Simulation modelling in accounting and finance: current practices and advances in input modelling

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One-page summary Dissertation Daniel Chorzelski: “Simulation modelling in accounting and finance: current practices and advances in input modelling”

This dissertation seeks to contribute to the understanding of simulation methods in Corporate Finance and Accounting with a focus on simulation input modelling. It is structured in six chapters applying different methods to investigate simulation methods in the discipline.
The first chapter uses bibliometrics to shed light on how simulation methods affected Finance and Accounting research, how they are used in the disciplines as well as quantifying the diffusion across a wide range of research clusters via a citation and CoCitation network analysis. Key findings are that several research clusters in Finance research embraced simulation methods, with less adoption in Accounting – despite noteworthy exceptions. Further, the methods are used primarily instrumentally rather than conceptually, suggesting untapped potential for theory-building research. Finally, we observe that simulation crossed the ‘chasm’ into the methodological mainstream in many research clusters in finance and is on the cusp of crossing this chasm for several accounting research clusters as well – notably around costing. The second chapter turns toward simulation input modelling and analyzes the state-of-the-art methods in simulation input modelling through a structured literature review of both the academic literature and practitioner publications. This is complemented in the third chapter through a unique perspective on simulation input modelling based on a series of in-depth semi-structured interviews with experts. The fourth chapter presents a simulation input modelling method based on Bayesian Updating of prior distributions aggregating data and expert-based methods. Thereby the method addressed several challenges as demonstrated through a case study. The fifth chapter proposes and discusses a novel metric, *Simulation Output at Risk (SOaR)*, that quantifies modelling risk stemming from epistemic and aleatoric uncertainty of input modelling parameters in a single metric and thereby generalizes the method used in chapter 4. The sixth and final Chapter builds onto chapter 4 by analyzing and discussing conditions under which Bayesian input modelling represents a viable alternative input modelling method along input modelling desiderata concluding that it represents a viable method. The results prove relevant for a readership in both academia as well as professional simulation modelers.

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Abbreviation	
ABM	Agent-based-models
ARMA	Auto-regressive moving average
CF&A	Corporate finance and accounting
CFO	Chief financial officer
DCF	Discounted Cash Flow
DGP	Data generating process
DOE	Design of experiments
GARCH	Generalized auto-regressive conditional heteroskedasticity
GDP	Gross domestic product
KPI	Key performance indicator
MCMC	Markov Chain Monte Carlo
NPV	Net present value
PDF	Probability density function
RQ	Research question
SME	Subject matter expert
SOaR	Simulation output at risk
VaR	Value at risk

General introduction

Simulation modelling has become a vital method in the field of accounting and finance, for researchers and practitioners alike. Its applications are myriad and its impact noticeable across the disciplines. Simulation is here understood per Kelton and Law (2000) as “models evaluated numerically to estimate their true characteristics” and thereby aid decision-making in a variety of contexts. Researchers in accounting and finance, and beyond, apply simulation modelling for many purposes including lowering the need for infeasible or prohibitively expensive experiments, approximating or evaluating otherwise intractable systems (e.g. via evaluating partial differential equations) or by providing an additional angle to corroborate or challenge theoretical or empirical contributions or even develop theory altogether. The method, all but new, has seen growing usage across the field, as we will demonstrate below, thereby contributing towards the evolution of accounting and finance research. In the process simulation methods diffused into various accounting and finance research clusters – however to starkly varying extents.

Use cases range widely including financial modelling, risk management and many others (Kelton & Law, 2000; Glasserman, 2003; Hertz, 1964). Simulation is a powerful method providing a quantitative perspective that complements experience and intuition in decision making. Yet the accuracy of these simulation models hinges upon the quality of the input parameters and distributions used (e.g. Vose, 2008; Rees, 2015). While a rich and structured research dialogue around input modelling is evidenced for capital market finance by various research clusters observed in the quantitative assessment of the discipline, this research dialogue appears to be less rich and differentiated for corporate finance and accounting research – thereby constituting a core motivation toward the second focus of this dissertation.

This research is motivated by the desire to fully grasp and quantify the methods impact on our field of inquiry as well as further advance simulation modelling in accounting and finance by contributing towards empirical analysis of simulation input modelling methods as well as theoretical contributions of input modelling and quantification of input modelling risk. The first objective of this dissertation, pursued in chapter one, is to provide a thoroughly quantified angle towards how simulation methods affected the research agenda and analyze its diffusion across research clusters to derive fruitful avenues of future research. The second objective, pursued in chapter two to six, is to advance simulation input modelling – both empirically as well as theoretically and the quantification of input modelling uncertainty, specifically parameter uncertainty.

In the following, each chapter's contribution is briefly summarized before the main body of this dissertation follows. The first chapter applies bibliometrics to shed light on how simulation methods affected finance and accounting research, how they are used in the disciplines as well as quantifying the diffusion across a wide range of research clusters in these two disciplines. The method used is a citation and CoCitation network analysis of the relevant research field. Key findings are that several research clusters in finance research embraced simulation methods, whereas accounting has seen much less adoption – despite note-worthy pioneering simulation-based research. Further, the methods are used primarily instrumentally rather than conceptually, suggesting untapped potential for theory-building simulation-based research in finance and accounting. Finally, we observe that simulation crossed the 'chasm' into the methodological mainstream in many research clusters in finance and is on the cusp of crossing this chasm for several accounting research clusters as well – notably around costing. One of the findings, that also sparked further research interest was the lack of evidence for a structured research dialogue on simulation input modelling, especially in corporate finance. In other words, of the many research cluster in simulation methods in accounting and finance, it appeared that remarkably little research focused on simulation input models beyond data-driven econometric models based on financial market data – despite widespread agreement on the importance of this topic to simulation modelling in the wider simulation methods research community as we will argue in the next chapter. The second chapter turns toward simulation input modelling and analyzes the state-of-the-art methods in simulation input modelling through a structured literature review of both the academic literature and practitioner publications. Reviewing the literature broadly, including various methodological treatments on simulation modelling in further disciplines such as operations research, we capture the consensus as well as disagreements on input modelling methods. Finally, we deduce a decision-tree for input modelling methods, that captures the consensus view of the input modelling methods. This is complemented in the third chapter through a unique perspective on simulation input modelling based on a series of in-depth semi-structured interviews with experts in applied simulation modelling in corporate finance and accounting and contrasts their point of view with the previously derived literature-based consensus. We find notable areas of agreement, though also divergent opinions on topics like aggregation of input sources and fundamental input modelling methods. Delving further into methods to aggregate input modelling sources, the fourth chapter presents a simulation input modelling method based on Bayesian updating of prior distributions aggregating data-based as well as expert-based methods in stochastic simulations. This method can address several challenges derived from the literature review and expert interviews whilst fulfilling several input modelling desiderata. Further, it is applied to an actual case study of a simulation model in a corporate finance context underscoring its relevance for practitioners. However, as this application to one case study does not provide proof of the superiority of Bayesian input modelling as the actual one-shot realization of the modelled process is stochastic and thus no sufficient benchmark therefore it is not feasible to determine if Bayesian modelling is indeed superior to conventional input modelling methods in the sense of providing a mean parameter estimate closer to the “true” underlying parameter than other input modelling methods as this “true” parameter is not known. Yet, there is firstly a very strong theoretical argument, laid out in the chapter, for this methods validity, and indeed superiority, that should inspire confidence. Further, though it is possible to provide proof of the method’s desirable properties through an extension to the way uncertainty is modelled and understood that will emphasize the uncertainty reducing properties of Bayesian input modelling. This is the motivation and objective of the next chapter.

The fifth chapter proposes and discusses a novel metric that quantifies modelling risk stemming from stochasticity of input modelling parameters in a single metric. Stochastic simulations tend to focus on either aleatoric uncertainty, inherent to the process modelled such as a coin toss, or epistemic uncertainty, uncertainty stemming from imperfect knowledge of the stochasticity of a variable – or modelers do not explicitly state which uncertainty is modelled. The metric discussed, Simulation Output at Risk (SOaR), allows for a straightforward joint modelling of aleatoric and epistemic uncertainty and thereby quantifies uncertainty in a novel way in a setting of Bayesian input modelling. Further, this method illustrates a key advantage of Bayesian input modelling by highlighting its uncertainty-reducing properties.

The sixth and final chapter builds onto chapter 4 by analyzing and discussing conditions under which Bayesian input modelling represents a viable alternative input modelling method along input modelling desiderata and challenges derived from the pre-ceding chapters. It concludes that Bayesian updating for simulation input modelling represents a viable method for applications meeting key assumptions that fulfills a majority of, though not all, modelling desiderata. Due to the plurality of methods used, each chapter contains a dedicated method section. The appendix provides further clarifications, additional data and analysis.

Chapter 1: Simulation methods in accounting and finance: A bibliometric study

1.1: Introduction

Scientific fields advance through the methods they apply (National Research Council, 2007). Simulation represents such a class of methods and is widely used in scientific fields as diverse as physics (Binder, Heermann, Roelofs, Mallinkrodt & McKay, 1993), genetics (e.g. De Jong, 2002) or chemistry (e.g. Gillespie, 2007) by providing estimates of otherwise intractable systems or reducing the need for costly or infeasible experiments. We study the usage and diffusion of simulation methods in accounting and finance research via the bibliometric methods of citation and cocitation analysis. This accomplishes a number of objectives, that we discuss here in turn. In finance, simulation is used extensively in select research clusters like stochastic asset pricing. Other research clusters and simulation methods like agent-based-modelling remain small yet promising. In contrast, in accounting simulation methods are not widely used and cannot be considered a mainstream method.

For accounting and finance, there appears to be broad scope for these methods. It is a versatile method to model uncertainty and complex systems, solve analytically intractable equations and models, yet it also helps researchers modelling human cognition and interactions through agent-based models (defined following Polhill et al. 2019; ABM henceforth) or model complex systems via system dynamic simulation, the "computer-aided approach to policy analysis" (System Dynamics Society, 2019). Several researchers underscore the potential of simulation for theory-building applications (Balakrishnan & Penno, 2014; Axelrod, 1997; Davis, Eisenhardt & Bingham, 2007; North & Macal, 2007; Kelton et al., 2000). In this quantitative literature review we use bibliometric and adjacent methods to analyze simulation methods' diffusion in and impact on accounting and finance research as well as provide a high-level overview of topical research clusters and strands within simulation-based accounting and finance research.

Methods can thoroughly affect a discipline. A good example are Ball and Brown (1968), who are acknowledged for contributing to shifting methodological paradigms in accounting research from normative theory to data-driven empirical research (Ball & Brown, 2013). While case studies had been used for normative policy prescriptions, Ball et al.'s research (1968) was positive and based on 'regular' companies rather than, e.g. recent bankruptcies. In the *Journal of Accounting Research,* the ratio of normative theory prescriptions to empirical, analytical and normative theory prescriptions declined from 0.64 in 1963-66 to 0.09 in 1971-75 underscoring the impact on methods used. Empirical, positive research continues to prevail in accounting and finance (Moser, 2012; Beattie, 2005; Ryan, 2002). Hopwood (2007) argues that, more recently, innovation in accounting research is partly held back by risk-aversion and methodological conformity as well as lacking intellectual curiosity, a sentiment mirrored elsewhere (Moser, 2012) as well as for finance (Gippel, 2015). It would be of interest to observe how simulation fits into this methodological discussion as we will discuss below.

From a frame of diffusion theory, the spread of empirical accounting research represents a case of swift and thorough diffusion through Rogers' (2010) stages from innovators like Ball et al. through early adopters and eventually a majority of publications. Adopting a related frame, Polhill et al. (2019) analyze how ABM adoption could be furthered. Polhill et al. conclude that ABM is not yet part of the methodological mainstream in the disciplines surveyed that were subjectively chosen to include a "human decision-making element" like in sociology or economics. They put forth a set of guiding principles for the method to further advance. Among the critical factors for diffusion of scientific methods, they argue along (1) finding a niche, (2) building alliances, (3) defining the agenda and (4) deliver on promises. Whilst steps (2)-(4) are normative prescriptions and not straightforward to examine empirically it is possible to analyze which niche or research cluster simulation found, or put differently: which research clusters and applications adopted simulation methods? We follow a related frame though remain positive in analyzing in which research clusters the chasm has been crossed. We seek to uncover the field's central simulation-based research strands through a bibliometric citation and cocitation analysis and contrast them with the core topics of the wider, non-simulation focused, accounting and finance literature to understand how the method has affected the discipline and what future trends might be. We find two contrasting states of diffusion of simulation between accounting and finance research. Simulation crossed the chasm into the methodological mainstream in *stochastic asset pricing, term structure models* and adjacent research clusters, to model stochastic assets in a method that is complementary to the prevailing data-driven empirical research paradigm per Gippel (2015). Yet simulation is not as well-established in other finance research clusters nor are simulation methods such as ABM or system dynamics. From a bibliometric perspective, the field's evolution can be described as that of a 'normal science' (Schäffer et al. 2011), sub-fields emerge around a differentiating core with gradually increasing network density. For accounting, fewer breakthrough applications appear to exist, although examples of pioneering simulation-based accounting research exist in our sample. We corroborate these results via approximating percentage shares of articles related to simulation in accounting and finance research clusters that indicate the state of diffusion per cluster. Our results imply that there is untapped potential for simulation research in select research strands, especially in accounting, and points towards areas where simulation can be applied fruitfully.

Our contribution is threefold. Via a broad, quantitative and non-selective literature review, we contribute to the dialogue around accounting and finance research clusters (Chenhall & Smith, 2011; Gaunt, 2014; Meyer, Schäffer & Just, 2010; Schäffer, Nevries, Fikus & Meyer, 2011) and uncover which research clusters of general accounting and finance research are complemented with structured simulation-based research. We show that instrumental (defined per Beyer, 1997) use, that is closely aligned with prevalent research paradigms, is most common thereby expanding on noteworthy prior research (Grisar & Meyer, 2015; Balakrishnan et al., 2014; Labro, 2015). Secondly, we point to future research directions in simulation-based accounting and finance research reflecting prevailing research paradigms and methodological discussions in the field. Finally, we contribute to a better understanding of diffusion of novel methods (Polhill et al., 2019) by quantifying diffusion across research clusters finding that several researcher clusters adopted simulation methods into their methodological mainstream – both in finance and to some extent in accounting, contrary to expectations of low adoption. Our analysis suggests that untapped potential remains in both disciplines for simulation methods and suggests promising opportunities for future research, such as more theory-building research.

1.2: Literature review

Hertz (1964) has been credited with introducing simulation methods to finance (Hall, 1975) focusing on risk in capital budgeting. Since then the method has been applied broadly, notably in asset pricing (Boyle, 1977). At first, simulation was applied in corporate finance before being introduced to capital markets where it was broadly adopted to simulate stochastic assets (Boyle, Broadie & Glassermann, 1997). Similarly, the merits of simulation for accounting were recognized early, e.g. to simulate budgeting spreadsheets (Mattessich, 1961; Murphy, 1997). However, neither management (Labro, 2015; Wall, 2016; Grisar et al., 2015) nor financial accounting, where just ~1% of articles in leading financial accounting journals use the method (Beattie, 2005), appear to have embraced simulation methods despite many use cases (Balakrishnan et al., 2014). Barriers to usage include the lack of familiarity of many researchers and their readers with simulation methods (Labro, 2015; Harrison, Lin, Carroll & Carley, 2007) and the absence of universally agreed methodological standards (Lorscheid, Heine & Meyer, 2012). Thus, there appears to be a discrepancy between simulation's adoption in accounting and finance research with a clear application in finance though not in accounting. We set out to uncover if this discrepancy can be confirmed through a broad, quantitative and not selective research design encompassing accounting and finance analyzing which structural reasons may explain such a divergence.

To understand simulation methods' impact on research clusters in accounting and finance, we first review bibliometric and qualitative reviews of research clusters in the disciplines to gauge potential applications for simulation. Starting with accounting research, Chenhall & Smith (2011) provide a topical overview of the research foci across ten leading accounting journals. Beattie (2005) identifies research clusters in financial accounting with little overlap with the research clusters in Chenhall et al. (2011). Benson, Clarkson, Smith & Tutticci (2015) review recent accounting research with a geographic focus on the Pacific Basin. Gaunt (2014) analyzes articles published in accounting and finance and deduces the main research clusters.

#	Beattie, 2005	Chenhall & Smith, 2011	Gaunt, 2014	Benson, Clarkson, Smith & Tutticci, 2015	Linnenluecke et al., 2017b
1	Normative	Capital budgeting	Acc. education	Auditing	Acc. standards
2	Financial behavioral acc. research	Incentives	Auditing	Acc. Education	Environmental Acc.
3	Market-based Acc.	Mngmt. control systems	Corporate governance	Financial Analysis	Earnings management
4	Disclosure incl. CSR and intangibles	Performance measurement	Financial accounting	Financial Reporting	Disclosure
5	Other business reporting issues	Budgeting	Mngmt. Acc.	Governance	Conservatism
6	Earnings mngmt.	Pricing/transfer pricing	Research methods / methodology	Mngmt. Acc.	Auditing
7	Acc. choice	Costing		Public Sector Acc.	Impairment
8	Economic consequences	Activity-based costing		Social and Environmental	Cost of capital
9	Failure prediction	Informal controls		Taxation	Corp. governance
10	Standard setting	MCS in inter-firm relationships			
12		Methodology			

Table 1 - Topical accounting research clusters in selected review publications (omitting "other" research areas)

Within these research clusters, though seemingly disparate, several may yield applications for simulation methods, e.g. stochastic simulation in, *costing* or *financial analysis*, as well as ABM for *governance* topics or *management accounting* generally – as Chenhall et al. (2011) noted, research moved away from a “mechanistic view” embracing that it takes place within organizations with “complex interactions”, a setting conducive to ABM with its ability to capture human interaction.

Analogous for finance we review research clustering from authoritative sources. Schäffer et al. (2011) analyze cocitation networks of four core finance journals. Further, we show the finance clusters from Gaunt (2014) and Benson et al. (2014). Linnenluecke, Chen, Ling, Smith & Zhu (2017a) analyze the contributions of the top 50 articles from the leading finance journals via bibliographic mapping.

#	Schäffer et al., 2011	Gaunt, 2014	Benson et al., 2014	Linnenluecke, Chen, Ling, Smith & Zhu (2017a)
1	Financial intermediation	Financial institutions	Financial institutions and markets	Factor Models
2	Asset pricing	Asset pricing / valuation	Investments	Asset pricing
3	Term structure	Derivatives	Options, futures and other derivatives	Conditional asset pricing
4	Market microstructure	Market microstructure Governance		Market micro-structure
5	Agency conflicts	Capital budgeting	International finance	
6	Corporate diversification and internal capital markets	Capital structure, payout policy	Special topics	Anomalies & Empirical Regularities
7	Initial public offerings	Incentives and compensation	Corporate finance	Corporate finance
8	Mutual funds	Mutual/hedge funds		
9		Behavioral finance		

Table 2 – Topical finance research clusters in selected review publications (omitting 'other' research areas)

Within the seemingly slightly more homogenous research topics within finance, several topics such as asset pricing or derivatives lend themselves to stochastic simulation. Further fields such as *Market microstructure*, *agency conflicts* or *governance* might yield fruitful applications for ABM as human behavior and its impact is at their core. This review of non-simulation focused research clusters in accounting and finance lays the groundwork, that we will cross-reference to understand where simulation methods are well-established in the disciplines thereby showing where simulation-based research has entered the methodological mainstream.

We seek to build onto and go beyond the research discussed here. We achieve this through this broad quantitative literature review with a simulation-focused research strategy, which is so far lacking in the literature. As simulation-based research in accounting and finance is distributed across journals and geographies, we build a broad, non-selective yet simulation-focused sample to capture the relevant literature. Through this uniquely broad yet focused sample we aim to find clusters of pioneering simulation-based research in accounting and finance that might have remained subdued in previous research.

Next, we review literature on the type of usage of simulation methods in accounting and finance to gain a firmer understanding of the purposes for which simulation is used with the objective of specifying fruitful future research. From a technical perspective, simulation methods can be broken down into sub-methods, for example Kelton et al. (2000) classify simulation models as either stochastic or deterministic, continuous or discretely-timed and static or dynamic – along to further prominent simulation methods like ABM, system dynamics or discrete event simulations. Beyond the technique, it can be insightful to understand the purpose for which simulation is applied, a useful distinction here is 'conceptual' vs. 'instrumental' (Beyer, 1997; Pelz, 1978). Per Beyer instrumental use seeks to "apply results in specific, direct ways" whereas conceptual use strives for "general enlightenment". Instrumental use refers to using information directly for decision-making such as simulating for asset prices with the aim of determining the 'correct' price. Conceptual use refers to using information to gain a better, deeper understanding, this entails theory-building via simulation. The prevalent research paradigms in accounting and finance appear to be complementary to instrumental use focusing on empirical research rather than new theory development. Grisar et al. (2015) lend some support to this hypothesis by analyzing uses of simulation in German management accounting research revolving around planning and risk management, thus instrumental uses. Through our sample we extend the analysis both to other geographies as well as to finance research.

To further quantify the field, we turn towards an in-depth analysis of the diffusion of simulation methods. We briefly review two central contributions of the diffusion literature along which we will describe the method's development in a similar vein as Polhill et al. (2019). Rogers (2010) is one of the central documents of the diffusion literature in which the stages of adoption are described and parallels of diffusion processes for various innovations highlighted. Notably, innovations are adopted among minority innovators at first, then followed by early adopters before the early and late majority. Moore (1991) expands onto this framework through the analogy of "crossing the chasm" arguing, originally in a Marketing context, that the strategy to drive adoption depends on the state of diffusion of, in this case, a method. Innovators and early adopters respond to similar incentives and 'cracks' between the two groups are bridged relatively swiftly as both groups are made up of "enthusiasts and visionaries". There is, however, a harder to cross 'chasm' between early adopters and the pragmatic majority where diffusion may not spread as swiftly.

Based on a thoroughly quantified analysis of current simulation-based research practices as well as the method's diffusion, we point toward potentially fruitful avenues of research. This discussion takes place against the backdrop of ongoing methodological discussions within the field of accounting and finance that we review here. Per Hopwood (2007) accounting research's innovation is held back for several reasons, notably a supposedly narrow set of methods that seeks to exploit available data to create publishable results yet with insufficient intellectual curiosity or even detachment of practical relevance of research findings. Moser (2012) argues that parts of accounting research may show signs of stagnation in its choice of methods and perceived lack of innovation therein. Although both Hopwood and Moser provide further nuanced arguments, their focus on lacking methodological innovation raises the question if simulation could be considered among the methods contributing toward breaking this perceived mold. Accounting scholars have mused about said perceived lack of usage of simulation (e.g. Labro, 2015) and argued in simulation's favor. Balakrishnan et al. (2014) provide an overview of applications of simulation including quantifying effect sizes, robustness checks of analytic results, analyzing necessary and sufficient conditions for phenomena of cost accounting systems or reducing “the set of factors (…) to consider”. Davis et al. (2007), argue cross-disciplinarily for the strength of simulation research to develop and shape theory as a method capable of “creative and systematic experimentation”.

Analogously to methodological discussions among accounting researchers, there have been reflections on of finance research from within the field such as Gippel (2015) who invites reflections of leading mainstream finance researchers on their discipline. They argue that signs toward methodological stagnation are observable in finance research as well, going as far as asserting “we all use the same data, methods, and theory”. Gippel (2015) argues that finance research largely follows an empirical data-based paradigm, an assertion mirrored in other reviews as well (e.g. Brooks, Fenton, Schopohl & Walker, 2019). Further, it is argued that theoretical contributions mostly test or moderate existing theory, rather than suggesting new theories with less than 1% of top three journal publications offering ‘pure’ theory (Gippel, 2013). They do not, however, mention simulation among the methods “not currently applied in the core”, thus either considering it part of the empirical data-driven paradigm or outside the scope of methods entirely. Further, it will be of interest to observe if simulation contributes to finance research also beyond the above-mentioned paradigm of empirical data-driven research as would be the case for theory-building finance research.

1.3: Method and Data

We analyze simulation’s adoption in the accounting and finance research mainstream via bibliometrics. According to Pritchard (1969) bibliometrics “shed light on the process of written communication and (…) development of a discipline”. Bibliometrics and particularly cocitation analysis stand apart from methods like qualitative literature reviews as it incorporates many experts' judgments as opposed to a small group (Schäffer et al., 2011). A cocitation occurs when one piece of research cites two earlier pieces of research. If two publications are cited together their content is likely closely related, the more they are cited together the stronger this link likely is. A key assumption is that a citation represents significance that the citing researchers attach to the cited material, for a critical discussion see Meyer, Waldkirch, Duscher & Just (2018). cocitation networks are built from cocitation links and constitute the 'intellectual base' of a field (Persson, 1994) that is made up of the central publications. Further Gmür (2003) posits that cocitation analysis is a "dominant method" of bibliometrics. Yet, it takes time for citations to accumulate, delaying visibility of trends (Meyer et al., 2009). Total cocitation counts show how often publications are cited together resulting in simple networks. Yet the Matthew effect (Merton, 1968) leads to networks of documents that are cited frequently in general but may not be closely related – text books are among the most cited documents. Against this backdrop, Gmür (2003) analyzed different approaches to building cocitation networks and concludes that the most reliable results are achieved through cocitation Scoring. Here each pair A and B of two publications gets assigned a score of its cocitation strength:

$$CoCit_{AB} = \frac{(number\ of\ Cocitations_{AB})^2}{minimum(citations_A;citations_B)*mean(citations_A;citations_B)} \tag{1}$$

Networks based on cocitation Scores "demonstrate considerably higher robustness" than other methods without obvious restriction (Gmür, 2003). We construct networks with score above 0.30. The resulting nodes and interconnections form clusters and networks that can be interpreted as representations of research strands. We follow Meyer, Lorscheid & Troitzsch (2009) and distinguish between clusters and groups. A cluster is defined as the set of all nodes that are connected via an unbroken chain of links and is thus determined solely via the strength of the links with other nodes. A group is defined via the Newman grouping algorithm and tends to have a more narrowly defined topical scope (Newman, 2006); the alternative Louvain algorithm delivered comparable results in a robustness check.

Clusters usually represent a topic, method or research question within the broader context. Typically, the more interconnected a cluster or group is, the denser it is, the more straightforwardly it can be assigned to a research strand (Iacobucci, 1994). In our qualitative analysis, we assign a name to each cluster that is intended to describe the overall topic covered in the publications that constitute the cluster nodes. Here we again follow Meyer et al. (2009) and start this analysis from the nodes with the highest degree centrality. Yet labelling clusters in cocitation analysis is not an exact science involving subjectivity. We chose the descriptor that most closely matches the topics of the nodes. To avoid bias, we discussed labels with several scholars in accounting and finance who provided feedback and confirmed labelling choices.

In large data sets, the full detail cluster network can grow large thereby hampering readability. For a high-level perspective, we show a summarized network where clusters are shown as nodes that following Hauke, Lorscheid & Meyer (2017) are referred to as Intercluster networks and enhance understanding of network dynamics. The first focus is on the nodes in the subsequent networks that represent the topical clusters that we derive as described above. Here we can observe from a high-level perspective how topics evolve over time. The size of the nodes corresponds to the number of nodes in the clusters they represent. Thusly we can analyze how linkages between clusters develop following. Link strength is calculated as:

$$LinkScore_{AB} = \frac{\sum_{i=1}^{n} CoCit*100}{count(nodes_A)*count(nodes_B)} \quad (2)$$

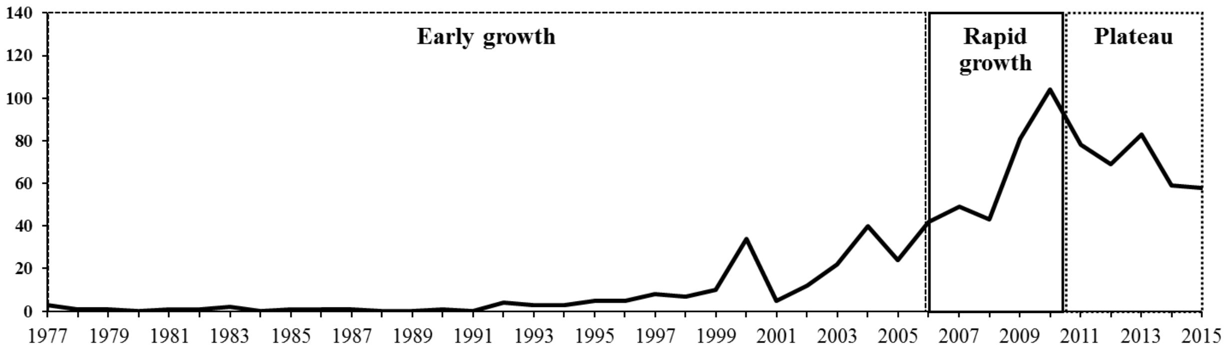
We only show links with a LinkScore above 1. Beyond readability, the Intercluster analysis also provides a perspective of the interrelation between clusters and reveals how groups of clusters revolve around related topics. We apply the Newman grouping algorithm to distinguish "hyper-clusters": groups of closely related cocitation network clusters that reveals connections between clusters that remain subdued otherwise.

Several options are available to compile citation data. One can use citation data from specific publications or journals if they capture a complete or unbiased view of a field. Another approach is keyword search in data bases which is most promising if the relevant contributions are not concentrated in a small set of journals, as this is not the case here we use a key word search. Per Falagas, Pitsouni, Malietzis & Pappa (2008) SCOPUS contains a broader selection of journals than other databases like Web of Science and is preferable to Google Scholar that has only forward-looking citation data. Hence, we decided to use data from SCOPUS. To capture the field broadly and without bias, we searched all articles and conference Papers including "simulation" or "Monte Carlo" classified as "Business, Management and accounting" or "Economics and finance" in SCOPUS. This set of articles still contains undesired articles outside our focus, we thus filter again by only including (1) journals with "finance" or "accounting" in their title, (2) journals titled with closely related terms (e.g. "auditing", "credit") and (3) additional journals from trusted lists[1] of accounting and finance journals. As a validity check we screen all keywords of the selected papers, although this did not yield keywords warranting inclusion in our sample. In our sample there are 861 articles with a total of 22,571 citations of 16,613 individual sources. These articles are published in 153 journals and proceedings, the Journal of Quantitative finance is most prevalent with 7.5% of articles. 84% were published in journals with the remainder published in conference proceedings. From the

[1] These include: the list published by the VHB (German Academic Association for Business Research, Beattie & Goodacre 2010, Beattie & Goodacre 2004 and Bradbury, Weightman, Morgan & Turley 2009

cited publications, we exclude all documents with fewer than three citations as these are unlikely to constitute the research core. At first, output grows slowly until a noticeable acceleration in the mid-90s that further accelerates and peaks in 2010 at 100+ publications around which level it stabilizes. This suggests three distinct periods, 'early growth' (1977-2007), 'rapid growth' (2008-2011) and 'plateau' (2012-2015). Although the periods are of different length, they are similarly sized in terms of citations (6,657 vs. 7,809 vs. 8,105). Output grew at 12.7% annually in the Scopus database and 10.9% in WoS. Per Bornmann & Mutz (2015) general scientific output grew between 8% and 9% per year in the period from 1945 to 2012 putting our research field above average.

Figure 1 - Number of articles in our sample per year and period

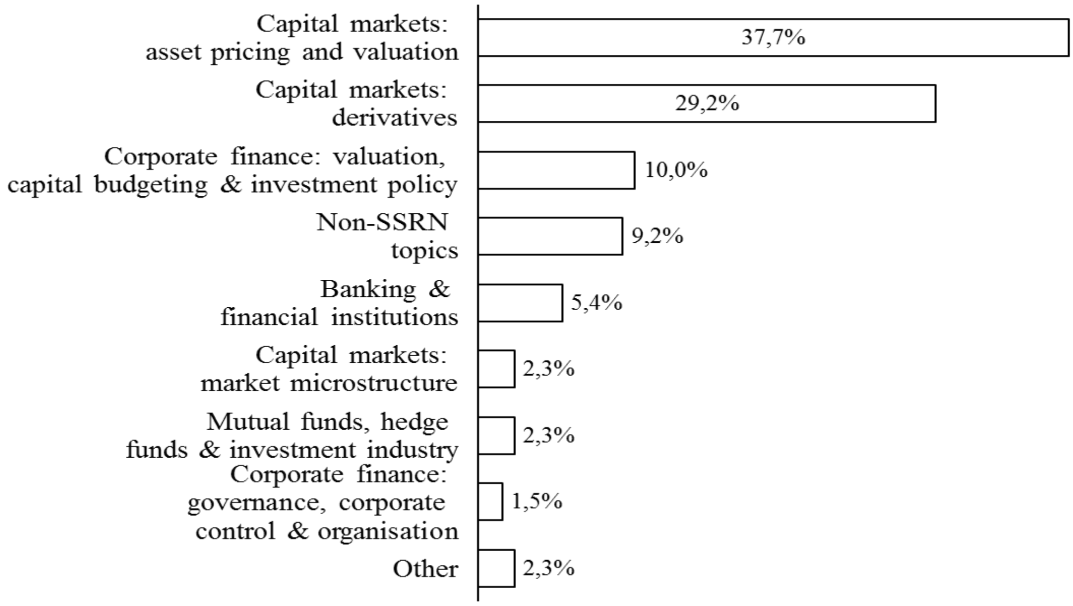


1.4: Results

1.4.1 Qualitative sample analysis

To obtain a perspective on the research topics in our sample of SCOPUS documents, we analyze a randomized subsample covering 15% of the 861 citing documents qualitatively.

Figure 2 – Classification of articles in sub-sample[2]



We follow Gaunt (2014) in using SSRN classifications and find capital market asset pricing and valuation dominate with over three quarters of articles wherein papers use simulation to model volatile assets. We provide further topical analysis of the subsample below. This corroborates the literature review's findings in that simulation has diffused in a focused research cluster in finance yet not in accounting research.

1.4.2 Citation analysis

Next, a citation analysis reveals journal, author and topical trends of our sample laying the groundwork for the cocitation network analysis. Journal articles (78%) and textbooks (19%) are cited most often with stable proportions over time - the remainder being working papers, conference proceedings etc. The most cited journals representing the mainstream are:

[2] 'Other' includes Behavioral and experimental finance, Corporate finance: capital structure & payout policy, Managerial accounting; Non-ssrn topics included simulation methodology, taxation, macroeconomics

Table 3 – Most cited journals and respective share of citations

Rank	Journal	Citation share%
1	Journal of finance	10,0%
2	Review of Financial Studies	6,8%
3	Econometrica	6,6%
4	Journal of Financial Economics	6,2%
5	Mathematical finance	4,2%
6	Journal of Econometrics	4,1%
7	Journal of Political Economy	3,0%
8	finance and Stochastics	3,0%
9	Management Science	2,5%
10	Journal of Derivatives	2,5%

For each period, we find the most cited documents, the "trending topics" among the top 15 most cited sources to understand which topics were driving the research agenda at the time. Many of these sources focus on a set of core topics:

Table 4 – Recurring topics of top 15 most cited sources per period (# of citations in brackets)

Topic	Period I	Period II	Period III
Asset pricing	- Black, 1973 (26) - Longstaff, 2001 (17) - Boyle, 1997 (18) - Barraquand, 1995 (10) - Cox, 1979 (10) - Press, 1992 (10) - Heston 1993 (9)	- Longstaff, 2001 (35) - Black, 1973, (31) - Heston, 1993 (27) - Hull, 1987 (15) - Tsitsiklis, 1999 (13) - Carriere, 1996 (13) - Clément, 2002 (13) - Duffie, 2000 (13) - Press, 1992 (12)	- Longstaff, 2001 (25) - Heston, 1993 (16) - Black, 1973 (15) - Merton, 1976 (12) - Tsitsiklis, 1999 (10) - Carriere, 1996 (9) - Andersen, 2004 (9)
Term structure of interest rates	- Cox, 1985 (15) - Vasicek, 1977 (10)	- Cox, 1985 (18)	- Cox, 1985 (13)
Volatility and risk	- Bollerslev, 1986 (12) - Engle, 1982 (10) - Artzner, 1999 (10)	- Bollerslev, 1986 (20) - Engle, 1982 (20)	- Bollerslev, 1986 (11) - Engle, 1982 (9) - Jorion, 2000 (9)
Other	- Glasserman, 2003 (13) - Boyle, 1977 (11) - Hull, 2000 (10)	- Glasserman, 2003 (42) - Hull, 2000 (18) - Kloeden, 2000 (11)	- Glasserman, 2003 (40) - Hull, 2000 (11) - Kloeden, 2000 (10) - Karatzas, 1991 (9)

Perhaps surprisingly, it may appear that topical foci remain somewhat similar throughout the three periods. In the cocitation network analysis we will further examine if claims of topical stagnation can be supported.

1.4.2 a) Simulation methods

To obtain a better understanding of how simulation methods are used, we first analyze the specific methods applied followed by their purpose. Within the sub-sample 38% use dynamic simulation methods with the remainder using static simulations. Furthermore, almost all simulations rely on discrete or discretized time steps, instead of continuous time, and are stochastic, instead of deterministic, with the respective shares at 99% and 98%. ABM accounts for 4% of the articles with the earliest paper published in 2009, all in the finance literature, occupying a narrow niche among simulation methods. We expect that instrumental usage prevails, particularly in asset pricing within the finance literature. Labro (2015) argues that the lower rate of simulation usage in accounting stems partly from the fact that "guidance from fields in which simulation methods are commonly used often does not translate straightforwardly, as these tend to have a more pragmatic focus" suggesting a higher share of conceptual simulation research in accounting. In line with previous research, we find in the subsample that both disciplines use simulation more instrumentally (88 papers) rather than conceptual (42) by a wide margin thereby suggesting that simulation has so far been mostly used narrowly to address specific research questions rather than broader theory-building sense.

1.4.3 Cocitation analysis

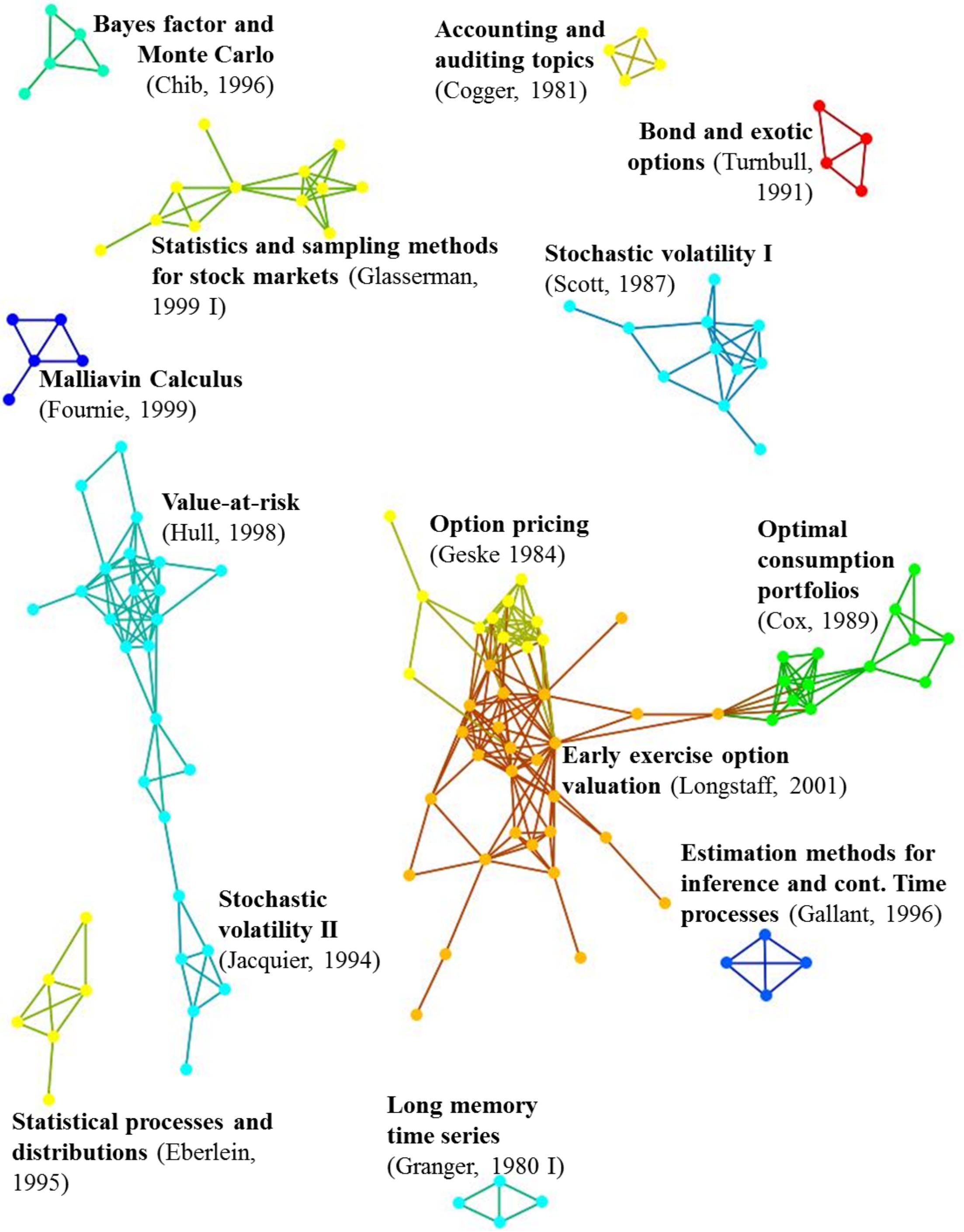
Across the three periods we observe that the density and interconnectedness of clusters shows a slightly increasing trend per the Newman modularity metric that rises from 0.581 to 0.663[3] indicating a solidifying research core. Emerging scientific subdisciplines can have high topical concentration signifying a narrow topical focus, we follow Schäffer et al.'s (2011) analysis of the network's Herfindahl index measuring concentration of individual shares within a network, concluding they are stable at ~0.1 suggesting low topical concentration.

Period I's (1977-2007) biggest cluster revolves around *Early exercise option valuation*, a simulation-based method to price non-European options before maturity, a case for which analytic methods are not applicable. Longstaff et al. 2001 is the most central node, introducing

[3] As measured at a CoCitation score of 0.00.

the method of Least-Squares-Monte-Carlo (LSM) connecting simulation methods with a regression to obtain computationally efficient valuations. *Option pricing* and *Bond and exotic* options are closely related asset pricing clusters. A densely connected *Value-at-risk* cluster has an applied focus as the risk metric can be determined through simulation methods. It is worth noting, that these cocitation clusters revolve around instrumental simulations complementary to the empirical paradigm in finance research. Period I reveals the only accounting cluster on financial planning models, closely related to finance topics, prediction of accounting number as well as methodological contributions to aggregation of time-series accounting data, strikingly with publication dates between 1978 and 1993 – against expectation of gradual adoption and potentially higher diffusion of the method in later periods.

Figure 3 - Period I – Cocitation networks based on a cocit score of 0.30[4]

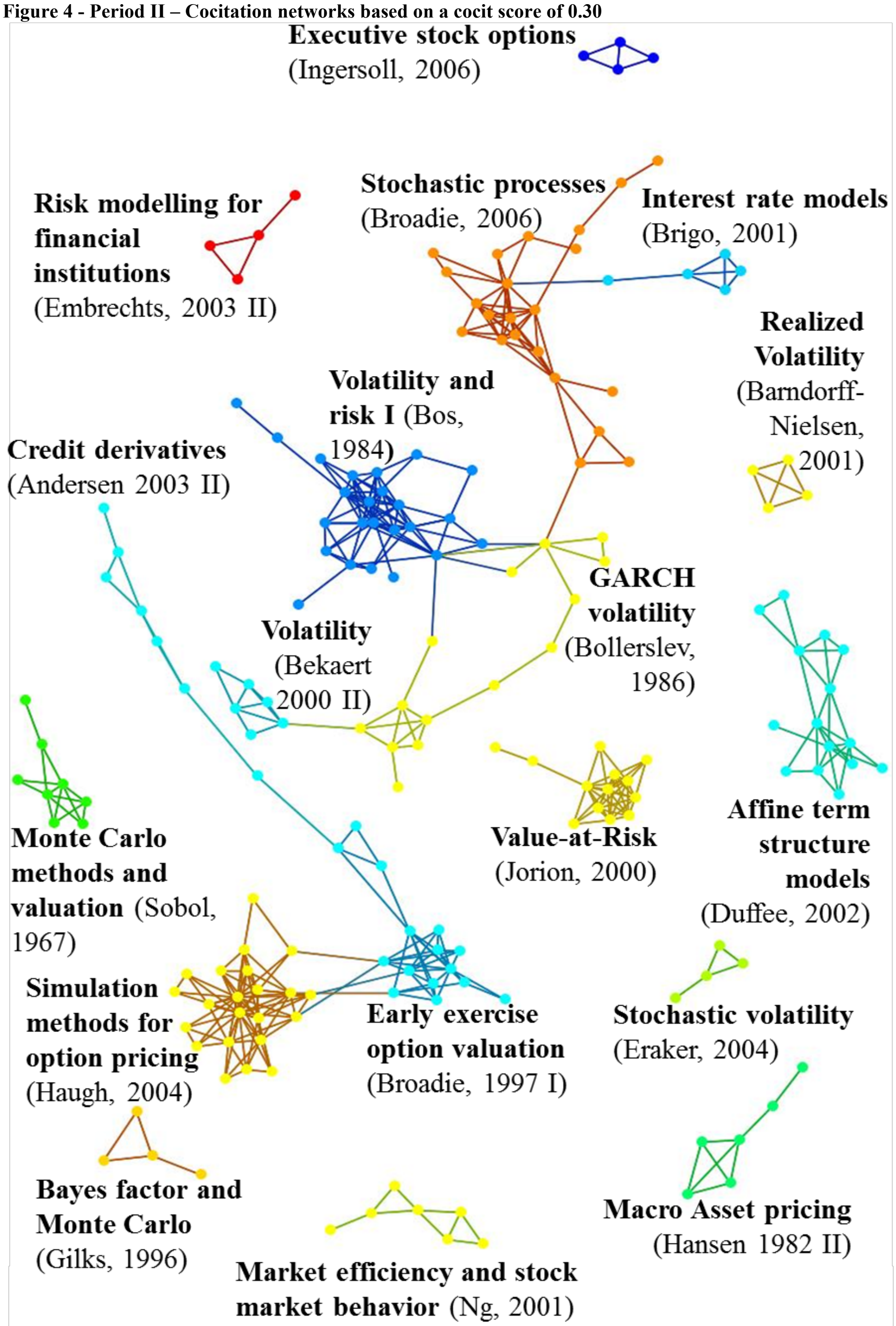


The six largest clusters of Period II (2008-2011) revolve around a central application of simulation methods in finance, pricing stochastic assets. These clusters tend to focus on two distinct aspects, model volatility and methods to price assets. While the largest cluster, *volatility*

[4] Cluster labels in bold and an abbreviation for each cluster's most central node in brackets

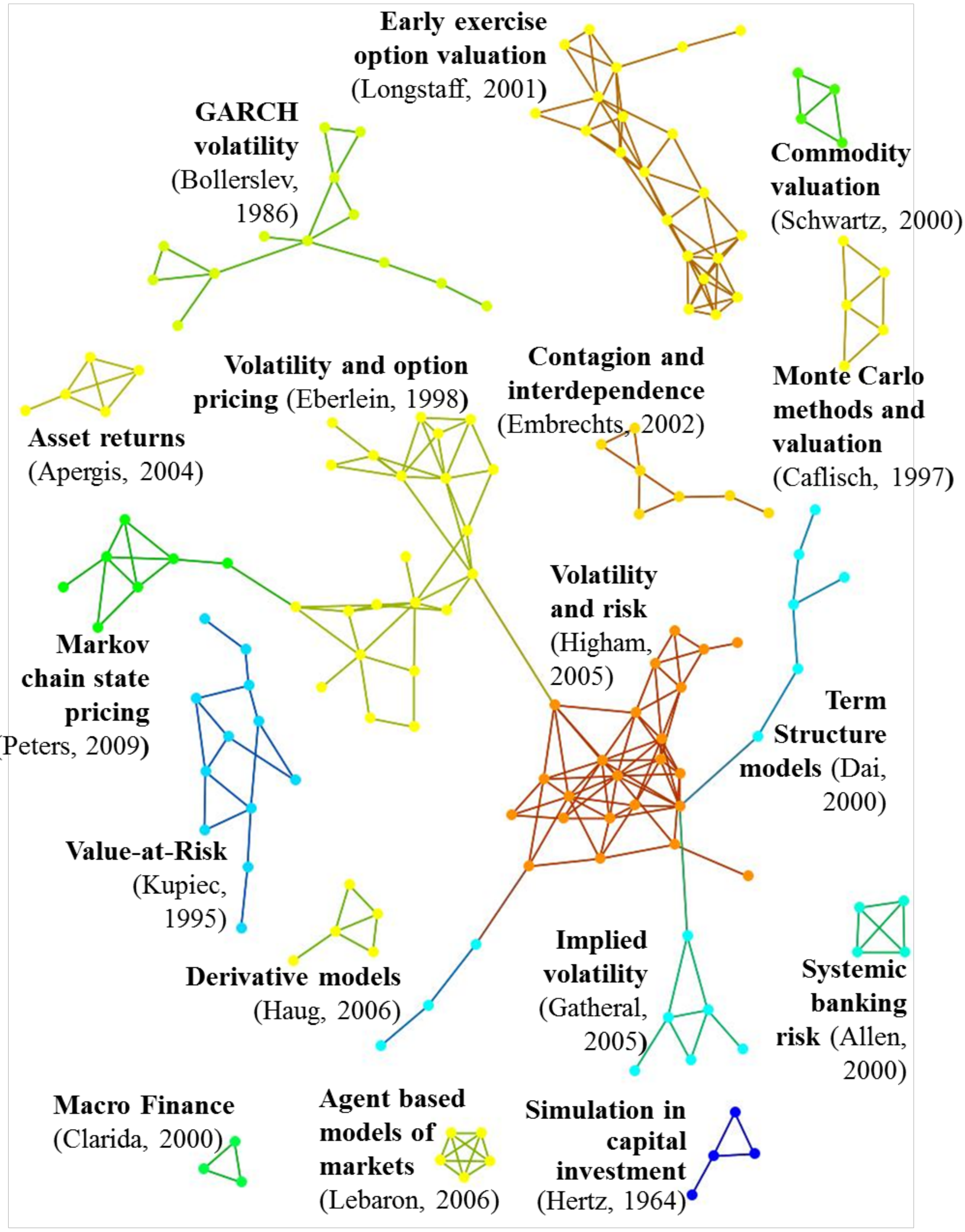
and risk, provides methods of modelling volatility, the second largest, *simulation methods for option pricing*, provides the technical methods to price assets. Further, *Stochastic processes, Affine term structure models* and *GARCH volatility* revolve around methods to model stochastic assets or interest rates whereas *Early exercise option valuation* is applied. *Value-at-Risk* recurs sharing six nodes with the VaR cluster of Period I. *Market efficiency and stock market behavior* constitutes a paper from outside the core simulation research. The papers citing its nodes are focused on abnormal returns, one key discussion associated with the Efficient Market Hypothesis. The field appears to be differentiating as we observe that topics captured through one cluster in Period I are addressed through multiple clusters on distinct aspects, e.g. the clusters on *credit derivatives* and *executive stock options.* The latter also addresses a topic from outside the core research agenda that took center stage due the Enron and WorldCom scandals (Hall & Murphy, 2003). Likewise, *Risk modelling for financial institutions* is likely driven by the financial crisis of 2008.

Figure 4 - Period II – Cocitation networks based on a cocit score of 0.30



Period III (2012-2015) is characterized by both continuity with former periods as well as new topics. The first four clusters can be described analogously to the largest clusters of Period II as they provide theory and applications for pricing stochastic assets. The largest cluster revolves around *volatility and risk*, followed by a cluster on *volatility and option pricing* and familiar topics of *Early exercise option valuation* and *GARCH volatility*, all sharing nodes with equivalents of the preceding periods. The continuity is further evidenced through *Value-at-risk* and *term structure* clusters sharing nodes with preceding clusters and other recurring topics. However, Period III also features clusters on new methods in *Agent based modelling of markets*, new applications in *macro finance* and c*ommodity valuation* or trends from outside the research field in *Systemic banking risk* and *Contagion and interdependence.* The latter two clusters reflect increased attention after the crisis of 2008 as simulation is used for stress-testing under the Basel solvency rules (Peura & Jokivuolle, 2004). More generally, new clusters from outside the core paradigm can be interpreted as evidence against a strictly narrowing research field (Schäffer et al., 2011). Further, the *value-at-risk* cluster illustrates how core literature develops as the size of the cluster decreases in size from 19 nodes to a core of 12. *Early exercise option* builds on the research cited in its previous manifestations reflecting the advances on this topic.

Figure 5 – Period III – Cocitation networks based on a cocit score of 0.30



1.4.3 a) Topical evolution

We observe above, that many of the most cited papers in our sample can be grouped topically into what appear to be stable research clusters constant in time. This could be interpreted as a sign of perceived stagnation; however, the following examples refute on this interpretation:

- **Asset pricing**: In Period I there are clusters on *Options pricing* via simulation and *Early exercise option* with papers on early-exercise options via LSM method. Nodes in the first cluster were on average published in 1986 and 1998 for the second, with articles in the latter building on the former reflecting the fields evolution.
- **Term structure of interest rates** is present throughout all periods grouped around a central node (Cox, Ingersoll & Ross, 1985), although with evolving foci. These clusters are *Optimal consumption portfolios, Affine term structure models* and *Term Structure models.* The research focus shifts from portfolio choice toward pricing of derivatives on interest rates that follow term structure models.
- **Volatility and risk**: Clusters evolve from Value-at-Risk toward modelling with GARCH-volatility. VaR clusters remain central throughout all periods, yet they shrink from 19 nodes in Period I to 13 and 12 in Period II and III evidencing an emerging core of the literature that specializes simultaneously.

These examples illustrate how the research front evolves within its research clusters.

1.4.3 b) Intercluster analysis

Intercluster analysis reveals topical clusters' proximity to one another and their evolution.

