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D-21073 Hamburg

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APPLICATIONS OF WAVE RESISTANCE THEORY TO PROBLEMS OF SHIP DESIGN

By Prof. Dr.-Ing. G. P. WEINBLUM*

27th January 1959

SYNOPSIS

The Author surveys recent work on the mathematical studies of wave resistance and discusses the results obtained when this is applied to surface displacement and other types of ship.

INTRODUCTION

In his classical paper on wave resistance of ships,[1] J. Michell calculated a numerical example for a particular example. He left it to others to discover whether some agreement exists between results of his computation and available experimental data. The evaluation and application of the important theory was started more than 20 years later by Sir Thomas Havelock[2] and C. Wigley.[3] Quite recently distinguished scholars have expressed their views on the practical usefulness of work accomplished in this direction. The Japanese scientist Inui[4] pointed out that great efforts had been made in applying ship hydrodynamics to the solution of problems presented in practice over the past 40 years but that the results achieved were rather poor compared with the effort expended till he himself succeeded in reaching a definite progress. Another hydrodynamicist of world-wide reputation who has made valuable theoretical contributions to problems of wave resistance expressed the opinion that this work, notwithstanding its scientific interest, has more or less ornamental value only as far as practice is concerned.

* Of the University of Hamburg.

[1] See bibliography, p. 150

Obviously, those who support the idea that there is a wide gap between theory and practice in the shipbuilding profession are eager to deny any use whatsoever in applying wave resistance theory to actual design. In fact, they are probably correct in thinking that so far methods to calculate the wave resistance are not used in design offices at all.

In these circumstances, the Author felt greatly honoured by the request made by this Institution to give his opinion on the influence which the application of the theoretical work in question has had and may have on shipbuilding practice.

This paper, therefore, should be judged from the viewpoint as to how far it is able to "enlighten on practical subjects."[5] It is not a treatise on hydrodynamics: its purpose is to show to what extent theory has already succeeded in furnishing results valuable for design and how the scope of applications can be extended. Notwithstanding its limited size, it is hoped that the study may fill some gaps in Capt. Saunders' book,[6] and Prof. Inui's comprehensive report[4] which is the most important pertinent contribution.

The aim of a consistent theory is firstly to establish laws from which a general understanding of the subject can be derived. There is no question that in this respect analytical investigations in the shipbuilding field have already fulfilled their purpose. However, an exhaustive theory should enable one to calculate with sufficient accuracy the wave resistance curve for a given hull shape, yield expressions for the wave resistance as a function of basic form parameters, and thus furnish the possibility of deriving optimum lines for given conditions. Such analytical procedures would dispense with the need of model investigations as far as wave-making problems are concerned. Obviously, however, there is still a great distance to go before this goal is reached. In fact, at present there are two practical purposes in developing and applying the hydrodynamic theory, namely, a direct one which refers immediately to design, and an indirect one which aims at stimulating and clarifying model research work. Up to now the second purpose is the more important.

The scope of the survey is rather wide. In principle it will review results of the wave resistance for:

(1) All known classes and sub-classes of ships. As such the Author lists the displacement ships, embracing surface vessels

and submerged floating bodies and the hydrodynamic craft represented by planing and hydrofoil boats.

(2) Various kinds of motion, for example, uniform rectilinear, uniform circular, rectilinear accelerated and motion in a regular and irregular seaway.

(3) Various boundary conditions for the water, that is, infinite depth, shallow water and restricted water like rectangular channels.

The paper will deal primarily with the steady speed problem of surface displacement vessels in deep water as the technically and economically most important problem. It may appear surprising that in addition attention should be paid to more complicated examples as long as the seemingly simplest one has not yet been satisfactorily solved. This is not to the point, however, since the application of results dealing with intricate conditions may sometimes be more easily justified, especially since here a lesser degree of accuracy may be permissible.

It is a characteristic of the subject that the evaluation of the complicated analytical solutions requires a large amount of calculation. The use of digital computers is here extremely helpful. Reference will be made to experimental checks and to attempts to reach a better agreement between theoretical and experimental work by semi-empirical methods.

When discussing the applicability of theory to practical work, different points of view can prevail. The furthest reaching requirement postulates a satisfactory quantitative agreement between theory and facts. On account of the high accuracy necessary in the calculation of ship resistance, such crucial tests have often failed, which is natural in the light of the restrictions underlying theory.

There is another school of thought (of which the Author is a protagonist)[6] according to which a " functional " agreement is itself considered valuable. This means, for example, that changes in resistance due to changes in form are predicted correctly by theory as to the sign and possibly as to the order of magnitude. By such an agreement as to functional dependency (which means somewhat more than qualitative agreement), theoretical results may already become valuable means for improving ship lines and searching for optimum forms. Quantitative statements must be checked and corrected by experiment.

On the other hand, the application of wave resistance theory has contributed considerably to the weeding out of inconsistencies in the analysis of model tests and it is gradually developing into a foundation for experimental research. Attempts to reverse Froude's method, that is, to determine the viscous drag by subtracting calculated values of the wave resistance from the measured total resistance are significant in this respect. Clearly, such a procedure requires quantitative agreement between theoretical and experimental results.

Summarizing, it is the Author's contention that the pessimistic attitude referred to above is not justified. Prof. Inui has slightly underrated earlier work, especially that based on directions of thought to which he himself has paid less attention; but he is completely justified in considering his own work as almost the beginning of a new era in the field of practical application.

There are two well-known types of limitation on which theory is based, namely, approximations made in solving the boundary problems in ideal fluid, and neglect of friction. When applying theory to practice, the basic assumptions in both respects are violated, thus reducing practical application of theory to a heuristic approach. The Author stresses this point to avoid incriminations by representatives of rigorous science.

Survey of Theoretical Solutions for Wave Resistance

The first successful attempt to base the investigation of ship wave phenomena on hydrodynamics theory is due to Lord Kelvin.[5] He " made it a condition that no practical results were to be expected " from this publication. The introduction of the pressure point concept, namely, the " forcive," led, however, very soon to a useful theory of planing phenomena.[7]

Michell's paper[1] represents the most important progress in dealing with the wave resistance of displacement ships moving with constant speed on a rectilinear path.

Further, the method of images was applied: primarily source and sink or doublet systems by Havelock, and later vortex systems. Since there is a simple correspondence between classes of ships and images suitable for the investigation of their hydrodynamic properties, the basic solutions following essentially the practical aspect are given below.

Surface Displacement Ships. Uniform rectilinear motion (constant speed) in deep water. The fundamental theory given by Michell[1] has been interpreted using the method of images by Havelock[2] and generalized.[7] Valuable side lines of research are Hogner's interpolation formula[9] and Guilloton's[10] and Inui's[4] investigations. The pertinent solutions are obviously the most important contributions from the point of view of practice.

Constant speed in shallow and in restricted water (rectangular channel). Solutions by Sretenski,[11,12] Keldysh and Sedov.[13]

Non-uniform rectilinear motion in deep, shallow and restricted water (Havelock[14] and Lunde[15]), and motion in a seaway, (Maruo[16]).

Multiple bodies (uniform motion [17,18]).

A large number of results quoted can be found in reference.[15]

Submerged Bodies Moving Horizontally Under a Free Surface.

Uniform rectilinear motion in deep water, Havelock,[2] Bessho.[19]

Uniform rectilinear motion in shallow and restricted water, (Wigley,[20] Haskind[21]).

Uniform circular motion (turning circle) Havelock.[22]

Planing Systems (hydrogliders), (Hogner,[7] H. Wagner[23] and Maruo[45]).

Hydrofoils. Uniform motion, two- (infinite span) and three-dimensional case (finite span), Keldysh and Laurentiev,[24] Kochin,[2] Breslin,[25] Wu.[26]

Uniform motion in waves (Kaplan,[27] Nishiyamo[28]).

A very complete synopsis of the theoretical work is being prepared by J. Wehausen.[29]

From the rather sketchy list given above it appears that impressive work has been done and is going on in this field.

In the following sections the Author will try to summarize what use has been made and can be made of the rich theoretical information available.

DISCUSSION OF RESULTS OBTAINED FOR SURFACE DISPLACEMENT VESSELS

General Remarks. As stated in the introduction the process of application involves in general three steps, namely:

The evaluation of the intricate resistance integrals. This comprises methods of systematic computation and results derived therefrom, for the purpose of determining good ship lines.

Experimental checks needed because of the restrictions of theory with respect to forms and neglect of viscosity.

Development of semi-empirical formulae to cope with the restrictions of theory and the resulting shortcomings in resistance determination.

For surface displacement ships the following problems are presented by shipbuilding practice:

(*a*) Calculation of the wave resistance curve for a given hull form.

(*b*) Representation of wave resistance diagrams for systematically varied hull shapes and establishment of laws for the wave resistance as function of characteristic form parameters.

(*c*) Improvement of a given set of lines.

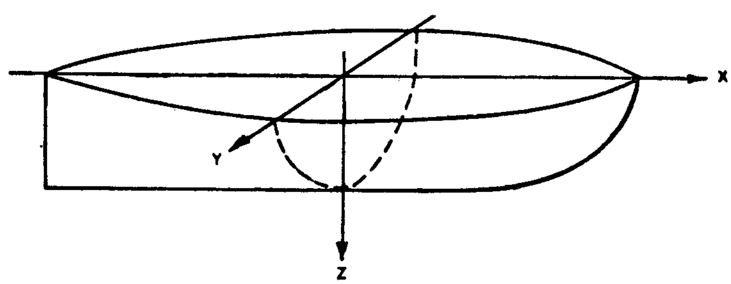

Fig. 1.—Axes of reference.

(*d*) Calculation of optimum forms for given conditions.

To deal with these problems something must be said about the representation and the geometry of the hull. In a broad way one distinguishes narrow and broad forms by the ratio B/L; flat and deep forms by B/H (the latter by H/L also); fine and full ships by $C_p = \varphi$; fat and slender ships by the parameter $C_v = \nabla/L^3$ or Ⓜ $= L/^3\sqrt{\nabla}$, where ∇ denotes the volume displacement; and thin ships by $B/L \ll 1$; $B/2H < 1$. Michell's theory applies to thin ships only.

