Hamburg University of Technology

Project Thesis

Comparison of Different Types of Stress Gradient Methods Belonging to the Theory of Critical Distances

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December 15, 2021

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I certify that the work presented here is, to the best of my knowledge and belief, original and the result of my own investigations, except where otherwise indicated. It has not been submitted, either in part or whole, for a degree at this or any other University. I gratefully acknowledge the supervision and guidance I have received from Prof. Sören Ehlers and Dr. -Ing. Moritz Braun.

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List of Acronyms

TCD Theory of Critical Distances

PM Point Method

LM Line Method

AM Area Method

VM Volume Method

FEA Finite Element Analysis

FEM Finite Element Method

LEFM Linear Elastic Fracture Mechanics

Abstract

Notches are almost unavoidable in material production. A notched geometry may cause crack initiation which can be followed by crack growth and in the end material failure. Gradient methods are one of the easiest and most useful analytical methods to calculate stress concentrations around the notches. In this project thesis, different gradient methods are compared for a simple plane stress example with different notch geometries. FEM analysis is done for the notched geometries to find resulting stresses. Stress concentrations around the notches are calculated with different stress gradient methods. These stress gradient methods, point method, line method and are method are compared for different critical distances. Up to 45% difference is found between calculated effective stresses between AM and the other two methods which may lead to critical differences in fatigue life.

1 Introduction

Fracture mechanics and fatigue strength are one of the most important subjects of engineering mechanics. Different methods of these subjects are used to know how reliable and durable a structure is in the fields of mechanical engineering, naval architecture, offshore engineering, and aerospace engineering, etc. Welded structures are used in fields such as naval architecture, offshore engineering, and civil engineering.

Weld's toes are very critical for fatigue strength since most of the time structure starts to crack and eventually fails from the notches on these areas. Notches increase local stress and strain concentrations. Thus it is essential to analyze how notch affects the fatigue limit state design of a structure.

There are several methods to calculate the effective stress of a notched structure. In this project gradient methods are used to analyze the affective stress of notched plates with different notch apparture angles and notch radii. These methods are Point Method, Line Method and Area Method. These methods in principe use different critical distance L to average stress concentrations. To understand these methods, the first basics of fatigue strength and fracture mechanics have to be understood so this project thesis begins with an explanation of these basic theories. Then basics of TCD Methods are explained. Finally, a series of FEM analyses are done to compare these methods. Results of TCD Line Method and Point Method are compared with a previous research paper (Braun, Müller, et al., 2020). To compare the results same model and variables were used. In the end, those methods are compared with TCD Area Method and AM is also validated with a basic numerical calculation.

2 Fatigue in Materials

Although detailed researches about fatigue crack propagation began in the 1960s, fatigue in metals started to be investigated in the late 1800s and early 1900s (Pook, 2007). Wöhler's famous railway axle tests are published in the early 1860s(Schütz, 1996). Statistical and dynamical analysis in engineering are well known before fatigue analysis, but especially with industrialization, more and more machines started to fail because of fatigue. Much of the work done by fatigue was usually expressive in early works and they were executed often by test specimens in the laboratory. This has changed in the last 5 decades with the development of computers(Pook, 2007). Today with the help of numerical methods, it is easier to calculate and test new approaches about fatigue strength such as gradient methods.
The phenomenon of fatigue occurs under cyclic loads. The resultant force of fatigue failure can be extremely smaller than the critical static load. The cyclic load might be even so small that it sometimes can not be visible without proper equipment. A mechanical failure can occur after two important stages(Schijve, 2008):

- crack initiation period
- crack growth period.

Different stages and required stress concentration factors are given in figure 2.1. Crack initia-

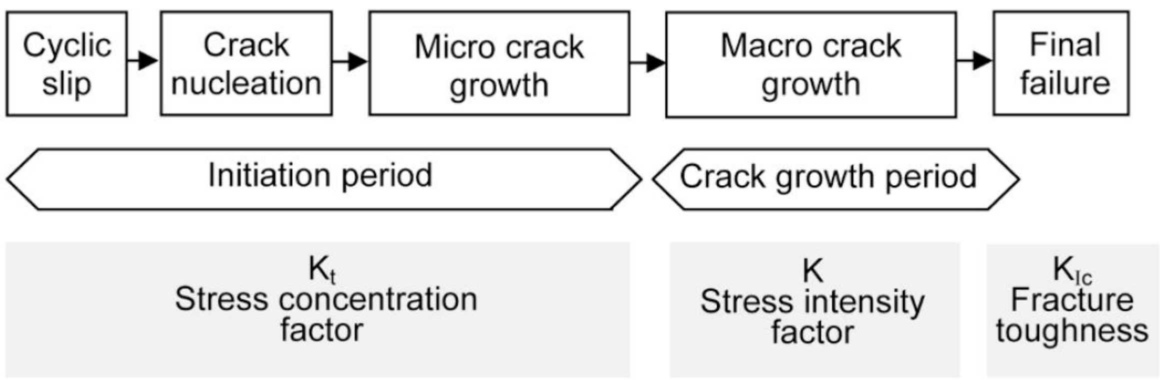


Figure 2.1: Different Stages of Fatigue (Schijve, 2008)

tion starts with a cyclic slip. Shear stresses may cause slip in material grains which results in microcracks and inhomogeneous stress concentration. A crack may grow continuously with the cyclic load and eventually material starts to fail. The crack growth rate is not constant along the grains. The growth rate decreases when the crack passes through the grain boundary. After crack growth continues eventually material starts to fail. Figure 2.2 shows a crossection of a microcrack.

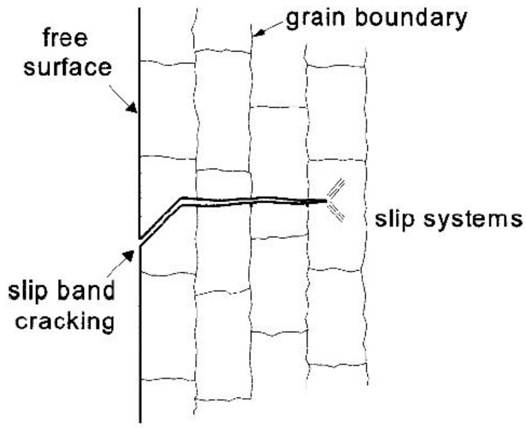


Figure 2.2: Crossection of a Microcrack (Schijve, 2008)

Variety of techniques are used to predict fatigue life of a material. The most common method is S-N curve, which is based on stess-life method, also called as Wöhler curves. S-N curves created with a series of specimen tests, in which a range of loads exposed to the specimens untill their complete breakage. Lifetime N is determined by following equation (Haibach, 2006):

$$N = N_R \left(\frac{\Delta\sigma}{\Delta\sigma_R} \right)^{-k} \tag{2.1}$$

in which N_R is the reference number of cycles untill failure, $\Delta\sigma_R$ is the corresponding reference fatigue strength, $\Delta\sigma$ is the effective stress and k is the slope exponent. Figure 2.3 shows how a typical S-N curve. Stress amplitude is the half of the total stress range, which is shown by Y-axis. The parameters affecting Wöhler curve are given below (Bai and Jin, 2016):

- Wall thickness
- Corrosion
- Material type and condition
- Test environment, specimen surface, alignment of the test machine, etc.
- Local stress peaks, residual stress, mean stress

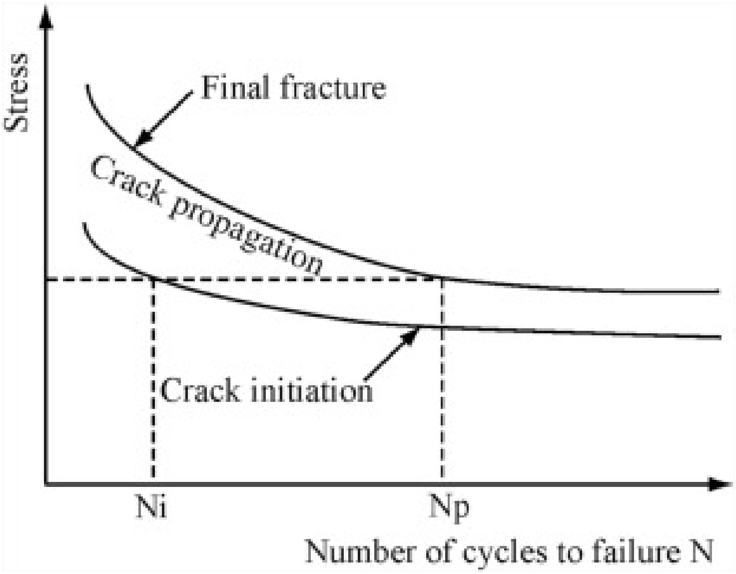


Figure 2.3: Illustration of fatigue life (initiation and propagation stages). (Bai and Jin, 2016)

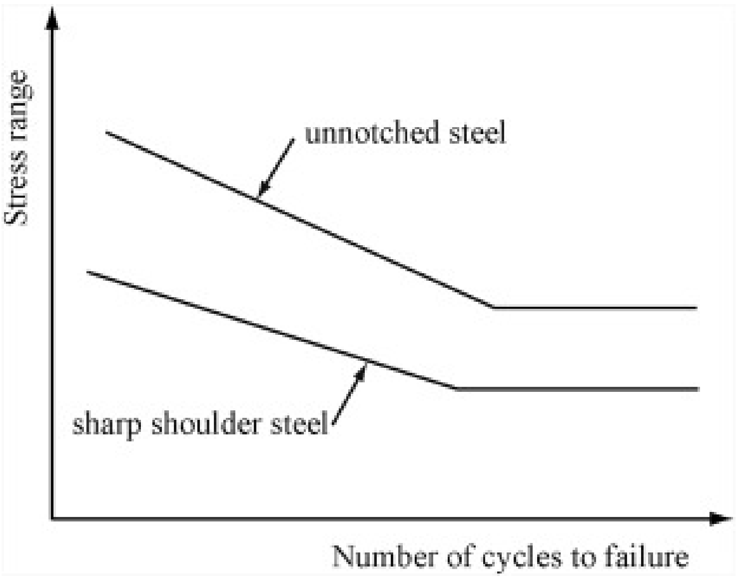


Figure 2.4: Wöhler's S-N curves for Krupp axle steel. (Bai and Jin, 2016)

3 Notch Fatigue

Notches and holes increase the stress concentration of a structural component which can lead to fatigue crack and failure of the structure. Nevertheless, it is almost impossible to avoid notches and holes in engineering structures. Keeping that in mind, it is important to understand how notches affect the fatigue limit and serviceability of the structure. Fatigue limit can be defined as the lowest stress amplitude capable of initiating a microcrack that can grow to failure (Schijve, 2008). Peterson has defined notch sensitivity in 1959 as (Peterson, 1959) ;

$$q = \frac{K_f - 1}{K_t - 1} = \frac{1}{1 + a^*/\rho}. \tag{3.1}$$

Here q indicates sensitivity factor, K_f and K_t are fatigue stress reduction factor and stress concentration factor respectively. High notch sensitivity is achieved if $q = 1$ and no sensitivity with $q = 0$. ρ is the notch radius and a^* as a material constant. K_f coincides K_t if $a*$ is small and ρ is large and vice versa.