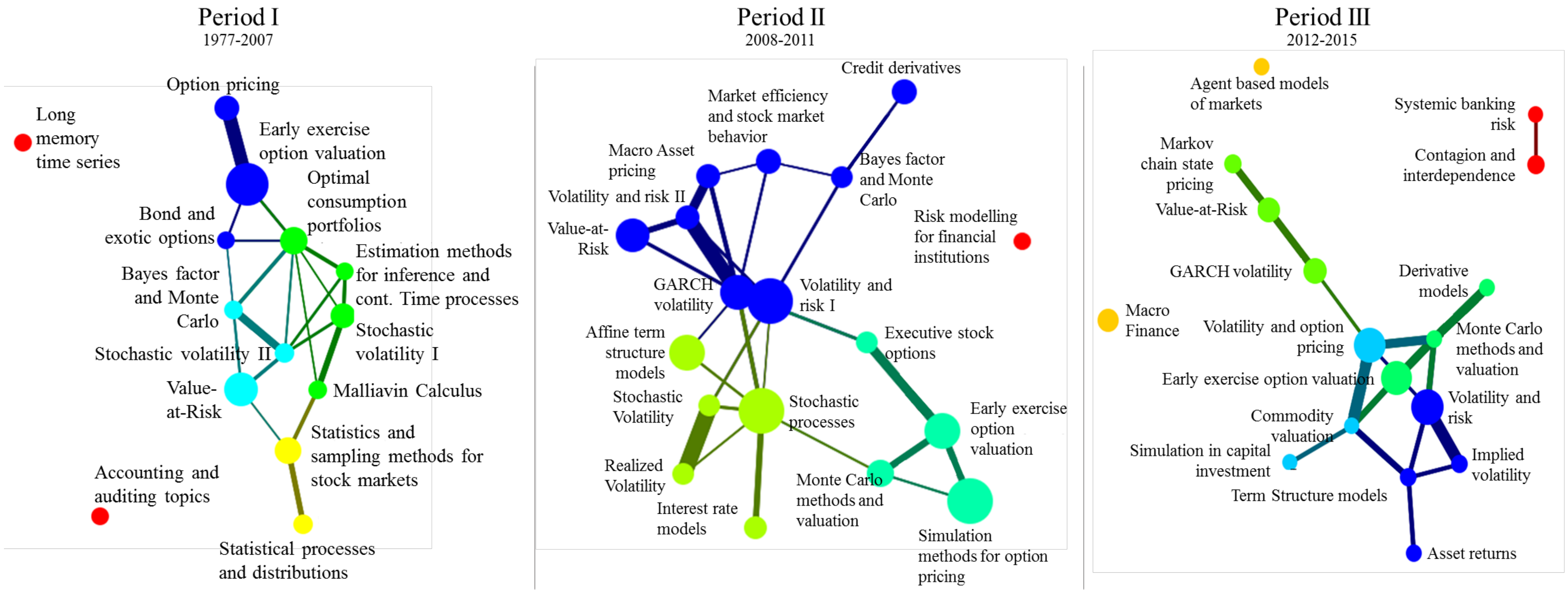


Figure 6 - Intercluster networks for periods I, II and III

Period I's network shows four distinct groups of methodological or applied clusters, such as the blue and yellow groups of clusters revolving around stochastic asset pricing. The green group of clusters presents theory of simulated processes, thus a methodological cluster. The turquoise group combine elements of applied and methodological clusters providing techniques for modelling *stochastic volatility* as well as *value-at-risk.* Finally, we observe disconnected clusters on accounting and auditing topics and others. Period II is characterized by three groups of clusters. The largest, shaded in blue, has both applied clusters like *value-at-risk* and *credit derivatives* as well as methodological ones such as *volatility and risk* and *GARCH volatility.* The group of clusters shaded in green revolves around application of simulation for stochastic asset pricing. In contrast, the final group of clusters features methodological topics and provides theory around stochastic volatility and interest rates. The isolated cluster on *risk modelling for financial institutions* appears to be among the first clusters to emerge from the financial crisis of 2008 and will be followed by more differentiated perspectives in the next period. Again, three large groups are discernible centering on recurring topical foci. We observe a large group of clusters on the methods and theory of simulation with contributions to volatility and risk, asset returns and term structure models. A second group of clusters combines both methodical and applied clusters in addressing applied topics like early exercise options, derivative models or value-at-risk while also covering volatility pricing. The last group of clusters addresses commodity valuation, simulation in capital investment and volatility and option pricing, thus more applied. Period III is more differentiated with three individual topical clusters outside the core group. The Intercluster analysis highlights the proximity between the *Contagion and interdependence* and *Systemic banking risk* clusters and the isolation of *Agent-based models of markets*. Observing the Intercluster networks over time, one can observe some differentiation despite overall continuity in research foci. As noted above, closer examination is necessary to discern newly emerging trends and the evolution of the discipline.

1.4.4 Comparative analysis of accounting and finance research

In this section, we contrast simulation's use between accounting and finance research across related research to show where the method has crossed the chasm into the mainstream. Data gathering aimed at neutrality between accounting and finance, yet a larger share of finance publications was expected – it is striking that ~95% of publications in our dataset were found via search terms more closely associated with finance.

1.4.4 a) Finance

We compare our cocitation networks with the networks from Schäffer et al. (2011) and Gaunt (2014) that focus on finance researches clusters generally. Schäffer et al. analyze bibliometric data from four leading finance journals between 1988 and 2007 divided into four periods. Among the resulting clusters, we expect parallels with overlapping nodes to the most prevalent clusters observed in our sample revolving around research on asset pricing, Term structure of interest rates and volatility/risk and this is in fact the case. The overlapping clusters from Schäffer et al. are labelled *Term structure of interest rates, asset pricing and their anomalies* and *methodological issues* as shown in table 4. For some clusters in Schäffer et al. all nodes are present in the clusters of our data set (e.g. *Term structure of interest rates* in 1998-2002). Over time the number and relative share of nodes present in both data sets grows from 7% of nodes in Schäffer's data in the first period to 19% in the last – we interpret this as an emerging core literature. In these research clusters, simulation has entered the methodological mainstream. Next, we cross-reference research clusters identified by Gaunt (2014) where many of the finance topics feature amongst the clusters in our cocitation networks: *Banking & financial institutions, asset pricing / valuation, derivatives, Corporate finance: valuation* and *capital budgeting & investment policy, incentives and compensation* thereby confirming the methods diffusion in these research clusters. Yet again, other research strands do not have equivalents in our networks: *Behavioral / experimental finance, Capital markets: market microstructure, Corporate finance: capital structure & payout policy, Corporate finance: governance, corporate control & organization, Governance* and *Mutual funds, hedge funds & investment industry* underscoring a lack of diffusion in other research clusters. To aggregate the analyses of Gaunt (2014) and Schäffer et al. (2011) we match the topics by proximity and show the overlap with the topical clusters of our data set in Table 4. Gaunt classified topics per SSRN subjects whereas Schäffer et al. assign labels, leading to imperfect matching.

Table 5 – Topical clusters and overlapping nodes between Gaunt (2014), Schäffer et al. (2011) and this data set (number of overlapping nodes in brackets)

Gaunt, 2014	Schäffer et al., 2011	Period I	Period II	Period III
Banking & financial institutions	Financial intermediation			
Behavioural and experimental finance	N.A.			
Capital markets: derivatives	Asset Pricing (Macro Factors, general models, anomalies)	Optimal consumption portfolios (1) Stochastic volatility II (1) Estimation methods for inference and cont. Time processes (1)	Volatility and risk I (3) GARCH volatility (1) Macro Asset pricing (2) Stochastic volatility (1) Stochastic processes (3)	Term Structure models (4) Asset returns (1) Volatility and option pricing (4)
Capital markets: asset pricing and valuation	Term structure	Optimal consumption portfolios (3)	Stochastic processes (3) Affine term structure models (4)	Term Structure models (4)
Capital markets: market microstructure	Market microstructure			
Corporate finance: capital structure & payout policy	N.A.			
Corporate finance: governance, corporate control, organisation	Agency conflicts (Market for control, Ownership, Capital Structure)			
Corporate finance: valuation, capital budgeting & investment policy	Corporate Diversification and internal capital markets			
N.A.	Initial public offerings (Underpricing, Long Term return)			
Governance, incentives and compensation	N.A.			
Mutual funds, hedge funds & investment industry	Mutual Funds			

Strikingly, all overlapping nodes belong to the *Asset Pricing* and *Term structure* clusters from the networks identified by Schäffer et al. Both research clusters apply models built around stochastic partial differential equations that can become intractable mathematically, though have numeric solutions via simulation. We interpret this finding in the sense that for these research clusters, simulation is applied complementarily to the prevailing research paradigm of data-driven empirical research (Gippel, 2015) with the resulting structured research dialogue and cocitation clusters. Outside this core, there is evidence for simulation-based research as we will show below jointly for accounting and finance clusters, though not to the extent that notable cocitation clusters arose. However, the question remains if structured research dialogue exists that shows how simulation can help breaking the methodological mold pertaining to the data-based empirical paradigm. First suggestive evidence comes in the of clusters on *Contagion and interdependence* and *Systemic Banking Risk* researched through simulating systemic risk and contagion in financial markets providing evidence for research dialogue employing methods outside the established core on research foci closely connected to the global financial crisis. While representing noteworthy examples of methodological diversity, these applications do not stride far from stochastic modelling, thus instrumental use. Yet, the cocitation cluster on Agent-based models of markets is cited by papers using ABM to research financial stability thereby showing how the research community uses progressive simulation methods conceptually as well. Finally, we shortly review how system dynamics simulation is applied within our sample. In contrast to ABM, with dedicated clusters and various examples of published research, System dynamics shows fewer examples such as Ding, Zhu & Xu (2013) to model decisions within companies.

In summary, this constitutes evidence that finance researchers took up methods outside their core paradigm. Simulation crossed the chasm into the methodological mainstream in distinct finance research clusters, though not in others. Simulation appears to work as a complementary method to the positive, empirical research paradigm, in addition, notable examples exist of conceptual theory-building simulation research in finance.

1.4.4 b) Accounting

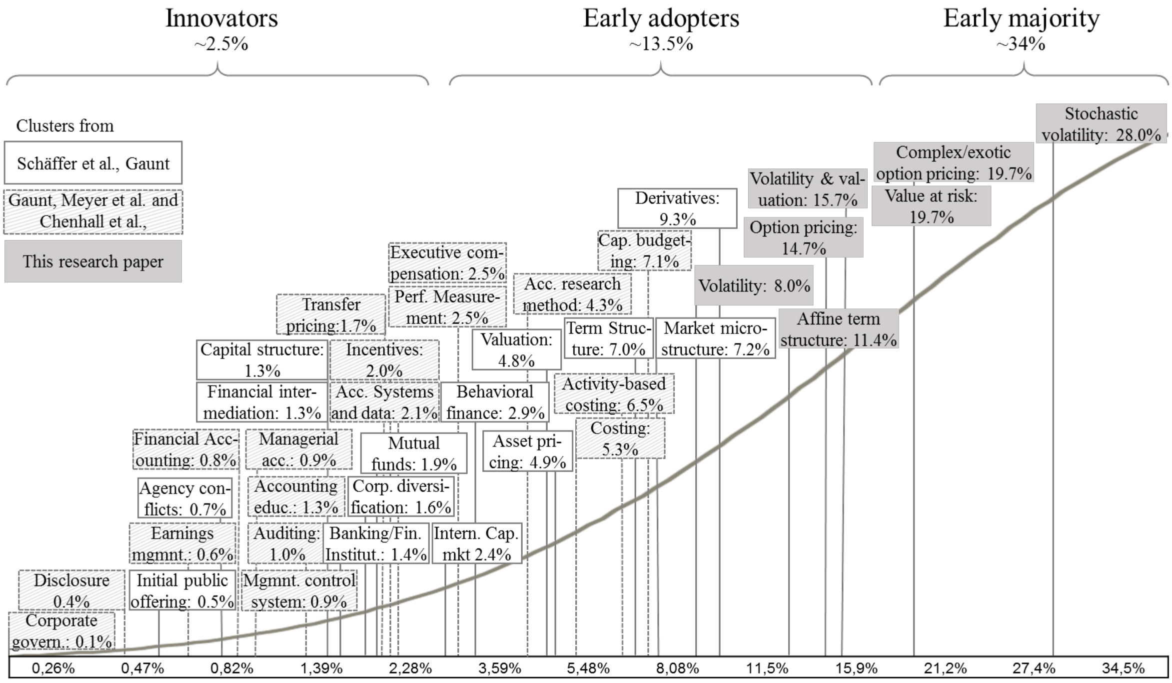
The equivalent analysis for accounting research paints a contrasting picture as overlaps with our networks are scarce. Meyer, Schäffer & Just (2010) share topical clusters on *Executive compensation*, though they do not share any nodes. The cluster on *executive stock options* from our data set is methodologically close to *Asset pricing* as it values stock options as part of executive compensation through simulation whereas Meyer et al.'s nodes revolve around *agency conflicts* and *performance measurement*. Major research strands from Meyer et al. like *Earnings research, Disclosure, Accruals* or *accounting systems and Data* are not present in our data set. *Auditing services* represents the only exception. Cross-referencing the research clusters obtained with Chenhall et al. (2011), only two are among the cocitation clusters from our data set: *capital budgeting*, which can be considered part of finance, and *Incentives*. Two other, *Costing* and *ABC-costing* increasingly profit from simulation methods, as we show further below, though without being represented through cocitation clusters in our data set. Similarly, the clusters identified by Beattie (2005) share almost no overlap with our data set, except for *Market-based accounting research* that shares some overlap as it relates to valuation. Based on this lack of overlap we conclude, that simulation-based accounting research is not part of the mainstream of accounting research. As expected, Linnenluecke et al.'s (2017b) clusters, that align well with Beattie (2005), do not show overlap with those from the simulation-based literature reviewed here. As a further robustness check we also scan all cocitation networks for 'emerging' clusters of two nodes or clusters with fewer connections than nodes, criteria that lead to exclusion. This showed, however, that also all emerging clusters belonged to finance research rather than accounting.

1.4.4 c) Research cluster diffusion

As noted above, the absence of cocitation clusters does not preclude the existence of noteworthy simulation-based research in a given cluster as we show in a further two-pronged analysis. Firstly, we approximate diffusion shares among research clusters beyond our sample and, secondly, cross-reference this with examples of pioneering simulation-based research from within our sample.

We approximately quantify the diffusion of simulation across clusters following the method put forth by Polhill et al. (2019) as the ratio of simulation articles pertaining to a research cluster to the total number of articles in the Scopus database. As Polhill et al. noted, this method entails inaccuracies as it is based on search terms risking false positives and negatives. It does, however, provide a high-level perspective on how far simulation diffused in each research cluster to cross-examine results from the preceding section. Research clusters are defined here as either research clusters identified by Schäffer et al. (2011), Gaunt (2014), Meyer et al. (2010) and Chenhall et al. (2011) or research clusters in this article (with more than 10 nodes) thereby covering both a general as well as a simulation-focused perspective. Per research cluster, e.g. *Management control systems* from Chenhall et al., we extract the number of articles in the Scopus database as well as the share of those articles containing the terms "simulation", "Monte Carlo" or "numerical experiment" to calculate approximate shares of simulation research per research cluster. The first group of research clusters stems from the general finance literature, thus the cocitation clusters and research clusters per Schäffer et al. and Gaunt. Here we expect both research clusters with low levels of adoption as well as outliers such as *term structure* or *asset pricing* with higher adoption. The second group of research clusters stems from the general accounting literature, thus Gaunt, Meyer et al. and Chenhall et al., where we expect relatively low adoption with few outliers. Finally, the third group of research clusters consists of cocitation clusters with at least 10 nodes identified in this research paper; here we expect by far the highest level of adoption as the research design focuses on the intersection of simulation-based research in accounting and finance. We exclude several research clusters if their label will lead to 100% diffusion per definition (e.g. *Least Squares Monte Carlo*), if their label is broad and thus likely to contain a large proportion of false positives / negatives (e.g. research clusters like *informal controls* or *governance*) or if analogous research clusters are represented.

Figure 7 - Relative spread of simulation per research cluster, diffusion percentage shown on the horizontal axis; calculated as number of published articles per research cluster containing "simulation", "Monte Carlo" or "numerical experiment" divided by total number of articles per research cluster (additional research clusters shown in table in the appendix)



As expected, we observe groups of low-adoption clusters from accounting sources on the left-hand side where simulation can be characterized as an emerging method used by innovators. We can also confirm the above observation that simulation diffused in distinct finance research clusters whereas it shows low adoption in others. Simultaneously, we observe a second cluster on the right-hand side of the distribution stemming from the clusters of this research paper where simulation, as expected, crossed the chasm into the methodological mainstream. Per Rogers, the cut-off between e.g. 'early adopters' and 'early majority' are defined as standard deviations from the mean of a Gaussian distribution. Interpreting cut-offs strictly, simulation has crossed the chasm into the mainstream for research clusters related to *Value-at-risk, complex/exotic option pricing* and *stochastic volatility* with more than ~16% adoption. However, due to the plurality of methods used even in research clusters where simulation is most advanced it is unrealistic to expect anything close to 'full' adoption, i.e. 100% of research publications using simulation. Even if simulation would be fully 'diffused' there will still be empirical and theoretical research applying a plurality of other methods. Thus, we argue for a range of research clusters, that simulation has crossed the chasm into the methodological mainstream, including *derivatives, option pricing, term structure, asset pricing, capital budgeting* and *volatility & valuation* from finance, as well as *costing* and *activity-based-costing*, from the accounting literature, with shares of simulation publications close to or above 5%. The research cluster *valuation* is both part of the finance as well as accounting literature and thus constitutes another outlier on the right-hand side of the distribution. Otherwise, most accounting research clusters exhibit low adoption, additional research clusters not shown to improve readability, typically had low adoption as well. This analysis provides further supportive evidence of the claim that finance has been more thorough in its adoption of simulation methods than accounting research.

Yet, even low-adoption clusters of both fields, have notable examples of simulation-based research suggesting substantial potential for the method. In table 6 we show evidence of pioneering simulation-based research in our sample within the same set of research clusters pictured in figure 7 there are examples of simulation-based research. We analyze all abstracts within the sample publications to find simulation-based research for an explorative perspective into pioneering simulation research:

Table 6 - simulation based research for research clusters from Figure 7

Source / discipline	Research cluster (percentage diffusion)	simulation-based research in accounting clusters
Accounting: Gaunt (2014), Meyer et al. (2010), Chenhall et al. (2011)	Costing (5.3%); activity-based costing (6.5%)	Simulation of cost accounting systems sparked pioneering research like Labro & Vanhoucke (2007) or Balachandran, Balakrishnan & Sivaramakrishnan (1997) who simulate costing systems and their errors; Kee & Matherly (2013) simulate target costing with "product and production interdependencies"
	Management control systems (0.9%)/	Fritsche & Dugan (1997) use simulation in a comparative analysis of errors in accounting and internal rate of return calculations
	accounting research method (4.3%)	Leitch & Chen (1999) simulate monthly accounting data where only annual data is available for use in empirical time-series accounting research
	Auditing services (1.0%)	Grimm & White (2014) use simulation to analyze the influence of regulation on audit processes; Chen & Leitch (1998) simulate financial statements to analyze accuracy of "prediction and error detection" of auditing procedures; Krauskopf & Prinz (2011) use simulation to test econometric results of tax compliance audit research
	Disclosure (0.4%)	Koh & Reeb (2015) research disclosure of R&D investments and apply simulations to "evaluate methods of dealing with missing R&D in empirical research"
	Executive compensation (2.5%)	Several research papers in our sample apply simulation methods to value executive stock options (e.g. Cheung & Corrado 2009; León & Vaello-Sebastià 2009; León & Vaello-Sebastià 2010), however these papers use simulation methods from the stochastic asset pricing paradigm
	accounting Systems & Data (2.1%)	Amen (2007) simulates different system of accounting for budgeting of unfunded defined benefit pension plans, similar simulation-based research was conducted by Morrill, Morrill & Shand (2009); Bikker & Vlaar (2007) use simulation to analyze pension plans in the Netherlands; Ouksel, Mihavics & Chalos (1997) investigate accounting information systems' effect on organizational learning through an agent-based simulation model
	Incentives (2.0%)	Bargain (2012) uses simulation to research the effects of incentives like tax and benefit changes on the labor market
	Financial accounting (0.8%)	Various simulation papers address solvency requirements, adjacent accounting rules and their implications in financial markets, particularly to stress-test financial institutions (Alm, 2015; Bauer, Reuss & Singer, 2012; Hermsen, 2010; Joshi, 2010; Peura et al., 2004; Rodriguez & Trucharte, 2007; van den End, 2012; Valencia, Smith & Ang, 2013)
	Earnings management (0.6%)	Friberg & Ganslandt (2007) simulate forex risk's stochastic impact on earnings and earnings management
	accounting Education (1.3%)	simulation is widely used in accounting Education as evidenced by a range of publications in our sample (Albright, Ingram & Lawley, 1992; Everaert & Swenson, 2014; Galitz, 1983; Miller & Savage, 2009; some educate via Monte Carlo simulation: Kelliher, Fogarty & Goldwater, 1996) – it has to be noted however that educational simulations are not necessarily in the scope of simulation-based accounting research as they aim at education rather than furthering advances in the science of accounting
Finance: Schäffer et	Agency conflicts (0.7%)	Monte Carlo methods are applied by Siddiqi (2009) to model capital structure that minimized agency costs of debt; Levesque, Phan & Raymar (2014)

al. (2011), Gaunt (2014)		model CEO's investment decision into R&D in relation to their bonus payments and solve intractable analytic models via simulation;
	Initial public offering (0.5%)	Mispricing of initial public offerings is investigated by Koop & Li (2001) with, among other methods, simulation
	Capital structure (1.3%)	To circumvent problems of statistical inference in empirical capital structure research, Chang and Dasgupta apply simulation instrumentally to approximate the data-generating process under varying management behavior
	Financial intermediation (1.3%) / Banking/Financial institutions (1.4%)	simulation-based research on bank stress-testing has been noted above in the section on financial accounting; Serguieva, Liu & Date (2011) use ABM to model contagion during financial crises and Georg (2013) model an interbank market including a central bank via ABM showing stabilizing effects of central bank actions on interbank contagion; Upper (2011) reviews the literature on simulation-based analysis of contagion in interbank markets broadly; Prorokowski (2013) models contagion of financial institutions via simulation of asset price time-series
	Mutual funds (1.9%)	Terhaar, Staub & Singer (2003) use simulation instrumentally improve fund valuation of non-traded assets; simulations of active fund management are used by Dichtl & Drobetz (2009) to evaluate performance of forecasting-based tactical asset allocation; further examples of simulation-based research around mutual and pension funds within our sample include Morton, Popova & Popova (2006); Kumara & Pfau (2013) and others
	Behavioral finance: 2.9%	Stochastic and historic simulations are used by Dichtl & Drobetz (2011a) to explain preference of portfolio insurance and investment timing decisions (Dichtl & Drobetz, 2011b) for investors in a context of prospect theory
	Market microstructure: 7.2%	To model salient properties of the 2010 'Flash Crash', Paddrik et al. (2012) deploy ABM models to accurately analyze market microstructure's impact; Mizuta, Izumi & Yoshimura (2013) address Market microstructure through ABM

We exclude research clusters that showed substantial cocitation clusters around them, such as *asset pricing, term structure models, valuation* and *derivatives* as these would show large amounts of simulation-based research. Yet, some low-adoption research clusters from Figure 7 appear not to exhibit simulation-based research in our sample, these include, from finance, *internal capital markets and corporate diversification* and, from accounting, *transfer pricing, corporate governance, performance measurement, managerial accounting* (though sub-topics of managerial accounting are represented).

The relative absence of ABM in accounting research is the exception that proves the rule: even where simulation is used in accounting, it tends to be stochastic numerical simulation rather than agent-based-models, thereby confirming the analysis of Polhill et al. (2019) – there is however noteworthy ABM research in accounting (e.g. Davis, Hecht & Perkins, 2003), though not represented in our sample. Yet for researchers in the field, these are encouraging prospects. We observe low adoption though promising research, thereby suggesting potential to build on current efforts. The clear implication for the practice of research is to consider applying simulation methods more broadly in the research clusters where we already observe research output based on simulation. It is worth pondering, cautiously, if even the research clusters with no simulation research in our sample might have fruitful avenues for simulation methods; examples could be ABM-based research in *corporate governance* or simulation to assess the impact of stochasticity of costs on *transfer pricing*.

We conclude in this comparative analysis, that finance research has shown greater adoption of simulation methods, both quantitatively as well as qualitatively, notably with relatively more research outside its core paradigm applying pioneering simulation methods conceptually. For accounting, despite the absence of evidence for a differentiated core literature, there is simulation-based research in several research clusters, for some simulation may already have crossed the chasm in the methodological mainstream with promising research opportunities.

1.5: Discussion and conclusion

We analyzed the citation and cocitation data of simulation-based research in accounting and finance, the type of simulation research conducted as well as quantified approximate diffusion shares of the method in each discipline's research clusters. Through this research we contribute to quantitative studies in simulation-based accounting and finance research, avenues for future research against the backdrop of methodological discussions in the field as well as its diffusion. We discuss these contributions here in turn.

Over the time period observed, simulation-focused accounting and finance research exhibits several noteworthy properties. All but one cocitation cluster stem from finance, rather than accounting research. Large and dense cocitation clusters are observable around *asset pricing* and adjacent topics, specifically under risk and uncertainty, where simulation is applied complementarily to prevailing paradigms in finance. Fewer and smaller cocitation clusters exist for research clusters less closely associated with this domain. Moreover, the field exhibits several traits of a 'normal' science in that it grows more differentiated over time, shows low topical concentration and research topics evolve by building onto previous research and incorporate trends from outside the core of the discipline. Simulation methods had an impact on accounting and finance, though uneven, that will continue to affect the discipline's evolution. Despite this level of adoption, there is little application of relatively newer methods such as ABM or other simulation methods such as system dynamics that may be less complementary to prevailing empirical research, though conducive to theory development. This research also provides a unique topical overview of research clusters and strands across simulation-based research in accounting and finance. A key takeaway here is the perceived low level of research dedicated to simulation input modelling, especially in a context different from capital market research, the only dominion with notable input modelling research clusters. We find that in both accounting and finance in our sample, simulation is mostly used instrumentally indicating that Davis et al. (2007) and Balakrishnan et al.'s (2014) suggestions for theory-building, conceptual simulation research in accounting and finance show untapped potential. Stochastic simulations used to model asset prices and adjacent topics fit well into the empirical data-driven paradigm and diffused widely there, whereas less established simulation-based research such as ABM, system dynamics and generally conceptual, theory-building simulation research represent divergence from this paradigm thereby explaining the relatively lower level of adoption. It is, however, noteworthy that finance research exhibits substantially more examples of conceptual, methodologically innovative simulation research in our sample than accounting research which showed lower adoption across the all realms of simulation research.

Quantifying the method's diffusion in the disciplines research clusters, we find an uneven distribution with higher shares for research clusters more closely associated with finance, especially those closer to risky asset pricing, rather than accounting research, however various exceptions prevail. Unlike Polhill et al. (2019), who observe no niches with high adoption of their method of interest, we observe several research clusters in which we argue that simulation crossed the chasm into the methodological mainstream. Despite concerns about lacking methodological creativity (Hopwood, 2007; Gippel, 2015), pioneering simulation-based research is present in finance and accounting. Notably in costing, an exception to low-adoption accounting clusters, simulation is on the cusp of crossing the chasm into the methodological mainstream. ABM could be used to capture behavioral and social influences within accounting that per Dyckman & Zeff (2015) are among the discipline's most salient though sometimes overlooked aspects.

Finally, we discuss avenues for future research by reviewing promising applications of simulation research specifically in research clusters without widespread use of simulation methods. We show that even low-adoption research clusters have scope for simulation methods as evidenced by various examples within our sample. Further, we point to areas of research where simulation can be applied fruitfully, e.g. to model human behavioral components via ABM or complex systems via system dynamics.

Bibliometric research is at no point completed (Cooper, Hedges & Valentine, 2009) but rather a snapshot, thus it will be insightful to observe how the method will continue to shape accounting and finance research, both where it is already widely diffused and where it is not and describe its effect on prevailing paradigms. Further, it would be of interest to delve deeper into the qualitative question of why accounting researchers, especially in areas like costing where promising research takes place, have been more cautious to adopt simulation methods.

Is this driven by lacking familiarity with the method due to overly similar educational curricula, as suggested by Hopwood (2007) or are there more structural reasons, that as well can be overcome as has been put forth by Labro (2015). Analogously, for finance research the most exciting question surrounds the slow adoption of ABM where a more qualitative analysis of research opportunities may be promising to support or refute our claims.

Chapter 2: A structured review of the literature on simulation input modelling in corporate finance and accounting

2.1: Introduction

After obtaining a robust picture of the research clusters and strands in simulation-based research in accounting and finance, this chapter now turns towards a core research focus of this dissertation: simulation input modelling. This chapter aggregates and presents the state-of-the-art approaches to simulation input modelling, also referred to as simulation model parameterization, in corporate finance and accounting (CF&A hereafter). Through a structured literature review this chapter provides a comprehensive treatment of simulation model parameterization that crosses disciplinary borders illustrating the state-of-the-art simulation input modelling. We commence with a review of the academic literature and then review the practitioner literature combining two distinct though interlinked viewpoints. We present a comprehensive review of the state-of-the-art in simulation input modelling. As simulation input modelling is context dependent, this chapter first reviews the question where or when simulations shall be used in CF&A and, secondly, which risk factors shall be modelled. The following chapter 3 complements this through expert interviews. We find that the methods discussed in the academic and practitioner literature are well-aligned for the most part, though with different emphases. The review suggests that simulation shall be applied to core functions of CF&A such as Capital Budgeting, Profitability or Portfolio analysis. On risk assessment, the process of mapping risks an organization faces prior to simulation, we find that both academics as well as practitioners use various frameworks and categorizations to structure risk factor, though remain vague when determining well-defined cut-off criteria. We derive a ranking of preferred input sources and find that aggregation methods that combine different information sources are not recommended widely nor discussed in detail in large parts of the literature.

Long after the merits of simulation-based financial management were established by Hertz (1964), corporate decision-makers still use less sophisticated methods as we discuss in a separate section. One may argue that practitioners require time to adapt to the state-of-the-art reasoning and methods advocated by academics as methods take time to spread in practice. While simulation is applied extensively by practitioners in financial markets it is much less widely applied in corporate settings (e.g. Boyle, Broadie & Glassermann, 1997; Verbeeten, 2006; Grisar & Meyer 2015). Yet many academics agree on the merits of simulation methods to support decision making in settings characterized by uncertainty and volatility. A significant amount of research has analyzed how practitioners use simulation for capital budgeting, risk management and others (e.g. Farragher, Kleiman & Sahu, 1999; Graham & Campbell, 2001; Verbeeten, 2006; De Andres, De Fuente & San Martin, 2015; Horn, Kjærland, Molnár & Stehen, 2015; Linder & Torp, 2014; Grisar et al., 2015; Moore & Reichert, 1983). While it has been shown that usage varies across industry, organization size and other factors it is widely found that adoption is relatively low, although increasing slightly over time. Notably many of these studies conclude that simulation methods are found complex and resource intensive despite recognition of their merits. We argue that some barriers to usage of simulation have come down through progresses in the method and technological advances in computing power and software. In this chapter, we seek to address one of these barriers by analyzing the state-of-the-art method. Both the technical benefits and the acceptance of simulation models are driven by their accuracy. Barring computational or structural model errors, simulation accuracy is driven by input modelling as illustrated by one of the most well-known proverbs regarding simulation modelling: "Rubbish in, rubbish out" (Anderson, 2004; Damodaran, 2012). The pivotal importance of input modelling has wide support (e.g. Cheng, 1994; Law, McComas & Vincent, 1994; Vose, 2008; Damodaran, 2012). This may suggest a large body of research and practical guidance on input modelling. However, publications specifically addressing input modelling for the context of CF&A are scarce. Despite input modelling's vital importance, their theoretical underpinning remains less solidly footed in CF&A publications on simulation than in other disciplines, notably operations research. We thus also review publications that treat simulation more generally and stem largely from operations research.

Despite a well-developed theoretical underpinning on the selection of input parameters for simulation models in general many CF&A publications advocate pragmatic solutions. We aim to provide an approach balancing academic rigor and pragmatism. Throughout this chapter, we will refer to parameters and distributions used as input to simulation models as simulation model parameterization or simulation input modelling (e.g. see Law & McComas, 1996).

In this chapter, we seek to provide an overview of the state of the art of simulation input modelling for the most important applications of simulations and risk factors in CF&A. An improved understanding of how various groups of input parameters shall ideally be derived supports both the accuracy of results as well as the level of acceptance of the method. Prior to addressing the key objective of this chapter, we derive which type of simulation is most commonly used and which parameters are required for these as input modelling is context-dependent (Johnson & Mollaghasemi, 1994). This chapter addresses three questions. Firstly, which applications and methods for simulation are considered most important in CF&A (RQ1)? This lays out the context in which input modelling is conducted and is critical to ensure a clear scope for the ensuing research foci. Secondly, which risk factors shall be modelled explicitly and how shall be decided what to include in the risk assessment (RQ2)? Finally, we approach the question of which input modelling methods are propagated for these applications and risk parameters per the leading experts in the field (RQ3)? The major interest is RQ3. We lay the groundwork to address this question through RQ1 and RQ2. We build on a parallel triumvirate from Johnson et al. (1994) where simulation input modelling is structured along Models/Applications and Data (risk factors) that in turn determine optimal input modelling methods. This chapter is organized as follows, firstly the research method is presented, secondly, the central applications of simulation in CF&A are derived and, thirdly, it is reviewed which risk factors should be modeled explicitly and finally we discuss the state-of-the-art methods to derive parameters for simulation models that are intended to serve as reference to simulation modelers.

2.2: Method

We derive our findings based on a review of academic literature across scientific disciplines that we complement with the practitioner perspective. We follow the structure of a meta-analysis of insights and methods developed both from academics and practitioners (Cooper, Hedges & Valentine, 2009). We pursue this approach to aggregate and contrast both perspectives. While practitioners as well as academics recognize the perennial challenge of determining input parameters (Winter simulation conference, Fox et al., 1990), it has been argued that there is a discrepancy between the views of academics and practitioners (Johnson et al., 1994) and we seek to contrast the opinions present in both realms via this research design.

Our results are to be understood in a positive sense while we follow a method that captures a normative perspective. Put differently, we capture the normative perspective of how simulation input modelling shall be conducted, although we do not evaluate the adequacy of the responses and rather assess these in a positive sense. Notably we seek to establish a consensus on the recommendations for simulation input modelling and not the most common practices.

2.2.1 Review of the academic literature

This literature review comprises of textbooks and journal articles and serves as a starting point for the review and contrasting comparison of the academic literature with practitioner publications and expert interviews in chapter 3. The objective of this literature review is to achieve progressive coherence of academic literature as defined in (Golden-Biddle & Locke, 2007). To this end and to avoid bias we follow the methods of a structured literature review by Tranfield, Denyer & Smart (2003). Several steps were taken to ensure an unbiased sample:

- A broad set of textbooks on CF&A was reviewed and relevant discussions of the topics of interest were taken into the sample. This includes general textbooks from leading scholars on CF&A as well as specialized literature on simulation methods in finance. Among the most cited general corporate finance textbooks there are Brealey, Myers, Allen & Mohanty (2012) with over 15,000 Google Scholar citations as of March 2019 and Damodaran (2012) with almost 4,000. We complement this with two somewhat less cited textbooks with the objective of achieving a diverse set of opinions among these textbooks by including Vernimmen, Quiry, Dallocchio, Le Fur & Salvi (2014) and Hillier, Ross, Westerfield, Jaffe & Jordan (2010) with about 282 and 222 citations respectively. Of the specialized Simulation textbooks, many had to be excluded as they focused exclusively on financial market topics such as derivatives and asset pricing and not on CF&A. Again, we use two of the most cited textbooks in Glasserman (2003 over 4,800 citations) and Jackel (2001; 749 citations) and one randomly selected less widely cited textbook that nonetheless maintains a general scope (Brandimarte, 2014; 65 citations). To our knowledge no textbooks exists that is fully dedicated to simulation input modelling in CF&A. we include textbooks as, firstly, we are interested in the consensus among academics that can be well-represented in text-books and, secondly, we are seeking broad and holistic treatments as opposed to more focused/specialized journal articles.

- Complimentarily we review published scientific articles to capture the debate, latest findings and viewpoints. To this end, multiple variations of key words were used to identify disparate though connected literature streams (e.g. Simulation input model-ling, model parameterization, Input Data Modelling etc.) in different databases like Google Scholar, Web of Science and Scopus.
- Further sources were obtained from the references through the method referred to as constrained snow balling (Lecy et al. 2012).
- To avoid bias from exclusion of unpublished articles, we include conference proceed-ings in the academic literature review as suggested by Tranfield et al. (2003). Sources used stem from reputable sources, notably the panel discussions at the Winter Simula-tion Conference.
- Experts interviewed for Chapter 3 were asked for literature to be included as suggest by Tranfield et al. (2003).

Hence, we create a transparent and reproducible body of literature and limit the risk of omitting central works of scholarship with relevance for this review.

2.2.2 Practitioners publications

Focusing on publications and theory from practitioners may seem counterintuitive. Oftentimes theoretical advances are developed in academia from where they spread to practitioners and are customized to their respective needs. Yet many scholars proclaim a divide between the views of academics and those of practitioners, particularly in management science (Buckley, Ferr, Bernardin & Harvey, 1998; Rynes, Bartunek & Daft, 2001). There is a trade-off between accuracy and effort regarding simulation methods as evidenced by findings that knowledge of simulation's benefits is more widespread than usage, which is often considered too complex to implement (e.g. Verbeeten, 2006; Horn et al., 2015). Practitioners take pragmatic short-cuts to avoid the most cumbersome methods. Our research methods thus rests on the attempt to understand what practitioners consider relevant theory and applications and which new methods they contribute to the body of knowledge. Our objective is to establish the state-of-the-art recommendations in model parameterization by aggregating practitioners' advice. To this end, we analyze recommendations from simulation software providers, associated consultants, simulation professionals and industry associations.

The analysis of the practitioner literature follows the structure common literature review used in academic studies (Creswell, 2013). One difference is that the literature under review is aimed at practitioners and thus may take a different, more applied angle at many of the topics addressed. Notably a lot of material tends to be application focused thus furthering our research objective of providing a distinct perspective. We provide a quantitative angle to this literature analysis by reviewing the frequency with which various points of view are expressed throughout the literature. This method results in a structure that we build onto in the next chapter, beginning with research question 1 (RQ1) from the academic and practitioner perspective, followed by RQ2 and RQ3, both reviewing first the academic and then the practitioner literature.

2.2.2 a) Empirical research on simulation in corporate finance and accounting

In the following section, we shortly review the quantitative research on how companies use simulation methods and what drives this usage. We review the positive literature on where it is applied before reviewing the normative literature on where it should be applied in the next section. This preceding review will provide a solid positive context to the normative question of where simulation modelling shall be applied. As we will show there are results from this research that are relatively consistent across studies that typically follow a questionnaire design. One central result is that despite growing adoption of advanced valuation methods many large and sophisticated companies use basic methods to decide about significant capital investments. A second central finding is that there is a growing level of adoption of simulation that is confirmed across studies, yet the absolute level of adoption is still considered low (De Andres et al., 2015; Horn et al. 2015; Linder et al., 2014; Farragher et al., 1999; Graham et al. 2001; Hasan, 2013; Baker, Dutta & Saadi, 2011). Notably many recognize the benefits of advanced methods yet choose not to implement them. This result is crucial as it illustrates the willingness to use advanced methods and hints at barriers to usage due to complexity and effort.

Two caveats are noteworthy when discussing this empirical research. Firstly, it is largely questionnaire based and thus subject to risk of misreporting due to various sources of error such as having the questionnaire filled out by ill-informed staff or by receiving responses biased by recall or social desirability biases (Krosnick & Presser, 2010). Secondly, the studies in our review are independent of one another and do not follow the same questionnaire design hampering comparability of results. A panel data study with a fixed set of companies and research design over time would be more insightful on trends over time.

We first review simulation usage among finance professionals. Moore & Reichert (1983) are among the first to review sophisticated capital budgeting techniques, they find an unexpectedly high level of usage at above 30% of frequent users of simulations among large American firms. Due to the design of the questionnaires it is not necessarily comparable across studies due to differences in wording of questionnaires and varying sample composition. In one of the most widely cited of studies on this subject Graham et al. (2001) conduct a large survey on the capital budgeting methods and methods used through a survey of almost 400 CFOs. This survey takes a particularly broad angle at the decisions on a CFO's agenda and thus serves well to illustrate their use of simulation methods. They find that about 14% of CFOs use value-at-risk or other simulation techniques in their capital budgeting processes. Financial leverage and industry seem to drive the use of simulation in the capital budgeting process which appears to be used primarily as a risk management method. Farragher et al. (1999) review three earlier studies of corporate capital budgeting processes and find adoption of Monte Carlo simulation to vary between 10% and 13%. More recent studies investigate the use of advanced capital budgeting techniques. These studies show that the use of simulation has spread notably in the last decade. Verbeeten (2006) analyzed the use of sophisticated capital budgeting techniques that go beyond a simple NPV calculation among a sample of Dutch companies. This includes simulation techniques, real option analysis and game theoretic approaches. They find that firm size and capital intensity of the industry have a positive effect of adoption. Notably financial uncertainty is a strong predictor while social, market and input uncertainty are no strong predictors. Furthermore, Verbeeten's research suggests that most companies that adopt advanced methods use their results to supplement simpler methods that continue to be used[5]. In a yet more recent study De Andres et al. (2015) show that 28.5% of Spanish non-financial firms "always" use simulation analysis in their investment decision process. This relatively high percentage of simulation users is notable though the specific way in which simulation is defined is not specified making it hard to compare this result across other studies that show far lower adoption. Horn et al. (2015) focus their research on real options and simulation[6] and find that roughly 6% of large Nordic companies use this capital budgeting method. Among non-users 70% are aware of the

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[5] Some survey research results suggest that companies do engage in scenario analysis (base case vs. best case vs. worst case). Although this is akin to a simplified simulation analysis we do not consider this basic level of scenario analysis to constitute simulation.

[6] Despite the focus on real options they survey the use of simulation in real option analysis which can be understood as proxy for simulation analysis more generally.

technique and apply it non-formally thus confirming their acknowledgment of the benefits. Complexity in implementation is given as the prime reason not to implement it more formally. Consistent results are furthermore reported by Hasan (2013) and Baker, Dutta & Saadi (2011) who find low adoption of simulation in finance departments in their respective samples.

The empirical literature on the adoption of simulations in accounting is less extensive. Linder et al. (2014) show that in line with findings in capital budgeting that simulation is used infrequently in their large sample of Danish companies although the method is well-known. In similar vein Grisar & Meyer (2015) survey the use of Monte Carlo simulations in the management accounting departments of companies in German-speaking countries. They find that the use of Monte Carlo simulation is not yet widespread despite knowledge of the method among the companies surveyed, yet adoption is growing quickly. Again, industry and organization size factors are shown to have predictive power. Strikingly this research stream confirms that practitioners recognize the benefits of simulation analysis to a much larger extent than they use the method in practice.

Despite a growing level of adoption by practitioners across both finance and accounting we conclude that the literature supports the claim of a theory-practice gap in the application of simulation in finance and accounting.

Per Rees (2015), key drivers of simulation modelling are complexity and number of decision variables, scale / size of the decision, opportunity to drive mitigation measures, corporate governance requirements, the need to support decisions with quantitative analysis and the need to reflect risk tolerances. Throughout the expert interviews (see chapter 3), there were several further drivers. These are internal and external uncertainty affecting organization performance and entrepreneurial flexibility/existence of real options[7]. Vose (2008) provides a brief overview on the reasons for "risk analysis", thereby already underscoring the perceived focus on modelling risk in his approach to simulation modelling.

2.3: Results

2.3.1 Applications for simulation in corporate finance and accounting

Here we take a normative perspective and review the state-of-the-art views are on where simulation should be applied. Monte Carlo simulation can be applied to a variety of tasks in corporate finance and accounting to support decision making through a holistic assessment of volatile and uncertain situations. We set out to analyze which applications exist for Monte Carlo simulation and where this method should be applied.

Firstly, based on the academic literature we derive where simulations should be applied in CF&A. Secondly, we review what practitioners recommend. This allows us to aggregate the academic and practitioner's views.

2.3.1 a) Review of academic literature

Hertz (1964) already discussed the potential benefits of evaluating major capital investments through simulation models long before large parts of today's state-of-the-art understanding of simulation modelling and the related real option theory had been conceived. These benefits still stand today and suggest that simulations shall be used in a variety of decisions made in the finance departments of large firms or entities.

Textbooks

In their classic textbook Brealey and Myers introduce simulation analysis and apply it to a capital budgeting decision without treating other applications. Similarly, there is a compact

[7] the opportunity to affect the outcome of decisions after an initial commitment has been made; this flexibility can also be created strategically in response to a certain risk

treatment of simulation by Damodaran (2012) where simulation is discussed as a method of coping with effects of "continuous risks". Vernimmen et al. (2014) discuss a short example of a Monte Carlo simulation for a profitability analysis. Hillier et al. (2010) also provide an example of a Monte Carlo simulation for a capital budgeting decision. Here it is also discussed that the method is not widely used in practice despite having been introduced in the field many years ago; it is argued that this is due to complexity. Such relatively short treatments of simulation are common across general corporate finance textbooks. There is a more in-depth treatment of simulation in textbooks on financial engineering and textbooks more closely focused on simulation methods directly. Glasserman's (2003) leading textbook on simulation in Financial Engineering focuses on applications in financial markets and derivatives thereby offering fewer insights on where Monte Carlo simulation should be used in CF&A. Similarly, McLeish (2011) and Jäckel (2001) discuss simulation and its applications in depth yet largely for financial markets, not corporate finance.

The discussed textbooks are primarily concerned with corporate finance applications and much less with management accounting. Labro's new monograph (Labro, 2019) on costing is a noteworthy addition to the accounting literature as it strongly builds on simulation methods and is written by an accounting scholar deeply rooted within the simulation research community. While this contribution builds on simulation-based research, it does not specifically discuss simulation-methods in accounting, research or practice.

Among the publications surveyed few textbooks focus on simulation in CF&A in depth. Particularly an in-depth treatment of simulation input modelling was lacking. This scarcity has been noted for textbooks in operations research (Schmeiser, 1999). Thus, we would conclude that the academic textbook literature does not offer complete guidance on where simulation shall be applied in CF&A.

Academic journal publications

Examples abound in publications that highlight the merits of simulation analysis for various applications in CF&A. Notable examples include:

- Salazar & Subrata (1968) provide one of the earliest applications as they describe a simulation model for capital budgeting under uncertainty
- Spedding & Sun (1999) illustrate how "Discrete Event Simulation may be used to evaluate activity-based costing" of manufacturing companies
- Rode, Fischbeck & Dean (2001) discuss valuation through simulation of financial flows of a power plant
- Gatti, Rigamonti, Saita & Senati (2007) apply simulation to analyze project finance particularly from the equity holder's point of view
- Labro & Vanhoucke (2007) simulate costing systems to draw inferences of interaction effects of costing errors

Such a list could be continued; however, it is not straightforward to infer a consensus on the most central applications for simulation in CF&A from the number or quality of published research papers. The number of research papers dedicated to one application is not necessarily indicative of its importance but rather suggests an ongoing debate. To find the consensus on where simulation should be applied in CF&A we need to look beyond this literature and focus on literature less determined on uncovering new methods and findings but rather tailored to simulation modelers overcoming their challenges and offering concrete guidance.

2.3.1 b) Practitioner publications

Practitioners publications are expected to provide pragmatic yet methodologically sound advice on where to use simulation methods in CF&A. Yet we are unaware of publications that claim to offer a comprehensive prioritized list of applications, methods and tools of simulation analysis in CF&A. We thus take a different approach by aggregating the topics discussed in various publications and gain understanding through their joint contributions. In this section, we focus on practitioner publications that approach the topic from a technical angle through the lens of the simulation modeler in leading software environments. To this end, we review several publications focused on simulation modelers and aggregate which applications and purposes for simulation are discussed.

Software providers and simulation professionals

Several companies have developed tailored software solutions for Monte Carlo simulations that are designed for practitioners. Klein (2010) offers a comprehensive review of leading software including Crystal Ball, @Risk, ModelRisk and Risk Solver. Although MATLAB is not reviewed in Klein (2010) and tends to be used even more ubiquitously for applications in quantitative finance we still include it here[8]. Our interest is not the adequacy of the software environment, but how these software providers advise their users on where to use simulation. As these companies engage in a dialogue with practitioners their advice can offer a perspective into the applications simulation modelers are most concerned with. We attempt to condense the provided advice to provide insight into industry best practices. For completeness, we review the offering of each of these above-mentioned providers on financial modelling in their software.

- **Crystal Ball:** Charnes (2012) provides a guideline for financial modelling with simulation in Crystal Ball. The general approach to building simulations for financial modelling is explained through theory and applied examples serving as introduction to some of the most common applications. We take the applications treated here as starting point

[8] Notably R is used extensively in quantitative finance for example to compute valuations of derivatives or similar applications; our focus is CF&A where it is less widespread

of our analysis. Charnes suggests that income statements and balance sheets are the most widely used financial models though also highlighting other applications.

- **@RISK**: Rees (2015) gives a recent comprehensive perspective via a practically focused and carefully argued treatment of simulation as business decision support tool. This treatment goes beyond basic guide to simulation modelling and provides comprehensive advice on simulation.
- **ModelRisk:** Vose (2008) is among the most widely cited guides on quantitative risk analysis that offers practical advice on solid academic footing with applications and examples.
- **Risk Solver:** Solver offers a comprehensive technical guide focused on implementing simulation in this software.
- **MATLAB:** Anderson (2004) takes a similar approach and introduces the reader to advanced simulation methods in finance and accounting. Here also some of the most central tasks of any firm's finance and accounting departments are addressed.

We focus on the above set of core publications for two reasons. Firstly, we focus on publications that address both simulation methods and the application of these methods in CF&A. Thereby we explicitly exclude part of the literature that addresses simulation in a financial market context thus achieving a more insightful sample. Secondly, we show that the applications covered exhibit increasing conformity across publications thus indicating that we reach a saturation point through coverage of the sources mentioned (Charmaz 2006). This is evidenced as only few or no additional applications emerge through the addition of further sources. Only three of the five sources reviewed include unique applications. These add informational value though are not considered core applications due to their rareness. These applications include simulation modelling in Real estate finance, Asset depreciation and Revenue management; all applications that are not mentioned in the above discussion of Simulation methods in textbooks.

In table 7 we show the frequency with which applications are treated and thereby capture their perceived importance. We show all applications discussed in the practitioner literature that is mentioned at least twice in the literature:

		Charnes (2012)	**Rees (2015)**	**Vose (2008)**	**Solver (2010)**	**Anderson (2004)**
1	Income statement / Profitability	x	x	x	x	x
2	Asset / risk portfolios	x	x	x	x	x
3	Net present value (NPV) / Capital Budgeting	x	x	x	x	x
4	Correlations	x	x	x	x	x
5	Real options	x	x		x	x
6	Credit analysis and risk	x	x	x		x
7	Financial options / derivatives	x	x			x
8	Risk management (e.g. Value at risk)	x	x	x		
9	Internal rate of return (IRR)	x		x	x	
10	Balance sheet	x		x		
11	Cash flow statement	x		x		
12	Insurance risk (i.e. risk the insurer assumes)		x	x		
13	Project finance / Project costs			x	x	

Table 7 – Most discussed applications of simulation in CF&A

Cash Flow statements and balance sheet simulation tend to be discussed more rarely despite the paradigm of integrated financial management which recommends a joint analysis of all financial statements. However, the analyses of profitability and other oft-mentioned applications have basic financial statements at their core and these are thus not mentioned. Per Charnes (2012), simulating balance sheets is likely one of the most common application, though it is not mentioned here explicitly, potentially reflecting the fact that the balance sheet may be run in the back ground of an e.g. real options or profitability analysis. Few applications of simulation from management accounting are mentioned.

In summary, the most frequently discussed applications of simulation center around Profitability analysis, Capital budgeting, which can be considered part of management accounting as well, and Portfolio analysis with less discussion of applications purely rooted in management accounting.

2.3.2 Risk assessment and prioritization

After discussing which applications and situations in simulation modelling in CF&A can be considered most central, we now review how simulation modelers should decide which specific risks shall be modelled and which risk factors to simulate. Risk factors describe granular groups of risks such as operational risks, price volatility of production inputs and demand volatility. This process is typically referred to as risk assessment or risk-breakdown. It consists of quantifying magnitude, likelihood and interrelation or correlation between risk factors. This can then be complemented via identifying potential countermeasures to individual risk factors (Lam 2014, Ch. 23). Lam defines risk assessment as "to identify, quantify, and prioritize an organization's key risks". Put differently a risk assessment framework helps simulation modelers decide which risk factors are sufficiently important to warrant inclusion in simulation models and the risk management process and which can be considered negligible. Resource constraints forbid explicit modelling of all existing risk factors. Particularly small risk factors that do not have strategic importance or low visibility are typically excluded from simulation analyses.

2.3.2 a) Review of the academic literature

To understand the academic perspective on risk assessment frameworks we commence our literature review in the risk management literature. A significant part of this literature revolves around risk management for financial institutions and the surrounding regulations. As we are researching how non-financial services organizations use simulation we exclude this literature. Much of the literature on risk management revolves around reducing the "variability of firm value or cash flows" (Stulz 1996), yet more recent approaches acknowledge the benefits of risk taking and build frameworks taking this perspective into account. Nocco & Stulz (2006) argue that companies shall not seek to minimize risk but rather “optimize the firm’s risk portfolio by trading off the probability of shortfalls and the costs with the gains” that can be achieved through taking on risks.

Companies start their risk identification processes via a bottom-up or top-down approach (Nocco et al., 2006). Both approaches then proceed to categorize risks along a framework or categorization scheme. For historic reasons, financial institutions tend to categorize risks into market, credit and operational risks. However, this classification does not necessarily reflect the requirements in other industries well (Nocco et al., 2006). Simple frameworks for non-financial institutions include the categorization per environmental, industry and organization intrinsic variables as presented in Miller (1992) or market, operational, credit, and reputational risks (Nocco et al., 2006) which again mirrors practice from financial institutions. In a recent review article from Dionne (2013), corporate risks are categorized into pure risk, market risk, default risk, operational risk and liquidity risk. Other frameworks exist that for instance categorize risks analogously to the classification common in financial markets as pertaining to market, credit, and operational risks (Lam 2014, Chapter 4). Lam also discusses a detailed risk categorization scheme that includes a list of risk categories including: Credit risk, Market risks and hedging, Stock price risk, Investment risks, hedging risks, Secondary risks, Operational and Insurable risks, Catastrophic failures, Business risk, Cultural risks, Pension risks, Outsourcing and Reputational risks. Yet another categorization scheme (Chapman 2011) makes a high-level division between internal process and business operating environment with many subdivisions. This illustrates that many different well-argued approaches coexist for risk assessment and categorization. The categories are usually further broken down in a next step into individual risk factors of increasing degree of specificity. Miller (1992) offers one of the early examples of such a framework. This categorization step in the risk assessment process is also commonly referred to as risk taxonomy (Lam 2014, chapter 23). Although these shortly presented categorizations are distinct they all present approaches that cover the major risks pertaining to an organization. Categorization schemes do not affect which risk groups an organization faces specifically risks. This insight and the level of interrelated risks across categorizations led to the emergence of Enterprise risk management (Stulz 1996).