In practice the shape is fixed by the set of lines or offsets; in principle the latter can be used to compute the wave resistance. A suitable approach has been developed, for example, by Guilloton;[10] methods are available to make use of electronic computers. For systematic work, however, it is preferable to

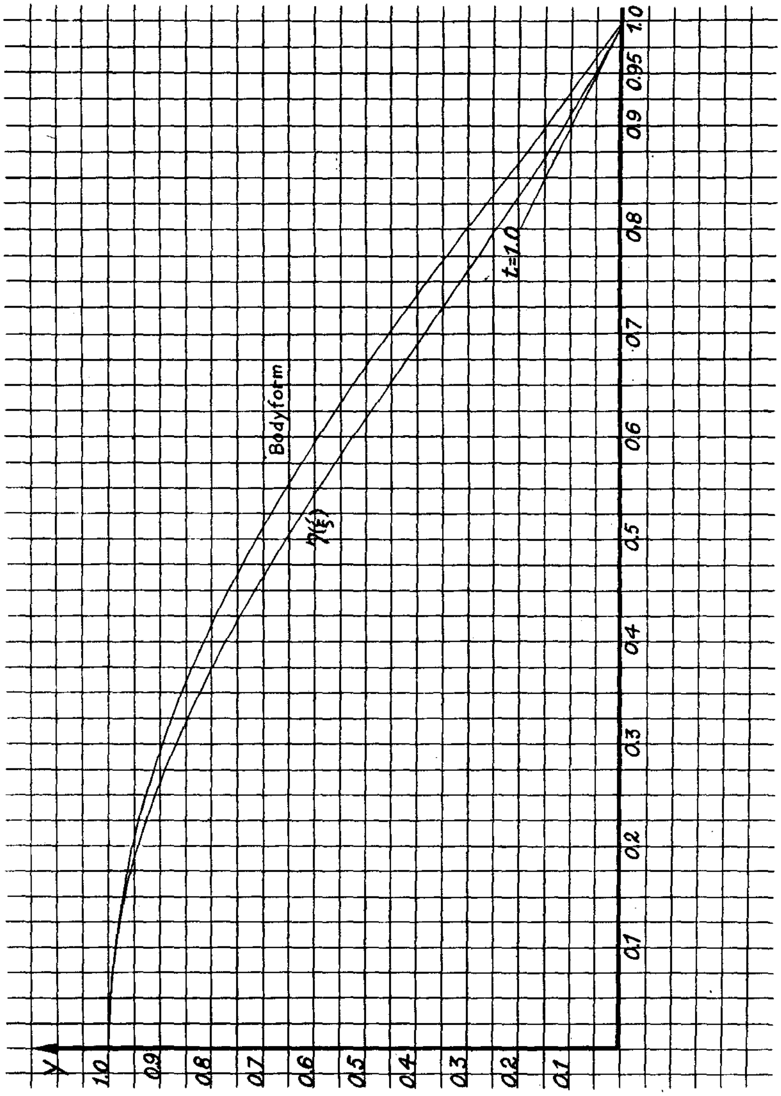

Fig. 2.—Cylinder y(x) generated by a doublet distribution $m(x) = m_0 (1 - 1{\cdot}5\xi^2 + 0{\cdot}5\xi^4)$.

express the geometrical ship surface by a suitable equation $y(x, z)$ and to vary the parameters of this equation in a prescribed manner. As such the well-known form parameters of the ship form serve directly, or simple relationships between the two sets of parameters are established.

No essential difficulties are encountered in " mathematizing " the geometrical surface of a ship, as already shown by D. W. Taylor.[30] Polynomials are especially suitable for this purpose on account of their simplicity and their kinship to spline curves underlying the design of lines. When tables of appropriate auxiliary integrals for the computation of the wave resistance are available the work is straightforward.[31]

In general it is advantageous for the computation and discussion of results to split off in the equation [1] a principal dimension and to use dimensionless co-ordinates.

Thus we may put $y(x, z) = B/2\ \eta(\xi, \zeta)$ [1]

with $\xi = x/(L/2)$ $\zeta = Z/H$; η represents the dimensionless or pure form of the hull. A similar procedure is frequently used for collecting samples of body lines of ships.

The basic form coefficients $C_p = \varphi$, C_B, C_x, C_w, are not changed by the affine transformations referred to, but unfortunately angular magnitudes vary. Taylor's t value is a differential parameter made invariant with respect to such a transformation

$$t = -\frac{L}{B}\frac{dy}{dx}\bigg|_{x=L/2} = -\frac{dy}{dx}\bigg|_{t=1} \qquad [2]$$

Other authors prefer to use a similarity transformation for the surface.[4]

The design of ship forms is based on the sectional area curve $A(x)$ which embodies the longitudinal displacement distribution; it is used as a foundation in systematic model work also. Fortunately, the area curve which represents purely geometrical relationships yields good criteria for the wave resistance and can be chosen as the departing point for the analytic evaluations. In fact, the most important information the present theory can give refers to the influence of longitudinal displacement distribution on wave resistance. From this point of view it is advantageous to introduce a simplified ship form called an elementary ship characterized by the equation

$$\eta(\xi, \zeta) = X(\xi)\, Z(\zeta) \qquad [3]$$

For such forms the shape of the L.W.L. and of the sectional area curve coincide; further $C_B = C_p$, C_x and the area coefficients $C_x(x)$ for all sections (which generally will be U-shaped) are equal, $C_x(x) = C_x$.

The next step is to investigate the resistance of U- and V-shaped forms with identical sectional area curves, but with different load waterlines. Thus the following relationships must be investigated:

Dependence of R_w upon B and H (B/L, H/L, B/H);

Dependence of R_w upon the sectional area curve;

Derivation of optimum forms, especially of optimum sectional area curves and improvement of given lines;

Dependence of R_w upon U- and V-shaped sections.

In addition to the geometrical approach there exists the well-known hydrodynamic method to generate ship-like body forms by sources and sinks or doublets (images) in an unbounded fluid. Errors due to the neglect of the free-water surface have been discussed by Pond.[32]

Simple approximate relationships have been established between generating doublets m and generated body forms (offsets) y in an unbounded fluid:

For a cylinder $m(x) \doteq 2vy$ [4]

For a body of revolution $m(x) \doteq vA(x)$ [5]

where $A(x) = \pi y^2$.

For a thin ship $m_A(x, z) \doteq 2vy(x, z)$ [6]

Examples are shown in Figs. 2 to 4.

The accuracy of the approximate relationship [4] and [5] depends essentially upon the ratio B/L. Further, the exact expression for the doublet distribution $m_A(x, z)$ generating an ellipsoid is known:

$$y(x, z) = b\sqrt{1 - x^2/a^2 - z^2/c^2} \quad [7]$$

$$m_A(x, z) = m_{A_0}\sqrt{1 - x^2/(a^2 - b^2) - z^2/(a^2 - c^2)} \quad [8]$$

Equation [8] represents an ellipsoidal surface over the focal conic. The integral curve

$$\int m_A(x, z)dz = M(x)$$

is a parabola as well as the sectional area curve $A(x)$ of the generated ellipsoid.

A distribution over the midship plane (x, z plane) following equation [8] leads to forms for which the draught $H = c > b$;

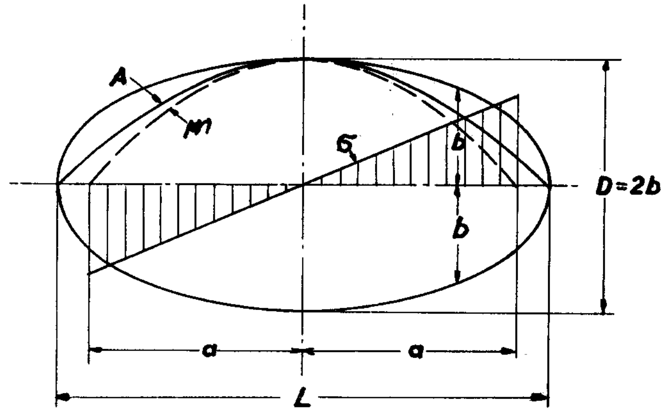

Fig. 3.—Body of revolution (spheroid) $y(x)$ and its sectional area curve A *generated by a doublet distribution m.*

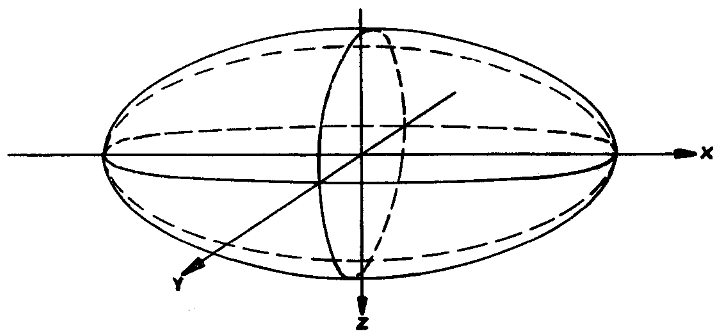

Fig. 4.—General ellipsoid generated by a surface doublet distribution, equation (7), over the focal conic.

generalizing to ship-like shapes it is seen why equation [6] is valid for thin ships only, that is, $B/L \ll 1$ and $B/2H < 1$.

Inui has calculated the actual body form generated by a doublet distribution m_A which is parabolic in the x-direction and constant in the z-direction. Therefore the difference in the shape of the distribution $m_A(x, z)$ and the body $y(x, z)$ surface is tremendous in the range of proportions of B/L and H/L in normal use. But from the ellipsoid it is supposed that the difference in the integral curves

$$\int m_A(x, z)dz = M(x) \text{ and } 2\,{}_0\!\int^{c(x)} y(x, z)dz = A(x)$$

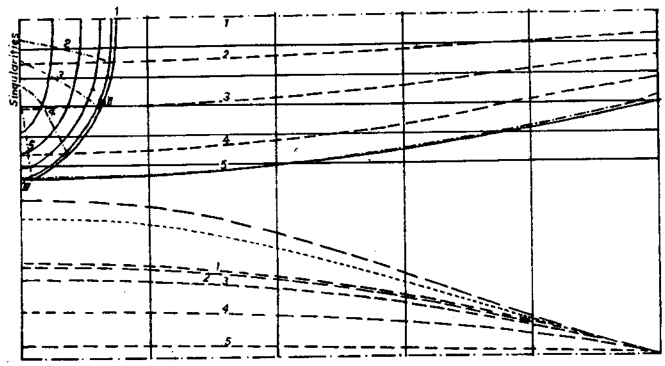

Fig. 5.—" Inui " body generated by a surface doublet distribution $m_A = m_{A_0}(1 - 1{\cdot}5\xi^2 + 0{\cdot}5\xi^4)$ *over a rectangular longitudinal section.*

may be much more moderate, as has been ascertained by extended calculations performed in the author's Institute.[18] This means that by basing the discussion of resistance results on the sectional area curve as done by the author useful results can be expected as to the functional dependency between resistance and ship form. Inui's analysis appears to show that the interpretation of Michell's integral on the base of doublet (source-sink) systems following Havelock is superior to the original one. This is true as long as the application of the resistance integral is considered as a heuristic approach, although in principle objections can be raised against distinguishing between the image and geometrical approach in the present example.

Hogner has developed a method to evalulate his so-called interpolation formula which in a limited number of instances fits the expression for a submerged ellipsoid, a thin ship and a form which can be adequately described by a pressure system. Results applied to ship forms which varied in the usual range of B/L and T/L ratios indicate that resistance results are higher than derived from Michell's integral. Further conclusions must be left till more numerical work is available.

Impressive work is being done by the Japanese to develop further improvements of the theory. Some approximate considerations will be dealt with later under the heading of semi-empirical approach.

When the more ambitious goal is pursued to obtain quantitative agreement between calculated and measured results the division into pure form and main dimensions is losing a little of its importance. The substitution of the sectional area curve for the actual hull form is no longer permitted, for example, when dealing with the merits of U- and V-shaped forms. At low Froude numbers especially the shape of the load waterline may become as important as the sectional area curve.

The investigation of resistance effects due to the vertical displacement distribution (described approximately by the shape of the waterline area curve with its area coefficient $C_{pv} = C_B/C_W$) is formally simpler and its results can be to a certain extent easily estimated. The position of humps and hollows in the wave-resistance curve does not change appreciably with the draught and the vertical prismatic coefficient.