Similarly, Neuber has also defined a similar equation for sensitivity factor (Neuber, 1958);

$$q = \frac{K_f - 1}{K_t - 1} = \frac{1}{1 + \sqrt{A/\rho}}. \tag{3.2}$$

As Neuber was averaging the linear elastic stress over a line, Peterson was suggested to take the reference stress at a point along the notch bisector. These methods are later on referred to as the Line Method and Point Method (Louks, 2016). K_t values can be found by two different methods such as (Schijve, 2008):

- by calculations: analytical methods, FEM
- by measurements: strain gauge measurements, photo-elastic measurements.

Notches can be found in different shapes and geometries. In this project u- and v-shaped notches are used with different radii and opening angles.

4 Theory of Critical Distances

It is possible to the assessment of fatigue by transforming the stress field into a scalar value, which is often referred to as the "effective stress" and this procedure is essential, since the maximum stress in the notch is not responsible for the fatigue strength, but rather a reduced effective value instead (Baumgartner et al., 2015). Various methods have been developed for the determination of effective stress parameters for the stress fields in the vicinity of notches. Among those effective stress methods, two versions of stress gradient methods have been widely used in recent the last years (Braun, Müller, et al., 2020). These methods are most of the time called the theory of critical distances.

The theory of critical distances is based on the works of Peterson and Neuber. Neuber suggested that to predict the notched fatigue strength at high cycles, effective stress could be calculated by averaging the linear-elastic stresses over a line and Peterson made this approach simpler as he proposed that the effective stress could be calculated directly by simply using the stress at a given distance from the notch tip (Louks, 2016). These approaches were the first forms of the line and point methods respectively. Neuber and Peterson faced two problems while implementing their methods of critical distance (Louks, 2016) ;

- The value of the critical distance was unknown. According to Peterson critical distance was correlated with grain size, which unfortunately caused several measurement issues. Therefore critical distance had to be calculated empirically by matching the predictions to the data.
- Maintaining precise stress-distance curves. There were some solutions proposed by Neuber, however, these were only approximations.

Later on, Whitney and Nuismer formulated the hypothesis that the critical distance is a material property linked to both tensile strength and fracture toughness under plane loading (Whitney and Nuismer, 1974).

TCD methods can be used for fracture mechanics of the materials such as steels, ceramics, composites, and polymers. They can be used to calculate the fatigue limit both for crack, notches, and circular holes while accounting for the size effect (Taylor, 1999). TCD methods are based on a material constant with a critical distance L. Standard stress-based and stress intensity-based methods are described with this constant to calculate the effect of several phenomena, such as notch root radius, the short fatigue crack effect, and size, scaling effects for the body under stress (Taylor and Hoey, 2009).

TCD gradient methods are related largely to classical LEFM. The brittle fracture appears if the stress intensity K is the same as the fracture toughness K_c and as K_c is a material constant, which means a relationship among K_c and constant in PM exist (Taylor, 2007). This relation can be shown with the following equations:

$$L = \frac{1}{\pi}\left(\frac{K_c}{\sigma_0}\right)^2 \tag{4.1}$$

,

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2. \tag{4.2}$$

Here L and σ_0 show critical distance and nominal stress. Equation 4.1 shows the relationship between fracture toughness and material constants of TCD. Equation 4.2 shows the appropriate critical distance for fatigue threshold ΔK_{th}.

To apply TCD methods stresses and strains around the notch have to be known. Only then fatigue behavior of the body can be analyzed. The method of calculating the stresses is independent of the TCD method. TCD methods can be divided into four, such as Point Method(PM), Line Method(LM), Area Method(AM), and Volume Method(VM).

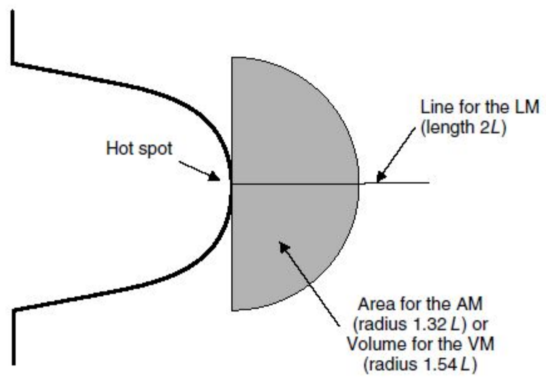


Figure 4.1: The Line, Area and Volume Methods appled to an edge notch (Taylor, 2007)

4.1 Point Method

According to the Point Method, ”Failure will occur when the stress at a distance L/2 from the notch root is equal to σ_0.”(Taylor, 2007). It can be described as:

$$\sigma(L/2) = \sigma_0, \tag{4.3}$$

in which σ and σ_0 are stresses at the $\frac{L}{2}$ distance from the notch tip and nominal stress respectively. Most of the time stress at the given point can be calculated linearly. If nonlinearities exist, then stress distance curves have to be calculated. Critical distance L can be calculated as:

$$L = \frac{1}{\pi}\left(\frac{K_\kappa}{\sigma_t}\right)^2 \tag{4.4}$$

in which K_κ is plane strain fracture toughness and σ_t is the tensile strength. Material constants L and σ_0 can be found by testing on specimens with two different stress concentrations which can be done by using two different notches (Taylor, 2007).

4.2 Line Method

For LM relevant parameter is the average of the stresses along with the focus patch from the notch tip to the crack growth direction. The starting point is chosen as $r = 0$. Brittle fracture can be defined as (Taylor, 2007):

$$\frac{1}{d}\int_0^d \sigma(r)\,dr = \sigma_0 \tag{4.5}$$

In addition, distance d can be calculated as:

$$d = \frac{2}{\pi}\left(\frac{K_c}{\sigma_0}\right)^2. \tag{4.6}$$

The equation gives $2L$ which means PM uses $L/2$ and LM uses $2L$. Further equation of LM becomes:

$$\frac{1}{2L}\int_0^{2L} \sigma(r)\,dr = \sigma_0. \tag{4.7}$$

According to Taylor LM and PM give similar results in most cases, especially for long cracks and plain tensile specimen (Taylor, 2007).

4.3 Area Method

Area and volume methods are based on the average area and volume of stresses around the notch tip. Area or volume has to be chosen wisely since the result is dependent on their shape. Most of the time a semicircular area or hemispherical volume is chosen. Failure occurs for AM when;

$$\frac{4}{\pi L}\int_{-\pi/2}^{\pi/2}\int_{0}^{L}\sigma(\theta,r)\,d_{\theta}d_{r}=\sigma_0. \tag{4.8}$$

The effective lengths of AM and VM are $1.32L$ and $1.52L$ respectively. Effective stress is the integration of stresses in chosen area or volume. This means for AM double integration and for VM third integration of the stresses with relevant area and surface.

5 FEM Model

FEM is done similarly to a previous study(Braun, Müller, et al., 2020). ANSYS software is used to perform FEM analysis to find relevant stresses. Parameters of notch radius r and opening angle w are different to understand their effects. Opening angles of 0° and 135° are chosen because they exist mostly for u and v-shaped weld toes respectively (Braun, Müller, et al., 2020). As a representation of sharp notches radius of 0.05 mm and for blunt notches radius of 0.5 mm is chosen. Typical steel has chosen for analyzing and its properties are given in table 5.2 A distributed unit force acting on the right-hand side of the specimen with a value of 1 N/mm^2. Notch depth d and distance g(the distance between notch root and neutral axis) is assumed to be 10 mm. Specimen length is 200 mm. In table 5.1, red-colored parameters indicate input parameters and blue-colored parameters are dependent parameters. Their values change depending on other input parameters. The depth of the area is shown by $r1$ and is equal to 5 mm. Angle ϕ is a parameter, which is a function of the opening angle. e_g represents global element size, which is a function of element size e_1. Further information about the FEM analysis can be found in the study of (Braun, Müller, et al., 2020).

To perform the FEM analysis, a square 2D element with 8 nodes is used, which is referred to

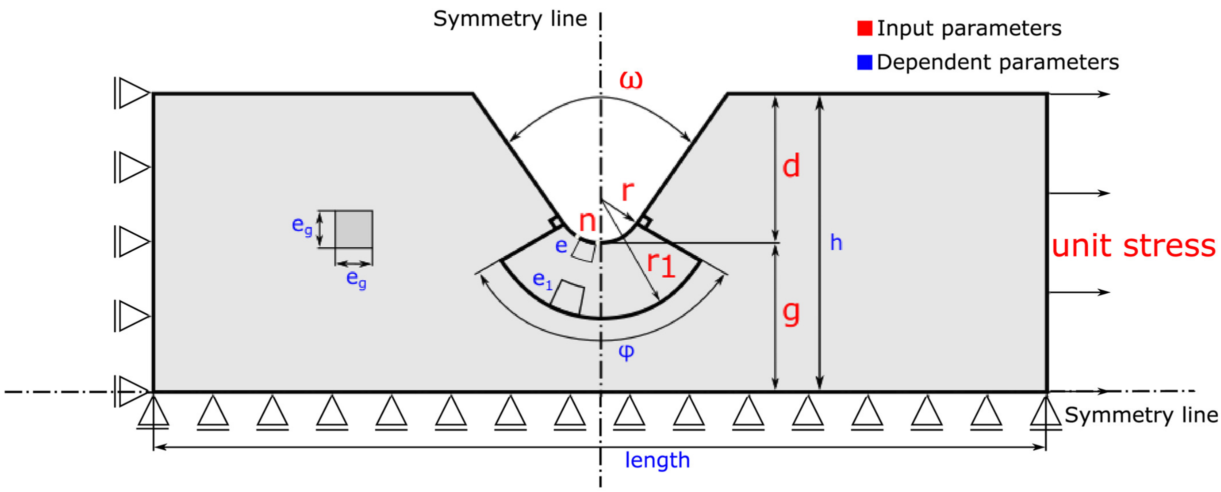


Figure 5.1: Model Geometry with Parameters (Braun, Müller, et al., 2020)

Table 5.1: Notch Parameters

Notch radius [mm]	Opening Angle [°]
0.05	0
0.05	135
0.5	0
0.5	135

Table 5.2: Material Properties

Young's Modulus [$\frac{N}{m^2}$]	Shear Modulus [$\frac{N}{m^2}$]	Poisson Ratio [-]
206000	$\frac{20600}{2+1.\nu}$	0.3

as plane 183. Figure 5.2 shows a stress contour for 72 elements ANSYS analysis with notch angle 135° and notch radius 0.05 mm. In addition, stress contours are plotted in the MATLAB program. Stress contours for all the examples that have been calculated in this project thesis can be seen in Figure 5.2 and Figure 5.3. It can be seen from the figures that maximum stress concentration is on the Notch Tip. As the distance from the tip increases, the stress amplitudes decrease.

