}$ for ship-like bodies and very elongated streamlined bodies of revolution. It is not evident that the same conclusion holds by analogy for the generally small factors $\overset{\infty}{k}_x$, $\overset{\infty}{k}_{xx}$. The damping coefficient is moderately increased.

Carrier model

In the present investigation some preliminary experimental studies were made in which the model was lifted above the water surface and dropped in order that the damped motions could be compared with and without slamming.

The heaving motion of a carrier model was investigated for three initial draft conditions (2.8-,

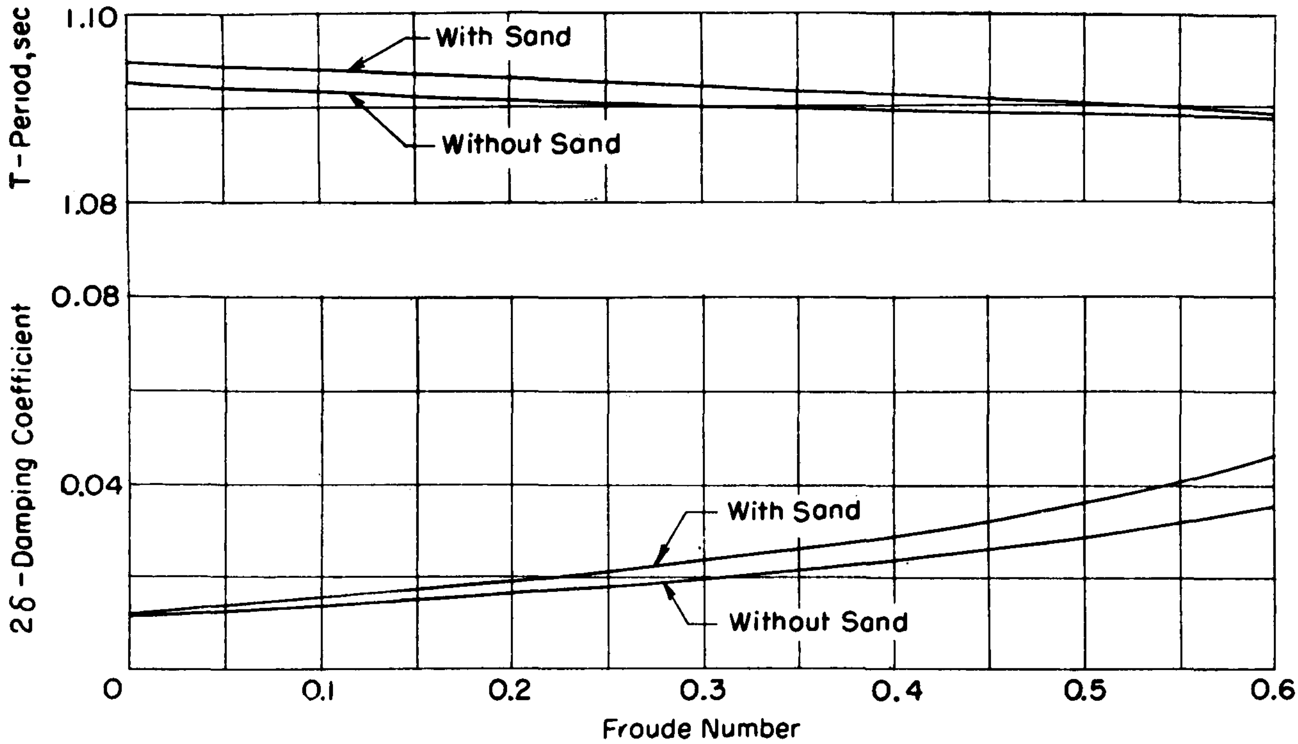

Fig. 15. *The Effect of Surface Roughness on Damping Coefficient and Period of Oscillation of Spheroid*