Although the resistance calculation using auxiliary tables[31] is straightforward, other methods must be considered to obtain a better understanding of properties of the wave-resistance curve in the range of extremely low Froude numbers. Two asymptotic laws for the wave resistance at very low and very high Froude numbers $F = v/\sqrt{gL}$, are

for $F \to 0 \quad R \sim t^2$ [9]

$F \to \infty \quad R \sim C_p{}^2$ or more generally, $R \sim A^2$ [10]

indicating that in the first instance the angles of entrance should be reduced to zero (extremely hollow lines). It appears, however, that the validity of equation [9] is below the range of Froude numbers of practical interest. Equation [10] points out that within reasonable assumptions the wave resistance becomes independent of the shape of lines for ∇ = const. and L = const. Again, direct calculations show that a Froude number of unity (an approximate limit beyond which planing effects may become decisive) cannot yet be considered as infinite since an appreciable dependency of resistance upon form remains.

More far-reaching is a development proposed by Inui for the calculation of elementary ships generated by the distribution at low and medium Froude numbers. Already Wigley[33] has proposed splitting the resistance curve into a monotonic rising part and oscillating terms $R = R_1 + R_2$.

Introducing a coefficient $C_w = R/(\rho/2\ v^2L^2)$
and following Wigley, putting

$C_{w1} = R_1/(\rho/2\ v^2L^2)$ for the monotonic
and $C_{w2} = R_2/(\rho/2\ v^2L^2)$ for the oscillating part.

Inui has developed approximate formulae for calculating these coefficients, which give excellent results when the Froude number $F \leqslant 0{\cdot}34$.

The first terms of these formulae are of the type:

$$C_{w1} = A_4\,F^4 + A_8\,F^8 \qquad [9a]$$

$$C_{w2} = A_5\,F^5 \cos(1/F^2 + \pi/4) - A_7 \sin(1/F^2 + \pi/4) \qquad [9b]$$

Formulae and graphs are presented by Inui[4] to compute the factors $A_4\,A_5$ etc.; A_4 and A_5 are proportional to t^2.

It is thought that equations [9*a*] and [9*b*] will enable the wave resistance at low Froude numbers to be estimated and the speed ratios beneath which wave effects can be neglected fixed.

Dependency upon Beam and Draught. The straightforward application of Michell's theory yields the results that, keeping all other conditions unchanged,

$$R \sim B^2 \qquad [11]$$

Although this simple relationship is subject to the condition that B/L remains small it has been widely used for practical purposes in a range of B/L, which lies outside of the validity of theory. Wigley[33] has suggested an empirical relationship

$$R \sim B^n \qquad [12]$$

where following his experiments $n = n(F)$ is increasing if not monotonously with F, reaching $n \doteq 1{\cdot}7$ at $F \doteq 0{\cdot}6$. Similar values have been found by Sretenski and Girs.[34] Again, these results must be re-analysed using the viscous drag concept to determine the wave resistance.

An interesting analytic relationship for an exponent n_h (F) has been established by Inui[4] for infinitely deep ships (with a cosine waterline) moving in an ideal fluid. It shows roughly that n_o (F) is smaller than 2 when $F < 0{\cdot}33$ with a sharply pronounced minimum at $F \doteqdot 0{\cdot}30$. For $F > 0{\cdot}33$ $n_o(F) > 2$. Some experimental evidence can be quoted: Taylor's standard series as well as recent British Shipbuilding Research Association investigations show that in the range of F not too far from 0·30 the influence of the ratio B/H on the resistance is small, and this can be interpreted as due to a small exponent n (F). There are further indications from Taylor's and Bragg's work that in the range of high F n (F) can become slightly larger than 2. It is thought that by extending calculations to three-dimensional hull forms following Inui and re-evaluating good experimental material useful estimates will be obtained for the resistance dependency upon beam. For the dependency upon draught an asymptotic formula is obtained at very high Froude numbers

$$F \rightarrow \infty \quad R \sim H^2 \qquad [13]$$

Re-writing Michell's formula in the form

$$R = 8/\pi\ \rho g\ B^2\ H^2/L\ E \text{ with } E = E(H/L, \eta, F) \qquad [14]$$

we put $R \sim H^{mo\,(F)}$ where the " theoretical " exponent m_o (F) is smaller than 2 in the whole range of useful F, and can be easily evaluated. Systematic experimental checks are lacking. Introducing, as in the case of beam dependency an empirical relationship $R \sim H^{m(F)}$ to describe results of experiments m (F) is obviously smaller than n (F) except perhaps in the region $F \doteqdot 0{\cdot}3$. When both B and H are varied, keeping $B/H = \text{constant}$ (similarity transformation) as in Taylor's and Bragg's experiments we put $R \sim B^n H^m$ where the asymptotic law for $F \rightarrow \infty$ by Michell's integral is $R \sim B^2\ H^2$ or $R \sim B^4$ and for finite F, $R \sim B^2\ H^2\ E$. Calculations appear to over-estimate the resistance at large B/L appreciably. Attempts[6, 34] have been made to compare calculated and measured wave resistance values (or some coefficients c_W) as functions of L/B and B/H based on systematic model series by Taylor and Kent. The results are encouraging for Taylor's work at high Froude numbers, and very unsatisfactory for fuller forms at lower speed. However, these attempts are open to question because of a rather arbitrary way of " mathematizing " the tested hull forms for the corresponding calculations,[34, 35] and because of defects in experimental results and their analysis.

It is therefore suggested that a new approach should be made in a similar direction after the present state of knowledge has been improved in many respects. Thus one can expect that within the near future better information will be available on the dependency of the wave resistance upon beam and draught.

Dependence of Wave Resistance upon the Shape of the Sectional Area Curve. Up till now investigations in this field of application of wave-resistance theory have been the most interesting and fruitful ones.

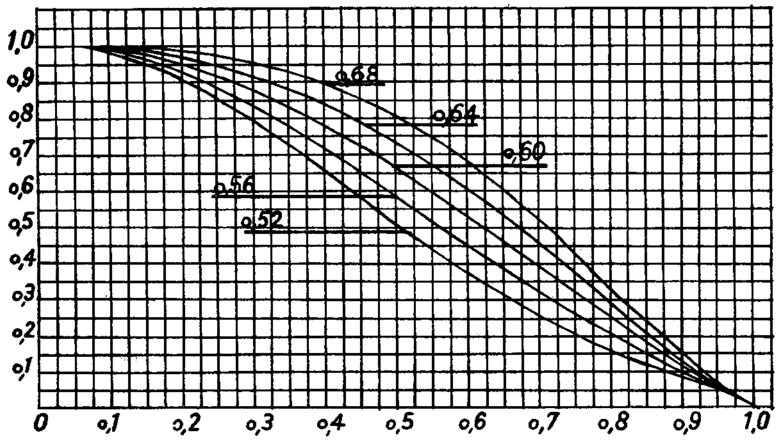

Fig. 6.—Ship lines (doublet distribution) generated by the polynomial family $X(\xi) = \langle 2\ 4\ 6;\ \varphi;\ 1\rangle$; *numbers on curves denote the prismatic coefficient* $c_p = \varphi$.

Restricting ourselves first to elementary ships for which the equation of the ship surface

$$\eta(\xi, \zeta) = X(\xi)\, Z(\zeta) \qquad [3]$$

or more rigorously the doublet distribution

$$m(\xi, \zeta) = m_1(\xi)\, m_2(\zeta)$$

with

$$A(\xi) \sim X(\xi) \sim m_1(\xi)$$

simple methods have been developed to compute complete resistance curves for families of mathematical lines which allow to be represented almost all " normal " section area curve shapes met in practice.

General information is obtained by investigating basic families of lines with two form parameters C_p and t. By using computed tables of auxiliary integrals resistance diagrams as shown in Fig. 6 are readily computed for different t values. So far auxiliary tables have been prepared for three H/L ratios; they admit further

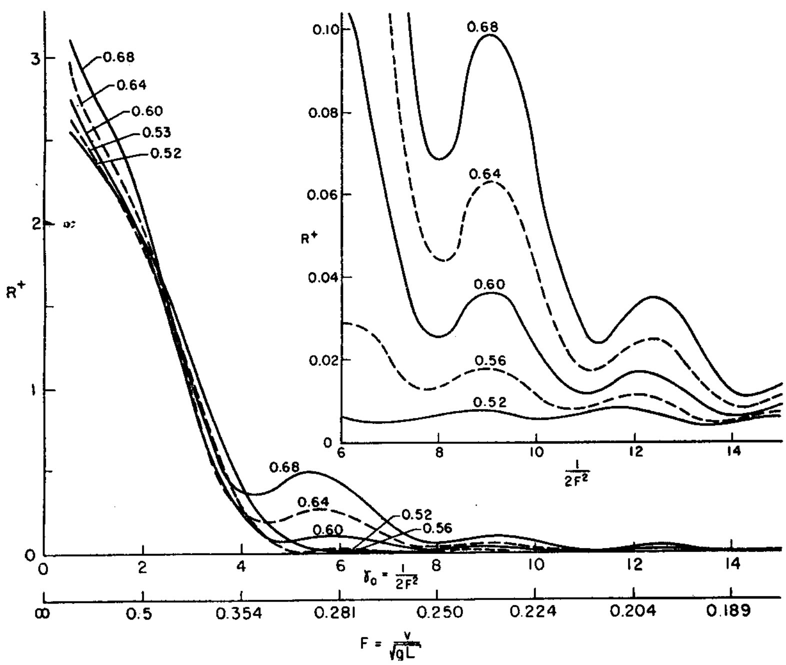

Fig. 7.—Wave resistance coefficients $R^+ = 7{\cdot}85R/\rho g B^2 H$ *of elementary ships generated by the family of distributions* $m_A = X(\xi)\ Z(\zeta)$ *with* $X(\xi) = \langle 2\ 4\ 6;\ \varphi;\ 1\rangle$ *and* $Z(\zeta) = 1$. $H/L = 1/20$.

a variation of C_x between 0·8 and higher than unity. Although variations in H/L and C_x influence appreciably the magnitude of the resistance values they change only slightly the general shape of the resistance curve. Therefore discussion is restricted to examples with H/L = const = 1/20, C_x = const = 1 only. By plotting further diagrams the dependence upon C_p and t for the given family can be established.

Fig. 7 discloses many interesting features, for example, that the average wave resistance of suitable fine forms may not increase much over wider ranges of the Froude number up to, say, about 0·3.

Using similar diagrams problems like "hollow versus straight" waterlines can be solved.[2, 4] The general conclusions are nicely supported by experimental work.

New and surprising results were obtained by selecting lines with equal values of C_p and t from different basic families (polynomials); generally they differ in shape. Apparently quite slight changes in the latter may lead to appreciable changes in

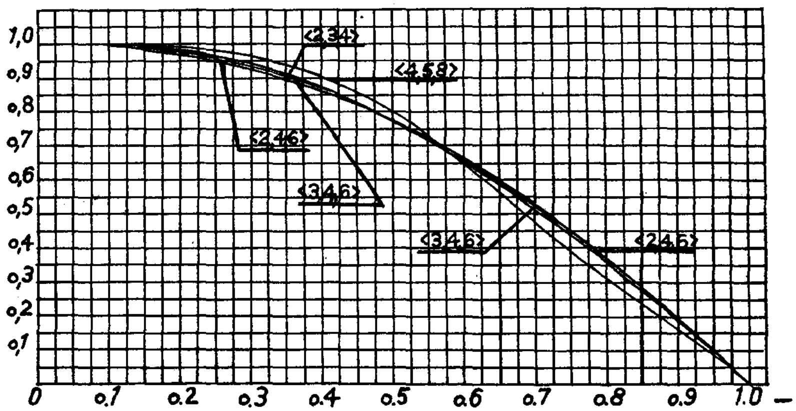

Fig. 8.—Ship lines (distributions) corresponding to various basic families with $\varphi = C_p = 0{\cdot}68$, $t = 2$.

the corresponding wave resistance; this is one reason why, notwithstanding the great amount of experimental work model research did not come to conclusive results.[6] (Figs. 8 to 11.)

