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Fleet Based Schedule Optimisation for Product Tanker Considering Ship's Stability

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Purpose: Scheduling a fleet of product tankers in a cost effective and robust way to satisfy orders is a complex task. A variety of constraints and preferences complicate this attempt. Manual solutions as common in tramp shipping are not sufficient to deliver optimal and robust schedules.

Methodology: For this, we present a mixed integer linear programming formulation of the scheduling problem. Additionally intact stability calculations for each ship of the fleet are implemented in a separate program that checks the feasibility of MILP solutions and creates new cuts for the integer program.

Findings: Usually the checking of the stability criteria is done before an order will be accepted and the schedule of the ship is planed accordingly. This requires the selection of a ship a priori. Checking the admissibility of a voyage gives access to a wider variety of possible combinations.

Originality: To our knowledge fleet scheduling under consideration of intact stability requirements has received little attention in the literature. Previous works make very simple assumptions on the capacity of the ships and do not include in their linear pro- grams any stability models.

Keywords: Cargo Scheduling in Tramp Shipping, Optimization, Ship Stability, Mixed Integer Linear Programming

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1 Introduction

Shipping even if not obvious is one of the most important industries with a big impact in our everyday life. Around 90% of the world trade is transported by the shipping industry (Ronen, 2019). Deep-sea shipping is the only way for transportation of a large amount of goods over the world. For many countries shipping belongs to one of their key industries. The financial crisis 2007 has changed this industry drastically. The reduction of profit margins on one side and the vessel as a huge capital investment for a shipping company with thousands of dollars daily total operation costs on the other side have increased the focus on optimization for cost savings. The efficiency of fleet schedules has a significant financial impact for the company.

Tramp shipping as described in (Lawrence, 1972) is one of the three main modes of maritime operations which include additionally industrial and liner shipping. Tankers usually do not follow fixed itineraries as container ships. They travel according to customers' orders and needs and are very often compared with taxi services. Shipping companies have long- term contracts as well as spot cargoes. The main aim of shipping companies working in this mode is to maximize their profits by appropriately scheduling and routing the fleet for the long-term contracts which implicitly limit the choice of spot cargoes.

Scheduling a fleet of product tankers in a cost effective and robust way to satisfy as many orders as possible is a complex task. This challenge can be defined as the problem of allocating the right assets to the right cargoes, such that all orders are delivered on time, at the right ports and by taking into account customer preferences as well as legislative requirements.

These requirements vary from minimum age of the ship to its vetting records. A robust scheduling and routing of the long term contracts has a direct impact on the online scheduling problem of assigning spot cargoes to ships. In this paper the focus will be on scheduling long-term orders in an optimal way under consideration of cargo, port and especially fleet and ship stability requirements. This problem is also known as the problem of cargo routing based on the classification and descriptions made in (Al-Khayyal and S.-J. Hwang, 2007). In contrast to this, in the inventory scheduling problem the cargo demand is determined by the schedules.

Nowadays in the shipping companies the very time consuming task of fleet scheduling is done manually and based on planners' experience. This process as described in (Trottier and Cordeau, 2019) and also confirmed during an interview with a tanker shipping company from Hamburg is a very iterative, manual and difficult one. Basically, the planners try to assign the mandatory orders first and then fill up the gaps with spot cargoes in a profitable way.

Obviously it is impossible even for a very small fleet to create optimal schedules in a manual way such that all constraints and requirements are taken into consideration. The suboptimal manual solutions lead to dead locks in the assignments of orders due to the lack of robustness or to a relatively high number of idle days and therefore to considerable financial loss. Safety and security are relevant for all mode of maritime transportation, but specially tanker ships have to fulfill high standards and requirements in security, which affect directly the company operations. Ship stability is at this point of high relevance and needs to be considered when optimizing the schedules in order to guarantee safety and save costs and reputation loss due to accidents. Nowadays this process is done also in a manual way and involves the engineering department that checks individual solutions proposed by the chartering department and revokes them if the stability of the ship is not guaranteed. The main aim is to schedule a given fleet in such a way that it delivers as much cargo as possible as a whole by maximizing the profit and on the same time complying with all cargo and ships stability requirements and restrictions. The problem is formulated as a mixed integer linear problem (MILP) with lazy constraints being generated by a separate module that calculates the stability of each ship given the cargo load from the MILP, as shown in Figure 1. Including this step in the optimization leads to more realistic results then existing MILP formulations, that are limited only to draft restrictions for ports and do not take ship design into consideration. Numerical experiments have shown that a large number of feasible solutions of the MILP do not comply with the stability requirements. A detailed description of the problem with all constraints and the mathematical formulation for the MILP and ships stability calculation can be found in section 3.

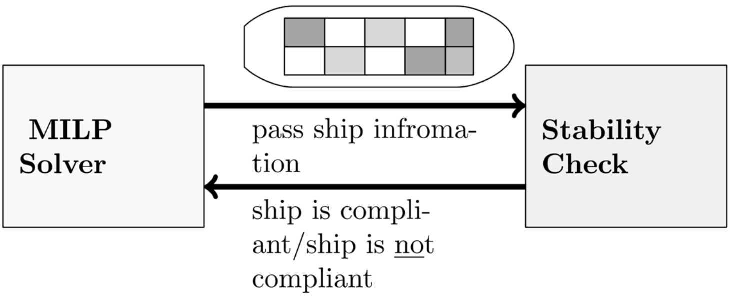

Figure 1: Program interaction

This paper is organized as follows. In section 2 the relevant literature is presented. The problem is described in detail in section 3. In section 4 first numerical experiments are presented and their results are analyzed. Concluding remarks and future work can be found in section 5.

