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Institute for Ship Structural Design and Analysis (M-10)
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NUMERICAL SIMULATION OF FATIGUE CRACK GROWTH IN LOAD- AND NON-LOAD-CARRYING FILLET WELDED JOINTS

Project Thesis

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Declaration

Hamburg,

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I am thankful for the opportunity to write my project thesis at the Institute for Ship Structural Design and Analysis M10. It was a great chance to practice my technical skills and getting an exposure to the scientific writing.

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Abstract

Fatigue is one of the major causes of failure in cyclically loaded structures such as, bridges, locomotive parts, rotating machinery components, etc. Welded structures are common in ship building. These welded structures are subjected to fluctuating loads in offshore environment, which requires proper attention for fatigue failure in the design phase of such structures. Although, lab tests can be conducted for concerned materials under controlled environmental conditions, but it is a cost and time intensive approach, and is not always possible. A number of assessments have been carried out over the years for a number of common structural configurations to standardize the design for fatigue and make it less cost-intensive. However, the amount of established standards is still very small relative to the number of possible design configurations. Hence, there is a constant need for numerical models which aim at estimating the fatigue life without the need for tests to be conducted.

Crack Propagation is one of the established numerical approaches used for assessing remaining fatigue life of structures which are already in service. This approach is based on the assumption of an initial crack already existing in the structure, and consequently, the remaining fatigue life is considered to be dominated by crack propagation stage. The same can be considered true for welded structures, where weld imperfections and weld gaps act as micro-cracks and the service life of such structures can be considered to be dominated by crack growth stage. This study aims at evaluating the ability of crack propagation approach to predict the fatigue life of fillet welded joints. For this purpose, a numerical simulation tool called Franc2D is used to calculate the stress intensity factors (SIF) from 2D models. These values along with other factors are used as inputs for Paris and Erdogan Law to predict the fatigue life of load carrying and non-load carrying fillet welds. These results are then compared to the lab test results and the conclusions are drawn.

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PROJECT THESIS

Numerical simulation of fatigue crack growth in cruciform joints failing from weld toe and root

The fatigue life of structures can be divided in two stages. While the crack initiation stages can occupy up to 90% of the lifetime of smooth specimen (Clark and Knott, 1975), the fatigue life of welded structures is generally propagation-dominated. Once a crack can be detected in a smooth specimen, the lifetime until final fracture is seemingly short. However, there is also a wide range of situations that are neither initiation- nor propagation-dominated. Such situations mainly cover structures with stress concentrations less severe than at welded joints. Thus, fatigue assessment and test methods vary significantly for those three cases.

Recommendations for using material data for fatigue crack growth analysis in failure assessment procedures and codes like BS 7910:1999 or API 579-1, are usually given regardless of the service conditions like temperature. However, fatigue assessment based on crack growth simulation is a difficult procedure due to many influencing factors. Hence, this project is concerned with the investigation of temperature effects on fatigue crack growth in welded joints under low temperatures.

For this purpose, finite element simulations will be applied.

1) Modelling of the fatigue crack growth in cruciform joints under tensile loading in FRANC2D.
2) Assessment of the effect of temperature on crack growth rate curves based on literature data and relevant industry standards
3) Calculation of fatigue life for different test temperatures and
4) Comparisons of the numerical results with the small-scale test results.

Literature studies of specific topics relevant to the thesis work shall be included. The work scope may prove to be larger than initially anticipated. Subject to approval from the supervisors, topics may be deleted from the list above or reduced in extent. In the thesis the candidate shall present his personal contribution to the resolution of problems within the scope of the thesis work. Theories and conclusions should be based on mathematical derivations and/or logic reasoning identifying the various steps in the deduction.

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The thesis shall contain the following elements: An executive summary, list of symbols and acronyms, followed by the main body of the thesis consisting of a brief background introduction, a state of the art defining the knowledge gaps defining the scope or work and limitations, the actual contribution chapters, conclusions with recommendations for further work, references and (optional) appendices. All figures, tables and equations shall be numerated. The supervisors require that the candidate, in an early stage of the work, presents a written plan for the completion of the work. The plan may include a budget for the use of computer and laboratory resources if applicable, which will be charged to the department. Overruns shall be reported to the supervisors.

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Thesis supervisors
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Acronyms

SIF	Stress Intensity Factor
IIW	International Institute of Welding
FAT	Fatigue Classes
LEFM	Linear Elastic Fracture Mechanics
BS	British Standard
FEM	Finite Element Methods
GPa	Giga Pascal
MPa	Mega Pascal
2D	Two Dimensional
3D	Three Dimensional
Franc2D	Numerical Solver
Casca	Pre-Processing Software Tool
MIZ	Marginal Ice Zones
SED	Strain Energy Density

List of Symbols

σ_{max}	Maximum Stress
σ_{mean}	Mean Stress
σ_{min}	Minimum Stress
R	Stress Ratio
m	Constant
N	Number of Cycles for Fatigue
$\Delta\sigma_{applied}$	Net Applied Stress
σ_m	Membrane Stress
σ_b	Bending Stress
σ_{nlp}	Non-Linear Part of Stress
K_t	Stress Concentration Factor
σ_K	Notch Stress
σ_n	Nominal Stress
r_{ref}	Reference Radius
r_{real}	Actual Root Radius
ρ^*	Microstructural Length
s	Support Factor
a_c	Crack Length
K_c	Fracture Toughness
ΔK	Net Stress Concentration
$\frac{da}{dN}$	Rate of Crack Growth
C	Constant
M_k	Magnification Factor
Y	Geometry Factor
$M_{k,m}$	Magnification Factor for Membrane Stress Component
$M_{k,b}$	Magnification Factor for Bending Stress Component
Y_m	Geometry Factor for Membrane Stress Component
Y_b	Geometry Factor for Bending Stress Component
H	Weld Leg Length
t	Plate Thickness

1. Introduction

1.1 Problem Description

Fatigue life is a major design criterion when designing for cyclic loaded structures. Although, fatigue life can be estimated by conducting lab tests but these tests are cost intensive and not always possible. Therefore, various mathematical models and approaches have been developed over the years which aim at estimating the fatigue life of the structures in the design phase [1].

In addition to nominal stress approach, structural hot spot approach is also widely used because of its capability to effectively take into consideration the stress increase due to structural configuration [2]. Further structural stress approaches have also been proposed in the literature and compiled by Radaj et. al [3]. However, structural approaches bring a limitation as the need for establishing different S-N curves for different design configurations and also these approaches primarily focus on weld toe failures. Therefore, a need still exists for a refined approach to predict fatigue life of welded structures with improved accuracy and reduced complexity [1].

Crack propagation approach is used to assess the structures for the remaining life and is based on a well-established crack growth law proposed by Paris and Erdogan [4]. This approach assumes the existence of an initial crack in the structure and that the life of the structure is dominated by crack propagation stage. This assumption also holds true for welded structures where, welding imperfections act as micro cracks. Therefore, crack propagation approach can be applied to the welded structures in the design phase to assess the fatigue life of the structures to be designed.

Another factor which could affect the fatigue life are the sub-zero service temperatures. There has been little work done to assess the effects of sub-zero temperatures on the fatigue life. The fatigue life approaches mentioned above provide no consideration.

Present study evaluates load-carrying and non-load carrying fillet welds using 2D crack simulator Frnac2D to obtain the stress intensity factors (SIF). These SIF values have been then used in crack propagation approach to predict the fatigue life. A comparison has been provided for the estimated fatigue life values against the test results. Furthermore, SIF values obtained from Franc2D have also been validated against SIF's obtained from analytical formulae proposed in the standards.

1.2 Goals and Purpose

As part of an effective fatigue design strategy for welded structures, the data such as initial crack length, final crack length, crack growth curves, and S-N curves should be established accurately. Only then an accurate estimation for fatigue life can be made. Unfortunately, in most cases, the actual crack lengths and crack growth properties are not known or established.

Although, some common design configurations and joints have been standardized and the data has been published in standards like BS 7910 [5] by British standard institution and fatigue design recommendations by International Institute of Welding (IIW) [6]. These standards provide design recommendations and data for a number of possible geometries. However, if the geometry to be designed is not found in these standards, the establishment of an S-N curve requires a number of tests to be conducted and a lot of resources to be employed. Hence, there is a lot of research been conducted to identify the numerical methods which could provide an accurate estimation of fatigue life without the need of conducting resource intensive tests.

Over the years, various numerical and analytical models have been developed to estimate the fatigue life. However, there are some gaps for a method to provide a close estimation for some welded structures. Fillet welds both load-carrying and non-load carrying (discussed in detail in section 2.5) are of main focus in this study. There has been some research conducted specifically for fatigue life estimation of fillet welds. Radaj et al. [3] has provided an overview of local approaches for fatigue assessment of welded structures. In another study by Fricke et al. [1] on fillet welds, it was concluded that crack propagation approach provides too conservative and small values of fatigue life which was considered to be associated with two dimensional 2D crack simulation.

This study aims at further investigating the capabilities of crack propagation approach to predict the fatigue life of fillet welded joints. For this purpose, a simplistic 2D numerical simulation tool (Franc2D) has been used to propagate the crack from an initial crack length to final fracture. The SIF values obtained from the tool have then been used as an input for crack propagation formulas to predict the fatigue life. The fatigue life values are then compared against the actual test results obtained from the laboratory tests to determine the accuracy of crack propagation approach in predicting the fatigue life.

In order to validate the SIF values and the numerical model, SIF values obtained from Franc2D are compared against the values obtained from analytical formulae present in the literature. Finally, the study also aims to look at the temperature dependence of fatigue life in order to fill the research gap for fatigue life dependence on sub-zero temperatures.

2. Theory

Metallic components in dynamic structures can fail under cyclic stresses even at magnitudes well below the tensile strength of the material. These cyclic stresses lead to microscopic imperfections which can eventually grow into cracks leading to the structural failure. This phenomenon of structural failure under cyclic loading is known as fatigue [7].