The decision which risk factors to simulate, is considered in a process commonly referred to as risk prioritization. In this process risk factors, can be structured around properties such as probability of occurrence, severity of impact on the entity and effectiveness of controlling measures (Lam 2014, chapter 23). These can then be used as inputs to risk matrices or heat maps that illustrate the risk profile of the entire entity along said dimensions that represent the axes of such matrices. In the next step the organization's risk appetite, key risk indicators and the overall risk profile's impact determine the response. Risk prioritization via matrices, risk radars or heat maps are limited for several reasons:

- Within heatmaps or matrices there is usually no clear criteria to define cut-off points and thresholds for inclusion. Thus, enabling the simulation modeler to prioritize the importance of risk factors yet not answering the question which risks including in a simulation model. This method has merit prioritizing risk factors under scarce resources, thus in a scenario where not all risk factors can be assessed and modeled.
- Factors like risk appetite or key risk indicators (KRIs) remain inherently hard to quantify and thusly a matter of subjective decisions. Yet one of the core ideas in simulation modelling is quantitative decision-making with little room for subjectivity.

In short, risk prioritization lacks clear cut-off criteria for risk modelling decisions thus remaining subjective. Stulz (1996) argues for inclusion based on whether a risk factor "affects the firm's ability to implement its strategy". Chapman (2011) analogously states that risks shall be organized along the threat they pose toward an organization's goals without specifying how to assess if this condition is met.

Hence, despite the advantages of risk assessment and prioritization there is a lack of well-grounded quantitative reasoning to decide which risks should be modeled prior to their quantification. For lack of clear guidance in the academic literature we turn to the practitioner community in the following section.

2.3.2 b) Practitioner literature

Like the academic literature, Vose (2008) also categorizes risk classes or groups, though concedes that such prompt lists are never exhaustive mainly serving as a check list of risks to include yet not offering decisive guidance on which variables to analyze and manage. Likewise, Kaplan & Mikes (2012) view compartmentalized thinking in risk modelling critically as organizations can be threatened by "combinations of small events that reinforce one another". Thereby they mirror core arguments of enterprise risk management. Instead of a categorization of risks by source or mapping by expected severity they put forward a categorization by its optimal response. The first group describes preventable risk factors that should be managed and prevented through the rules and processes of an entity. Secondly, strategic risks an entity willingly and rationally assumes to achieve its objectives in line with its risk tolerance, here entities shall actively work on containment strategies. Thirdly, external risks from outside the organization should be managed through risk identification and mitigation. However, challenges remain as this classification still leaves up to the modeler to decide which magnitude of risks to simulate explicitly.

Including all risks that threaten an organization's strategy or its survival has been voiced as a rule of thumb among practicing risk managers (Stulz, 1996). A broader definition supported by industry associations includes factors that may impair an organization's ability to reach its goals

(Ballou, 2005). We argue that these heuristics turn the risk assessment process on its head as the threat risk factors pose to an organization's survival or goals can only be assessed via a holistic risk management analysis rather than being its input. Put differently, risk analysts need to analyze a risk factor before its impact can be confidently judged. Rees (2015) supports our argument. 'A priori' risk exclusion is deemed inappropriate as inclusion decisions hinge upon context- and objective-dependent criteria. Further, exclusion of individual risk factors leads to an understatement of total risk and to the potential omission of mitigation measures. We fully support this argument, though concede that for practical purpose it can be necessary to prioritize and exclude risk factors. Below, we condense the risk factors analyzed in this set of practitioner literature that offers more granularity than the academic literature:

		Charnes (2012)	**Rees (2015)**	**Vose (2008)**	**Solver (2010)**	**Anderson (2004)**
1	Variable production costs (COGS)	x	x	x	x	x
2	Purchasing price / cost of inputs (e.g. commodities)	x	x	x	x	x
3	Demand	x	x		x	x
4	Sales prices (incl. discounts)	x	x	x	x	
5	Production output / operational / technical factors	x		x		x
6	Market share	x	x	x		
7	Fixed costs / SG&A	x	x	x		
8	Stock returns	x				x
9	Interest rates	x				x
10	Market size	x		x		
11	Sales / Revenue	x				x
12	Product quality		x			x
13	Exchange rate		x	x		
14	Personnel fluctuations		x	x		
15	Policy changes		x	x		

Table 8 - Frequency of specific risk factors modelled the publications reviewed mentioned at least twice

Most frequently discussed are central variable determinants of the income statement or profitability analysis reflecting the applications discussed above. Further, there is a wider range of risk factors in line with our expectations as risk factors stem from diverse sources. Risk factor as defined above can contain various volatile input factors for simulation models.

In addition, several industry associations of accountants and risk managers publish on risk assessment and prioritization that complement the practitioner literature.

The Institute of Management Accountants (Shenkir & Walker, 2007) provides yet a new perspective on categorizing risks into financial, strategic, hazard and operational risks while acknowledging the importance of linking these categories back towards enterprise risk. A risk mapping approach is suggested for prioritization of risk factors with impact and likelihood as dimensions, the typical heatmaps discussed above. The Professional Risk Manager's International Association, another leading industry association, follows the framework of market, credit and operational risk likely rooted in risk management of financial institutions (PRMIA 2011). Hence it appears that the industry associations are well-aligned in their advice with the broader practitioner literature. Similarly, they provide various frameworks for structuring and prioritizing risks yet the central question where to place the cut-off for inclusion is not directly addressed.

2.3.3 Parameterization of simulations in CF&A

This section addresses the major research question of this chapter: state-of-the-art simulation input modelling and model parameterization.

Various methods for simulation input modelling exist and can be categorized into a new framework that we devise to provide structure to the discussion. Input modelling methods can be distinguished and groped into a classification of input modelling methods by data source as these are distinctive of each modelling method. This simple classification is mutually exclusive, as no method belongs to more than one category, and collectively exhaustive, as all methods found in this review can be categorized into one of these classes. We distinguish the following methodical groups:

- **Data driven methods:** parameters from empirical data

- **Expert driven methods:** parameters from expert opinion
- **Theory driven methods:** parameters based on theory
- **Aggregation methods**[9]**:** parameters from a mix of the above methods, also including combining multiple expert opinions and the use of fundamental models that attempt to replicate the full DGP (as opposed to the resulting parametric distribution as done in theory driven methods) and resulting distribution

Within the academic and practitioner literature review this section is structured along these classes. Other frameworks for structuring sources of input parameters for simulation in CF&A exist that use other traits and properties of methods for classification. These frameworks or treatments though typically comprise of the above-mentioned categories (e.g. Rees, 2015) and are thus consistent.

2.3.3 a) Review of the academic Literature

Again, we review wide-ranging sources including both the generalist simulation literature on parameter selection as well as the CF&A-focused literature.

Data driven methods

For simulation input modelling in general the standard work of Law et al. (2000) is a good starting point that focuses on data-driven methods. Law stipulated three ways to obtain stochastic input parameters that assume access to historic data:

1. **Trace-driven:** this entails using actual past data from the process modelled. This method is also called bootstrapping for its similarity to bootstrap sampling (Cheng, 1994). This method faces limitations as many simulations are driven by the properties

[9] This is not to be confused with the research method of mixed methods as discussed in e.g. Creswell 2013

of the input distribution's tails that represent extreme events and are not regularly observed thus might be missing finite sample of empirical data (Kelton & Law, 2000). Capital market finance applies this regularly to back-test portfolios (Morgan, 1996).

2. **Empirical distribution:** Actual data is used to define an empirical distribution from which to draw random variables (Kelton et al., 2000). Discrete or continuous distributions are derived directly along empirical data. This method can involve adding tails to the empirical distribution to allow for sampling of values outside the observed range (Cheng, 1994). This then constitutes an aggregation method as the information for the appended tails requires a distinct source.
3. **Fitted standard theoretical distribution**: Statistical techniques are used to find the best-fitting theoretical distribution to the observed values: this approach entails approximating the empirically observed data via the best fitting theoretical distribution. This is also referred to as the "parametric bootstrap" (Cheng, 1994). The statistical fitting procedure is typically based on least-squares, method of moment or maximum likelihood estimation (Leemis, 1995; Kelton et al., 2000). In this process the modeler fits an initial distribution to the data. Unless there is a perfect fit between empirical and theoretical distribution the modeler decides if the discrepancy is due to sampling error or represents key characteristics of the distribution that need modelling (Leemis, 1995). Fitting standard theoretical distributions to data is supposed to overcome key weaknesses of trace driven distributions, in that they can fall short of adequately capturing the properties of a distribution's tails. Barton (2002) argues that also parametric distributions can be susceptible to this problem and may lack sufficient realism. Further, a fully specified theoretical distribution can create a false sense of security.

If the data to be simulated, follow a theoretical distribution (3) reasonably closely the fitted standard distribution "will generally be preferable to using an empirical distribution (2)" (Kelton et al., 2000). Theoretical distributions smooth empirical irregularities inherent to samples of limited sizes for approaches 1 and 2. Additionally, empirical data observations tend to be discrete while the underlying distributions are continuous, leading to unrealistic distributions if observation periods are short. Another major advantage of this technique is that it is generalizable and allows for manipulation of its parameters. In addition to the above approaches, Kelton et al. (2000) also mention theoretical derivation of distributions, however these may suffer from shortage of information.

Cheng (1994) provides a tutorial focused on the statistics of fitting theoretical distributions to the data. Cheng posits that fitting standard distributions is also the most commonly used method among simulation modelers. Law provides an extensive treatment of theoretical distributions and their statistical properties. Though he does not provide guidance on which classes of variables shall be used or how one shall decide if the statistical fitting procedure is inconclusive.

In a similar vein Leemis (1995) provides detailed examples of using maximum likelihood estimation to approximate a Weibull distribution addressing some of the major challenges.

Bratley (1987), a leading simulation textbook, expresses preference for data driven input modelling methods. Conceding that absence of perfect data is common, it presents experts opinion as a fallback option if fitting data cannot be found. Bratley favors using empirical distribution if the theoretical shape of a variate's distribution is not known and cannot be inferred from the data alone as statistical tests for fitting distributions have low power to reject the fit, potentially leading to a false sense of certainty regarding the choice of a distribution's shape. He also points to the general difficulty of estimating distribution's tails precisely from limited data sets and the conditions under which it is reasonable to append a theoretical distribution to cover the tail of an empirical distribution. The difference in viewpoints with Law is noteworthy.

Biller & Nelson (2002) address frequent questions in simulation input modelling and again consider deriving distributions from data to be the preferred option, advising on the use of experts only where data is not available. Interestingly, they prefer theory driven methods and advise to choose a distribution if there is a "strong physical basis" for it even if its goodness of fit measure is not the best.

Johnson et al. (1994) review simulation input modelling methods for operations research however, the depth and scope of the discussion merits inclusion. Here, again the need to start from empirical data is stressed. Various methods of deriving distributions and parameters from data are discussed as well as shortcomings of data like disadvantageous formatting like grouping. Johnson et al. also briefly mention expert-based methods though only if data is not available and dismiss them largely on the grounds of cognitive biases. Expert input may function as a "veto power" to reject implausible distributions. Vincent (1998) discusses input modelling in OR and expresses preference for data driven methods. Data driven methods are considered optimal despite the assertion not to take data "too seriously" for the risk of imperfect data quality. Deriving distributional characteristics from the data with dedicated software is discussed in-depth. In a related publication, Vincent & Law (1991) discuss a simple method to use software for distribution fitting in the absence of data where it shall be fit to data points such as the minimum, mean or maximum that may be known, however it is not elaborated from which sources these should stem. Preference for data over expert opinion is further supported in Fox et al. (1990) among a panel of leading simulation researchers. In summary, it appears that the perspective from operations research is strongly data driven with less scope for expert-based and theory-driven methods.

In addition to preference for data based-methods, most dedicated simulation literature places substantial focus on selecting the right distributions and discusses various methods of deriving distributions from data and assessing their goodness of fit. The dedicated chapters in Vincent (1998), Bratley (1987) and Kelton et al. (2010) and the references therein offer detail on these methods. It is noteworthy, that the discrimination between candidate distributions is based on reason rather than strict rules (e.g. Vincent 1998).

However, other simulation scholars differ and argue against the use of empirical data. Schruben argues (Barton 2012) that "most simulation studies desire to learn what would happen if we were to change a system". Moreover, Schruben & Schruben (2001) discuss the limitations of data that can be "distorted, dated, deleted, depleted or deceptive" thus rendering methods to derive input modelling parameters from data problematic. Schruben concludes that real-world data is "not important to the success" of simulation studies in dynamic environments.

Analogously to the previous sub-section we commence the review of parameterization of simulations in finance and accounting with the leading corporate finance textbooks. Vernimmen et al. (2014) advise on risk assessment and parameterization by recommending to "identify influential factors" and "to look at available information to determine the uncertainty profile" without further specifying the information's source. Damodaran (2012) introduces simulation suggesting two methods. Firstly, the use of historical data for macroeconomic and comparable data; thus, implicitly assuming constant stochastic parameters. Secondly cross-sectional data of e.g. comparable organizations shall be used for more specific variables.

Expert-based methods

Law et al. (2000) discuss approaches to be used in the absence of historical data that rely on estimates of SMEs. Law also discusses the limitations of SMEs and potential biases without an in-depth discussion of de-biasing techniques. Law also argues in favor of using theory to understand which family of distributions a process could follow based on preexisting knowledge of the process. Henderson (2003) addresses the topic of uncertainty about the true value of a stable input parameter as opposed to known fluctuations of a stochastic parameter. A further level of uncertainty is introduced through the uncertainty about the level of volatility of input parameters. Throughout this chapter, we assume that the non-observable parameters of distributions are fixed and can be known – however, this assumption is relaxed for the chapters revolving around Bayesian input modelling. Biller et al. (2002) view expert-based methods as a back-up option if data is unavailable. It is worth noting, that they point toward simple de-biasing methods that shall be applied to derive probability distributions from experts' statements. Although Vincent (1998) points to the difficulty to detect "deviations from stability" of distributions, he does not directly argue for obtaining insights from experts who may have access to data that allows them to make forward looking statements.

The textbooks reviewed (Brealey et al. 2012, Vernimmen et al. 2014, Damodaran 2012, Hillier et al. 2010) do not discuss expert-based methods in-depth. However, insight can be gleaned from the discussion in the corporate finance literature. Damodaran (2012) suggests making assumptions about the distribution if data is not available or of inferior quality also pointing toward some of the difficulties of selecting consistent probability distributions when parameterizing models without solid data access but does not discuss more general challenges of expert elicitation such as cognitive or organizational biases. Brealey et al. (2012) dedicate a short paragraph to the selection of probabilities which in this case represents their treatment of simulation input modelling. In the example, they elicit expert input from the marketing department of the example organization. In Hillier et al. (2010) market share is estimated based on expert input. Market size is estimated based on external data from an industry publication combined with judgment from the simulation modeler, thus an aggregation method.

In summary, the simulation-focused literature views eliciting distributions from experts as a fall back option if data is not available and does not discuss the advantages and drawbacks in depth. There is however a literature on expert elicitation with relevance for simulation modelling. Cooke (1991) constitutes a broader perspective on expert opinions under conditions of uncertainty. Although the discussion is not specifically aimed at simulation modelling in CF&A it touches on many aspects of importance in this context. The book covers topics from probabilistic thinking and biases to elicitation and scoring of expert opinion to methods of combining divergent expert opinions. These situations are analogous to situations faced by simulation input modelers. We do not review the methods in depth, yet this is advised for applied modelers looking for a robust elicitation guidance.

Theory driven methods

By Theory Driven Methods we refer to those that harness the theoretical foundation or understanding of the DGP to derive input parameters or distributions for simulation models. This approach is also referred to as the scientific or conceptual approach (Rees 2015). Law et al. (2000) discuss hypothesizing on distributions based on theoretical arguments yet concede that in practice “we seldom have enough prior information” for this approach to be precise. In a similar vein, Barton (2002) affirms that there is rarely a strong theoretical argument for a specific distribution thus arguing in favor of empirical distributions. Johnson et al. (1994) mention the appeal of theory-driven distributions like the Bernoulli for a Coin toss, though do not discuss when to use this method. They use the term of “physical plausibility” for input parameters that are derived from information about the DGP. Their discussion suggests preference for data driven methods. Biller et al. (2002) argue in similar vein for the theory driven methods. In summary, the academic literature advocates this class of input modelling methods, though doubts their practical applicability.

Aggregation Methods

By aggregation methods we refer to those using a combination of the discussed methods to derive parameters. The reviewed textbooks that do cover simulation input modelling (Brealey et al. 2012, Vernimmen et al. 2014, Damodaran 2012, Hillier et al. 2010) do not discuss aggregation methods in depth.

While Cooke (1991) discusses combining expert opinions at length, there is no discussion of combining information sources of different formats such as combining expert opinion and empirical data. We argue that a broader discussion of combining different information sources would be beneficial for two reasons. Firstly, there is no generally agreed and theoretically derived hierarchy of preferred input sources in dynamic modelling environments as became evident in this literature review and is supported as well by simulation experts interviewed in the next chapter who do not exhibit consistent preferences. Secondly, simulation input modelling challenges can be characterized by a lack of data and well-fitting experience for the specific situation to be modelled (Barton et al. 2002) as simulations are often applied in dynamic circumstances.

As we will show in the following chapter through expert interviews, aggregation methods are important for simulation modelers as situations with lacking data sources or theory and divergent expert opinions commonly arise. Leading simulation modelers routinely combine data sources typically in pragmatic ways.

2.3.3 b) Practitioner publications

Below we provide an overview of the most commonly advanced opinions amongst practitioner publications on simulation input modelling. Rees (2015) supports practitioners through an overview of academic and pragmatic approaches that provides a deeper background into the reasoning for simulation modelers than most other sources. We review this in the subsection on hierarchy of input modelling methods.

Data driven methods

Palisade (Rees 2009) offers a pragmatic framework on input distributions; similar guidance is provided in Rees (2015, Chapter 9) who also represents Palisade. Anderson (2004) discusses importing data into a simulation model for parameter estimation as the default case underscoring the central role of data driven methods. Beyond these publications there is a range of secondary sources like Charnes 2012 who presents two methods: data-driven methods or "other" methods such as experts. Like the treatment in Law et al. (2000) a clear distinction is made between different approaches like using actual historical data via bootstrapping or fitting distributions to data. Historical data for distribution fitting is advised if available thereby displaying preference for data over experts. Despite the purported advantages of historical data some caveats are offered. Data may not be available or sufficiently recent to be accurate, moreover data may be biased, or that the DGP changes over time. In such situations Charnes (2012) advises on using input from experts.

Moreover, several software packages offer distribution fitting applications that provide the best fitting distribution based on historical data. This is evidently a practical way to parameterize a distribution if historical data is available and parameter drift is unlikely (Charnes 2012). Notably to our knowledge none of the software packages offer tools specifically designed to address structural breaks.

In summary, the practitioner literature appears well-aligned with the academic literature in that it prioritizes deriving distributions from data over experts.

Expert-based methods

Vose (2008) reviews methods for eliciting subjective probability distributions from experts. Non-parametric distributions are highlighted in this section as being especially adept at modelling expert opinion. The quality of expert's assessments is crucial for the simulation and aggregation process because objective information or historical data are often unavailable or costly (Vose 2008).

We borrow the term pragmatic distributions from Rees (2015) for distributions that approximate real-life processes and distributions while being straightforward to communicate and parameterize. Per the accuracy-complexity trade-off, practitioners apply "pragmatic or easy to communicate" distributions (Charnes; 2012; Rees, 2015; Vose 2008). Pragmatic approaches include uniform, triangular or PERT distributions. PERT[10] is a distribution intended to raise the realism of triangular distributions if limited knowledge about the distribution is available.

Through the prevalent cognitive biases some parameters and distributions may be more prone to biases than others. Vose (2008) discusses some of the challenges of extracting expert input with respect to the distribution used. Some non-parametric distributions such as the triangular or uniform distribution are straight-forwardly captured though few intuitive parameters. In this context, non-parametric distributions are understood as distributions without an underlying probability model (Vose 2008). In other words, non-parametric distributions do not make any assumptions about the DGP and can therefore be defined with great flexibility. Furthermore, many commonly used parametric distributions do not have upper and / or lower bounds,

[10] This distribution stems from the Program Evaluation and Review Technique approach where distributions of project lengths had to be estimated without prior experience. The estimation of a PERT distribution requires the minimum, mode, maximum and a shape parameter that determines the peakedness, and thereby the curvature, of the distribution.

whereas boundedness is more common in non-parametric distributions (Vose, 2008). Among the most flexible distributions is the “relative” or “custom” (Charnes, 2012) distribution consisting of an array {x} of values and an array {p} of the respective probabilities with full flexibility. Vose (2008) thus advocates the use of non-parametric distributions when eliciting distributions from experts as they may lack familiarity with abstract parameters of existing distributions. This includes drawing distributions or specifying their density across their range in small intervals. Sampling from this distribution can be achieved in common modelling software environments (Charnes, 2012) although no general closed-form solutions exists for its descriptive statistics.

While these methods may be common in elicitation of expert opinion, the extracted information is then typically transformed into a parametric distribution (Gigerenzer et al., 2003; expert interviews, 2016). Vose argues to leave this step out and use a non-parametric distribution directly. Yet, combining multiple non-parametric estimates is not directly addressed in Vose (2008).

In summary, the practitioner publications are again well-aligned with the academic literature. Significant attention is directed toward the intricacies of eliciting subjective probabilities.

Theory driven methods

Rees (2015) posits that theory-based methods are the most accurate way to derive input parameters that is based on knowledge of the DGP. Such models must be individually derived for each process (Rees, 2009) thus requiring extensive customization. Gleißner et al. (2014) describe a scenario of “complete information” of the modeled process as the ideal basis for parameter derivation. This can be likened to a theoretical understanding of a process that results in complete knowledge of the modeled parameters. This scenario is discussed as a textbook case without in-depth discussion of how it could occur in realistic settings.

In summary, we again find analogous viewpoints as both practitioners and academics view theory driven methods favorably if they can be applied.

Aggregation Methods

Vose (2008) discusses Bayesian methods that combine prior information with new data for simulation input modelling. Here, the prior can have various sources like empirical data, expert estimates or theory and the new information takes the form of observations of the distribution or process to be modelled.

Crystal Ball advises on using historical data to select distributions or if not available to "use judgment based on experience" to gather all knowledge accessible about a distribution (Oracle, 2013).

In summary, Aggregation Methods are not explored in great depth as a simulation input modelling method. This is also reflected in the low rank of preference expressed in the following hierarchical discussion of simulation input modelling methods.

2.3.4 Hierarchy of input modelling methods

We discussed which type of simulations are used in CF&A and which Risk Factors to be modelled. When faced with broad modelling challenges the question arises which input modelling technique is adequate under which circumstances ideally with an implied hierarchy ranking input source per modelling situation.

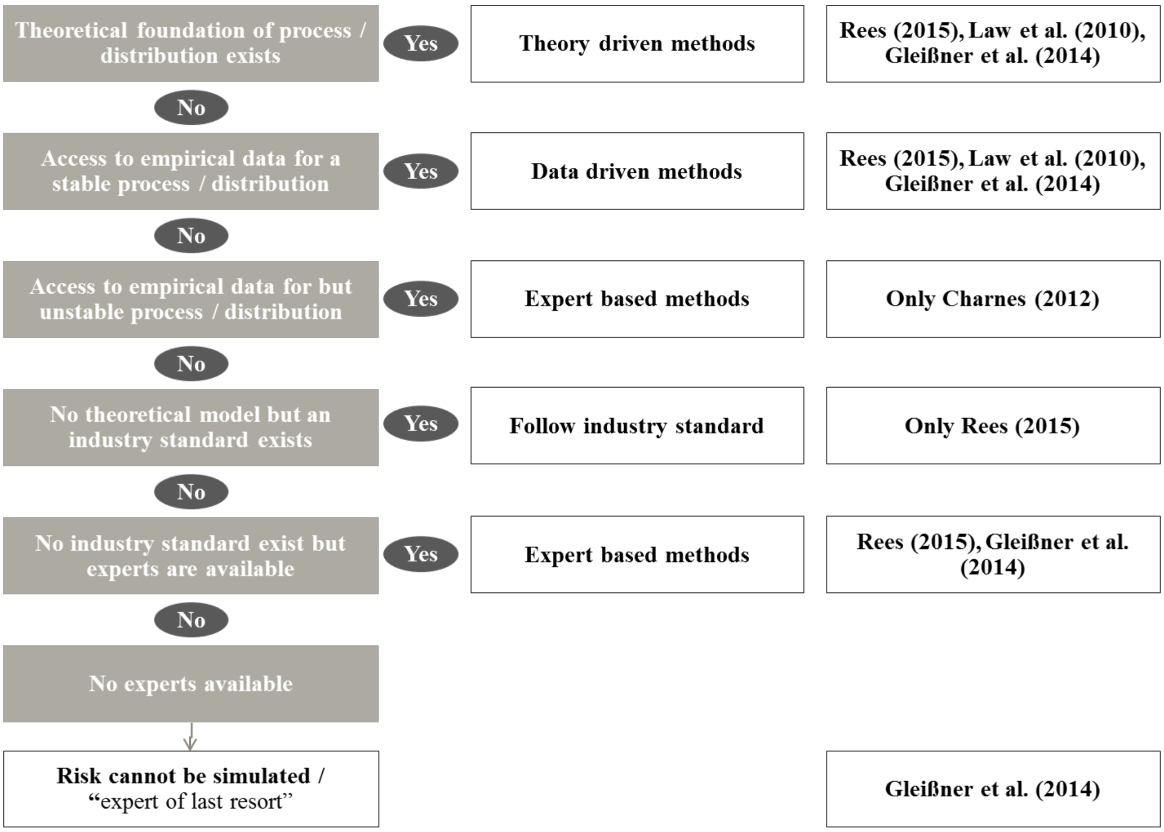
Rees (2015) discusses a framework that follows a set of considerations made by simulation modelers. This framework provides some degree or hierarchy for simulation input modelling methods. Theory driven methods are preferred, followed by data driven methods and lastly pragmatic approaches where no data or theoretical foundations are available. While this guidance is valuable to simulation modelers, it also remains vague and does not explicitly consider varying modelling challenges such as changes in the DGP.

Another candidate is provided by Gleißner et al. (2014), who break down the options for model parameterization available to simulation modelers depending on which kind and quality of empirical information or data is available. This overview cascades down from a situation of close to perfect availability of appropriate data to a situation where not even experts can make reasonably accurate estimates. Implicitly this reflects a preference for parameters based on empirical data. The next chapter will show that this preference is not common to all experts (interview transcripts, 2016). Damodaran (2012) expresses preference for data sources depending on the variable to be modelled, though this is based solely on empirical data and does not constitute a full hierarchy of sources. Charnes (2012) supports the view that expert input is preferable to historic data if parameters can be expected to change over time. Based on this brief review we argue that among the methods used there is a lack of a generally agreed hierarchy under dynamic modelling conditions.

We condense the hierarchical logic discussed here in a simple decision tree. Decision trees are a useful method to structure decisions along a chain of multiple points (Safavian & Landgrebe, 1990). Hierarchical classifiers are decision trees that support multilayered non-linear decisions (Magee, 1964) and is ideally suited here. The decision tree is constructed per the most relevant sources used for input modelling. Other methods for decision trees attempt to “minimize uncertainty from each level to the next level” (Safavian et al., 1990) or, put differently, the decision nodes with the highest informational entropy form the root of the decision tree. We follow the order of preference as we do not have quantitative data to calculate the information gain per decision node in each step. In the decision tree, we provide the hierarchical order of simulation input modelling methods based on circumstances as shortly described in the grey shaded boxes. To the right, we place the recommended input modelling method or group of methods and references.

The resulting decision tree below here is noteworthy as it underscores the solid level of alignment between academic and practitioner community. Charnes (2012) recommendation to harness expert knowledge if DGPs are unstable is not recommended in most other publications, though the limitation of relying on data is mentioned elsewhere (e.g. Rees 2015).

Figure 8 - Decision tree of parameterization methods depending on data availability; notes: theoretical foundations include scenarios of complete information, see Gleißner et al. (2014)



Within several nodes more detailed modelling specifications arise. Within node "Derive distribution from data" a variety of methods exists as reviewed above (e.g. Kelton et al., 2010); depending on data availability. Further, within node "Elicit Expert opinion and derive distribution" we suggest the methods discussed above from Vose (2008) and Cooke (1991). From a viewpoint of information theory (Winkler, 1981), it is noteworthy that aggregation methods are not recommended widely as situations with non-perfect data are common (e.g. Gleißner, 2014). This could lead to situations where it is desirable to harness all available information into a combined parameter estimate with a theoretically sound weighting. This is explored in Cooke (1991) for expert opinions, though not other sources. Finally, this decision tree puts forth, that if no adequate input modelling source is present, a risk factor cannot be simulated. However, in practice one can work with what we call here an "expert of last resort" that can provide however ad-hoc risk estimates to preclude a risk from being considered at all. Yet, this ad-hoc decision tree does not provide an answer to all modelling challenges as this requires more detailed modelling scenarios as well as judgment from simulation modelers.

2.3.5 Distributions and parameters

Input distributions for simulations can mathematically be described via their probability density functions (PDF) and its parameters (McLeish, 2011). We differentiate between two factors, that is firstly the family the distribution belongs to and secondly the parameters that fully define the specific shape of the distribution. A further treatment is provided in the appendix.

Analogously to the previous representation of most frequently modelled applications and risk factors in simulation analysis we show the most frequently mentioned distributions in table 9[11].

		Charnes (2012)	**Rees (2015)**	**Vose (2008)**	**Solver (2010)**
1	Standard normal distribution	x	x	x	x
2	Log normal distribution	x	x	x	x
3	Beta Distribution	x	x	x	x
4	Weibull	x	x	x	x
5	Triangular (symmetric & skewed)	x	x	x	x
6	Uniform continuous	x	x	x	x
7	Uniform discrete	x	x	x	x
8	Exponential	x	x	x	x
9	Poisson distribution	x	x	x	
10	PERT	x	x	x	
11	Mixture distribution (standard normal with Excess Skewness or Kurtosis)	x	x	x	
12	Bernoulli	x	x	x	
13	Binomial	x	x	x	
14	Gamma / Chi Square	x	x	x	
15	Pareto	x	x	x	

[11] We exclude Anderson (2004) from the depiction of this analysis as this publication is not focused on the choice of stochastic distribution and thus is not insightful for this analysis

16	Logistic	x	x	x
17	Log-Logistic	x	x	x
18	Students-T	x	x	x
19	Maximum extreme	x	x	x
20	Minimum extreme	x	x	x
21	Negative binomial	x	x	x
22	Geometric	x	x	x
23	beta - PERT	x		x
24	Hypergeometric (incl. Inverse)		x	x

Table 9 - Most frequently used distributions per leading sources

Table 9 highlights the conformity regarding the set of statistical distributions that find application in the practitioner literature. As part of the expert interviews we focus on whether practitioners likewise support the use of this number of statistical distributions.

2.3.5 d) Correlation

Unlike univariate simulation models, multivariate models require an estimate for the dependence or correlation between the input parameters (e.g. Damodaran, 2012). Brealey and Myers (2012) point out that "specifying the interdependencies is the hardest and most important part of a simulation" and that simulations would rarely be necessary if all input factors were unrelated. Simulations are most powerful when multiple dependent stochastic variables are modelled as otherwise analytic methods could often be used. Higher level dependence in multivariate settings can be captured through Copulae (Rüschendorf, 2013). Copulae allow "more explicit control of the way in which joint percentile samples of distributions are to be drawn" (Rees, 2015) than correlation coefficients if comovement between variates is not constant across the range of their variance. A common empirical example is that correlation between stocks has been shown to be stronger in market turmoil (Chiang et al., 2007; Rees, 2015).

Generally, a sampling dependence like correlation or copulae appear to be the most recommended method for capturing comovement of variables. Other approaches like a structural model that implicitly assume a causal and / or directional dependence structure are not recommended in the sources surveyed.

Kuritzkes et al. (2003) show that correlation parameters between risk factors are in many cases based on estimates of other organizations for practical reasons. This method can of course also be extended toward other parameters and constitutes a viable data-driven alternative if own data is not available or of insufficient quality. This reveals preference for own data despite challenges of this approach such as small sample bias that are discussed in the actuarial literature (e.g. Longley-Cook, 1962).

2.4: Discussion and conclusion

In this chapter, we conducted a structured literature review of the academic and practitioner literature on simulation input modelling in general and with a focus on CF&A. The objective was a consolidated understanding of the state-of-the-art consensus view of how simulation models in CF&A shall be parameterized.

We find that the most central applications of simulation in CF&A are core analyses like profitability or basic financial statements. Risk assessment is advised to be conducted through different frameworks and methods that remain vague when it comes to defining specific risk inclusion cut-off points. We conclude that both academic and practitioner literature recommend conducting risk assessment along a categorization scheme to guide thinking on potential risks. Due to the wide array of potential risks organizations may face, both literature fields abstain from determining definitive rankings of risk factors. Furthermore, we argue that a major limitation of the frameworks in both the academic as well as practitioner literature is the lack of well-defined cut-off points for inclusion of risk factors into a formal simulation model.

We observe that simulation input modelling methods can be organized by their data source including methods based on data, experts, theory or aggregation of these sources. A decision tree shows that theory-driven methods are preferred, though hard to implement. In order of preference this is followed by data driven methods. Expert-based methods are usually only recommended if other methods are unavailable. This ranking is also reflected in the fact that a considerable part of the literature and the software providers (Barton et al., 2002) assume data-driven methods as the base case of model parameterizations. A further notable finding is that aggregation methods, in which different information sources are combined do not receive in-depth discussions in the simulation input modelling literature.

This review relies largely on analyzing previous research and is therefore necessarily limited by its sample. Given the growth in scientific output (Bornmann et al., 2015) there is a risk of missing out on relevant research, particularly if it has not yet achieved broad recognition. Notably, this review is subject to bias as only published literature was considered. This limitation was addressed by taking a broad angle to the sample selection, as described above, thereby minimizing the risk of inadvertently excluding research that would merit inclusion.

To develop this review further, we complement it with in-depth semi-structured interviews with leading experts in simulation input modelling in the next chapter.

Chapter 3: Expert interviews on simulation input modelling in corporate finance and accounting

3.1: Introduction

Leading thinking on simulation input modelling is dynamic and context dependent. To build onto the comprehensive structured review of the academic and practitioner literature, we aggregate and contrast the viewpoints on simulation input modelling in corporate finance and accounting of leading practicing simulation experts through a series of in-depth semi-structured interviews. We present a comprehensive review of what experts consider the state-of-art in simulation input modelling. It has been argued that there is a disconnect between academia and practitioners, commonly labelled the theory-practice gap or a disconnect between rigour and relevance (Kieser & Leiner, 2009). Contrasting the literature and leading practicing experts may illuminate in how far such a disconnect exists in simulation input modelling. This chapter follows a parallel structure as the previous one. This research is positive whilst the recommendations from experts are normative and yields a unique perspective on challenges in simulation input modelling.

We find that modelling dependence via correlation and related methods tends to be considered only if direct modelling of causal dependence is infeasible. We find strong preference for theoretically grounded models over empirical data consistent with the structured literature review. Moreover, there is widespread awareness of expert biases causing sophisticated yet pragmatic de-biasing strategies to be applied. Further we observe pragmatism toward the number of statistical distributions required, a relatively small number of distributions offers sufficient realism for most purposes. Contrary to the findings of the structured literature review, there appears to be no clear consensus regarding preference of data sources to derive input parameters as different interviewees prefer theory, data and expert opinion. In the same vein, we recognize that Aggregation Methods are supported amongst the experts in our sample, in contrast to the literature reviewed in chapter 2.

3.2: Method

Qualitative research methods are adept to capture the complexity of processes such as simulation input modelling choices (Creswell, 2013) as they offer flexibility to gain understanding through discourse. We conduct a series of semi-structured interviews with subject matter experts (SMEs) to formalize knowledge from experts following an analogous structure as a traditional literature review. The line of questioning for the interviews is based on the preceding chapter. Subject matter experts add a new angle to this research through possession or access to “contextual knowledge” (Meuser & Nagel, 2009) which is used complimentarily to the literature review (Bogner & Merz, 2009).

We chose this method for its unconventional angle and flexibility. Through this approach, we can address specific research questions while leaving room for experts to address further dimensions (Kvale 1996). We take Bogner & Menz (2009) as guidance for our interviews as we seek to delve deeply into topics as conversations unfold beyond predetermined questions (DiCicco-Bloom & Crabtree, 2006; Creswell, 2013). Questionnaire research ensures a uniform design of the same questions to achieve independent, comparable answers and eliminate the interviewer as a source of error (Groves, 1989). Semi-structured interviews do not necessarily follow this structure (Galletta, 2013). We aggregate expert knowledge with learning on behalf of the interviewer through the interviews. While this would be problematic from a standpoint of comparability of responses it can be beneficial as in Grounded Theory (Burkard & Knox, 2014) that incorporates learning on behalf of the interviewer into its research design. This is also referred to as emergent design where the data collection process may evolve throughout the study in “response to what is learned in the earlier parts of the study” (Morgan, 2013; Creswell, 2013). We take a case-based view of our subject rather than deducing theory and testing its predictive ability subsequently as in Grounded Theory, building theory inductively based on data as opposed to building theory deductively (see Glaser & Strauss, 1967). We deviate from Grounded Theory as it is designed for slightly different research objectives, namely the objective to deduce a “general, abstract theory” (Creswell, 2013) from the views of participants whereas our objective is to access their expertise on a set of research questions to construct the status quo of views held by experts. To this end, we harness Grounded Theory’s general framework without following it in detail.

3.2.1 Interview design

To strike the balance between achieving an unconstrained interview situation and the downside of obtaining incomparable responses we developed a set of questions for guidance derived from Chapter 2. Per Creswell (2013) the distinction of qualitative and quantitative research is not binary, we thus quantify findings where possible for readability and transparency. Additionally, the interview and its general structure were tested and hence calibrated through a test interview. Per Wrona & Gunnesch (2016) there are two schools of thought on using pre-existing theoretical knowledge throughout qualitative research studies, particularly case studies. As explained, the research design of this chapter builds onto a previous theoretical foundation which per Wrona et al. is common in the "analytical-empirical" tradition of qualitative research approach that is concerned with testing hypotheses. In this chapter, we take a positive view, which is more closely associated with a view unconstrained by previous knowledge. However, we follow Wrona et al. in arguing that previous knowledge can further the ability to interpret new data whilst maintaining an open approach.

3.2.2 Sample selection

We attempt to capture a broad perspective of leading simulation modelers to interview from various functional roles, industries and backgrounds (Mays & Pope, 1995) including simulation consultants, risk managers, corporate finance and strategy consultants.

Firstly, experts were selected based on publication of books and articles with a practitioner focus and practical experience. Secondly, we use a snowball sampling technique (Marshall, 1996) where we ask interviewees for references of other potential interviewees thereby growing our sample to saturation. As we only included recommended expert interviewees where the expert status could be confirmed through other sources, we label the method "selective snowballing”. This sampling method is not without risks if relied upon as the only source of interviewees as it could lead to a non-representative group of closely networked experts that does not represent an unbiased view. Yet, only a quarter of the sample was built through selective snowballing, thus reducing this risk. Although random sampling of interviewees has attractive properties it was not applied here. We are interested in the opinions of leading practitioners, not random ones per Mays et al. (1995) for situations where the sampling shall “identify specific groups of people who possess characteristic” traits. In addition, leading experts available for interviews are scarce.

Further we include practitioners who publish regularly on applied simulation in CF&A. It is common for practitioners in the German speaking countries to publish in dedicated practitioner journals (Grisar & Meyer, 2015), thus the prevalence of publishing practitioners may be owed to the number of German-speakers in the sample. Interviewee’s academic background is important ensuring strong theory background of simulation input modelling, particularly on PhD-level, ensures understanding of the academic perspective. Within our sample, 58% of interviewees held a PhD or equivalent, at least 50% published articles or books on simulation modelling while 75% worked as advisors or consultant with the remainder having a background in risk management. However, there are minor limitations. While the sample does include various nationalities, German-speaking researchers are overrepresented in the sample. Further, due to the scarcity of experts, it cannot be ruled out that those declining interviews were the ones most in demand.

3.2.3 Sample saturation

We try to saturate our sample (Charmaz, 2006) and build a sample complete enough that the marginal interviewee provides only little additional informational value. Further there is a trade-off that limits sample size growth as adding less well-selected interviewees dilutes. Our research design combines elements of case study research and grounded theory. Creswell (2013) aims for as many as 15-20 interviews for grounded theory though acknowledging that this is highly dependent on the research design and access to experts. We obtained 13 full length interviews, putting us slightly below the range indicated by Creswell. However, we show that we obtained saturation nonetheless through a standard approach. After the first nine interviews, we analyzed the emerging themes. Further we conducted four more in-depth interviews and examined the additional insights generated. As the themes expressed were mirroring the themes uncovered in the analysis we considered the sample as saturated.

3.2.4 Avoiding bias

The risk of subjective assessments in qualitative interviews requires a transparent research method (Flick, von Kardoff & Steinke, 2004). Interpretation of qualitative data and especially semi-structured interviews is necessarily “colored by the researcher’s experiences and biases” (Given, 2008). Transparency though can achieve inter-subjectivity, i.e. shared understanding of how the researcher arrived at the presented conclusions and interpretations.

The open format of semi-structured interviews creates a risk for interviewer bias to be avoided or minimized (Mays et al., 1995). We reduce interviewer bias via non-leading questions, it is crucial to strike the balance between guiding the interview toward its objective whilst avoiding leading questions that may trigger specific responses. One challenge in semi-structured interviews is to obtain focused data. We first elicit responses where we do not provide any predefined answers. In a second step, we provide these through follow-up questions.

Various steps were taken to avoid interviewee bias. Gigerenzer & Fiedler (2003) suggested that experts generally perform best when the interview is “ecologically consistent” with the environment they are used to, both physical as well as the conversation and questioning style. We ensure ecological consistency by creating interview situations close to the daily experience of the practitioners. When interviewing managers Trinczek (2009) advocates a conversational or even informal (Connaway & Powell, 2010). Furthermore, reactions to interviewee statements were avoided to reduce social desirability bias. Moreover, the freedom of not having to act on the statements and views expressed allows the experts to articulate their views freely. We provide anonymity and confidentiality to interviewees. In total the transcribed Interviews cover over 35,000 words in addition to the non-transcribed interviews where interviewees objected to the recordings (transcripts available on request). When discussing relative prevalence of methods and concepts propagated in the interviews, we distinguish whether experts support a concept or apply it.

3.2.5 Data analysis and coding of themes

Qualitative interviews contain information that needs to be aggregated to be accessible (Creswell, 2013). To this end qualitative analysis to categorizes and codifies information. These categories and codes represent the core recurring *themes*. Methodically, such codes can either be predetermined ahead of the interview analysis phase based on expectations or prior knowledge of prevailing codes on behalf of the researcher or the codes emerge through the research process directly. We follow the latter for a broader set of codes (Creswell, 2013). By having already reviewed the literature one cannot avoid having unconsciously formed approximate themes and codes mentally. Yet this is not problematic as it is common for codes to develop over time to reach their final state upon the research's completion (Creswell, 2013). The coding of themes is derived from the transcribed interviews first and subsequently applied to them (Schmidt, 2004). The coding was aided by the QCAmap software that minimizes bias, enhances transparency and reproducibility of the research method by making the information accessible for readers interested in the details of the coding process. The choice for this software was motivated by the strong theoretical footing of the method that is based on the work of Mayring (2014).

While most interviewees consented to having, their interviewees recorded there were exceptions where the interview touched upon topics of their job responsibilities they deemed sensitive, e.g. corporate risk managers can made be liable for statements made. To prevent worries about detrimental effects of interview recordings from affecting their responses we still conducted the interviews and took notes in writing. Although this constitutes a limitation it still represents additional informational value and thus included in the research design. Notably there were risk managers who agreed to recordings and their responses did not deviate systematically from the non-recorded interviews.

Yet anonymity is required to ensure unbiased responses: the information's retrievability must be balanced with interviewee confidentiality (APA, 1994). We follow the APA suggestion that retrievability can only be provided as far as the interviewee's confidentiality is preserved.

3.3: Results

3.3.1 Applications for simulation in CF&A

Semi-structured Interviews were conducted to access expert opinions on where and why simulation should be applied in CF&A. We present the key themes in condensed form with references to the literature review. We present the themes in declining order of the support received.

3.3.1 a) Most central applications

Assertion of the most central or important application of simulation in CF&A will be contentious or inspire debate. Nevertheless, we aim to provide a perspective on this question. This list of applications is not to be understood as definitive proof yet rather an introduction to the views the experts take. Later discussions in this chapter on other topics are to be read with these applications in mind. Further, these applications are not mutually exclusive and occur simultaneously. Further, most expert stressed that simulations' strength lies in its flexibility underscoring the difficulty of ranking the importance of applications. The ensuing discussions are to be understood in the context of these applications:

Table 10 - Prevalence of themes and number of mentions

Theme	Number of mentions
Income statement	9
Balance sheet	8
Cash-Flow statement	8
Strategic decisions / investments / M&A	6
Profitability	4
Real options analysis	3
Value / Earnings at risk	2
Credit, Portfolio, insurance and cost modelling	1 (each)

3.3.1 b) Drivers of simulation analysis

Although treatments of the drivers of simulation analysis have been discussed for example by Vose (2008) or Rees (2015), a deeper understanding of what should drive decision makers to use simulation analysis sharpens understanding of its applications. Hence, we seek to understand which attributes lend themselves to simulation modelling.

Table 11 - Prevalence of themes and number of mentions

Theme	Number of mentions
Uncertainty / risk	13
Scale / size	6
Strategic Importance / financial stability	6
Managerial flexibility	3
Regulatory	2
Complexity	2
Constrained Budgets, cognitive limitations, lack of experience, authorization to mitigate risk, reflect risk tolerance, requirement of decision process	1 (each)

- **Uncertainty:** unanimously experts underscored the importance of uncertainty affirming that high visibility of risk in company history supports awareness thereby driving simulation use. Some simulation modelers in practice distinguish common risks such as price volatility and event risks, the latter following compound distributions[12]

[12] This also includes modelling where the is a primary binary risk, such as the risk of a policy change and a secondary continuous stochastic variable describing the impact of such a change if it occurs

- **Scale / size:** as expected the relative scale of a decision or project should drive application of simulation analysis
- **Strategic importance / Financial stability:** including the need to quantify the risk of bankruptcy which can drive the regulatory need for simulation (Hüffer & Koch, 1993) Several experts argued not in favor of a static framework or decision guideline on how to decide if decisions should be supported with simulation analysis. Rather they suggested the straightforward notion of applying simulation for projects of major strategic importance without specifying how to assess strategic importance
- **Managerial flexibility:** the potential for the explicit valuation of flexibility in future decisions shall drive the use of simulation
- **Regulatory requirements:** in some jurisdictions and industries the use of simulation analysis can be driven by regulatory bodies
- **Complexity:** closely related to the level of uncertainty

3.3.1 c) Benefits of simulation modelling

We discuss the benefits of simulation modelling in CF&A.

Theme	Number of mentions
Calculation of ranges	13
Improved strategic thinking	9
Accurate calculation of avg. Outcomes	7
Risk aggregation	4
Non-linearities / options	4
Low probability risks, asymmetries, customizability, risk identification	2 (each)
Ease of implementation, complete modelling of capital markets	1 (each)

Table 12 - Prevalence of themes and number of mentions

- **Calculation of ranges:** as expected universal support was expressed for beneficial effect of probabilistic output ranges instead of point estimates of decision variables.
- **Improved strategic Thinking:** it was argued that decision makers profit from the process of building simulation models. Hillier, Ross, Westerfield, Jaffe & Jordan (2010)

support this claim. Although no substitute to quantitative analysis several experts advocated conceptual use of simulation to improve strategic thinking. This view treats simulation as an enhanced form of scenario analysis that can advance strategic thinking through the analysis of risk factors and company strategy whereby decision-makers can be led to consider future scenarios (e.g. Gerber, Arms, Wiecher & Danner, 2014).

- **Accurate estimation of mean / average outcomes:** it was stressed how simulation analysis improves estimation of mean decision variables; static calculations using the most likely input parameters do not yield the most likely outcome (Rees, 2015).
- **Risk aggregation / dependencies:** experts argued in line with the academic literature (Temnov & Warnung, 2008) that simulation is a powerful tool to aggregate interrelated risks with complex co-dependence structures
- **Non-linearities and options:** non-linearities and options can be modelled accurately through simulation. Moreover, simulation is argued to be more flexible in terms of distributional assumptions than analytic approaches
- **Low probability risks:** explicit calculation of low probability risks and their effects on average outcomes benefit from explicit modelling. Per interviewees these risks are often not included in static calculations. For risk factors with a detrimental impact this leads to an overestimation of profitability or other KPIs – and vice versa for upside risks
- **Asymmetries:** simulations can capture asymmetric probability distributions
- **Customizability:** analogously the flexibility of simulation methods that allows for detailed customization was praised
- **Risk identification:** simulation methods achieve superior risk identification through a quantification of impact on decision variables

3.3.1 d) Barriers to usage

Better understanding of barriers to more widespread usage of simulation is not the main objective of this research. However, the heterogeneity of expert opinion is noteworthy, thus this brief discussion. As laid out technical barriers to such applications have come down markedly in recent years through the widespread availability of computing power and tailored software. However, a lack of technical knowledge was still an oft-cited barrier. Yet, equally vocally the role of organizational barriers was highlighted.

3.3.1 e) Integrated financial management

Nine of the surveyed experts supported a holistic approach to simulation modelling analogous to Enterprise Risk Management (ERM) where the risk management process is conducted enterprise wide. ERM "strengthens a company's ability to carry out its strategic plan" (Nocco & Stulz, 2006). The literature on risk management reached a consensus on this over more compartmentalized risk management that may not account for interrelated risks (Shapiro & Titman, 1986; Miller 1992) or cross-entity effects. Integrated financial management, as understood by the experts interviewed, is the extension of this approach towards all major financial processes and analyses. Smith (1964) was early to recognize the benefits of financial planning and risk management with the wider range of financing analyses and capital budgeting decisions. Trigeogis (1991) points out that "corporate strategic planning, capital budgeting, incentive schemes, and control mechanisms should form an integrated system" as seemingly unrelated projects can be linked via channels including taxation, bankruptcy risk and financing conditions (Grob, 1989). Translating this concept into simulation modelling includes building fully integrated financial simulation models covering all financial statements and investment decisions that are capable of simulating risk profiles with flexible input parameters for all relevant risk factors and interdependent effects.

3.3.2 Risk assessment and prioritization

3.3.2 a) Risk assessment based on simulation

Any simulation model that models risk requires a previous assessment of the relevant risks and cut-off for risk inclusion to decide which risk factors to simulate. It is not generally clear how to assess and prioritize the major risks an entity faces before accurately modelling these risks as their interrelations and impact on decision variables are not known a priori. This theme came up repeatedly. While pragmatic approaches that build on experience and risk assessment workshops prevail, some apply simulation methods in this step already. A suggested best practice is to simulate a wide and comprehensive array of risks to build understanding of which factors are the most impactful for a company's risk tolerance. Thus, the risk assessment itself is based on simulation analysis. While SMEs agreed with the merit of this approach they highlighted constraints with its implementation mirroring barriers that hold back simulation generally.

3.3.2 b) Big data and machine learning approaches

Through digitalization, many companies have access higher quality data (Manyika et al. 2011) that big data and machine learning help to put to productive use (McAfee, Brynjolfsson, Davenport, Patil & Barton, 2012). Several experts speculated about potential uses of techniques that harness access to company-specific data for input modelling presenting interesting research avenues.

3.3.3 Parameterization of simulations in CF&A

3.3.3 a) Sampling vs. parameter dependence

Co-dependence between risk factors is cited as a major reason to run simulation models and is covered in all reviewed textbooks (e.g. Rees, 2015; Vose, 2008; Lam, 2014; Charnes, 2012) as we saw in the previous chapter. Correlation coefficients are commonly discussed to account for co-movement and dependence between stochastic risk factors by academics and practitioners.

Another cross check with the discipline of operations research reveals a similar focus on correlation and related methods (Schmeiser, 1999). However, despite their favorable mathematical properties they present challenges and shortcomings.

- Many different risks lead to large correlation matrices, complex to derive and handle
- Correlation coefficients can tend to be less straightforwardly communicated to non-technical management than direct structural dependence
- Correlation is directionless and may not capture a relationship accurately if in fact one variable has a causal effect on another
- Directional models allow for more flexible treatment of e.g. non-constant co-movement across the full variable's range; e.g. for conditional events like insurance, contracts

This finding stands out as it is the sole theme that was supported through all expert interviews and represents a stark contrast to the academic literature. Consequently, the SMEs argued for the use of directly modeled dependence structures. This is supported in the literature where Rees (2015) differentiates between sampling dependence and parameter dependence and argues that there are situations were pure sampling dependence is inferior to parameter dependence for reasons analogous to the ones above. Parameter dependence describes situations where "parameters of a distribution are determined from the samples of other distributions" and thus notably occur directionally. Other methods included:

- Some experts suggest building a correlation matrix of underlying risk drivers that affect individual risk factors. The intuition behind this approach is that there is typically only a small number of underlying risk drivers that in turn reflect a much larger number of risk factors (e.g. a risk driver is GDP growth that affects many risk factors such as demand, input costs, labor costs etc.). To capture dependence in the risk factors it is thus sufficient to capture the sampling dependence between the underlying risk drivers and define a causal dependence structure of these risk drivers on individual risk factors

- A further modelling choice builds causal models in the structural form following the Capital Asset Pricing Model, or CAPM, a regression model: a set of underlying risk drivers is identified, and historical beta-coefficients are calculated.

Several factors contribute to the discrepancy between the acceptance of correlation methods in academic texts and their low acceptance among practitioners. Correlations have mathematical expressions that can be manipulated in larger models flexibly. Correlations can be measured empirically if data is available, reducing the need to assume causal dependence.

3.3.3 b) Preference for input sources

It appears possible to derive an order of preference where theory driven methods are preferred where possible then followed by data-driven and expert methods. The SMEs in our sample appear to agree on the preference for theory, although have dispersed views on the preference between data and experts.

Perhaps contrary to expectations, seven SMEs recommended theoretically derived distributions. Contrary to arguments of infeasibility (Barton et al. 2002; Kelton & Law, 2000) several SMEs found ways in practice to implement. Examples were simulations of stock price behavior, that follow a statistically well-established process, or physical regularities such as the output of a power plant. Theory driven methods offer advantages over empirically derived distributions that mirror the advantages of using a fitted theoretical distribution over historical data:

- Robustness with respect to empirical data irregularities, particularly likely if the data set is small or there is reason to believe that quality may be hampered
- Available in the absence of data
- More reliable in the extremes of the distribution (see Law et al. 2000)

Yet it is striking that some experts have a strong preference for data and almost always prefer it to experts' judgment, while others prefer experts' judgments over historical data categorically across parameter classes. In other words, some experts always prefer to work with data while others prefer to always work with experts. In our sample four experts strongly leaned toward data, two leaned strongly toward expert judgments with the rest expressing more nuanced views. Proponents of expert judgment raise several challenges to the use of historical data, arguing that one can hardly be sure whether historical data can in fact be used due to quality and availability problems. Further, data generating processes can change and this may go undetected. However, proponents of the use of historical data point to bias in human judgment particularly with regards to probabilities despite the widespread use of de-biasing strategies.

In conclusion, a preference for theory driven methods is shared if only narrowly applicable, but SMEs differ from the reviewed literature as they do not exhibit clear preference for data or expert-driven methods for simulation input modelling.