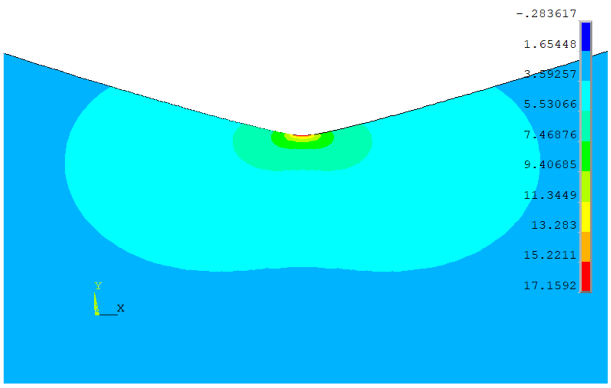
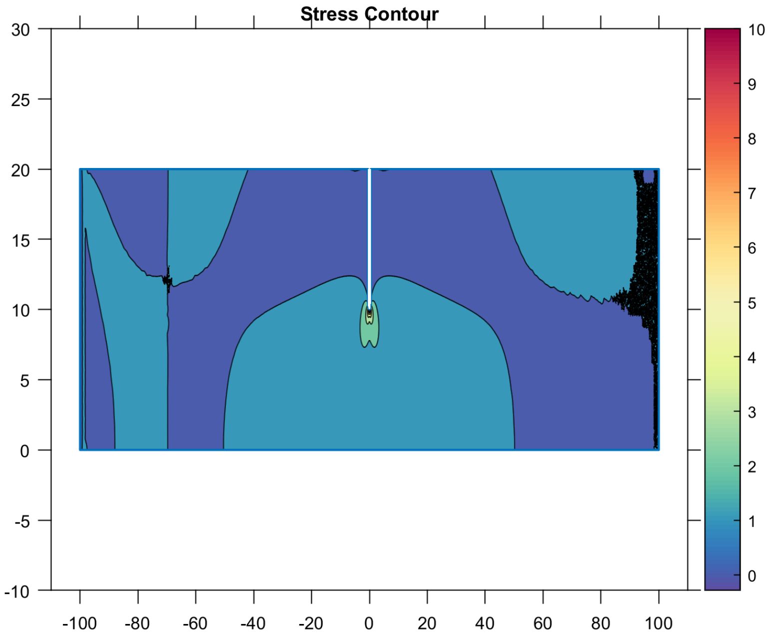
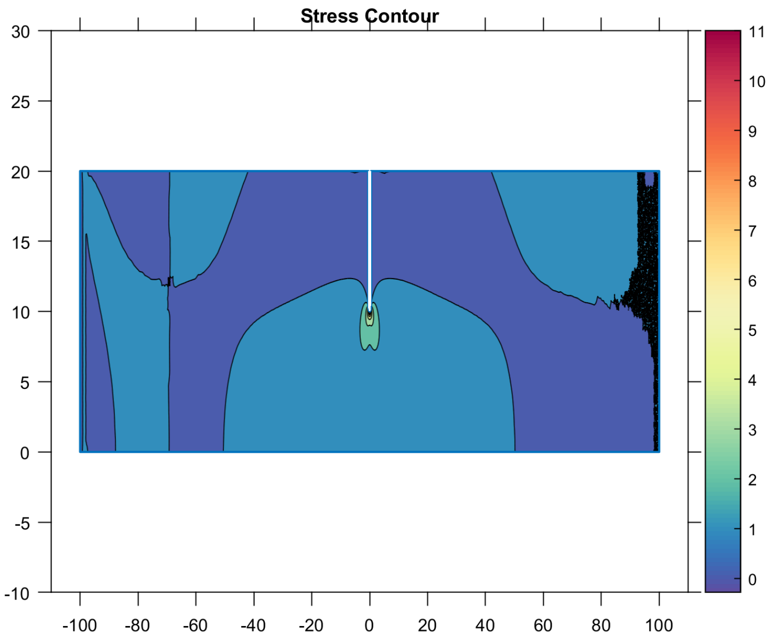


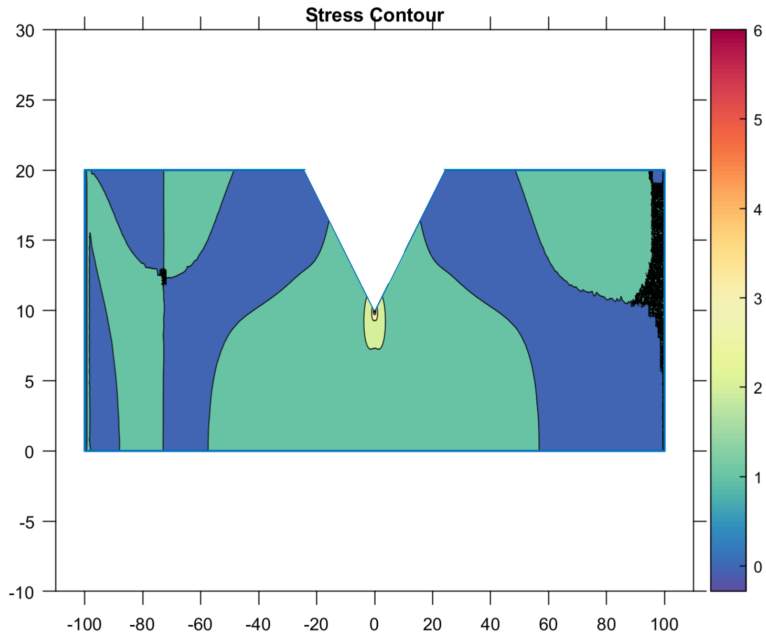
Figure 5.2: Stress Contour with ANSYS for r=0.05 mm, w=135°and n=72