In the light of the Author's results "classical" assumptions on the position of hollows and humps in the resistance curve (speed ranges of low and high resistance) must be revised. So far the approach has not included abnormalities in the sectional area curve like the bulb, swellings due to bossings and possible other peculiarities. By simple means Wigley[36] and the Author[37] succeeded in explaining the effect of the bow bulb and an additional stern bulb. There was some concern over the applicability

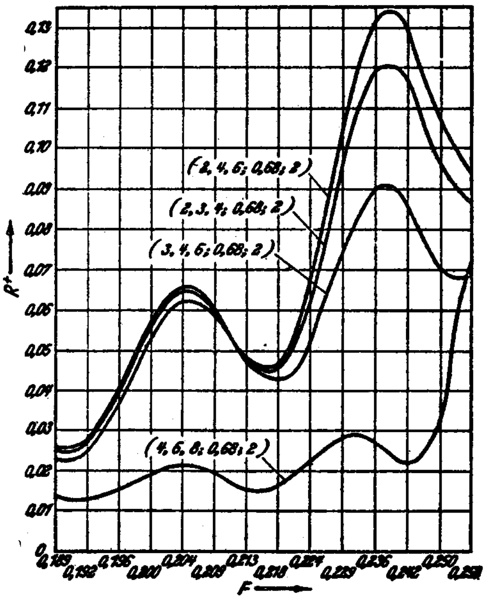

Fig. 9.—Wave resistance coefficients R $^+$ *of elementary ships generated by distributions following Fig. 8.*

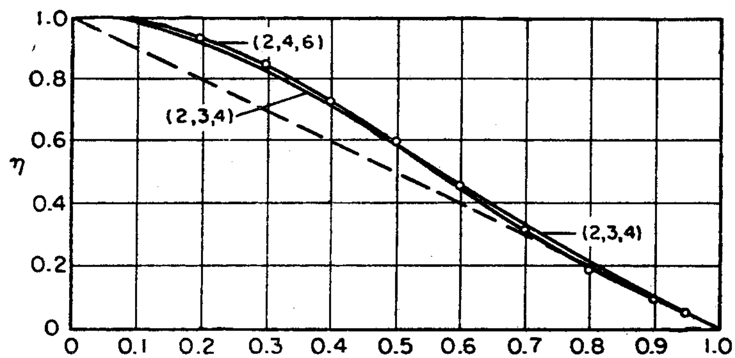

Fig. 10.—Distributions (sectional area curves) for $\varphi = C_p = 0{\cdot}56$ *and* $t = 1$ *corresponding to basic families* $\langle 2\ 3\ 4\rangle$ *and* $\langle 2\ 4\ 6\rangle$.

of the resistance integral to such blunt forms, but experimental checks justified the heuristic approach.

Further, it has been shown that especially in the range of lower and medium Froude numbers extremely bad results can be obtained when using inadequate lines like, for example, Chapman's parabolas of higher degree (Figs. 12 and 13). However, we do not wish to get lost in studying the properties of bad form; the essential task is to find ships of least wave resistance for given conditions, especially within the present context to derive optimum sectional area curves.[38]

Surprising scientific difficulties have been pointed out by

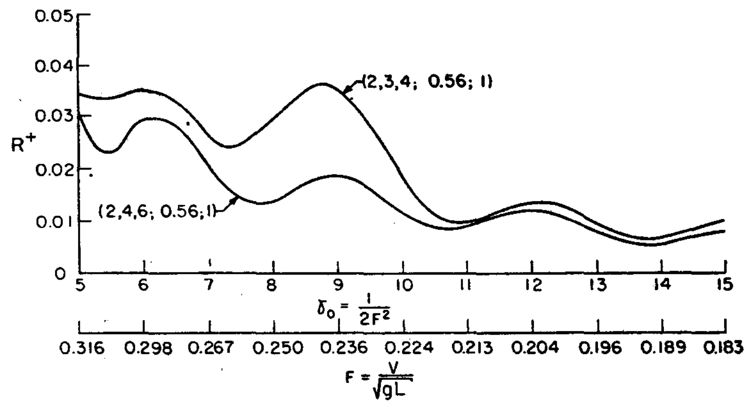

Fig. 11.—Wave resistance coefficients corresponding to Fig. 10.

different authors as to the possibility of solving exactly the underlying problem of the calculus of variation. It appears that satisfactory methods are now being developed to overcome the purely mathematical difficulties.

The general optimum problem goes into the core of this subject, but it is impossible to discuss here all formulations involved, which in principle go far beyond the determination of the sectional area curve.

In the meantime by using approximate methods a number of results referring to the optimum shape of sectional area curves, or better of distributions, have been derived. The limitations introduced by choosing polynomials with a small number of terms are still more serious than in the case of systematical variations

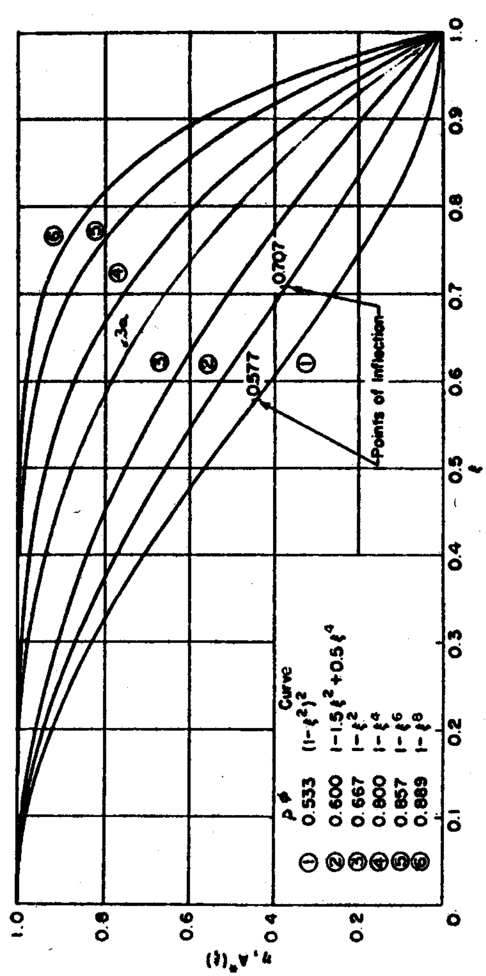

Fig. 12.—Chapman's parabolas and two fine parabolic curves; curve 3a corresponds to the equation $1 - |\xi|^s$.

already referred to. The numerical approach, which at the beginning admitted a very limited accuracy only, has been much improved by the use of the tables mentioned above.[31] Applying

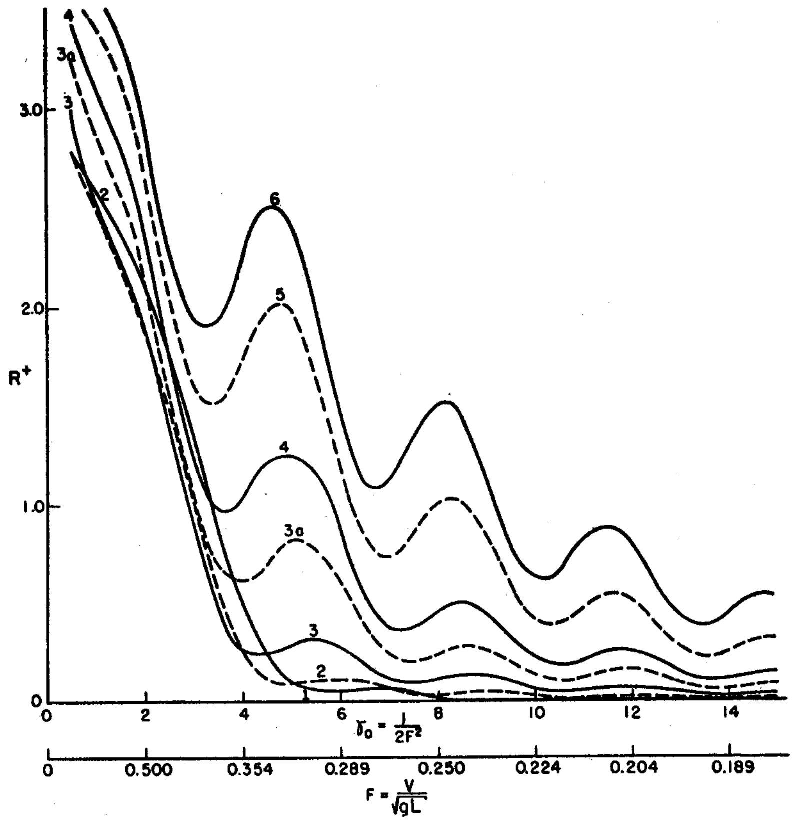

Fig. 13.—Wave resistance coefficients R^+ *of elementary ships generated by distributions following Fig. 12.*

electronic computers more general expressions than Michell's integral can be optimized. From the structure of the theoretical wave resistance formulae it follows that the optimum forms in ideal fluid are symmetrical with respect to the midship section,

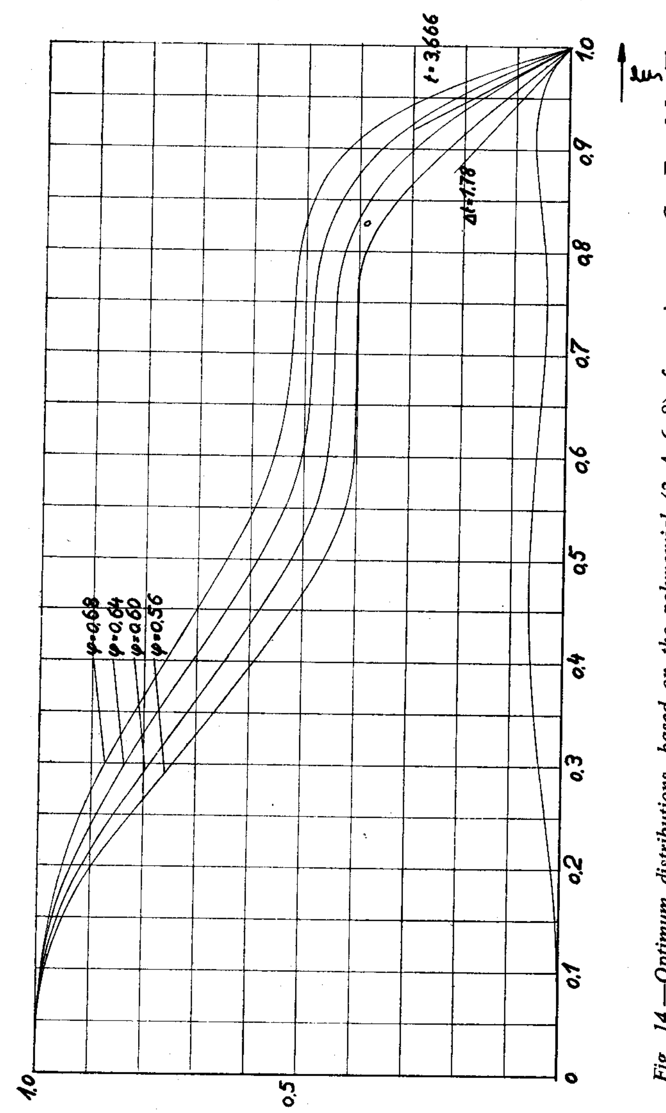

Fig. 14.—Optimum distributions, based on the polynomial ⟨3 4 6 8⟩ *for various* $\varphi = C_p$; F = 0·5. *The curves at the bottom of Figs. 14 and 15 represent the difference in offsets corresponding to a difference in* $\varphi = C_p = 0{\cdot}04$.