Considering the steel structures, Ohme conducted a study on the types of structures and their causes of failure. Railway and road bridges have been shown to be the most vulnerable steel structures in the study. Fatigue failure has been ruled out as the third most relevant cause of failure [8]. It is also one of the common failure phenomenon in offshore structures and ships [1]. Fatigue life in general is divided into three phases as follows.

- **Crack Initiation Period:** This is the stage where crack is nucleated either from the surface imperfections or notches with high stress concentrations. The crack initiation happens as a result of plastic deformations or slips at the grain boundaries on microscopic levels. This crack can then continue to grow into a micro crack which is still a surface phenomenon.
- **Crack Growth Period:** Once the crack has grown to a level where it is no longer a surface phenomenon but depends on the material as bulk property, it can be termed as crack growth stage. This is the stage which consists most of the service life of various components.
- **Final Fracture:** This is the stage where stress intensity become so large that it crosses the fracture toughness of the material and it leads to ultimate fracture [9].

2.1 General Concepts

For some applications the higher and lower end of the loads are same, which leads to a constant amplitude loading. This constant amplitude fatigue loads can vary between a maximum σ_{max} and minimum σ_{min} stress values with a stress range $\Delta\sigma$ (Eq. 2.2) and mean stress σ_{mean}, as shown in Eq.2.1 [7]. The constant amplitude loading is shown in Figure 1.

However, in many dynamic structures like, a railway bridge, wheel of a vehicle, and landing gear of a plane, the dynamic loads can vary quite frequently and it is almost unrealistic to obtain a standard load spectrum for these applications. To perform a fatigue analysis of such structures, the actual load spectrum is obtained with the help of real time data acquisition. These complex load spectrums are then converted into an equivalent constant amplitude spectrum with the help of methods such as: level crossing method, rainflow count method, range count method etc. [9].

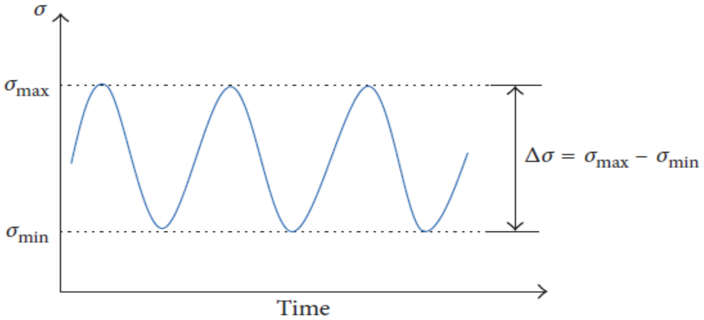


Figure 1: Constant amplitude cyclic loading for fatigue assessment [4].

$$\sigma_{mean} = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{2.1}$$

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \tag{2.2}$$

Previous discussion is based on one type of constant amplitude loading where mean stress σ_{mean} is zero, i.e., σ_{max} is equal to σ_{min}. However, this is not the case for all the loading applications even when converted to constant amplitude loadings. This is where the stress ratio R comes into consideration as shown in Eq.2.3. Stress ratio is the ratio between the maximum stress to minimum stress, which takes into account the situations where the magnitude compressive stresses are not the same as tensile stresses, and vice versa, see Figure 2.

$$R = \frac{\sigma_{min}}{\sigma_{max}} \tag{2.3}$$

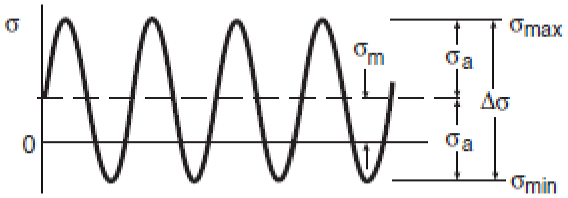


Figure 2: Constant amplitude cyclic load with non-zero stress ratio [1, p. 419] .

2.2 S-N Curve and FAT Classes

S-N curve is a plot of stresses against the number of stress cycles that a particular structural element can undergo before failure. The number of cycles varies according to the maximum cyclic stress. The S-N curve is developed by conducting a series of laboratory tests for various stress amplitudes for a specific geometry and material. Once the data for a specific structure is obtained from laboratory tests, a standard S-N curve is established for that particular specimen and material [10]. There are many S-N curves already published for common applications in standards. A typical S-N curve for mild steels and other materials which harden by strain ageing is shown in Figure 3. The solid line hits a plateau beyond 10^6 cycles and this point is called endurance limit. If the maximum stresses are below this endurance limit, the structure is considered to exhibits unlimited fatigue life [11].

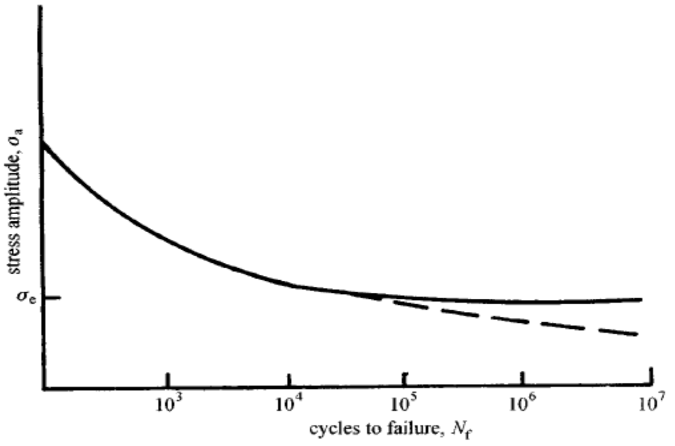


Figure 3: A typical S-N curve for smooth ferrous and nonferrous alloys [5, pp. 222].

A number of specimens including welded joints have been tested for fatigue life and the S-N curves have been established in various standards. For instance, IIW recommendations use FAT classes to classify the welded joints and BSK 99 uses a C-value to determine the right type of joint for references. Both of these methods determine the classes based on stress range MPa for a fatigue life of N=2x 10^6 cycles with a 2.7 % probability of failure. This means that a joint with FAT =x or C =x will have a 97.7% probability of survival for N=2x 10^6 number of cycles [6]. The equation for FAT class can be written as shown in Eq. 2.4.

$$FAT = \Delta\sigma_{applied} \sqrt[m]{\frac{N}{2 \times 10^6}} \tag{2.4}$$

2.3 Fatigue Assessment Methods

The various fatigue assessment methods that exist today can be classified into two major categories, global or local approaches. Global approaches are usually based on the macro geometry and principal stresses in the cross section of the geometry, while neglecting local stress concentrations. On the contrary, local approaches address the local stress concentrations and local effects of changes in geometry, with each method presenting limitations in the application as well as prediction accuracy[12]. There are essentially three kinds of local assessment methods, elastic structural stress or strain methods, notch stress or strain methods, and linear elastic fracture mechanics [13]. The focus of this study is linear elastic fracture mechanics or crack propagation approach, hence the other methods have been discussed briefly in this section.

2.3.1 Nominal Stress Method

Nominal stress method is the most established approached and is supported by various standards. The model is based on the stresses in the cross section of the geometry under consideration while ignoring the local stress concentrations. This method does take into consideration the effects of local geometry changes. Although, S-N curves have been established for various common structural elements, the application is only limited to the specimens for which S-N data already exists. This is one of the major limitations of this approach. The other main limitation is for the complex geometries for which the analysis must be done on the local stress concentrations. For this kind of geometries, one of the local approached must be used [12].

2.3.2 Structural Stress or Strain Approaches

Structural stress and strain approaches are primarily based on the consideration that the structural configurations lead to higher stresses near the welds or material discontinuities in the structural elements. The S-N curved are used only on the basis of weld types, however, the exact estimation of the stresses in the vicinity of notches is challenging. Hence, there are a number of approaches which have been developed to estimate the structural stresses [13].

For non-welded structures, the results are in the form of qualitative assessments rather than a quantitative fatigue strength [14]. As far as non-welded structures are considered, “The aim of these approaches is to shield the notches from crack initiation and shift the critical areas of the structural design to the notch free parent material” [3]. Once the structural stress is calculated for non-welded structures, the newly designed structural element can be compared against a similar geometry for which the crack initiation data is already known provided that the benchmark geometry’s material, environment and geometry match with the newly designed element. The two designs can be compared such that the calculated structural stress amplitudes in the new design are always below the crack initiation stresses for the already tested benchmark [3].

As for welded structures, structural stress approach can give some quantitative assessments for the fatigue life of critical areas so that the expensive methodologies (e.g., notch stress approach) can be avoided [3]. The approach is largely based on structural stress values σ_s in the vicinity of the notches calculated from structural theories. However, this stress doesn’t take into consideration the local notch effects directly. For instance, if we take into consideration a structural approach named as structural hot-spot stress for non-load carrying fillet welds, the weld acts as a notch and the stresses in the vicinity of this notch are non-linear as shown in Figure 4. In the figure, σ_{nlp} The non-linear part of the stress field is addressed by the designer when selecting an S-N curve, while the linear portions of the stress field, σ_m (membrane stress) and σ_b (bending stress), are added up as structural stress, mentioned above [15].

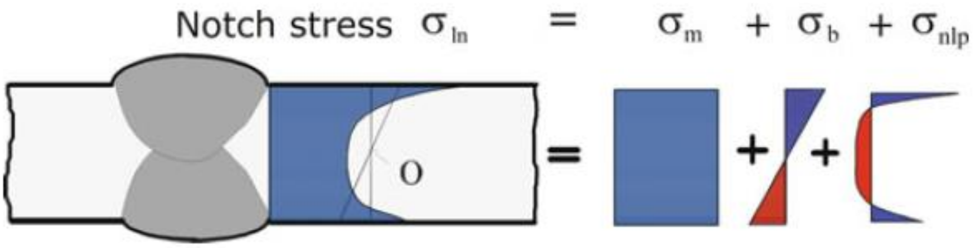


Figure 4: Nonlinear notch stresses in the vicinity of weld toe in non-load carrying joints (An example of structural hot spot method).