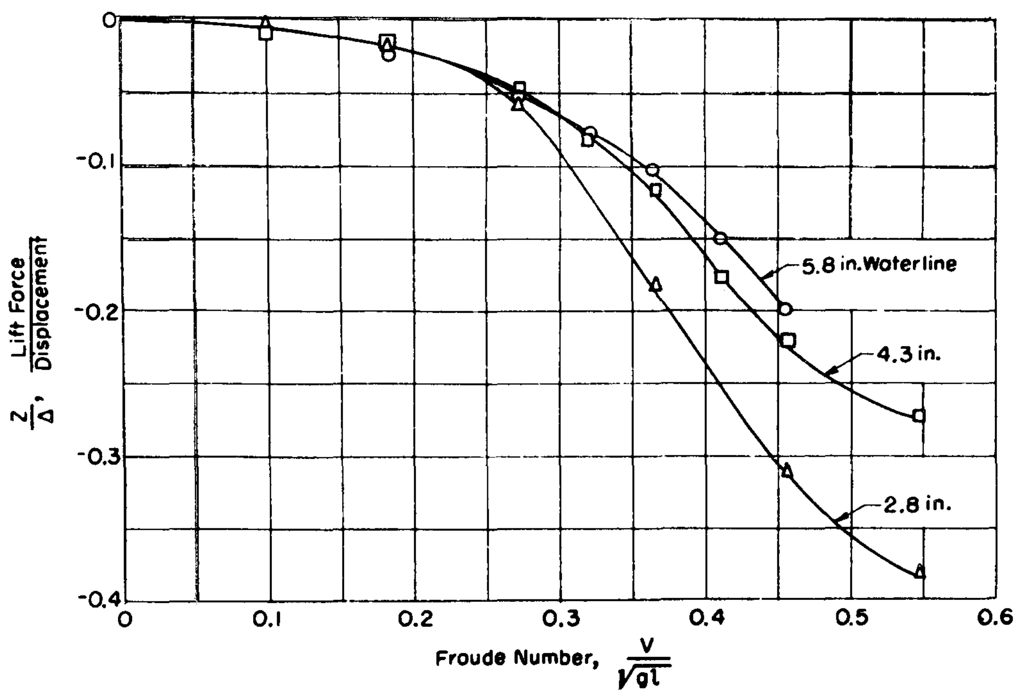

Fig. 16. *Lift Force Versus Froude Number for Surface Ship*

4.3- and 5.8-inch waterlines). The model was supported by the 55 pound per inch spring for the first two conditions and no spring was used for the 5.8-inch waterline condition. Several initial vertical displacements were tested at each draft. Two initial displacements for the 2.8-inch waterline condition corresponded to an elevation of the body above the freewater surface of 1/4 and 1 1/4 inches. An impact was obtained on the bottom after releasing the model.

Incidental to the main purpose of these experiments, namely the determination of the effects of slamming on the oscillatory motion of a surface ship, was the measurement of the lift force on the body due to the free surface during non-oscillating runs at various forward speeds. The force is non-dimensionalized by the appropriate displacement for the particular draft and is plotted against Froude number in Fig. 16.

The results of the heaving tests of the carrier model are summarized in Fig. 17. The frequency parameter Ω_B, damping coefficient 2δ, and added mass coefficient $\overset{\infty}{k}_z$ are plotted against Froude number with the results of several initial displacements averaged. Not much importance should be attached to the maximum draft because of the heavy flare of the sections. The most important results are those for the design draft of 4.32 inches. The variability of the period is not pronounced, and the same applies to the added masses which, within the accuracy of our experiments, could be averaged by a straight line.

Following Lewis' method [22], St. Denis calculated the added mass in this case and obtained $\overset{\infty}{k}_z =$ $= 1.63$. This may be compared with the experimental value of 1.25 for the 4.32-inch waterline. Note that the latter refers to a higher frequency than that corresponding to natural conditions. However, since we expect an increase in $\overset{\infty}{k}_z$ with frequency, it can be assumed as probable that calculations based on Lewis' method are liable to give too high a value for $\overset{\infty}{k}_z$.

Comparison of the extinction curves for slamming and non-slamming conditions revealed no significant difference. Values for the damping coefficient for the two slamming tests are plotted in Fig. 17 with special symbols, and it can be seen that the difference between these points and the averaged faired data are small. This contradicted our preconseived ideas. However, the explanation of the phenomenon is simple. Although the momentary force during impact is large, its time of duration is very short so that the resulting impulse is only moderate. However, high accelerations are reached. These cannot be determined from the extinction curves, but they have been investigated subsequently by accelerometers. The experimental results and their discussion following Wagner's theory are given by Szebehely in Reference 23. We hope that our preliminary study of impact phenomena, which is only a sideline in the present investigation, will stimulate further research in this field of theoretical naval architecture.