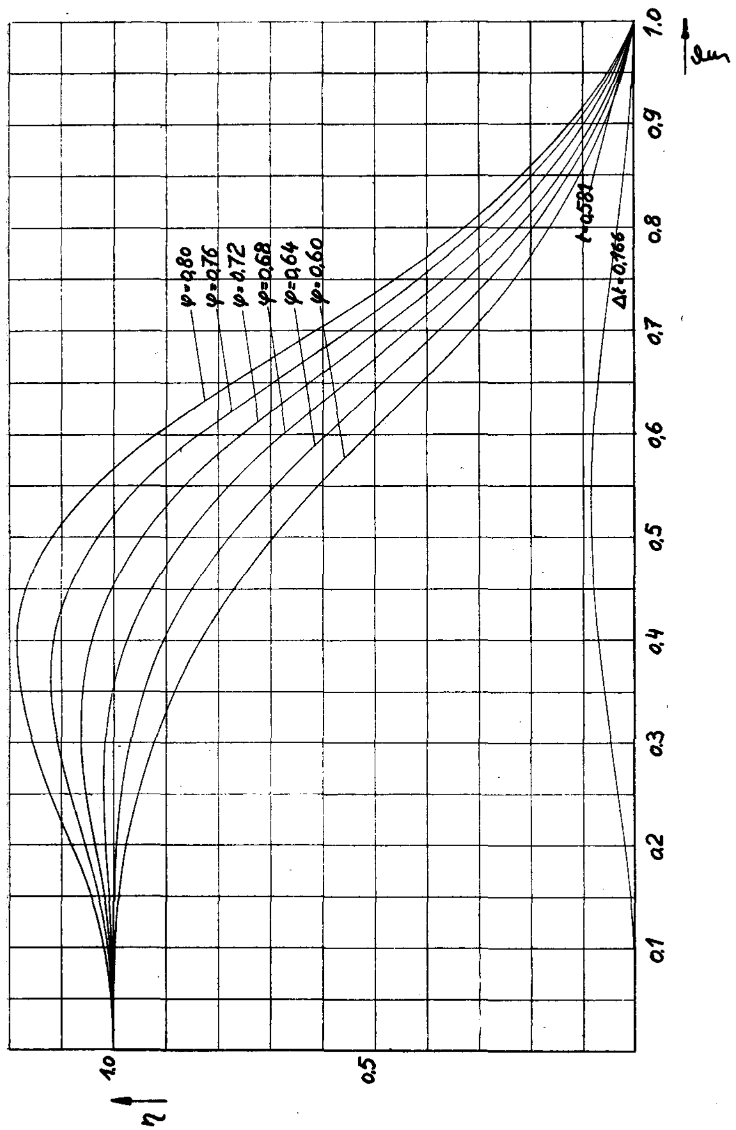

Fig. 15.—Optimum distributions, based on the polynomial ⟨3 4 6 8⟩ *for various* $\varphi = C_p$; F = 0·183.

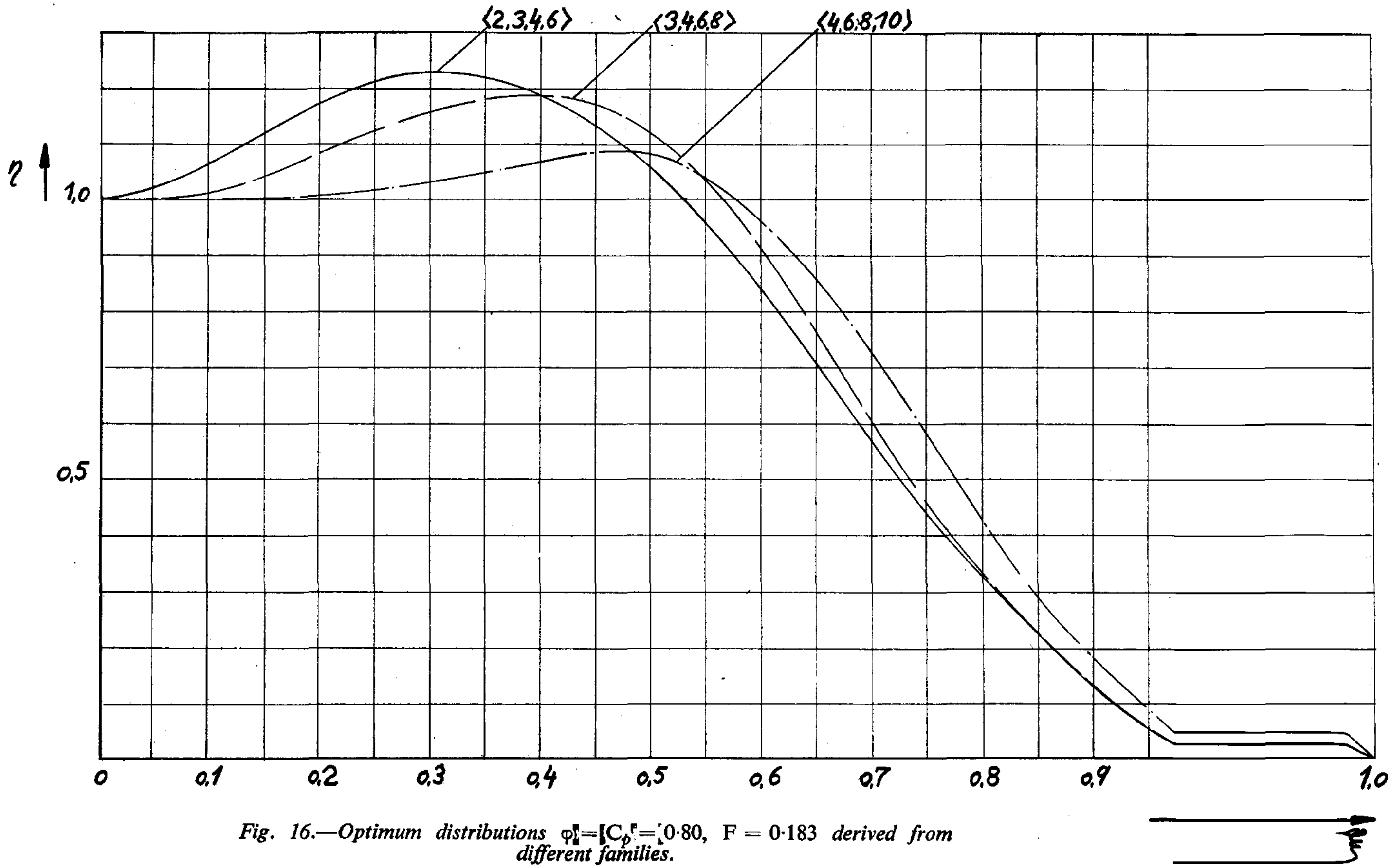

Fig. 16.—Optimum distributions $\varphi = C_p = 0{\cdot}80$, $F = 0{\cdot}183$ *derived from different families.*

that is, their fore- and after-body are identical. It was unfortunate for the prestige of theory from a practical point of view that the optimum properties of swan-neck forms at high Froude numbers predicted by theory had been discovered earlier in an intuitive way by D. W. Taylor (and even before). A rather comprehensive study[38] has been published recently dealing primarily with the domain of medium and low speeds keeping C_p = constant for optimization. The results can be summarized as follows:

Simple procedures were found to derive the optimum shapes of distributions for arbitrarily prescribed C_p values if such were calculated for two C_p.

The optimum shape and the corresponding minimum resistance for given Froude numbers depend greatly upon the basic form of the polynomial used. Unfortunately one must restrict oneself to a small number of terms to avoid a loss of accuracy in computation. Thus one cannot actually speak of optimum forms in a general way but only of optima derived for a certain polynomial distribution, or in a more practical way of forms of low resistance.

Forms derived for higher Froude numbers (in the range of the large hump) agree nicely with Taylor's results. However, the percentage gain obtained by such swan-neck forms is not too high when compared with good orthodox forms. Some experiments by D. W. Taylor indicate that in certain ranges of higher Froude numbers forms with lower t values may be superior, contrary to his systematic experimental findings and theoretical evidence. The neglect of trim underlying theoretical calculations appears to limit to a certain extent the generality of results obtained.

The resistance calculated for optimum forms, even with high C_p, at low Froude numbers is small, frequently almost negligibly small for usual ship proportions.

The optimum distribution curves for high C_p at low Froude numbers disclose some kind of unpleasant swellings in front of the midship section.

At this point the necessity to distinguish between the geometrical form (sectional area curve) and the distribution curve to which the optimization refers becomes especially obvious. Referring to computations[39] made first for bodies of revolution and quite recently for cylindrical and " Inui " bodies,[40] it can be established that the generating uniform flow has a smoothing influence upon

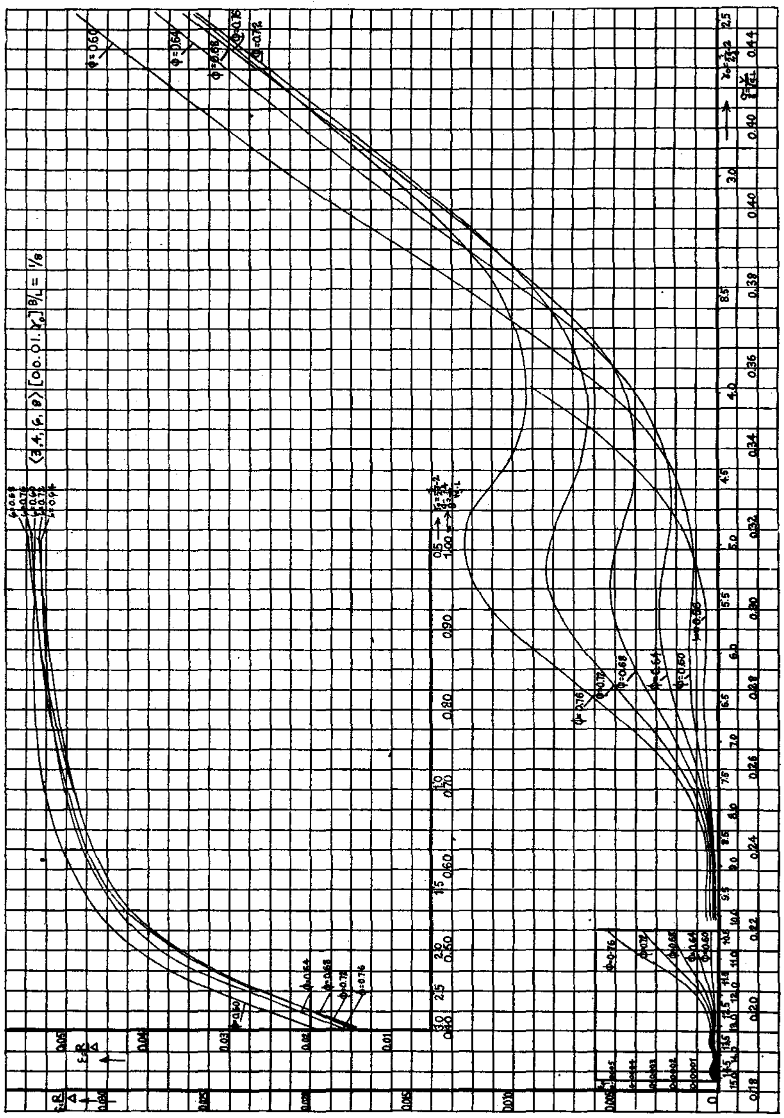

Fig. 17.—Unit wave resistance curves $\varepsilon = R/\Delta$ *corresponding to optimum distributions derived from the family* ⟨3 4 6 8⟩ *for various Froude numbers. The curves represent the minimum resistance attainable by the hull shapes derived from the mentioned polynomial for* $\varphi = C_p$ = constant *at pertinent Froude numbers.* L/B = 8.

the resulting body form as compared with that of the distribution; one of the most urgent needs is to extend the computations to a large number of distributions and resulting bodies. In the meanwhile the obtained optimum distributions are intermediate results.