The structural stresses for welded joints can be either calculated using numerical methods as proposed by Niemi et al [15] or the stresses can be measured with a strain gauge by constructing a sample weld in the design phase as proposed in [3] by Radaj et al. A comparison of fatigue life estimates obtained from different structural hot spot methodologies against test results is given by by Frike et al. [16]. However, different methodologies were not able to correlate with each other to a satisfactory level, but the fatigue life results obtained from these methodologies correlate to the test results within an acceptable level. Hence, structural approaches can be used for making conservative assessments for welded joints [15].

2.3.3 Effective Notch Stress Approach

Structural stress methods discussed in 2.3.2 are based on linear stress increase from far field stress to the local stresses, where non-linear part of local stress is considered in fatigue design curves. Due to large diversity in local stresses for weld toes and weld roots, there is a need for large number of fatigue design curves [14]. In order to effectively consider the local notch stresses within the fatigue analysis, notch stress method can be used [6].

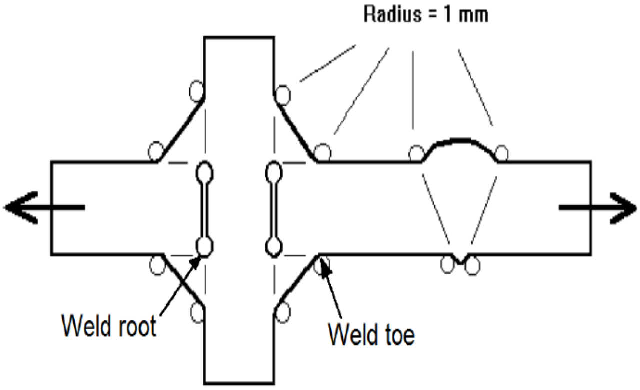


Figure 5: Effective notch radius of 1mm for structural steels [6].

Notch stress method is based on the local notch stress concentration factor K_t, which is the ratio between the maximum notch stress σ_k and nominal stress σ_n, see Eq 2.5. The underlying assumption of the method is that material behaves in linearly elastic manner. Notch stress can be calculated using functional analysis methods based on theory of elasticity, numerical methods, or measuring methods [3].

To address the singularities and non-linarites at the base of the roots, the actual sharp notch (or weld root) radius is replaced by a so called effective notch radius as can be seen in Figure 5. The assumption for effective notch stress is that actual notch radius approaches zero $r_{real} \rightarrow 0$. The relationship between r_{real} and r_{ref} , as shown in Eq. 2.6, is defined with respect to a material dependent microstructural length ρ^* and a support factor s which addresses multiaxial stresses based on von Mises stress hypothesis [14, 17].

$$K_t = \frac{\sigma_K}{\sigma_n} \tag{2.5}$$

$$r_{ref} = r_{real} + \rho^* \times s \tag{2.6}$$

2.4 Crack Propagation Approach

Crack propagation approach is based on linear elastic fracture mechanics principles, which characterises the behaviour of cracks in a cyclic loaded structure with the assumption that the material behaves in a linear-elastic manner. As discussed in section 2.1, crack propagation is usually divided into three stages. The LEFM approach predicts the crack growth over the three stages until final failure is reached. The number of cycles that a structure can undergo, under the applied loads, until final fracture is referred to as fatigue life *N [6]*.

The crack propagation approach characterises the crack growth with the stress intensity factor range ΔK_I at the crack tip. The crack types are usually divided into three modes in the literature, Mode I, Mode II, and Mode III, as indicated in Figure 6. Mode I, also known as crack opening mode, is where the stresses are applied perpendicular to the crack propagation plane and is the focus of this study. Mode II and Mode III are shear and tear modes respectively, which are out of the focus for this study. The subscript *I* in the stress intensity factor range ΔK_I signifies the stress intensity factor for mode I. However, this subscript will not be mentioned in the later parts of the study for the sake of simplicity, since all the following discussion will be based on Mode I [18].

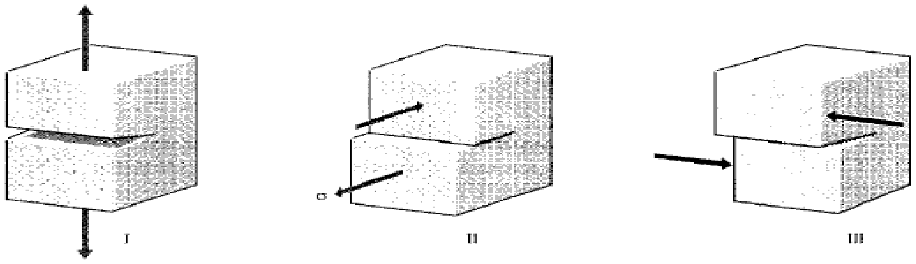


Figure 6: Crack modes in crack propagation approach. I) Opening mode. 2) Shear mode. 3) Tear mode [14]

Crack propagation approach is of main focus in this study. In this study the approach has been applied to the cruciform load carrying and non-load carrying joints. An FEM solver developed by Cornell Fracture Group [19] has been employed to calculate the stress intensity factors (SIF) and these values are then used to predict the fatigue life for these joints. A detailed explanation of the Stress Intensity formulae, Paris Erdogan Law, and Fatigue Life formula is given in state of the art, Section 3 of the study.

2.5 Cruciform Joints

The cruciform joints, load-carrying as well as non-load carrying, are vulnerable to high stress concentrations near the location of welds which behave as notches. The welding gaps and residual stresses result in elimination of the crack initiation phase and the major part of fatigue life for welded joints then considers of crack propagation approach

2.5.1 Non-Load Carrying Cruciform Joints

Non-load carrying fillet welds are the joints where the loads are not transmitted into the main member through the weld. As seen in Figure 7, the load lines from the tensile load are being transmitted through the weld and across the continuous structural member. Even though the weld is not transmitting the load across the members, it is still constrained by the load in the stressed member. The crack initiation for non-load carrying fillet weld always occurs at the weld toe regardless of the weld size [20].

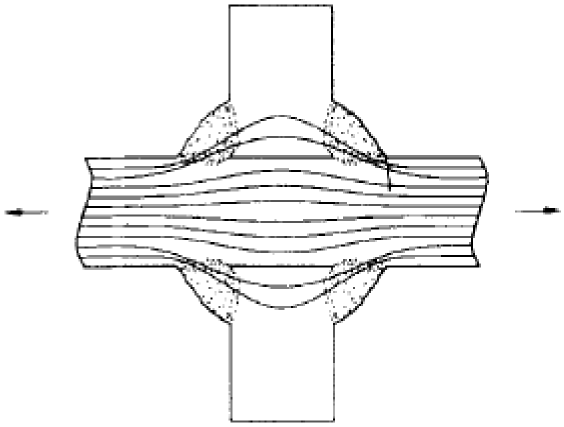


Figure 7: Non-load carrying joint under tensile loading [16] .

2.5.2 Load Carrying Cruciform Joints

Load carrying joints are designed to transmit load from one member to another. As shown in Figure 8, the significant part of the load is transmitted through the welds and across the continuous member. For these joints, there are two possible locations for high stress concentrations, the weld root gap due to the lack of penetration or the weld toe. Crack can initiation at either of these points depending on the weld size to plate thickness ratio. If the weld throat size to plate thickness ration is small, the stress at the weld root can be high

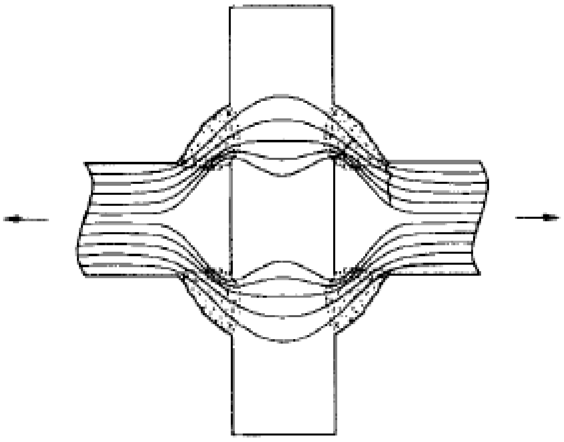


Figure 8: Load carrying fillet weld under tensile loading [16].

enough to cause root failure before the weld toe fails. This case has been the focus of investigation in this study where the load-carrying fillet weld fails at the weld root, as discussed in the later parts of the study [20].

3. State-of-the-Art

3.1 Analytical Formulae for SIFs

The formulae presented in this section have been used to validate the SIF results obtained from Franc2D simulations.

3.1.1 Fracture Mechanics (Paris Law)

Crack propagation approach predicts the number of cycles (Fatigue Life *N*) it takes for an initial crack of length a_i to grow to a final crack length of a_f. The three stages of crack propagation are represented by the graph shown in Figure 9. Stage II of crack growth curve occupies most of the fatigue life and considers the crack growth from the point where crack is no longer a surface phenomenon (threshold value of SIF K_{th}) until the crack reaches a critical crack length a_c. or the stress intensity factor becomes equal to the fracture toughness of the material K_c. The crack propagation approach in the graph is described by the Paris Erdogan Law as given in Eq. 3.1[1, 3, 6].

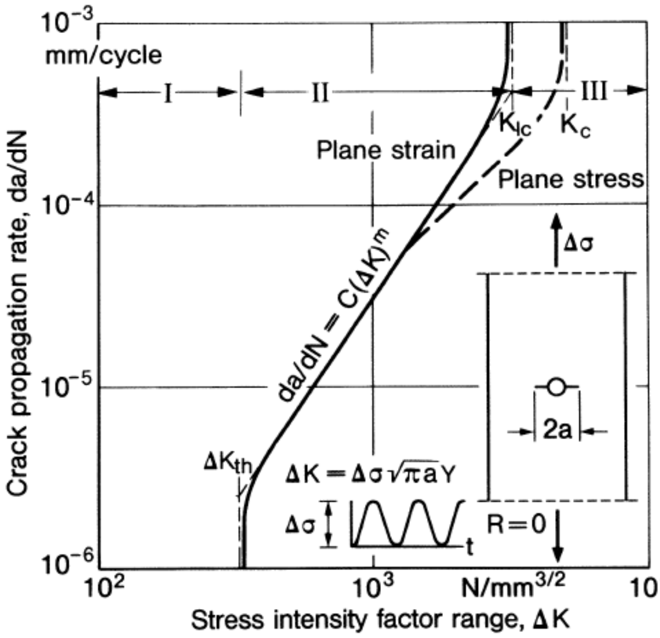


Figure 9: Crack growth curve according to Paris Erdogan Law. [9]

$$\frac{da}{dN} = C(\Delta K)^m \tag{3.1}$$

For welded structures with undetectable imperfections, it is recommended to consider the presence of an initial crack. The welding locations are sensitive spots with higher stress concentrations and vulnerability for undetected presence of imperfections in the form of weld gap or sharp notches. For this reason, it is recommended to use an initial crack of 0.1 mm for fillet welds failing from weld toe. In case of root failure for cruciform load-carrying joints, the actual root gap is to be considered as initial crack [6].