3.3.3 c) Expert bias

Throughout all interviews the complex problem of expert bias was stressed. It is well-established in the literature that experts tend to err systematically when dealing with probabilities, risk, volatility and related concepts. This understanding was well-reflected as all, but one expert described their approach to handling expert bias. Notably the only interviewee who did not raise the issue did not deny the existence of bias but rather expounded on the difficulty of de-biasing. Typical biases include overconfidence, anchoring, status quo bias and various more. Per our expert interviews one frequent source for bias is that the type of thinking required is not common in the daily experience of many experts resulting in a lack of calibration. This may include creating an optimal response to prevent a detrimental outcome, that do not overlap with statistical assessments of risk factors. Notable exceptions are experts who receive continuous timely feedback on their predictions, a common example in the literature are weather forecasters who tend to be well-calibrated due to timely and regular feedback on their forecasts (Murphy & Winkler, 1977). Secondly a lack of formal training in statistics and possible sources of bias contributes to the risk of obtaining biased parameter and input estimates (Clemen & Lichtendahl, 2002). Notably this type of bias is not necessarily contradicting the expert status (see discussion in chapter 6). Despite a growing body of knowledge on de-biasing strategies this remains challenge as it is not possible to be certain that bias is eliminated entirely (Meyer, Grisar & Kuhnert, 2011). Various de-biasing strategies were used, sometimes in combination:

- The most prevalent approach, applied by seven experts, was to derive inputs from descriptions in the experts' known terminology to achieve ecological consistency (Gigerenzer & Fiedler, 2003). Hereby SMEs do not have to engage in formalized statistics. This approach can be considered an ex-post de-biasing strategy as data is de-biased after it is elicited (McClelland & Bolger, 1994)
- A further ex-post strategy used by four of the experts was the aggregation of expert opinions to reduce perceived uncertainty, especially through multi-disciplinary teams. These aggregations are done ad-hoc without emphasis on which method to use to weight opinions. This can be likened to the Delphi method where a group of experts is presented with additional data and factors considered important by other experts (Dalkey & Helmer, 1963). Related methods are recommended in the literature (e.g. Liebsch, 2003).
- Another approach, applied by four experts, was to graphically illustrate parameter estimates and question experts' model of thinking about their estimates. This approach results in non-parametric distributions and is described as in Vose (2008). It is known from the academic literature as an approach that "centers on improving the elicitation process and countering bias a priori" (McClelland et al., 1994). A well-established bias-

reducing method from Winman, Hansson & Juslin (2004) does not ask expert to estimate confidence intervals but rather presents intervals of potential outcomes repeatedly and asks experts to judge the confidence level with which an interval would capture the true value of a risk factor.

- Further approaches included making the experts accountable on their estimates, hence an ex-ante de-biasing approach.

Overall experts used sophisticated de-biasing strategies to counteract bias. It thus appears that the practitioner and academic community are well-aligned regarding the risk of bias. Finally, it is noteworthy how a large fraction of the experts interviewed used de-biasing strategies without being explicitly aware of the literature on the topic or even the term "de-biasing".

3.3.3 d) Aggregation Methods

Experts in our sample advocated Aggregation methods of combining data sources more generally, albeit with differing levels of certainty. It is striking that Aggregation Methods enjoy wider support amongst the experts surveyed than in the literature review. This could be explained as SMEs may approach challenges more pragmatically.

3.3.3 e) Enriching data with expert judgment

Empirical data was in many cases identified as the starting point for model parameterization. However, empirical data is inherently backward looking and thus error-prone in dynamic environments. This downside can be addressed through the combination of empirical data with exogenous inputs such as expert opinions, thereby constituting another aggregation method. Trends and structural breaks can be captured through the adjustment of distributional properties based on expectations as described above. However, practitioners pointed toward the challenge of how different inputs can adequately be combined or aggregated. Accurate methods shall consider the uncertainty inherent to different parameter estimates, i.e. must also capture the risk of error on behalf of the experts.

- A simulation and risk management consultant used this method on a commodity price process where an expert panel adjusted historical parameters
- Another consultant argued that this is feasible though expressed less confidence in the method due to a higher level of trust in data
- Two further widely published simulation consultants argued in favor of adjusting data based on expert opinion

However, these methods remain ad-hoc as will be further discussed in chapter 4.

3.3.3 f) Appending distributional tails

Data is enriched in the extremes of the distribution through appending tails. The actual statistical distributions of extremely rare events can be difficult to approximate for a lack of robust empirical data. This challenge is addressed by consulting with subject matter experts and adding this information to the empirically observed data. As has been suggested in the academic literature (Lambert, Matalas, Ling, Haimes & Li, 1994; Kelton et al. 2010) experts' model extreme events separately from the distribution of less extreme events by e.g. attaching a longer tail.

3.3.3 g) Fundamental models

While theory-driven methods are based on theoretical understanding of the data generating process fundamental models take a pragmatic approach to approximate the observed behavior of a stochastic process according to experts interviewed. Such models can take different forms. One mode, used in the utility industry, models demand and supply of electricity to simulate electricity prices. Models can be calibrated using past data to enable scenario analysis. Seven of the experts in our sample had worked with such models. Fundamental models constitute a combined approach of historical data and theory. A major advantage of such models is the ability to make out of sample forecasts to account for structural changes in the data generating process. Structural breaks in data generating processes have first been empirically analyzed by Chow (1960) and describe situations where data generating process are unstable over time and their parameters shift. Experts in our interviews agreed that fundamental models generate superior parameters if the context is rule driven and follows a forecastable pattern.

3.3.3 h) Distributions used

Statisticians have put forth many statistical distributions for use in simulations. There was strong consensus that sufficient accuracy can be achieved for most applications without the use of the full spectrum of distributions as improved modelling in detail would rarely lead to improved decision making once an approximately fitting distribution is used. Some rely almost exclusively on (in some cases truncated) log-normal distributions:

Distribution	Prevalence
Normal (incl. Truncated and log normal)	100%
Weibull	60%
PERT	40%
Triangular	30%
Compound distributions (various families), Poisson & Binomial	20% (each)
Pareto, Exponential, Uniform, Beta, Exponential, Uniform & Beta	10% (each)

Table 13 - prevalence of distribution families

This result stands in contrast to the academic literature that emphasizes choice of distributions.

3.4: Discussion and conclusion

In summary, we find substantial areas of agreement as well as ongoing debate between the literature review in chapter 2 and the expert interviews. In accordance with the literature review, experts argued for theoretically-grounded input models, strong emphasis of addressing cognitive biases consistently as well as using the same set of input sources and modelling concepts prevalent in academia. In contrast to the literature, experts argued against using correlation but rather structural dependence, argued for the use of only a handful of distributions and argued for the use of aggregation methods. Finally, applied experts did not exhibit consistent preferences for data vs. expert-based input models.

Our analysis paints a differentiated picture of how experts view the most important applications of simulation in CF&A. SMEs interviewed strive for customized pragmatic solutions whilst displaying discipline to use simulation only where considerable rewards can be reaped. They rather advocate flexible approaches to decide on the use of simulation analysis such as value tree analysis. Yet they advocated for integrated simulation modelling instead of a compartmentalized application. These two views may appear to be at odds, yet they argued in favor of a holistic deep analysis where resources allow. Per the experts interviewed the full benefits of simulation are reaped if applied in a fully integrated manner.

Practitioners are both pragmatic and forward thinking. For pragmatic reasons risk assessment methods are recommended that have methodical weaknesses such as following experience-based risk assessment and identification strategy or using heat maps for prioritization. Yet organizational constraints are oft-cited reasons for such methods beyond technical complexity.

Practitioners may have a reputation for pragmatic and empirical solutions. This is not strictly reflected in the recommendations aggregated for the parameterization of simulation models in CF&A in our sample of SMEs. They favored non-pragmatic, theory-driven approaches like theoretically derived parameters, fundamental models and de-biasing strategies. Notably the treatment of interdependent risk factors goes beyond the academic consensus on correlation. Finally, SMEs argued in favor of using Aggregation Methods though without robust theoretical footing.

Throughout the interviews there was a recurring theme attributing the perceived lack of simulation analysis in practice to organizational constraints rather than technical ones. It was argued that it is rather a lack of acceptance than a technical capability constraint that holds back a more widespread usage of simulation, Rees (2015, Chapter 5) devotes a full chapter toward organizational challenges. Future research could explore this multi-causality in more detail. Further, the amount of data accessible to simulation modelers is likely to continue to increase (McAfee, Brynjolfsson, Davenport, Patil & Barton, 2012). Future research could explore the potential effects of this and related trends on simulation input modelling. A more speculative conjecture is that this will lead to changing management culture further embracing quantitative analysis thus furthering simulation's acceptance. Machine learning algorithms could be used to simulate scenarios based on analysis of a company's historic data without the necessity to formally define a structural model preventing model specification errors.

A limitation of this research is the risk of researcher bias, referring to the bias introduced by the researcher that can be critical in qualitative research (Mays et al., 1995). Despite taking multiple steps to avoid bias qualitative research remains affected by the researcher's background (Given, 2008). Measures were taken to ensure a broad and balanced sample, yet the reliance on a select number of leading experts remains a limitation.

Chapter 4: Bayesian estimation for simulation input modelling

4.1: Introduction

Precise and forward-looking simulation input modelling is pivotal to achieve accurate simulation modelling results. As the preceding chapters underscored, simulation input modelling represents a rich and nuanced research strand within simulation modelling in corporate finance and accounting. We build onto this research by presenting an input modelling method that allows the aggregation of quantitative empirical data and quantified expert opinions via the process of Bayesian updating of prior distributions. Through this combination of input sources, the informational value of each input source is utilized through a formal method to reduce uncertainty about the unknown estimated input parameters. While various methods exist to parameterize simulation models, many face limitations under realistic assumptions. Simulation modelers use empirical data for simulation model parameterization, yet this method faces limitations if the modeled process undergoes changes or when there are various viable sources of data on the modelled process. Furthermore, data quality may be imperfect thus needing to be enhanced to serve as simulation input. By using input from forward looking experts for model parameterization simulation modelers can attempt to overcome such limitations. The parameterization method presented here seeks to harness the advantages of both methods drawing on an extensive body of research from academic sub-fields as diverse as actuarial sciences, reliability engineering and signal processing. Through a comparison of simulation results based on different simulation input modelling methods like empirical data, expert input and a “naïve” or Bayesian aggregation we illustrate the effects of the suggested method.

The motivation for this method rests on common challenges in simulation modelling where Bayesian simulation input modelling may help addressing these. Firstly, there are challenges posed by non-constant distribution parameters as various input modelling methods, implicitly or explicitly, assume that the parameters of the underlying data generating process stay constant through time. A range of methods (e.g. Kelton et al., 2000), rely on distributions based on historic data. These methods have limitations (e.g. Bratley, Fox & Schrage, 1987) if this assumption does not hold as evidenced by the literature on structural breaks (e.g. Chow, 1960). Harnessing expert opinion presents an opportunity to address this challenge through enriching data with expert judgment. Secondly, the aggregation and weighting of different inputs is not straightforward (Cooke, 1991). It is common among simulation modelers to be presented with imperfect data (e.g. Kelton et al., 2000) creating a need to use multiple data sources aggregated robustly. Expert judgments have a level of certainty or credibility that should affect the decision weight of their input in an aggregate estimate that must be reflected in any method aggregating different input sources. Thirdly, data quality can be poor in applied simulation contexts (Schruben & Schruben, 2001; Kelton et al., 2000; Bratley et al., 1987), thus necessitating the use of different imperfect input sources that each have some advantage in their data properties. Leading practitioners confirm that data quality tends to be poor (interview transcripts, 2016), e.g. due to small samples with little data in the extremes of a distribution (Kelton et al., 2000).

These challenges underscore limitations of traditional, single data source-based parameterization methods. In Chapter 2 we presented the state-of-the-art simulation input modelling methods. One of the key conclusions was that aggregation Methods that combine different classes of information sources for input modelling are not as widely recommended as methods relying on a singular source. This is despite theoretical arguments to harness all available information and the realization that simulations are oftentimes run in dynamic environments of incomplete information.

Per Oberkampf (2019), a key measure for accuracy of applied simulation modelling for decision support, is predictive ability rather than "agreement with empirical results". This is, of course, exacerbated in situations where no direct empirical comparison is possible to verify accuracy, or further, where it is reasonable to assume that past data cannot be used as the sole input to predict future system behavior, as is common in simulation modelling. Schruben has argued that most simulations analyze what happens in systems if "something changes" (Barton et al., 2002), thereby underscoring the need for a dedicated method. These challenges are varied yet may be addressable in part through the versatile method of recursive Bayesian estimation. Both quantitative input modelling based on historic data and more qualitative approaches based on expert input have distinct advantages and drawbacks that can be viewed as complimentary. While parameters based on historic data can accurately capture the central properties of distributions they may fail to recognize the dynamic nature of data generating processes as well as their extremes (e.g. Vose, 2008). Further, data is vulnerable to quality problems or measurement errors that cannot be addressed from within the data set. Experts on the other hand may be able to recognize the dynamic nature of the process but may be subject to cognitive biases (Rees, 2015; Kahneman & Tversky, 1972). Our approach thus is to combine both methods to arrive at an alternative parameterization for simulation models. More generally this approach can be beneficial when empirical data and expert opinion need to be aggregated.

Following Weber, Schmid, Pietz & Kaserer (2011) we investigate the impact of input modelling methods through a case study in the waste incineration and adjacent industries that is based on an actual application of the method. This chapter contributes to the literature on simulation input modelling for CF&A as it provides an analysis of the sensitivity of simulation results to the proposed input modelling method. The remainder of this chapter begins with the literature review, the method used and case application, followed by a critical discussion of the results, limitations, research outlook as well as concluding remarks.

4.2: Review of the literature

Utilizing Bayesian estimation for simulation input modelling builds on a broad and deep literature spanning various disciplines and specializations. These include operations research, actuarial sciences and signal processing where researchers have been concerned with questions related or analogous to simulation model parameterization. The methods discussed apply Bayesian statistics and therefore a general introduction is provided. Hence, we also structure the literature review in four sub-sections per the most relevant literature strands. We first review the Bayesian approaches to simulation input modelling, then review the foundations of credibility theory followed by a brief discussion of methods to aggregate expert opinions and finally discuss recursive Bayesian updating. Though disparate, these fields exhibit coherence in their view of the merits of Bayesian Statistics for information aggregation. In the sense of Golden-Biddle

& Locke (2007) we seek to build a "synthesized coherence" of these literature strands that are not generally considered closely related.

4.2.1 Bayesian methods in simulation modelling

We henceforth describe methods in the literature that harness Bayesian statistics for simulation input modelling and related purposes. Using Bayesian methods has been described by Cheng as "powerful method for injecting human opinion into an analysis" (Barton et al. 2002) though also a tool that has not received the wide attention it deserves. Cheng also touches upon the potential of Bayesian methods to combine different data sets to reduce informational uncertainty in an operations research context.

Chick (2000) provides an excellent starting point for Bayesian methods in simulation. The method presented seeks to manage uncertainty about simulation input "parameters, sensitivity analysis, and the selection of the best of several simulated alternatives" if structural and parameter uncertainty exist – as is common in simulation modelling. If a simulation model is parameterized by fitting a theoretical distribution to empirical data via maximum likelihood estimation, it is not straightforward to derive a robust confidence interval of the estimate due to the inherent properties of the estimation procedure. Confidence intervals are important particularly when dealing with small data sets with uncertainty. Via Bayes rule it is shown how to explicitly quantify uncertainty about input parameters. Chick furthermore presents and references a substantial part of Bayesian methods in simulation analysis. Despite the wide range of uses of Bayes Theorem in simulation modelling, most of the research does not explicitly address simulation input modelling and the potential to use Bayesian methods to aggregate information sources with entropy-reducing positive information value as understood by Shannon (1948).

Vose (2008) discusses Bayesian inference for simulation input modelling under general assumptions where a prior opinion about a distribution is updated through new data. For situations where, new data is available this constitutes a hands-on approach to use Bayesian updating to introduce this information. This approach is related to the one we discuss here, though differs as it focuses on aggregating empirical data rather than aggregating different data sources. The case where expert opinion is treated as an observation and constitutes new information used for updating is not discussed.

Another branch of research shows how Bayesian methods can be applied to de-bias expert judgment. Clemen & Lichtendahl (2002) propose to de-bias expert judgments that are subject to overconfidence in the context of estimating confidence intervals. Using past expert estimates of probabilities that exhibit overconfidence the model illustrates how Bayesian statistics can be used for de-biasing. It is assumed that each expert has an intrinsic unknown bias factor that is constant across estimates. Across a data set of estimated confidence intervals and realizations an estimate is obtained of the inherent bias via a Markov Chain Monte Carlo algorithm. MCMC algorithms can approximate the posterior distribution when it is not known ex-ante what distributional family the posterior belongs to. Where such data is available, and the assumptions of the method are met, this method of Bayesian de-biasing can in fact be combined with the method we discuss here. Armstrong, Galli, Bailey & Couët (2004) incorporate Bayesian updating into a real options analysis. Their methods provide a way to explicitly value newly obtained information in context of the decision to invest in an oil field. Additional information about an oil field and its prospects can reduce the uncertainty inherent in the investment decision and thus more precise estimates of project values and risks. Yee (2008) applies Bayesian updating in Valuation and the DCF context to construct a posterior evaluation of asset values via a Bayesian triangulation of different valuations that combine analytic valuations (DCF, comparables, multiples) and market valuations. The central result of the article is the weighting function that uses Bayes Theorem to formalize the uncertainty-weighted averaging of different valuation methods. Yee utilizes two advantageous properties of Bayesian statistics. Firstly, a Bayesian framework aggregates different estimates and aggregates informational value, thereby reducing entropy, across sources: the resulting valuation estimate has less uncertainty than any individual estimate. Secondly, this aggregation is inverse-uncertainty weighted thereby giving higher weight to less uncertain source – a key property of Bayesian updating. Whilst the method discussed in this chapter is focused on simulation input modelling, it harnesses these two specific traits of Bayesian statistics as well.

This review illustrates that Bayesian methods have several applications in Simulation modelling, even some in simulation input modelling (e.g. Vose, 2008), that serve versatile purposes. However, it also underscores the need for a straightforward method to be applied to aggregate information sources in general input modelling situations.

4.2.2 Methods to combine expert opinions

Per Kelly & Smith (2011) a common assertion is that the method of aggregating expert opinions shall not be more sophisticated than the experts that provide the estimates. We argue against this as errors of potentially inaccurate expert estimates would only be exacerbated by non-optimal aggregation methods. The extensive literature on combination of expert opinions appears to support our claim through the level of sophistication of its methods. Cooke (1991) discusses expert input into decision processes broadly under conditions of uncertainty, though not focusing specifically on requirements of simulation input modelling. Although this work does not address the challenges identified above it does provide a deep treatment of expert judgment in stochastic settings that closely align with the core of this chapter. Several chapters in Cooke (1991) are dedicated to a review of techniques and models to combine or aggregate multiple expert inputs. Both Bayesian and non-Bayesian models to combine expert opinions are discussed.

The classical non-Bayesian model is discussed as a practical tool that relies on weighted averages of all experts. A central objective of the literature is to assign optimal weights for averaging of diverse input sources. Various weighting schemes are discussed including assigning each expert equal weight, ranking per preference to assign weights accordingly and recursive self-ranking by the experts. The latter comes closest to the variance- or uncertainty weighting we advocate via Bayesian updating as it captures the expert's self-assessed uncertainty around the estimate provided. However, it is more laborious in practice and requires greater access to experts.

In the context of eliciting probability estimates, the term scoring refers to a numerical evaluation of an estimate's accuracy. Scoring values are obtained through repetitive comparisons between estimates and realizations revealing the average accuracy of an expert. Hence, scores can be used to combines expert inputs as weighting terms. The weighting increases with the amount of relevant information an expert has and the level of calibration. Formal definitions for numerical values of entropy and calibration are treated in Cooke (1991). This framework results in robust expert scores and subsequent weightings in the combination of experts. However, it assumes wide access to the experts to calibrate their scores that may prove unrealistic.

The Bayesian models are in fact analogous to the Bayesian updating procedure discussed in this chapter where each expert is viewed as an "observation" or "realization" that provides additional information. This view has already been proposed e.g. by Winkler (1968). However, experts are required to provide a prior distribution and thus deviate from the approach we discuss here that derives its prior distributions from empirical data.

Bayesian Methods have been used in Probabilistic Risk Assessment in reliability engineering (e.g. Kelly et al. 2011) for analogous reasons as for simulation input modelling. While parameter estimation is fundamentally different in the settings of risk assessment in reliability engineering and simulations in CF&A, there are some parallels that underscore the advantages of Bayesian Methods in combining information sources to quantify uncertainty. Notably, they also harness the opportunity to combine multiple quantitative and qualitative sources to estimate risk metrics such as rate of aging for engine parts.

We conclude that the merits of Bayesian methods in aggregation of input sources has broad acclaim. We build onto this by adapting, illustrating and benchmarking Bayesian simulation input modelling for use in corporate finance and accounting.

4.2.3 Credibility Theory and Bayesian methods from actuarial sciences

Actuaries emphasize risk modelling and are on the forefront of methodical advancements in simulation methods. Adjusting simulation input parameters in a Bayesian framework is considered a solid tool for parameter adjustment, particularly in situations where two or more input sources are used for model parameterization (Temnov & Warnung, 2008). Actuaries have traditionally worked with multiple data sets, e.g. one internal set of insurance claims and one of pooled data that is used jointly with other insurance providers with a trade-off between specificity and robustness. Internal data being more specific to the expected uncertainty but with low robustness due small sample sizes whilst the external data is less specific though more robust. This tradeoff is well-suited for a Bayesian approach of data aggregation through methods of Credibility Theory. In actuarial sciences, the term credibility is used to describe the level of credence attached to data (Longley-Cook, 1962). By the law of large numbers, one can infer that small samples have lower credibility than large ones. However, small data sets may be more specific or targeted to the modelling challenge and thus still be valuable. Credibility Theory estimates the level of credence of data inputs. This level of credibility of data depends of course on how data is to be used and not only on properties of the data. Credibility is higher the more similar data is to the case one is inferring about, e.g. when an insurance seeks to set the premium for a car insurance it will attach higher credibility to past damage data of similar drivers by age, car type etc. than to more heterogeneous data. This also illustrates the key trade-offs: more similar data has high credibility, yet it also has lower sample sizes, thereby lowering credibility. Credibility Theory provides methods to aggregate information across sources and weight them per credibility.

Arthur Bailey has been credited with advancing Credibility Theory in the actuarial sciences (Norberg, 2006). The reasoning of actuaries described by Bailey (1950) is strikingly Bayesian in that it emphasizes the importance of prior knowledge:

> *"[…] Underwriters belief that they are not devoid of knowledge before they acquire statistics. […] When statistics […] are acquired, the problem is not 'what should the rate have been?' but 'how much should the existing rate be changed […]?'"*

This is analogous to Bayesian updating of a prior distribution in the face of new data to obtain a posterior distribution. Not only are the intuition of using prior data analogous, so are some formal results. Venter (2003) shows a simple derivation of a primary result in *Least squares credibility* of minimizing the variance of the "posterior" estimator in a weighted average of two previous estimates that is equivalent to the inverse-variance weighted result obtained for updating the mean of a normal distribution in Bayesian statistics.

Credibility of empirical data can oftentimes not be captured in a single number as it is highly context dependent thereby introducing an element of subjectivity or judgment (Longley-Cook 1962; Behan, 2009).

Jewell (1991) uses a Bayesian framework with an independent prior information set about a compound distribution of the severity and frequency of excess losses in an insurance setting.

The effects on insurance loss estimates of using different informational input into the parameterization process is illustrated here as well. Further, Hesselager (1993) builds onto this work by using Bayesian updating in an approach with parallels to the method presented in this chapter. It is assumed that the reinsurer has "sparing knowledge" of the insurance contract and seeks other data sources to incorporate into its risk assessment. This is formalized via the Bayesian framework that estimates the compound distribution of losses. The compound distribution follows a Poisson process with Pareto-distributed loss amounts. A further critical assumption is that the true parameter of the distribution of losses is itself a random variable that is approximated via the combination of multiple input sources in a Bayesian setting. This assumption is in line with Bayesian theory that holds that the true parameters of statistical processes or distribution follow some distribution themselves. Thus, a Bayesian posterior estimate of the distribution of excess losses is generated. The example provided in this article illustrates in how far Bayesian updating represents an uncertainty-weighted average of two data sources about insurance losses that reduces the overall parameter uncertainty.

A further insightful illustration of such a problem is provided in Temnov et al. (2008). They discuss three risk aggregation approaches based on simulation, Fourier transformation and recursion for assessment of operational losses of financial institutions. Parameters for the simulation are obtained via Bayesian updating from external and internal data sources. Bayesian statistics offer a way to aggregate the information from both sources and weight them per their specificity or uncertainty. Here, as in the method we present below there are two or more data inputs with idiosyncratic strengths and weaknesses, maintaining the strengths whilst alleviating the weaknesses.

4.2.4 Recursive Bayesian updating

Researchers in various fields use recursive Bayesian updating algorithms that incorporate multiple measurements or data points for e.g. signal processing or time-series econometrics (Grewal & Andrews, 2001). Recursive Bayesian estimation refers to the process of updating a prior estimate in multiple steps where the posterior after the first updating becomes the prior for the subsequent update. These methods seek to overcome the filtering problem where the true state of a dynamic system is unknown and only incomplete or imperfect data is available. These algorithms filter the underlying signal from its noise (Grewal et al., 2001) by aggregating multiple sources of information to approximate the unobservable true system state. The analogy to the problem of simulation model parameterization lies in the infeasibility to observe the future system to be simulated and the imperfection of experts' estimates of the parameters for simulation modelling. In addition, past data for use in simulation input modelling may be prone to measurement error, likewise a challenge that can be addressed through Kalman or Bayes filtering, including concepts applied here as further discussed in the appendix.

This extensive literature review underscores the various usages of Bayesian Statistics in simulation modelling, risk assessment and aggregation of information sources. We build onto this work by extending the scope of application that we will introduce in detail hence.

4.3: Method

Our core application for Bayes Theorem is to quantitatively incorporate expert opinion into the parameterization of a simulation model. Thus, this method constitutes an aggregation method of deriving input parameters, as discussed in chapter 2. Bayesian statistics allows us to quantify this intuitive concept of combining historical data with new information in the form of expert opinions. Bayesian updating can be used to incorporate new information into existing statistical distributions to reduce informational uncertainty. A general basic introduction to Bayesian statistics is provided in the appendix and describes a discrete binary probability example and how updating is applied there. A key contribution of this chapter is the focus on applicability of Bayesian updating to defined simulation input modelling environment. The perceived lack of in-depth discussion of aggregation methods and a guide on how to apply them motivate the method presented here.

4.3.1 Assumptions

Recursive Bayesian updating is built on a set of assumptions that we shortly discuss here, focusing on their realism and generalizations that do not necessarily build on these assumptions.

4.3.1 a) Conjugate prior distributions

Throughout this chapter, we assume conjugate prior distributions. This represents the assumption that the prior and the likelihood take such a form that the posterior distribution follows the same functional form as the prior thus belonging to the same family of distributions. The prior and likelihood are then said to be conjugate (Lynch, 2007). Thereby the Bayesian updating can be represented in analytical form rather than being approximated through Markov Chain Monte Carlo (MCMC) methods. While this assumption represents a minor loss of generality, it is common in actuarial practice (Bailey, 1950) and the oil and gas industry where Bayesian methods are used to value new information explicitly (e.g. Armstrong et al., 2004). In fact, in the applications we consider the case that a statistical process changes parameter without changing functional form and distributional characteristics. Assuming conjugate priors also facilitates the interpretation of the impact of new information on the posterior. In situations where this assumption is in doubt there are well-established MCMC methods to model non-conjugate priors (Gelman, Carlin, Stern & Rubin, 2014)

4.3.1 b) Informative prior

Informative priors are based on empirical data of the distribution in question rather than being based merely on assumptions. *Uninformative* priors on the other hand are applied in the absence of empirical data when there is some other, possibly vague, information on the distribution, such as an upper or lower limit or a range. Hence, the priors used are informative in the understanding of Bayesian statistics. More generally we assume that the past data of a statistical process contains meaningful informational value and can thus improve a standalone expert estimate.

4.3.1 c) Knowledgeable experts

Perhaps one of the most critical assumption is that experts have access to information that can improve upon the empirically observed data. Put differently we assume that experts can exogenously determine changes to the data generating process. One example is based on the notion of event-induced structural breaks. Practical examples for this include competitive market dynamics like the bankruptcy of a competitor, a new mining development in a commodity market or policy decisions such as tax or subsidy changes. However, there are various applications that do not require the assumption of structural change. This includes the need to combine data from different, relevant sources or the benefits of enriching data with expert opinion when dealing with measurement error or incomplete data. While this is a strong assumption we consider it realistic for our purposes and the suggested application.

Uncertainty of parameters needs to be quantifiable, this quantification may not be straightforward especially for expert input, although this has been standard practice when working with expert input (Cooke, 1991) and we will explain in detail below the method used in the applied simulation model.

We further assume that experts are statistical frequentists in the sense that they provide independent assessments given the data they have access to. If the experts where full Bayesians, situations could arise where the formal updating process replicates a process the experts have already run through as they arrived at their estimates. In other words, every expert would have their own prior giving rise for an adjustment of these priors, this is discussed in Gelman (2012).

4.3.1 d) Observed data variance proxies for parameter variance

In this chapter, we assume that the historically observed variance can be interpreted as a reasonable approximation of the uncertainty of the historically observed mean. This variance is then used as the prior variance of the mean for the normal distribution. This assumption is critical yet common for the closed form solution (Fink, 1997) with known variance. Without this assumption, one would obtain a different proxy for the variance of the empirically observed mean. In the next chapter we discuss an extension to this method that investigates this assumption in further depth.

4.3.1 e) Uncorrelated estimation errors

In Kalman filtering theory the measurement errors are generally assumed not to exhibit autocorrelation, yet methods exist to circumvent these problems if this assumption does not hold (e.g. Wang, Li & Rizos, 2012; Jazwinski, 2007). We assume the expert's estimation error is not correlated with the variance of the estimated parameter. We discuss the threat posed by correlated expert opinions in chapter 6.

4.3.2 New information

In Bayesian statistics new information arrives in the form of new observations of a stochastic, potentially noisy process. We substitute actual observations with expert opinions that we treat like observations as is common in Bayesian statistics (e.g. Kelly et al., 2011). A key distinction here is that actual observations become only accessible as new realizations of a stochastic process occur, whereas expert opinion is forward looking in nature thereby enabling a distinct input modelling approach. The process of deriving numerical estimates based on beliefs is referred to as elicitation in the literature (Cooke, 1994). Rees (2015) acknowledges that soliciting expert judgment is prone to statistical inconsistencies and offers some basic questioning tips to counteract this. We discuss several more cognitive and related biases in the analysis part of chapter 6. Notably, the elicitation is a complex process that has been researched in considerable depths (e.g. Cooke, 1991 and references therein), we touch upon various aspects of this process in the discussion of biases. In this section, we discuss which parameters experts are required to estimate.

Expert opinion must be provided in a form suitable for Bayesian analysis. For the case of the normal distribution with fixed variance, that we treat here, this means that estimates must be provided of the future mean as well as self-assessed uncertainty. The latter can be provided, e.g. via estimation of confidence intervals. Crucially, experts must quantitatively estimate their own uncertainty about statistical parameters they are estimating. Here, we follow Cooke (1991, Ch. 11) where experts estimate their own uncertainty which is critical for a correct weighting of inputs yet is subject to various cognitive biases, most notably overconfidence / overprecision bias and the generally observed difficulty of quantifying uncertainty (Spiegelhalter, Pearson & Short, 2011). Despite these challenges, the de-biasing methods from the literature provide a reasonable countermeasure to potential biases.

4.3.3 Derivation for the Standard normal distribution

In simulation modelling, continuous distributions are commonly used thus we show how Bayesian updating can be applied here. As we showed in the preceding chapters, the normal distribution continues to be among the most prevalent distribution and is thus chosen for this example.

Further, the log-normal distribution can be derived directly from the normal, thereby addressing additional modelling scenarios.

We seek to update μ, the unobservable mean of some population data. This closed-form solution assumes that σ is known and fixed, we show an example of how the variance can be updated separately in the appendix. Historically we have observed the mean and standard deviation and will denote them:

$\bar{\mu}$ = historically observed sample mean

$\bar{\sigma}$ = historically observed sample standard deviation

As outlined above we assume that the historically observed squared standard deviation proxies for the uncertainty of the mean. Next, we obtain some new information about said parameters via additional data or expert input. We denote the new data X and now seek to obtain the likelihood of observing μ given our previously held believes:

$$p(\mu|X) \propto p(X|\mu)p(\mu) \tag{4}$$

The operator ∝ stands for proportionality and is read as „is proportional to “. This operator allows us to simplify the notation by omitting variables that are constant with respect to μ as these do not affect the proportionality of the statement (Rachev, Hsu, Bagasheva & Fabozzi, 2008). P(X | μ) denotes the likelihood of the new data given the mean and P(μ) denotes the prior believe about the population mean. For simplicity, we assume here that the new data follows a normal distribution with mean μ and variance σ_x^2. The observed variance of empirical data can be interpreted as the expected spread around the mean of a variable (Fink, 1997). Under these assumptions, the likelihood function of X is:

$$p(X|\mu) = \sum_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left\{-\frac{(x_i-\mu)^2}{2\sigma_x^2}\right\} \tag{5}$$

Where n is sample size of new information and “exp” the exponential function.

To solve (1) we need to derive the likelihood function of the second element on the right-hand side of the equation, the prior distribution of μ.

$$p(\mu) = \frac{1}{\sqrt{2\pi\bar{\sigma}^2}} \exp\left\{-\frac{(\mu-\bar{\mu})^2}{2\bar{\sigma}^2}\right\} \tag{6}$$

Where $\bar{\mu}$ represents the prior mean and $\bar{\sigma}$ represents the standard deviation of the prior. By substituting (2) and (3) into (1) we obtain:

$$p(\mu|X) \propto \frac{1}{\sqrt{\sigma_x^2\,\bar{\sigma}^2}} \exp\left\{\frac{-(\mu-\bar{\mu})^2}{2\bar{\sigma}^2} + \frac{-\sum_{i=1}^{n}(x\_1-\mu)^2}{2\sigma_x^2}\right\} \tag{7}$$

As shown in Lynch (2007) this can be rearranged to show that $\mu \mid X$ is normally distributed with mean:

$$\mu = \frac{\bar{\mu}\sigma^2 + n\bar{\sigma}^2\mu_x}{\sigma_x^2 + n\bar{\sigma}^2} \tag{8}$$

This result can be reformulated to further highlight its intuition:

$$\mu = \frac{\frac{1}{\bar{\sigma}^2}}{\frac{1}{\bar{\sigma}^2}+\frac{n}{\sigma_x^2}}\,\bar{\mu} + \frac{\frac{n}{\sigma^2}}{\frac{1}{\bar{\sigma}^2}+\frac{n}{\sigma_x^2}}\,\mu_x \tag{9}$$

Note that the mean is a weighted average of the prior empirical mean, the believe $\bar{\mu}$ and μ_x, the mean of the new information. Each is weighted by its inverse variance. Thereby a highly uncertain prior raises the weight of the new information in the posterior and vice-versa. The prior mean's ($\bar{\mu}$) weight is proportional to the inverse of its variance ($\frac{1}{\bar{\sigma}^2}$) and likewise the data / new information mean (μ_x) weight is proportional to the ratio of its sample size (n) and its variance ($\frac{n}{\sigma_x^2}$). Intuitively a new source of information with a high level of certainty moves the posterior towards this new information whereas analogously a highly certain prior will take a larger weight.

Similarly, we follow Lynch (2007) to obtain the result of the variance:

$$\sigma^2 = \frac{\bar{\sigma}^2\,\sigma_x^2}{n\bar{\sigma}^2+\sigma_x^2} \tag{10}$$

Note that the variance of the estimate of the posterior $\mu \mid X$ has the noteworthy property of being strictly smaller than both the variance of the prior and the variance of the new data. This makes intuitive sense as the combination of two sources of evidence of positive, entropy-reducing informational value allows for a more precise estimate of the unknown population parameters. This is one of the core results of Bayesian statistics and has appealing properties that we further explore in the next chapter.

4.4: Case application

The objective of these simulations is a classical impact analysis of varying input modelling specifications or parameterizations that constitute the factor levels in this experimental design. In short, this constitutes a benchmark of various input modelling methods via one simulation model. This simulation experiment is based on an actual application of the method and therefore offers a level of realism that is not necessarily present in comparable research (e.g. Weber et al., 2011). As we address simulation input modelling for CF&A, we model common challenges revolving around profitability and risk (e.g. Weber et al., 2011; Meyer et al., 2011).

4.4.1 Case study

The case application discussed here revolves around the financial position and short-term earnings forecast of a mid-sized German waste incineration facility. A sale of the facility had been agreed one year hence and the current management was focused on forecasting key financial metrics to address any potential funding gaps before the eventual sale one year after the current base year 2018. The objective was thus to forecast financial metrics over a one-year time based on current actuals and knowledge of the entity's assets. The entity can be considered part of the German Mittelstand. The willingness of banks to provide short-term credit to Mittelstand companies has been shown to be low (Hansmann, Höck & Ringle, 2003) thereby necessitating a diligent scrutiny of the short-term financial position and potentially resulting financing needs.

The entity in this case study incinerates waste to generate district heating and electricity earning most revenues from three distinct sources:

1. Revenue Electricity: electricity is generated from the waste incineration process and fed into the network at current market rates; this typically accounts for ~23% of their revenues (depending on relative per unit prices of different revenue streams); electricity rates are constant across all assets
2. Revenue District heating ("Fernwärme"): further exhaust heat is used for district heating accounting for a further ~28% of revenues; contrary to electricity rates, there can be differences in district heating rates across assets/locations, however prices are highly correlated
3. Revenues from incinerating waste: the largest part of revenue is generated through the fees obtained from waste collection companies for incinerating waste accounting for ~48% of revenues; it is noteworthy how unusual this case is as this entity is not paying for the primary input into its production process but rather is getting paid

For this entity, historically, prices for district heating and electricity are positively correlated whereas prices for waste are negatively correlated with both district heating and electricity prices resulting in a natural hedge, this is assumed constant in the model. The full correlation matrix is provided in table 14.

Historical Correlation coefficients between unit prices of revenue sources	Waste price	Electricity rate	District heating rate: Location 1	District heating rate: Location 2
Waste price	1,00	-0,67	-0,85	-0,69
Electricity rate	-0,67	1,00	0,59	0,91
District heating rate: Location 1	-0,85	0,59	1,00	0,50
District heating rate: Location 2	-0,69	0,91	0,50	1,00

Table 14 - Correlation matrix of output prices

The entity owns two major assets, both waste incineration plants, of varying size, that we will label simply 'location 1' and 'location 2' with annual waste incineration capacities of 450.000 and 270.000 tons respectively and historical utilization of typically ~95%.

It must be noted here, that the situation the entity was in, was characterized by a discrepancy between what experts predicted and what historical analysis suggest would be the development of factor prices – thereby underscoring the need for precise input modelling. Expert input was used to obtain the following data points for the year following the base year.

Factor	**Expected mean price**	**Self-assessed variance**
Waste Prices	64.00 €	7.00 €
Electricity prices	39.00 €	1.00 €
District heating location 1	19.00 €	12.00 €
District heating location 2	21.00 €	12.00 €

Table 15 - Expert opinion / estimate for input modelling

Expert input was solicited through experts' assessment of confidence intervals of the variables to be modelled through the method discussed in Winman, Hansson & Juslin (2004) as well as in the appendix. This method entails providing intervals and asking experts to assign probability judgments rather than the other way around. This method has been shown to reduce cognitive bias in the form of overconfidence/overprecision bias. The input was obtained from experienced experts in the commodities and purchasing department of the entity. From confidence intervals it is straightforward to infer self-assessed uncertainty / variances by inverting the steps undertaken to construct confidence interval and solving for the variance.

The prior is based on historical data intrinsic to the entity in question and simple extrapolation of trends.

Factor	Historic 7-year mean price	Historic 7-year variance
Waste Prices	67.25 €	13.64 €
Electricity prices	34.75 €	2.89 €
District heating location 1	21.08 €	6.84 €
District heating location 2	29.16 €	6.74 €

Table 16 – Historic data for model input variates

The case application presented here is stylized for several reasons. As it is based on an actual entity it is bound to strict confidentiality standards that require to make certain discretionary changes to the entity to ensure its anonymity without changing the salient features of the method's application. To ensure generalizability, a key objective of case study research is to discuss cases that can be considered “typical” for the research strand (Seawright & Gerring, 2008); this can be achieved as evidenced by the review of typical applications in Chapter 2 where simulation of profitability is among the most frequent applications. Further, we simplify policy factors like the government subsidies for eco-friendly electricity as part of the German “Erneuerbare Energien Gesetz” that are not essential to the method.

4.4.2 Benchmarking

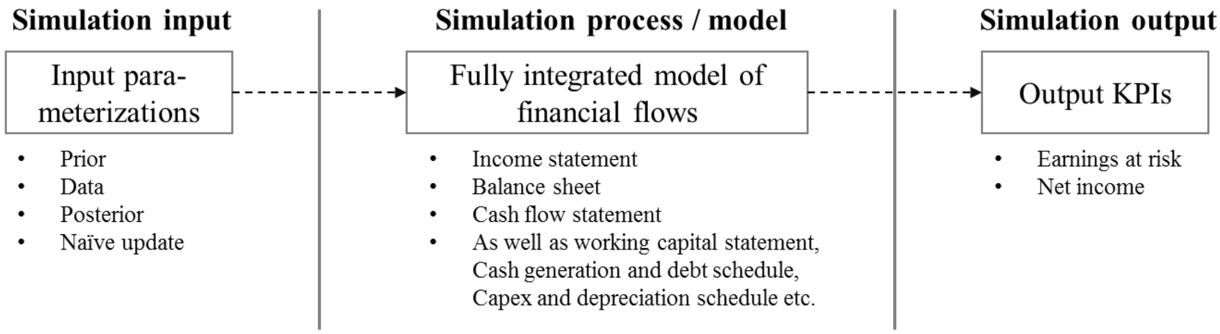
In a benchmarking analysis, we compare simulation results based on the proposed method of Bayesian updating with alternative input modelling parameterizations. Empirical data and expert opinions are most commonly used and subject to uncertainty or imprecision as we showed in the previous chapter and are therefore used as benchmarks. Bayesian input modelling is on the other hand not compatible with theory-based input modelling if those theories are deterministic with respect to the parameters they define. We therefore refrain from comparative analysis of theory-based input modelling in this chapter as Bayesian input modelling is aimed at situations with multiple informative input sources.

Finally, non-Bayesian methods of data aggregation can combine multiple data sources (e.g. Cooke, 1991). As the expert interviews showed (2016) this can take a pragmatic form such as averaging of sources. We Benchmark our simulation results with results obtained from these ad-hoc or naïve methods. We exclude methods requiring extensive access to experts for calibrations or violate other assumptions.

4.4.3 Design of Experiment

The analysis and communication of simulation results follows the Design of Experiment (DoE) principles presented in Lorscheid, Heine & Meyer (2012) aiming at systematically structured analysis and transparency. Following Hocke, Lorscheid & Meyer (2015) we also provide a simplified representation of the simulation model in Figure 10. This simplicity of the model underscores the focus on input modelling which utilizes the model to illustrate input modelling impact on the response variables.

Figure 9 - Simulation process overview



I: Formulate objective of simulation experiment

As touched upon above, the objective of this simulation is a quantification of the effect of different simulation input modelling specifications, thus different input parameterizations. Our objective is to research the viability of Bayesian updating for simulation input modelling in simulation environments in CF&A. As pointed out in Lorscheid et al. (2012) simulation experiments are likely to uncover all effects of input distributions if a proper DoE is provided. Thus, it may seem a foregone conclusion that a different input parameterization will result in a different simulation output. Yet simulation modelers typically face trade-offs between accuracy and model complexity (Weber et al., 2011) as increasingly realistic or detailed models become more resource intensive in terms of time, computing power and modelling know-how. We extend this to the discussion of model parameterization via Bayesian updating. Hence, we analyze the effects of alternative model parameterization against the backdrop of this trade-off. Moreover, we investigate the sensitivity of simulation results to different input parameterizations.

More generally, there is no established method to proof superiority of subjective parameters as used in simulation input modelling as subjective statements cannot generally be judged right or wrong (Keren, 1991). As mentioned in the introduction, this simulation experiment was applied in a stochastic setting with only a single real-world realization and therefore no definitive proof if the Bayesian input modelling is superior in the sense of being closer to the "true" mean parameter than alternative parameterizations. Hence in a Bayesian setting, it is not generally feasible to know the unique parameters of a distribution as these are random variables themselves. An empirical comparison is infeasible as there is no data source containing the required data points that would include many simulations under the described conditions with additional data on the future realizations of the simulated variables. However, this simulation succeeds in demonstrating the desirable properties Bayesian updating can have for simulation input modelling in CF&A.

II: Classification of variables

Following Lorscheid et al. (2012) we assign each stochastic variable to the groups independent, dependent and control variables.

Independent variable	Control variables	Dependent variables
1) Input model: waste incineration prices	1) Number of simulation runs	1) Earnings-at-risk at 5%
2) Input model: electricity prices	2) Utilization rates per location	2) Net income
3) Input model: district heating prices for location 1	3) Heating value per waste unit	3) Probability of incurring a loss
4) Input model: district heating prices for location 2	4) Electricity and district heating shares	
	5) Cost of disposal of burnt waste	
	6) Combustibles	
	7) Additional consumables, raw materials	
	8) Maintenance	
	9) Additional services consumed	
	10) Salaries and wages	
	11) Social security contributions	
	12) Other SG&A (incl. Professional services)	
	13) Depreciation	
	14) Interest payments & financing conditions	
	15) Taxes	
	16) Net working capital	

Table 17 - Classification of variables

III: Definition of response variables and factors

This step is of utmost importance for sensitivity of simulation to input parameters. Input modelling methods represent factor levels for the independent variables. The parameters resulting from these modelling methods represent the factor's levels that differ between the applications. The dependent variables are profitability and risk KPIs of the modelled entity such as or Earnings-at-risk (Lorscheid et al., 2012). We follow Meyer et al. (2011) in choosing Earnings-at-risk (EaR) as one of the response variables of interest, a widespread risk metric (Viemann 2005). EaR is based on the concept of Value-at-risk that is commonly defined as "the worst loss over a target horizon that will not be exceeded with a given level of confidence" (Jorion 2001). Table 18 presents the independent variables, factors and factor level ranges for this simulation experiment.

Independent variable	Factors	Factor level range
Waste incineration prices	Input modelling method	{Prior, Data, Posterior, Naïve update}
Electricity prices	Input modelling method	{Prior, Data, Posterior, Naïve update}
District heating prices location 1	Input modelling method	{Prior, Data, Posterior, Naïve update}
District heating prices location 2	Input modelling method	{Prior, Data, Posterior, Naïve update}

Table 18 - Definition of factors, factor level ranges and response variables

IV: Selecting a factorial design

This step determines an experiment's factors that influence the independent variable and potential interactions between these variables. In the simulations, we isolate the effects of input distributions and do not alter other variables making our factorial design straight-forward with a single factor: the choice of input modelling method and thus no interactions between factors[13]. This experiment follows a 4x1 factorial design with the four factors representing input modelling specifications.

[13] Although Bayesian and naïve updating constitute aggregation methods of the other two factors, we do not view these as factor level combinations but rather new factors altogether

V: Estimation of experimental error variance

To ensure simulation results are not driven by unintended randomness one must determine the number of simulation runs that is sufficient. Lorscheid et al. (2012) suggest using the stability of the coefficient of variation of as a stopping criterion. The coefficient of variation stabilizes after 100.000 simulations runs and is plotted on a logarithmic scale in figure 11.

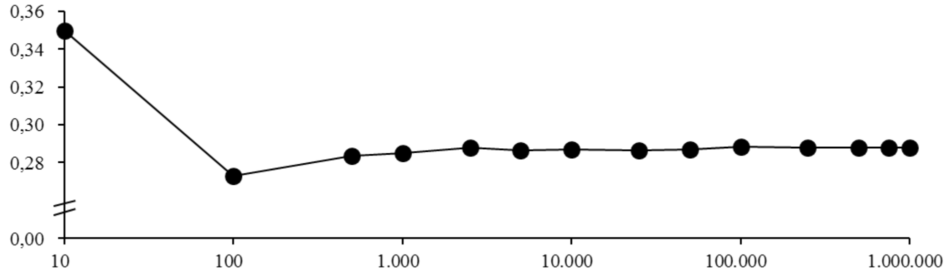


Figure 10 - Coefficient of variation, logarithmic scale of # of simulation runs, in %

In addition, we ran the simulation in increments from 10 to 1.000.000 runs and observe that the standard error of the response variable net income only decreases slightly after 100.000 simulation runs as is plotted in figure 12.

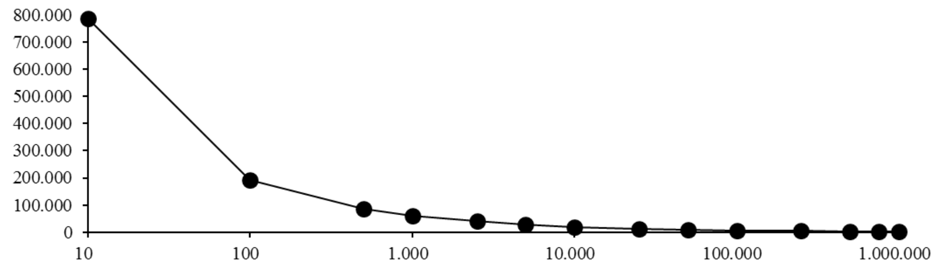


Figure 11 - Standard deviation of net income, logarithmic scale of # of simulation runs, in €

In a related analysis, Weber et al. (2011) conclude that 100.000 simulation runs offer sufficient stability in a similar modelling context. We thus conclude that for our purposes 100.000 simulation runs are sufficient. Furthermore, Crystal Ball's *Precision Control* tool allows for setting a threshold of accuracy and then runs the simulation until a specified level of accuracy (e.g. +/- 1%) is reached with 95% certainty, as a further layer to ensure to stay within +/- 1% of the desired accuracy.

VI: Simulation experiment

This simulation experiment is implemented in the crystal ball software environment; in this choice, we follow, among others, Meyer et al. (2011) who argue that this is commonly used choice among simulation modelers, especially in CF&A. We further discuss modelling assumptions in the appendix.

VII: Analyzing effects

We run the simulation model in the crystal ball environment as described above and obtain the following response variables summarized in table 19.

Factor levels	Response variable I: Earnings-at-risk 5%	Response variable II: average net income	Response variable III: Probability to incur loss
Factor level 1: Prior	Mn 1.91 €	Mn 5.14 €	0.45%
Factor level 2: Data (expert input)	Mn (3.07) €	Mn 0.94 €	29.15%
Factor level 3: Bayesian Posterior	Mn 0.87 €	Mn 4.10 €	1.85%
Factor level 4: Naïve updating	Mn (0.19) €	Mn 3.08 €	5.70%

Table 19 - Factor levels and response variables of the 4x1 simulation experiment (negative numbers in brackets)

Through the three separate response variables we obtain a differentiated picture of the entity's risk profile and the varying input modelling methods, especially Bayesian input modelling compared to *naïve updating*. While both input parameterizations build onto the same aggregated input sources, their mean net incomes are estimated at Mn 4.10 €, for the Bayesian update, and Mn 3.08 €, for the naïve update, thereby constituting a ~33% difference. In addition, we observe negative skewness for all distributions of net income (see table 20).

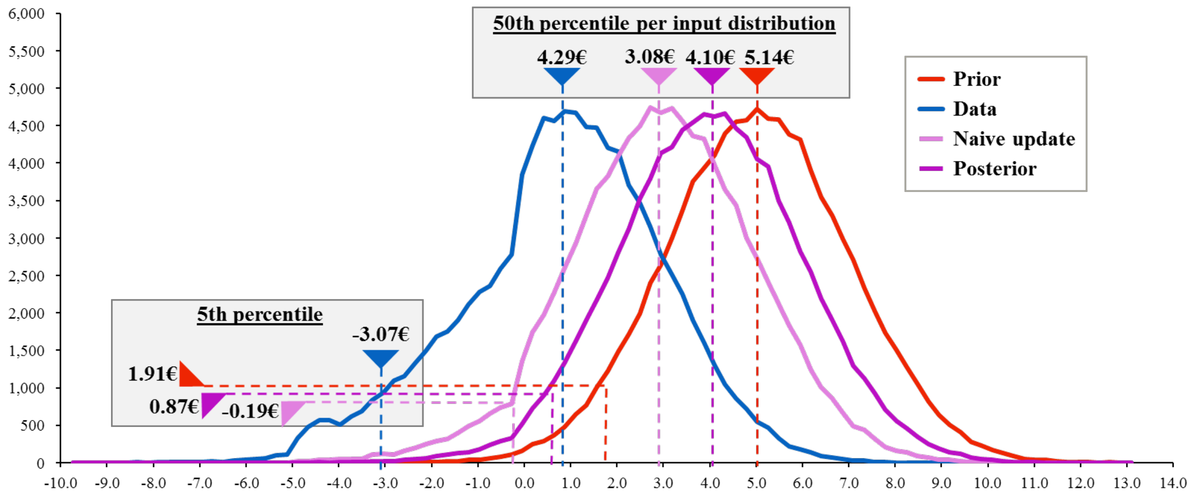


Figure 12 - Probability density functions per input modelling specification with 5th and 50th percentile / mean highlighted

Figure 13 shows the simulation-based probability density functions of the four input modelling methods overlaid on one another. In addition, it shows response variables including the 5th percentile of net income, which corresponds to the response variable of *Earnings-at-risk at 5%*, as well as the mean of simulation net incomes (probability to incur losses is not shown here). It must be noted here, that the updating focused on the mean of the distribution assuming the standard deviation fixed which leads to the similarity in shape of the output distributions. Whilst the distributions based on Bayesian and naïve updating share significant overlap, as is expected as both are based on data sources, albeit aggregated differently, it becomes apparent that they differ substantially. This will become even more apparent in the following figure.