With regard to the distributions with swellings, already referred to, it can be estimated that as long as the latter are moderate they lead to sectional area curves with approximately parallel middle bodies, the advantageous properties of which were established experimentally long ago for low Froude numbers. But in the light of what has been stated about the possible sensitivity of the wave resistance to slight changes of form definite steps must be taken to put estimates on a safer ground.

Crucial tests are suggested by the law of symmetry with respect to the midship section as optimum condition, and more generally by the rule following which the wave resistance of forms unsymmetrical with respect to the midship section is the same when going ahead and astern in an ideal fluid. Limitations have been indicated by R. E. Froude in his well-known empirical statement that in the range of the most useful Froude numbers the forebody is more active in generating waves than the afterbody.

As in good forms and at low and medium F the wave resistance is a small fraction only of the viscous drag such a rule appears to be natural.[45, 46] However, consistent comparisons can be obtained only when the viscous drag is determined with a high degree of accuracy, that is, from careful experiments with deeply submerged double bodies or with surface models (towed at low Froude numbers) of such a large size that turbulent flow conditions are reached in a range of negligible R_w. The latter method may be extended to somewhat higher F when, following Inui, the viscous drag can be determined by deducting calculated wave-resistance values from the total resistance.

Progress has been reached in understanding the much discussed problem of the optimum longitudinal position of the centre of buoyancy by applying wave-resistance theory. Notwithstanding the fact that forms with optimum total resistance must be asymmetric with respect to the midship section to reduce viscous pressure resistance the influence of strong asymmetry on wave resistance is borne out by experiments.

Tables have been prepared to deal with more general shapes of distributions or ship surfaces than represented by " elementary ships." Using such tables the wave resistance of U- versus V-shaped forms with identical sectional area curves can be immediately calculated. In agreement with experimental findings the advantage of U-shaped forms has been established. More work has to be done to obtain reliable quantitative predictions.

Empirical Corrections. Contradictions between calculated and experimental results are partly genuine, that is, due to shortcomings of theory, and partly to the inadequate analysis of resistance experiments. Further, comparatively few experiments are based on models of thin ship forms, which enable one to separate the departure from theory due to viscous effects.[47] Most comparisons should be listed under the heading " heuristic approach," that is, attempts to derive immediate practical results in the range of actual ship forms. Thus much remains to be done to investigate separately the divergence between theory and experiment due to approximations in boundary conditions (body shape) and viscosity of the medium. The most pronounced discrepancies in computed and measured resistance curves are the exaggerated interference effects (humps and hollows) found by theory, a slight shift towards higher F of the experimental curves and perhaps exaggerated mean resistance values in poor forms and diminished in optimum forms.

Proposals have been made by Havelock, Wigley, Guilloton, Emerson and Inui to correct these discrepancies and the following is a short account of the present state of pertinent suggestions due to Inui.[4]

(1) Consider a wave height correction β due to viscosity which influences the fundamental resistance term C_{w1} and the oscillating term C_{w2}.

(2) A phase shift due to an increase of the wavemaking length to $L + \delta L$, where δ is $\leqslant 0{\cdot}1$, say, influencing the oscillating term only.

(3) A sheltering effect on the bow wave system due to the body (hull interference) which is taken care of by a coefficient.

(4) A finite wave height correction.

Both (3) and (4) are especially important for large B/L and low F. Further information has been given by Inui on the magnitude of these correcting factors, and it has been proved that by using them good agreement can be obtained between results of calculation and measurements. The usefulness of this semi-empirical approach obviously depends upon the possibility to assess in advance the values of empirical factors for given conditions.

Shallow and Restricted Water Effects. Theory has succeeded in explaining the striking shallow-water wave effects.[4] Difficulties have been stated in the range of $F_h = u/\sqrt{gh} \doteq 1$ where steady state conditions are obtained under exceptional conditions only. Similar remarks apply to restricted water effects. Especially for low ratios water depth:length h/L experimental resistance values are much higher than predicted by theory in the neighbourhood of $F_h \doteq 1$, so that caution is recommended when applying quantitative results. Thus theory cannot yet claim to have solved the problem in an exhaustive way. But at h/L ratios valid for deep-water tanks theory furnishes excellent means to correct the influences of a finite cross-section on the wave resistance.[4]

Ship model tanks have finally realized the importance of theoretical solutions available in this field. There was a well-founded uneasy feeling about model experiments with high-speed ships which are operated in a range of F_h in the neighbourhood of and above $F_h = 1$. Evaluations are now available at $h/L = 0{\cdot}5$ and various draught:length ratios H/L, and further data will be presented in the near future. It appears that errors admitted by converting such experimental results to full size are not prohibitive. This could be gathered from a summary physical reasoning as long as $h/L \geqslant 1$, because of the prevalence of the diverging waves at very high Froude numbers.

The theoretical investigation of the accelerated rectilinear motion on shallow water has yielded surprising results. Maruo[16] has shown that the required length for the initial run is rather high in the range of depth Froude numbers $0{\cdot}7 < F_h < 1{\cdot}1$ as the resistance curve shows strong fluctuations over a much longer distance than earlier thought. It turns out that in shorter tanks it may be impossible to reach approximately steady state conditions. The impact of these findings on practice was decisive: so, for example, the newly-built shallow-water tank at Duisburg was considerably lengthened, at high cost, after the theoretical studies became known.

Maruo's investigations[16] on the increase of resistance in a seaway ΔR will have still more far-reaching practical consequences although it may take more time before the results will be applicable on a wider scale. Obviously, there exist relationships between motions of the ship (especially heave and pitch) and the resistance increase ΔR. However, it took a long time to understand them properly. Experimental evidence led to the conclusion that hull shapes may be superior as to motion and under the same conditions inferior with respect to the resistance. Starting from Havelock's and Hanaoka's work Maruo has considerably promoted shipbuilding science by showing that ΔR is a rather complicated function of heave and pitch even in the case of a ship moving with the waves or heading into them, although the influence of pitching is predominant. The maximum increase ΔR occurs close to a Froude number for which there is pitching synchronism. In a first approximation the resistance increase in waves does *not* depend upon the wave resistance in calm water. In the light of these statements earlier suggestions by Kent based on model experiments appear doubtful following which the use of a block coefficient $C_b > 0{\cdot}74$ appears detrimental with respect to the resistance in a seaway. Although the present results do not exhaust the problem it is thought that they will inspire the work of towing tanks.

Wave Resistance of Other Classes of Ships

Submerged Bodies of Revolution. The results obtained are in many respects similar to those for surface ships. Rather comprehensive evaluations based on tables of auxiliary integrals are available, and these answer a large number of questions presented by practice. One would expect that agreement between computed and measured resistance values is even better than in the case of surface ships; this question is, however, still open because of the shortcomings of the experimental material. Good qualitative agreement has been reached with regard to the dependence of resistance upon the prismatic coefficient of bodies of revolution especially in the range of the second hump, the adjacent hollow and the first hump of the resistance curve.[48] The wave resistance of bodies with other than circular cross-sections can be estimated with good accuracy from pertinent results with bodies of revolution having the same sectional area curve as proved by Havelock for the general ellipsoid.

Of special interest are such solutions for motions of submerged bodies which are not yet known for surface ships since they admit some conclusions by analogy. Such is the resistance of a spheroid in steady circling (Havelock[22]) and on a rectilinear path in a yawed condition. Both will contribute to improve the extremely poor knowledge of resistance phenomena experienced by a manoeuvring ship.

Planing Vessels and Hydrofoil Craft. The application of foil theory leads to a better understanding of the principles involved. It has been shown that for very high Froude numbers wave or gravity effects do not contribute much to the resistance although the lift force is affected, but in an intermediate range they are important. Impressive work is going on to determine unsteady flow effects on hydrofoil performance.

Hogner's formula[7] leads to an estimate of the wave resistance of planing bodies. More recently solutions have been given by Maruo which enable the spray as well as the wave effects to be estimated.

Propeller and Ship Propulsion

Dickmann[50] points out that by choosing a simple hydrodynamical model (a sink) for the propeller the wave system created by the latter is not strong. Thus the wave resistance of the propeller as such is in general almost negligible; however, the interference effects between the wave fields created by the ship and the propeller can be appreciable. It follows that it is advantageous to locate the propeller in the region of a wave crest created by the hull, as suggested earlier by Horn. It has been further deduced that the thrust deduction effects due to wave phenomena are small. However, experimental findings are still lacking.

Conclusions

It is the Author's hope that he has succeeded to some extent in emphasizing the importance to practice of the theoretical results so far obtained, notwithstanding the numerous limitations of the latter. Still more essential is the outlook for the future. At present experimental tanks have to base their work on a theoretical foundation when solving practical problems. In this respect the substantial work due to Inui appears to be significant. It is further expected that beyond the indirect way mentioned an immediate impact of theory on design methods will be felt within the near future.

Acknowledgment. The greater part of this paper was prepared during the Author's stay, as visiting professor, at the Department of Naval Architecture of the University of California, and it is a pleasant duty to express his thanks to this Alma Mater. Further, he dedicates his study, which is of transitory value only, to Sir Thomas Havelock as a sign of indebtedness and admiration for his lasting achievements in this field.

Bibliography

(1) " The Wave Resistance of a Ship," by J. Michell. *Phil. Mag.*, 5th series, 1898, vol. 45, p. 106.

(2) " Studies in Wave Resistance: Influence of the Form of the Water-Plane Section of the Ship," by T. H. Havelock. Proc. Royal Soc., ser. A, 1923, vol. 103, p. 571.

(3) " Ship Wave Resistance. A Comparison of Mathematical Theory with Experimental Results," by W. C. S. Wigley. Trans. Inst. Naval Arch., 1926, vol. 68, p. 124.