The integrated form of Eq.3.13.3 can be used to predict the fatigue life of the crack provided a reasonable initial crack length is assumed. The integral is given in Eq.3.2. For this study, Franc2D solver has been used to calculate the SIF values. The SIF values are then used in Eq. 3.2 to predict the fatigue life results presented in section 5.1.

$$N = \frac{2(1-R)}{(m-2)C(M_k Y \Delta\sigma\sqrt{\pi})^m}\left[\left(\frac{1}{\sqrt{a_i}}\right)^{m-2} - \left(\frac{1}{\sqrt{a_f}}\right)^{m-2}\right] \qquad 3.2$$

C and m factors recommended in BS 7910 [5] for marine structures have been employed in the study for estimation of fatigue life. The aforementioned study recommends two stage crack propagation path to estimate the fatigue life. Figure 10 shows stage A and B with transition points given with certain stress values for steels in marine environment. Stage A and B are differentiated with different slopes and hence different C (signified by $A_1$and A_2 in Figure 10) and m factor values. These values have been used later in the study to estimate the fatigue life of fillet welds under consideration.

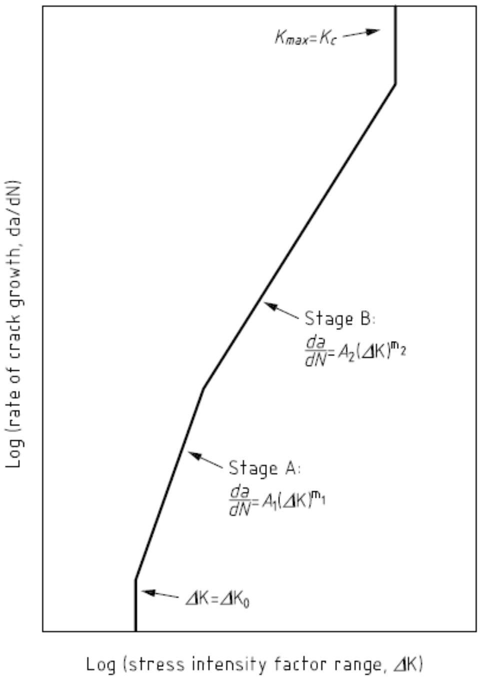


Figure 10: Two stage crack propagation recommended in BS 7910.[9]

3.1.2 Weld Throat Equations

The main formula for calculating the SIFs for weld throat is given in Eq.3.3. This formula is proposed in IIW recommendations for fatigue life calculations [6]. The magnification factor M_k given in Eq.3.4 to Eq.3.6, are also given in IIW recommendations. Although, the geometry correction factor Y_m for double edge crack under tensile loading, as given in Eq.3.7, is applied for non-load carrying joints.

$$K = \sqrt{\pi}.a.\left(\sigma_m.Y_m(a).M_{k,m}(a) + \sigma_b(a).Y_b(a).M_{k,b}(a)\right) \tag{3.3}$$

$$M_k = C.(\frac{a}{T})^k \ , \qquad M_k \leq 1 \tag{3.4}$$

$$C = 0.8068 - 0.1554\left(\frac{H}{T}\right) + 0.0429\left(\frac{H}{T}\right)^2 + 0.0794\left(\frac{W}{T}\right) \tag{3.5}$$

$$k = -0.1993 - 0.1839\left(\frac{H}{T}\right) + 0.0495\left(\frac{H}{T}\right)^2 + 0.0815\left(\frac{W}{T}\right) \tag{3.6}$$

$$Y_m = 1.98 + 0.36\left(\frac{2a}{T}\right) - 2.12\left(\frac{2a}{T}\right)^2 + 3.42\left(\frac{2a}{T}\right)^3 \tag{3.7}$$

3.1.3 Weld Root Equations

Weld root formulas proposed in IIW recommendations have been employed to calculate the SIFs for load-carrying cruciform joints. These formulas

$$K = \frac{\sigma.\left(A_1 + A_2.\frac{a}{w}\right).\sqrt{\pi}.(a.\sec(\pi.\frac{a}{2w}))}{1 + 2.H/t} \tag{3.8}$$

where, $w = H + t/2$
$X = H/t$

$$A_1 = 0.528 + 3.287(x) - 4.361(x)^2 + 3.696(x)^3 - 1.875(x)^4 + 0.415(x)^5 \tag{3.9}$$

$$A_2 = 0.218 + 2.717(x) - 10.171(x)^2 + 13.122(x)^3 - 7.755(x)^4 + 1.783(x)^5 \tag{3.10}$$

Formulas mentioned in section 0 and 3.1.3 have been used in the study to calculate SIF values to validate the results obtained from simulations, in later parts of the study.

3.2 Fatigue Assessment at Sub-Zero Temperatures

The continuous ice breaking events in the Arctic regions are creating so called Marginal Ice Zones (MIZ) which have led to increased activities in these cold offshore environments. The structures and ships are however not fully designed according to these extreme environments. The interaction between the ice and waves can create complex loading conditions on the structures, which in turn can not only affect the material properties but also the long term fatigue phenomenon. Hence, the need for methods to assess the sub-zero temperature effects on fatigue life is imminent. However, there are a large amount of knowledge gaps related to ice mechanics and fracture mechanics at sub-zero temperatures which still exist, there have recently been some studies conducted where different methods have been proposed to incorporate the temperature effects in the design for fatigue [21].

3.2.1 Strain Energy Density (SED) Method

This method extends the Stress Energy Density method to welded joints to directly incorporate the temperature changes in the fatigue assessment. Unlike other stress methods which require modification factors to account for temperature effects, SED method directly takes into account the changes in the material support effects and Young’s Modulus. The method is based on a temperature modification function which is developed to extend SED method for welded joints. The method requires an estimation of suitable control radii, which can also be derived by balancing the room temperature and sub-zero temperature deviations in the design curves. An investigation has also been conducted which provides an agreement between the test results and the literature for sub-zero welded joints [22].

3.2.2 Stress Averaging Approach

Different studies have established a relation between the high temperature and fatigue life as well as sub-zero temperatures and fatigue life. [23][24] The changes in fatigue life with temperature are thought to be linked to the variations in material support effects with temperature. In typical stress based approaches, temperature effects can only be taken into consideration with the help of modification factors. [14] Stress Averaging Approach overcomes this problem by taking directly into consideration the changes in the material support effects at the notches. The method relies on the estimation of notch stress values by averaging the stress gradient over a material characteristic length, which takes into consideration the support effect of the material in the vicinity of the notch hence, accounting for the changes in material properties. [25]

4. Methodology

In this section, the organization of the study has been discussed. The approach followed in the study is presented in Section 4.1 which enlists the sequence of major activities performed in the study to arrive at the conclusions. The software tools and the analytical formulas employed in the process have been described in Section 4.2. Finally, section 4.3 contains a detailed description of the geometrical models as well as the technical specifications of the cruciform joints analysed in this study.

4.1 Organization of the study

The approach of the work primarily consisted of three major stages starting from the geometry construction to the final calculation of fatigue life. These stages are shown in Figure 11. In the first stage named as simulation, the geometry and the mesh was created using a mesh creating program called Casca. This geometry was then imported in the simulation software known as Franc2D, where material parameters, inputs, and boundary conditions were defined (further details in section 4.3). The results obtained from Franc2D simulations were in the form of SIF values for non-load carrying and load carrying joints.

In second stage, analytical formulas were employed to calculate the SIF values for the specimens used in Franc2D simulations. The results of analytical formulas were validated against the SIF values obtained from the simulations, as discussed in detail in section 5.2.

In the final stage of post processing, the validated SIF values were used as main input for predicting the fatigue life of the load-carrying and non-load carrying joints. This post processing has been done in excel by implementing the crack propagation formulas proposed by Paris and Erdogan.

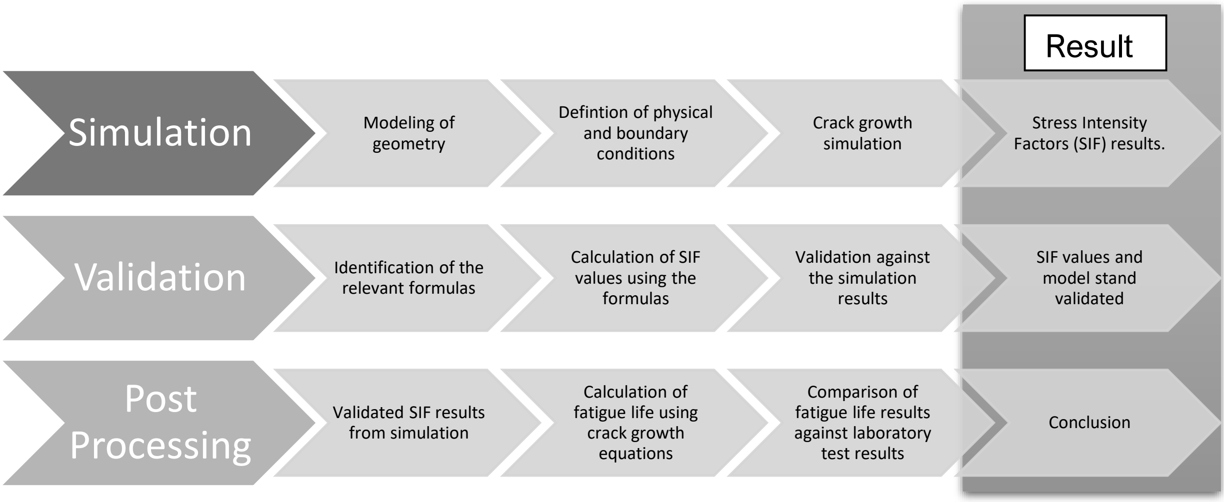


Figure 11: Organization of the study.