2 Literature Review

Optimization models and methods for ship routing and scheduling problems are relatively new for this old and traditional transportation industry. Thus problems are considered to be complex and not easy to classify and solve. In comparison to air, train or public transportation shipping is characterized by a high uncertainty in the voyage and cargo information, which limits the usage of deterministic models. Nevertheless in the last years a significant growth in the number of scientific publications in this field has been recognized. The problem of ship routing and scheduling presented in this paper is considered as a special case of the general vehicle routing problem, see (Desrosiers et al. 1995). Methods and formulations used for this class of problems can not be overtaken for the cargo scheduling problems in tramp shipping due to the special properties of maritime transportation including the fact that ships operate 24 hours a day and have special physical and legislative constraints. General surveys on ship routing and scheduling can be found in (Ronen, 1983 and Christiansen, Fagerholt, and Ronen, 2004). In (Ronen, 1983) one of the first detailed classification schemes for ship routing and scheduling problems is introduced. This paper presents also a first analysis of the differences between the vehicle and ship scheduling and routing problems. It includes almost all modes of shipping like short and long-term as well as liner, tramp and industrial shipping and further criteria that support the classification of the problem and the choice of the appropriate solution methods. The proposed classification schema focuses on the cost optimization and includes only port entry constraints for the ship and no further ship design or stability constraints. Same holds for the classification schema presented in later papers as for example (Christiansen, Fagerholt, and Ronen, 2004). One of the latest general survey papers is (Christiansen, Fagerholt, Nygreen, et al., 2013). (Appelgren, 1969) and (Appelgren, 1971) are two of the first papers dealing with the problem of the optimal assignment of cargoes to a given fleet under consideration of all tramp shipping specific requirements. In both papers the author presents a mixed integer linear program formulation of the scheduling problem and uses Dantzig-Wolfe decomposition and branch and bound to solve small instances of the problem. In the following years the formulation of the cargo routing and scheduling problem has evolved by including more parameters and making the models more realistic as it is being stated in (H.S. Hwang, Vi- soldilokpun, and Rosenberger, 2008). Varying the ship's speed and using it as a decision variable (Norstad, Fagerholt, and Laporte, 2011) has lead to significant changes in the results and cost saving (Fagerholt et al., 2013). On the other side the introduction of this variable has increased the complexity of the MILPs as well as the solution time. Beside this sailing much slower than service speed may look good in theory but often forces the ship to put the helm at big angles which causes a major increase in resistance and thus implifying an increase in operational cost and risk of casualty. For solving large and real world data instances different problem specific heuristic methods like the ones proposed in (Malliappi, Bennell, and Potts, 2011 and Jetlund and Karimi, 2004) as well as known local search methods including tabu search, as presented in (Korsvik, Fagerholt, and

Laporte, 2010), have been developed. A further parameter being considered in later works as in (Brønmo et al., 2007) is the parcel size, which in most cases is considered as fixed. In this work a genetic algorithm that takes advantage of the problem's specific structure and properties is implemented and enables the solution of large data instances within reasonable computation time. In (Cóccola and Méndez, 2015) a combination of a small instance MILP and a heuristic that delivers near-optimal solutions for large-scale cargo scheduling problems is presented. The focus of the mathematical model lies in costs minimization and the constraints do not include any ship design requirements from a stability point of view. The benchmark paper (Hemmati et al., 2016) gives a state of the art overview of the formulations for cargo routing and scheduling problems and proposes a benchmark suite for different real world size problems. It is one of the first papers dealing with the lack of standardized data for scheduling problems in tramp shipping and proposing data for benchmarks. In this paper a modern implementation of the adaptive large neighbourhood search (ALNS) heuristic method is presented which has been developed firstly by (Ropke and Pisinger, 2006). One of the latest papers (Homsi et al., 2018) presents a branch and price algorithm as an exact method and a hybrid metaheuristic for large instances considering 50 ships and 130 cargoes. To our knowledge, in actual existing formulations and proposed solution methods for the cargo scheduling problem in tramp shipping none of them are really considering the stability or the design of the ships as an integral constraint of the MILP. In (Fagerholt et al., 2013) the stability of ships that transport very special cargo is mentioned as a constraint to be taken into consideration. It is checked manually by the engineering department which can revoke the solution if the ship is not stable with the given load.

3 Problem Description and Formulation

The main purpose of this paper is to present a MILP formulation of the cargo scheduling problem in tramp shipping under consideration of ships stability constraints. The problem considers long-term orders to be charged and discharged in different ports within a given time frame. Each order has its own profit value. The fleet of tankers is homogeneous and each ship has a different design and number of cargo tanks. Compatibility constraints for different cargo types are taken into account. For safety reasons not all chemicals can be charged to adjacent tanks as well as in succession in the same tank. These requirements are formulated as constraints in the MILP. Cargo can also be charged and discharged between ships. In maritime terms, this is called ship-to-ship operation. The velocity of the ships is assumed to be constant.

The assignment of cargo to a ship from the fleet is checked for intact stability based on the ships design, its cargo hold configuration and the filling level of each tank. The rules from (IMO, 2002) that apply to tankers are checked by taking free surfaces of the liquid cargo into account. This means that the solution of the MILP solver could assign only partially filled tanks what represents common practice in tanker operation. Because the free surface of a fluid within any tank aboard causes huge reduction of the stability of the ship the model for stability calculation has to be precise. The gain one obtains is a much greater set of possible solutions for the fleet optimisation problem.

The objective function of the problem consist of finding the most profitable assignment of cargoes to the ship tanks compliant with the ships stability requirements and time constraints of the orders. The interplay between the stability checker and the MILP leads to reduction of the feasible region. Nevertheless the problem is very hard to solve and even small data instances take a very long time to achieve an optimal solution with a zero gap. In this section the problem formulation as a MILP and the stability calculations are presented in detail.

3.1 Integer Program

The aim of this integer program is to determine the fleet coordination while maximizing the number of fulfilled orders and while minimizing the costs, which arise from loading and unloading cargo in ports and from the ship travelling from one port to another.

Parameters

λ_{spq}	The cost per journey of a ship s from port p to q.
μ_{sp}	The cost per loading process of ship s in port p.
P	The set of all ports.
S	The set of all ships.
T	The time horizon.
Y	The set of all types of goods.
C	The set of all tanks.
O	The set of all orders.
c_{size}	The size of tank c.
s_{size}	The size of ship s.
τ	The loading time, i.e. how long a ship has to be in a port to finish the loading (same for all ships)
C_s	The set of all tanks on ship s.
γ_{zy}	is one, if good $y \in Y$ cannot be carried next to good $z \in Y$ on a ship, otherwise zero.