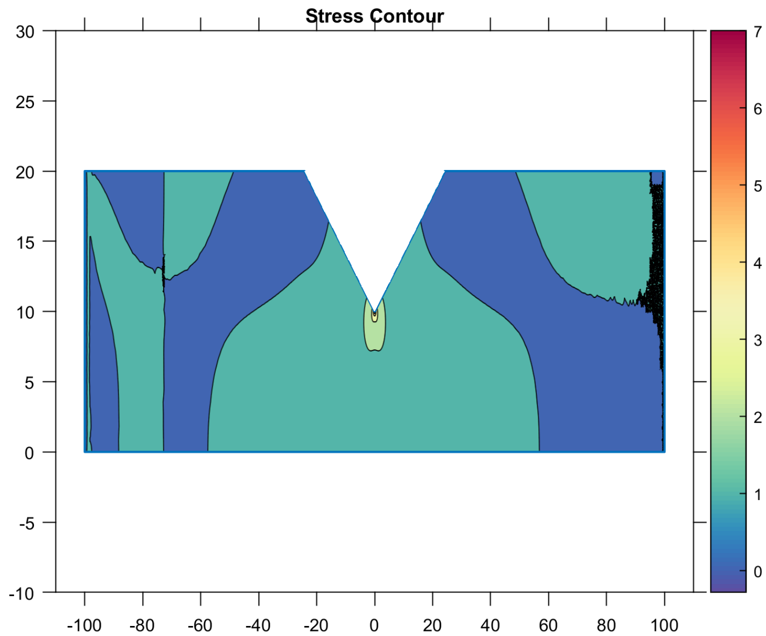
(a) radius 0.05 mm and angle 0 °



(b) radius 0.5 mm and angle 0°

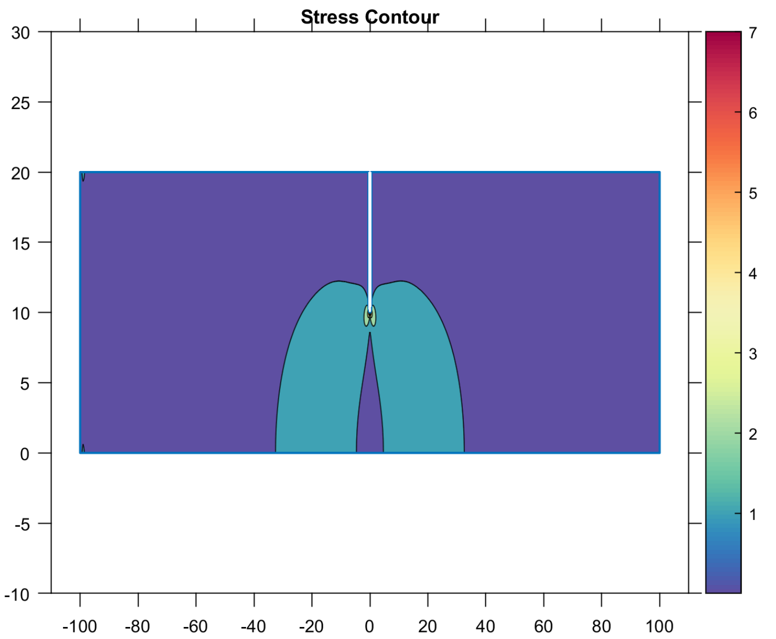


(c) 0.05 mm and angle 135°

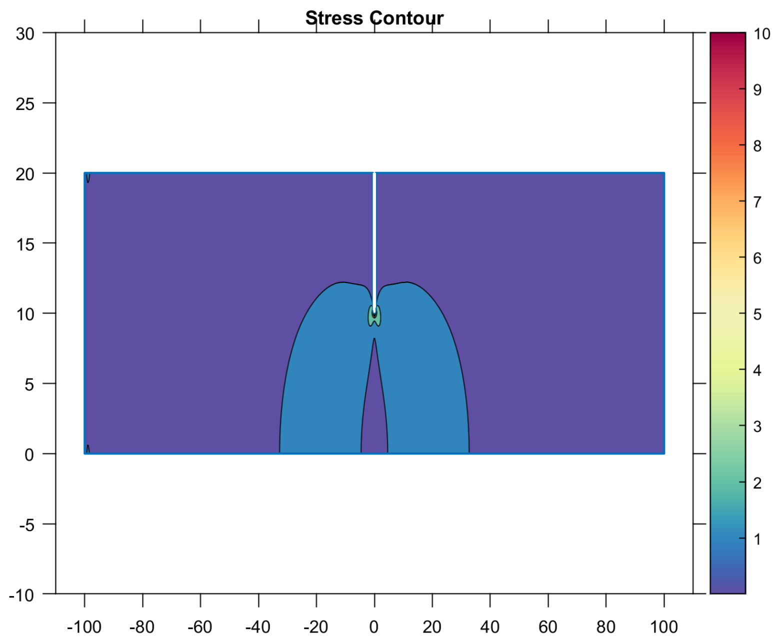


(d) radius 0.5 mm and angle 135°

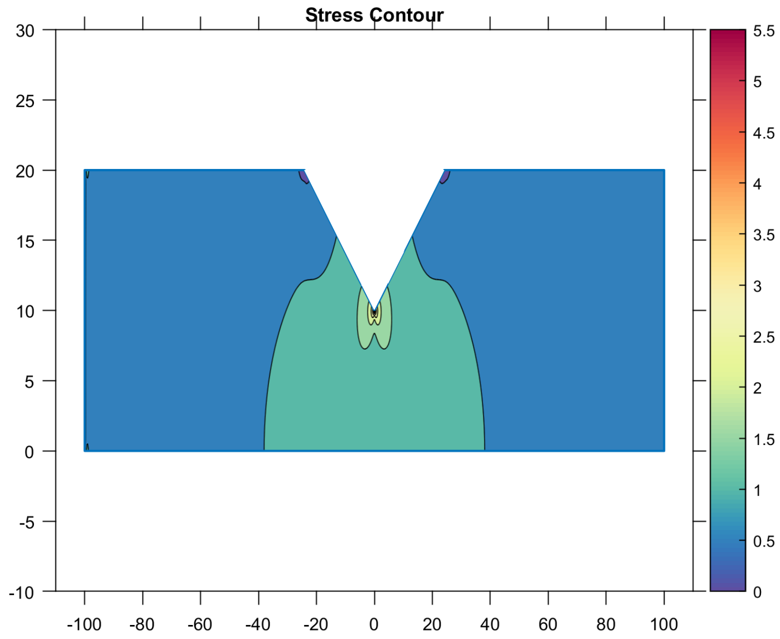
Figure 5.3: First Principal Stress Contours with MATLAB Program



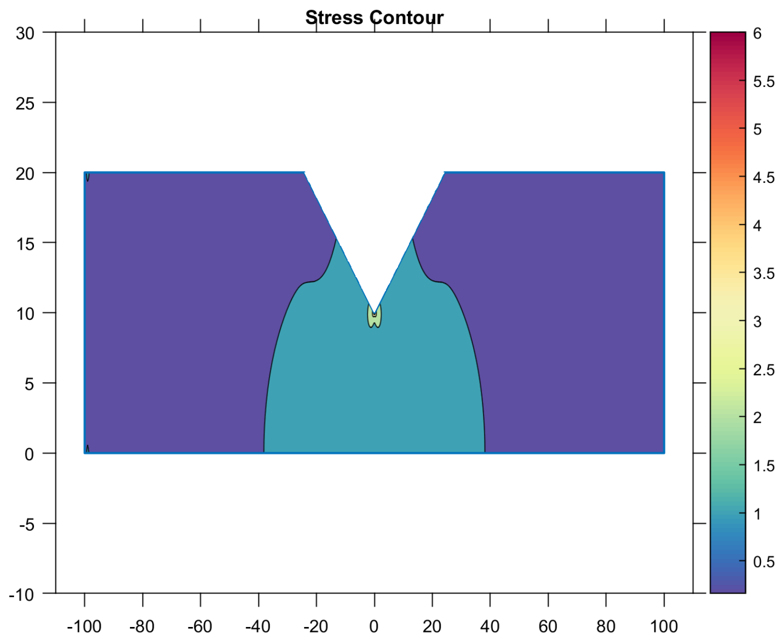
(a) radius 0.05 mm and angle 0°



(b) radius 0.5 mm and angle 0°



(c) 0.05 mm and angle 135°



(d) radius 0.5 mm and angle 135°

Figure 5.4: Von Mises Stress Contours with MATLAB Program

6 Validation of Area Method

There are no comparable data for AM for this example, since a simple numerical example is used to validate the code that has been used for AM. To define an example as close as possible for AM calculation, stress concentration has been calculated for a circle shape. Stress magnitudes inside the circle are given as the following function:

$$\sigma(X, Y) = X + Y, \tag{6.1}$$

whereas $\sigma(X, Y)$ is the stress in X, Y positions. As mentioned before AM is calculated by averaging the stresses over the critical area. Thus stress concentration can be calculated as:

$$Effective\ Stress = \iint_A \frac{\sigma(X, Y)}{A}\, dx\, dy \tag{6.2}$$

in which A is equal to the area of the calculated circle. This equation also can be expressed as:

$$\int_{min(X)}^{max(X)} \int_{min(Y)}^{max(Y)} \frac{(X + Y)}{\pi R^2}\, dx\, dy \tag{6.3}$$

or

$$\int_0^{2R} \int_0^{2\pi} \frac{(rcos\theta + rsin\theta)}{\pi r^2}\, dr\, d\theta. \tag{6.4}$$

Here min(X), min(Y), and max(X), max(Y) indicate the minimum and maximum positions of the coordinates which equal "center of the circle ± radius". r is the radius and θ is the angle that indicates the direction of R. Diameter of the circle equal to $2R$.

Figure 6.1 shows the stress contour of this function. The radius of this circle is assumed to be 1 mm, and the center of the circle is (1,1). Stress concentration can be calculated numerically 2.54 by using integral rules. After applying the MATLAB code, the same result has been found, thus AM calculation with this code is validated.

Figure 6.1: Stress Contour for the Example Geometry

7 Comparison of Gradient Methods

The theory behind the gradient methods is explained in the previous chapters. To compare the results of different gradient methods. Results of the FEM analysis are imported to the MATLAB with a help of a script.

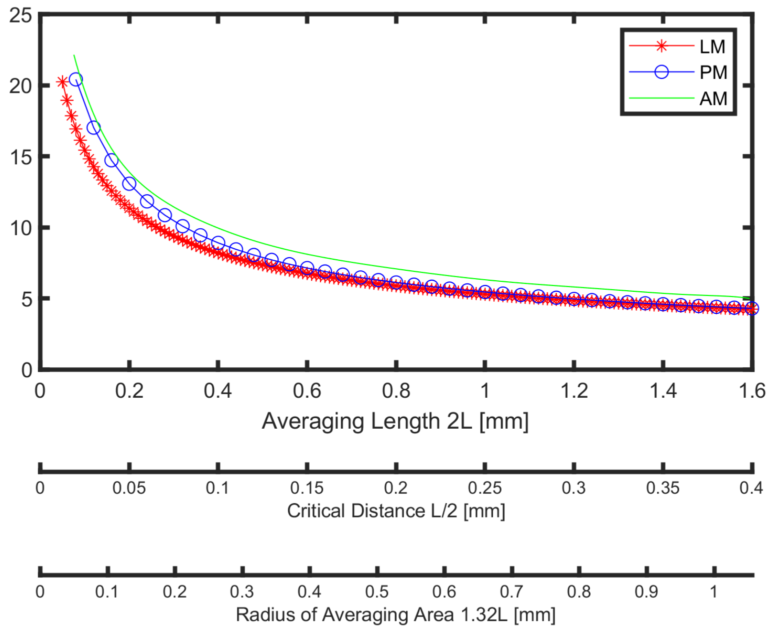
7.1 Analysis with Differient Gradient Methods

As mentioned before comparison of PM and LM are executed before in the previous study of (Braun, Müller, et al., 2020). In this project also both methods compared with a different script and stress contour is displayed with the help of MATLAB software. In addition, effective stresses are calculated with AM. As expected results of the PM and LM are very similar. Stress concentration is calculated for different effective lengths. Effective length is different for each method. As mentioned before even though not have the same effective lengths, their effective lengths have a relation. A correlation between all three methods can be set up. Critical distances for PM, LM, and AM can be written as:

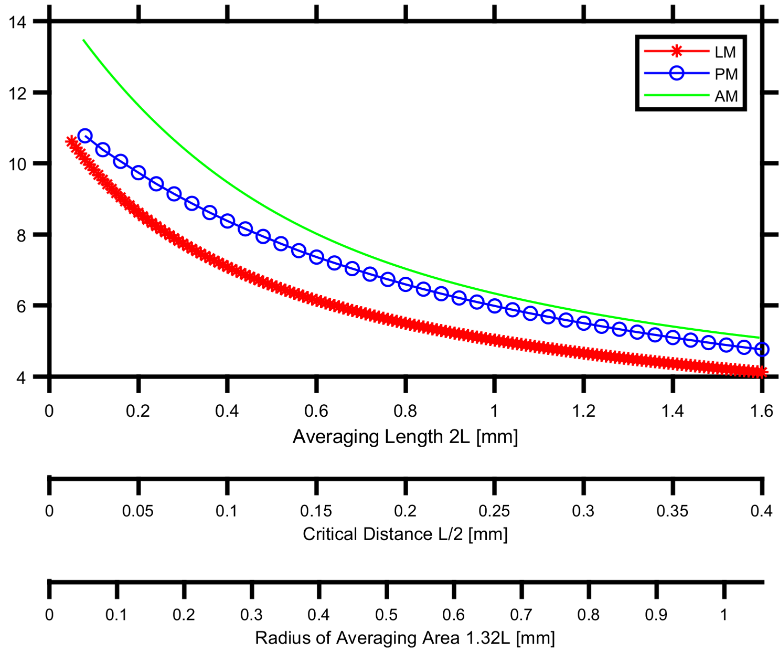
$$\begin{aligned} a &= L/2, \\ \rho^* &= 2L, \\ R &= 1.32L. \end{aligned} \tag{7.1}$$

Here a indicates critical distance for PM, ρ^* indicates averaging length for LM and R indicates critical radius for AM. L is changing from the value 0.01 mm to 0.4 mm. The geometry of the area, which is calculated to obtain the stress concentration, is severely important for AM. For V-shaped notch geometries, it is advised to calculate AM with a half-circle area. Especially for AM, it is extremely important to have more elements in mesh to define the geometry. Otherwise, interpolation between nodes can be difficult and unreliable.