4.2 Tools Employed in the Study

This section discusses the software used to perform the FEM analysis as well as the formulas used to validate the results. These software tools were already specified within the scope of the study since one of the goals of the study is to established if Franc2D is able to provide realistic estimations of the stress intensity factors (SIF).

4.2.1 Casca

Casca is a pre-processing platform to generate the geometry. It also provides the options to divide the geometry into different segments. The output file from Casca is the input file which can be imported in Franc2D to further apply the boundary conditions and run the simulations [19].

4.2.2 Franc2D

Fracture Analysis Code (Franc2D) is a freeware software developed by Cornell Fracture Group [19]. The program uses Linear Elastic Fracture Mechanics (LEFM) concepts and modified crack closure techniques to perform the numerical analysis for stress intensity factors. This approach corresponds to the recommended approach defined within the scope of this study, i.e., crack propagation theory.

4.3 Steps of the simulation

FEM analysis was required to obtain the SIF values for the crack propagation path. The analysis was performed on Franc2D. Following sections explain the step-by-step process used to obtain the SIF values.

4.3.1 Geometry Definition

The geometry construction is done using pre-processor called Casca (see section 4.2.1). A single quarter model is used for all the non-load carrying joints since the throat thicknesses for all the specimens are the same (Table 1). However, different models are constructed for load-carrying joints because of varying throat thicknesses (see Table 2). Quarter models are then divided into three different segments to incorporate different mesh densities throughout the specimen. A refined mesh is introduced in the vicinity of weld throat for non-load carrying specimens and around weld root for load-

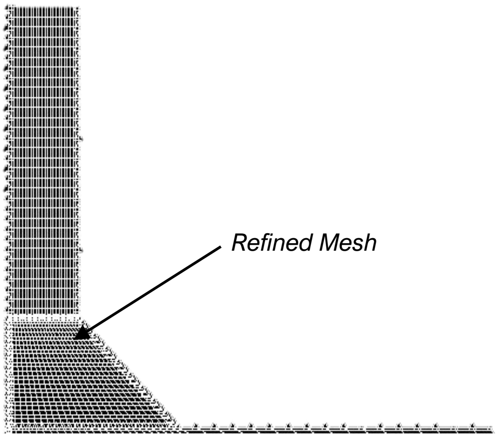


Figure 12: Meshed non-load carrying specimen.

carrying joints (Figure 12). The mesh elements used in the study are simple quadrilateral Q4 elements which are enough for the analysis of cruciform joints.

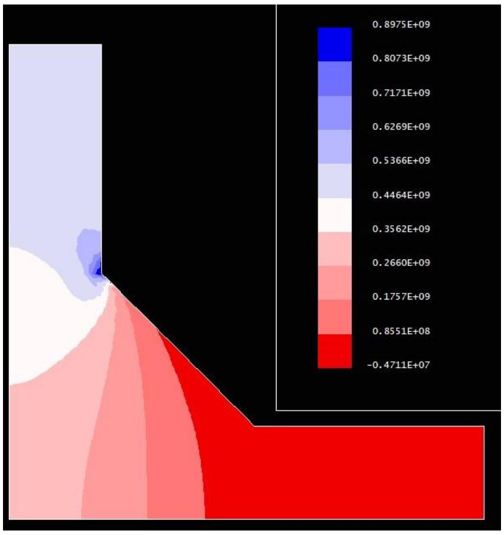
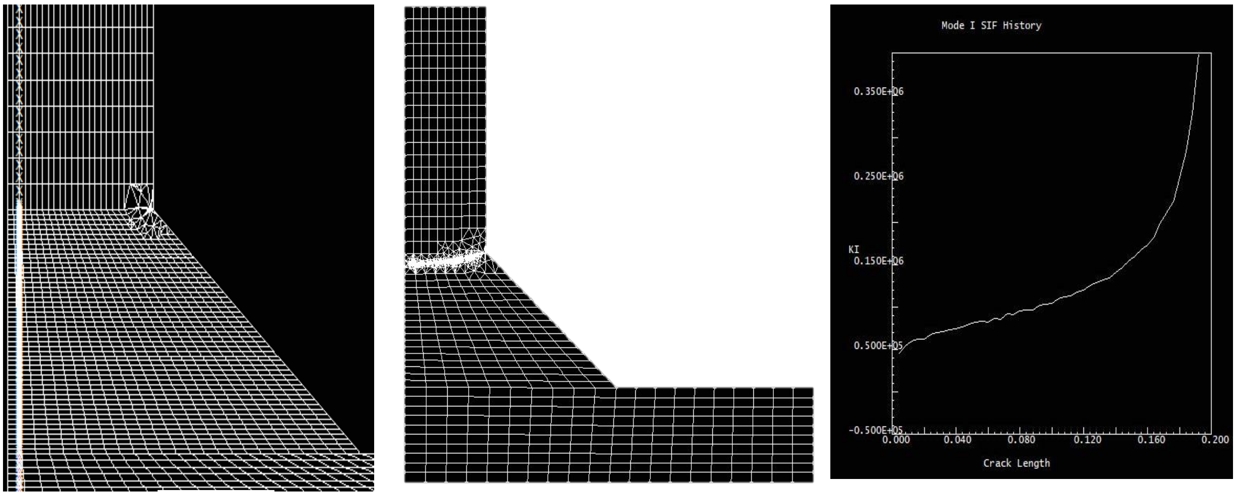


Figure 13: Tensile stresses in a loaded non-load carrying specimen.

The model generated by Casca was imported in Franc2D. The loads, material, and boundary conditions were defined as discussed in section 4.4. After the definition of the boundary conditions, analysis was performed for the un-cracked non-load carrying specimen. The maximum tensile stresses (in Y direction) were obtained at the weld toe, as indicated in Figure 13. Hence, the weld toe, in line with the literature studies, has been used as the stress intensive location and also the spot for crack initiation in the study.

In accordance with the maximum stresses, a crack of 0.1 mm was induced at the location of weld toe. This crack was then grown for 49 steps with each step of 0.01mm of crack increment. For crack growth, automatic crack propagation option was used in Franc2D which grows the crack in the direction of maximum hoop stresses around the crack tip (Section 4.4.1). Once, the cack is induced, the program automatically refines the mesh to better predict the SIF values [26], see Figure 14a. The automatic crack propagation is also followed by automatic analysis after each step of crack growth, resulting in an SIF value for each iteration of crack growth. The crack growth path has been shown in Figure 14b. A sample SIF curve generated by Frnac2D has been shown in Figure 14c.



14a) *14b)* *14c)*

Figure 14: Non Load Carrying Joint. a) Crack initiation. b) Crack propagation path. c) SIF history.

The SIF values given by Franc2D were then exported out of the program in the form of .txt file to do the post-processing in excel. The integrated form of Paris Law, as discussed in section 3.1.1, was applied to obtain the fatigue life results. These results were then compared to the laboratory test results, as discussed in next sections in detail.

For load-carrying joint, an initial crack of 5mm was introduced at the left edge of the specimen in order for the crack tip to lie at the weld root. The crack tip served as non-penetrated weld gap creating a location for high stresses at the weld root, which is also the case for load carrying cruciform joints. The high stress values are shown in Figure 15.

From the crack tip of the initial crack, as indicated in Figure 16a, the crack was grown over 60 small steps with each steps leading to a crack growth of 0.01 mm. Like non-load carrying specimen, automatic crack growth option was used to propagate the crack. As seen in Figure 16b, the crack path curved at around 45° as the crack grows further towards fracture.

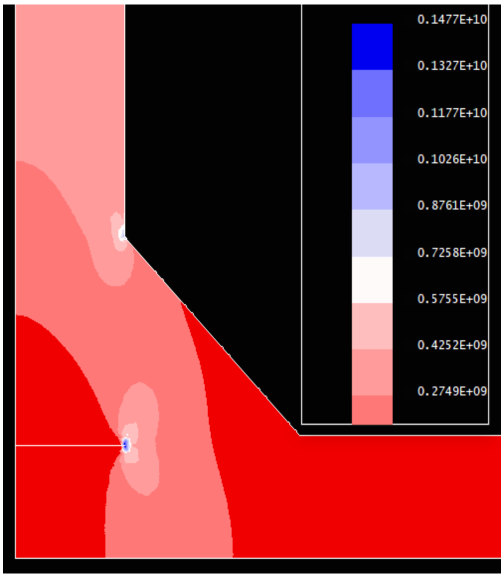
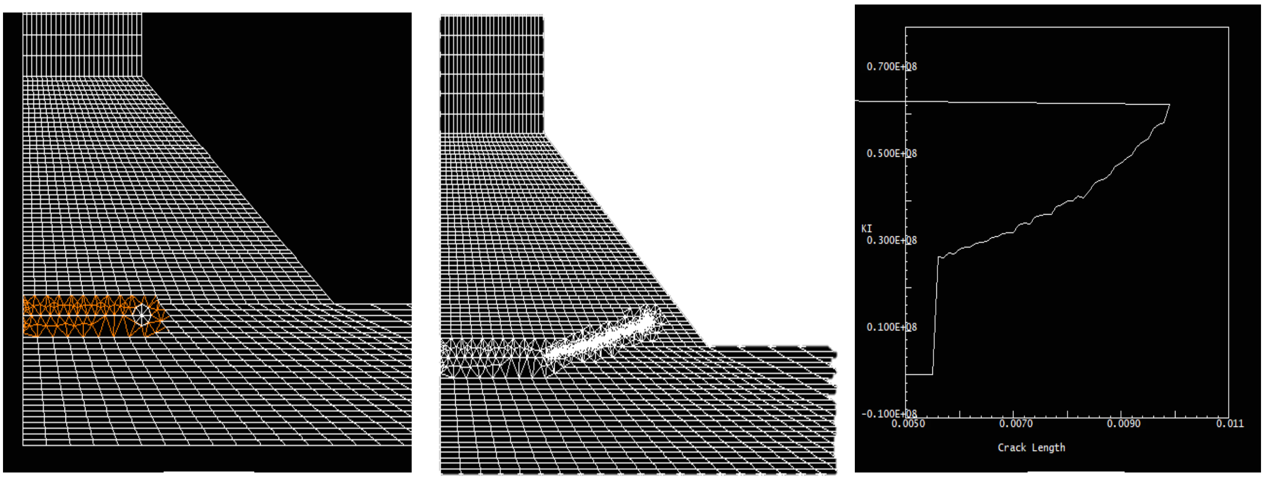


Figure 15: High stress location at weld root for load-carrying specimen.