A_c	The set of all tanks, which are adjacent to tank c.
δ_{zy}	is 1, if good y can be carried immediately after good z in a tank.
PB_o	The pickup begin of order o. Similar for PE (pickup end), DB (delivery begin) and DE (delivery end).
$amount_o$	The amount of the good, which is ordered in order o.
r_o	The amount of money which is earned when fulfilling order o.
$type_o$	The type of good which is ordered in order o.
α_{spq}	The time it takes ship s to travel from p to q.
s_c	is the ship s which carries tank c.
C_L^S	the set of all tanks on ship s which have some sort of loading in a given solution.

Variables

p_{spt}	is 1 if ship s is at port p at time t.
ℓ_{spt}	is 1 if ship s is at port p at time t and just finished loading.
a_{spqt}	is 1 if ship s just ended a cruise from port p to port q at t.
h_{cyt}	is the amount of type y tank c contains at time t.
b_{cyt}	is 1 if tank c contains at least 1 unit of type y at time t.
x_{cyat}	is 1 if tank c contains exactly the amount a of type y at t.
f_{sypt}	is the amount of type y which flows from ship s to port p at t.
g_{ot}	the amount of order o to be picked up at time t.
d_{ot}	the amount of order o delivered at time t.
e_o	is one if and only if order o is completed.

The following function represents the objective function to be used in the problem setting. It contains two terms that represent the earning maximization and the port as well as journey cost minimization leading in total to a profit maximization.

$$\max \sum_{o \in O} e_o \cdot r_o - \left(\sum_{s \in S} \sum_{p \in P} \sum_{t \in T} \left(\mu_{sp} \cdot \ell_{spt} + \sum_{q \in P} \lambda_{spq} \cdot a_{spqt} \right) \right) \quad (*)$$

Constraints

$$\begin{gathered} 0 \le p_{spt} \le 1 \ \forall s \in S, p \in P, t \in T \\ 0 \le \ell_{spt} \le 1 \ \forall s \in S, p \in P, t \in T \\ 0 \le a_{spqt} \le 1 \ \forall s \in S, p, q \in P, t \in T \\ 0 \le h_{cyt} \le c_{\text{size}} \ \forall c \in C, y \in Y, t \in T \\ 0 \le b_{cyt} \le 1 \ \forall c \in C, y \in Y, t \in T \\ 0 \le x_{cyat} \le 1 \ \forall c \in C, y \in Y, t \in T, 0 \le a \le c_{\text{size}} \\ -s_{\text{size}} \le f_{sypt} \le s_{\text{size}} \forall s \in S, \forall y \in Y, \forall p \in P, \forall t \in T \\ 0 \le g_{ot} \ \forall o \in O, t \in T \\ 0 \le d_{ot} \ \forall o \in O, t \in T \\ 0 \le e_o \le 1 \ \forall o \in O \end{gathered} \quad (1)$$

The constraints in equation 1 set the basic domains of the variables.

$$\sum_{p \in P} p_{spt} \le 1 \forall s \in S, t \in T \quad (2)$$

Constraint 2 ensures that a ship s is at no more than one port at a time.

$$\ell_{spt} = 0 \qquad \forall s \in S, p \in P, t < \tau, t \in T \quad (3)$$

Constraint 3 ensures that in the beginning no ship finishes loading before it was in the port for at least τ time slots.

$$\sum_{u=t-\tau}^{t} p_{spu} \geq (\tau + 1)\ell_{spt} \qquad \forall s \in S, p \in P, t \geq \tau \quad (4)$$

Constraint 4 ensures that the ship was in port p for at least τ time slots before it finishes loading. Note that ℓ_{spt} doesn't have to be equal to one if the ship was at the port for τ time slots.

$$\sum_{p \neq q \in P} a_{spqt} \geq p_{sqt} - p_{sq(t-1)} \; \forall s \in S, q \in P, t \geq 1 \qquad (5)$$

Constraint 5 ensures that if a ship s is in port q in time t, it either was in port q at time $t-1$ or it just arrived, i.e. ended a cruise from port p to q for some p.

$$a_{spqt} = 0 \quad \forall s \in S, p \neq q \in P, t < \alpha_{spq} \qquad (6)$$

where α_{spq} is the distance between p and q divided by the speed of ship s. Constraint 6 sets all variables α_{spq} equal to zero, if there is no possibility, that a ship arrived from p at q at time t.

$$p_{sp(t-\alpha_{spq})} - \alpha_{spq} a_{spqt} + \alpha_{spq} - 1 \quad \geq \sum_{r \in P} \sum_{u=t-\alpha_{spq}+1}^{t-1} p_{sru}$$

$$\forall s \in S, p \neq q \in \mathrm{P}, t \geq \alpha_{spq} \qquad (7)$$

If α_{spq} equals zero, constraint 7 has no impact since the left hand side is greater or equal to $\alpha_{spq} - 1$ and the right hand side is at most $\alpha_{spq} - 1$ because of constraint 2. If α_{spqt} = 1 (which means ship s just arrived from a cruise from p to q), ship s must have been in port p at time

$t - \alpha_{spq}(p_{sp(t-\alpha_{spq})}$ = 1), because otherwise the left hand side would be equal to minus one and the right hand side is not negative. So in this case the left hand side is equal to zero and therefore the ship couldn't be somewhere else other than on the cruise from p to q in the mean time, which means each p_{sru} on the right hand side has to be equal to zero.

$$h_{cyt} \leq c_{\text{size}} \cdot b_{cyt} \quad \forall c \in C, y \in Y, t \in T \tag{8}$$

Constraint 8 ensures that the variable b_{cyt} is set to one if tank c is loaded with some cargo type y at time t.

$$\sum_{y \in Y} b_{cyt} \leq 1 \quad \forall c \in C, t \in T \tag{9}$$

Constraint 9 ensures that a tank carries just one type of good at a time.