(4) " Study on Wave-Making Resistance of Ships," by T. Inui. Soc. Naval Arch. Japan, 60th Anniv. Series, 1957, vol. 2, p. 173.

(5) Scientific Papers of Lord Kelvin, vol. IV. 1937.

(6) " Analysis of Wave Resistance," by G. P. Weinblum. 1950. Report No. 710. Taylor Model Basin, Washington.

(7) " Notes on some new Contributions to the Theory of Ship-Waves," by E. Hogner. Arch. Matematik Astronomi och Fysick, 1923, vol. 18, p. 1.

(8) " Theory of Wave Resistance," by T. H. Havelock. Proc. Royal Soc., ser. A, 1932, vol. 183, p. 339.

(9) " Schiffsform und Wellenwiderstand," by G. P. Weinblum. Discussion by E. Hogner. Jb. Schiffbautech. Ges., 1932, vol. 33, p. 419.

(10) " Further Notes on the Theoretical Calculation of Wave Profiles, and of the Resistance of Hulls," by R. Guilloton. Trans. Inst. Naval Arch., 1946, vol. 88, p. 308.

(11) " Sur la Détermination de la Résistance Ondulatoire d'un Navire se Déplaçant à la Surface de l'eau d'une Profoundeur Finie," by L. Sretenski. Doklady Acad. Nauk U.S.S.R., Comptes Rendus de l'Acad. des Sciences de l'U.R.S.S., 1936, vol. II (XI), p. 247.

(12) " Theory of Wave Resistance," by L. Srentenski. (In Russian.) Trudi zentralnogo aero-gidrodinamitscheskogo Instituta, 1939, No. 458.

(13) " Theory of Wave Resistance in a Channel of Finite Depth," by M. Keldysh and L. Sedow. (In Russian.) Wave Confer., Moscow, 1937.

(14) "The Wave Resistance of a Cylinder Started from Rest," by T. H. Havelock. *Quart. Journ. Mech. Appl. Math.*, 1949, vol. 2, p. 325.

(15) "On the Linearized Theory of Wave Resistance for Displacement of Ships in Steady and Accelerated Motion," by J. K. Lunde. Trans. Soc. Naval Arch. & Mar. Engrs., 1951, vol. 59, p. 25.

(16) "Modern Developments of the Theory of Wave-Making Resistance in the Non-Uniform Motion," by H. Maruo. Soc. Naval Arch. Japan, 60th Anniv. Series, 1957, vol. 2, p. 1.

(17) "Wave Resistance: the Mutual Action of Two Bodies," by T. H. Havelock. Proc. Royal Soc., ser. A, 1936, vol. 155, p. 460.

(18) "Ueber Widerstandsverhältnisse von Zweikörperschiffen," by K. Eggers. Jb. Schiffbautech. Ges., 1955, vol. 49, p. 516.

(19) "On the Wave Resistance Theory of a Submerged Body," by M. Bessho. Soc. Naval Arch. Japan, 60th Anniv. Series, 1957, vol. 2, p. 135.

(20) "L'état Actuel des Calculs de Résistance de Vagues," by W. C. S. Wigley. Bull. de l'Assoc. Tech. Mar. et Aeron., 1949, vol. 48, p. 533.

(21) "Wave Resistance of a Solid in Motion through a Fluid of Finite Depth," by M. Haskind. (In Russian.) Prikladnaja Matematika I Mekanika, 1945, vol. 9, p. 257.

(22) "The Forces on a Submerged Spheroid Moving in a Circular Path," by T. H. Havelock. Proc. Royal Soc., ser. A, 1950, vol. 201, p. 297.

(23) "Ueber Stoss- und Gleitvorgange an der Oberflache von Flüssigkeiten," by H. Wagner. *Zeit. Angew. Math. Mech.*, 1932, vol. 12, p. 193.

(24) "On the Wave Resistance of Hydrofoils," by — Keldysh, — Lavrentiev and — Kochin. (In Russian.) Wave Conf., Moscow, 1937.

(25) "Application of Ship-Wave Theory to the Hydrofoil of Finite Span," by P. Breslin. *Journ. Ship Research*, 1957, vol. 1, p. 27.

(26) "Viscous Effect on Surface Waves generated by Steady Disturbance," by T. Yao-tsu Wu and R. E. Messick. Cal. Inst. Tech., Eng. Div. Report 85-8. 1958.

(27) "Application of Slender Body Theory," by P. Kaplan. *Journ. Ship Research*, 1957, vol. 1, p. 40.

(28) "Study on Submerged Hydrofoils," by T. Nishiyama. Trans. Soc. Naval Arch. Japan, 60th Anniv. Series, 1957, vol. 2, p. 95.

(29) "Waterwaves," by J. Wehausen. Handbuch der Physik: Springer & Co., Berlin.

(30) "Some Model Basin Investigations of the Influence of Form of Ships upon their Resistance," by D. W. Taylor. Trans. Soc. Naval Arch. & Mar. Engrs., 1911, vol. 19, p. 59.

(31) "Ein Verfahren zur Auswertung des Wellenwiderstandes vereinfachter Schiffsformen," by G. Weinblum. *Schiffstechnik*, 1956, vol. 3, p. 278.

(32) "The Moment Acting on a Rankine Ovoid Moving under a Free Surface," by H. L. Pond. 1951. Report No. 795. Taylor Model Basin, Washington.

(33) "Calculated and Measured Wave Resistance of a Series of Forms defined Algebraically, the Prismatic Coefficient and Angle Entrance being varied Independently," by W. C. S. Wigley. Trans. Inst. Naval Arch., 1942, vol. 84, p. 52.

(34) "Michell's Theory and its Application to the Calculation of the Wave Resistance," by K. Antimonov. (In Russian.) Trudy Wnitoss, Leningrad, 1934, vol. 4, p. 19.

(35) "The Influence of the Main Dimensions of a Ship on its Wave Resistance," by L. Srentenski and I. Girs. Prikladnaja Matematika I Mekanika, Moscow, 1946, vol. 10, p. 25.

(36) "The Theory of the Bulbous Bow and its Practical Application," by W. C. S. Wigley. Trans. N.E. Coast Inst. Eng. & Shipbldrs., 1935, vol. 52, p. 65.

(37) "Theorie der Wulstschiffe," by G. Weinblum. *Schiffbau*, 1936, vol. 37, p. 55.

(38) "Schiffe geringsten Widerstandes," by G. Weinblum, D. Wustrau and G. Vossers. Jb. Schiffbautech. Ges., 1957, vol. 51, p. 175.

(39) "Rotationskörper geringsten Wellenwiderstandes," by G. Weinblum. *Ing.-Archiv.*, 1936, vol. 7, p. 105.

(40) "Ueber die Ermittlung der schiffsähnlichen Umströmungskörper vorgegebener Quell-Senken-Verteilungen mit Hilfe Elektronischer Rechenmaschinen," by K. Eggers and W. Wetterling. *Schiffstechnik*, 1957, vol. 4, p. 284.

(41) "Wellenwiderstand von Rotationskörpern," by G. Weinblum. Schiffstechnik, 1958, vol. 5, p. 43.

(42) Discussion by — Eaves on paper by J. Wehausen at Symposium on Ship Hydrodynamics, Washington, 1956.

(43) "Research Trends in Naval Hydrodynamics," by P. Eisenberg. *Journ. Ship Research*, 1958, vol. 2, p. 3.

(44) "The Wave Resistance of a Floating Slender Body," by W. E. Cummins. Dissertation, American Univer., Washington, May 1956.

(45) "Comparison of Calculated and Measured Wave Resistances for a Series of Forms not Symmetrical Fore and Aft," by W. C. S. Wigley. Trans. Inst. Naval Arch., 1944, vol. 86, p. 41.

(46) "Rechnerische Entwicklung von Schiffsformen," by G. Weinblum. *Schiffbau*, 1938, vol. 39, p. 129.

(47) "Investigation of Wave Effects produced by a Thin Body," by G. Weinblum, J. Kendrick and A. Todd. 1952. Report No. 840. Taylor Model Basin, Washington.

(48) "Tests on Wave Resistance of Immersed Bodies of Revolution," by G. Weinblum, H. Amtsberg and W. Bock. 1950. Translation 234. Taylor Model Basin, Washington.

(49) "Two Dimensional Theory of the Hydroplane," by H. Maruo. Proc. 1st Nat. Congress Appl. Mech., 1952, p. 409.

(50) "Schiffskörpersog, Wellenwiderstand eines Propellers und Wechselwirkung mit Schiffswellen," by J. Dickmann. *Ing.-Archiv.*, 1938, vol. 9, p. 452.

Discussion

Mr. S. TENNANT, B.Sc.: Wave resistance theory is a highly mathematical subject and beyond the scope of the average practising naval architect, so that it is to their benefit to have an expert on the subject like the Author to bring home to them the salient points on the progress of research into the subject.

How near to agreement are the theoretical and actual values of residuary resistance at the present moment, and how long will it be before the method takes its place as a tool of the naval architect in the same way as the Taylor series of curves do at the moment? Will it ever be possible to reverse the Froude approach and derive the true skin friction line?

For calculation work it is obvious the ship form must be capable of exact mathemetical representation, which incidentally would be valuable in many fields of practical ship construction apart from the field at present under consideration. To what extent can a good waterline be represented mathematically? Can it be represented by one polynomial expression, or must it be divided into segments, each of which must be represented by a different equation?

The whole scope of the practical application of this work hinges upon complex mathematics which would be very laborious without the use of the electronic computer. Looking ahead, will it be necessary to have access to computers, or will it be possible to apply factors and tables, as in what might be considered a companion sphere to this, at least mathematically, the pressure distribution on the propeller blade?

The Author has provided a very meaty sandwich which will require much chewing over, in association with the bibliography, to provide adequate digestion. For his guidance on this complex subject, naval architects owe him a great debt of gratitude.

Prof. A. M. ROBB, D.Sc., LL.D. (Past President): It may be desirable that this extensive—and persuasive—paper should be brought under review against a background of actual circumstance. On that consideration it is permissible to argue that the most important development in the fairly recent approach to the problem of ship resistance was the discarding of the Froude evaluation of frictional resistance; the importance lay in the fact that the evaluation had been accepted without question or challenge for nearly half-a-century, and then had survived, although under challenge, for nearly another quarter-century. The mathematicians had no part in bringing about the departure from long-established practice. Indeed they were content to check their results by reference to the original Froude dichotomy long after it had become reasonable to suspect that somewhere, somehow, there was an error. For the record, it is justifiable to point out that suspicion was first generated somewhere between 1912 and 1916 by a slightly older naval architect of enquiring mind. The ground for suspicion was consideration of published claims for reductions in ship resistance as a result of model experiments; the claims indicated that if the basis of calculation were sound the residuary resistance, even at service speeds, had been reduced almost to vanishing point. Ground for suspicion was confirmed early in 1924 by consideration of a paper in which an attempt had been made to correlate propeller thrust and tank prediction, and a relative rotative efficiency around 0·80 to 0·85 seemed to be necessary. That led to a first, and admittedly timid, expression of doubt. It was embodied in a paper* on ship design presented by Sir John Biles in 1924; there was, at least, reference to gaps in our knowledge.