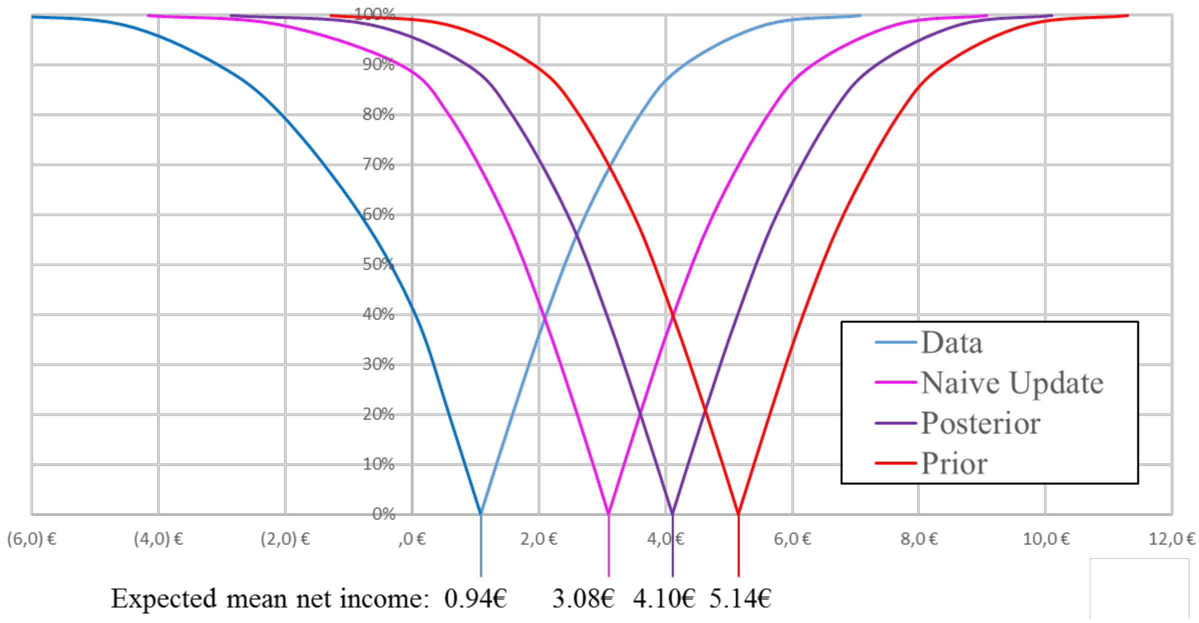


Figure 13 – Simulation-based confidence intervals for net income projections (axis in € millions)

Figure 14 shows smoothed simulation-based confidence intervals for net income projections thereby further highlighting the impact on response variables to the four different factor levels, most notably between Posterior and the "Naïve Update" process based on simple averages. Note, how the increasing confidence levels on the vertical axis correspond to increasingly wide intervals that center on the average net income per the simulation model.

Finally, we provide additional simulation data in table 20.

Statistic	Data	Naive update	Posterior	Prior
Simulation runs	100.000	100.000	100.000	100.000
Average (mean)	Mn 0.94 €	Mn 3.08 €	Mn 4.10 €	Mn 5.14 €
Standard deviation	Mn 2.19 €	Mn 2.01 €	Mn 1.98 €	Mn 1.97 €
Skewness	-0,344	-0,167	-0,074	-0,008
Kurtosis	3,18	3,36	3,30	3,29
Minimum	Mn (9.73) €	Mn (7.20) €	Mn (6.29) €	Mn (4.64) €
Maximum	Mn 9.10 €	Mn 11.12 €	Mn 12.78 €	Mn 13.32 €
Standard error of the mean	6,940 €	6,369 €	6,265 €	6,239 €

Table 20 - Additional descriptive statistics for simulation experiment

In the appendix we provide multivariate regression outputs for the simulation of the four factor levels. As expected these analyses support our conclusions drawn above and results are highly statistically robust across all simulations.

4.5: Discussion and conclusion

This chapter derived and applied a straightforward and computationally inexpensive method to aggregate prior data with expert input for the purpose of simulation input modelling in corporate finance and accounting. We showed the impact the method had in a case application and analyzed and interpreted the simulation model's general results. We conclude that this method can be applied in simulation settings with various imperfect input modelling sources to efficiently aggregate information sources.

As argued above, it is not possible in this context to proof the superiority of Bayesian input modelling over naïve aggregation methods. However, the solid theoretical foundation of more accurate weighting of input sources and thereby more precise parameter estimates should inspire confidence in the method. Beyond the theoretical foundations, there are ways to further illustrate the uncertainty-reducing properties of Bayesian estimation that unequivocally show the advantages of this method in quantitative form. This will be the objective of the next chapter.

Future research may focus on the application of this input modelling method to other realms of simulation modelling. While we focus our analysis on stochastic simulations in CF&A there is a wider range of simulation methods such as Agent-Based Models or System Dynamics where input parameters are critical. Future research could analyze how the methods discussed here can be transferred to these methods. We limit the applications to a set of core distributions as identified by an expert sample in line with the literature as discussed in the previous chapter. Future research may replicate the analysis and illustration with yet more distributions.

Chapter 5: *Simulation Output at risk (SOaR)*: quantifying parameter input stochasticity

5.1: Introduction

The accuracy of simulation models in CF&A hinges upon their input parameters and distributions which oftentimes are not known precisely and can even be considered stochastic variables themselves in a Bayesian setting. The epistemic uncertainty about simulation input parameters is not always quantified intuitively although it can constitute a substantial modelling risk. We present here a straightforward method to quantify and communicate modelling risk stemming from stochasticity in distributional parameters called *Simulation Output at Risk (SOaR)*. The concept is analogous to *Value at risk (VaR)*, among the most important and widely used risk metrics in finance and accounting thus widely understood in a potential target audience. The *SOaR* metric quantifies modelling risk due to input parameter variability intuitively in a single metric. It thereby contributes to the theory of sensitivity analysis of stochastic simulation and to improved communication of simulation methods more generally. After discussing the metric in general, we apply it to an adaptation of the business case application from chapter 4 in a simulation experiment highlighting both the metric and its ease of communication as well as the uncertainty reducing properties of the Bayesian input modelling approach

Input error can induce a simulation modeler to under- or overestimate risks (Lam, 2016). Input error refers to the uncertainty of simulation input modelling parameters and their subsequent effects on simulation outputs (e.g. Henderson, 2003). Input error remains an important challenge in simulation input modelling. As the effects of uncertainty of input parameters do not decline with the number of simulation runs, simulation models can lead to a false sense of security in simulation outputs if indicators like the coefficient of variation decline and stabilize with increasing numbers of simulation runs. The objective of this chapter is to present and discuss a novel metric to communicate risk of input error called *Simulation Output at Risk (SOaR)*.

SOaR enables the communication of a sensitivity analysis of parameter uncertainty in a single number. Further, the close conceptual relation to the *Value-at-risk* metric ensures that a wide group of simulation modelers can be expected to be acquainted with the interpretation of this metric (Jorion, 2007).

One objective of simulations in CF&A can be to provide a probability distribution of possible outcomes of a stochastic variable or KPI such as a Net present value or future net income. However, a simulation model will only yield potential outcomes given its input parameters. Assuming non-negligible risk of error from the input parameters, simulation results can be skewed and erroneous. Simulation input parameters that fail to account for their own uncertainty can present modelling risk.

Extensive efforts in the literature on simulation modelling have sought to improve the communication of simulation results through standardized reporting formats (Lorscheid, Heine & Meyer, 2012; Hocke, Meyer & Lorscheid, 2015)**.** One challenge noted by Lorscheid et al. was the lack of generally understood standards that are straightforward to communicate and understood by a wide audience. *SOaR* represents a step in this direction, seeking to improve communication on modelling risk from stochastic input parameters. While the consideration and quantification of input modelling uncertainty is not new, it's spread may have been hindered by the complexity of its communication. As Henderson (2003) emphasizes, any method used to capture input uncertainty must be, among other factors, transparent, implementable as well as efficient. A key contribution of this metric lies in the ease of estimation, communication and understanding. Beyond the core contribution of the *SOaR* metric, this chapter also serves to establish and illustrate one of the key benefits of Bayesian input modelling, namely its uncertainty reducing properties, thereby underscoring a feature of the method that is not straightforwardly replicated with different input modelling method.

5.2: Review of the literature

In this section, we briefly review modelling risk quantification methods, sensitivity analysis in simulation modelling, the Value-at-risk metric.

5.2.1 Modelling risk / Model uncertainty

Vose (2008) discusses Model uncertainty broadly encompassing the structural model as well as the input parameters. If the stochastic model that drives a variable is uncertain, Vose argues for an inclusion of more than one modelling structure in the simulation with a stochastic choice within each simulation run for one of the candidate models and thus distributions again being stochastic. Per Vose (2008) it is rare not to observe deviations between stochastic distributions and processes used in simulations and reality, he argues, however, that this need not be "terrible" for the model. Vose suggests testing the model's robustness or sensitivities to such specifications yet without providing a primer on how such analyses shall be quantified generally and communicated in a widely interpretable way. To this end, Lam (2016) provides a tutorial on input uncertainty in simulation experiments generally. He distinguishes between *simulation error* and *input error*. The former referring to errors from finite simulation runs whereas the latter stem from inaccuracies in the probabilistic assumptions that serve as input to simulation models. Through a robust DOE (e.g. Lorscheid et al., 2012) it is possible to minimize risk of *simulation error.* Input error can be further broken up into the *parameter uncertainty*, concerning erroneous distributional parameters, and *model uncertainty*, concerning incorrect choice of distribution family (e.g. Gaussian instead of log normal) and its correlation over time with itself and across other variates. This dichotomy implicitly assumes a correct functional form of the modelled process as it does not mention modelling errors such as incorrectly modelled relationships between variables. As Song, Nelson & Pegden (2014) show, it is possible to dissect variance owing to simulation and input errors. While simulation error declines with increasing number of simulation runs and approaches zero, input error is independent of the number of simulation runs and thus remains a significant threat to simulation outputs even as declined costs of computation have lowered the threat of simulation errors in practice. This underscores the importance of a clear communication of input error.

Henderson (2003) introduces input model uncertainty along two examples, in one case uncertainty is known and inherent to the model whereas in the other, the parameters are stable though not known with certainty resulting in a perceived probability distribution around the unknown parameter. This illustrates well the dichotomy between aleatoric and epistemic uncertainty, that we discuss in further depth in the next section.

Lam (2016) identifies two objectives of understanding input uncertainty. The first objective is to quantify the “sensitivity of output variability from the uncertainty of the input”. This objective motivates the *SOaR* metric. *SOaR* summarizes output uncertainty stemming from input uncertainty in a single number for straightforward communication. The second objective is to “generate an interval that covers the true performance measure with high confidence”. This harks back to Vose’s (2008) recommendation of using different models stochastically in case of uncertainty regarding the functional form of the DGP and not only its parameterization. In this Bayesian setting where the parameter (vector) is considered a stochastic variable, this would translate into drawing from the population of possible parameters given our estimate of its stochastic value for each simulation run. This, of course, leads to a wider distribution of input parameters and thereby independent variables and thus its dependent variables as well.

For the rest of this chapter we will use the following structure and definitions. Building onto Lam’s definition, we divide modelling risk into three branches with the first two in line with

Lam (2016). We can differentiate between choosing an incorrect distribution or process, referred to as *model uncertainty,* and using erroneous parameters for these distributions, referred to as *parameter uncertainty*. The third branch can be labelled *model specification error* or *functional form uncertainty* and is analogous to its equivalent in econometrics (MacKinnon, 1992).

- *Functional form uncertainty* describes an incorrectly specified model and can lead to a series of biases and errors, including omitted variable bias, simultaneity bias, or the inclusion of unnecessary or the exclusion of necessary variables. This is also described as Structural uncertainty (Draper, 1995).
- *Model uncertainty*: It is not always straightforward to determine what type of distribution created an empirical data set and simulation outputs are not always sensitive to the specification of the "correct" distribution (Rees, 2015; Kelton & Law, 2000; Vose, 2008). However, in dissecting various risk factors to accurate modelling it is important to distinguish different sources of modelling risk or uncertainty. Choice of distribution tends to be discrete whereas a distribution's parameters are usually defined on a ratio scale.
- *Parameter uncertainty* remains the focus of this chapter that the *SOaR* metric seeks to capture. It describes uncertainty regarding the parameters of a pre-specified distribution and *SOaR* captures its effect on the simulation output.

Barton (2012) uses a similar dichotomy by differentiating errors resulting from incorrect execution logic and errors from incorrect input models, the latter comprising both model and parameter uncertainty.

5.2.2 Sensitivity analysis

Attempts to quantify parameter uncertainty can be described as a part of sensitivity analysis of a simulation model. Parameter uncertainty is captured through sensitivity analysis by some applied simulation modelers. Bratley, Fox & Schrage (1987) advocates the use of sensitivity analysis to determine a models' sensitivity to input modelling specifications, especially if the data source contains "little or wrong, but related" data giving rise to suspicions. This shall include the parameters of a distribution but also the choice of distribution, thereby covering parameter as well as model uncertainty, though not uncertainty regarding the functional form. The *SOaR* metric is congruous to a sensitivity analysis of simulation results in relation to changes in the input parameterization.

It is not generally possible to conclude the effect of input uncertainty on output uncertainty in a simulation model without running the simulation model as the relationship between input and output uncertainty are not necessarily linear or otherwise predictable. Non-linearities can arise from many model specifications in CF&A such as explicit modelling of non-linear credit covenants (e.g. credit ratings), assignment of cost pools depending on amount of allocated costs or modelling of future decisions (e.g. real options analysis). Where these occur, there is a distinct non-linearity in the relationship between input uncertainty and output distribution.

Hofer, Kloos, Krzykacz-Hausmann, Peschke, & Woltereck (2002) discuss pathbreaking applications dissecting aleatoric and epistemic uncertainty (see 'Method and result'-section) in reliability engineering. They propose a modelling method that can account for two separate sources of uncertainty around a single variate that circumvents the need to run nested simulations that would entail prohibitive computational effort. Instead, they propose two methods of either sampling from a joint distribution, effectively a compound distribution, or running a simulation focusing on one source of uncertainty, the aleatoric part, and keeping the other fixed, in this case the epistemic. There are clear analogies to the *SOaR* metric that we will point out as we discuss the metric and build on this reasoning and develop it further.

In a similar vein, Guo & Du (2007) apply a method of jointly analyzing aleatoric and epistemic uncertainty through simulations in reliability engineering that they label "Unified Uncertainty Analysis".

5.2.3 Value at risk

Value-at-risk inspires the *SOaR* metric and is a frequent measure of risk in simulation models in CF&A (Jorion, 2007) thus meriting a brief review, Wipplinger & Jorion (2007) review in-depth. VaR remains a debated risk measurement metric as it does not capture the severity of potential losses beyond its pre-defined thresholds (e.g. 1% or 5%). Thereby, critics argue, it over-simplifies the risk profile of potentially irregular or asymmetric distributions to a single number (Einhorn & Brown, 2008). This perceived shortcoming can be addressed through the metric of conditional Value at risk, commonly labelled cVaR (Rockafellar & Uryasev, 2002). It is defined as the average loss or deviation of a distribution's mean beyond the defined threshold, put differently, cVaR describes the average of the response variable beyond the 5% cut-off threshold. Thereby the variability in the response variable's tail distribution is captured. This metric can be used in addition to the unconditional VaR as well as a substitute. It can straightforwardly be calculated for earnings-at-risk as we will show in the simulation model below. To calculate *conditional Earnings-at-risk* all sources of uncertainty are modelled jointly. Additional steps need to be taken to calculate *conditional simulation-output-at-risk* (*cSOaR*) that we will discuss below. We calculate the conditional Simulation output at risk for all relevant factor levels.

The metric's simplicity is also a major advantage as it supports communication and is widely understood (Jorion, 2007). It is this simplicity of communication in a stochastically complex environment that we aim for in the metric of *Simulation Output at risk*.

Simulations are a tool to estimate VaR which is driven by a variety of risk factors. We can distinguish between real risk factors, e.g. the variability of returns associated with political or operational risk, and modelling risk, notably parameter uncertainty. Modelling risk describes the variation in simulated VaRs that arises due to the risk of errors in the model. The second objective as formulated in Lam (2016) strives to capture the probability distribution of the simulated dependent variables including their parameter uncertainty and by extension also the VaR including parameter uncertainty.

One key differentiation to VaR is that SOaR is defined via the input distribution threshold, rather than purely the output of the distribution. In other words, the input distribution defines the cut-off thresholds rather than the output distribution as would be the case for VaR. This innovation enables the metric to capture risk from different sources of uncertainty and holds as long as an approximately linear relationship between input model and output distributions can be assumed.

5.3: Method and results

In a Bayesian setting, that we continue to follow here, we use Bayesian updating to reduce the uncertainty of input parameters and thereby SOaR. To this end, it is necessary to differentiate two sources of uncertainty relevant to this context. We are confronted with two levels of uncertainty (note that both aleatoric as well as epistemic uncertainty are distinct from the above-mentioned *model* and *functional form* uncertainties):

1. **Aleatoric or physical** uncertainty (Der Kiureghian & Ditlevsen, 2009; Fox & Ülkümen, 2011): describes randomness in the realizations of the stochastic process that cannot be

reduced through more information and are stochastic for each simulation run or physical realization; even if assuming stable distributional parameters each realization is random just like a coin toss is random even if the process of tossing the coin is perceived to be the same for each toss.

2. **Epistemic** uncertainty (Der Kiureghian et al., 2009; Fox & Ülkümen, 2011): describes uncertainty about input modelling parameters due to incomplete information; this uncertainty could be reduced through improved information or modelling. Consider the example of the fair and unfair coins being tossed (see appendix) and the application of Bayesian statistics to make inferences about the coins being tossed, here we observe epistemic uncertainty as we do not know the distributions pertaining to the coins. Similarly, consider any situation with limited access to data from the process to be modelled, resulting in imperfect knowledge of distributional characteristics.

It is generally not necessarily clear which variables are subject to aleatoric or epistemic uncertainty, or both (Der Kiureghian et al., 2009). Furthermore, in simulation research and practice, it is oftentimes not straightforward to distinguish epistemic and aleatoric uncertainty, thus simulation modelers may not specify which one they focus on (Hofer, Kloos, Krzykacz-Hausmann, Peschke & Woltereck, 2002). Assuming perfect information, one would seek to model strictly the aleatoric or physical uncertainty that is inherent in the process to be modelled. However, under circumstances of imperfect information and aleatoric uncertainty, simulations should model both sources of uncertainty (Hofer et al., 2002) as incorrect assessment of these two uncertainty sources can lead to inconsistent risk assessment including over- or underestimation of variability, depending on the modelling context (Der Kiureghian et al., 2009).

Examples abound for variables whose perceived variation can be driven by both epistemic as well as aleatoric uncertainty. Previously we discussed major risk factors commonly modelled and construct straightforwardly conceivable scenarios of how these can be subject to both classes of uncertainty. An input distribution can "contain" both aleatoric as well as epistemic uncertainty if the simulated variable is both stochastic in its realizations and knowledge about it is imperfect.

1. **Variable costs** are amongst the most commonly modelled variables; aleatoric uncertainty can stem from e.g. input cost factors, stochastic production processes etc.; epistemic uncertainty may stem from imperfect knowledge due to errors in cost accounting systems (e.g. Labro et al. 2007)
2. **Purchasing prices** of input factors can fluctuate due to aleatoric uncertainty such as a stochastic commodity price process and simultaneously due to epistemic uncertainty if e.g. price negotiations are involved with unknown outcomes (that are still based on fluctuating base rate prices) that represent epistemic uncertainty; this can be denoted as structural breaks where an input distribution that is undergoing structural change with imperfectly known consequences for the distribution's parameters, as is common in simulation modelling (Barton et al. 2002)
3. **Demand** factors are the third most commonly modelled risk factor whose variation can be driven by both sources of uncertainty, e.g. considering an aleatorically stochastic demand forecast that is subject to epistemic uncertainty in the form of measurement error in the data used to construct the forecast (Der Kiureghian et al., 2009)

These examples underscore that both sources of uncertainty are common in the input modelling factors of simulations in corporate finance and accounting.

For this application we assume that the risk factors modelled, that will be introduced below, are driven by both aleatoric and epistemic uncertainty. Further, we assume uncorrelated and normally distributed aleatoric and epistemic uncertainty. The resulting compound distribution, accounting for both sources of uncertainty, is also normally distributed with a strictly increased standard deviation of $\sigma_{compound} = \sqrt{\sigma^2_{Aleatoric} + \sigma^2_{Epistemic}}$. Via this compound distribution, it is thus possible to calculate a simulation output that accounts for parameter uncertainty within the distributional assumptions of this example. This compound distribution simplifies the simulation model as it circumvents the need to run a nested model that is a feature of analyses concerned with capturing aleatoric and epistemic uncertainty in simulations (Henderson, 2003). Further, the assumption of normality allows us to fully define the input distributions by their first two central moments, its mean and standard deviation. A stylized visualization of the compound distribution is shown in figure 15.

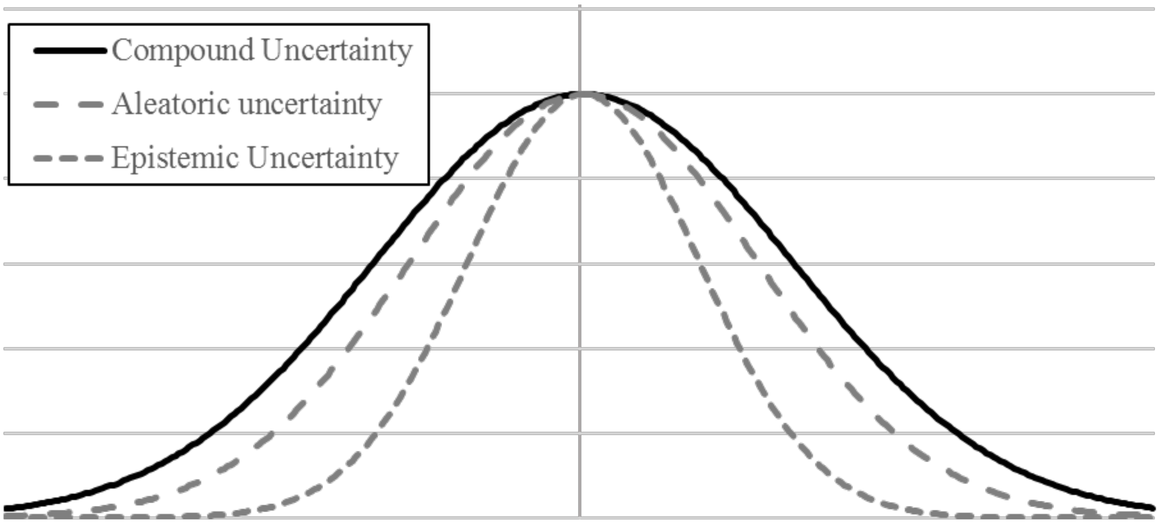


Figure 14 - Compound uncertainty containing both aleatoric and epistemic uncertainty for a univariate normal distribution (Probability mass not normalized to 1 for illustrative purposes)

The compound uncertainty leads to a strictly wider distribution in the input variates and therefore per the central limit theorem also to a higher level of variability in the simulation model. Per definition, epistemic uncertainty can theoretically be reduced through obtaining more accurate information, though usually at a cost (Oberkampf, 2019) which would reduce the compound uncertainty and lead to a narrower distribution in the stylized figure 15**.** As epistemic uncertainty approaches zero, the compound uncertainty approaches the aleatoric uncertainty (see chapter 6). A key challenge with both aleatoric and epistemic uncertainty in a modelling context is touched upon in Oberkampf (2019): while it is necessary, for some purposes or objectives pertaining to the simulation model, to model both sources of uncertainty jointly, it may lead to simulation outputs with non-straightforward interpretations for other objectives. Relating this to the objective put forth by Lam (2016) and discussed above, for objective 1, quantifying output uncertainty stemming from epistemic uncertainty, a separate modelling is necessary whereas a joint modeling achieves objective number 2. The following two-staged simulation model will encompass joint as well as separate modelling of both sources of uncertainty and show how simulation modelling can adequately capture this uncertainty.

5.3.1 Bayesian updating with aleatoric and epistemic uncertainty

In chapter 4 we did not distinguish between aleatoric and epistemic uncertainty but introduce this distinction here. We assume that the compound uncertainty as discussed in chapter 4 can be decomposed into aleatoric and epistemic uncertainty; the next section outlines this decomposition and shows a derivation applicable here. The aleatoric uncertainty represents the variance of the modelled variable that we continue to assume fixed for the Bayesian updating. The epistemic uncertainty represents the uncertainty around the aleatorically stochastic input parameter, that we use for weighting in the updating process thereby reducing uncertainty. It is critical to emphasize that only the epistemic uncertainty shall determine the prior or new information's weight in the posterior due its information content and not the aleatoric uncertainty that represents mere variability. Here, it is of course critical to achieve consistent decomposition for all input sources contributing to the update as inaccurate approximation of epistemic and aleatoric uncertainty shares would imply skewed and inaccurate posterior parameter estimates as we discuss further below. For the Bayesian updating we continue to assume conjugate prior distributions to the normally distributed input variates.

5.3.2 Derivation

After discussing epistemic and aleatoric uncertainty in this Bayesian context, we proceed to the core of this chapter, the derivation of the SOaR metric. The objective of the *SOaR* metric is to quantify the impact of epistemic uncertainty on simulation outputs whilst simultaneously modelling aleatoric uncertainty. We define *SOaR* analogously to the definition of VaR from Jorion (2007):

> *Simulation-Output-at-Risk is the maximum expected deviation of simulation outputs due to epistemic uncertainty that will not be exceeded with a low, specified probability.*

Analogously, we define:

> *Conditional Simulation-output-at-risk is the average deviation of simulation outputs due to epistemic uncertainty beyond a low, specified probability threshold.*

Just like the definition of VaR, *SOaR* is nonconstructive as it specifies the properties of the metric, though not how it is derived. However, in a context of simulation modelling, one method is to run the simulation model itself with varying input parameterizations. Obtaining the (downside) *SOaR* metric via this method can be done in the steps outlined in Table 21:

Step	Procedure
1	Derive stochastic input model (assuming known and non-stochastic functional form)
2	Derive (or make assumptions for) aleatoric and epistemic uncertainty contribution to the compound uncertainty input model and keep aleatoric uncertainty fixed
3	Obtain threshold percentiles of the distribution of epistemic uncertainty at defined cut-off points, e.g. 5th or 95th; 1st or 99th etc.; ensure consistent modelling of 'downside' and 'upside' risk
4	Run simulation model with fixed aleatoric uncertainty centered on its mean (corresponding to the mean or 50th percentile of the epistemic uncertainty distribution)
5	Run simulation model again with fixed aleatoric uncertainty though now centered on the pre-defined percentile of the epistemic uncertainty distribution thereby capturing the uncertainty related to epistemic uncertainty at the corresponding percentile
6	Calculate deviation in simulation output metrics between input modelling specifications to obtain the simulation output at risk per metric

Table 21 - Step-by-step procedural approach for SOaR calculation

Figure 16 visualizes how the *SOaR* parameterization relates to the full probability distribution of a modelled variate. The distribution on the left-hand-side is centered on the 5th percentile of the epistemic uncertainty distribution. It shows thus the downside deviation of the input modelling variate that will not be exceeded with a probability of 5%. This distribution corresponds to a probability mass of 5% in the Bayesian sense

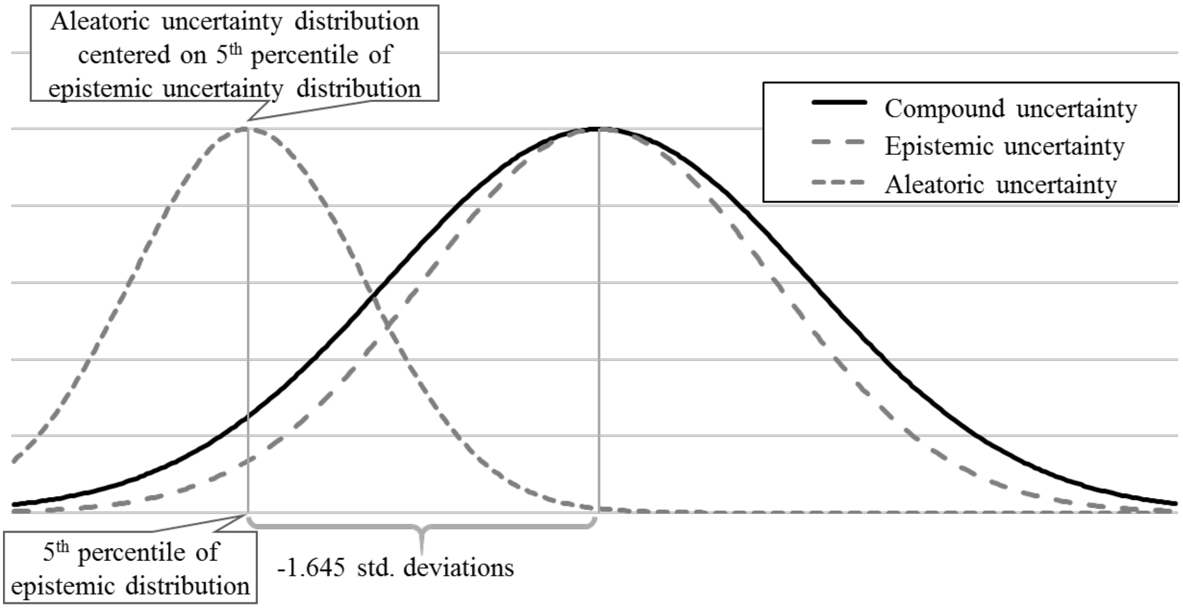


Figure 15 – Epistemic uncertainty modelled on fixed percentile of the aleatoric uncertainty distribution for a univariate distribution

For a multivariate distribution, one would estimate the threshold percentiles for the joint distribution considering co-dependencies as we show in the application below.

5.3.2 a) Variance decomposition

In the case application we discuss here, SOaR is applied in a scenario where it is possible to approximate a variance decomposition via the method described below. To generalize the method, the question arises how simulation modelers shall apply the metric if this decomposition is not as straightforward as in the scenario below where a clear distinction of sources of uncertainty can be made. While decompositions, including the method presented below, are approximate and thus have inherent imperfections, they can still advance prudent simulation input modelling. Bayesian statisticians tend to favor approximations where precise sources of information are unattainable. Further, in Bayesian contexts it is often viewed acceptable to work with assumptions, even if these entail imprecisions, where no data exists as echoed, e.g. in Bayesian methods in credibility theory (Bailey, 1950; Longley-Cook, 1962) where statisticians assume they are “not devoid of knowledge before acquiring data”. This is reflected in choices of priors that include “weakly informative” and “objective” or “uninformative” priors, a somewhat misleading name, that describes vague or imprecise prior knowledge (Jaynes, 2003) that is nonetheless a useful element in the Bayesian updating process to aggregate information. The objective of using less informative priors is referred to as regularization and prevents overfitting posteriors to data, or in this case expert opinion, and represents a standard approach in Bayesian modelling. This context notwithstanding, it remains a challenge to decompose variance prudently and robustly. Bayesian estimation of posterior input modelling distribution is only accurate if the decomposition of aleatoric and epistemic uncertainty can be considered consistent for all input sources of the updating process, in this case historic data and expert opinion.

Henceforth, a short discussion follows of decomposition approaches for both empirical data as well as expert input, the two sources considered here.

For expert input, one has to rely on the experts’ own estimation of the epistemic and aleatoric uncertainty shares of their estimates. Here again Cooke (1991) provides useful guidance in the discussion of expert calibration and especially normative goodness (Winkler & Murphy, 1968). Normative goodness refers to the consistency of an expert’s probabilistic estimates with general probability theory as opposed to substantive goodness that refers to actual subject matter expertise. As we discuss in the next chapter, normative goodness is critical in Bayesian updating based on expert opinion generally to ensure consistent estimation of posteriors. This extends further to self-assessed variance decomposition that requires experts to be able to distinguish sources of variance within their own estimates. This results in the need to increase awareness and sensitize experts of the distinction, akin to de-biasing methods building onto creating awareness of cognitive biases, thereby counteracting it – a method that has been shown to be effective, though without necessarily reducing bias to zero (see next chapter). Note however that this is not a structural bias with a typically 'known direction', such as overconfidence, as it is not a priori known if experts tend to overestimate aleatoric or epistemic uncertainty shares. The challenge to rely onto experts' self-assessment represents a limitation of this modelling approach.

For historic data there is more flexibility of methods for variance decomposition as both expert-based as well as empirical methods exist. One method, that we shortly touch upon here, is decomposition purely via expert assessment. An expert's estimate of the epistemological and aleatoric share of compound variance of empirical data may be imperfect however represents an improvement over otherwise available methods to quantify uncertainty. For this, it would be necessary to ensure a high level of statistical calibration on behalf of the expert, as discussed above. Variance decomposition via empirical data, as used in vector-auto-regressions (e.g. Lütkepohl, 2005), can be applied if consistent interpretations of epistemic and aleatoric uncertainty exist, as in our example below.

5.3.3 Case description

We build here on the case example from chapter 4. This setting contains a natural extension that lends itself to an application of *simulation output at risk*. Here in fact, we model the Simulation-Output-at-Risk for district heating rates that were introduced in the previous chapter. The entity in question still has two major locations / assets that provide, among other sources of revenue, district heating. While electricity and waste (for incineration) prices are equal across different asset locations, this is not the case for district heating for the entity in question. Prices per MW/h of district heating per location are correlated but can diverge substantially, even over sustained periods of time. The entity in question had access to a forecast for average district heating rates for southern Germany, though not for each of the two areas it serviced. This forecast for the whole region has proved to be reasonably accurate, superior to time-series auto-regressive forecast models[14].

Hence, we treat uncertainty stemming from the (stochastic) average price rate for southern Germany as *aleatoric uncertainty* and treat uncertainty stemming from the deviation from each location to this forecast as *epistemic uncertainty*. More precisely, we decompose the compound variance into its approximate constituent parts of aleatoric and epistemic uncertainty. The aleatoric uncertainty is approximated through the explained part of a linear regression of each location's district heating price on average for Southern Germany whilst the epistemic uncertainty is approximated by the residual sum of squares of the same regression (the full model of this derivation is provided in the appendix). This is mathematically equivalent to variance decomposition as used in vector-auto-regressions (Lütkepohl, 2005).

Here, the prior mean is based on a simple forecast model that uses the exogenous forecast of the Southern German average district heating price to model each location's price based on a simple regression model. In this setting the resulting prior suggested a steep increase in district heating rates for the simulated one-year period. This prior represents one factor level in the simulation and is based on actual data from the application of the Bayesian input modelling method discussed in chapter 4.

The prior is updated with new information in the form of expert opinion through Bayesian input modelling as shown in the previous chapter. Expert input was derived in the same format as in chapter 4 as we apply the equivalent context once more here. It is crucial to note here, that there was a discrepancy between the suggested prices for district heating based on the prior and the

[14] Straightforward regression analysis showed that this was in fact the case

expert opinion as shown in table 22. In other words, the prior suggested steep increases in prices whereas experts from the case company were less optimistic and cautioned against planning with prices based on these priors.

Parameter	Prior	Data (expert input)	Posterior
Mean: District heating location 1	25,29	21,00	23,03
Epistemic Variance: District heating location 1	3,34	3,00	1,58
Mean: District heating location 2	36,03	25,00	32,00
Epistemic Variance: District heating location 2	1,73	3,00	1,10

Table 22 - simulation input modelling data for both stages of the simulation experiment

A key deviation from chapter 4 is that we assume, as explained above, that we can differentiate aleatoric and epistemic uncertainty. Here only the epistemic uncertainty is interpreted as the uncertainty associated with the (data-based) prior and expert assessment and thus used for weighting in the updating process. The aleatoric uncertainty is assumed fix and compounded in a separate step. The posterior compound uncertainty is strictly smaller than its prior and thusly leads to a strictly reduced compound posterior uncertainty.

Table 22 shows only the approximated epistemic variance per each variate, as we discussed above. This epistemic variance is used for weighting of input sources rather than the aleatoric variance, which is fixed, and shall not determine a sources' weight in the posterior in this setting. Note the uncertainty-reducing features of the Bayesian update: the posterior epistemic variance is strictly smaller than its priors. This is a salient feature of Bayesian statistics and the following application will underscore its benefits for simulation modeling under parameter uncertainty.

The case application described here, operates within the context of Bayesian updating assuming conjugate priors via expert opinion with aleatoric and epistemological uncertainty. Within the Bayesian updating process, we assume a fixed variance or standard deviation – though this assumption is not necessary and can be circumvented via Markov-Chain Monte Carlo methods.

As discussed, this variance corresponds to the aleatoric variance rather than the epistemological, which per definition cannot be fixed. This means, that input providing experts must be aware of the critical distinction of the two sources of uncertainty in the input model. For the weighting in the Bayesian update, only the experts' epistemic uncertainty is elicited and required. The key objective remains eliciting the distribution's mean as well as the uncertainty of this estimate, which in this case is restricted to the epistemological uncertainty pertaining to the expert themselves. As the aleatoric uncertainty is assumed fix, it can be based on data from the prior rather than needing to be estimated.

To compute the SOaR metric, we model epistemic uncertainty at a pre-defined threshold point defined at 5%. As the two location's district heating prices are correlated, we model a multivariate distribution and compute the 5th percentile of the epistemic uncertainty at their joint cumulative density function at equidistant points from their mean as measured in each process's epistemic uncertainty. As district heating has an approximately positively linear effect on net income, we estimate the downside SOaR at the 5% level by running the simulation with a parameterization on the 5th percentile of the epistemic uncertainty distribution that we show below. The derivation for cSOaR is slightly different as it builds onto parameterization not at the threshold but rather along the average parameterization below the defined threshold that we show below in the case application as well.

An extension of this model could assume the input modelling distribution's aleatoric variance not fixed and incorporate the expert's estimate of the aleatoric variance into the posterior of the epistemic variance. This would work analogously to the input modelling specification described in the appendix to chapter 4.

5.3.4 Simulation model and DoE protocol

We extend the application from chapter 4 to capture how *SOaR* will be applied in a two-staged model, the first stage is a 1x2 experimental design and the second is a 2x2 experimental design. We present the analysis of *Simulation Output at Risk* based on a further development of the application from chapter 4 of a simulation model in the context of a decision-making scenario in a corporate finance setting. It must be noted however, that while the application is based on an actual case application of the method, the *SOaR* metric was not used in this context.

I: Formulate objective of simulation experiment

The objective of this simulation model is twofold. Firstly, the first and second stages both show, in distinct form, the uncertainty-reducing properties of Bayesian updating for simulation input modelling with aleatoric and epistemic uncertainty. Thereby, addressing both objectives of quantifying parameter uncertainty put forth by Lam (2016). Secondly, the second stage of this simulation model introduces the calculation of the *SOaR* metric within this Bayesian modelling context. For stage I, the parameterization and thereby factor levels are illustrated graphically in figure 17:

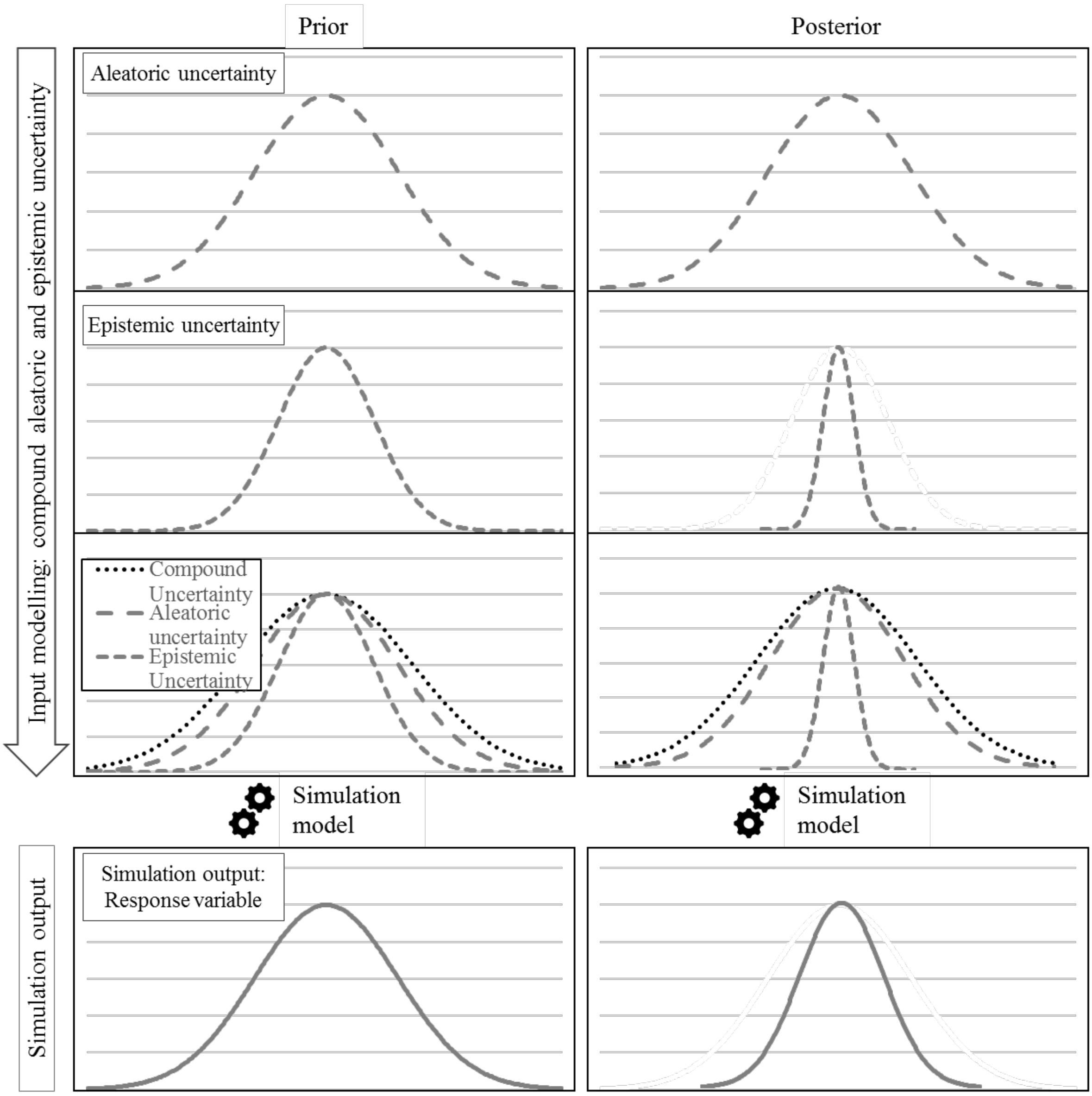


Figure 16 - Stage I of the simulation model

The posterior epistemic uncertainty is reduced due to the Bayesian update, leading to a lower compound variance, as well as an expectation of lower variation in the simulation model output. Joint modelling however, precludes distinguishing the two types of uncertainty, representing a potential shortcoming of the modelling approach depending on the purpose and objective of the simulation model (Oberkampf, 2019). Stage I with its joint modelling of epistemic and aleatoric uncertainty seeks to achieve Lam's (2016) second objective.

We will seek to overcome this limitation in stage II of the model through the *SOaR* metric as well as *conditional SOaR.* The objective of stage II of the model is to introduce the *Simulation Output at risk metric* in a setting of Bayesian updating as illustrated in figure 18**:**

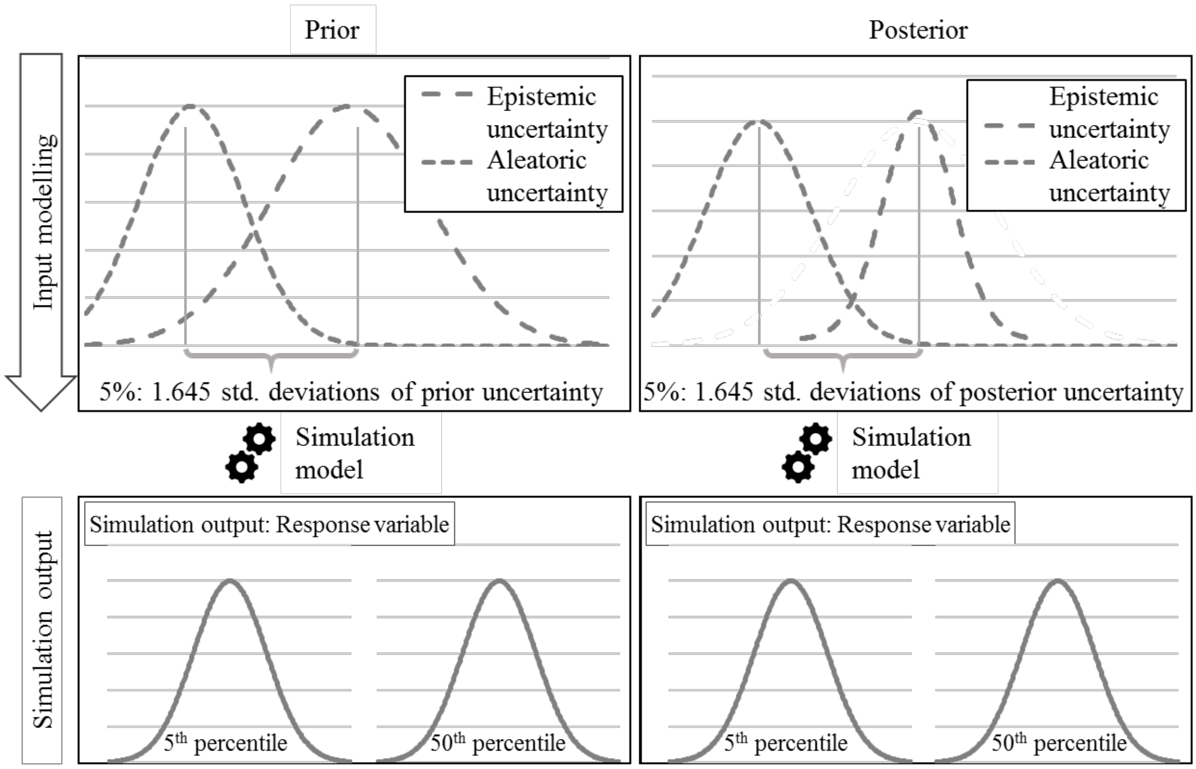


Figure 17 – Representation of stage II of the simulation model with prior and posterior epistemic and aleatoric variances simulated independently

The input modelling of stage II of the simulation model models epistemics and aleatoric uncertainty separately and through centering the fixed aleatoric uncertainty along different percentiles of the prior and, strictly lower, posterior uncertainty, captures the effect of each source of uncertainty.

Here we will show several of the desirable properties of the SOaR metric including distinguishing aleatoric and epistemic uncertainty and its straightforward communication. This contribution then underscores unequivocally one of the salient properties of Bayesian input modelling, its uncertainty reducing features, which are illustrated through the SOaR metric and go beyond the benchmarking of chapter 4 by establishing its superiority in this setting of joint modelling of epistemic and aleatoric uncertainty.

II: Classification of variables

The classification of variables follows to some extent the previous chapter, we thus show only were we deviate from the corresponding table shown in chapter 4. In fact, only the dependent variables change in this classification of variables.

Stage I: Dependent variables	Stage II: Dependent variables
I) Earnings-at-risk at 5%	I) Net income (with epistemic variance centered at different percentiles of the aleatoric uncertainty distribution to calculate Simulation-output-at-risk)
II) Conditional Earning-at-risk at 5%	II) Earnings-at-risk at 5% (only for input distribution centered on 50th percentile)
III) Net income	
IV) Standard deviation of net income	

Table 23 - Classification of dependent variables for stage I & II (independent and control variables equivalent to chapter 4)

As response variables we choose a similar set of metrics as in chapter 4 if they have straightforward interpretations for the compound modeling of epistemic and aleatoric uncertainty. For Earnings-at-risk we also show its conditional variation, conditional-earnings-at-risk. One exception is that we do not show Earnings at risk metrics for simulation models with input distributions of district heating rates centered on the 5th percentile of the epistemic uncertainty distribution. These do not have straightforward frequentist statistical interpretations and would require additional simulation models of the aleatoric uncertainty centered along all percentiles of the epistemic distribution which would be tantamount to running a model with fully compounded input distributions as in stage I of the simulation experiment. In addition, we include dependent variable IV, standard deviation of net income, that highlights the uncertainty-reducing properties of Bayesian updating. We do not include the dependent variable “probability to incur losses” as this risk is quite low due to the expectation of increasing district heating rates.

III: Definition of response variables and factors

We show here the response variables and factors for stage I and II respectively. Table 24 shows independent variables, factors and factor level ranges for stage I.

Stage I: Independent variable	Factors	Factor level range
Mean of District heating prices location 1	Input modelling method	{Prior, Posterior}
Standard deviation of District heating prices location 1	Input modelling method	{Prior (compound of aleatoric, prior epistemic), posterior (compound of aleatoric, posterior epistemic)}
Mean of District heating prices location 2	Input modelling method	{Prior, Posterior}
Standard deviation of District heating prices location 2	Input modelling method	{Prior (compound of aleatoric, prior epistemic), posterior (compound of aleatoric, posterior epistemic)}

Table 24 - Definition of independent variables, factors, factor levels for stage I

Note that in the factor level range for the independent variable we vary the input model used for the standard deviation. In the first factor level we compound the assumed fixed aleatoric variance with the prior of the epistemic variance whereas in the second we use the strictly reduced Bayesian posterior of the epistemic for compounding. This is predicted to lead to a narrower range of response variables of interest.

For stage II the independent variables and factors are shown in table 25. As we model only fixed aleatoric uncertainty through the independent variates' standard deviation in all factor levels, the standard deviation of the input price distributions becomes a "mere" control variable. Epistemic uncertainty is modelled through centering of the mean along thresholds of the epistemic uncertainty distribution and is thus an independent variable however modelled via the mean of the price input distributions.

Stage II: Independent variable	Factors	Factor level range
Mean of District heating prices location 1	Input modelling method	{Prior (centered on 50^{th} percentile of epistemic uncertainty distribution), posterior (centered on 50^{th} pctl.), prior (centered on 5^{th} pctl.), posterior (centered on 5^{th} pctl.)}
Mean of District heating prices location 2	Input modelling method	{Prior (centered on 50^{th} percentile of epistemic uncertainty distribution), posterior (centered on 50^{th} pctl.), prior (centered on 5^{th} pctl.), posterior (centered on 5^{th} pctl.)}

Table 25 - Definition of independent variables, factors, factor levels for stage II

The response variables for both stage I and II are the standard deviation of net income and the Simulation Output at Risk respectively. Throughout this simulation we keep the input modelling parameters for waste and electricity prices unchanged (i.e. we model their stochastic effect, though do not change their factor levels) at its prior values to isolate the effect of modelling choices around district heating rates. The priors for independent variables are based on the OLS-regression model analogous to the one used for the variance decomposition and further elaborated upon in the appendix.

The SOaR method allows us to model aleatoric uncertainty while keeping epistemic uncertainty fixed at pre-defined points (50th and 5th percentile of the distribution of district heating prices). Note that the aleatoric uncertainty, in this set of assumptions, is not altered through the updating process and we thus consider the aleatoric variance fixed throughout the scenarios of the second stage of the simulation experiment for highlight the effects of epistemic uncertainty that is in fact altered through the Bayesian update. The key response variable is then defined as the difference in the output variable(s) of interest between the simulation points on the epistemic uncertainty distribution.

IV: Selecting a factorial design

As each input modelling choice represents an independent factorial design we cannot model interaction effects as would be common in more complex simulation experimental settings (Kelton et al. 2000). Thus, the factorial design is straightforward as we model each factorial design independently as is common in related benchmarking simulation experiments (e.g. Weber, Schmid, Pietz & Kaserer, 2011). Hence for stage I there is a 2x1 factorial design, as well as for stage II both modelling the effects of the two factor levels: input modelling based on the Bayesian prior and posterior parameter estimate.

V: Estimation of experimental error variance

As the model follows an analogous structure to the simulation model in chapter 4, we do not show the coefficient of variation and standard deviation of simulated net incomes as the graphs are almost equivalent. We run again 100,000 simulations per factorial design.

VI: Simulation experiment

We build again on the model developed for chapter 4 and run it in the Crystal Ball simulation environment and run the model as per the defined number of simulation runs from the analysis of experimental error variance. Beyond the basic analysis provided through Crystal Ball, it is also possible to extract the full experimental data set to run further advanced analyses that we discuss below.

VII: Analyzing effects

Table 26 shows the dependent and response variables for stage I of the simulation model.

Factor levels	Dependent variable I: Earnings-at-risk 5%	Dependent variable II: Conditional-Earnings-at-risk 5%	Dependent variable III: Average net income	Dependent variable IV: Standard deviation of net income
Factor level 1: Prior	Mn 4.07€	Mn 3.25€	Mn 7.30€	Mn 1.96€
Factor level 2: Bayesian Posterior	Mn 2.81€	Mn 2.09€	Mn 5.71€	Mn 1.76€

Table 26 - Dependent/response variables stage I

As expected the Bayesian posterior input model leads to linearly reduced expected average net income as well as a risk of lower earnings per the Earnings-at-risk metric because the expectations from experts on price developments of district heating rates were less optimistic than the prior. The reduced modelling uncertainty through the lower epistemic uncertainty can already be observed here through the difference between dependent variables III vis-à-vis dependent variables I and II. Unconditional earnings-at-risk for the factor level parameterized based on the Bayesian prior deviate Mn 3.23€ (Mn 7.30€ -Mn 4.07€) from the corresponding mean net income (conditional EaR: Mn 4.05€) whereas the factor level based on the Posterior deviates Mn 2.89€ (conditional EaR: Mn 3.62€) thereby highlighting the reduced variability of dependent variable III, average net income. This is of course underscored in dependent variable IV which shows reduced standard deviation of net income.

The output for stage I of the simulation model shows the joint modelling of epistemic and aleatoric uncertainty thereby achieving Lam's (2016) second objective. However, as stated above, this model does not enable us to differentiate the two sources of uncertainty. We observe different variances of the output variables, yet the sources of the uncertainty cannot be decomposed as the input distributions are already compounded.

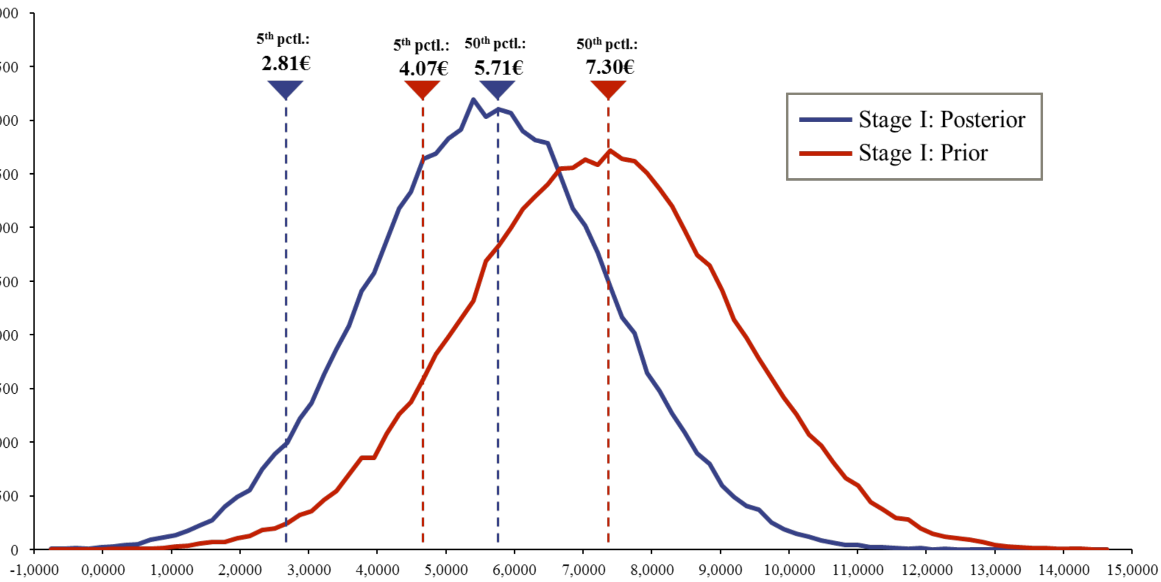


Figure 18 - Scaled Density functions for stage I of the simulation experiment

Figure 19 visualizes the probability distributions of net income simulations based on both input modelling methods of stage I. To improve readability of overlaying density plots, we transformed the discrete histograms into approximate PDFs along 100 intervals of width €181,042. Visual inspection confirms a continuous distribution without non-linearities, thereby ensuring that the estimation of *SOaR* is indeed valid here. The first salient feature of this simulation model is the uncertainty reducing effect of Bayesian updating that we observe. The distribution of net incomes based on Bayesian updating is narrower due to this reduced uncertainty. This is further illustrated through confidence intervals shown in figure 20.