$$\sum_{c \in C_s} \left(h_{cyt} - h_{cy(t-1)}\right) + \sum_{p \in P} f_{sypt} = 0$$
$$\forall s \in S, y \in Y, t \geq 1 \tag{10}$$

Constraint 10 ensures that the change in the amount of good of type y on ship s from t to $t - 1$ equals the flow of good y from ship s to some port at time t.

$$\left|f_{sypt}\right| \leq s_{\text{size}} \cdot \ell_{spt} \qquad \forall s \in S, p \in P, t \in T, y \in Y \tag{11}$$

Constraint 11 ensures that the flow f_{sypt} of type y from the ship s to port p is at most the size of the ship at each time.

$$\left|h_{cyt} - h_{cy(t-1)}\right| \leq c_{\text{size}} \cdot \sum_{p \in P} \ell_{s_c pt}$$
$$\forall c \in C, y \in Y, t \geq 1 \tag{12}$$

where s_c denotes the ship s which carries tank c. This constraint 12 ensures that the amount of a good y in a tank just changes if the ship is at least τ time slots in a port to finish loading and that the change in the amount of y in c from $t-1$ to t is at most the size of this tank.

$$\sum_{s \in S} f_{sypt} + \sum_{\substack{o: from_o = p \\ \wedge y_o = y}} \left(g_{o(t-1)} - g_{ot}\right) = \sum_{\substack{o: to_o = p \\ \wedge y_o = y}} \left(d_{ot} - d_{o(t-1)}\right)$$

$$\forall p \in P, y \in Y, t \geq 1 \tag{13}$$

Constraint 13 ensures that the flow of good y from all ships into the port at each time t plus the change in the amount of y which is to be picked up (i.e. the flow of good y from the port onto the ship) equals the change in the amount of y which is to be delivered. Note that f_{sypt} can be negative, i.e. a flow from the port onto a ship.

$$d_{ot} \geq d_{o(t-1)} \quad \forall o \in O, t \in T \tag{14}$$

$$g_{ot} \leq g_{o(t-1)} \quad \forall o \in O, t \in T \tag{15}$$

Constraints 14 and 15 ensure that the amount of a delivered good y within an order just increases and similar the amount of a good y to be picked up within an order just decreases. The binary variables x_{cyat} are introduced solely for the reason to exclude a specific loading configuration of a ship. With the following two constraints we set x_{cyat} equal to 1 if and only if exactly the amount $a > 0$ of type y is in tank c at time t:

$$\sum_{a=1}^{c_{\text{size}}} x_{cyat} = b_{cyt} \qquad \forall c \in C, y \in Y, t \in T \tag{16}$$

$$\sum_{a=1}^{c_{\text{size}}} a x_{cyat} = h_{cyt} \quad \forall c \in C, y \in Y, t \in T \tag{17}$$

With these variables we are able to tell the program to forbid a specific loading configuration if the ship stability constraints are hurt. Assume we have a solution which is feasible for our program, but does not satisfy the stability constraints. Let *s* be the ship which is instable when leaving port *p*. Our aim is to forbid exactly the given loading configuration for ship *s*. C_L^s is the set of all tanks on ship *s* which have some sort of loading in our solution. Let y_c be the type which is loaded in $c \in C_L^s$ and similar a_c the amount of type y_c in c. From constraints 16 and 17 we know that $x_{cyat} = 1$ for $y = y_c$ and $a = a_c$ and zero otherwise. We then are able to add a constraint which ensures that the loading configuration for ship *s* is changed:

$$\sum_{c \in C_L^s} \left(1 - x_{c y_c a_c t}\right) + \sum_{c \in \left(C_s \setminus C_L^s\right)} \sum_{y \in Y} h_{cyt} \geq p_{sp(t-1)} - p_{spt} \quad \forall t \in T \tag{18}$$

The right hand side of constraint 18 just defines the time slot *t* at which the ship *s* leaves port *p*, since for *t* the right hand side equals 1 and for all other time slots it is at most zero. The left hand side is zero for our given solution with which ship *s* is instable. To change the loading configuration we have two options. Either we change the amount of the loading in our tanks $c \in C_L^s$ so that in the first sum of the left hand side we get that at least one $x_{c y_c a_c t}$ is zero. Or additional tanks are loaded so that in the second sum at least one h_{cyt} gets at least one.

Another problem we need to address is the specific adjacencies of tanks on the ship. There are goods which are not allowed to be carried next to each other on a ship for various safety reasons. Therefore our feasible solution has to ensure that these adjacencies are considered. This adjacency constraint is constructed via the parameter γ_{xy}:

$$b_{cyt} \leq 1 - \sum_{\tilde{c} \in A_c} \sum_{\tilde{y} \in Y} \gamma_{\tilde{y}y} b_{\tilde{c}\tilde{y}t} \quad \forall c \in C, y \in Y, t \in T \tag{19}$$

The right hand side of constraint 19 is less or equal zero if and only if there is a tank adjacent to c, which is loaded with a good $\tilde{y}$ that is not allowed to be carried next to good y. Therefore b_{cyt} has to be equal zero, which means that tank c is not allowed to load good y at time t.

Similarly there are goods which are not allowed to be carried in a tank directly after another good, because for example there have to be two separate cleanings between carrying those two goods.

$$b_{cyt} \leq 1 + \sum_{\tilde{y} \in Y} \left(\delta_{\tilde{y}y} - 1\right) b_{c\tilde{y}(t-1)} \forall c \in C, y \in Y, \geq 1 \tag{20}$$

The right hand side of constraint 20 equals one if and only if either the tank c didn't carry any good at time $t-1$ or good y is allowed to be carried immediately after the good which was in the tank at $t-1$. Otherwise the right hand side equals zero and therefore b_{cyt} has to be zero, meaning good y is not allowed to be in tank c at time t.