The SIF values generated from Franc2D, as shown in Figure 16c, were used as inputs to the Paris integration equation (see section 3.1.1) along with C and m values from BS7910 [5] to calculate the fatigue life for load carrying joints. These results are then compared to the laboratory test results to obtain a correlation, if possible. The detailed results are discussed in later parts of the study.



16a) *16b)* *16c)*

Figure 16: Load carrying cruciform joint. a) Crack initiation. b) crack growth path. c) SIF history.

4.4 Specifications of the Model

The scope of this study involves investigation of two types of joints. Non-load carrying joints, as discussed previously in section 2.5.1, fail at weld toe where the stress concentrations are high. On the other hand, load carrying joints, see section 2.5.2, fail from the weld root. The material and geometry parameters of non-load carrying joints are given in 4.4.1. The model specifications of load carrying joints are described in 4.4.2.

4.4.1 Non-Load Carrying Joints

The material used in the study is a steel alloy with Modulus of Elasticity of "*E"* 206 *GPa* and poison ration "*v*" of 0.3. The material specifications remain consistent throughout the study and do not change. Non-load carrying joints are modeled in such a way that a distributed tensile load is applied at the top edge. Crack is initiated at the weld toe. The direction of the load and the crack propagation path are indicated in Figure 17. Due to the symmetry of the joint, only a quarter model is used. The node on the bottom left is contrained in both x and y directions, whereas, the bottom and left edges are constrained in y and x directions, respectively, as shown in Figure 18.

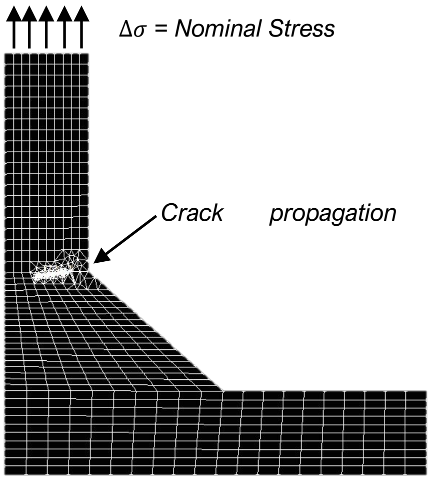


Figure 17: Non Load Carrying Specimen

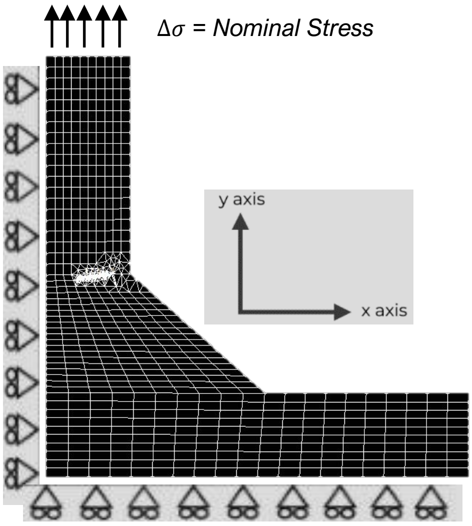


Figure 18: Quarter model with x and y constraints.

Four models with varying nominal stresses and throat length have been employed for the study. The specimens are shown in Table 1.

Table 1: Specifications Non-Load Carrying Joints

Parameter	**Unit**	**S235-T-1-1**	**S235-T-1-2**	**S235-T-1-3**	**S235-T-1-4**
Nominal Stress	MPa	241.14	159.94	260.30	134.14
Throat Thickness	mm	5.0	5.0	5.0	5.0
Test Temperature	°C	-20	RT	-20	RT

4.4.2 Load Carrying Joints

The material properties for load carrying joints are same as mentioned in 4.4.1. Also, the quarter model and the constraints mentioned in section 4.4.1 have been used.

Like load carrying joints, non-load carrying joints are also modelled with tensile loads applied at the top edge. As discussed in 2.5.2, load carrying joints have high stress intensities at the weld throat due to lack of penetration. To incorporate the lack of penetration in the model, an initial crack is induced at the left edge in such a way that the crack tip of the induced initial crack is located at the weld throat. Crack is then propagated in small steps from initial crack yielding a crack path at 45° of angle, as shown in Figure 19.

Four specimens with different nominal stress values and throat thickness are analysed for the study, as shown in Table 2.

Table 2: Specifications of Load Carrying Joints.

Parameter	**Unit**	**S235-C-1-1**	**S235-C-1-14**	**S235-C-3-14**	**S235--1-2**
Nominal Stress	MPa	295.73	111.13	126.30	175.59
Throat Thickness	mm	5.70	5.96	5.48	5.77
Test Temperature	°C	-20	-20	-50	RT

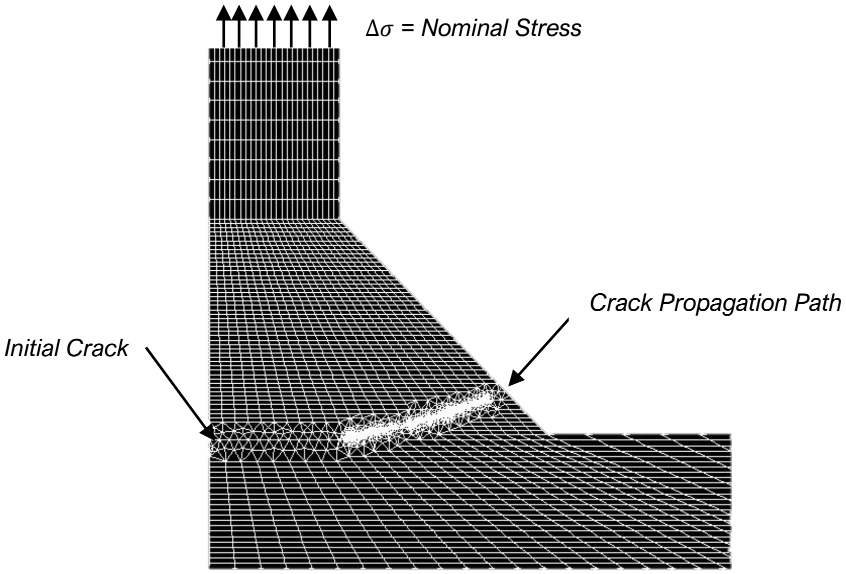


Figure 19: Load Carrying Cruciform Joint.

4.5 Measurement of Throat Thickness for Weld Geometries

Welded structures are prone to stress concentrations at the weld locations which directly influence the fatigue life of the welded components. In order to arrive at accurate estimations of stress concentration factors, a precise measurement of geometry is vital. One of the most relevant geometry factors for such fatigue life investigations of fillet welded joints is the throat thickness. Hence, it is felt important to mention the exact method used to measure the exact throat thicknesses for the welded joints under consideration in this study. The throat thicknesses for the joints referred in this study have already been presented in Table 1and Table 2.

The fillet welded joints under consideration have been measured using a manual method at institute M10 of the Technical University of Hamburg (TUHH). This method is based on collecting the dense point cloud using a 3D scanning device which is imported into a scan-to-3D software called rapidform. The point cloud is further converted into a closed space with 3D triangulation approach leading to polygon meshes. Flank angle and toe radius make the basis to characterize the specimen geometry in the concerned method. Toe radius is measured by making equally spaced cuts (Figure 20) on the surface of the created model. Flank angle is determined by fitting a circle at the weld toe, whereas, the flank angle is determined by two tangent lines which are linearly fitted to the base plate and the weld flank, see Figure 21 [27].

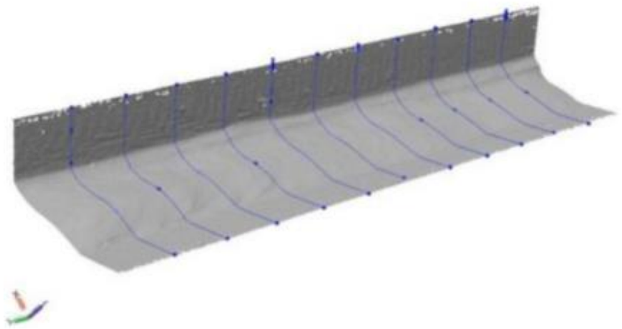
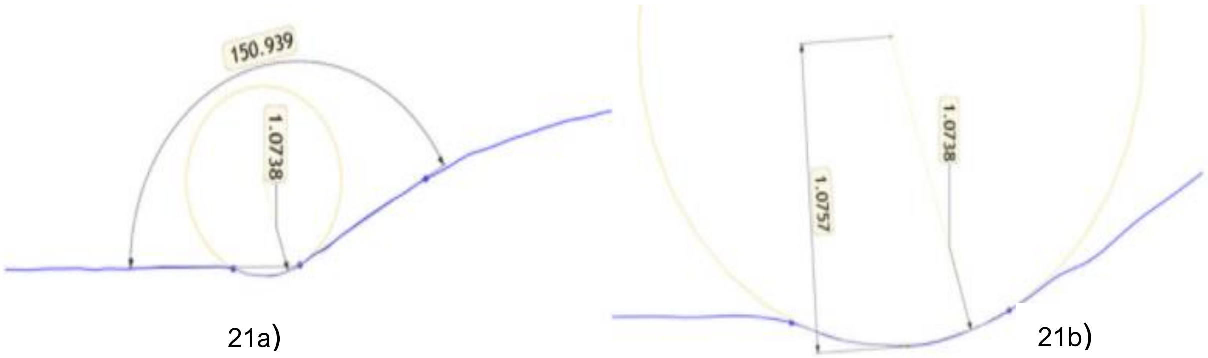


Figure 20: Equally spaced cuts made after triangulation. [27]



21a) 21b)

Figure 21: Weld toe radius and flank angle determination. 21a) Circle fitting for toe radius. 21b) Tangent lines fitting for flank angle. [27]

5. Results and Discussion

In this section, the results of the study have been discussed in detail. The stepwise method followed for FEM analysis to obtain SIF values is given in section 4.3. The results for SIF values as well as fatigue life have been presented 5.1 followed by validation of SIF values through the of formulas in 5.2.