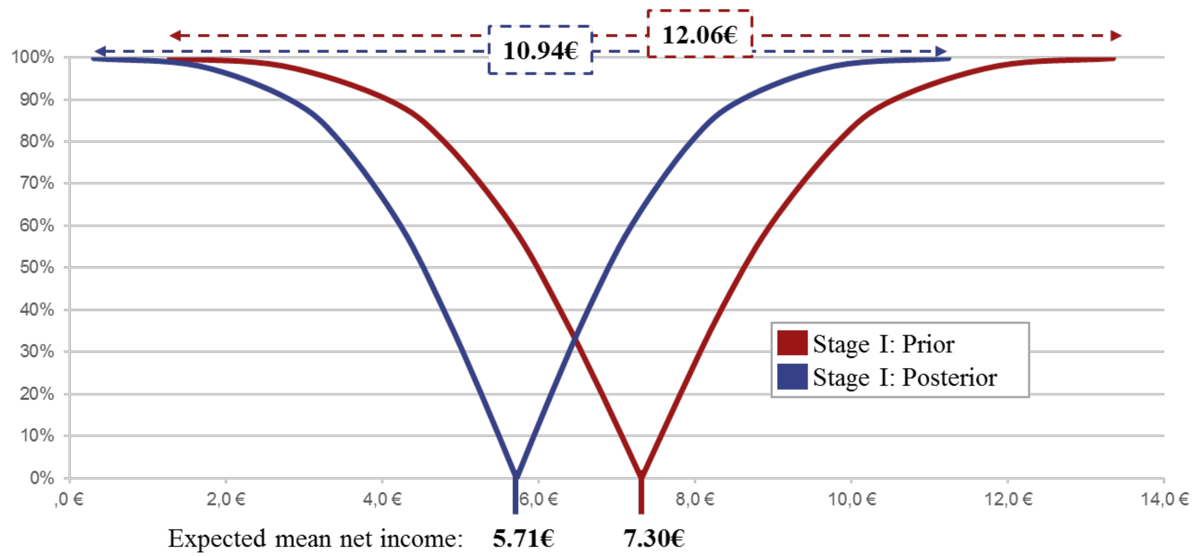


Figure 19 - Simulation-based Confidence intervals of expected net income for stage I and II

Through the Bayesian update we reduce epistemic uncertainty by leveraging all available information in the prior as well as the expert opinions. In the simulation model output this can be observed through a reduced compound variance consisting of the fixed aleatoric and the reduced epistemic variance. The result is a narrower distribution of net income and adjacent metrics in the second factorial design. This is an important result as it represents a methodologically robust way to reduce overall or compound uncertainty through reducing epistemic uncertainty within the assumptions of this model.

Table 27 shows the dependent and response variables for stage II of the simulation model.

Factor levels	Response variable I: average net income	Response variable II: Earnings-at-risk 5%	Response variable III: conditional Earnings-at-risk 5%
Factor level 1: Prior at 50^{th} percentile	Mn 7.30€	Mn 4.81€	Mn 4.15€
Factor level 2: Prior at 5^{th} percentile	Mn 5.33€	N.A.	N.A.
Factor level 3: Bayesian posterior at 50^{th} percentile	Mn 5.72€	Mn 3.21€	Mn 2.59€
Factor level 4: Bayesian Posterior at 5^{th} percentile	Mn 4.29€	N.A.	N.A.

Table 27 - Response variables for second stage of the simulation model

Table 28 shows the calculation of the *simulation output at risk* metric for the input models based on the prior as well as the Bayesian posterior based on the four factorial designs.

Factor levels	Response variable I: net income 5^{th} percentile	Response variable I: net income 50th percentile	**Delta between 50^{th} and 5^{th} percentiles** (column 3- column 2)
Prior	Mn 5.33€	Mn 7.30€	Mn 1.97€
Bayesian Posterior	Mn 4.29€	Mn 5.72€	Mn 1.43€

Table 28 - Simulation output at risk estimation

In short, this metric now allows a straightforward communication of modelling risk in terms of the response variable of interest in the simulation output. It follows the following statement than now communicates epistemic uncertainty in one variable.

> *"Based on input modelling following the prior (posterior), the maximum expected downside deviation of simulation-based net income forecasts due to epistemic parameter uncertainty that will not be exceeded with a probability of 5% is Mn 1.97€ (Mn 1.43€)"*

An alternatively specification reads:

> *"Based on input modelling following the prior (posterior), the probability with which net income will fall short of Mn 5.31€ (Mn 4.29€) due to epistemic parameter uncertainty is 5%"*

In addition, Figure 21 shows the probability density functions of all four input modelling choices of stage II.

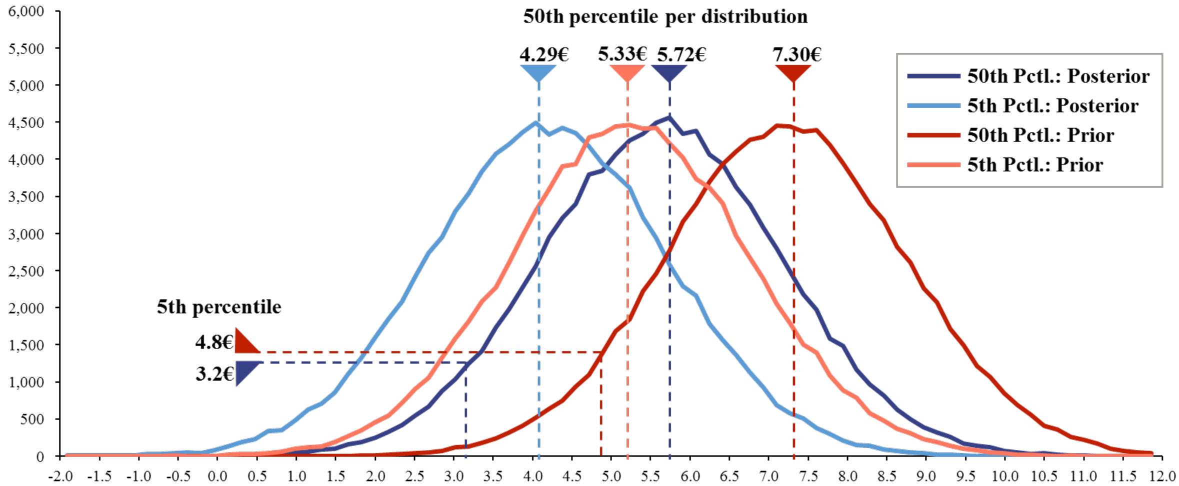


Figure 20 - Probability density functions of stage II

As above we show the simulated net incomes in the form of approximated probability density functions to improve readability and show percentiles of the distributions. The distribution colored in lighter shades correspond to those with input distributions centered on the 5th percentile of the epistemic uncertainty distribution. The visual interpretation is, that there is a 5% chance that epistemic uncertainty about the aleatoric variability of input distributions leads to downside deviations from the aleatoric uncertainty at least as extreme as this distribution. Hence, this distribution has a probability mass as understood in the Bayesian sense of 5%, we show it here scaled to the same magnitude as the other distributions to improve comparability.

5.3.5 Discussion of modelling properties

Henderson (2003) discusses various methods for dealing with parameter uncertainty and defines four requirements: transparency, validity, implementability as well as efficiency. We shortly discuss this illustration of the *SOaR* metric along these dimensions.

- In the context of aleatoric and epistemic uncertainty, *SOaR* can create **transparency** between these sources of uncertainty in a straightforward manner. Further, its simplicity of capturing parameter uncertainty contributes to its transparency.

- This metric's **validity** can be ensured as it is based on a set of foundations in simulation modelling and Bayesian statistics that provide a solid footing of the method as long as the underlying set of assumptions, as discussed above, is satisfied and if the output distribution has an approximately normal shape.
- **Implementability** is a key focus of the method that can be implemented without significant computational expense beyond the simulation model. This however would change if assumptions such as conjugate prior distributions would not hold necessitating Markov-Chain-Monte-Carlo methods for the Bayesian update. Again, within the assumptions discussed, this method is designed to be straightforwardly implemented.
- **Efficiency** in the computational sense is assured through the low computational expense incurred in the implementation as shown here. This, however is again dependent on the assumptions of the model.

In short, this application of the *SOaR* metric appears to fulfill the basic requirements put forth by Henderson (2003). It must be noted, however, that this is a stylized application with the intention of illustrating the method. Several assumptions may not hold in other scenarios. For instance, the *SOaR* metric, as shown here, can only be applied if an approximated decomposition of aleatoric and epistemic uncertainty is feasible. As pointed out above, this is a strong assumption as evidenced by the fact that simulation models do not necessarily distinguish the source of uncertainty (Henderson, 2003; Hofer et al., 2002). Further, the assumption of conjugate priors does not always hold leading to considerably larger computational effort and reduced implementability and efficiency in the Bayesian updating part of this method. Finally, as any method that rests on elicitation of subjective probabilities and distributions, this method will be subject to the risk of residual cognitive bias, notably overconfidence/-precision bias, that cannot be guaranteed to be fully eliminated regardless of the sophistication or thoroughness of the de-biasing strategy (Cooke, 1991; see discussion in next chapter).

Beyond the assumptions, that may not hold, there are limitations to the methods. Similar to *Value-at*-risk, using a single metric to communicate risks or variability has drawbacks as irregular distributions, that do not belong to the known families of distributions, pose a challenge to SOaR as the extreme tails of the distribution beyond the cut-off thresholds (e.g. 5%) are not necessarily modelled (Einhorn & Brown, 2008). This includes asymmetric, discontinuous, non-parametric or otherwise irregular distributions. *SOaR* can be error prone for models or applications containing non-linearities or optionalities as these can lead to discontinuous distributions of output variables. While *conditional-SOaR* alleviates this shortcoming, it does not rule out vulnerability entirely as the tails of simulated distributions depend on potentially error-prone tails of input distributions. While both *VaR* and *SOaR* can be calculated correctly from an analytical standpoint, distributional irregularities can invalidate the interpretation of the metric beyond the thresholds. Put simply, downside risk can be unpredictable beyond defined thresholds and this is exacerbated by discontinuous distributions. This can partially be addressed through visual inspection of probability density functions of response variables.

5.4: Discussion and Conclusion

SOaR enables simulation modelers to communicate modelling risk stemming from stochastic input parameters in this simulation setting in a corporate finance and accounting context. In this Bayesian setting, it further highlights the uncertainty-reducing properties of Bayesian input modelling. The method generally fulfills the requirements of methods seeking to capture input modelling uncertainty, however it is dependent on several assumptions that are critical to this assessment. We propose to include this metric into the communication of simulation experiments' treatment of parameter uncertainty.

Future research could delve into the following topics. Contingent non-linearities are not straightforwardly described with the *SOaR* metric. Contingent non-linear effects can emerge through various channels like explicit modelling of future decisions based on future developments of unpredictable variables within contractual obligations or real options analysis. Resulting distributions can be non-continuous or of non-parametric shape with the resulting problems mentioned above. Future research could describe the resulting effects in further depth or offer metrics that can more accurately capture simulation risk for non-parametric distributions.

Generally, this method can be extended to additional distributions straightforwardly. It would be of interest to simulation modelers to observe how to design input thresholds for compound distributions of, e.g. compounds of binary distributions such as a Poisson process with normal or log-normal distributions and thereby extend *SOaR* to this environment. Further, Variance decomposition is critical to accurate modelling of *SOaR* and represents a viable area for future research, particularly the elicitation of variance shares.

In more complex experimental designs, simulations can result in emergent effects or behaviors that depend on specific input parameter values, especially in fields such as Agent-Based-modelling (Miller & Page, 2009). Some effects may only occur if input parameters are within specific ranges or exceed some threshold value. Thus, the uncertainty of input values can be considered of high importance for Agent-based modelling as the occurrence of emergent effects may be affected by uncertainty of parameter values. The approach described above may not be able to capture non-linear effects like these, thus calling for a different quantification of simulation input modelling risk.

Chapter 6: Bayesian input modelling desiderata

6.1: Introduction

Bayesian updating for simulation input modelling has several desirable properties for simulation input modelling as the previous chapters showed. However, various other factors determine in how far these properties come to bear or even raise new challenges. This chapter further analyzes the method under a broad set of circumstances and conditions and discusses counterarguments against the method, common challenges in simulation input modelling and how Bayesian updating copes with these. Where previous chapters introduced and illustrated the method, Chapter 6 provides a sensitivity analysis of the behavior or performance of Bayesian updating for simulation input modelling under challenging and general though still practically relevant circumstances. Through literature analysis and empirical modelling, we discuss the circumstances in which the method is fulfilling the input modelling desiderata along which the argument is structured.

This chapter is structured in three parts. The first section establishes a set of criteria, input modelling desiderata, that can be used to assess the properties of input modelling and reflect the various trade-offs within the assessment of input modelling methods. The second section derives the challenges to Bayesian simulation input modelling if, in addition to data, one expert provides input. Based on the challenges derived, individual methods like modelling and literature analysis are used to analyze the properties of simulation input models based on Bayesian updating. In this second section, we also analyze the behavior of input parameters with different assumptions on the prior and data used for the Bayesian updating. The third section analyzes challenges arising if multiple experts provide input.

We conclude that simulation input modelling based on Bayesian updating fulfills several input modelling desiderata and thus presents a useful addition to a simulation modeler's toolkit.

However, there are challenges that can adversely affect this method and could result in inaccurate parameter estimates where the necessary assumptions of this method are not met. Therefore, the method must be applied prudently and thoroughly.

6.2: Analysis and Results

Formally we structure the analysis along a derivation of input modelling desiderata followed by an analysis of challenges with at first one expert and then multiple experts.

6.2.1 Simulation input modelling desiderata

We define what properties the ideal input modelling method shall have. The literature on input modelling methods provides clear guidance here, although there is some discrepancy as to the priorities put forth to evaluate input modelling methods. In the preceding chapter Henderson's (2003) four criteria were already briefly mentioned: transparency, validity, implementability as well as efficiency. Johnson & Mollaghasemi (1994) discuss desiderata for simulation input modelling methods that go beyond the objective of accurately capturing a physical phenomenon: physical plausibility, flexibility, generality, legal precedence, ease and efficiency of parameter estimation as well as ease and speed of variate generation. The latter two overlap with Henderson's efficiency desideratum. Table 29 shows their overlap:

	Source	
Desideratum	**Henderson, 2003**	**Johnson & Mollaghasemi, 1994**
1. Transparency	Yes	No
2. Validity	Yes	No
3. Implementability	Yes	No
4. Physical plausibility	No	Yes
5. Ease and efficiency of parameter estimation	Yes	Yes
6. Ease and speed of variate generation		Yes
7. Flexibility	No	Yes
8. Generality	No	Yes
9. Legal precedence	No	Yes

Table 29 - Input modelling desiderata per Henderson (2003) and Johnson & Mollaghasemi (1994)

6.2.1 a) Transparency

Transparency here refers to the objective that a method "shall be understood by its users" (Henderson 2003). Despite the emphasis on making the method accessible, Bayesian updating builds on statistical methods that may not be accessible to all simulation modelers. It may yet still be possible to apply the method even without full grasp of the underlying statistics, though this raising risks of methodological errors in the implementation. This does, however, represent a restriction and potentially a barrier to the diffusion of this input modelling method.

6.2.1 b) Validity

Bayesian updating and its application to simulation input modelling as described in the context of chapter 4 rests on solid technical foundations ensuring its general validity. It remains however necessary to ensure that none of the assumptions underlying the method are violated if it is to be applied.

6.2.1 c) Implementability

This refers to the desideratum that a method shall be implementable for a variety of challenges without the need the for "expert intervention" (Henderson, 2003). As this chapter shows, Bayesian updating can be implemented in a variety of settings despite existing challenges within these settings. However, Bayesian updating entails information gathering requirements that may go beyond what simpler input modelling methods require, especially regarding the quantification of uncertainty within the elicitation of expert opinion and beyond.

6.2.1 d) Physical plausibility

Physical plausibility is achieved if a model parameterization or its method accurately captures the physical properties of the modelled process (Johnson et al., 1994). Through the assumption of conjugate priors, the updating process does not alter a variate's distribution and thereby its physical plausibility. It remains, however, important that the prior already takes a shape that is physically plausible. More generally, without the assumption of conjugate priors, the posterior can take on many distributional shapes and is thus flexible to achieve physical plausibility conditional on all available data and the prior. As Bayesian updating is flexible in its choice of distribution (Lynch, 2007) there is no general limitation to physical plausibility of the method.

6.2.1 e) Ease and efficiency of parameter estimation

This desideratum refers to the ease and efficiency with which parameters can be precisely estimated. Parameter estimation is analogous to data-driven or expert based methods that rely on a single data source and thus Bayesian input modelling does not affect this input modelling desideratum beyond the requirements of its simpler constituent input parts. The only restriction is the aforementioned information requirement during the updating process.

6.2.1 f) Ease and speed of variate generation

Some “esoteric” (Johnson et al. 2004) distributions do not generate variates efficiently constituting a computational downside, however of decreased importance with faster computers. Yet, as Bayesian updating does not tamper with the distribution but rather it’s parameterization, it does not affect the ability to generate variates from an updated distribution.

6.2.1 g) Flexibility

Here flexibility refers to a modelling method having exceptional cases or additional variations. Again, as Bayesian updating is possible for a variety of distributions (Lynch, 2007) the method is flexible with respect to various distributional shapes. However, the necessary assumptions for the closed form solutions are curtailing flexibility by limiting its scope. For example, no conjugate prior exists for the Weibull distribution if both the scale and shape parameter are unknown. Yet through the application of Markov-Chain-Monte-Carlo methods, this limitation can be overcome.

6.2.1 h) Generality

Generality refers to the ease with which a parameterization can be transformed from univariate to multivariate. Extending parameter estimates to multivariate settings was not treated in chapter 4. Generally, multivariate normal posteriors exist (Marin & Robert, 2007) and can be updated through new data. However, for more complex multivariate distributions the full proofs are not always available, necessitating a derivation of and solution to the likelihood function to derive the posterior and its parameters.

6.2.1 i) Legal precedence

Legal precedence refers to previous use by reputable sources. Following these may give credence to the choice of a distribution beyond its practical merits and the above desiderata. While precedence should not drive the choice of a distribution, Johnson et al. point toward the importance of being able to communicate and defend the method or distribution. As the literature review in the previous chapter showed, there is some precedence for related methods that show how Bayesian updating can be applied for data aggregation and in simulation environments.

In conclusion, the short discussion along these nine simulation input modelling desiderata reveals a favorable view on Bayesian input modelling. However, Bayesian input modelling is limited in terms of its transparency, implementability, generality as well as legal precedence.

6.2.1 j) Alternative desiderata for simulation input modelling

As a further robustness check we analyze alternative sets of desiderata. Schmeiser argues for three criteria to evaluate input distributions (Fox et al. 1990): generality, ease of generating variates and ease of parameter estimation. In the same source, Wilson argues for a set of six properties of input distributions: flexibility (of shape), generalizability in one dimension, extendibility to higher dimensions / multivariate distributions, tractability, good parameterization and ease of variate distributions. These criteria appear generally in line with the above discussed, however are designed to evaluate distributions directly rather than input modelling methods. For completeness, we provide a short perspective of the two properties not included above.

Flexibility of shape.

Input models that are flexible in shape accommodate markedly different distributional shapes in a stable distributional family (Fox et al. 1990). Again, Bayesian updating for simulation input modelling is flexible with regard to the use of distributions and does not alter a distribution's shape beyond the updating process thereby allowing flexibility of shape within one distributional family as well as beyond.

Good parameterization.

Here, Wilson refers to interpretable parameters that govern separable properties of a distribution (Fox et al. 1990). This argument holds here again, as the method as used here does not affect a distribution's shape and thus does not affect the property of good parameterization.

6.2.2 Challenges with a single input providing experts

In the following we discuss challenges using Bayesian input modelling with one expert including cognitive biases, dishonest experts, organizational bias, elicitation of non-parametric distributions and resulting input parameter surfaces.

6.2.2 a) Cognitive Bias

Cognitive bias can represent a challenge in simulation input modelling when working with expert judgment (Cooke, 1991; Meyer, Grisar & Kuhnert, 2011; Vose, 2008) despite existing de-biasing methods. We seek to understand in how far such de-biasing strategies are compatible with and should be used complimentarily with Bayesian updating. There is a solid theoretical foundation for the prevalence of cognitive biases in general and their impact on simulation input modelling. Experts can be subject to a range of well-documented biases (Kahneman & Tversky, 1972). These biases include anchoring (Tversky and Kahnemann, 1974), overconfidence (Brenner, Koehler & Libermann, 1996), availability and representativeness heuristics (Vose, 2008) and others reviewed in Tversky (1982). Kahnemann and Tversky suggest that biases arise because people deviate from a calculating approach to decision making and decide based on simple cognitive heuristics. Besides these generally well-established cognitive biases, simulation input modelers are faced with a related though overlapping challenge that arises when people deal with probabilities, distributions and related concepts. Spiegelhalter, Pearson & Short (2011) discuss biases in probability judgment and confidence intervals that seem to amplify existing cognitive biases. Low level of numeracy and statistical literacy have been shown among experts that translate into inaccuracies when quantifying stochastic variables (e.g. Fagerlin, Ubel, Smith & Zikmund-Fisher, 2007; Spiegelhalter et al., 2011). Expertise in one subject domain does not preclude risks of inaccurate translation of expertise into probabilities and distributions.

When discussing cognitive biases in the context of probability or distributional judgment, there is a helpful dichotomy between normative and substantive goodness (Winkler & Murphy, 1968). The former describes how well an expert's estimate fits with probability theory and is conceptually distinct from the latter that concerns the expert's subject matter expertise. As introduced in the preceding chapter, normative goodness refers to how well an expert can translate their subjective probability believes into numerical quantities whereas substantive goodness refers to the expertise itself. The following discussion emphasizes biases affecting normative goodness due to its structural effect on Bayesian estimation via its effect on self-assessed statistical confidence, further discussion of de-biasing methods to improve substantive goodness can be found in Soll, Milkman & Payne (2014).

Biases can be counteracted through ex-ante and ex-post methods. Ex-ante methods attempt to counteract the bias of the subject by e.g. making it aware of its bias and let it correct such behavior or questioning strategies as judgment is elicited. Ex-post methods on the other hand try to eliminate bias from judgment after they provided estimates. The larger part of the literature on counteracting bias "centers on improving the elicitation process and countering bias a priori" (Meyer, Grisar & Kuhnert, 2011; McClelland & Bolger, 1994), in other words ex-ante. Pivotal elements of ex-ante de-biasing methods are training and calibration through repeated feedback on experts' assessments (Jolls & Sunstein, 2006). Welsh, Begg, Bratvold & Lee (2004) provide an ex-ante de-biasing strategy for the oil and gas industry where capital commitments depend on expert judgments of probabilities relating to efficiency of oil wells. They suggest a strategy that recognizes experts' tendency to think in terms of heuristics rather than probabilities. This reasoning has also been proposed by Gigerenzer (1991) to provide a more "ecologically consistent" environment closer to the expert's experience than questioning techniques that may be beyond the expertise of the experts interviewed. Remarkably similar methods have been applied by the experts interviewed for chapter 3, notably without their explicit knowledge that these are well-established methods indeed. Ex-ante de-biasing also entails the method applied for expert elicitation in the case study of chapter 4 based on Winman, Hansson & Juslin (2004, see chapter 4 or appendix). This method has been confirmed by Teigen & Jørgensen (2005). Yet it must be stressed that ex ante de-biasing strategies have not proven to fully eliminate bias (Morgan, Henrion & Smal, 1992). More generally, sophisticated expert judgments and simulation models are usually not needed for situations where well-calibrated experts are plentiful but rather in situations that go beyond the "day-to-day experience"

(Clemen & Lichtendahl, 2002) as calibration is task and context dependent (Klayman, Soll, González-Vallejo & Barlas, 1999). Hence, the effectiveness of ex-ante de-biasing is limited as many methods are not applicable due to lacking expert calibration or may risk residual bias. Ex-post de-biasing methods have been applied in simulation modelling to reduce the impact of cognitive bias by Meyer et al. (2011). They show how biases can lead to inaccurate expert judgments and that such biases are non-additive and simple de-biasing strategies are often insufficient to fully eliminate bias leading to potential underestimation of risk in aggregate. We conclude that despite the merits of ex-ante and ex-post de-biasing methods, cognitive biases remain a challenge that merits further discussion.

Overconfidence has been considered the most important cognitive bias when eliciting information from experts (Cooke, 1991; Plous, 1993; Meyer et al., 2011) and a challenge to expert opinions (Vose, 2008). Overconfidence bias takes different shapes, the one we are concerned with here is *overprecision*[15], its most predictable (Moore & Healy, 2008) and persistent (Capen, 1974) manifestation that affects primarily normative goodness or calibration. Statistical calibration is a quality of experts not closely correlated with their level of expertise that has been shown to be worse for questions considered difficult (Cooke, 1991). Simulation studies tend to be conducted in non-standard situations (Barton et al., 2002; Clemen et al., 2002) thereby adding to the difficulty of achieving calibration via training.

Overprecision or expert calibration thus has a structural effect on Bayesian updating through the expert's quantification of their estimate's uncertainty. Experts providing confidence intervals for a variate consistently estimate these intervals too narrowly implying "excess cer-

[15] The other two are overestimation (deeming one's skills better than they factually are) and overplacement (deeming one's performance better than it is relative to others) and do not directly affect the arithmetic mechanics of Bayesian updating

tainty” (Winman et al., 2004; Alpert & Raiffa, 1982). By construction, the width of a confidence interval is inversely proportional to its variance. Due to variance-weighting in the aggregation process an input source is assigned weight per its perceived certainty and thus overconfidence bias causes artificially high decision weights in the Bayesian posterior. To counteract this effect, we briefly review three methods:

- With ample access to experts, one shall obtain individual calibration scores (Cooke, 1991) that are preferable to generic adjustments as overconfidence bias varies (Klayman et al., 1999).
- With less access, one shall de-bias according to the literature, e.g. per Winman et al. (2004) as discussed in previous chapters and the appendix.
- If only confidence intervals are provided, self-assessed variances shall be adjusted upwards to de-bias overprecision. Teigen et al. (2005) find that 90% confidence intervals typically have a 50% or even lower chance of containing the “correct” value, approximately consistent with other studies (e.g. Alpert et al. 1982). Note, that the implied upward adjustment of the variance is of a factor of 2.44 based on the ratio of z-scores of the 50% and 90% confidence intervals and thus implies a sizable underestimation of uncertainty. We provide a table of adjustment scores for over- and underconfidence:

For a Gaussian, a confidence interval is calculated as *standard deviation*z-score*$_{\alpha}$, for 50% and 90% intervals, the z-scores are 0.674 and 1.645 respectively. Transforming 90% confidence intervals into 50% CIs is achieved by maintaining its width and adjusting the z-score and scaling standard deviation by the inverse amount, which is equal to 1.645/0.674=2.44.

...actual confidence	Stated confidence level vs. … 90%	80%	70%	60%	50%	40%	30%	20%	10%
90%	1,00	0,78	0,63	0,51	0,41	0,32	0,23	0,15	0,08
80%	1,28	1,00	0,81	0,66	0,53	0,41	0,30	0,20	0,10
70%	1,59	1,24	1,00	0,81	0,65	0,51	0,37	0,24	0,12
60%	1,95	1,52	1,23	1,00	0,80	0,62	0,46	0,30	0,15
50%	2,44	1,90	1,54	1,25	1,00	0,78	0,57	0,38	0,19
40%	3,14	2,44	1,98	1,60	1,29	1,00	0,73	0,48	0,24
30%	4,27	3,33	2,69	2,18	1,75	1,36	1,00	0,66	0,33
20%	6,49	5,06	4,09	3,32	2,66	2,07	1,52	1,00	0,50
10%	13,09	10,20	8,25	6,70	5,37	4,17	3,07	2,02	1,00

Table 30 – Implied over- and underconfidence scores based on stated and actual confidence levels of estimated confidence intervals following Teigen et al. (2005)

In summary, overconfidence bias has a structural effect on Bayesian updating that must be counteracted as described.

6.2.2 b) Dishonest experts

Dishonesty or agency conflicts among agents can obliterate the variance weighted averaging on purpose by the experts. A related question is how uninformative experts affect the method. Dishonest or uninformative experts that provide new information invalidate Bayesian updating as it violates the assumption that new information contains positive information value. This is detrimental in two dimensions. Firstly, it creates an expected parameter value that is structurally biased toward the falsely provided expert estimate. Secondly, by reducing the variance of the biased posterior it creates "excess certainty". This argument can also be based on theory underlying Kalman filters. Throughout this chapter, we treat expert inputs analogously to observations in Bayesian (Kalman) filtering. In Kalman filtering it is assumed that new observations stem from the same process as previous observations**.** By analogy, a dishonest expert does not constitute an observation from the same data generating process and thus does not yield information about the true parameters of the process.

6.2.2 c) Elicitation of non-parametric distributions

In Chapter 2 we discussed the preference for eliciting expert opinion on non-parametric distributions as recommended by, e.g. Vose (2008). Bayesian updating is not generally applicable for non-parametric distributions as the lack of parameters that define the distribution prevents the derivation of posterior hyperparameters via conjugate priors. However, in individual cases it is possible to construct PDFs and likelihood functions for, e.g. truncated distributions that can approximate non-parametric distributions.

6.2.2 d) Input parameters with varying assumptions on prior and new information

Figure 22 shows a surface plot of a posterior estimate of a mean parameter of a normal distribution with known variance based on a fixed prior and a new information. The new information is varied in its self-assessed variance or uncertainty and the deviation between prior estimate and new information. This surface plot is based on the first independent variable of the case application in chapter 4, namely the price of waste for incineration. This visualizes the posterior by varying both the mean as well as the self-assessed variance of new information. With increasing uncertainty or variance, the weight of the new information in the posterior decreases and the posterior approaches the prior. With high assigned variance, the weight of the new information in the posterior approaches zero. On the other hand, for artificially low variance the posterior approaches the mean provided as new information. With increasing deviation between prior and new information, the delta between the prior and the posterior increases linearly.

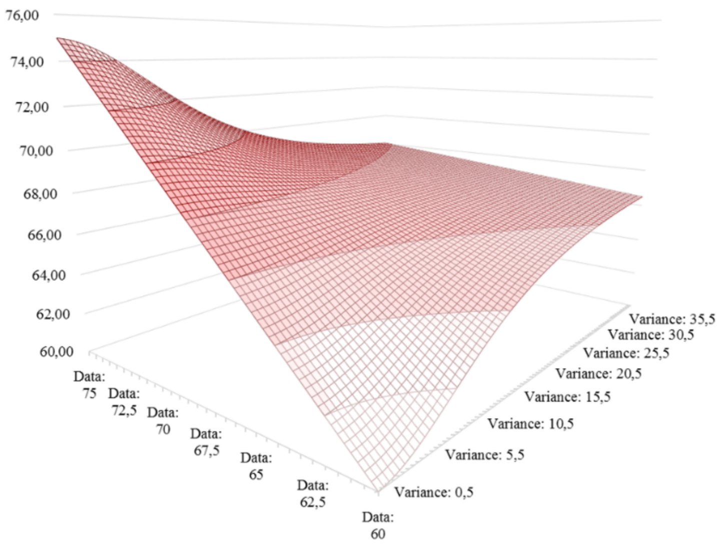


Figure 21 – Surface plot of posterior estimate of the man of a normal distribution with known variance and varying properties of the new information

6.2.3 Analysis with multiple input providing experts

Here we assume two or more experts who provide input and discuss the effects. In a recursive update the posterior as derived above takes the place of the prior and an analogous calculation for a secondary posterior follows with no general limit to the number of recursive updates. The properties of recursive estimators are symmetric standing in contrast to methods used by practitioners (interview transcripts, 2016; section on expert bias) as shown in the preceding chapter. Here methods were advocated were a final estimate based on expert opinions was deduced through arithmetic averaging without explicit weighting of the associated uncertainty risking mis-calibrated weights of data points. We shortly discuss challenges based on chapters 2 and 3, including: Uneven calibration, Correlation between experts' opinions and Expert heterogeneity.

6.2.3 a) Recursive updating

The following application briefly captures the simplicity and power of recursive Bayesian updating in the context of aleatoric and epistemic uncertainty. Recursive updating follows analogous arithmetic as one-staged updates, notably with the constantly decreasing uncertainty around parameters estimates as highlighted in figure 22 that is based on hypothetical numbers for illustrative purposes. Suppose here a prior for the normally distributed mean parameter of 110.0 with variance of 10.0 and new information in the form of expert opinion of 100.0 with equal variance. For simplicity, assume further additional experts providing the same input recursively. The posteriors of higher order converge gradually to a value of close to the mean of the new information with strictly decreased variance. These parameters represent input modelling specifications to be used as input parameters in simulation models.

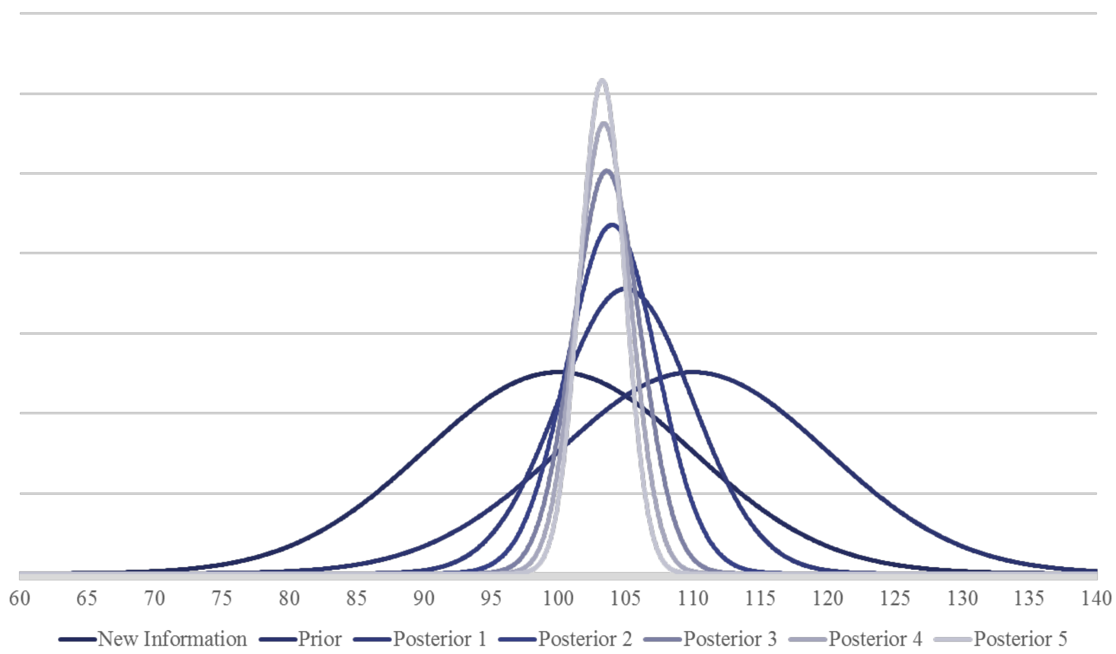


Figure 22 - Probability density functions of parameter estimates of Prior, new information and multiple posteriors

Table 31 shows numerical values for each plotted input modelling specification:

Parameter	New Information	Prior	posterior 1	posterior 2	posterior 3	posterior 4	posterior 5
Mean	100,00	110,00	105,00	104,00	103,60	103,39	103,26
Aleatoric variance	10,00	10,00	5,00	3,33	2,50	2,00	1,67

Table 31 – Numerical values of parameter estimates per Bayesian input modelling specification

Figure 22 above represents the uncertainty around the mean parameter, interpreted as the aleatoric uncertainty that decreases through information aggregation thereby improving precision. In the context of aleatoric and epistemic uncertainty we can extend the above example and introduce epistemic uncertainty as well, that we assume to be normally distributed. Figure 23 then illustrates the effect on the compound uncertainty, where we observe a compound-normal-normal distribution, which is itself again a normal distribution, though of increased variance as shown in the preceding chapter. This compound distribution is effectively to be used in a simulation input modelling context as argued before. Note that aleatoric variance is fixed across Bayesian updates and therefore does not changes as more information is aggregated. The compound variance follows the formula introduced in the previous chapter.

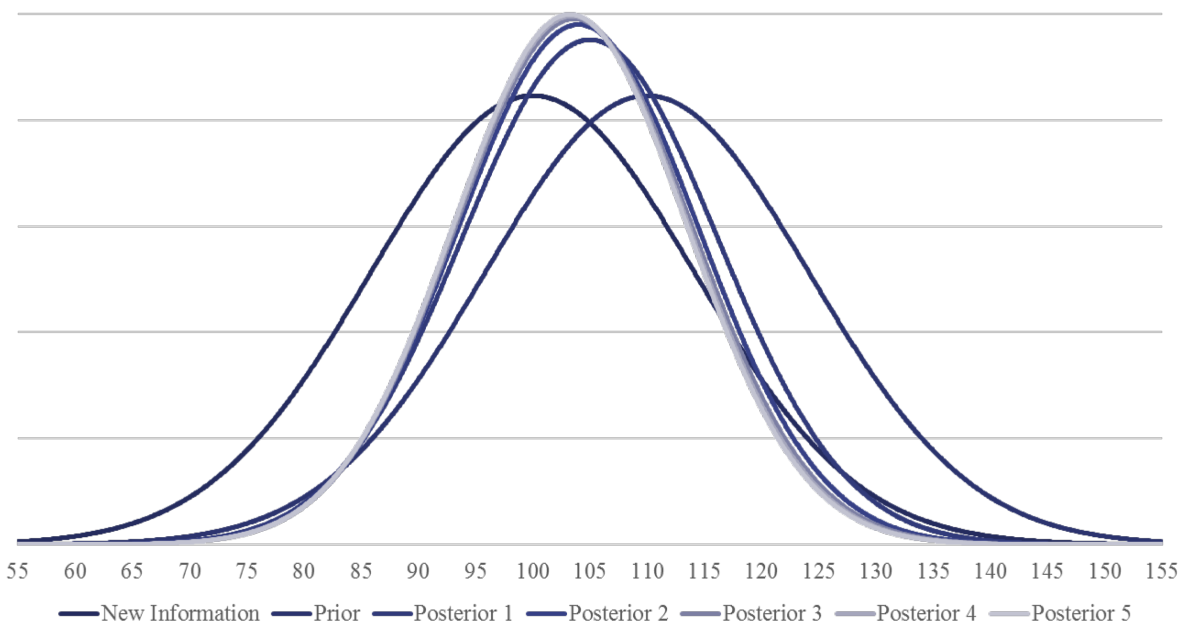


Figure 23 – Probability density functions of compound distribution of estimates of prior, new information and multiple posteriors of aleatoric and epistemic uncertainty

Several properties of figure 23 are noteworthy. As in the figure above, successive updates lead to a shift of the distribution towards the mean value of new information provided. Higher order posterior distributions also show decreasing variance, however not to the same extent as above as the aleatoric or physical uncertainty remains unchanged through Bayesian updates. Numerical values in table 32 further illuminate this point:

Parameter	New Information	Prior	Posterior 1	Posterior 2	Posterior 3	Posterior 4	Posterior 5
Mean	100,00	110,00	105,00	104,00	103,60	103,39	103,26
Epistemic variance	10,00	10,00	5,00	3,33	2,50	2,00	1,67
Physical variance	10,00	10,00	10,00	10,00	10,00	10,00	10,00
Compound variance	14,14	14,14	11,18	10,54	10,31	10,20	10,14

Table 32 – Numerical values of parameter estimated per Bayesian input modelling specification and full distribution parameters

Through the uncertainty-reducing recursive Bayesian updating, a swiftly decreasing weight of the epistemic uncertainty is observable in the compound variance that approaches the physical variance through the recursive updates – as is desirable for many modelling contexts. In short, Bayesian updating and especially recursive updating, can present a viable method for input modelling in a context with both aleatoric and epistemic uncertainty.

6.2.3 b) Uneven calibration or overprecision bias

Posterior parameters estimates are affected by experts' calibration. This effect can be exacerbated in settings with unevenly calibrated experts which is equivalent to uneven levels of overprecision bias per Cooke (1991). This leads to Bayesian posteriors biased towards the most overconfident experts and hence biased simulation outputs. In situations with unevenly calibrated experts, the objective shall be to assign each expert the appropriate level of credibility and thus the appropriate level of weight by de-biasing individual experts. If calibration levels are known and uneven, the approach is straightforward. If calibration levels of multiple experts are not known it is necessary to follow the above-mentioned de-biasing steps.

6.2.3 c) Correlation between experts' opinions and errors

Shared information sources used by experts can lead to shared errors and represents a challenge. Here we are discussing more specifically correlated biases and thus estimation errors among experts. Cooke (1991) raises the issue of correlation among expert opinion, both "across" and "within" experts considering them "unavoidable but usually benign" in practice without elaborating why it should be considered benign. We consider the information theoretical implications of correlated expert judgments. Shared sources among experts lead to correlated expert opinions with implications for their informational value. Aggregation of experts is only beneficial if new information is contained in the experts' opinions. If recursive updates of a parameter estimate are conducted with expert opinions built on identical data, this would give excess credibility to these observations and skew the posterior. Each observation is considered a new stochastic realization of the unobservable DGP (Grewal & Andrews, 2001) that carries new information. By extension each expert is assumed to carry new information, where this assumption does not hold it is not valid to aggregate additional expert opinions at full weight. Winkler (1968) argues that if experts all come from a common subpopulation their weights should be chosen so that the total number of "observations" assigned to them should be equal to one. If experts are "independent" their individual weights should be equal to one and its sum equal to the number of experts. More generally, the sum of all expert's weights must be larger or equal to 1 and smaller or equal to the number of experts involved depending on the level of independence of their information sources. Yet cases in between these extremes are less straightforward to address and leave the optimal assignment of decision weight at the discretion of the simulation input modeler. In summary, correlation among estimation error of expert opinion and expert judgment generally presents challenges to Bayesian updating for simulation input modelling that can be most effectively addressed through weighting of inputs.

6.2.3 d) Expert heterogeneity

Expert interviewees affirmed that differing experts' views exacerbate challenges in input modelling while others noted that a panel of experts is strictly preferable to a single expert opinion. Thus, the question arises if particularly heterogenous opinions present any challenges to the method. Again, an analogy to Kalman filtering is insightful. We view expert opinions as observations of a stochastic process. Hence, considerations of auto-correlated errors notwithstanding, heterogeneous expert opinions are akin to heterogeneous observations in a Kalman filtering process. One of the pivotal reasons for the success of Kalman filters is their ability to extract information from noisy or heterogeneous data (Grewal et al., 2001). Thus, in principle Bayesian updating can extract data from heterogeneous experts.

6.3: Discussion and conclusion

This chapter analyzed the properties of Bayesian updating for simulation input modelling along general input modelling desiderata as well as for challenges if working with one input providing expert as well as multiple.

Despite not fulfilling some of the desiderata, like transparency, implementability, generality or legal precedence, Bayesian input modelling meets most of the desiderata identified in the literature and thus represents a viable addition to the simulation modeler's tool set.

Amongst the challenges discussed, most notably the overprecision bias has a structural effect on Bayesian updating requiring simulation modelers to take measures to counteract this bias. This effect translates as well to multiple experts with uneven calibration which necessitates de-biasing. Further, correlated expert judgment must be counteracted via discretionary decision weighting through assignment of observation numbers reflecting the information content and independence of expert opinion.

7 General conclusion

Simulation has thoroughly affected finance and accounting research whilst the method is honed and advanced simultaneously. As a research method as well as a method applied by practitioners it has had profound impact on both the science and theory of finance and accounting as well as its practice. Simulation input modelling in corporate finance and accounting affects the method of simulation along various levels from the technical implementation of simulation models to the acceptance and thereby diffusion of the method in academia and practice. Likewise, this dissertation contributes to the understanding of simulation modeling, and input modelling in particular, along several dimensions, from positive bibliometric research and literature reviews to theoretical and methodological contributions in input modeling and the quantification and communication of input parameter stochasticity. This dissertation makes several contributions. It analyzed the current use and diffusion of simulation methods in finance and accounting research, highlighting how simulation crossed the 'chasm' into the methodological mainstream in various finance research clusters though diffused much less in accounting research, albeit with exceptions such as costing that apply simulation methods. There appears to be promising simulation-based research in various niches of accounting research as well. These findings contribute to several streams of literature, notably to critical reflections on finance and accounting research, the type of simulation-based research conducted in these disciplines as well as the diffusion of scientific methods more generally. A further finding of the bibliometric study points to the relative lack of research dialogue on simulation input modelling specifically for corporate finance and accounting as opposed to the substantial research efforts directed towards input modelling for capital market and especially derivative and asset pricing.

The resulting research on the state-of-the-art input modelling methods for corporate finance and accounting provides a structured overview of the field and contrasts this with perspectives of leading simulation modelers in the field. A 'consensus' view of input modelling sources is derived that can help guide simulation researchers in their modelling choices. This consensus is challenged through semi-structured expert interviews where a more nuanced and occasionally divergent preference for input modelling sources prevails. Notably, there appears to be a lack of formal discussions of aggregation methods that combine different and potentially diverging input sources into one coherent input modelling distribution. Further, the interviews shed light on several specialized modelling topics contrasting with the purported consensus and thereby contributing to the literature on simulation input in corporate finance and accounting.

Within the field of aggregation methods focused on corporate finance and accounting we discuss and illustrate a method of aggregating historical data and expert opinion, two potentially divergent sources that tend to be available in common modelling environments, based on

Bayesian updating to fully utilize all available information in a robust aggregation scheme. The method is illustrated through a case study applying the method to a real-life modelling environment. This represents a contribution to simulation input modelling for CF&A that strives for robust yet implementable method that address several common input modelling challenges such as dynamic data generating processes. However, due to the nature of this benchmarking, it is not straightforward to prove the superiority of Bayesian updating in this context, this is however achieved in the next chapter that emphasizes and illustrates the uncertainty reducing properties of Bayesian updating.

Finally, the modelling metric *Simulation output at risk* is developed and applied to the above-mentioned case study. Within the context of Bayesian input modelling in an environment of both aleatoric and epistemic uncertainty the metric is demonstrated. It seeks to facilitate communication of simulation output variability due to input parameter stochasticity by capturing its impact in a single figure with a parallel interpretation to commonly used risk metrics. This contributes to the literature on quantification of input uncertainty as well as the literature around standards for the communication of simulation experiments. Moreover, this application serves to highlight how Bayesian input modelling can in fact reduce simulation model output variability by lowering the uncertainty pertaining to its input models thereby unequivocally establishing a key desirable property of Bayesian input modelling. Finally, this dissertation critically reviews its main methodological contributions in terms of Bayesian input modelling via a series of challenges to the method that highlights its strength and notably its limitations.

By advancing simulation input modelling for corporate finance and accounting we contribute to the robust foundation on which the method is built and further underscore the vital role input modelling should play in simulation modelling generally – perhaps a subfield that can profit from further theoretical contributions in a similar vein.

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9 Appendix

Chapter 1

Most central nodes

Table 33 provides full references for each cluster's most central node ordered by cluster size.

Period	Cluster label	Short reference / node	Reference
Period I	Early exercise option valuation	Longstaff 2001	Longstaff, F. A., & Schwartz, E. S. (2001). Valuing American options by simulation: a simple least-squares approach. The review of financial studies, 14(1), 113-147.
Period I	Value-at-risk	Hull 1998	Hull, J., & White, A. (1998). Incorporating volatility updating into the historical simulation method for value-at-risk. Journal of risk, 1(1), 5-19.
Period I	Optimal consumption portfolio	Cox 1989	Cox, J. C., & Huang, C. F. (1989). Optimal consumption and portfolio policies when asset prices follow a diffusion process. Journal of economic theory, 49(1), 33-83.
Period I	Statistics and sampling methods for stock markets	Glasserman 1999 I	Glasserman, P., Heidelberger, P., & Shahabuddin, P. (1999). Asymptotically optimal importance sampling and stratification for pricing path-dependent options. Mathematical finance, 9(2), 117-152.
Period I	Option pricing	Geske 1984	Geske, R., & Johnson, H. E. (1984). The American put option valued analytically. The Journal of Finance, 39(5), 1511-1524.
Period I	Stochastic volatility I	Scott 1987	Scott, L. O. (1987). Option pricing when the variance changes randomly: Theory, estimation, and an application. Journal of Financial and Quantitative analysis, 22(4), 419-438.
Period I	Stochastic volatility II	Jacquier 1994	Jacquier, E., Polson, N. G., & Rossi, P. E. (2002). Bayesian analysis of stochastic volatility models. Journal of Business & Economic Statistics, 20(1), 69-87.
Period I	Statistical processes and distributions	Eberlein 1995	Eberlein, E., & Keller, U. (1995). Hyperbolic distributions in finance. Bernoulli, 1(3), 281-299.
Period I	Bayes factor and Monte Carlo	Chib 1996	Chib, S., & Greenberg, E. (1996). Markov chain Monte Carlo simulation methods in econometrics. Econometric theory, 12(3), 409-431.
Period I	Malliavin Calculus	Fournie 1999	Fournié, E., Lasry, J. M., Lebuchoux, J., Lions, P. L., & Touzi, N. (1999). Applications of Malliavin calculus to Monte Carlo methods in finance. Finance and Stochastics, 3(4), 391-412.
Period I	Estimation methods for inference and cont. time processes	Gallant 1996	Gallant, A. R., & Tauchen, G. (1996). Which moments to match?. Econometric Theory, 12(4), 657-681.
Period I	Bond and exotic options	Turnbull 1991	Turnbull, S. M., & Wakeman, L. M. (1991). A quick algorithm for pricing European average options. Journal of financial and quantitative analysis, 26(3), 377-389.
Period I	Long memory time series	Granger 1980 I	Granger, C. W., & Joyeux, R. (1980). An introduction to long-memory time series models and fractional differencing. Journal of time series analysis, 1(1), 15-29.
Period I	Accounting & auditing topics	Cogger 1981	Cogger, K. O. (1981). A time-series analytic approach to aggregation issues in accounting data. Journal of Accounting Research, 285-298.
Period II	Volatility and risk I	Bos 1984	Bos, T., & Newbold, P. (1984). An empirical investigation of the possibility of stochastic systematic risk in the market model. Journal of Business, 35-41.
Period II	Simulation methods for option pricing	Haugh 2004	Haugh, M. B., & Kogan, L. (2004). Pricing American options: a duality approach. Operations Research, 52(2), 258-270.
Period II	Stochastic processes	Broadie 2006	Broadie, M., & Kaya, Ö. (2006). Exact simulation of stochastic volatility and other affine jump diffusion processes. Operations research, 54(2), 217-231.
Period II	Affine term structure models	Duffee 2002	Duffee, G. R. (2002). Term premia and interest rate forecasts in affine models. The Journal of Finance, 57(1), 405-443.
Period II	Early exercise option valuation	Broadie 1997 I	Broadie, M., & Glasserman, P. (1997). Pricing American-style securities using simulation. Journal of economic dynamics and control, 21(8-9), 1323-1352.
Period II	GARCH volatility	Bollerslev 1986	Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. Journal of Econometrics, 31, 307-327
Period II	Value-at-Risk	Jorion 2000	Jorion, P., (2000). Value-at-Risk, McGraw-Hill: New York
Period II	Monte Carlo methods and valuation	Sobol 1967	Sobol', I. Y. M. (1967). On the distribution of points in a cube and the approximate evaluation of integrals. Zhurnal Vychislitel'noi Matematiki i Matematicheskoi Fiziki, 7(4), 784-802.
Period II	Credit derivatives	Andersen 2003 II	Andersen, L., Sidenius, J., & Basu, S. (2003). All your hedges in one basket. RISK-LONDON-RISK MAGAZINE LIMITED-, 16(11), 67-72.
Period II	Market efficiency and stock market behavior	Ng 2001	Ng, S., & Perron, P. (2001). Lag length selection and the construction of unit root tests with good size and power. Econometrica, 69(6), 1519-1554.
Period II	Volatility	Bekaert 2000 II	Bekaert, G., & Wu, G. (2000). Asymmetric volatility and risk in equity markets. The review of financial studies, 13(1), 1-42.