The following constraints set the variable e_o to one if and only if order o is completed, which is important for our objective function:

$$\begin{gathered} g_{oPE_o} = 0 \ \forall o \in O \\ d_{o(DB_o+\tau-1)} = 0 \ \forall o \in O \\ d_{oDE_o} = amount_o \cdot e_o \ \forall o \in O \\ g_{o(PB_o+\tau-1)} = amount_o \cdot e_o \ \forall o \in O \\ g_{o0} = amount_o \cdot e_o \ \forall o \in O \end{gathered} \tag{21}$$

For the sake of running time we constructed a few extra constraints, which are direct consequences of the constraints presented above, but help the program to exclude infeasible solutions faster.

$$\tau\left(p_{spt} - p_{sp(t-1)}\right) \leq \sum_{i=1}^{\tau} p_{sp(t+i)} \;\forall s \in S, p \in P, t \geq 1 \quad (22)$$

The statement of constraint 22 is, that if a ship arrived at a port, it stays at this port for at least the τ time slots it takes to (un)load. This ensures that a ship doesn't go to a port without doing anything there.

$$\sum_{t=1}^{T} \sum_{p \in P} f_{sypt} = 0 \quad \forall s \in S, y \in Y \quad (23)$$

Constraint 23 is a consequence from the fact that a ship is empty in the beginning and in the end, so the sum of the flow from each ship to all ports equals zero for each good over the time span.

$$\sum_{y \in Y} max\{0, \sum_{\substack{o:\mathrm{PB_o} \geq a_i \\ \wedge \mathrm{PE_o} \leq u_j \\ \wedge \mathrm{type_o} = y}} amount_{\mathrm{o}} - \sum_{\substack{o:\mathrm{DB_o} \leq u_j \\ \wedge \mathrm{DE_o} \geq a_i \\ \wedge \mathrm{type_o} = y}} amount_{\mathrm{o}}\} \leq \sum_{t \in [a_i, u_j]} \sum_{s \in S} s_{\mathrm{size}} \ell_{spt}$$

$$\forall p \in P, a_i, u_j \in T, a_i \leq u_j \quad (24)$$

Constraint 24 ensures that for each time interval at a port the sum of the sizes of ships that load in this port are large enough, meaning the sizes of the ships are able to fit all orders.

3.2 Ship stability calculation

In general the stability of a ship is the ability to come back to a stable equilibrium after any perturbation caused the ship to heel or trim. Because usually the trim does not cause safety issues this is not a matter of safety. To make sure that all ships on duty do withstand such influences there are certain rules that apply to the ship. These rules are issued by the *International Maritime Organisation* (IMO) and apply to any ship of more than 24 m length. Before a ship will leave the harbour, the master shall ensure that the ship is compliant to these rules for the current loading condition as well as for the condition when arriving at the destination. Accidents due to lack of stability lead to cost and reputation loss which has a great impact in company's future contracts and therefore a long-term effect.

Even though the stability of a tanker ship might not be a major issue in the first place, in particular because bulk carriers for any kind are well known for their high stability, it is necessary to check whether a ship does fulfill the stability criteria from the IMO, 2002 (2008 IS-Code) or it will not be allowed to leave the harbour. Because the *MILP* solver does not know if a ship would be capable of sailing at a given loading condition the stability has to be taken into account when choosing an admissible solution for the fleet scheduling problem.

Therefore the results of the *MILP* solver, where its output is the loading condition of each ship of a fleet at departure, have to be evaluated. A separate program has been written to perform stability calculations when it has been given a ship and its loading condition as input. The output of the program is a boolean *true/false* that indicates the stability of the loaded ship. As shown below this is an admissible way to involve the requirements evaluation because of the interlaced quantities and the non linearity of the problem. There is no way of formulating the ship stability problem in terms of linear inequalities.

Because a ship always has the ability of using ballast water to become stable enough, this has to be taken into account. Therefore the model contains ballast water tanks and every possible combination of full and empty ballast water tanks is tested until stable conditions are reached. If this was successful the program returns *true* to mark that the ship is compliant. If all combinations are checked but the rules are not fulfilled for any of the ballast water tank combinations, the program returns *false* to mark that the ship is not be compliant. All rules described in the following subsection are well known and only a brief introduction shall be given here.

3.3 General aspects of ship stability

In ship building the following approach to perform the stability evaluation is required and commonly used. The 2008 IS-Code asks for certain properties of the lever arm curve ($\overline{\mathrm{GZ}}$ curve) that have to be fulfilled at any situation at sea. The $\overline{\mathrm{GZ}}$ curve is defined as the distance between the alignment of the vector of the gravity force G and the vector of the buoyancy force B. The $\overline{\mathrm{GZ}}$ curve of one of the example ships used for the numerical experiments described in section 4 is shown in Figure 2. While the ship is upright we have $\overline{\mathrm{GZ}} = 0$. As soon as the ship heels, B moves towards the side of the ship. Close to the favourable stable equilibrium the forces cause a torque that pushes the ship back into the upright position. For a more detailed description see e.g. (Biran, 2003 and Tupper, 2013).

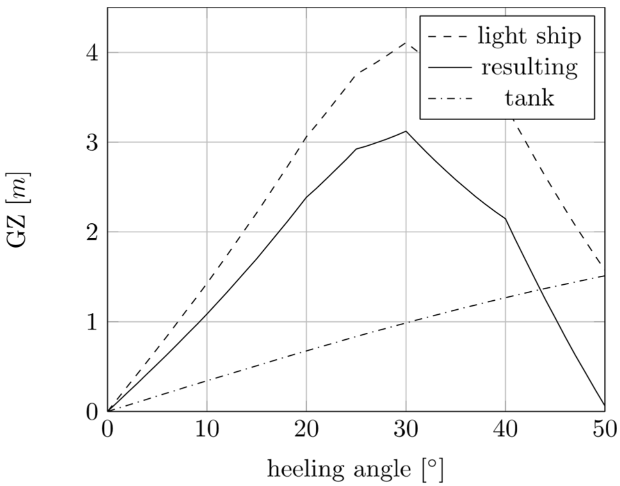

Figure 2: $\overline{\mathrm{GZ}}$ curve of one of the example ships

In general, the gravity force G and buoyancy force B have to be at an equilibrium when the ship does not move, viz. $\mathrm{B} = \mathrm{G}$, which is obvious due to the principle of linear momentum. The principle of angular momentum leads to the fact that the righting and the heeling lever arm have to be equal at an equilibrium. Therefore, it is common practice to only look at the lever arms for evaluating the ships hydrostatics. $\overline{\mathrm{GZ}}$ depends on the heeling angle and on the displacement of the ship. Because the lever arm is always calculated for one particular displacement this dependency is neglected in the following. The righting lever arm $\overline{\mathrm{GZ}}_{\mathrm{LS}}$ (the dashed line in Figure 2) for the light ship is defined by