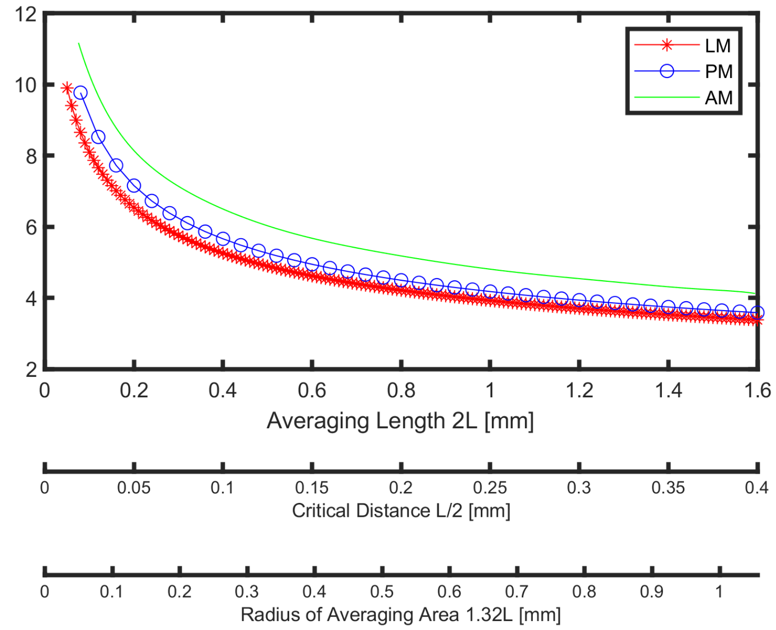
Stresses are calculated with the help of the ANSYS program for the element nodes. Stresses between nodes are calculated with interpolation methods of a MATLAB function. Thus a complete stress contour can be drawn. Piecewise Linear Cubic Interpolation method is used for PM and LM. For LM a trapezoidal integration method is used for averaging stress concentration. In the case of AM, integration is more challenging, since a double integration between integration points is needed. Integral2 integration function is used for calculating this double integration, but first, the area had to be calculated. The measured Area for AM is calculated with a help of a MATLAB script.



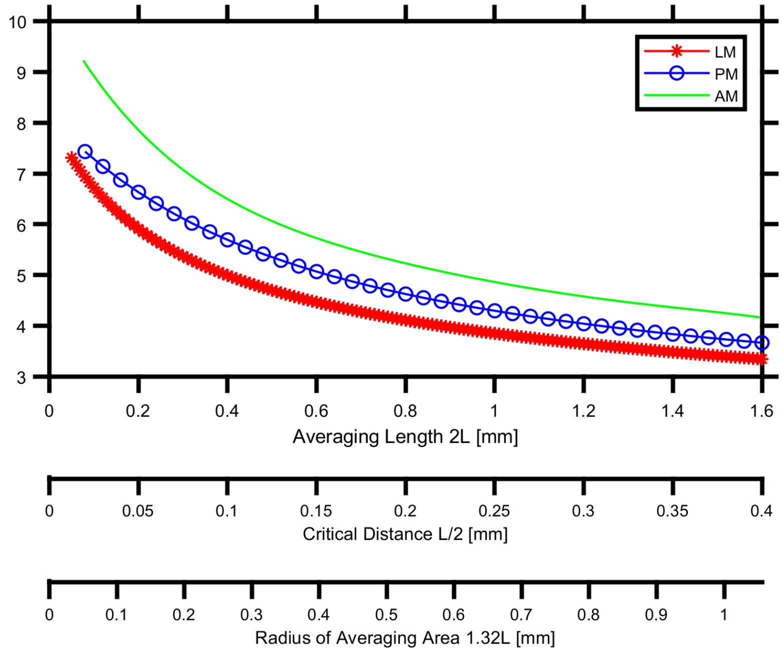
(a) radius 0.05 mm and angle 0°



(b) radius 0.5 mm and angle 0°



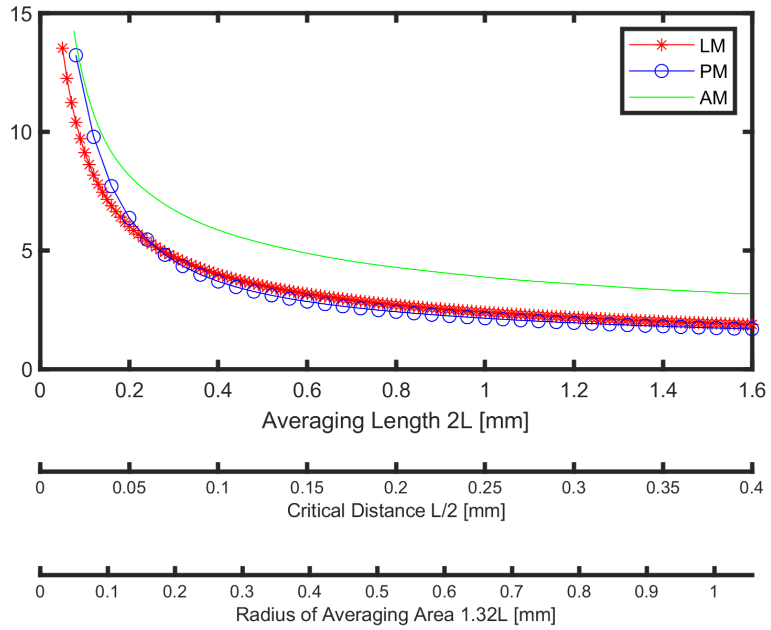
(c) radius 0.05 mm and angle 135°



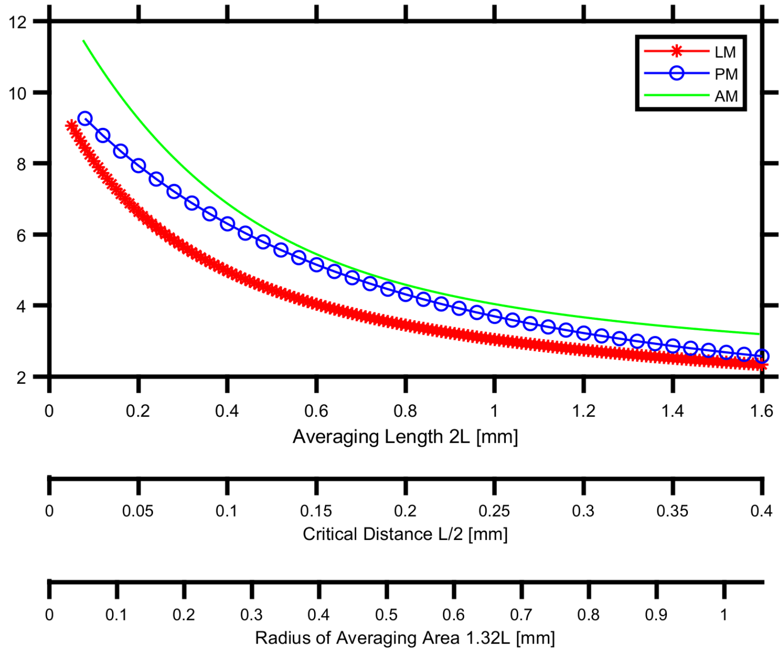
(d) radius 0.5 mm and angle 135°

Figure 7.1: Comparison of Gradient Methods for 1.Principal Stresses

It can be seen from the figures that even though PM gives higher results, PM and LM have the almost same result for all the cases. On the other hand, AM gives a much higher result for all the cases. It can be seen from Figure 7.1 and Figure 7.2 that for blunt notches (notch radius 0.5 mm) the difference between PM and LM is higher than sharp notches (notch radius 0.05 mm). The closest results for all the methods are obtained from the geometry with radius 0.05 mm with

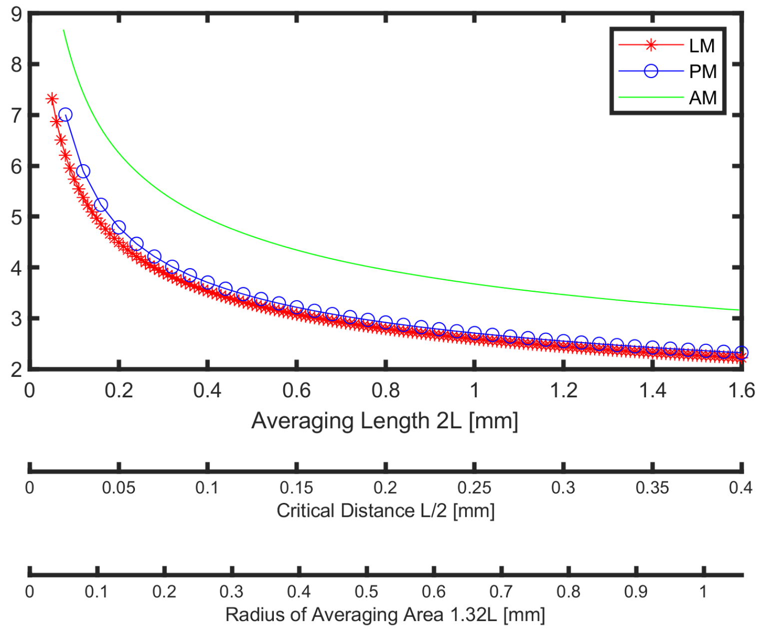


(a) radius 0.05 mm and angle 0°

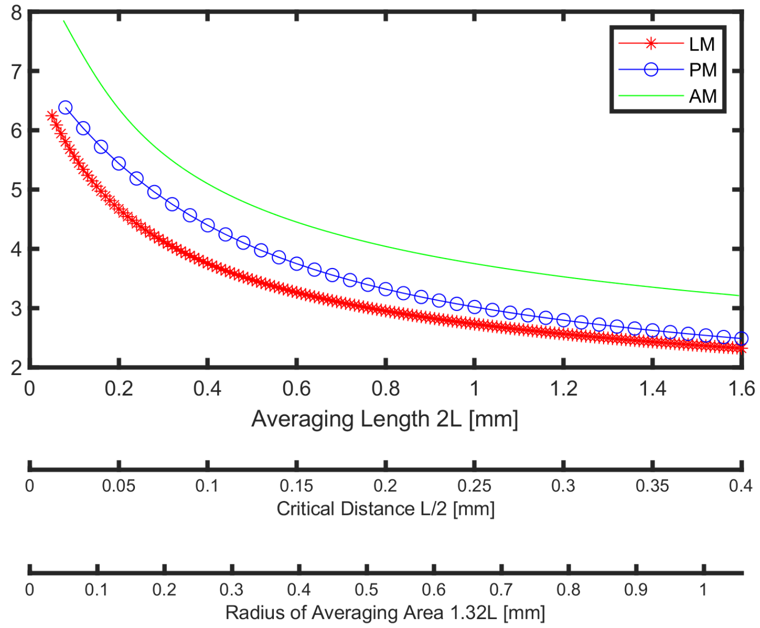


(b) radius 0.5 mm and angle 0°

an opening angle of 0°. In all cases, AM is giving much higher results in smaller distances. It is likely occurring because of the numerical integration. In smaller distances calculated area is also very small and the tolerance of the integration becomes more important. Comparison between different gradient methods in detail has done next section with the help of a deviation factor.