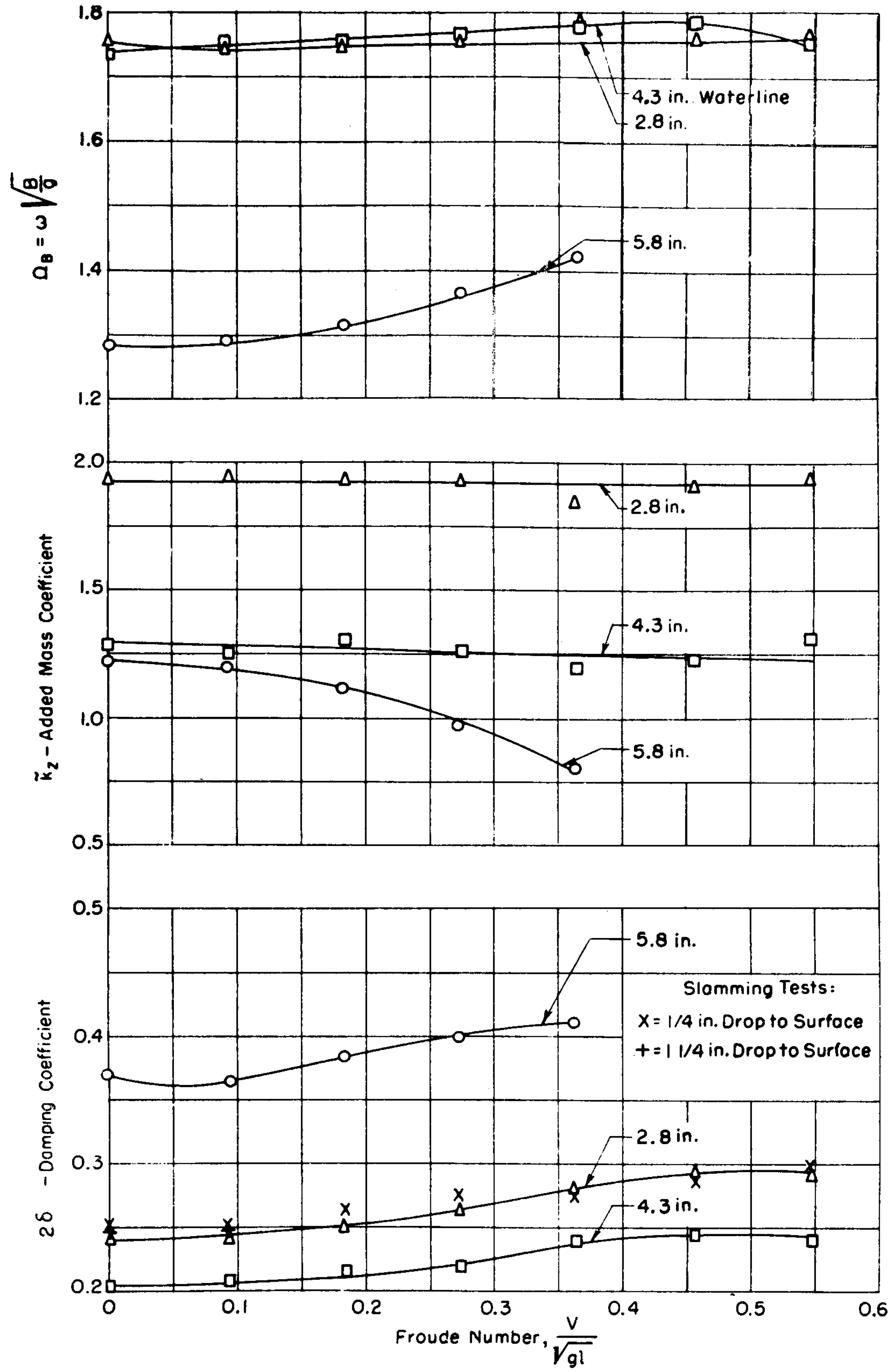

Fig. 17. *Froude Number Versus Damping Coefficient, Added Mass Coefficient, and Frequency Parameter Ω_B for Surface Ship*

Acknowledgment

The senior author inspired the research project, directed the experimental program and wrote the first draft of the report. The second author carried out the experimental program. After Dr. Weinblum left the D.T.M.B., the third author revised the first draft into its final form.

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Notation

A = Characteristic area
$2a$ = Length of spheroid
B = Beam of surface model
$2b$ = Maximum diameter of spheroid
C' = Stiffness coefficient
C_L = Coefficient of lift
C_L' = $\partial C_L/\partial \alpha$
c = Speed of the free plane wave corresponding to a frequency ω, $c = g/\omega$
d = Maximum diameter of spheroid
F = Froude number $V/\sqrt{gl}$
g = Acceleration of gravity
H = Draft of ship
h = Depth of submergence to longitudinal axis of spheroid
$\tilde{k}_z$ = Added mass coefficient for free surface condition
k_0 = ω^2/g
L = Lift force
l = Length of body
m = Mass of the model
$\tilde{m}_z$ = Added mass for free surface condition
m_0 = Mass of displaced water
m^* = Apparent mass
$2N$ = Damping coefficient
$2N'$ = $2N/(\Delta/\sqrt{gl})$, Dimensionless damping coefficient
$2n$ = $2N/m^*$
r = Radius of cross section of spheroid
T = Period of oscillation of damped motion
t = Time
V = Speed of advance
V = Volume of the body
Z = Lift force
z = Displacement in the direction of the gravity axis, positive downward
a = Amplitude of the regular waves
α = Angle of attack
Δ = Displacement of the model
2δ = Dimensionless damping coefficient
ϱ = Density of the fluid medium
Ω_d, Ω_h, Ω_B = Dimensionless frequency parameters
ω = Circular frequency of damped motion

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