$$\overline{\mathrm{GZ}}_{\mathrm{LS}}(\varphi) = \overline{\mathrm{KN}}(\varphi) - \overline{\mathrm{TCG}}_{\mathrm{LS}}\cos(\varphi) - \overline{\mathrm{VCG}}_{\mathrm{LS}}\sin(\varphi) \quad (25)$$

where $\overline{\mathrm{TCG}}_{\mathrm{LS}}$ is the transverse and $\overline{\mathrm{VCG}}_{\mathrm{LS}}$ the vertical distance of the center of gravity of the light ship to the keel and

$$\overline{\mathrm{KN}}(\varphi) := \overline{\mathrm{TCB}}(\varphi)\cos(\varphi) + \overline{\mathrm{VCB}}(\varphi)\sin(\varphi) \tag{26}$$

with the transverse distance $\overline{\mathrm{TCB}}$ and the vertical distance $\overline{\mathrm{VCB}}$ of the center of buoyancy in global coordinates. $\overline{\mathrm{KN}}$ is also known as the cross curves of stability when evaluated for different displacements. In Figure 3 this is illustrated. There the center of gravity, marked as CG_0 and CG_ϕ, as well as the center of buoyancy, marked as GB_0 and CB_ϕ, are shown. The index stands for either no heeling angle ($\varphi = 0°$) or at some heeling angle ($\varphi > 0°$).

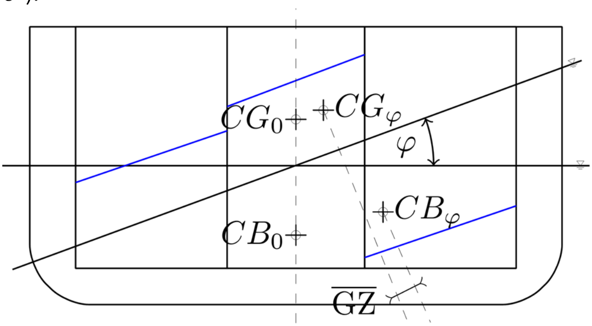

Figure 3: Midship section

The position of the center of gravity of the ship has to be calculated according to the loading status of the ship by:

$$\overline{\mathrm{TCG}}(\varphi) = \overline{\mathrm{TCG}}_{\mathrm{LS}} \frac{\Delta_{\mathrm{LS}}}{\Delta} + \frac{1}{\Delta} \sum_{l=1}^{n_l} y_{G,l}(\varphi) V_l \rho_l \tag{27}$$

$$\overline{\mathrm{VCG}} = \overline{\mathrm{VCG}}_{\mathrm{LS}} \frac{\Delta_{\mathrm{LS}}}{\Delta} + \frac{1}{\Delta} \sum_{l=1}^{n_l} z_{G,l}(\varphi) V_l \rho_l \tag{28}$$

where $y_{G,l}(\phi)$, $z_{G,l}(\phi)$ are the transverse and vertical position of the center of gravity of each single cargo hold, bunker and stores compartment and ballast water tank. In the case of a free surface the position of the center of gravity of the fluid within the tank changes due to the heeling angle of the ship.

The effective righting lever arm $\overline{\mathrm{GZ}}_{\mathrm{eff}}$, which is the $\overline{\mathrm{GZ}}_{\mathrm{LS}}$ of the light ship corrected by the tank lever, as shown in Figure 2 (dashed/dotted line), is calculated by

$$\overline{\mathrm{GZ}}(\varphi) = \overline{\mathrm{KN}}(\varphi) - \overline{\mathrm{TCG}}(\varphi)\cos(\varphi) - \overline{\mathrm{VCG}}(\varphi)\sin(\varphi) \tag{29}$$

which leads to the continuous curve in Figure 2. Thus the difference between the lever arm of the light ship (dashed line in Figure 2) at the draught matching the displacement the ship in loaded condition and the effective lever arm is a reduction caused by the cargo and in particular because of the free surface. This effect is described in detail in e.g (Biran, 2003 and Tupper, 2013).

3.4 Criteria for intact stability

The criteria which the 2008 IS-Code demand can be written as

$$\int_{0^\circ}^{30^\circ} \overline{\mathrm{GZ}}_{\mathrm{eff}}(\varphi)\mathrm{d}\varphi \geq 0.055\mathrm{mrad} \tag{30}$$

$$\int_{0^\circ}^{40^\circ} \overline{\mathrm{GZ}}_{\mathrm{eff}}(\varphi)\mathrm{d}\varphi \geq 0.09\mathrm{mrad} \tag{31}$$

$$\int_{30^\circ}^{40^\circ} \overline{\mathrm{GZ}}_{\mathrm{eff}}(\varphi)\mathrm{d}\varphi \geq 0.033\mathrm{mrad} \tag{32}$$

$$\overline{\mathrm{GZ}}_{\mathrm{eff}}(\varphi^*) := \max \overline{\mathrm{GZ}}(\varphi) \text{ with } \varphi^* \leq 25^\circ \tag{33}$$

$$\overline{\mathrm{GZ}}(\varphi \geq 20^\circ) \geq 0.3\mathrm{m} \tag{34}$$

$$\overline{\mathrm{GM}}_0 \geq 0.15\mathrm{m} \tag{35}$$

where $\overline{\mathrm{GM}}_0$ is the metacentric height of the upright floating ship. The criteria 30 to 35 aim at ensuring a certain amount of energy a ship is able to take up without risking to capsize. In addition to the mentioned criteria the weather criterion has to be fulfilled. The basic idea is, that at the worst situation at sea a seafarer may think of, in terms of intact stability of the ship, the ship shall withstand without being at risk of capsizing. For detail see (Meier-Peter, 2012).