(c) radius 0.05 mm and angle 135°



(d) radius 0.5 mm and angle 135°

Figure 7.2: Comparison of Gradient Methods for Von Mises Stresses

7.2 Deviation between Different Gradient Methods

To compare different methods with each other a deviation factor has to be defined. The deviation between LM and PM is done in a previous project (Braun, Müller, et al., 2020). In addition to these in this project deviations between AM and the other methods are calculated. The deviation factors for AM can be defined as:

$$\begin{aligned} \Delta_{PM} &= \sigma_{eff,PM}/\sigma_{eff,AM}, \\ \Delta_{LM} &= \sigma_{eff,LM}/\sigma_{eff,AM}, \end{aligned} \tag{7.2}$$

in which Δ and σ_{eff} show deviation factor and effective stress respectively. Indices indicate which gradient method does the symbol belongs to. Deviation percentage can be calculated as:

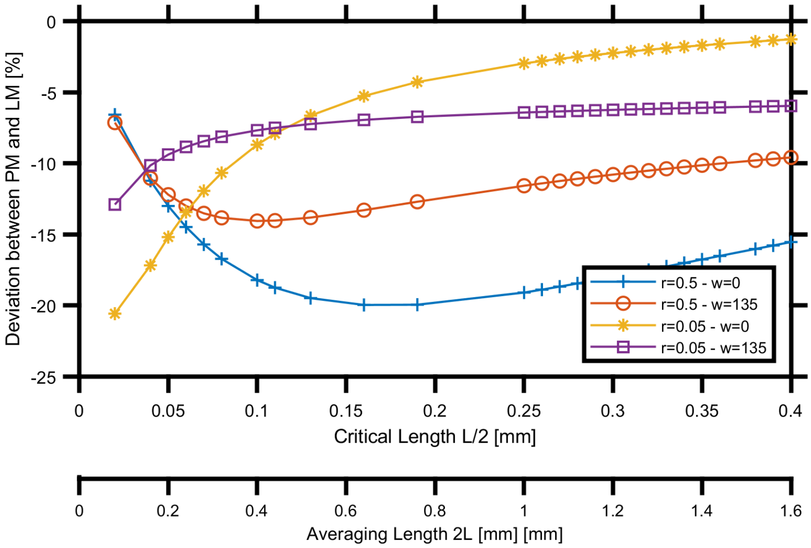
$$\%Deviation = (1 - \Delta)100. \tag{7.3}$$

Figure 7.3 shows changing of the percentage deviation (defined as $1 - \sigma_{eff,PM}/\sigma_{eff,LM}$) depending on the critical length and averaging length between PM and LM. Minimum deviations are observed for both cases von mises and the first principal stresses in this figure with the opening angle 0° and notch radius 0.05°. For the first principal stresses this value reaches something around 1.28% at ρ^*=1.6 *mm* and a=0.4 *mm*, and for the von mises stresses it reaches 0% at a=0.062 *mm* and ρ=0.25 *mm*. The maximum absolute deviation value for the first principal stress is equal to -20.58% with the same combination at a=0.02 *mm* and ρ=0.08 *mm*. If both cases are compared, it can be seen that von mises stresses have resulted higher deviations. The maximum absolute value for von mises is equal to -27.17% at the same position and the same combination. It can be seen from Figure 7.3, that the same radii have the same reactions as the critical length changes. For sharp notches with radii 0.05 *mm*, the difference between PM and LM decreases as the distance increases. This is vice verse for the blunt notches, as it is observed with radius 0.5 mm.

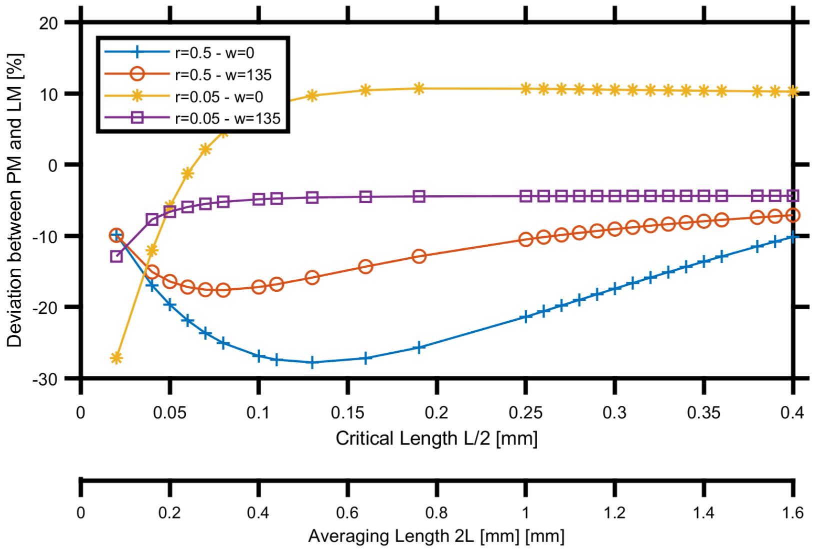
Figure 7.4 shows the percentage deviation factor between PM and AM for different critical distances and averaging radii. The curves are similar to the previous graphic except the combination with r=0.05 *mm* w=0°, in this combination deviation starts to decreas at a=0.29 *mm*. In addition the deviations are higher compared to previous figure. This is an expected result since the difference between AM and PM are higher than the difference between PM and LM. Deviation never becomes zero for the first principal stress. The smallest value it takes 5.287 % with r=0.5 *mm* and w=0°. On the other hand for the von mises stresses deviation percentage reaches zero at the notch tip for r=0.05 *mm* and w=0°. Similar to the previous figure we can conclude here aswell same radii have same tendencies and again von mises stresses have higher deviations. The maximum deviation percentage observed with the combination r=0.05 *mm* and w=0° which is interesting because the minimum deviation is also observed in this combination. The maximum daviation is 46.48% at a=0.34 *mm*.

In the last figure for deviation percentage factor (Figure 7.5), the deviations between AM and

LM methods are shown. The graphics are different than previous figures since in these figures there are no similarities observed for same radii. The minimum deviation percentages are observed both for first principle stress and von mises stresses with combination r=0.05 *mm* and w=0°. The minimum values are 18.79% at the notch tip, 13.36% at ρ=1.6 *mm* for von mises and first principal stresses respectively. The maximum deviation percentage is observed for von mises stresses with r=0.05 *mm* and w=0° at ρ=1.44 *mm*. For the first principal stress the maximum value is reached with combination r=0.5 *mm* and w=0° at ρ=0.18 *mm*. The maximum values are 24.75% and 40.26% for first principal and von mises stresses respectively. The curves with same angles with different radii have asymetrical tendencies. The asymettry can be better observed for von mises stresses.

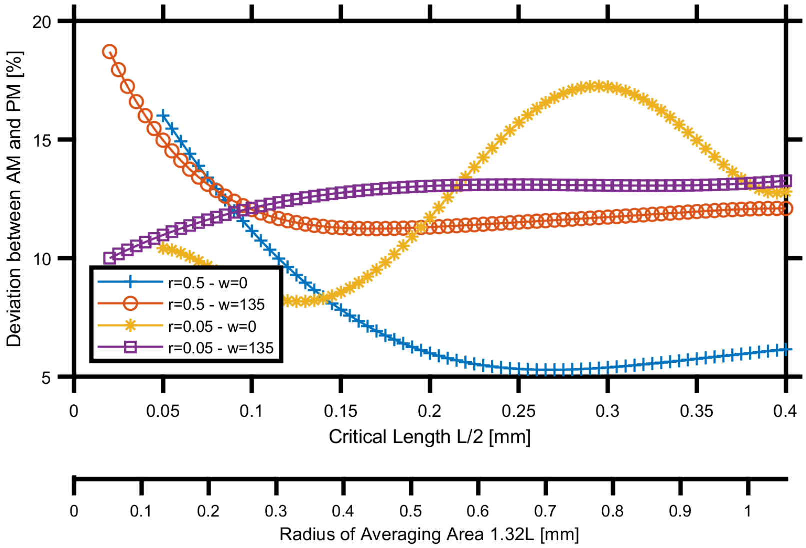


(a) First Principal Stress

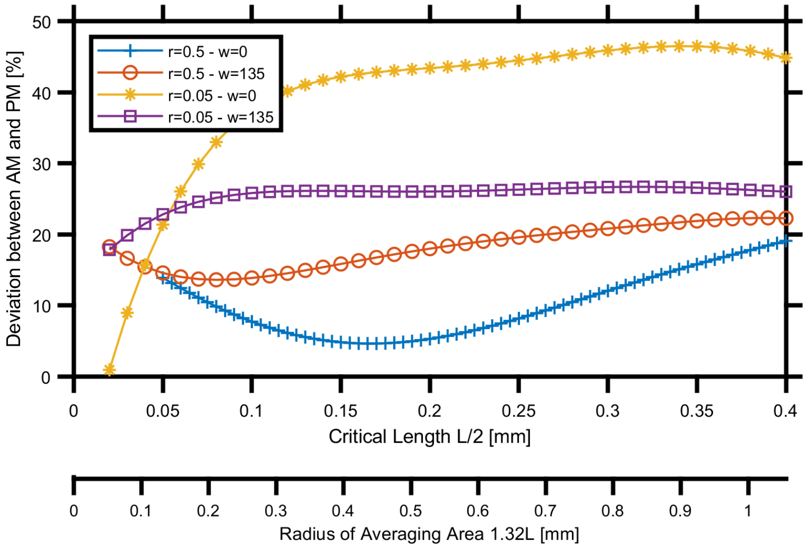


(b) Von Mises Stress

Figure 7.3: Deviation between Point Method and Line Method for First Principal and Von Mises Stresses