Period II	Macro Asset pricing	Hansen 1982 II	Hansen, L. P., & Singleton, K. J. (1982). Generalized instrumental variables estimation of nonlinear rational expectations models. Econometrica: Journal of the Econometric Society, 1269-1286.
Period II	Interest rate models	Brigo 2001	Brigo, D., & Mercurio, F. (2007). Interest rate models-theory and practice: with smile, inflation and credit. Springer Science & Business Media.
Period II	Stochastic volatility	Eraker 2004	Eraker, B. (2004). Do stock prices and volatility jump? Reconciling evidence from spot and option prices. The Journal of Finance, 59(3), 1367-1403.
Period II	Bayes factor and Monte Carlo	Gilks 1996	Gilks, W. R., Richardson, S., & Spiegelhalter, D. (1995). Markov chain Monte Carlo in practice. Chapman and Hall/CRC.
Period II	Risk modelling for financial institutions	Embrechts 2003 II	Embrechts, P., Furrer, H., & Kaufmann, R. (2003). Quantifying regulatory capital for operational risk. Derivatives Use, Trading and Regulation, 9(3), 217-233.
Period II	Executive stock options	Ingersoll 2006	Ingersoll, Jr, J. E. (2006). The subjective and objective evaluation of incentive stock options. The Journal of Business, 79(2), 453-487.
Period II	Realized Volatility	Barndorff-Nielsen 2001	Barndorff-Nielsen, O. E., & Shephard, N. (2001). Non-Gaussian Ornstein–Uhlenbeck-based models and some of their uses in financial economics. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 63(2), 167-241.
Period III	Volatility and risk	Higham 2005	Higham, D. J., & Mao, X. (2005). Convergence of Monte Carlo simulations involving the mean-reverting square root process. Journal of Computational Finance, 8(3), 35-61.
Period III	Volatility and option pricing	Eberlein 1998	Eberlein, E., Keller, U., & Prause, K. (1998). New insights into smile, mispricing, and value at risk: The hyperbolic model. The Journal of Business, 71(3), 371-405.
Period III	Early exercise option valuation	Longstaff 2001	Longstaff, F. A., & Schwartz, E. S. (2001). Valuing American options by simulation: a simple least-squares approach. The review of financial studies, 14(1), 113-147.
Period III	GARCH volatility	Bollerslev 1986	Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. Journal of Econometrics, 31, 307-327
Period III	Value-at-Risk	Kupiec 1995	Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. FEDS Paper, (95-24).
Period III	Markov chain state pricing	Peters 2009	Peters, G., Shevchenko, M., Wüthrich, P. (2009). Model uncertainty in claims reserving within tweedie's compound poisson models. Astin Bulletin, 39 (1), 1-33
Period III	Contagion and interdependence	Embrechts 2002	Embrechts, P., McNeil, A., & Straumann, D. (2002). Correlation and dependence in risk management: properties and pitfalls. Risk management: value at risk and beyond, 1, 176-223.
Period III	Term Structure models	Dai 2000	Dai, Q., & Singleton, K. J. (2000). Specification analysis of affine term structure models. The Journal of Finance, 55(5), 1943-1978.
Period III	Implied volatility	Gatheral 2005	Gatheral, J. (2011). The volatility surface: a practitioner's guide (Vol. 357). John Wiley & Sons.
Period III	Monte Carlo methods and valuation	Caflisch 1997	Caflisch, R. E., Morokoff, W. J., & Owen, A. B. (1997). Valuation of mortgage backed securities using Brownian bridges to reduce effective dimension. Department of Mathematics, University of California, Los Angeles.
Period III	Asset returns	Apergis 2004	Apergis, N., & Miller, S. M. (2004). Consumption asymmetry and the stock market: further evidence.
Period III	Derivative models	Haug 2006	Haug, E., (2006) The Complete Guide to Option Pricing Formulas, 2nd ed, New York, NY: McGraw-Hill
Period III	Commodity valuation	Schwartz 2000	Schwartz, E., & Smith, J. E. (2000). Short-term variations and long-term dynamics in commodity prices. Management Science, 46(7), 893-911.
Period III	Systemic banking risk	Allen 2000	Allen, F., & Gale, D. (2000). Financial contagion. Journal of political economy, 108(1), 1-33.
Period III	Simulation in capital investment	Hertz 1964	Hertz, D. B. (1964). Risk analysis in capital investment. Harvard Business Review, 42, 95-106.
Period III	Macro Finance	Clarida 2000	Clarida, R., Gali, J., Gertler, M., (2000). Monetary policy rules and macroeconomic stability: evidence and some theory. Quarterly Journal of Economics, 115, 147-180

Table 33 - Most central node per cluster per period including full reference

Full tables including all references

In the following all clusters are listed in tables following the period of the cluster as well as the order of size.

Period I

Table 34 - Early Exercise option valuation, 27 nodes

Short citation / node	Full reference
Longstaff 2001	Longstaff, F., Schwartz, E., Pricing American Options by Simulation: A Simple Least Square Approach (2001) Rev. Financial Stud., 14, pp. 113-147
Glasserman 2003	Glasserman, P., (2003): Monte Carlo Methods in Financial Engineering, New York: Springer
Barraquand 1995 I	Barraquand, J., Martineau, D.,: Numerical valuation of high dimensional multivariate american securities (1995) J Finan Quant Anal, 30, pp. 383-405
Carriere 1996	Carriere, J., Valuation of Early-Exercise Price of Options Using Simulations and Nonparametric Regression (1996) Insur.: Math. Econ., 19, pp. 19-30
Broadie 1997 I	Broadie, M., Glasserman, P.,: Pricing American-style securities using simulation (1997) J Econ Dyn Control, 21, pp. 1323-1352. , 8-9
Andersen 2004	Andersen, L., Broadie, M.,: A primal-dual simulation algorithm for pricing multi-dimensional American options (2004) Management Science, 50 (9), pp. 1222-1234
Rogers 2002	Rogers, L.,: Monte Carlo valuation of American options (2002) Math. Finance, 12 (3), pp. 271-286
Broadie 2004	Broadie, M., Glasserman, P.,: A stochastic mesh method for pricing high-dimensional American option (2004) J. Comput. Finan, 7, pp. 35-72
Tilley 1993	Tilley, J.,: Valuing American options in a path simulation model (1993) Trans. Soc. Actuaries, 45, pp. 83-104
Haugh 2004	Haugh, M., Kogan, L., Pricing American options: A duality approach (2004) Oper. Res., 52, pp. 258-270
Tsitsiklis 1999 I	Tsitsiklis, J., Van Roy, B.,: Regression Methods for Pricing Complex American Style Options (1999) IEEE Trans. Neural. Net., 12, pp. 694-703. , and
Clement 2002	Clément, E., Lamberton, D., Plotter, P.,: An analysis of a least squares regression method for American option pricing (2002) Finance and Stochastics, 6, pp. 449-471
Duffie 1996 II	Duffie, D., (1996): Dynamic Asset Pricing Theory, (Princeton University Press: Princeton, NJ)
Carr 1998	Carr, P.,: Randomization and the American put (1998) Review of Financial Studies, 11, pp. 597-626
Broadie 1997 II	Broadie, M., Glasserman, P., Jain, G.,: Enhanced monte carlo estimates for American options prices (1997) Journal of Derivatives, 5, pp. 25-44
Raymar 1997	Raymar, S., Zwecher, M.,: A Monte Carlo valuation of American call options on the maximum of several stocks (1997) Journal of Derivatives, 1, pp. 7-23
Harrison 1979	Harrison, J.M., Kreps, D.,: Martingale and arbitrage in multiperiod securities markets (1979) J. Econ. Theory, 20, pp. 381-408
Tsitsiklis 1999 II	Tsitsiklis, J., van Roy, B., Optimal stopping of Markov process: Hilbert space theory, approximation algorithms, and an application to pricing high-dimensional financial derivatives (1999) IEEE Trans. Automat. Control, 44 (10), pp. 1840-1851
Andersen 1999 I	Andersen, L., (1999): A Simple Approach to Pricing Bermudan Swaptions in the Multi-Factor LIBOR Market Model, Geneva Re Financial Products. Working Paper
Jamshidian 1997	Jamshidian, F.,: LIBOR and swap market models and measures (1997) Financ. Stoch., 1, pp. 293-330
Kolodko 2006	Kolodko, A., Schoenmakers, J.,: Iterative construction of the optimal Bermudan stopping time (2006) Finance and Stochastics, 10, pp. 27-49
Bossaerts 1989	Bossaerts, P., (1989): Simulation Estimators of Optimal Early Execise, Working paper, Carnegie-Mellon University
Lamberton 1996	Lamberton, D., Lapeyre, B., (1996): Intoduction to Stochastic Calculus Applied to Finance, Chapman & Hall
Andersen 2000	Andersen, L., Andreasen, J.,: Volatility skews and extensions of the Libor market model (2000) Appl. Math. Financ., 7 (1), pp. 1-32
Ibanez 2004	Ibanez, A., Zapatero, F.,: Monte Carlo Valuation of American Options through Computation of the Optimal Exercise Frontier (2004) J. Finan. Quant. Anal., 39, pp. 253-275. , and
Carr 1992	Carr, P., Jarrow, R., Mynemi, R.,: Alternative characterization of American puts (1992) Mathematical Finance, 2, pp. 87-106
Longstaff 1999	Longstaff, F., Santa-Clara, P., Schwartz, E., (1999) Throwing Away a Billion Dollars: The Cost of Suboptimal Exercise Strategies in the Swaptions Market, working paper, University of California, Los Angeles

Table 35 - Value-at-risk, 19 nodes

Short citation / node	Full reference
Bollerslev 1986	Bollerslev, T.,: Generalized autoregressive conditional heteroscedasticity (1986) Journal of Econometrics, 31, pp. 307-327
Engle 1982	Engle, R.,: Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation (1982) Econometrica, 50, pp. 987-1008
Artzner 1999	Artzner, P., Delbaen, F., Eber, J., Heath, D.,: Coherent measures of risk (1999) Mathematical Finance June
Barone-Adesi 1999	Barone-Adesi, G., Giannopoulos, K., Vosper, L.,: VaR without Correlation for nonlinear Portfolios (1999) Journal of Futures Markets, 19, pp. 583-602
Barone-Adesi 1998	Barone-Adesi, G., Bourgoin, F., Giannopoulos, K.,: Don't look back (1998) Risk, 11., August

Duffie 1997	Duffie, D., Pan, J.,: An overview of value at risk (1997) Journal of Derivatives, 4, pp. 7-49
Hendricks 1994	Hendricks, D.,: Evaluation of value at risk models using historical data (1994), Federal Reserve Bank of New York, New York
McNeil 2000	McNeil, A., Frey, R., Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach (2000) J. Empir. Finance, 7, pp. 271-300
Bollerslev 1992 II	Bollerslev, T., Chou, R., Kroner, K., ARCH modelling in finance: A review of the theory and empirical evidence (1992) Journal of Econometrics, 52, pp. 5-59
Hull 1998	Hull, J., White, A.,: Incorporating volatility updating into the historical simulation method for value-at-risk (1998) J. Risk, 1 (1), pp. 5-19
Nelson 1991	Nelson, D., Conditional heteroscedasticity in asset returns: A new approach (1991) Econometrica, 59, pp. 347-370
Kupiec 1995	Kupiec, P.,: Techniques for verifying the accuracy of risk measurement models (1995) Journal of Derivatives, 3, pp. 73-84
Barone-Adesi 2002	Barone-Adesi, G., Giannopoulos, K., Vosper, L.,: Backtesting derivative portfolios with filtered historical simulation (2002) European Financial Management, 8, pp. 31-58
Dowd 2002	Dowd, K., (2002): Measuring Market Risk, John Wiley & Sons, Chichester
Black 1976 II	Black, F.,: The pricing of commodity contract (1976) Journal of Financial Economics, 3, pp. 167-179
Bollerslev 1987	Bollerslev, T.,: A conditional heteroscedastic time series model for speculative prices and rates of return (1987) Rev Econ Stat, 69, pp. 542-547
Boudoukh 1998	Boudoukh, J., Richardson, M., Whitelaw, R.,: The best of both worlds (1998) Risk, 11 (5), pp. 64-67
Barone-Adesi 2001	Barone-Adesi, G., Giannopoulos, K.,: Non-parametric VaR techniques. Myths and realities (2001) Economic Notes by Banca Monte dei Paschi di Siena SpA, 30, pp. 167-181
Jorion 1996	Jorion, P., Risk: Measuring the Risk in Value at Risk (1996) Financial Analysts Journal, 52, pp. 47-56

Table 36 - Optimal consumption portfolio, 13 nodes

Short citation / node	Full reference
Cox 1985 I	Cox, J., Ingersoll, J., Ross, S.,: A theory of the term structure of interest rates (1985) Econometrica, 53, pp. 385-408
Chib 1995 I	Chib, S., Greenberg, E.,: Understanding the Metropolis-Hastings algorithm (1995) Am. Statist., 49, pp. 327-335
Ait-Sahalia 1996	Ait-Sahalia, Y.,: Testing continuous-time models of the spot interest rate (1996) Review of Financial Studies, 9, pp. 385-426
Merton 1969	Merton, R., Lifetime Portfolio Selection under Uncertainty: The Continuous Time Case (1969) Rev. Econ. Stat., 51, pp. 247-257
Merton 1971	Merton, R.,: Optimum consumption and portfolio rules in a continuous-time model (1971) Journal of Economic Theory, 3, pp. 373-413
Chan 1992	Chan, K.C., Karolyi, G.A., Longstaff, F., Sanders, A.B.,: An empirical comparison of alternative models of short-term interest rates (1992) Journal of Finance, 47, pp. 1209-1227
Eraker 2001	Eraker, B.,: MCMC analysis of diffusion models with application to finance (2001) J. Bus. Econ. Stat., 19 (2), pp. 177-191
Brennan 1997	Brennan, M., Schwartz, E., Lagnado, R.,: Strategic asset allocation (1997) Journal of Economic Dynamics and Control, 21, pp. 1377-1403
Samuelson 1969	Samuelson, P.,: Lifetime portfolio selection by dynamic stochastic programming (1969) Rev. Econ. Stat., 51, pp. 239-246
Cox 1989	Cox, J., Huang, C.,: Optimal consumption and portfolio policies when asset prices follow a diffusion process (1989) Journal of Economic Theory, 49, pp. 33-83
Elerian 2001	Elerian, O., Chib, S., Shephard, N.,: Likelihood inference for discretely observed non-linear diffusions (2001) Econometrica, 69, pp. 959-993
Liu 1999	Liu, J.,: Portfolio selection in stochastic environments (1999) Working Paper, UCLA
Schroder 1999	Schroder, M., Skiadas, C.,: Optimal consumption and portfolio selection with stochastic differential utility (1999) Journal of Economic Theory, 89, pp. 68-126

Table 37 - Statistics and sampling methods for stock markets, 12 nodes

Short citation / node	Full reference
Glasserman 1999 I	Glasserman, P., Heidelberger, P., Shahabuddin, P.,: Asymptotically optimal importance sampling and stratification for pricing path-dependent options (1999) Math. Finance, 9, pp. 117-152
Nelsen 1999	Nelsen, R.B., (1999) An Introduction to Copulas, New York: Springer
Mandelbrot 1963	Mandelbrot, B.,: The variation of certain speculative prices (1963) J. Bus., 36, pp. 394-419

Morgan 1996	Morgan, J.P., (1996): RiskMetrics Technical Document, 4th Ed., New York
Moro 1995	Moro, B.,: The full Monte (1995) Risk, 8 (2), pp. 57-58
Embrechts 2002	Embrechts, P., McNeil, A., Straumann, D., Correlation and dependence in risk management: Properties and pitfalls (2002) In Risk Management: Value at Risk and Beyond, pp. 176-223.
Fama 1965	Fama, E.,: The behavior of stock market prices (1965) J. Bus., 38, pp. 34-105
Glasserman 2000 I	Glasserman, P., Heidelberger, P., Shahabuddin, P.,: Importance Sampling and Stratification for Value-at-Risk (2000) Computational Finance, 1999, pp. 7-24
Glasserman 1999 II	Glasserman, P., Heidelberger, P., Shahabuddin, P.,: Importance sampling in the Heafh-Jarrow-Morton framework (1999) The Journal of Derivatives, 7, pp. 32-50
Blattberg 1974	Blattberg, R., Gonedes, N.,: A comparison of stable and student distributions as statistical models for stock prices (1974) Journal of Business, 47, pp. 244-280
Press 1990	Press, W., Farrar, G.R.,: Recursive stratified sampling for multidimensional Monte Carlo integration (1990) Computers in Physics, 4 (2), pp. 190-195
Glasserman 1998	Glasserman, P., Heidelberger, P., Shahabuddin, P., Gaussian importance sampling and stratification: Computational issues (1998) Proc. 1998 Winter Simulation Conf., 1, pp. 685-693., eds. D. J. Medeiros, E. F. Watson, J. S. Carson and M. S. Manivannan (IEEE Computer Society Press)

Table 38 - Option pricing, 12 nodes

Short citation / node	Full reference
Boyle 1997	Boyle, P., Broadie, M., Glasserman, P.,: Monte-Carlo methods for security pricing (1997) Journal of Economic Dynamics and Control, 21, pp. 1267-1321
Boyle 1977	Boyle, P., Options: A Monte Carlo approach (1977) Journal of Financial Economics, 4, pp. 323-338
Cox 1979	Cox, J., Ross, S., Rubinstein, M., Option pricing: A simplified approach (1979) Journal of Financial Economics, 7, pp. 229-264
Barone-Adesi 1987	Barone-Adesi, G., Whaley, R.,: Efficient analytic approximation of American option values (1987) Journal of Finance, 1, pp. 301-320., June
Geske 1984	Geske, R., Johnson, H.,: The American put option valued analytically (1984) Journal of Finance, 39, pp. 1511-1524
Broadie 1996 II	Broadie, M., Detemple, J., American option valuation: New bounds approximations, and a comparison of existing methods (1996) The Review of Financial Studies, 9, pp. 1211-1250
Brennan 1977 I	Brennan, M., Schwartz, E.,: The valuation of american put options (1977) Journal of Finance, 32, pp. 449-462
Barraquand 1995 II	Barraquand, J.,: Numerical Valuation of High Dimensional Multivariate European Securities (1995) Management Science, 41, pp. 1882-1891
Jacka 1991	Jacka, S.D.,: Optimal stopping and the American put (1991) Mathematical Finance, 1, pp. 1-14
MacMillan 1986	MacMillan, L.W.,: An analytic approximation for the american put price (1986) Advances in Futures and Options Research, 1, pp. 119-139
Parkinson 1977	Parkinson, M., Option pricing: The american put (1977) Journal of Business, 50, pp. 21-36

Table 39 - Stochastic volatility I, 11 nodes

Short citation / node	Full reference
Heston 1993 I	Heston,: A Closed-Form Solution for Option with Stochastic Volatility with Applications to Bond and Currency Options (1993) Review of Financial Studies, pp. 327-343
Hull 1987	Hull, J., White, A.,: The pricing of options as assets with stochastic volatilities (1987) Journal of Finance, 42, pp. 281-300
Kloeden 2000	Kloeden, P., Platen, E., (2000): Numerical Solution of Stochastic Differential Equations, New York, NY: Springer
Wiggins 1987	Wiggins, J.B.,: Option values under stochastic volatilities (1987) Journal of Financial Economics, 19, pp. 351-372
Duan 1995	Duan, J.,: The GARCH option pricing model (1995) Mathematical Finance, 5, pp. 13-32
Scott 1987	Scott, L., Option pricing when the variance changes randomly: Theory, estimators and applications (1987) J. Finan. Quant. Anal., 22, p. 419
Stein 1991	Stein, E., Stein, J.,: Stock-price Distributions with Stochastic Volatility-An Analytic Approach (1991) Rev. Finan. Stud., 4, pp. 727-752
Schoebel 1999	Schoebel, R., Zhu, J.,: Stochastic volatility with an Ornstein-Uhlenbeck process: An extension (1999) European Finance Review, 3, pp. 23-46
Heston 2000	Heston, S.L., Nandi, S.,: A closed-form GARCH option valuation (2000) Rev. Financial Stud., 13, pp. 585-625
Melino 1990	Melino, A., Turnbull, S.,: Pricing foreign currency options with stochastic volatility (1990) Journal of Econometrics

Karlin 1981	Karlin, S., Taylor, H., (1981) A Second Course in Stochastic Processes, Academic Press: New York

Table 40 - Stochastic volatility II, 6 nodes

Short citation / node	Full reference
Kim 1998	Kim, S., Shephard, N., Chib, S.,: Stochastic volatility: Likelihood inference and comparison with ARCH models (1998) Rev. Econ. Stud., 65, pp. 361-393
Jacquier 1994	Jacquier, E., Poison, N., Rossi, P.,: Bayesian analysis of stochastic volatility models (with discussion) (1994) Journal of Business & Economic Statistics, 12, pp. 371-417
Harvey 1994	Harvey, A., Ruiz, E., Shephard, N.,: Multivariate stochastic variance models (1994) Review of Economic Studies, 61, pp. 247-264
Doornik 2001	Doornik, J., (2001): Object-oriented Matrix Programming Using Ox, Timberlake Consultants Press, London
Taylor 1986	Taylor, S., (1986): Modeling Financial Time Series, John Wiley & Sons, New York, USA
De 1995	De Jong, P., Shephard, N.,: The simulation smoother for time series models (1995) Biometrika, 82, pp. 339-350

Table 41 - Statistical processes and distributions, 6 nodes

Short citation / node	Full reference
Karatzas 1991	Karatzas, I., Shreve, S., (1991) Brownian Motion and Stochastic Calculus, (New York: Springer-Verlag)
Eberlein 1995	Eberlein, E., Keller, K.,: Hyperbolic Distribution in Finance (1995) Bernoulli, 1, pp. 281-299
Madan 1990	Madan, D., Seneta, E.,: The variance gamma model for share market returns (1990) J. Business, 63, pp. 511-524
Broadie 1997 III	Broadie, M., Glasserman, P., Kou, S.,: A continuity correction for discrete barrier options (1997) Mathematical Finance, 7 (4), pp. 325-348
Rydberg 1997	Rydberg, T., The normal inverse Gaussian Lévy process: Simulation and approximation (1997) Communications in Statistics: Stochastic Models, 13 (4), pp. 887-910
Rydberg 1999	Rydberg, T.H.,: Generalized hyperbolic diffusion processes with application in finance (1999) Mathematical Finance, 9 (2), pp. 183-201

Table 42 - Bayes factor and Monte Carlo, 5 nodes

Short citation / node	Full reference
Chib 1996	Chib, S., Greenberg, E.,: Markov chain Monte Carlo simulation methods in econometrics (1996) Econ. Theory, 12, pp. 409-431
Kass 1995	Kass, R., Raftery, A.,: Bayes factors (1995) J. Am. Statist. Assoc., 90, pp. 773-795
Casella 1992	Casella, G., George, E.,: Explaining the Gibbs sampler (1992) American Statistician, 46, pp. 167-174
Carlin 1996	Carlin, B., Louis, T., (1996): Bayes and Empirical Bayes Methods for Data Analysis, Chapman & Hall, London
Verdinelli 1995	Verdinelli, I., Wasserman, L.,: Computing Bayes factors using a generalization of the Savage-Dickey density ratio (1995) Journal of the American Statistical Association, 90, pp. 614-618

Table 43 - Malliavin Calculus, 5 nodes

Short citation / node	Full reference
Fournie 1999	Fournie, E., Larsy, J.M., Lebuchoux, J., Lions, P.L., Touzi, N.,: Applications of malliavin calculus to monte carlo methods in finance I (1999) Finance and Stochastics, 3 (4), pp. 391-412
Nualart 1995	Nualart, D., (1995): The Malliavin Calculus and Related Topics, Springer, Berlin
Broadie 1996 I	Broadie, M., Glasserman, P.,: Estimating security price derivatives using simulation (1996) Management Science, 42 (2), pp. 269-285
Fournie 2001	Fournié, E., Lasry, J.M., Lebouchoux, J., Lions, P.L.,: Applications of Malliavin calculus to Monte Carlo methods in finance, II (2001) Finance and Stochastics, 5, pp. 201-236
Ikeda 1989	Ikeda, N., Watanabe, S., (1989) Stochastic Differential Equations and Diffusion Processes, Amsterdam: North-Holland

Table 44 - Estimation methods for inference and cont. time processes, 4 nodes

Short citation / node	Full reference
Gallant 1996	Gallant, A., Tauchen, G.,: Which moments to match (1996) Econometric Theory, 12, pp. 657-681
Eraker 2003	Eraker, B., Johannes, M., Polson, N.,: The impact of jumps in equity index volatility and returns (2003) Journal of Finance, 58, pp. 1269-1300
Chacko 2003	Chacko, G., Viceira, L.M.,: Spectral GMM estimation of continuous-time processes (2003) Journal of Econometrics, 116 (1-2), pp. 259-292
Das 1999	Das, S., Sundaram, R., Of smiles and smirks: A term structure perspective (1999) Journal of Financial and Quantitative Analysis, 34 (1), pp. 60-72

Table 45- Bond and exotic options, 4 nodes

Short citation / node	Full reference
Jamshidian 1989	Jamshidian, F.,: An Exact Bond Option Formula (1989) Journal of Finance, 44, pp. 205-209
Geman 1995	Geman, H., El Karoui, N., Rochet, J.,: Changes of numeraire, changes of probability measures and pricing of options (1995) Journal of Applied Probability, 32, pp. 443-458
Ho 1986	Ho, T., Lee, S.,: Term structure movements and pricing interest rate contingent claims (1986) Journal of Finance, 41, pp. 1011-1029
Turnbull 1991	Turnbull, S., Wakeman, L.,: A quick algorithm for pricing European Average Options (1991) Journal of Financial and Quantitative Analysis, 26, pp. 377-389

Table 46 - Long memory time series, 4 nodes

Short citation / node	Full reference
Granger 1980 I	Granger, C.W.J., Joyeux, R.,: An introduction to long memory time series and fractionally differencing (1980) J. Time Series Analysis, 1, pp. 15-29
Geweke 1983	Geweke, J., Porter Hudak, S.,: The estimation and application of long-memory time series models (1983) J. Time Ser. Anal., 4, pp. 221-238
Dickey 1979	Dickey, A., Fuller, A.,: Distribution of the estimators for autoregressive time series with a unit root (1979) Journal of Statistical Association, 74, pp. 427-431
Diebold 1989	Diebold, F., Rudebusch, G.D.,: Long Memory and Persistence in Aggregate Output (1989) Journal of Monetary Economics, 24, pp. 189-209

Table 47 - Accounting & auditing topics, 4 nodes

Short citation / node	Full reference
Allen 1993	Allen, R.D., (1993) Analytical Procedures Using Financial and Nonfinancial Information: A Comparison of Alternative Methods, Unpublished monograph, University of Utah
Cogger 1981	Cogger, K.O.,: A Time-Series Analytic Approach to Aggregation Issues in Accounting Data (1981) Journal of Accounting Research, Autumn
Kaplan 1978	Kaplan, R.S., Developing a financial planning model for an analytical review: A feasibility study (1978) Proceedings of the Symposium on Auditing Research III, Champaign, IL: University of Illinois
Wild 1987	Wild, J.J.,: The Prediction Performance of a Structural Model of Accounting Numbers (1987) Journal of Accounting Research, Spring

Period II

Table 48 - Volatility and risk I, 24 nodes

Short citation / node	Full reference
Jacquier 1994	Jacquier, E., Poison, N., Rossi, P.,: Bayesian analysis of stochastic volatility models (with discussion) (1994) Journal of Business & Economic Statistics, 12, pp. 371-417
Kim 1998	Kim, S., Shephard, N., Chib, S.,: Stochastic volatility: Likelihood inference and comparison with ARCH models (1998) Rev. Econ. Stud., 65, pp. 361-393
Danielsson 1994	Danielsson, J., Stochastic volatility in asset prices: Estimation with simulated maximum likelihood (1994) J. Econometrics, 64, pp. 375-400

Harvey 1994	Harvey, A., Ruiz, E., Shephard, N.,: Multivariate stochastic variance models (1994) Review of Economic Studies, 61, pp. 247-264
Yu 2002	Yu, J.,: Forecasting volatility in the New Zealand stock market (2002) Applied Financial Economics, 12, pp. 193-202
Lintner 1965	Lintner, J.,: The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets (1965) Review of Economics and Statistics, 47, pp. 13-37
Harvey 1996	Harvey, A.C., Shephard, N.,: Estimation of an Asymmetric Stochastic Volatility Model for Asset Returns (1996) Journal of Business & Statistics, 14 (4), pp. 429-434
Jagannathan 1996	Jagannathan, R., Wang, Z.,: The conditional CAPM and the cross section of expected returns (1996) Journal of Finance, 51, pp. 3-52., and. pp
Tauchen 1983	Tauchen, G., Pitts, M.,: The price variability volume relationship on speculative markets (1983) Econometrica, 51, pp. 485-505
Sharpe 1964	Sharpe, W., Capital asset prices: a theory of market equilibrium under conditions of risk (1964) J. Finance, 19 (3), pp. 425-442
Clark 1973	Clark, P.,: A subordinated stochastic process model with finite variance for speculative process (1973) Econometrica, 41, pp. 135-155
Ritchken 1999	Ritchken, P., Trevor, R.,: Pricing options under generalized GARCH and stochastic volatility processes (1999) Journal of Finance, 54, pp. 377-402
Andersen 1996 I	Andersen, T., Return volatility and trading volume: an information flow interpretation of stochastic volatility (1996) J Finance, 51, pp. 169-204
Bollerslev 1987	Bollerslev, T.,: A conditional heteroscedastic time series model for speculative prices and rates of return (1987) Rev Econ Stat, 69, pp. 542-547
Engle 1995	Engle, R., Kroner, F.K.,: Multivariate simultaneous generalized ARCH (1995) Econometric Theory, 11, pp. 122-150
Engle 2002	Engle, R.,: Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models (2002) Journal of Business & Economic Statistics, 20 (3), pp. 339-350
Geman 1984	Geman, S., Geman, D.,: Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images (1984) IEEE Transactions on Pattern Analysis and Machine Intelligence, 6, pp. 721-41., and. pp
Melino 1990	Melino, A., Turnbull, S.,: Pricing foreign currency options with stochastic volatility (1990) Journal of Econometrics
Spiegelhalter 2002	Spiegelhalter, D.J., Best, N.G., Carlin, B.P., Van Der Linde, A.,: Bayesian measures of model complexity and fit (2002) J. R. Stat. Soc. Ser. B (Stat Methodol.), 64 (4), pp. 583-639
Gibbons 1989	Gibbons, M.R., Ross, S.A., Shanken, J.,: A test of the efficiency of a given portfolio (1989) Econometrica, 57, pp. 1121-1152
Bos 1984	Bos, T., Newbold, P.,: An empirical investigation of the possibility of stochastic systematic risk in the market model (1984) Journal of Business, 57 (1), pp. 35-41
Braun 1995	Braun, P., Nelson, D., Sunier, A.,: Good news, bad news, volatility and betas (1995) Journal of Finance, 50, pp. 1575-1604., and
Fabozzi 1978	Fabozzi, F.J., Francis, J.,: Beta as a random coefficient (1978) Journal of Financial and Quantitative Analysis, 13, pp. 101-115
Fama 1992	Fama, E., French, K.,: The cross-section of expected returns (1992) Journal of Finance, 47, pp. 427-465

Table 49 – Simulation methods for option pricing, 22 nodes

Short citation / node	Full reference
Longstaff 2001	Longstaff, F., Schwartz, E., Pricing American Options by Simulation: A Simple Least Square Approach (2001) Rev. Financial Stud., 14, pp. 113-147
Carriere 1996	Carriere, J., Valuation of Early-Exercise Price of Options Using Simulations and Nonparametric Regression (1996) Insur.: Math. Econ., 19, pp. 19-30
Tsitsiklis 1999 I	Tsitsiklis, J., Van Roy, B.,: Regression Methods for Pricing Complex American Style Options (1999) IEEE Trans. Neural. Net., 12, pp. 694-703., and
Clement 2002	Clément, E., Lamberton, D., Plotter, P.,: An analysis of a least squares regression method for American option pricing (2002) Finance and Stochastics, 6, pp. 449-471
Rogers 2002	Rogers, L.,: Monte Carlo valuation of American options (2002) Math. Finance, 12 (3), pp. 271-286
Haugh 2004	Haugh, M., Kogan, L., Pricing American options: A duality approach (2004) Oper. Res., 52, pp. 258-270
Andersen 2004	Andersen, L., Broadie, M.,: A primal-dual simulation algorithm for pricing multi-dimensional American options (2004) Management Science, 50 (9), pp. 1222-1234
Egloff 2005	Egloff, D.,: Monte Carlo Algorithms for Optimal Stopping and Statistical Learning (2005) Ann. Appl. Probab., 15, pp. 1-37
Andersen 1999 I	Andersen, L., (1999): A Simple Approach to Pricing Bermudan Swaptions in the Multi-Factor LIBOR Market Model, Geneva Re Financial Products. Working Paper
Broadie 2004	Broadie, M., Glasserman, P.,: A stochastic mesh method for pricing high-dimensional American option (2004) J. Comput. Finan, 7, pp. 35-72
Kolodko 2006	Kolodko, A., Schoenmakers, J.,: Iterative construction of the optimal Bermudan stopping time (2006) Finance and Stochastics, 10, pp. 27-49

Belomestny 2009	Belomestny, D., Bender, C., Schoenmakers, J.,: True upper bounds for Bermudan products via non-nested Monte Carlo (2009) Math. Finance, 19, pp. 53-71
Schoenmakers 2005	Schoenmakers, J., (2005) Robust Libor Modelling and Pricing of Derivative Products, Boca Raton London New York Singapore: Chapman and Hall - CRC Press
Lucia 2002	Lucia, J., Schwartz, E., Electricity prices and power derivatives: Evidence from the Nordic power exchange (2002) Rev. Deriv. Res., 5, pp. 5-50
Bender 2006 I	Bender, C., Schoenmakers, J., An iterative method for multiple stopping: Convergence and stability (2006) Adv. Appl. Probab., 38 (3), pp. 729-749
Glasserman 2004	Glasserman, P., Yu, B.,: Number of Paths Versus Number of Basis Functions in American Option Pricing (2004) Ann. Appl. Probab., 14, pp. 1-30
Meinshausen 2004	Meinshausen, N., Hambly, B.,: Monte Carlo methods for the valuation of multiple-exercise options (2004) Math. Finance, 14 (4), pp. 557-583
Bouchard 2004 I	Bouchard, B., Ekeland, I., Touzi, N.,: On the Malliavin approach to Monte Carlo approximation of conditional expectations (2004) Finan. Stochast, 8, pp. 45-71
Kolodko 2004	Kolodko, A., Schoenmakers, J.,: Upper bound for Bermudan style derivatives (2004) Monte Carlo Meth. Appl., 10, pp. 331-343
Gyoerfi 2002	Györfi, L., Kohler, M., Krzyzak, A., Walk, H., (2002) A Distribution-Free Theory of Nonparametric Regression, Berlin: Springer
Bally 2005 I	Bally, V., Pages, G., Printems, J.,: A quantization tree method for pricing and hedging multidimensional American options (2005) Math. Finan, 15, pp. 119-168
Belomestny 2004	Belomestny, D., Milstein, G.N.,: Monte Carlo evaluation of American options using consumption processes (2004) Int. J. Theoret. Appl. Finance (Forthcoming), WIAS-Preprint No. 930 Berlin

Table 50 –Stochastic processes, 22 nodes

Short citation / node	Full reference
Heston 1993 I	Heston,: A Closed-Form Solution for Option with Stochastic Volatility with Applications to Bond and Currency Options (1993) Review of Financial Studies, pp. 327-343
Cox 1985 I	Cox, J., Ingersoll, J., Ross, S.,: A theory of the term structure of interest rates (1985) Econometrica, 53, pp. 385-408
Hull 1987	Hull, J., White, A.,: The pricing of options as assets with stochastic volatilities (1987) Journal of Finance, 42, pp. 281-300
Duffie 2000	Duffie, D., Pan, J., Singleton, K.,: Transform Analysis and Asset Pricing for Affine Jump Diffusion (2000) Econometrica, pp. 1343-1376
Kloeden 2000	Kloeden, P., Platen, E., (2000): Numerical Solution of Stochastic Differential Equations, New York, NY: Springer
Broadie 2006	Broadie, M., Kaya, O.,: Exact simulation of stochastic volatility and other affine jump diffusion processes (2006) Operations Research, 54 (2), pp. 217-231
Vasicek 1977	Vasicek, O.,: An equilibrium characterization of the term structure (1977) Journal of Financial Economics, 5, pp. 177-188
Stein 1991	Stein, E., Stein, J.,: Stock-price Distributions with Stochastic Volatility-An Analytic Approach (1991) Rev. Finan. Stud., 4, pp. 727-752
Lord 2009	Lord, R., Koekkoek, R., Van Dijk, D.,: A comparison of biased simulation schemes for stochastic volatility models (2009) Journal of Quantitative Finance, 10 (2), pp. 177-194
Chan 1992	Chan, K.C., Karolyi, G.A., Longstaff, F., Sanders, A.B.,: An empirical comparison of alternative models of short-term interest rates (1992) Journal of Finance, 47, pp. 1209-1227
Carr 1999	Carr, P., Madan, D.,: Option valuation using the Fast Fourier Transform (1999) J. Comp. Finance, 2, p. 61
Scott 1987	Scott, L., Option pricing when the variance changes randomly: Theory, estimators and applications (1987) J. Finan. Quant. Anal., 22, p. 419
Andersen 2008	Andersen, L.,: Efficient simulation of the Heston stochastic volatility model (2008) Journal of Computational Finance, 11 (3), pp. 1-22
Kahl 2006	Kahl, C., Jackel, P.,: Fast strong approximation Monte Carlo schemes for stochastic volatility models (2006) Quant. Finan., 6, pp. 513-536
Schoebel 1999	Schoebel, R., Zhu, J.,: Stochastic volatility with an Ornstein-Uhlenbeck process: An extension (1999) European Finance Review, 3, pp. 23-46
Andersen 2001 I	Andersen, L., Brotherton-Ratcliffe, R.,: Extended libor market models with stochastic volatility (2001) Working Paper, Gen. Re Securities
Willard 1997	Willard, G.,: Calculating prices and sensitivities for path-independent derivative securities in multifactor models (1997) Journal of Derivatives, 5, pp. 45-61
Duffie 1995	Duffie, D., Glynn, P.,: Efficient monte carlo estimation of security prices (1995) Ann. Appl. Probab., 4, pp. 897-905
Abramowitz 1964	Abramowitz, M., Stegun, I., (1964) Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables, New York: Dover
Bates 1996	Bates, D., Jumps and Stochastic Volatility: Exchange Rate Process Implicit in Deutsche Mark Options (1996) Review of Financial Studies, 9, pp. 69-107
Revuz 1991	Revuz, D., Yor, M., (1991) Continuous Martingales and Brownian Motion, Springer Verlag: New York

Andersen 2007 I	Andersen, L., Piterbarg, V.,: Moment explosions in stochastic volatility models (2007) Finance and Stochastics, 11 (1), pp. 29-50
Heston 1993 I	Heston,: A Closed-Form Solution for Option with Stochastic Volatility with Applications to Bond and Currency Options (1993) Review of Financial Studies, pp. 327-343
Cox 1985 I	Cox, J., Ingersoll, J., Ross, S.,: A theory of the term structure of interest rates (1985) Econometrica, 53, pp. 385-408

Table 51 - Early exercise option valuation, 15 nodes

Short citation / node	Full reference
Broadie 1997 I	Broadie, M., Glasserman, P.,: Pricing American-style securities using simulation (1997) J Econ Dyn Control, 21, pp. 1323-1352., 8-9
Boyle 1977	Boyle, P., Options: A Monte Carlo approach (1977) Journal of Financial Economics, 4, pp. 323-338
Barraquand 1995 I	Barraquand, J., Martineau, D.,: Numerical valuation of high dimensional multivariate american securities (1995) J Finan Quant Anal, 30, pp. 383-405
Moreno 2003	Moreno, M., Navas, J.F.,: On the robustness of least-squares Monte Carlo (LSM) for pricing American derivatives (2003) Rev. Deriv. Res., 6 (2), pp. 107-128
Stentoft 2004 II	Stentoft, L.,: Assessing the least squares Monte-Carlo approach to American option valuation (2004) Rev. Deriv. Res., 7 (2), pp. 129-168
Tilley 1993	Tilley, J.,: Valuing American options in a path simulation model (1993) Trans. Soc. Actuaries, 45, pp. 83-104
Broadie 1997 II	Broadie, M., Glasserman, P., Jain, G.,: Enhanced monte carlo estimates for American options prices (1997) Journal of Derivatives, 5, pp. 25-44
Stentoft 2004 I	Stentoft, L.,: Convergence of the least squares Monte Carlo approach to American option valuation (2004) Management Science, 50 (9), pp. 1193-1203
Raymar 1997	Raymar, S., Zwecher, M.,: A Monte Carlo valuation of American call options on the maximum of several stocks (1997) Journal of Derivatives, 1, pp. 7-23
Stulz 1982	Stulz, R.,: Options on the minimum or maximum of two risky assets (1982) The Journal of Financial Economics, 10, pp. 161-185
Bossaerts 1989	Bossaerts, P., (1989): Simulation Estimators of Optimal Early Execise, Working paper, Carnegie-Mellon University
Margrabe 1978	Margrabe, W.,: The Value of An Option to Exchange One Asset for Another (1978) Journal of Finance, 1, pp. 177-186
Johnson 1987	Johnson, H.,: Options on the maximum or the minimum of several assets (1987) J. Financial Quant. Anal., 22, pp. 227-83
Brately 1992	Brately, P., Fox, B.L., Niederreiter, H.,: Implementation and Tests of Low-Discrepancy Sequences (1992) ACM Transactions on Modelling and Computer Simulation, 2, pp. 195-213
Tian 2003	Tian, T., Burrage, K.,: Accuracy issues of Monte-Carlo methods for valuing American options (2003) ANZIAM J, 44, pp. C739-C758

Table 52 - Affine term structure models, 15 nodes

Short citation / node	Full reference
Duffie 1996 I	Duffie, D., Kan, R.,: A yield-factor model of interest rate (1996) Mathematical Finance, 6, pp. 379-406
Dai 2000	Dai, Q., Singleton, K.J.,: Specification analysis of affine term structure models (2000) J. Financ., 55, pp. 1943-1978
Heath 1992	Heath, D., Jarrow, R., Morton, A.,: Bond pricing and the term structure of interest rates (1992) Econometrica, 60, pp. 77-106
Hull 1990	Hull, J., White, A.,: Pricing interest-rate derivative securities (1990) Review of Financial Studies, 3, pp. 573-592
Duffee 2002	Duffee, G.R.,: Term premia and interest rate forecasts in affine models (2002) Journal of Finance, 57, pp. 405-443
Diebold 2006	Diebold, F., Li, C.,: Forecasting the term structure of government bond yields (2006) Journal of Econometrics, 130 (2), pp. 337-364., DOI 10.1016/j.jeconom.2005.03.005, PII S0304407605000795
Dejong 2000	Dejong, F.,: Time series and cross-section information in affine term structure models (2000) Journal of Business and Economic Statistics, 18, pp. 300-314
Litterman 1991	Litterman, R., Scheinkman, J.,: Common Factors Affecting Bond Returns (1991) Journal of Fixed Income, 3, pp. 34-61
Nelson 1987	Nelson, C.R., Siegel, A.F.,: Parsimonious modeling of yield curves (1987) Journal of Business, 60 (4), pp. 473-489
Harvey 1989	Harvey, A.C., (1989) Forecasting Structural Time Series Models and the Kalman Filter, Cambridge, UK: Cambridge University Press
Rebonato 2005	Rebonato, R., Mahal, S., Joshi, M., Bucholz, L.-D., Nyholm, K., Evolving yield curves in the real-world measure: A semi-parametric approach (2005) Journal of Risk, 7, pp. 29-61

Ait-Sahalia 2002 I	Aït-Sahalia, Y., Kimmel, R., (2002): Estimating Affine Multifactor Term Structure Models Using Closed-Form Likelihood Expansions, Working paper, Princeton University
Bikbov 2009	Bikbov, R., Chernov, M., Unspanned Stochastic Volatility in Affine Models: Evidence from Eurodollar Futures and Options (2009) Management Science, 55 (8), pp. 1292-1305
Cheredito 2007	Cheredito, P., Filipovic, D., Kimmel, R., Market Price of Risk Specifications for Affine Models: Theory and Evidence (2007) Journal of Financial Economics, 83, pp. 123-170
Duffee 2004	Duffee, G.R., Stanton, R., (2004): Estimation of Dynamic Term Structure Models, Working paper, University of California at Berkeley

Table 53 - GARCH volatility, 14 nodes

Short citation / node	Full reference
Bollerslev 1986	Bollerslev, T.,: Generalized autoregressive conditional heteroscedasticity (1986) Journal of Econometrics, 31, pp. 307-327
Engle 1982	Engle, R.,: Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation (1982) Econometrica, 50, pp. 987-1008
Taylor 1986	Taylor, S., (1986): Modeling Financial Time Series, John Wiley & Sons, New York, USA
Duan 1995	Duan, J.,: The GARCH option pricing model (1995) Mathematical Finance, 5, pp. 13-32
Bollerslev 1994	Bollerslev, T., Engle, R., Nelson, D.,: ARCH models (1994) Handbook of Econometrics, 4, pp. 2959-3038., R.F. Engle and D.L. McFadden (Eds.), North Holland, Amsterdam, Netherlands
Engle 1993	Engle, R., Ng, V.,: Measuring and testing the impact of news on volatility (1993) J. Finance, 48, pp. 1749-1778
Gallant 1996	Gallant, A., Tauchen, G.,: Which moments to match (1996) Econometric Theory, 12, pp. 657-681
Bollerslev 1992 II	Bollerslev, T., Chou, R., Kroner, K., ARCH modelling in finance: A review of the theory and empirical evidence (1992) Journal of Econometrics, 52, pp. 5-59
Christoffersen 2004 II	Christoffersen, P., Jacobs, K.,: Which GARCH model for option valuation? (2004) Manag. Sci., 50, pp. 1204-1221
Heston 2000	Heston, S.L., Nandi, S.,: A closed-form GARCH option valuation (2000) Rev. Financial Stud., 13, pp. 585-625
Nelson 1990	Nelson, D.,: ARCH models as diffusion approximations (1990) Journal of Econometrics, 45 (1-2), pp. 7-38
Andersen 1999 II	Andersen, T., Chung, H., Sorensen, B., Efficient method of moments estimation of a stochastic volatility model: A Monte Carlo study (1999) J. Econometr., 91, pp. 61-87
Ball 1996	Ball, C., Torous, W.N.,: Unit roots and the estimation of interest rate dynamics (1996) Journal of Empirical Finance, 3, pp. 215-238
Ghysels 1996	Ghysels, E., Harvey, A., Renault, E.,: Stochastic volatility (1996) Handbook of Statistics, 14, pp. 119-191., North-Holland, Amsterdam

Table 54 - Value-at-Risk, 13 nodes

Short citation / node	Full reference
Jorion 2000	Jorion, P., (2000) Value-at-Risk, McGraw-Hill: New York
Kuester 2006	Kuester, K., Mittnik, S., Paolella, M., Value-at-Risk prediction: A comparison of alternative strategies (2006) Journal of Financial Econometrics, 4, pp. 53-89
Barone-Adesi 1998	Barone-Adesi, G., Bourgoin, F., Giannopoulos, K.,: Don't look back (1998) Risk, 11., August
Efron 1979	Efron, B., Bootstrap method: Another look at the jackknife (1979) Annals of Statistics, 7 (1), pp. 1-26
Barone-Adesi 2002	Barone-Adesi, G., Giannopoulos, K., Vosper, L.,: Backtesting derivative portfolios with filtered historical simulation (2002) European Financial Management, 8, pp. 31-58
Barone-Adesi 1999	Barone-Adesi, G., Giannopoulos, K., Vosper, L.,: VaR without Correlation for nonlinear Portfolios (1999) Journal of Futures Markets, 19, pp. 583-602
Christoffersen 1998	Christoffersen, P.,: Evaluating interval forecasts (1998) Int Econ Rev, 39, pp. 841-862
Duffie 1997	Duffie, D., Pan, J.,: An overview of value at risk (1997) Journal of Derivatives, 4, pp. 7-49
Hendricks 1994	Hendricks, D.,: Evaluation of value at risk models using historical data (1994), Federal Reserve Bank of New York, New York
Boudoukh 1998	Boudoukh, J., Richardson, M., Whitelaw, R.,: The best of both worlds (1998) Risk, 11 (5), pp. 64-67
Pritsker 2006	Pritsker, M.,: The hidden dangers of historical simulation (2006) Journal of Banking and Finance, 30 (2), pp. 561-582
Angelidis 2004	Angelidis, T., Benos, A., Degiannakis, S.,: The use of GARCH models in VaR estimation (2004) Statistical Methodology, 1 (1-2), pp. 105-128., DOI 10.1016/j.stamet.2004.08.004, PII S1572312704000103
Davison 1997	Davison, A.C., Hinkley, D.V., (1997): Bootstrap methods and their application, Cambridge University Press, Cambridge

Table 55 - Monte Carlo methods and valuation, 8 nodes

Short citation / node	Full reference
Jackel 2002	Jackel, P., (2002): Monte Carlo Methods in Finance, Wiley
Broadie 1996 II	Broadie, M., Detemple, J., American option valuation: New bounds approximations, and a comparison of existing methods (1996) The Review of Financial Studies, 9, pp. 1211-1250
Caflisch 1997	Caflisch, R.E., Morokoff, W., Owen, A.B.,: Valuation of mortgage backed securities using Brownian bridges to reduce effective dimension (1997) J. Comput. Finance, 1, pp. 27-46
Niederreiter 1992	Niederreiter, H., (1992): Random Number Generation and Quasi-Monte Carlo Methods, Philadelphia, PA SIAM
Moro 1995	Moro, B.,: The full Monte (1995) Risk, 8 (2), pp. 57-58
Paskov 1995	Paskov, S., Traub, J.,: Faster Valuation of Financial Derivatives (1995) J. Portfol. Manage., 21, pp. 113-120., and
Papageorgiou 2002	Papageorgiou, A.,: The Brownian bridge does not offer a consistent advantage in quasi-Monte Carlo integration (2002) J. Complex., 18, pp. 171-186
Sobol 1967	Sobol, I.M.,: On the distribution of points in a cube and the approximate evaluation of integrals (1967) USSR Computational Mathematics and Mathematical Physics, 7, pp. 86-112., 4
Jackel 2002	Jackel, P., (2002): Monte Carlo Methods in Finance, Wiley

Table 56 - Credit derivatives, 7 nodes

Short citation / node	Full reference
Broadie 1996 I	Broadie, M., Glasserman, P.,: Estimating security price derivatives using simulation (1996) Management Science, 42 (2), pp. 269-285
Li 2000	Li, D., On default correlation: A copula function approach (2000) J. Fixed Income, 9, pp. 43-54
Schoenbucher 2003	Schoenbucher, P., (2003): Credit Derivatives Pricing Models, Wiley Finance
Asmussen 2007	Asmussen, S., Glynn, P.W., (2007) Stochastic Simulation, New York: Springer Verlag
Andersen 2003 II	Andersen, L., Sidenius, J., Basu, S.,: All your hedges in one basket (2003) Risk, 16, pp. 67-72
Joshi 2004	Joshi, M., Kainth, D.,: Rapid and accurate development of prices and Greeks for nth-to-default credit swaps in the Li model (2004) Quant. Finance, 4, pp. 266-275
Hull 2004	Hull, J., White, A.,: Valuation of a CDO and an nth to default CDS without Monte Carlo simulation (2004) Journal of Derivatives, 12, pp. 8-23
Broadie 1996 I	Broadie, M., Glasserman, P.,: Estimating security price derivatives using simulation (1996) Management Science, 42 (2), pp. 269-285

Table 57 - Market efficiency and stock market behavior, 7 nodes

Short citation / node	Full reference
Elliott 1996	Elliott, G., Rothenberg, J., Stock, H.,: Efficient tests for an autoregressive unit root (1996) Econometrica, 64, pp. 813-836
Phillips 1988	Phillips, P., Perron, P.,: Testing for a unit root in time series regression (1988) Biometrika, 75 (2), pp. 335-346
Lo 1988 I	Lo, A., Mackinlay, A.C., Stock market prices do not follow random walks: Evidence from a simple specification test (1988) Review of Financial Studies, 1, pp. 41-66
Dickey 1979	Dickey, A., Fuller, A.,: Distribution of the estimators for autoregressive time series with a unit root (1979) Journal of Statistical Association, 74, pp. 427-431
Fama 1970	Fama, E., Efficient capital markets: A review of theory and empirical work (1970) Journal of Finance, 25, pp. 383-417
Ng 2001	Ng, S., Perron, P.,: Lag length selection and the construction of unit root tests with good size and power (2001) Econometrica, 69, pp. 1519-1554
Grossman 1980	Grossman, S., Stiglitz, J.,: On the impossibility of informationally efficient markets (1980) The American Economic Review, 70, pp. 393-408., June
Elliott 1996	Elliott, G., Rothenberg, J., Stock, H.,: Efficient tests for an autoregressive unit root (1996) Econometrica, 64, pp. 813-836

Table 58 - Volatility, 6 nodes

Short citation / node	Full reference
Nelson 1991	Nelson, D., Conditional heteroscedasticity in asset returns: A new approach (1991) Econometrica, 59, pp. 347-370
Glosten 1993	Glosten, L., Jaganathan, R., Runkle, D.,: On the relation between the expected value and the volatility of the nominal excess return on the stocks (1993) J. Finance, 48, pp. 1779-1801
Black 1976 I	Black, F.,: Studies in stock price volatility changes (1976) Proceedings of the 1976 Business Meeting of the Business and Economic Statistics Section, pp. 177-181., American Statistical Association
Bollerslev 1992 I	Bollerslev, T., Wooldridge, J.,: Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances (1992) Econometr. Rev., 11 (2), pp. 143-172
Bekaert 2000 II	Bekaert, G., Wu, G.,: Asymmetric volatility and risk in equity markets (2000) Rev. Financ. Stud., 13, pp. 1-42
Christie 1982	Christie, A., The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects (1982) Journal of Financial Economics, 10, pp. 407-432
Nelson 1991	Nelson, D., Conditional heteroscedasticity in asset returns: A new approach (1991) Econometrica, 59, pp. 347-370

Table 59 - Macro asset pricing, 6 nodes

Short citation / node	Full reference
Lucas 1978	Lucas Jr., R.E.,: Asset prices in an exchange economy (1978) Econometrica, 46 (6), pp. 1429-1445
Mehra 1985	Mehra, R., Prescott, E.C., The equity premium: A puzzle (1985) Journal of Monetary Economics, 15 (2), pp. 145-161
Hansen 1982 II	Hansen, L.P., Singleton, K.J.,: Generalized instrumental variables estimation in non-linear rational expectations models (1982) Econometrica, 50, pp. 1269-1286
Cochrane 2001	Cochrane, J., (2001) Asset Pricing, Princeton: Princeton University Press
Hansen 1983	Hansen, L.P., Singleton, K.J.,: Stochastic Consumption, Risk Aversion and the Temporal Behavior of Stock Returns (1983) Journal of Political Economy, 91 (2), pp. 249-265
Campbell 1999	Campbell, J.Y., Cochrane, J., By force of habit: A consumption-based explanation of aggregate stock market behavior (1999) Journal of Political Economy, 107, pp. 205-51

Table 60 - Interest rate models, 5 nodes

Short citation / node	Full reference
Brigo 2001	Brigo, D., Mercurio, F., (2001): Interest Rate Models Theory and Practice, (Springer: Berlin)
Brace 1997	Brace, A., Gatarek, D., Musiela, M.,: The market model of interest rate dynamics (1997) Math. Finance, 7, pp. 127-155
Jamshidian 1997	Jamshidian, F.,: LIBOR and swap market models and measures (1997) Financ. Stoch., 1, pp. 293-330
Gatheral 2005	Gatheral, J., (2005) The Volatility Surface: A Practitioners Guide, Wiley Finance
Miltersen 1997	Miltersen, K., Sandmann, K., Sondermann, D.,: Closed form solutions for term structure derivatives with log-normal interest rates (1997) J. Financ., 52, pp. 409-430

Table 61 - Stochastic volatility, 4 nodes

Short citation / node	Full reference
Eraker 2003	Eraker, B., Johannes, M., Polson, N.,: The impact of jumps in equity index volatility and returns (2003) Journal of Finance, 58, pp. 1269-1300
Eraker 2004	Eraker, B.,: Do stock market and volatility jump? Reconciling evidence from spot and option prices (2004) Journal of Finance, 59, pp. 1367-1404
Heston 1993 II	Heston, S.,: A closed form solution for options with stochastic volatilities with applications to Bond and Currency Options (1993) The Review of Financial Studies, 6, pp. 329-343
Barndorff-Nielsen 2004	Barndorff-Nielsen, O., Shephard, N.,: Power and Bipower Variation with Stochastic Volatility and Jumps (2004) J. Financ. Econom., 2, pp. 1-48