3.5 Application

In the program the above described criteria are implemented and evaluated for the *MOERI Tanker KVLCC2* which is widely known in research. The cross curves of stability as well as the displacements and other parameters are provided in tabular form for heeling angles from 0° to 50° and displacements from $141{,}000t$ to approximately $321{,}000t$. The values at an intermediate state are interpolated linearly to avoid overestimation of the capability of the ship. For reasons of simplification the free surface of fuel oil tanks and fresh water tanks is not taken into account. These tanks are small compared to the ships size and do not have an impact on the ships stability that is worth mentioning. But the free surfaces of a cargo hold that is not fully filled or empty shall be taken into account. Therefore a simplified tank model is chosen to determine the center of gravity when the ship heels (see Figure 3). A quadrilateral shape of the transverse section is assumed. This allows an analytical solution for the tank lever arms depending on the heeling angle and the filling height of the tank. The integration of the resulting $\overline{\mathrm{GZ}}_{\mathrm{eff}}$ is realised by a simple quadrature and the interpolation. The criteria may also be fulfilled not only at the departure condition but also at the arrival condition to ensure that at every intermediate state reached during the journey no unsafe condition occurred.

1. departure conditions with 100 % bunkers and stores
2. arrival conditions with 10 % bunkers and stores

The loading condition of each cargo hold of a ship within the fleet is set as a result of the MILP. Then the criteria described above are checked for each ship and a boolean true is returned if and only if all criteria at departure and arrival are fulfilled.

4 Experiments and Analysis

As real world data was not available to us, we produced some artificial instances. Due to the complexity of the problem and its exponential growth we could only test the model on small instances. This means there is a small number of orders and ports as well as a quite short time interval for few ships that made the input parameters.

Table 1: Instance data

#	ships	ports	orders	time steps
1	2	3	6	15
2	3	4	110	30
3	3	3	7	15
4	3	3	7	15
5	3	3	7	15

The MILP program is implemented in C++ and the GUROBI (8.1) solver (academical license) is used for solving the problem. We ran the solver on a quad-core processor (i5-6600 CPU @ 3.30GHz) with 16 GB random access memory. The stability calculations are also implemented in C++ and communicate directly with the MILP. Whenever a MILP solution is found, it is passed to a function that returns **true** or **false** depending on whether the stability constraints are satisfied.

All ships in all five problems are of the same type. They have 15 tanks with a volume of approximately 10, 000 litres or 15, 000 litres. They are arranged in a 3 X 5 grid, which is important for both the ship stability constraints as well as the adjacency constraints. The tank volumes were discretized into units of volume 5, 000 litres to reduce computation time.

Instance 1 was constructed to have a feasible solution with respect to the ship stability constraints. It was solved in 5 seconds. The optimal solution completed all orders.

Instance 2 turned out to be much harder to solve than the first one. After 20 minutes the solver found a solution, which completes 7 out of 10 orders.

After two hours in total, we stopped the program. It had not found a better solution in the meantime. The MIP gap was at 44%.
Instance 3 to 5 were the same with minor differences. Compared to instance 3, in instance 4 we set $\gamma_{z,y} = 0$ and $\delta_{z,y} = 1$ to remove all good type restrictions. Instance 5 in turn was the same as instance 3 except that we multiplied all densities of good types by 0.7, which relaxes the ship stability constraints. The lighter the goods, the smaller is the effect on the stability. During the two hours in which we let the solver tackle this problem, its best solution was found already after about half an hour. In the remaining time the solver was able to reduce the MIP gap to 17.7%. Removing the good type restrictions did not help the solver find a solution. In fact, because it increased the number of solutions, it only found its best solution after 36 minutes. The gap after two hours was again 17.7%. Finally, for instance 5 the changes we made significantly affected the solver. The solution that was also found for instances 3 and 4 was already found after 5 minutes. Half an hour later it found a slightly better solution with respect to the number of journeys and loading operations. After a total of two hours it had decreased the MIP gap to 17.5%.

Table 2: Results with stability

#	obj. value * of best solution	time (s)
1	589	5
2	681	1113
3	587	1899
4	587	2199
5	588	1992

Table 3: Results without stability

#	obj. value * of best solution	time (s)	Intermediate result
1	589	5	
2	981	92	683 after 38 s
3	684	213	588 after 19 s
4	686	108	589 after 7 s
5	684	213	588 after 19 s

As a comparison we also ran the solver on these instances without stability constraints, which show significant differences in run time and in the best solution found within two hours. The objective values can be interpreted as follows: Each completed order adds 100 to the value and each loading operation and ship cruise subtracts 1 from it, i.e. we set $O_r = 100$, $\mu_{sp} = 1$ and $\lambda_{spq} = 1$.

As shown in Table 3 and 4, adding the stability constraints significantly influences the run time of the program. Instance 1 was solved in both cases very fast. But as soon as the tested instance got more complicated, the stability constraints led to a big difference in the run time. This can be explained by the number of solutions of the MIP which have to be tested in order to find a feasible solution for the stability constraints. In the case that the progam finds a feasible solution with objective value k, it has to test whether this solution satisfies the stability constraints. If not, the program tries to find a different assignment to achieve an objective value k. If the stability constraints are excluded, the program directly tries to improve the objective value, which means that significantly fewer feasible solutions for the MIP have to be found. This also explains why the best solution with stability constraints takes way longer to find than to find a solution with this objective value without stability constraints. Unfortunately real world problems are much larger than our test instances. To conduct more meaningful numerical experiments, we need to further develop our approaches.