5.1 Fatigue Life Results

The SIF values obtained from numerical simulations in section 4.3 have been used in the crack propagation formulae (presented in section 3.1.1) to obtain the fatigue life values (results identified as Franc2D in Figure 20). The results obtained from crack propagation formula have been compared against the fatigue life values from destructive tests, data referred to as Real Time Tests.

As seen in Figure 20, there are high errors in the fatigue life values obtained from crack propagation approach and those of the test results. For specimen S235-T-1-1, the crack propagation approach underestimates the fatigue life by 70%. Similarly, for specimen S235-T-1-2 and S235-T-1-3, the fatigue life predicted by the formula is lower by 86% and 52%, respectively. Unlike former three specimens, for S235-T-1-4 the crack propagation formula overestimates the fatigue life by 82%. The major difference between the specifications of S235-T-1-4 with the rest of the specimens is the nominal stress value.

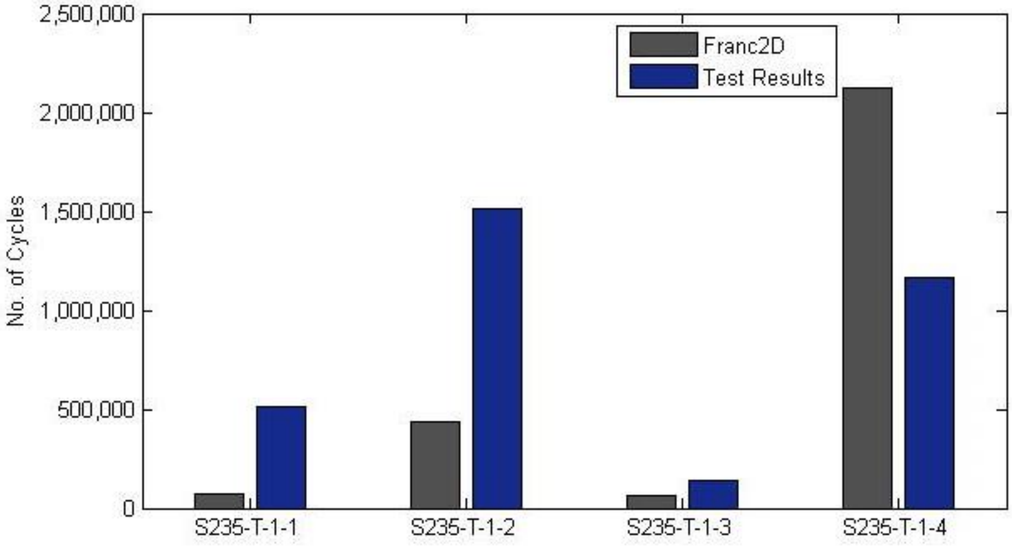


Figure 22: Fatigue life results of crack propagation approach (Franc2D) vs. destructive laboratory tests for non-load carrying joints.

The cyclic stress applied on S235-T-1-4 is significantly lower than the rest of the specimens as presented in Table 1. This lower stress value results in higher number of cycles towards lower crack lengths of the analysis, hence, resulting in overestimation of the number of fatigue cycles that this particular specimen can endure before fracture.

Fatigue life comparison of formula and test results for load carrying joints are presented in Figure 21. The trends shown in this comparison are similar to the non-load carrying specimens. Paris Law Eq. 3.2 underestimates the fatigue life for all the load-carrying specimens under consideration. The results calculated from the formula exhibit significantly lower fatigue life values with errors ranging 56-72 % as compare to the test results.

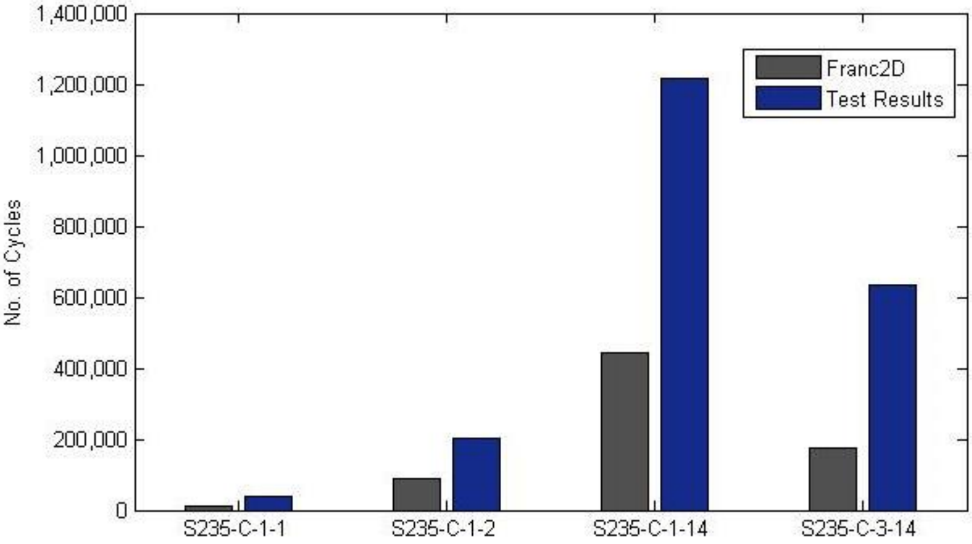


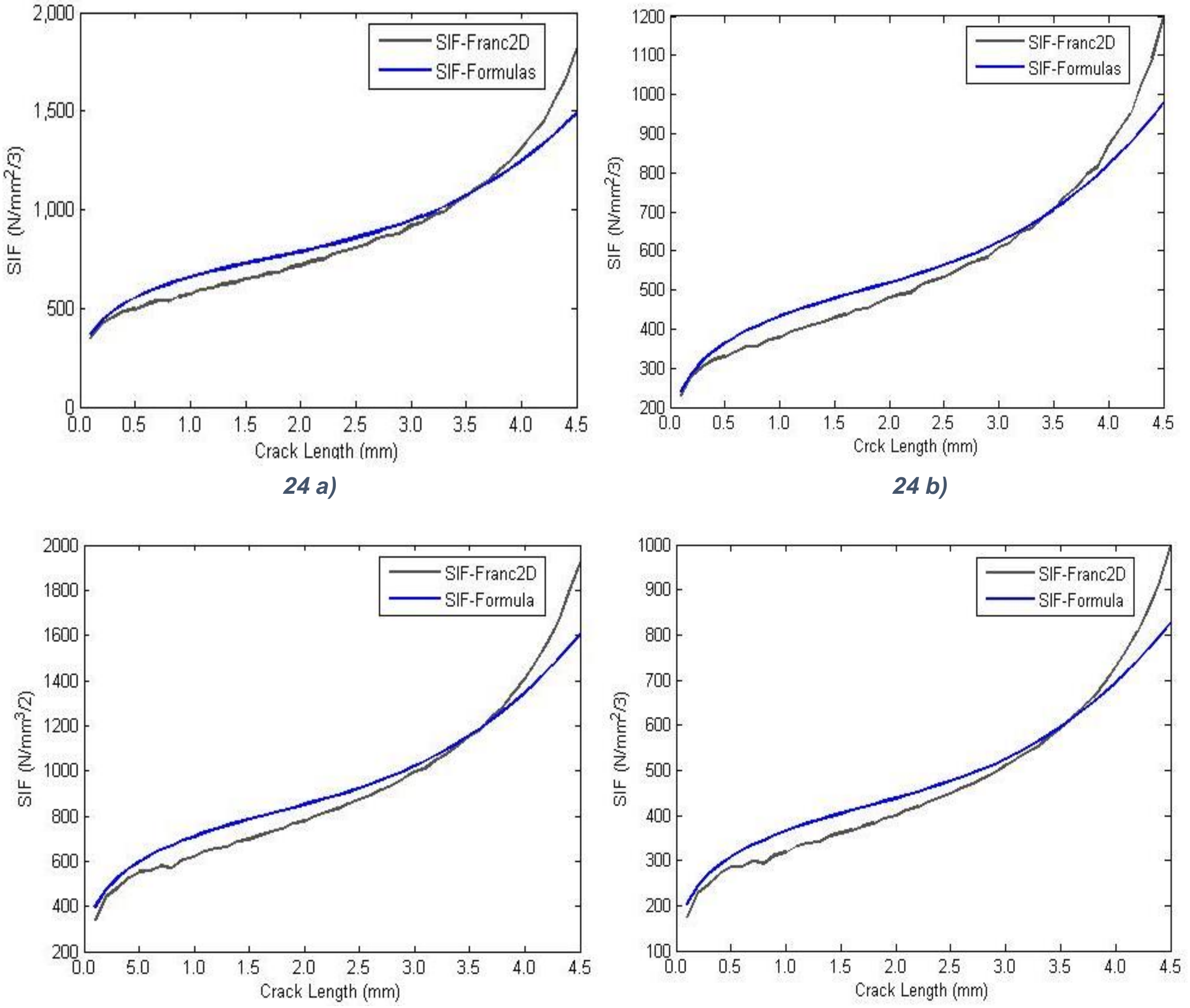
Figure 23: Fatigue life results of crack propagation approach (Franc2D) vs. destructive laboratory tests for load-carrying joints.