Table 62 - Bayes factor and Monte Carlo, 4 nodes

Short citation / node	Full reference
Gilks 1996	Gilks, W.R., Richardson, S., Spiegelhalter, D., (1996) Markov Chain Monte Carlo in Practice, (Eds.) (Chapman & Hall: London)
Gelman 1995	Gelman, A., Carlin, J., Stern, H., Rubin, D., (1995): Bayesian Data Analysis, Chapman & Hall, New York
Lando 2002	Lando, D., Skødeberg, T.,: Analyzing rating transitions and rating drift with continuous observations (2002) Journal of Banking and Finance, 26, pp. 423-444
Spiegelhalter 2003	Spiegelhalter, D., Thomas, A., Best, N., Gilks, W., (2003) WinBUGS User Manual (Version 1.4), Cambridge, UK: MRC Biostatistics Unit

Table 63 - Risk modelling for financial institutions, 4 nodes

Short citation / node	Full reference
Embrechts 1997	Embrechts, P., Kluppelberg, C., Mikosch, T., (1997): Modeling Extremal Events for Insurance and Finance, Springer, Berlin
Bank 2006	Bank for International Settlements,: International convergence of capital measurement and capital standards (2006), BCBS
Degen 2006	Degen Embrechts, P., Lambrigger, D., The Quantitative Modelling of Operational Risk: Between g-and-h and EVT (2006) Astin Bulletin, 37 (2)
Embrechts 2003 II	Embrechts, P., Furrer, H., Kaufmann, R.,: Quantifying regulatory capital for operational risk (2003) Deriv. Use, Trad. Regul, 9, pp. 217-233

Table 64 - Executive stock options, 4 nodes

Short citation / node	Full reference
Ingersoll 2006	Ingersoll, J.E.,: The subjective and objective evaluation of incentive stock options (2006) Journal of Business, 79, pp. 453-487
Carpenter 1998	Carpenter, I.,: The exercise and valuation of executive stock options (1998) Journal of Financial Economics, 48, pp. 127-158
Carr 2000 II	Carr, P., Linetsky, V.,: The valuation of executive stock options in an intensity-based framework (2000) European Finance Review, 4 (3), pp. 211-230
Hall 2002	Hall, B.J., Murphy, K.J.,: Stock options for undiversified executives (2002) J Account Econ, 33, pp. 3-42

Table 65 - Realized Volatility, 4 nodes

Short citation / node	Full reference
Barndorff-Nielsen 2001	Barndorff-Nielsen, O., Shephard, N.,: Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial economics (2001) J. R. Stat. Soc. Ser. B, 63 (2), pp. 167-241
Barndorff-Nielsen 2002	Barndorff-Nielsen, O., Shephard, N.,: Econometric Analysis of Realized Volatility and Its Use in Estimating Stochastic Volatility Models (2002) Journal of The Royal Statistical Society, 64 (2), pp. 253-280. Series B
Anderson 2001	Anderson, T., Bollerslev, T., Diebold, F., Ebens, H.,: The distribution of realized stock return volatility (2001) J. Financ. Econ., 61, pp. 43-76
Andersen 1997 I	Andersen, T., Bollerslev, T.,: Intraday periodicity and volatility persistence in financial markets (1997) Journal of Empirical Finance, 4 (2-3), pp. 115-158

Period III

Table 66 - Volatility and risk, 22 nodes

Short citation / node	Full reference
Heston 1993 I	Heston,: A Closed-Form Solution for Option with Stochastic Volatility with Applications to Bond and Currency Options (1993) Review of Financial Studies, pp. 327-343
Broadie 2006	Broadie, M., Kaya, O.,: Exact simulation of stochastic volatility and other affine jump diffusion processes (2006) Operations Research, 54 (2), pp. 217-231
Lord 2009	Lord, R., Koekkoek, R., Van Dijk, D.,: A comparison of biased simulation schemes for stochastic volatility models (2009) Journal of Quantitative Finance, 10 (2), pp. 177-194

Ninomiya 2008	Ninomiya, S., Victoir, N.,: Weak approximation of stochastic differential equations and application to derivative pricing (2008) Appl. Math. Finance, 15, pp. 107-121
Andersen 2008	Andersen, L.,: Efficient simulation of the Heston stochastic volatility model (2008) Journal of Computational Finance, 11 (3), pp. 1-22
Alfonsi 2010	Alfonsi, A., High order discretization schemes for the CIR process: Application to affine term structure and Heston models (2010) Math. Comp, 79, pp. 209-237
Ninomiya 2009	Ninomiya, M., Ninomiya, S.,: A new higher-order weak approximation scheme for stochastic differential equations and the Runge-Kutta method (2009) Finance and Stochastics, 13 (3), pp. 415-443
Carr 1999	Carr, P., Madan, D.,: Option valuation using the Fast Fourier Transform (1999) J. Comp. Finance, 2, p. 61
Kahl 2006	Kahl, C., Jackel, P.,: Fast strong approximation Monte Carlo schemes for stochastic volatility models (2006) Quant. Finan., 6, pp. 513-536
Revuz 1991	Revuz, D., Yor, M., (1991) Continuous Martingales and Brownian Motion, Springer Verlag: New York
Berkaoui 2007	Berkaoui, A., Bossy, M., Diop, A., Euler scheme for SDEs with non-Lipschitz diffusion coefficient: Strong convergence (2007) ESAIM: Probab. Stat., 12, pp. 1-11
Joe 2008	Joe, S., Kuo, F.Y.,: Constructing Sobol' sequences with better two-dimensional projections (2008) SIAM J. Sci. Comput., 30, pp. 2635-2654
Glasserman 2009	Glasserman, P., Kim, K.,: Gamma expansion of the Heston stochastic volatility model (2009) Finance and Stochastics, 15, pp. 267-296
Andersen 2007 I	Andersen, L., Piterbarg, V.,: Moment explosions in stochastic volatility models (2007) Finance and Stochastics, 11 (1), pp. 29-50
Willard 1997	Willard, G.,: Calculating prices and sensitivities for path-independent derivative securities in multifactor models (1997) Journal of Derivatives, 5, pp. 45-61
Johnson 1995	Johnson, N., Kotz, S., Balakrishnan, N., (1995) Continuous Univariate Distributions, 2., New York: Wiley
Alfonsi 2005	Alfonsi, A.,: On the discretization schemes for the CIR (and Bessel squared) processes (2005) Monte Carlo Meth. Appl., 11, pp. 355-384
Higham 2005	Higham, D., Mao, X.,: Convergence of Monte Carlo simulations involving the mean-reverting square root process (2005) J. Comput. Finan., 8, pp. 35-61
Pitman 1982	Pitman, J., Yor, M.,: A decomposition of Bessel bridges (1982) Probab. Theory Related Fields, 59, pp. 425-457
Yuan 2000	Yuan, L., Kalbfleisch, J.,: On the Bessel distribution and related problems (2000) Ann. Inst. Statist. Math., 52, pp. 438-447
Bossy 2007	Bossy, M., Diop, A., (2007): An efficient discretization scheme for one dimensional SDEs with a diffusion coefficient function of the form {pipe}x{pipe}a, a in [1/2, 1)., RR-5396, INRIA, December 2007
Lyons 2004	Lyons, T., Victoir, N.,: Cubature on Wiener space (2004) Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci., 460 (2041), pp. 169-198
Van 2010	Van Haastrecht, A., Pelsser, A.,: Efficient, almost exact simulation of the Heston stochastic volatility model (2010) Int. J. Theor. Appl. Financ., 13, pp. 1-43

Table 67 - Volatility and option pricing, 22 nodes

Short citation / node	Full reference
Black 1973	Black, F., Scholes, M.,: The pricing of options and corporate liabilities (1973) J. Political Economy, 81, pp. 631-654
Duffie 2000	Duffie, D., Pan, J., Singleton, K.,: Transform Analysis and Asset Pricing for Affine Jump Diffusion (2000) Econometrica, pp. 1343-1376
Eraker 2003	Eraker, B., Johannes, M., Polson, N.,: The impact of jumps in equity index volatility and returns (2003) Journal of Finance, 58, pp. 1269-1300
Madan 1990	Madan, D., Seneta, E.,: The variance gamma model for share market returns (1990) J. Business, 63, pp. 511-524
Bates 1996	Bates, D., Jumps and Stochastic Volatility: Exchange Rate Process Implicit in Deutsche Mark Options (1996) Review of Financial Studies, 9, pp. 69-107
Carr 2000 I	Carr, P., Geman, H., Madan, D., Yor, M., The fine structure of asset returns: An empirical investigation (2000) Journal of Business, 75 (2), pp. 305-332
Jacquier 1994	Jacquier, E., Poison, N., Rossi, P.,: Bayesian analysis of stochastic volatility models (with discussion) (1994) Journal of Business & Economic Statistics, 12, pp. 371-417
Abramowitz 1964	Abramowitz, M., Stegun, I., (1964) Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables, New York: Dover
Bakshi 1997	Bakshi, G., Cao, C., Chen, Z.,: Empirical Performance of Alternative Option Pricing Models (1997) J. Finance, 52, pp. 2003-2049
Hull 1987	Hull, J., White, A.,: The pricing of options as assets with stochastic volatilities (1987) Journal of Finance, 42, pp. 281-300
Barndorff-Nielsen 1998	Barndorff-Nielsen, O.,: Processes of normal inverse Gaussian type (1998) Financ. Stoch., 2 (1), pp. 41-68
Eraker 2004	Eraker, B.,: Do stock market and volatility jump? Reconciling evidence from spot and option prices (2004) Journal of Finance, 59, pp. 1367-1404

Cox 1976	Cox, J., Ross, S.,: The valuation of options for alternative stochastic process (1976) J. Financ. Econ., 3, pp. 145-166
Bates 2000	Bates, D.,: Post-.87 Crash Fears in the S&P 500 Futures Option Market (2000) Journal of Econometrics, 94, pp. 181-238
Madan 1998	Madan, D.B., Carr, P., Chang, E.,: The variance gamma process and option pricing (1998) Eur. Finan. Rev, 2, pp. 79-105
Carr 2003 I	Carr, P., Geman, H., Madan, D., Yor, M.,: Stochastic Volatility for Lévy Processes (2003) Mathematical Finance, 13, pp. 345-382
Eberlein 1998	Eberlein, E., Keller, U., Prause, K., New insights into smile, mispricing and value at risk: the hyperbolic model (1998) J. Bus., 71 (3), pp. 371-405
Spiegelhalter 2002	Spiegelhalter, D.J., Best, N.G., Carlin, B.P., Van Der Linde, A.,: Bayesian measures of model complexity and fit (2002) J. R. Stat. Soc. Ser. B (Stat Methodol.), 64 (4), pp. 583-639
Delbaen 1994	Delbaen, F., Schachermayer, W.,: A general version of the fundamental theorem of asset pricing (1994) Math. Ann., 300 (3), pp. 463-520
Andersen 2002	Andersen, T., Benzoni, L., Lund, J.,: An empirical investigation of continuous-time equity return models (2002) J. Finan., 57, pp. 1239-1284
Koop 2003	Koop, G., (2003) Bayesian Econometrics, London: Wiley-Interscience
Primiceri 2005	Primiceri, G.,: Time varying structural vector autoregressions and monetary policy (2005) Rev. Econ. Stud., 72, pp. 821-852

Table 68 - Early exercise option valuation, 21 nodes

Short citation / node	Full reference
Longstaff 2001	Longstaff, F., Schwartz, E., Pricing American Options by Simulation: A Simple Least Square Approach (2001) Rev. Financial Stud., 14, pp. 113-147
Tsitsiklis 1999 I	Tsitsiklis, J., Van Roy, B.,: Regression Methods for Pricing Complex American Style Options (1999) IEEE Trans. Neural. Net., 12, pp. 694-703., and
Carriere 1996	Carriere, J., Valuation of Early-Exercise Price of Options Using Simulations and Nonparametric Regression (1996) Insur.: Math. Econ., 19, pp. 19-30
Andersen 2004	Andersen, L., Broadie, M.,: A primal-dual simulation algorithm for pricing multi-dimensional American options (2004) Management Science, 50 (9), pp. 1222-1234
Broadie 2004	Broadie, M., Glasserman, P.,: A stochastic mesh method for pricing high-dimensional American option (2004) J. Comput. Finan, 7, pp. 35-72
Broadie 1997 I	Broadie, M., Glasserman, P.,: Pricing American-style securities using simulation (1997) J Econ Dyn Control, 21, pp. 1323-1352., 8-9
Clement 2002	Clément, E., Lamberton, D., Plotter, P.,: An analysis of a least squares regression method for American option pricing (2002) Finance and Stochastics, 6, pp. 449-471
Rogers 2002	Rogers, L.,: Monte Carlo valuation of American options (2002) Math. Finance, 12 (3), pp. 271-286
Haugh 2004	Haugh, M., Kogan, L., Pricing American options: A duality approach (2004) Oper. Res., 52, pp. 258-270
Tilley 1993	Tilley, J.,: Valuing American options in a path simulation model (1993) Trans. Soc. Actuaries, 45, pp. 83-104
Stentoft 2004 I	Stentoft, L.,: Convergence of the least squares Monte Carlo approach to American option valuation (2004) Management Science, 50 (9), pp. 1193-1203
Boyle 1997	Boyle, P., Broadie, M., Glasserman, P.,: Monte-Carlo methods for security pricing (1997) Journal of Economic Dynamics and Control, 21, pp. 1267-1321
Barraquand 1995 I	Barraquand, J., Martineau, D.,: Numerical valuation of high dimensional multivariate american securities (1995) J Finan Quant Anal, 30, pp. 383-405
Belomestny 2009	Belomestny, D., Bender, C., Schoenmakers, J.,: True upper bounds for Bermudan products via non-nested Monte Carlo (2009) Math. Finance, 19, pp. 53-71
Duffie 1996 II	Duffie, D., (1996): Dynamic Asset Pricing Theory, (Princeton University Press: Princeton, NJ)
Kolodko 2006	Kolodko, A., Schoenmakers, J.,: Iterative construction of the optimal Bermudan stopping time (2006) Finance and Stochastics, 10, pp. 27-49
Stentoft 2004 II	Stentoft, L.,: Assessing the least squares Monte-Carlo approach to American option valuation (2004) Rev. Deriv. Res., 7 (2), pp. 129-168
Bally 2003 I	Bally, V., Pages, G.,: A quantization algorithm for solving multidimensional discrete optimal stopping problem (2003) Bernoulli, 9, pp. 1003-1049
Gamba 2002	Gamba, A., (2002) Real options valuation: A Monte Carlo approach, Working Paper, University of Verona
Kan 2009	Kan, K., Reesor, R., Whitehead, T., Davison, M.,: Correcting the bias in monte carlo estimators of american-style option values (2009) Monte Carlo and Quasi-Monte Carlo Methods 2008, pp. 439-454
Schoenmakers 2013	Schoenmakers, J., Zhang, J., Huang, J.,: Optimal dual martingales, their analysis and application to new algorithms for Bermudan products (2013) SIAM J. Finance Math., 4, pp. 86-116

Table 69 - GARCH volatility, 13 nodes

Short citation / node	Full reference
Bollerslev 1986	Bollerslev, T.,: Generalized autoregressive conditional heteroscedasticity (1986) Journal of Econometrics, 31, pp. 307-327
Engle 1982	Engle, R.,: Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation (1982) Econometrica, 50, pp. 987-1008
Nelson 1991	Nelson, D., Conditional heteroscedasticity in asset returns: A new approach (1991) Econometrica, 59, pp. 347-370
Diebold 1995	Diebold, F., Mariano, R.,: Comparing predictive accuracy (1995) J Bus Econ Stat, 13 (3), pp. 253-263
Taylor 1986	Taylor, S., (1986): Modeling Financial Time Series, John Wiley & Sons, New York, USA
Glosten 1993	Glosten, L., Jaganathan, R., Runkle, D.,: On the relation between the expected value and the volatility of the nominal excess return on the stocks (1993) J. Finance, 48, pp. 1779-1801
Andersen 1998	Andersen, T., Bollerslev, T., Answering the skeptics: yes, standard volatility models do provide accurate forecasts (1998) International Economic Review, 39 (4), pp. 885-905
Andersen 2003 I	Andersen, T., Bollerslev, T., Diebold, F., Labys, P.,: Modeling and forecasting realized volatility (2003) Econometrica, 71, pp. 579-625
Zhang 2005	Zhang, L., Mykland, P., Ait-Sahalia, Y., A tale of two-time scales: determining integrated volatility with noisy high-frequency data (2005) J. Am. Stat. Assoc., 100, pp. 1394-1411
Kuester 2006	Kuester, K., Mittnik, S., Paolella, M., Value-at-Risk prediction: A comparison of alternative strategies (2006) Journal of Financial Econometrics, 4, pp. 53-89
Dowd 2002	Dowd, K., (2002): Measuring Market Risk, John Wiley & Sons, Chichester
Ding 1993	Ding, Z., Engle, R., Granger, C.,: A Long Memory Property of Stock Market Returns and A New Model (1993) Journal of Empirical Finance, 1 (1), pp. 83-106
Zumbach 2004	Zumbach, G.,: Volatility processes and volatility forecast with long memory (2004) Quant. Finance, 4, pp. 70-86

Table 70 - Value-at-Risk, 12 nodes

Short citation / node	Full reference
Jorion 2000	Jorion, P., (2000) Value-at-Risk, McGraw-Hill: New York
Kupiec 1995	Kupiec, P.,: Techniques for verifying the accuracy of risk measurement models (1995) Journal of Derivatives, 3, pp. 73-84
Christoffersen 1998	Christoffersen, P.,: Evaluating interval forecasts (1998) Int Econ Rev, 39, pp. 841-862
Engle 2004	Engle, R., Manganelli, S., CAViaR: conditional autoregressive value at risk by regression quantiles (2004) J. Bus. Econ. Statist., 22, pp. 367-381
Mandelbrot 1963	Mandelbrot, B.,: The variation of certain speculative prices (1963) J. Bus., 36, pp. 394-419
Morgan 1996	Morgan, J.P., (1996): RiskMetrics Technical Document, 4th Ed., New York
Geman 1984	Geman, S., Geman, D.,: Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images (1984) IEEE Transactions on Pattern Analysis and Machine Intelligence, 6, pp. 721-41., and. pp
Berkowitz 2011	Berkowitz, J., Christoffersen, P., Pelletier, D.,: Evaluating value-at-risk models with desk-level data (2011) Manage Sci, 57 (12), pp. 2213-2227
Barone-Adesi 1999	Barone-Adesi, G., Giannopoulos, K., Vosper, L.,: VaR without Correlation for nonlinear Portfolios (1999) Journal of Futures Markets, 19, pp. 583-602
Fama 1965	Fama, E.,: The behavior of stock market prices (1965) J. Bus., 38, pp. 34-105
Christoffersen 2004 I	Christoffersen, P., Pelletier, D., Backtesting value-at-risk: A duration-based approach (2004) Journal of Financial Econometrics, 2, pp. 84-108
Gelfand 1990	Gelfand, A.E., Smith, A.,: Sampling-based approaches to calculating marginal densities (1990) Journal of the American Statistical Association, 85, pp. 398-409

Table 71 - Markov chain state pricing, 7 nodes

Short citation / node	Full reference
Green 1995	Green, P.J.,: Reversible Jump Markov Chain Monte Carlo Computation and Bayesian Model Determination (1995) Biometrika, 82, pp. 711-732
Gilks 1996	Gilks, W.R., Richardson, S., Spiegelhalter, D., (1996) Markov Chain Monte Carlo in Practice, (Eds.) (Chapman & Hall: London)
Hastings 1970	Hastings, W.,: Monte Carlo sampling methods using Markov Chains and their application (1970) Biometrica, 57, pp. 97-109., pp
Wuethrich 2008	Wüthrich, M.V., Merz, M., (2008) Stochastic claims reserving methods in insurance, Chichester: Wiley

England 2002	England, P.D., Verrall, R.J.,: Stochastic Claims Reserving in General Insurance (2002) British Actuarial Journal, 8, pp. 443-518
Metropolis 1953	Metropolis, N., Rosenbluth, A.W., Rosenbluth, M.N., Teller, A.H., Teller, E.,: Equation of state calculations by fast computing machines (1953) J. Chem. Phys., 21, pp. 1087-1092
Peters 2009	Peters, G.W., Shevchenko, M.V., Wüthrich, P.V.,: Model uncertainty in claims reserving within tweedie's compound poisson models (2009) Astin Bulletin, 39 (1), pp. 1-33

Table 72 - Contagion and interdependence, 7 nodes

Short citation / node	Full reference
Nelsen 1999	Nelsen, R.B., (1999) An Introduction to Copulas, New York: Springer
Forbes 2002	Forbes, K., Rigobon, R.,: No Contagion, only Interdependence, Measuring Stock Market Co-movements (2002) Journal of Finance, pp. 285-297
Bekaert 2005	Bekaert, G., Harvey, C.R., Ng, A.,: Market Integration and Contagion (2005) Journal of Business, 78 (1), pp. 39-69., DOI 10.1086/426519
Rodriguez 2007	Rodriguez, J., Measuring financial contagion: A copula approach (2007) J. Empir. Finance, 14 (3), pp. 401-423
Embrechts 2002	Embrechts, P., McNeil, A., Straumann, D., Correlation and dependence in risk management: Properties and pitfalls (2002) In Risk Management: Value at Risk and Beyond, pp. 176-223.
Joe 1997	Joe, H., (1997): Multivariate Models and Dependence Concepts, London: Chapman & Hall
Bae 2003	Bae, K., Karolyi, G.A., Stulz, R.M.,: A new approach to measuring financial contagion (2003) Review of Financial Studies, 16, pp. 717-763

Table 73 - Term Structure models, 6 nodes

Short citation / node	Full reference
Cox 1985 I	Cox, J., Ingersoll, J., Ross, S.,: A theory of the term structure of interest rates (1985) Econometrica, 53, pp. 385-408
Vasicek 1977	Vasicek, O.,: An equilibrium characterization of the term structure (1977) Journal of Financial Economics, 5, pp. 177-188
Duffie 1996 I	Duffie, D., Kan, R.,: A yield-factor model of interest rate (1996) Mathematical Finance, 6, pp. 379-406
Dai 2000	Dai, Q., Singleton, K.J.,: Specification analysis of affine term structure models (2000) J. Financ., 55, pp. 1943-1978
Ang 2003	Ang, A., Piazzesi, M.,: A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables (2003) Journal of Monetary Economics, 50, pp. 745-787
Carter 1994	Carter, C.K., Kohn, R.,: On Gibbs sampling for state space models (1994) Biometrika, 81, pp. 541-553

Table 74 - Implied volatility, 6 nodes

Short citation / node	Full reference
Karatzas 1991	Karatzas, I., Shreve, S., (1991) Brownian Motion and Stochastic Calculus, (New York: Springer-Verlag)
Dupire 1994	Dupire, B.,: Pricing with a smile (1994) Risk Mag., January, pp. 18-20
Hagan 2002	Hagan, P., Kumar, D., Lesniewski, A., Woodward, D.,: Managing smile risk (2002) Wilmott Mag, September, pp. 84-108
Fang 2008	Fang, F., Oosterlee, C.W.,: A novel pricing method for European options based on Fourier-cosine series expansions (2008) SIAM J Sci Comput, 31, pp. 826-848
Gatheral 2005	Gatheral, J., (2005) The Volatility Surface: A Practitioners Guide, Wiley Finance
Gyoengy 1986	Gyoengy, I.,: Mimicking the one-dimensional marginal distributions of processes having an Itô differential (1986) Probab. Theor. Relat. Fields, 71 (4), pp. 501-516

Table 75 - Monte Carlo methods and valuation, 5 nodes

Short citation / node	Full reference
Niederreiter 1992	Niederreiter, H., (1992): Random Number Generation and Quasi-Monte Carlo Methods, Philadelphia, PA SIAM

Joy 1996	Joy, C., Boyle, P.P., Tan, K.S.,: Quasi-Monte Carlo Methods in Numerical Finance (1996) Manage. Sci., 42, pp. 926-938., and
LEcuyer 2009	LEcuyer, P.,: Quasi-monte carlo methods with applications in finance (2009) Finance and Stochastics, 13 (3), pp. 307-349
Caflisch 1997	Caflisch, R.E., Morokoff, W., Owen, A.B.,: Valuation of mortgage backed securities using Brownian bridges to reduce effective dimension (1997) J. Comput. Finance, 1, pp. 27-46
Owen 1995	Owen, A.B., Randomly permuted (t, m, s)-nets and (t, s)-sequences Monte and Quasi-Monte-Carlo Methods in Scientific Computing: Proceedings of a Conference at the University of Nevada, pp. 299-317., Las Vegas, Nevada, USA, June 23-25, 1994, edited by H. 5 Niederreiter and P.J-S. Shiue, 1995 (Springer: New York)

Table 76 - Asset returns, 5 nodes

Short citation / node	Full reference
Newey 1987	Newey, W., West, K.,: A simple, positive semi-definite, heteroscedasticity and autocorrelation consistent co-variance matrix (1987) Econometrica, 55, pp. 703-708
Apergis 2004	Apergis, N., Miller, S.M., (2004) Consumption asymmetry and the stock market: Further evidence, University of Connecticut, Department of Economics,,, and
Ng 2001	Ng, S., Perron, P.,: Lag length selection and the construction of unit root tests with good size and power (2001) Econometrica, 69, pp. 1519-1554
Campbell 1988	Campbell, J.Y., Shiller, R.,: Stock prices, earnings, and expected dividends (1988) Journal of Finance, 43, pp. 661-676
Ludvigson 2009	Ludvigson, S., Ng, S.,: Macro factors and bond risk premia (2009) Review of Financial Studies, 22, pp. 5027-5067

Table 77 - Agent based models of markets, 5 nodes

Short citation / node	Full reference
Lebaron 2006	Lebaron, B.,: Agent-based computational finance (2006) Handbook of Compulalional Economics, 2 (1), pp. 187-1233
Friedman 1993	Friedman, D., The double auction market institution: A survey (1993) The Double Auction Markel: Inslilu-lions, Theories, and Evidence, pp. 3-25
Guide 2012	Guide to Tse Trading Methodology, http://www.tse.or.jp/about/books/b7gje60000004q31-att/tradingmethod-ology.pdf, Tokyo Stock Exchange (2012)
Yagi 2010	Yagi, I., Mizuta, T., Izumi, K.,: A study on the effectiveness of short selling regulation using artificial markets (2010) Evolutionary and Institutional Economics Review, 7 (1), pp. 113-132
Yeh 2010	Yeh, C., Yang, C.,: Examining the effectiveness of price limits in an artificial stock market (2010) Journal of Economic Dynamics and Control, 34 (10), pp. 2089-2108

Table 78 - Derivative models, 5 nodes

Short citation / node	Full reference
Broadie 1997 III	Broadie, M., Glasserman, P., Kou, S.,: A continuity correction for discrete barrier options (1997) Mathemati-cal Finance, 7 (4), pp. 325-348
Feng 2008	Feng, L., Linetsky, V.,: Pricing discretely monitored barrier options and defaultable bonds in Lévy process models (2008) Math Finance, 18 (3), pp. 337-384
Clewlow 2000	Clewlow, L., Strickland, C., (2000): Implementing Derivative Models, Chichester, UK Wiley Publications
Haug 2006	Haug, E., (2006) The Complete Guide to Option Pricing Formulas, 2nd ed, New York, NY: McGraw-Hill
Korn 2010	Korn, R., Korn, E., Kroisandt, G., (2010): Monte Carlo Methods and Models in Finance and Insurance, Boca Raton FL Chapman and Hall

Table 79 - Commodity valuation, 4 nodes

Short citation / node	Full reference
Schwartz 1997	Schwartz, E., Stochastic behavior of commodity prices: Implications for valuation and hedging (1997) Journal of Finance, 52 (3), pp. 923-973
Schwartz 2000	Schwartz, E., Smith, J.,: Short-term variations and long-term dynamics in commodity prices (2000) Mgmt Sci., 46, pp. 893-911
Brennan 1985	Brennan, M., Schwartz, E.,: Evaluating Natural Resource Investments (1985) J. Business, 58, pp. 133-158

Casassus 2005	Casassus, J., Collin-Dufresne, P.,: Stochastic convenience yield implied from commodity futures and interest rates (2005) J. Financ., 60, pp. 2283-2331

Table 80 - Systemic banking risk, 4 nodes

Short citation / node	Full reference
Nier 2007	Nier, E., Yang, J., Yorulmazer, T., Alentorn, A.,: Network models and financial stability (2007) Journal of Economic Dynamics and Control, 31, pp. 2033-2060
Allen 2000	Allen, F., Gale, D.,: Financial contagion (2000) Journal of Political Economy, 108, pp. 1-33
Freixas 2000	Freixas, X., Parigi, B., Rochet, J.,: Systemic risk, interbank relations and liquidity provision by the central bank (2000) Journal of Money, Credit and Banking, 32 (3 PART 2), pp. 611-638
Iori 2006	Iori, G., Jafarey, S., Padilla, F.,: Systemic risk on the interbank market (2006) Journal of Economic Behavior & Organization, 61, pp. 525-542

Table 81 - Simulation in capital investment, 4 nodes

Short citation / node	Full reference
Hertz 1964	Hertz, D.,: Risk analysis in capital investment (1964) Harvard Business Review, 42 (1), pp. 95-106
Hoesli 2006	Hoesli, M., Jani, E., Bender, A.,: Monte Carlo simulations for real estate valuation (2006) Journal of Property Investment & Finance, 24 (2), pp. 102-122
French 2005	French, N., Gabrielli, L., Discounted cash flow: Accounting for uncertainty (2005) Journal of Property Investment & Finance, 23 (1), pp. 75-89
Kelliher 2000	Kelliher, C., Mahoney, L.,: Using Monte Carlo simulation to improve long-term investment decisions (2000) Appraisal Journal, 68 (1), pp. 44-56

Table 82 - Macro Finance, 3 nodes

Short citation / node	Full reference
Clarida 2000	Clarida, R., Gali, J., Gertler, M., Monetary policy rules and macroeconomic stability: evidence and some theory (2000) Quarterly Journal of Economics, 115, pp. 147-180
Fernandez-Villaverde 2007	Fernández-Villaverde, J., Rubio-Ramírez, J.F., Sargent, T.J., Watson, M.W.,: ABC's (and D)'s for Understanding VARS (2007) American Economic Review, 97, pp. 1021-1026
Smets 2007	Smets, F., Wouters, R., Shocks and frictions in U.S. business cycles: A bayesian DSGE approach (2007) The American Economic Review, 97 (3), pp. 586-606., JUNE

Additional data on niche diffusion

Table 82 below shows diffusion shares calculated analogously to Figure 7 in the first chapter for additional clusters

Discipline	Source	Niche	# simulation papers	# total papers	% of simulation papers
Finance	Schäffer et al.	Financial intermediation	13	1.024	1,3%
Finance	Schäffer et al.	Asset Pricing	204	4.176	4,9%
Finance	Schäffer et al.	Asset Pricing Macro Factors	-	50	0,0%
Finance	Schäffer et al.	Asset Pricing general models	183	3.321	5,5%
Finance	Schäffer et al.	Asset Pricing anomalies	7	187	3,7%
Finance	Schäffer et al.	Term structure	170	2.437	7,0%
Finance	Schäffer et al.	Market microstructure	89	1.230	7,2%
Finance	Schäffer et al.	Agency conflicts	9	1.341	0,7%
Finance	Schäffer et al.	Agency conflicts Market for control	1	69	1,4%
Finance	Schäffer et al.	Agency conflicts Ownership	-	264	0,0%
Finance	Schäffer et al.	Agency conflicts Capital Structure	1	97	1,0%

Finance	Schäffer et al.	Corporate Diversification	4	243	1,6%
Finance	Schäffer et al.	Internal capital markets	26	1.106	2,4%
Finance	Schäffer et al.	Initial public offerings	9	1.856	0,5%
Finance	Schäffer et al.	Initial public offerings Underpricing	2	523	0,4%
Finance	Schäffer et al.	Initial public offerings Long Term return	1	82	1,2%
Finance	Schäffer et al.	Mutual Funds	41	2.212	1,9%
Finance	Gaunt	Banking & Financial institutions	387	27.047	1,4%
Finance	Gaunt	Behavioural finance	25	877	2,9%
Finance	Gaunt	Experimental finance	2	36	5,6%
Finance	Gaunt	Derivatives	570	6.161	9,3%
Finance	Gaunt	Asset pricing and valuation	944	19.724	4,8%
Finance	Gaunt	market microstructure	89	1.230	7,2%
Finance	Gaunt	Capital structure	36	2.784	1,3%
Finance	Gaunt	Payout policy	143	5.380	2,7%
Finance	Gaunt	Governance	94	29.286	0,3%
Finance	Gaunt	Corporate control	3	855	0,4%
Finance	Gaunt	Organisation	1.671	131.548	1,3%
Finance	Gaunt	Valuation	740	15.336	4,8%
Finance	Gaunt	capital budgeting	74	1.047	7,1%
Finance	Gaunt	investment policy	31	892	3,5%
Finance	Gaunt	incentives	618	31.093	2,0%
Finance	Gaunt	compensation	234	10.982	2,1%
Finance	Gaunt	Mutual funds	39	2.212	1,8%
Finance	Gaunt	Hedge funds	30	1.246	2,4%
Finance	Gaunt	Investment industry	3	74	4,1%
Accounting	Gaunt	Accounting education	11	868	1,3%
Accounting	Gaunt	Auditing	107	11.238	1,0%
Accounting	Gaunt	Corporate governance	14	10.561	0,1%
Accounting	Gaunt	Financial accounting	12	1.467	0,8%
Accounting	Gaunt	Managerial accounting	5	363	1,4%
Accounting	Gaunt	Research methods and methodology in acc.	67	1.551	4,3%
Accounting	Just et al.	Earnings management	11	1.891	0,6%
Accounting	Just et al.	Disclosure	8	2.110	0,4%
Accounting	Just et al.	Executive Compensation	12	1.328	0,9%
Accounting	Just et al.	Auditing Services	107	11.238	1,0%
Accounting	Just et al.	Accounting Systems & Data	42	2.010	2,1%
Accounting	Just et al.	Analyst forecasts	22	896	2,5%
Accounting	Just et al.	Valuation	740	15.336	4,8%
Accounting	Just et al.	Corporate Governance	14	10.561	0,1%
Accounting	Chenhall	Capital budgeting	74	1.047	7,1%
Accounting	Chenhall	Incentives	618	31.093	2,0%
Accounting	Chenhall	Management control systems	15	1.613	0,9%
Accounting	Chenhall	Performance measurement	115	4.526	2,5%
Accounting	Chenhall	Budgeting	132	3.206	4,1%
Accounting	Chenhall	transfer pricing	8	468	1,7%
Accounting	Chenhall	Costing	45	857	5,3%
Accounting	Chenhall	Activity based costing	27	418	6,5%
Accounting	Chenhall	Informal controls	-	115	0,0%
Accounting	Chenhall	MCS in inter-firm relationships	-	7	0,0%
Accounting	Chenhall	Methodological aspects	92	3.747	2,5%
F&A	Chapter 1	Stochastic volatility	450	1.610	28,0%
F&A	Chapter 1	Volatility and option pricing	147	825	17,8%
F&A	Chapter 1	Monte Carlo Valuation	14	14	
F&A	Chapter 1	Volatility	1.493	18.711	8,0%
F&A	Chapter 1	Value at risk	416	2.107	19,7%
F&A	Chapter 1	Volatility and valuation	105	687	15,3%
F&A	Chapter 1	(Least squares) Monte Carlo valuation	42	77	
F&A	Chapter 1	Affine term structure models	18	158	11,4%
F&A	Chapter 1	Financial market statistics	-	-	
F&A	Chapter 1	complex / Exotic option pricing	23	117	19,7%
F&A	Chapter 1	Consumption optimal portfolios & interest rates	17	274	6,2%
F&A	Chapter 1	Multivariate stock market statistics	-	-	
F&A	Chapter 1	Option pricing	333	2.272	14,7%

Table 83 - Diffusion share of simulation research clusters obtained via the method described in chapter 1

Chapter 2

Probability density functions

The PDF describes the general shape of the distribution. Some common examples of families of PDFs are the standard normal / Gaussian distribution, the lognormal distribution or the Weibull distribution. Depending on the variable to be modeled different distributions are applicable to accurately model and capture the range of expected realizations of the given variable. The functional form of a distribution is typically rooted in its data-generating process, e.g. a lognormal distribution can emerge through accumulation of normally distributed small percentage changes that are additive on the logarithmic scale.

Over time a plethora of distributions has emerged that can capture many salient features of DGPs. Many of these distributions have clear theoretical or practical justifications. These include but are not limited to the standard normal, log normal, Mixture (standard normal with skew or Kurtosis) / skew normal, Beta, Weibull, PERT, beta - PERT, Triangular, Poisson, uniform (continuous / discrete), Bernoulli / Binomial, Gamma, Exponential, Pareto, Logistic, Log-Logistic, Students-T, Maximum / Minimum extreme, Negative binomial, Geometric, General discrete, Integer Uniform and Hypergeometric distribution. Through our analysis and the interviews with leading experts we can provide a strong framework for simulation modelers for which core distributions to keep in their toolbox.

Stochastic processes

If a simulation model requires successive draws multiple times from the same marginal distribution that exhibits some form of autocorrelation, then it becomes necessary to define a stochastic process that models this auto-dependence (Law et al., 2010). Put differently, if a simulation needs not only a single draw from a distribution but rather a series of draws that are interrelated in some way, then this requires the definition of how this process is defined stochastically. Stochastic processes include Brownian motion, autoregressive moving-average (ARMA or ARIMA) or generalized autoregressive conditional heteroskedasticity (GARCH).

Parameters of probability density functions

PDFs of various distribution families are characterized by a set of parameters that specify the exact attributes of the distribution (Law et al., 2000). Further below we provide an overview of distributions that are used regularly by practitioners. While the standard normal distribution is fully described by its mean and standard deviation other distributions may require additional or different parameters to further describe its exact shape. Furthermore, its defining parameters are also its descriptive statistics and median, mode and mean coincide. In other distributions, the parameters may not be as straightforwardly understood in an intuitive sense to simulation modelers as for the commonly used Weibull distribution that is defined by three parameters, scale, shape and location. These parameters are also common across many other distributions (Law et al., 2000). The mean of a Weibull distribution is a function of both the scale (α) and shape (β) parameter[16]:

$$mean = \alpha\Gamma(1 + \frac{1}{\beta}) \tag{3}$$

Where Γ represents the gamma function (see Artin, 2015). Thus, the defining parameters determine its descriptive statistics, though not as straightforwardly as for the Gaussian.

Chapter 4

Introduction to Bayesian statistics

[16] Assuming a location parameter of 0

As our method relies on Bayesian updating we provide a brief introduction to Bayesian statistics. The objective here is not to give a general treatment of Bayes Theorem as excellent resources are plentiful (e.g. Kruschke, 2014; Gelman et al. 2014). Bayesian statistics dates to Thomas Bayes who developed its first applications in the 18th century (Gelman et al. 2014). Despite the long intellectual history there has been a growing interest in applications of Bayesian statistics to a host of topics particularly in the finance literature (e.g. Rachev, Hsu, Bagasheva & Fabozzi, 2008). In part this renaissance is due to Markov Chain Monte Carlo methods that allow more efficient calculations if prior and posterior distributions do not necessarily follow the same functional form or follow an unknown functional form. Bayesian statistics at its core shows how a prior believe about statistical properties of data or a process can be updated or improved through incorporation of new information to form a posterior believe.

Bayesian updating of a binary probability

A simple coin tossing example is commonly used to illustrate how Bayes Theorem can be used to update a prior belief through the incorporation of new information (e.g. used in Kelly et al., 2011; Vose, 2008). This illustration assumes a binary probability distribution. Other distributional families can be treated analogously (Fink 1997). Through Bayes theorem we can invert conditional probabilities. For two binary events A and B it holds:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{11}$$

Here P denotes the probability of respective events A and B. This conditional probability allows us to reason about our prior believe based on newly obtained evidence. In this general form our prior of $P(A|B)$ is equal to $P(A)$ if we do not have any knowledge about the probability of occurrence of B. If we obtain knowledge about B (in this binary case this would simply mean B occurred or not) we can make a more precise estimate of $P(A|B)$ as shown in

(1). The process of Bayesian updating describes how an unconditional prior probability or belief can be updated through new information to become a conditional posterior probability or belief. A common illustration for Bayes theorem is a game of chance where a coin is tossed three times and one should guess based on the outcome what type of coin is tossed. Assume that there are four possible coins from which the tossed coin is drawn at random, three fair coins F with $P_F(Head) = 0.5$ and one unfair coin UF with $P_{UF}(Head) = 0.9$. Frequentist statistics allows the calculation of the likelihood of each possible outcome of the three tosses (e.g. the chance of three heads with the unfair coin is given by $P_{UF}(HHH) = 0.9^3 = 0.73$). Bayes Theorem allows us to calculate the relative likelihood that the game was played with each type of coin given an outcome. Here the *prior* estimate, the unconditional likelihood of any coin being a fair coin F is $P(F) = \frac{3}{4} = 0.75$. The *prior* is an unconditional estimate prior to any new information to condition an estimate on. Through Bayes theorem we can incorporate new information NI obtained through the three coin tosses. Assume that one coin is tossed three times each time showing head, thus $NI = HHH$. The posterior believe about $P(F|NI)$ is given by the likelihood function:

$$P(F|NI) = \frac{P(NI|F)P(F)}{P(NI)} = \frac{P(NI|F)P(F)}{P(NI|F)P(F)+P(NI|UF)P(UF)} \tag{12}$$

The middle segment of Equation (2) already reflects the general form that is used to derive the Bayesian posterior that one observes again in the case of continuous distributions:

$$posterior \propto likelihood * prior$$

By substituting the respective probability masses, we derive the likelihood of $P(F|NI)$ for three tosses of heads in a row:

$$P(F|NI) = \frac{P(NI|F)P(F)}{P(NI)} = \frac{P(0.5^3)P(0.75)}{P(0.5^3)P(0.75)+P(0.9^3)P(0.25)} = 34\% \tag{13}$$

In short, the probability of the coin tossed being a fair coin is just 34% given the observation that three heads have been tossed in a row. Comparing this to the unconditional probability of 75% illustrates the predictive power of Bayesian analysis that can be generalized for various distributions.

Experts input

Chapter 4 briefly mentioned the method used to extract expert input following the method described in Winman, Hansson & Juslin (2004). According their research, overconfidence bias can be reduced substantially by inverting the typical elicitation process. In elicitation it is common to extract subjective probabilities via confidence intervals. Experts are thus asked for their assessment of a variables mean, e.g. in our case the future electricity price, and then to ask them on their assessment of a confidence interval of, e.g. 90%. In other words, experts construct intervals of a given confidence level. It has been shown that such intervals are usually estimated too narrowly due to the overconfidence, or more precisely, the overprecision bias (Cooke, 1991). Winman et al. (2004) however have shown that this bias can be reduced by a different elicitation process. Here, elicitors construct confidence intervals and request experts to judge their confidence levels. Via recursion of this elicitation process the subjective probability estimates are derived. This process was applied in the elicitation for this case.

Multivariate regression analysis of simulation experiments

The following tables contain multivariate regression outputs for the simulation experiment from chapter 4.

Regression Statistics: Data input model	
R Square	0,9915
Adjusted R Square	0,9915
Observations	100.000

ANOVA

	df	**SS**	**MS**	**F**	**Significance F**
Regression	4	4,76966E+17	1,19241E+17	2900867,114	0
Residual	99995	4,11034E+15	41105438798		

Total	99999	4,81076E+17			

	Coefficients	Standard Error	t Stat	P-value
Intercept	-81.321.845	42.115	-1.931	0
District heating rate: Location 1 · 10	757.967	568	1.334	0
District heating rate: Location 2 · 10	467.815	762	614	0
Electricity rate · 10	611.574	1.109	552	0
Waste price · 10	534.055	448	1.192	0

Table 84 - Multivariate regression output for the data / new information input modelling specification

Regression Statistics: Naïve update input model	
R Square	0,9956
Adjusted R Square	0,9956
Observations	100.000

ANOVA

	df	SS	MS	F	Significance F
Regression	4	9,27876E+17	2,31969E+17	5633586,498	0
Residual	99995	4,1174E+15	41176085835		
Total	99999	9,31994E+17			

	Coefficients	Standard Error	t Stat	P-value
Intercept	-76.718.169	19.647	-3904,8277	0
District heating rate: Location 1 · 10	715.649	245	2926,836457	0
District heating rate: Location 2 · 10	442.318	247	1791,204447	0
Electricity rate · 10	576.672	378	1527,593447	0
Waste price · 10	503.683	174	2901,700081	0

Table 85 – Multivariate regression output for the naïve updating input modelling specification

Regression Statistics: Posterior input model	
R Square	0,9973
Adjusted R Square	0,9973
Observations	100.000

ANOVA

	df	SS	MS	F	Significance F
Regression	4	8,90572E+17	2,22643E+17	9152192,177	0
Residual	99995	2,43255E+15	24326728363		
Total	99999	8,93004E+17			

	Coefficients	Standard Error	t Stat	P-value

Intercept	-75.081.772	15.355	-4.890	0
District heating rate: Location 1 · 10	700.835	188	3.732	0
District heating rate: Location 2 · 10	433.063	190	2.281	0
Electricity rate · 10	565.034	290	1.945	0
Waste price · 10	493.495	133	3.698	0

Table 86 - Multivariate regression output for the posterior input modelling specification

Regression Statistics: Prior input model	
R Square	0,9986
Adjusted R Square	0,9986
Observations	100.000

ANOVA

	df	**SS**	**MS**	**F**	**Significance F**
Regression	4	8,70484E+17	2,17621E+17	18445727,84	0
Residual	99995	1,17973E+15	11797904067		
Total	99999	8,71663E+17			

	Coefficients	**Standard Error**	**t Stat**	**P-value**
Intercept	-73.913.919	10.505	-7.036	0
District heating rate: Location 1 · 10	690.785	131	5.272	0
District heating rate: Location 2 · 10	426.931	132	3.238	0
Electricity rate · 10	556.698	202	2.762	0
Waste price · 10	486.120	93	5.231	0

Table 87 - Multivariate regression output for the Prior input modelling specification

Modelling assumptions of the financial models

To focus on the properties of different input modelling approaches we keep the model simple where this does not affect accuracy. These assumptions include:

- We assume fixed percentages for annual investment volumes / capital expenditure and a constant rate of depreciation
- Further we assume an absence of non-tangible capital and thus no amortization

To simplify modelling further we assume that interests are calculated on final account balances of the balance sheet as opposed to averages over multiple periods

Bayesian updating with unknown variance

In the applications of Chapter 4, we assumed that the variance of the distributions of independent variables were known throughout the Bayesian updating process. This can be generalized to the case where their variances are also unknown and thus is updated as well. Hence, this represents a setting with a second change in the simulation input modelling environment. This illustration assumes conjugate priors for the variance as well. It can be shown (see e.g. Lynch 2007) that the conjugate prior for the variance follows an Inverse-Gamma distribution. Intuitively, as the variance is strictly positive the normal distribution cannot be its conjugate prior. It is common to work with a Gamma distribution instead of the Inverse-Gamma and invert the variance leading to the intuitive term of *precision* $\lambda = \frac{1}{\sigma^2}$. This inversion is used throughout the literature (e.g. DeGroot et al., 2012) as it leads to a more parsimonious mathematical representation. A reparameterization would enable us to use an Inverse-Gamma distribution and the usual variance term.

Hence, we are interested in the conjugate prior distribution of the mean and the variance that follow a joint Normal-Gamma distribution:

$$\mathrm{Normal} - \mathrm{Gamma}\ (\mu, \lambda | \mu_0, \kappa, \alpha, \beta) \tag{14}$$

That follows the joint Normal-Gamma distribution by definition:

$$\mathrm{Normal} - \mathrm{Gamma}\ (\mu, \lambda \mid \mu_0, \kappa, \alpha, \beta) = \mathrm{N}(\mu | \mu_0, (\kappa_0 \lambda)^{-1}) \mathrm{Ga}(\lambda | \alpha_0, \beta_0) \tag{15}$$

Where μ_0 describes the location parameter of the normal distribution, $(\kappa_0\lambda)^{-1}$ describes the variance, κ describes the number of pseudo-observations for the hyperparameters (prior parameters) that can be interpreted as sample sizes of the observations with properties defined by the prior parameters, α_0 describes the Gamma distribution's shape parameter and β_0 describes the Gamma distribution's rate or inverse scale parameter. The joint Normal-Gamma distribution implies μ is normally distributed conditionally on λ. The parameterization of the

Normal distribution is conditional on the Gamma distribution's parameters; hence we first derive its posterior parameters. It can be shown that the posterior parameters of the Gamma-distributed variance are:

$$\alpha_n = \alpha_0 + \frac{n}{2} \tag{16}$$

$$\beta_n = \beta_0 + \frac{1}{2}\sum_{i=1}^{n}(x_i - \overline{x})^2 + \frac{\kappa_0 n(\bar{x} - \mu_0)^2}{2(\kappa_0 + n)} \tag{17}$$

n stands for the number of observations (i.e. experts' inputs in our case) in the updating process. Subsequently we would derive the posterior values of the Normal distribution:

$$\mu_n = \frac{\kappa_0\mu_0 + n\bar{x}}{\kappa_0 + n} \tag{18}$$

$$\kappa_n = \kappa_0 + n \tag{19}$$

Full derivations for these posteriors are provided in Murphy (2007) and Degroot et al. (2012). Finally, we now obtain expected values for the mean and variance by taking expectations of mean or the Mode over the Gamma and Normal distribution. Hence, the posterior mean is given by (here E denotes the expectations operator):

$$E(\mu) = \mu_0 \tag{20}$$

And the posterior variance is given by:

$$\mathrm{Mode}(\lambda^2) = \frac{\beta_n}{(\alpha_n + 1)} \tag{21}$$

Kalman filter

Kalman filters use series of stochastic, noisy or incomplete measurements over time and gradually incorporate information into estimates of a dynamic system's state that is not directly observable. Mathematically the Kalman filter is based on a representation of Bayesian concepts of prior, likelihood and posterior (Grewal et al. 2001). Three objectives are achieved through these algorithms, filtering, smoothing of time series data and prediction. The Bayesian filter is in fact equivalent to the Kalman filter under the assumption of Gaussian noise and normally distributed measurement errors (Charles, 2011). The Bayesian filter is an application of Bayesian estimation to a setting where estimated parameters change over time thusly requiring recursive re-estimation as new data becomes available over time. Kalman filter is built on the assumption of a measurement error, i.e. it is not possible to directly observe the true state of a dynamic system (Grewal et al., 2001) assumed to be an unobserved Markov state. This is akin to challenges in simulation input modelling. In Kalman filters measurement noise refers to new observations. In input modelling setting described here, these are the errors of the experts' judgments that are assumed to be normally distributed. A derivation is shown in Reid (2001) that demonstrates that the Kalman filter's estimate is the minimum variance unbiased estimator of the state variable.

Chapter 5

Additional analysis of simulation experiment, chapter 5

Table 88 shows input parameters for all factor levels in stage I of the simulation experiment of chapter 5.

Stage I Factor levels	**Average of normal distribution** (1st central moment)	**Standard deviation of normal distribution** (2nd central moment)
Factor level 1: Prior - district heating location 1	22.25€	2.62€
Factor level 1: Prior - district heating location 2	32.39€	2.60€
Factor level 2: Posterior - district heating location 1	21.59€	2.25€
Factor level 2: Posterior - district heating location 2	29.69€	2.47€

Table 88 - Input parameters for stage I of the simulation experiment

Table 89 shows the input parameters for all factor level in stage II of the simulation experiment.

Stage II Factor levels	**Average of normal distribution** (1st central moment)	**Standard deviation of normal distribution** (2nd central moment)
Factor level 1: Prior at 50th percentile – district heating location 1	22.25€	1.87€
Factor level 1: Prior at 50th per-centile – district heating location 2	32.39€	2.24€
Factor level 2: Prior at 5th percentile – district heating location 1	20.24€	1.87€

Factor level 2: Prior at 5th percentile – district heating location 2	30.95€	2.24€
Factor level 3: Bayesian posterior at 50th percentile – district heating location 1	21.59€	1.87€
Factor level 3: Bayesian posterior at 50th percentile – district heating location 2	29.69€	2.24€
Factor level 4: Bayesian Posterior at 5th percentile – district heating location 1	20.24€	1.87€
Factor level 4: Bayesian Posterior at 5th percentile – district heating location 2	30.95€	2.24€

Table 89 - Input parameters for stage II of the simulation experiment

The correlation matrix used in the model remained unchanged from chapter 4. Further, the elicitation method to gain expert input remained unchanged as well.

Approximation of epistemic and aleatoric uncertainty

To approximate the aleatoric and epistemic uncertainty contribution to the compound variation observed in the district heating prices we applied the following method. Note that there existed a reasonably accurate forecast for the average prices across district heating locations. In the past this price has been a strong predictor for individual district heating locations, where the price can vary substantially over sustained periods of time. An ordinary least square (OLS) regression of the following form confirmed the predictive power of the average Southern German district heating rate:

$$Price_{district\_heating\_location\_1\,i} = \alpha_0 + \beta_1\,Price_{district\_heating\_average\,i} + \varepsilon \qquad (22)$$

For the price of district heating the regression output is given below:

Regression Statistics: Naïve update input model	
R Square	0.512
Adjusted R Square	0.415
Observations	7

ANOVA

	df	**SS**	**MS**	**F**	**Significance F**
Regression	1	21.03	21.03	5.25	0.070
Residual	5	20.02	4.00		
Total	6	41.05			

	Coefficients	**Standard Error**	**t Stat**	**P-value**
Intercept	-5.574	9.14	-0.61	0.56

Germany average prices	1.087	0.47	2.29	0.070

Table 90 - Regression output for annual prices series for location 1

Likewise, Table 91 shows regression outputs for the price series of location 2:

Regression Statistics: Naïve update input model	
R Square	0.74
Adjusted R Square	0.69
Observations	7

ANOVA

	Df	**SS**	**MS**	**F**	**Significance F**
Regression	1	30.07	30.07	14.50	0.013
Residual	5	10.37	2.07		
Total	6	40.43			

	Coefficients	**Standard Error**	**t Stat**	**P-value**
Intercept	-0.88	6.58	-0.13	0.909
Germany average prices	1.30	0.34	3.81	0.013

Table 91 - Regression output for annual prices series for location 2

Note that per the R^2 metric the regression models explain 0.415 and 0.969 of the variation observed. This part, the "explained" part of the variation, is here interpreted as aleatoric uncertainty and the remainder, the "unexplained" part as the epistemic variation. In other words, the epistemic variance is approximated through the regression's residuals and calculated as the variance of these residuals. Through this method we obtain a variance decomposition shown in table 92:

	Location 1	Location 2
Total Variance	6,84	6,74
Total Standard deviation	2,62	2,60
"Epistemic" variance	3,34	1,73
"Epistemic" SD	1,83	1,31
"Aleatoric" variance	3,50	5,01
"Aleatoric" SD	1,87	2,24

Table 92 - Variance decomposition for the price series of district heating in location 1 & 2

This method approximates the epistemic and aleatoric uncertainty shares in the variation of each location's district heating rates.