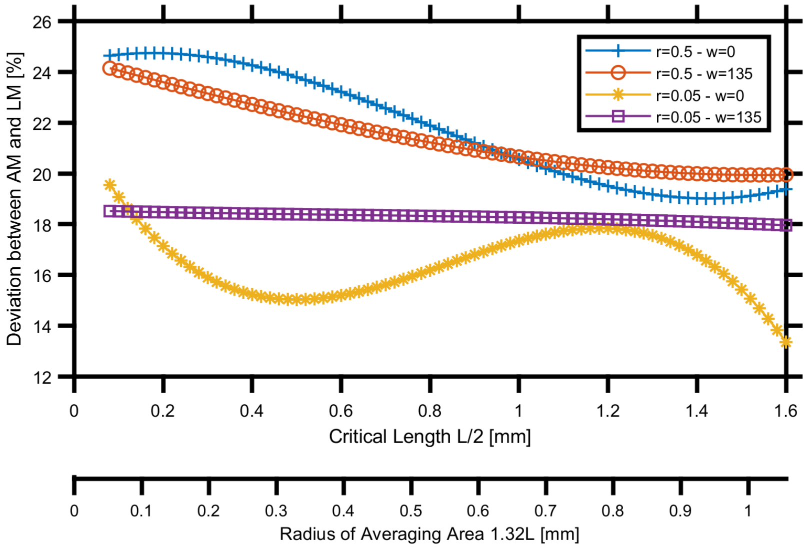


(a) First Principal Stress

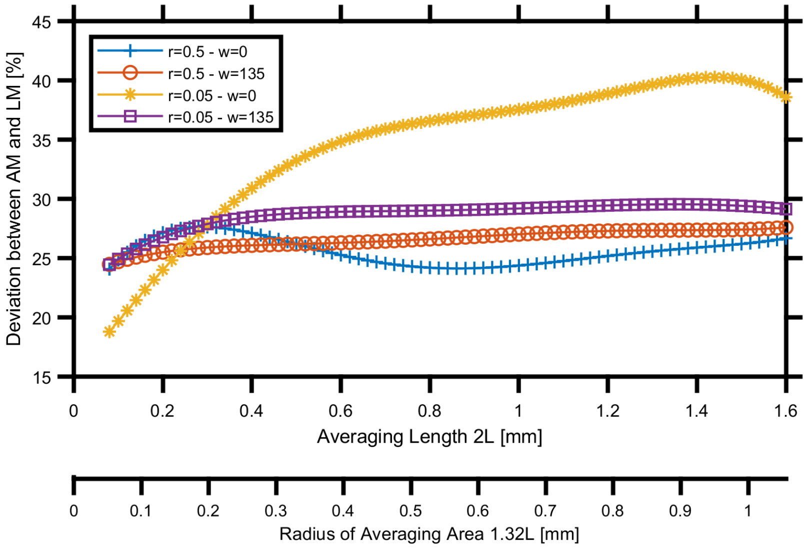


(b) Von Mises Stress

Figure 7.4: Deviation between Area Method and Point Method for First Principal and Von Mises Stresses



(a) First Principal Stress



(b) Von Mises Stress

Figure 7.5: Deviation between Area Method and Line Method for First Principal and Von Mises Stresses

7.3 Required Ratios for Different Gradient Methods

It can be seen from the previous chapter that the deviation percentage is rarely zero, which means critical distances can not be taken as suggested. To understand which critical distances should be used for different methods, a required ratio has been defined. Required ratio is the ratio between critical distances, which ensures two different methods have the same stress values. The required ratio between PM and LM can be shown as $q_{(PM,LM)} = a/\rho^*$. For the area method required ratios are $q_{(AM,PM)} = R/a$, and $q_{(AM,LM)} = R/\rho^*$, to ensure the same stress value with PM and LM respectively. To understand the effect of the notch radius and opening angle easily, the data set is fitted to polynomials.

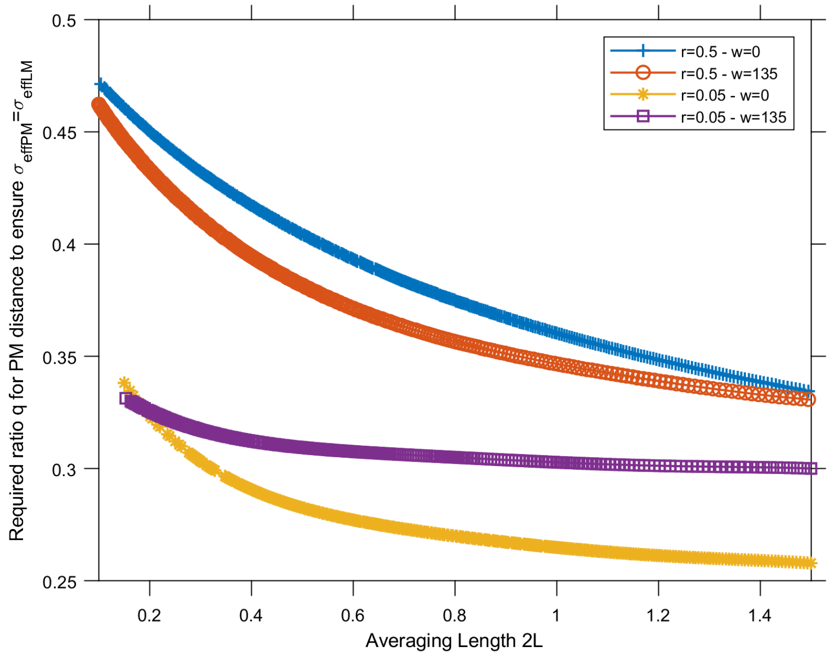
Figure 7.6 shows the required ratios q for PM and LM to ensure the same stress values with responding averaging lengths. The results comply with the previous study of Braun, Müller, et al., 2020. The suggested ratio for PM and LM methods was 1/4 with $a = 0.5L$ and $\rho^* = 2L$. However, this value has never reached the first principal stress, which can also be seen by the deviation percentage factor. The closest value reached is 0.254 with r=0.05 mm and w=0° at ρ^*=1.6 mm. As mentioned before maximum stresses were observed at the notch tip. As the distance from the notch tip increases, values come closer to the suggested value of 0.25. Von mises stresses reach a value of 0.25 with r=0.05 mm and w=0° at ρ^*=0.246 mm. Other combinations never reached 0.25, the closest value they take changes between 0.28-0.29 at ρ^*=1.6 mm.

Required ratios between AM and PM for the radius of the averaging area can be seen in Figure 7.7. The expected ratio for these two methods is 0.382. Distances closer to notch tip are too high for AM, thus the required ratio can only be calculated when AM values become smaller enough. This can be achieved with smaller distances for the first principal stresses compared to the von mises stresses. The expected ratio is reached only for the first principal stress with r=0.05 mm and w=0° at R=0.201 mm. This also complies with Figure 7.4. The maximum required ratio for von mises stresses reaches q=0.210 with r=0.5 mm and w=0° at R=1.056 mm, which is also the closest ratio to the expected one. The difference between recommended ratio and the calculated one is higher for von mises stresses. It can be seen from the figures that the slope is higher for w=135° compared to a combination with the same radius, which means the change of the ratio between AM and PM are higher for w=135° than w=0°.

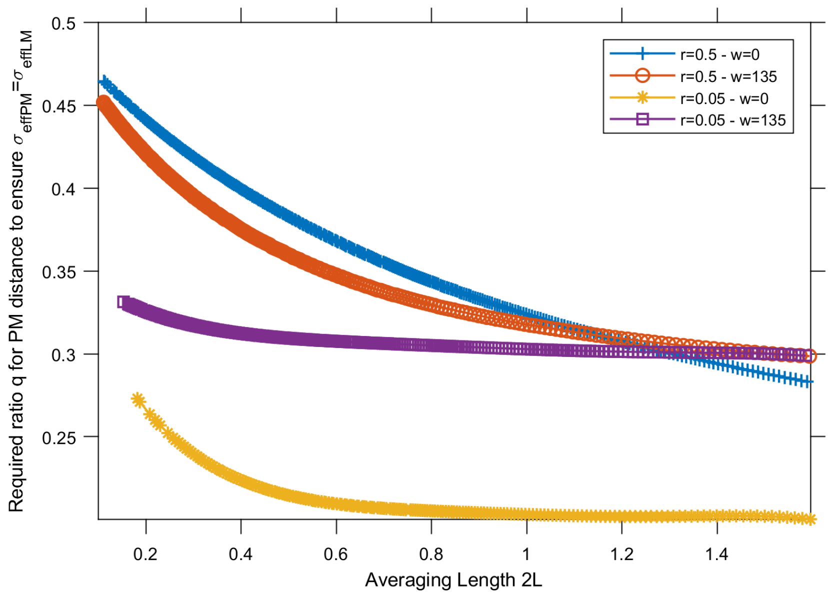
Figure 7.8 shows the ratios between AM and LM. Recommended ratio between AM distances and LM distances is 1.515 ($2L/1.32L$). This ratio is achieved only for first principal stress with r=0.05 mm and w=0° at R=0.345 mm. For von mises stresses the nearest ratio to the recommended one is 0.78 with the same combination at R=0.423 mm. For r=0.5 mm both angles for both stresses come closer to the recommended ratio with increasing critical distance. The combination r=0.05

mm and w=0° takes for every critical distance closest ratio to the recommended one. Required ratio q, for this combination, has an increasing trend for the first principal stresses and decreasing trend for von mises stresses. The notch with r=0.05 *mm* with angle 135° has the minimum slope for both stresses. It has a decreasing trend for von mises stresses on the other hand for the first principal stresses it has an increasing trend.

It can be seen from the figures all three figures that the required ratio is changing depending on the critical distance. Compared to combinations with w=135°, combinations with w=0° are effected more by critical distances. Almost in every combination closest values to the recommended ratio is reached with combination with r=0.05 *mm* and w=0°. Combination with r=0.05 *mm* and w=135° is always the least affected one by changing the critical length and its opposite, combination with r=0.5 *mm*, w=0° is the most affected by changing critical distances.