5 Conclusions and Future Work

This work shows that mathematical models and methods can improve the planning process significantly and give well-founded decision support for planners. With this first model it has been shown that the integration of non linear requirements as the ship stability check to common MILP formulations of the cargo scheduling problems is possible. This inclusion makes the model even more realistic and enables calculations of scenarios that are very time consuming done in a manual way. The constraints of this MILP formulation link many different aspects of the cargo scheduling problem making it on the one side quite complex to solve real-world instances exactly and more realistic on the other side. This formulation is a first draft and further analysis of the numerical results need to be made in order to identify performance potentials. The examination of appropriate algorithms and methods, that take advantage of special problem structure and properties, is missing and is considered to be one of the main factors with a big impact in the calculation time. The development of appropriate heuristics is also one key aspect to be taken into consideration for future work. Beside the performance issue the model itself can be made even more realistic by considering the speed as a decision variable and fuel consumption in dependence to loading. From a ship design point of view, in particular for bulk carriers of liquid or solid cargo, the longitudinal strength is a problem worth mentioning. In this work the view on mechanical requirements is neglected but for real world application an evaluation also has to be applied to the algorithm. Modeling the complex problem of cargo scheduling as realistically as possible and adapting methods for solving real data instances is one of the main future challenges.

References

Appelgren, L. H., 1969. A column generation algorithm for a ship scheduling problem. In: Transportation Science 3.1, pp. 53–68.

Appelgren, L. H., 1971. Integer programming methods for a vessel scheduling problem. In: Transportation Science 5.1, pp. 64–78.

Biran, A. B., 2003. Ship hydrostatics and stability. Includes bibliographical references and index. Amsterdam u.a.: Butterworth-Heinemann. isbn: 9780750649889 0750649887 1417502053 9781417502059.

Brønmo, G., M. Christiansen, K. Fagerholt, and B. Nygreen., 2007. A multi-start local search heuristic for ship scheduling—a computational study. In: Computers & Operations Research 34.3, pp. 900–917.

Christiansen, M., K. Fagerholt, B. Nygreen, and D. Ronen., 2013. Ship routing and scheduling in the new millennium. In: European Journal of Operational Research 228.3, pp. 467–483.

Christiansen, M., K. Fagerholt, and D. Ronen., 2004. Ship routing and schedul- ing: Status and perspectives. In: Transportation science 38.1, pp. 1–18.

Cóccola, M. E. and C. A. Méndez., 2015. An iterative MILP-based approach for the maritime logistics and transportation of multi-parcel chemical tankers. In: Computers & Industrial Engineering 89, pp. 88–107.

Desrosiers, J., Y. Dumas, M. M. Solomon, and F. Soumis., 1995. Time con- strained routing and scheduling. In: Handbooks in operations research and management science 8, pp. 35–139.

Fagerholt, K., L. M. Hvattum, T. A. V. Johnsen, and J. Eirik Korsvik., 2013. Routing and scheduling in project shipping. In: Annals of Operations Research - Annals OR 207. doi: 10.1007/s10479-011-0888-1.

Hemmati, A., L. M. Hvattum, K. Fagerholt, and I. Norstad., 2016. Benchmark Suite for Industrial and Tramp Ship Routing and Scheduling Problems. In: INFOR: Information Systems and Operational Research 52.1, pp. 28–38.

Homsi, G., R. Martinelli, T. Vidal, and K. Fagerholt., 2018. Industrial and Tramp Ship Routing Problems: Closing the Gap for Real-Scale Instances. In: arXiv preprint arXiv:1809.10584.

Hwang, H.-S., S. Visoldilokpun, and J. M. Rosenberger., 2008. A branch-and- price-and-cut method for ship scheduling with limited risk. In: Transportation science 42.3, pp. 336–351.

IMO., 2002. Code on intact stability for all types of ships covered by IMO instruments : resolution A.749(18) as amended by resolution MSC.75(69). 2nd ed. IMO publication. Sales no.: IA874E. London. isbn: 9280151177 9789280151176.

Jetlund, A. S. and I. Karimi., 2004. Improving the logistics of multi-compartment chemical tankers. In: Computers & Chemical Engineering 28.8, pp. 1267–1283.

Al-Khayyal, F. and S.-J. Hwang., 2007. Inventory constrained maritime routing and scheduling for multi-commodity liquid bulk, Part I: Applications and model". In: European Journal of Operational Research 176.1, pp. 106–130.

Korsvik, J. E., K. Fagerholt, and G. Laporte., 2010. A tabu search heuristic for ship routing and scheduling. In: Journal of the Operational Research Society 61.4, pp. 594–603.

Lawrence, S. A., 1972. International sea transport: the years ahead. Lexington Books.

Malliappi, F., J. A. Bennell, and C. N. Potts., 2011. A variable neighborhood search heuristic for tramp ship scheduling. In: International Conference on Computational Logistics. Springer, pp. 273–285.

Meier-Peter, H., ed., 2012. Handbuch Schiffsbetriebstechnik : Betrieb, Überwachung, Instandhaltung. 2., komplett überarb. Neuaufl. Literaturangaben. Hamburg: Seehafen-Verl. isbn: 9783877438299 3877438296. url: http://d-nb.info/1023883112/04.

Norstad, I., K. Fagerholt, and G. Laporte., 2011. Tramp ship routing and scheduling with speed optimization. In: Transportation Research Part C: Emerging Technologies 19.5, pp. 853–865.

Ronen, D., 1983. Cargo ships routing and scheduling: Survey of models and problems. In: European Journal of Operational Research 12.2, pp. 119–126.

Ronen, D., 2019. Key Facts. In: http://www.ics-shipping.org/shipping-facts/shipping- and-world-trade.

Ropke, S. and D. Pisinger., 2006. An adaptive large neighborhood search heuris- tic for the pickup and delivery problem with time windows. In: Transportation science 40.4, pp. 455–472.

Trottier, L.-P. and J.-F. Cordeau., 2019. Solving the vessel routing and schedul- ing problem at a Canadian maritime transportation company. In: INFOR: Information Systems and Operational Research, pp. 1–26.

Tupper, E. C., 2013. Introduction to naval architecture. 5th ed. Includes bibliographical references and index. Amsterdam Boston: Elsevier, Butter- worth Heinemann, p. 476. isbn: 9780080982724 0080982727 9780080982373 0080982379.