Hence, it can be said that the crack propagation approach can either underestimate the fatigue life or overestimate the values, depending upon the magnitude of cyclic stress applied on the model. The errors ranging from 56% to 86% for non-load carrying joints and 56 to 72 % for load-carrying joints suggest a further investigation into the SIF values obtained from Franc2D. The model specifications for FE analysis as well as the SIF values obtained from Franc2D are hence validated in the next section.

5.2 Validation of the FE Model and SIF values

As discussed in previous section, the fatigue life results obtained from Paris equation are significantly different from the lab test results, which leads to the question of validity of the SIF values obtained from Franc2D. These values are validated by calculating SIF values from the analytical formulas proposed in section 3.1 (Eq. 3.3 to Eq. 3.7) and then these values are compared against Franc2D values.

As seen in Figure 24, the trend is consistent for all the non-load carrying models, i.e., the values estimated by Franc2D remain slightly below those of obtained from analytical formulas and become higher towards the higher crack lengths. Hence, it can be said that the SIF values estimated by Franc2D are very close to the ones obtained from the formulas, at least for the crack lengths below 3.7 mm.



24 a)

24 b)

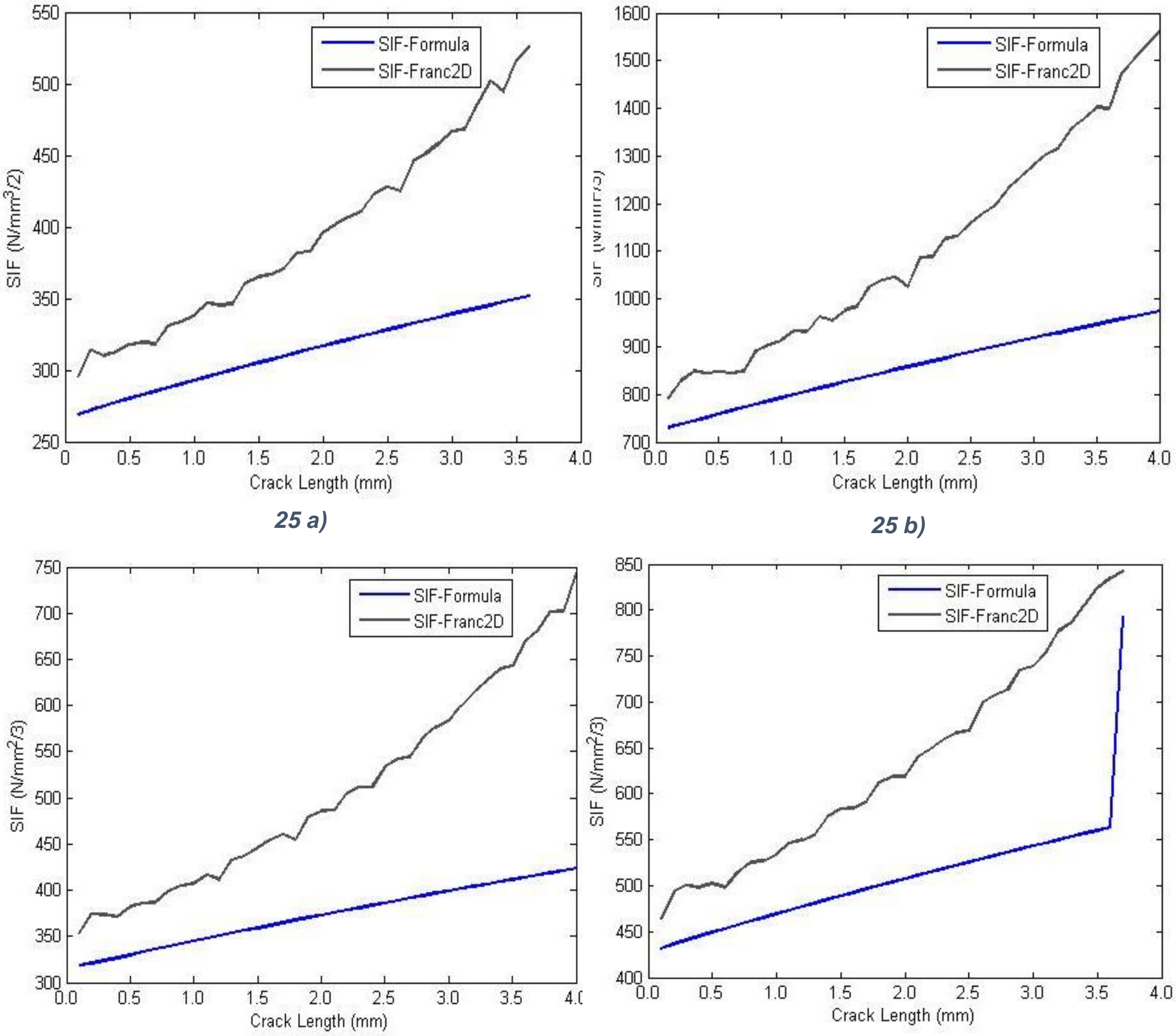
24 c)

24 d)

Figure 24: Crack growth trends from analytical formulae and Franc2D for non-load carrying models, 24a) Specimen S235-T-1-1, 24b) Specimen S235-T-1-2. 24c) Specimen S235-T-1-3. 24d) Specimen S235-T-1- .

The SIF values obtained from Franc2D for load carrying joints are compared against the results obtained from the equations 3.8 to 3.10, as mentioned in section 3.1.3. As evident from Figure 25, SIF values are constantly underestimated by the formulas for the first two specimens Figure 25a and Figure 25b. However, the error between the formula and Franc2D increases as the crack growth values approach final fracture. For specimens given in Figure 25c and Figure 25d, a sudden overestimation of the SIF values can be seen by the formulas. This upward trend gives an indication to the limitations of the formula used here.

Based on the results presented in Figure 24, it can be said that the SIF values for non-load carrying joints stand validated since the results from the formulas ´closely correlate with those of Franc2D. Although for load-carrying joints, Figure 25 shows larger errors towards bigger crack lengths, the errors are below 40% for almost 70 % of the crack growth path. Based on these results, the SIF values predicted by Franc2D for load carrying joints can also be considered valid and accurate.



25 a)

25 b)

25 c)

25 d)

Figure 25: Crack growth trends from analytical formulae and Franc2D for load carrying joints, 23a) Specimen S235-C-1-1, 23b) Specimen S235-C-1-14. 23c) Specimen S235-C-3-14. 23d) Specimen S235-C-1-2.

Although the validity of SIF values obtained from Franc2D is already established, the question of large errors for fatigue strength, raised in section 5.1, still remains open. The underestimation of the fatigue life by crack propagation approach can be linked to three factors related to the 2D simulation, temperature effects and the chosen values for C and m constants in Paris equation.

Regarding the effect of temperature on fatigue life values for both load-carrying and non-load carrying joints, some studies have been performed at sub-zero temperatures. In the studies by Braun et al [24] and Bridges et al [28], the fatigue strength of the welded steel joints have been found to increase with a decrease in temperature. This temperature factor can explain a small portion of the errors between the calculated and tested fatigue strengths indicated by the fatigue life results, as some of the specimens have been tested on sub-zero temperatures as indicated in Table 1 and Table 2. However, this increase in fatigue life has been reported to be around 8% for -20°C and 15% for -50°C [24], which means that the testing temperature is not the single factor for the large errors under discussion.

The other factor to which can be associated with the errors are the values of constants in Paris Law. As seen in Eq. 3.1, fatigue life is dependent upon range of SIF values, C and m factors. As SIF values have already been verified, the large errors can be associated with the values chosen for constants C and m.

Another factor that can be associated with the underestimation of fatigue life by crack propagation approach is the 2D simulation factor. A similar study of fillet welded joints conducted by Fischer et.al [1] proposed similar results, where crack propagation approach has been identified as predicting very small fatigue life as compared to the fatigue life tests. These conservative assessments have been linked to the simplified 2D simulations and crack closures in the study. The study argues that the crack shapes can be semi-elliptical which is neglected in 2D simulations leading to conservative fatigue life assessments.

6. Conclusion and Suggestions

In an effort to evaluate the crack propagation approach when applied in a 2D configuration to welded fillet joints, this study utilized a 2D numerical simulator called Franc2D to grow the cracks from an initial crack length a_i of 0.1 mm to a final fracture. The SIF values obtained from Franc2D along with C and m values from IIW recommendations were used in the integrated form of Paris Law to predict the fatigue life values. Based on the comparison of results from crack propagation approach against the test results, following conclusions can be withdrawn.

- Franc2D can predict accurate crack growth curves for both load carrying and non-load carrying fillet welds.
- SIF values obtained from Franc2D can be considered as valid after a very close correlation shown between the values obtained from Franc2D and that of analytical formulas proposed in the literature.
- It can be deduced from the comparison of fatigue values from crack propagation approach against test results that crack propagation approach underestimates the fatigue life and gives a very conservative number.
- Crack propagation approach underestimates or overestimates (in some cases) the fatigue life for non-load carrying joints with errors ranging from 52% to as high as 86%.
- For load-carrying joints, crack propagation approach has been found to be constantly underestimating the fatigue life from 56% to 72 %.
- Since the SIF values stand validated, one of the factors which could be linked to the underestimation of fatigue life by crack propagation approach is the values of C and m constants in Paris equation.
- Another reason for underestimation of fatigue life is considered to be an over simplification of the crack growth path in 2D simulation. The crack growth path is thought to be semi-elliptical in reality, which is ignored in 2D simulation.
- Since, fatigue life values obtained from crack propagation approach are not reasonable, further investigation into the temperature dependence of fatigue life was not plausible. However, because of the fact that some of the actual tests were conducted at sub-zero temperatures, this could be an additional factor in the increased fatigue life in actual tests.
- As to which factor contributes largest to the underestimation of fatigue life, further investigations are necessary. A 3D simulation of crack growth path could be a next step into validation of the elliptical crack path argument. Further investigations are needed regarding C and m values to be used for welded fillet joints. Finally, if the results show acceptable errors after the implementation of aforementioned steps, a further investigation into sub-zero temperature effects could be feasible.

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