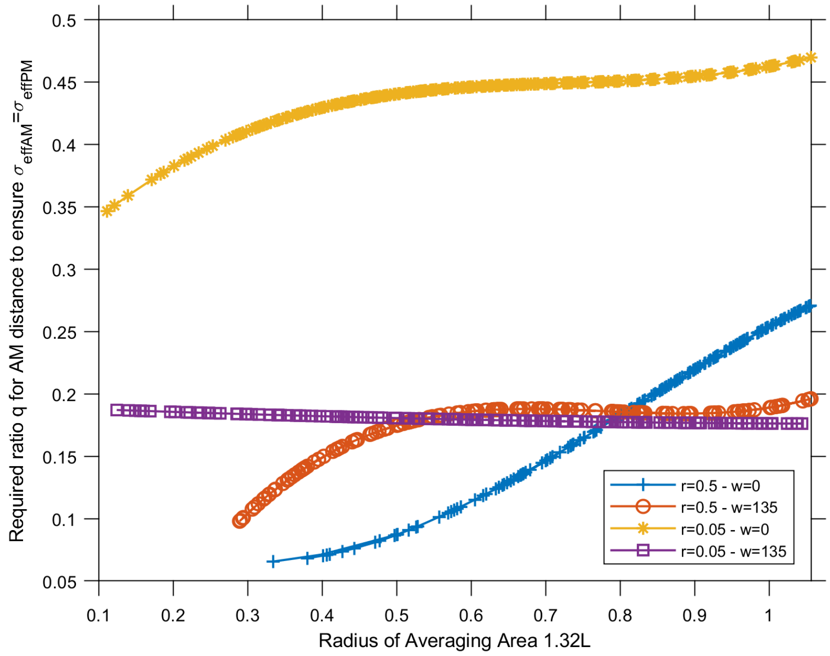


(a) First Principal Stress

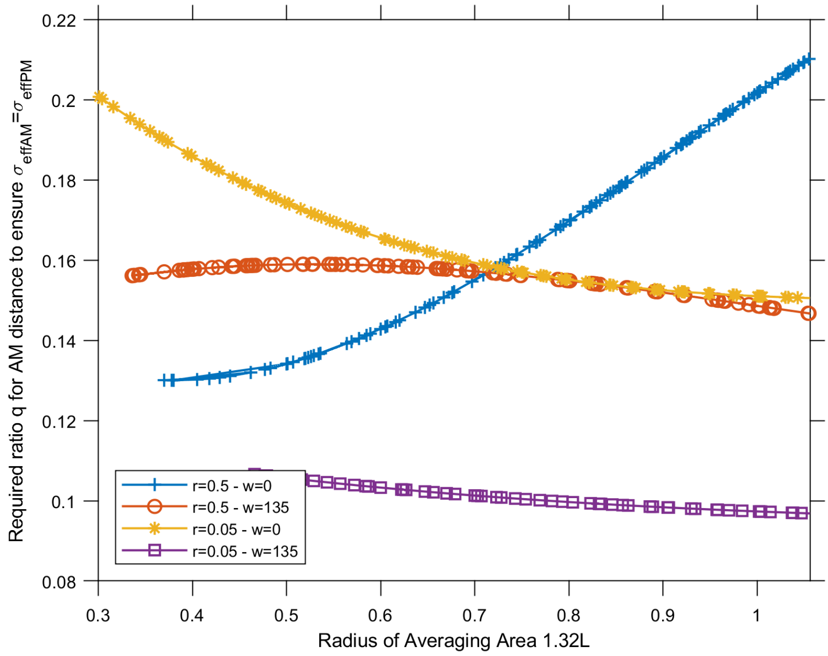


(b) Von Mises Stress

Figure 7.6: Required Ratio q for point method distance a to ensure same stress values for Point Method and Line Method for First Principal and Von Mises Stresses

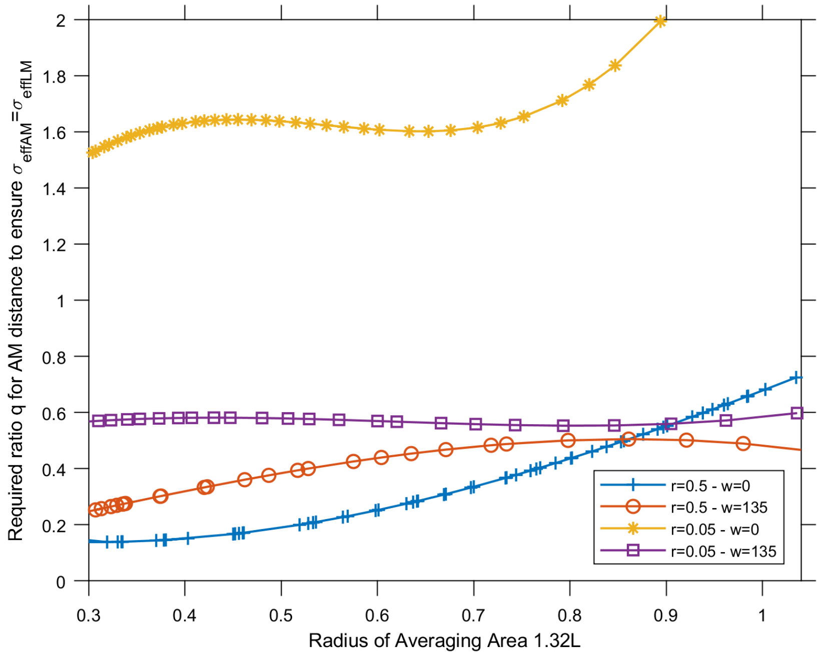


(a) First Principal Stress

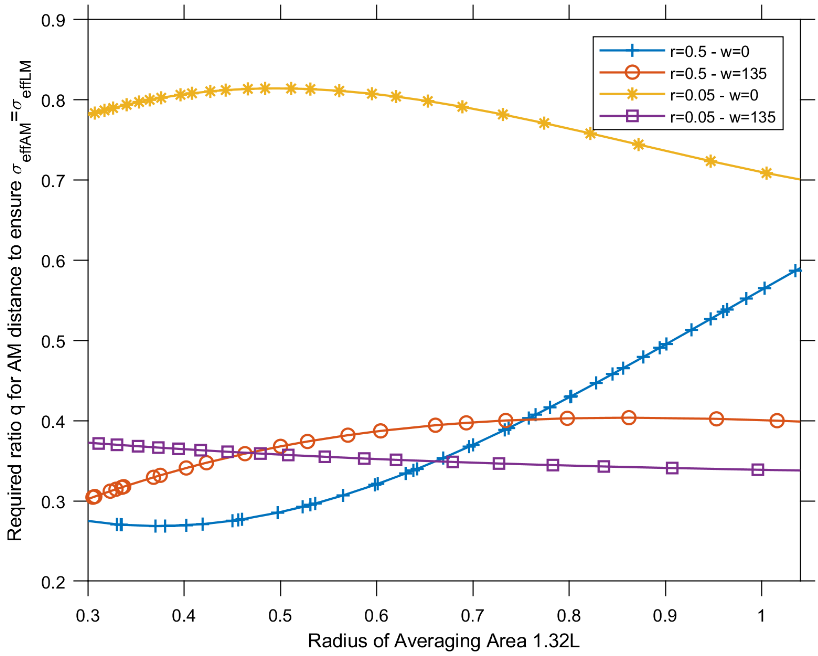


(b) Von Mises Stress

Figure 7.7: Required Ratio q for area method distance R to ensure same stress values for Point Method and Area Method for First Principal and Von Mises Stresses



(a) First Principal Stress



(b) Von Mises Stress

Figure 7.8: Required Ratio q for point method distance R to ensure same stress values for Area Method and Line Method for First Principal and Von Mises Stresses

8 Discussion

In previous sections, results are displayed and explained. A discussion about these results is necessary. As it can be seen from Figure 7.1 and Figure 7.2 AM gives most of the time higher stress concentration values compared to the other two methods. Especially as coming closer to the notch tip, this difference increases. This may result from the complications of the integration of the geometry. To have a more accurate result, the element number has to be increased so that errors caused by integration may be reduced. Furthermore, notch geometry and material type may cause this difference. Similar to the study of Braun, Müller, et al., 2020 difference between gradient methods are higher for von mises stresses.

To have a better understanding of the difference between different gradient methods deviation percentages are used. Deviations graphics can be seen in Figure 7.3, Figure 7.4, and Figure 7.5. Highest and the smallest deviations are given before. The deviation between PM and LM has very similar characteristics to the deviation between AM and PM. In both deviation graphics, the same radii have the same shape of curves, the only exception is the first principle stress with the combination r=0.05 mm and w=0° has different shapes. This may also be seen from Figure 7.1 for the same combination. In this combination for the first principle stresses LM and the PM have almost no difference after a=0.3 also AM comes closer to the others. This combination is probably the most basic geometry. Notch is very sharp and there is no opening angle, so it is probably not very important to which gradient method should be chosen to calculate stress concentration after some distance from the notch tip. The deviation between AM and LM can not be completely explained with differences in the radii. Still, the values of the same radii are closer to each other than the other ones, but opening angles are also playing a big role in the shape of the curves. Especially for notch geometry with no opening angle, deviations fluctuate relatively more than the notch geometry with opening angle w=135°. So it might be easier to find the error between two methods for the geometry with large opening angles.

Since there are deviations between different gradient methods, finding the real relationship between critical distances is an interesting subject. To do that a required rato r is defined. Graphics in Figure 7.6, Figure 7.7, and Figure 7.8 show required ratios for different critical distances. Ex-

pected required ratios reached most of the time only with r=0.05 mm and w=0°. For other combinations, the complexity of the notch geometry is most likely to prevent reaching the expected value. The required ratio for r=0.05 mm and w=135° is the one least affected by the changing of critical distance. According to this result, it can be assumed that having a recommended ratio for critical distances is more logical for v-shaped notch geometries because it is not changing much with different critical distances.
AM is one of the most complex gradient methods to calculate stress concentrations. There are a lot of depending parameters, such as notch geometry, integration methods, the geometry of chosen area, material distances, etc. The results found in this study include only a basic plain specimen with basic notch geometries. Further discussions can be made in the future with more researches about this subject. The calculated material can be supported with laboratory experiments or it can be applied on different material geometries such as welded joints.

9 Conclusions

It is really easy to use gradient methods with today's computer capacities. Gradient methods are one of the most useful methods for calculating stress concentration of a notched geometry, but their outcome is not always close as expected. The critical distance between different methods is changing depending on the notch geometry. For sharp notches with small opening angles suggested critical distances can be used with small errors, however as the geometry becomes more complicated the suggested critical distances may give higher results. The difference between PM and LM is smaller compared to AM. The maximum deviation between PM and LM is measured at around 27.17%. It is complying with Braun, Müller, et al., 2020 and other studies. On the other hand, the maximum deviation between PM and AM is 46.48%, and between AM and LM 40.26%. AM needs smother meshes because calculated stresses in different directions are also included in the calculation. In addition, area geometry is really important for AM. Depending on the chosen geometry this difference may change. In general von mises stresses result in higher deviations than the first principal stresses.

The calculations do not give the same results with recommended critical distances, thus required ratios have been calculated in order to understand the relationship between critical distances. For most cases, sharp notch with no opening angle is given the closest value to the expected ratio. As the geometry of the notch changes relationship between critical distances is also changed. The required ratios are affected extremely by opening angles. The higher opening angle (w=135°) is affected less by distance from the notch.

Required ratios are fluctuating most of the cases so there is no relationship between critical distances of different methods, which means different S-N curves are needed for each method.

10 Outlook

This project thesis is alone not enough to prove the real capacity and the limitations of gradient methods. To understand the relationship between different gradient methods, this study should be improved and supported with further researches. Gradient methods and their relationship is not a new subject but the application of these methods are relatively small than the theoretical part of the subject. As computer systems developed, it is far easier to use these methods in real-life examples.

There are really small examples for AM. In the future, there should be more examples of AM to understand its real capacity. This study can be improved in the future by laboratory tests or application on welded joints for different environmental conditions. These types of examples have previously been done in several studies, such as Santus et al., 2018, Karakaş et al., 2018, Braun, Milaković, et al., 2021 and Braun, Ahola, et al., 2022. Instead of using a basic plain stress specimen, different weld geometries with the combination of different materials and environmental conditions may be tested to ensure the results of this project thesis. Especially the comparison of AM with different methods in more complicated problems may be really interesting because AM has fewer examples in the literature.

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