Not only was there on the part of the mathematicians an absence of the heuristic approach commended by the Author, there was actual blindness to records of experiment. Older members may recall the Ⓟ criterion for the occurrence of humps and hollows on curves of resistance—a matter which gave rise to more rudeness in discussion than any other in naval architecture. On at least one occasion the criterion was justified by reference to mathematical treatment. But the criterion was based on seriously improper plotting, and not long after a discussion notable for rudeness it was quietly discarded. There was a more remarkable instant of blindness on the part of a high-priest of mathematical investigation. He plotted a curve of resistance in shallow water. The general trend shown by a solid line was quite in accordance with experimental record. Unfortunately the justifiable solid line was supplemented by a dotted line showing the succession of humps and hollows which, according to an attractive and seemingly rational theory, should precede the ultimate hump.

* Trans. Inst. Naval Arch. 1924, vol. 66, p. 285.

Most unfortunately there is not a shred of experimental evidence in support of the dotted line; it is a reasonable presumption that the steep rise in resistance for a ship in shallow water is due to something other than wave interference.

In view of these considerations it is surely permissible to question whether the mathematical approach to the problem of ship resistance has yet been put on the right line. Is it possible to justify the basic assumption that frictional and pressure resistance are completely independent? Incidentally, Prof. Lunde, as one of the mathematical investigators, has fairly recently indicated doubt on that matter. But once doubt is admitted the " gaps in our knowledge " visibly become immensely wider. There is, for example, a vast ignorance of the turbulent motion which must come under consideration if the doubt is admitted. Incidentally, is it not possible that a rational approach to the consideration of turbulent motion demands investigation of the basic assumption that there is no slipping between a flowing fluid and a stationary body in contact with it? There is a theoretical consideration, and there is some experimental evidence, which suggest that the assumption is not fully valid; it may be permissible to suggest that the initiation of turbulence marks the breakdown of an assumption valid for laminar motion.

All in all, there is some justification for the belief that the mathematicians are unduly optimistic. And it is permissible to suggest that the heuristic attitude which is completely desirable should find expression in a phrase recently applied to a prominent figure in our literature: he was classed as a " believer in disbelief."

Mr. H. VOLPICH, B.Sc. (Member): Up to the present both the tank authorities and the practitioners in the United Kingdom and Europe have in most instances worked to the R. E. Froude friction line. In 1957 at the International Towing Tank Conference at Madrid a new line was proposed, generally known as the I.T.T.C. 1957 line, in order to bring model predictions and ship data better into line. Although this new line has so far not been universally adopted the time is arriving when this change will have to be considered. The Author now proposes yet another modulation, which would increase the difficulties that tanks in general already experience.

The Author further makes reference to bulbous bow designs. Could the best bulbous bow be evolved mathematically without having to go to model tests? At the Denny tank some remarkable results were obtained with a bulbous bow form in recent times, but this was the outcome of some extensive model testing. The writer wonders whether this could be short cut to a certain extent by determining mathematically the optimum size of bulb and its longitudinal position (ordinary or ram).

Prof. J. F. C. CONN, D.Sc. (Member of Council): This excellent paper has been presented in masterly fashion and with a humour which has softened the rigour of the mathematical analysis. It gives a far-reaching review of wave resistance theory and its applications to ship design. The comments which follow are intended to elicit further information.

Firstly, the optimum sectional area curves in Figs. 14, 15 and 16 are less disturbing than they might seem to the casual observer. There is an unfortunate tendency to consider immersed area curves in the at-rest condition only. When the usual merchant vessel is in motion there is a tendency for the vessel to squat bodily and for trim by the head to develop. When this is kept in mind and considered in relation to the wave profile, the sectional area distributions in Figs. 14, 15 and 16 are quite reasonable.

Secondly, the Author is an acknowledged authority on mathematical lines for ships. In these days of computer facilities mathematical lines have attained a new importance. Any remarks which he may care to make on the latest methods of representing the surface of a hull by analytical expressions would be most welcome.

Thirdly, the majority of practical people still think in terms of the Froude division of total resistance into skin friction and residuary resistance components. An exposition of the latest views on this matter, particularly a discussion of the total viscous resistance as the sum of separate components, would enhance the interest and the value of the paper to the ordinary reader.

Fourthly, would the Author give his views on the separation of viscous and wave drags by means of the wake survey, or, as it is sometimes called, the Pitot traverse method? In an excellent paper,* Marshall P. Tulin has contended that this method would

* Taylor Model Basin Report No. 772. 1951.

give the total viscous drag of a ship model. If this is correct, and it appears so to the writer, then a great advance has been made, although the experimental work which is necessary is extremely laborious.

Fifthly, some remarks on Sretenski's work on the wave resistance of ships in a viscous fluid would be appreciated.

Finally, the writer wishes to express his appreciation of, and his thanks for, a most interesting, scholarly and stimulating paper.

Mr. J. M. FERGUSON (Member): It is rather interesting to note how the development of the wave resistance theory and calculations appear to be so dependent on the sectional area curve. In a paper* as far back as 1904, R. E. Froude stated: " It is important for it to be understood, therefore, that in our experience, so long as no unfair features are introduced, such as may cause serious eddy making, we may almost say that the resistance of a form is determined solely by the curve of cross sectional areas, together with the extreme beam and the surface water-line of the fore body; and if these are adhered to, the lines may be varied in almost any reasonable way without materially increasing or decreasing the resistance at any speed."

The writer is pleased that Prof. Conn has raised the question of the sectional area curve of a form in motion. Recently, while working on tests involving that particular problem, the sectional area curves derived from an analysis of the wave profile ordinates at various speeds of advance were more suggestive of a hybrid between the curves shown in Figs. 14 and 15 of the paper than the sectional area curve normally accepted as the basis of design.

Many years ago a young colleague of the writer began an investigation into the properties of sectional area curves. He carried out a harmonic analysis of a sectional area curve; taking the cosine series therefrom, he endeavoured to relate the coefficients of each cosine term to a particular speed/length ratio. Generally, the humps and hollows of this coefficient curve appeared to bear some relationship to the humps and hollows of the resistance coefficient curve. This led to a suggestion that the reduction of the coefficient of a harmonic at or near a hump

* Trans. Inst. Naval Arch. 1904, vol. 46, p. 64.

in the resistance curve might result in a reduction of the resistance at that point. The sectional area curve, of course, would be modified through an adjustment of the harmonic series, the reduction in one term being balanced by an increase in another term well away from the critical speed/length ratio.

One particular case was examined on these lines and, to our surprise and a degree of horror, it was discovered that the reconstituted sectional area curve had a very marked hollow at midships, much as shown in Fig. 16. Unfortunately, a model was not made to this curve.

The Yourkwitch patent leads to a similar type of curve for high speed/length ratios. How this type of sectional area curve could be applied to ship forms as we know them is a matter of conjecture and would present a serious practical problem to the shipbuilder.

The question of the sectional area curve under way leads to a very provocative thought. If one takes the curve of sectional areas for a normal ship shape form when it is advancing at a specified speed, and then attempts to design a form with that sectional area curve, would the humps and hollows of the form iron out the humps and hollows of the resulting wave system and thereby produce a form of exceptionally good performance. The appearance of such a form, however, would be rather staggering from the orthodox or traditional standpoint.

Mr. A. EMERSON, M.Sc.: The Author has presented an admirably balanced account of present work on wave resistance and has used results of the " polynomial representation," for which he is mainly responsible, to illustrate applications.

There is one comment to make on Inui's work. Inui's first major contribution was to show that the approximation implied in Michell's " thin ship " theory gave an equivalent ship form appreciably different from the actual ship, the so-called " equivalent ship " having fuller ends and fine midships, the differences being important even for long narrow ships. With this fundamental mistake in representation eliminated, the more baffling differences between calculated and measured results disappear. As Inui's papers have only recently been readily available in this country it seems to be well to emphasize this point in the discussion.

Mr. C. WIGLEY: This paper is of the nature of a textbook or dictionary of the recent progress in its subject. It will therefore be very useful to research workers and ship designers, but does not lend itself easily to comment. One or two points which occur to the writer may, however, be mentioned.

In the first paragraph the Author is not quite fair to Michell, who not only made the calculation referred to in the paper, but also compared his result with an approximate figure which he must have obtained from some technical source, and observed that the calculation gave a resistance of the right order.

The Author does not emphasize as strongly as he might that no direct comparison of absolute values is possible between calculated wave resistance and any physical measurement. The frictional and form resistances are still too much a matter of guesswork. The changes in resistance produced by slight changes in form are the only calculations that can be directly checked, on the assumption that the changes are too slight to affect either the frictional or form resistances. As these changes tend to be lessened in a viscous fluid, there is a great need for the establishment of an empirical correction to take into account the causes of discrepancy mentioned by the Author on p. 146. To this list the effect of the discontinuity of the potential at the bow when the bow angle is finite might be added.

In conclusion the writer hopes that this excellent paper will fulfil its purpose of helping to bridge the gap between the practical shipbuilder and the theoretical worker.

Author's Reply

Prof. WEINBLUM: Replying to Mr. Tennant, it has been emphasized that the present theory cannot replace model work, especially results of reliable series investigations. Much basic scientific development has to be done to reach this goal, but it is hoped that within the near future it will be possible to obtain reasonable estimates of the total viscous drag using guidance by wave resistance theory.

The representation of good waterlines and sectional area curves by algebraic expressions is a comparatively simple matter. It is advisable to divide the curves into segments when a parallel part is involved; in general the use of separate equations for the fore-and after-body will be recommendable in the process of approximating empirical lines.

For systematic investigations the calculation of wave resistance should be based on tables prepared by an extensive use of electronic computers. Ample work is going on in this field. Computers have been applied for solving particular cases also, but the Author has no experience in this matter.

According to Prof. Robb the most important development " was the discarding of the Froude evaluation of frictional resistance." The Author would agree to some extent with this opinion if Prof. Robb would kindly substitute " criticism of the Froude method " for " evaluation of frictional resistance." This involves the introduction of the viscous drag concept. He disagrees, however, with the severe criticism concerning the contribution made by mathematicians to the subject. W. Froude himself was an excellent physicist and engineer; over a long period the profession has failed in displaying a comparable amount of sound judgment and intuition. The rather unfortunate Ⓟ criterion has been put forward by naval architects; the case of mathematical support hinted at is not too conclusive as the scientist concerned has checked within a limited range a tentative hypothesis suggested by persuasive practical people. This is a legitimate procedure.

One cannot leave the solution of shipbuilding problems to mathematicians alone; naval architects and physicists have their share of responsibility. In the oral presentation the Author attempted to describe what he considers a fruitful attitude: modesty in claims put forward by theoreticians as to the applicability of their results and patience and grateful appreciation by practical people for the help given by mathematicians. The widespread habit in the profession to press research workers too hard for final solutions of difficult problems can be extremely detrimental.

The decisive step in promoting the naval architect's problem was made by the eminent engineer Foettinger, the father of the double model technique. When starting his research on wave resistance theory the Author made checks by this method and gave some thought to the interaction of frictional and wave phenomena by using smooth and extremely rough simple and double models.[9] Inui's recent work is a further proof that the mathematical theory has decisively stimulated the problem at stake.

As in turbulent motion there is a laminar sub-layer no reason exists to consider a departure from basic assumptions as to boundary conditions.

With regard to Mr. Volpich's first question, it is the Author's personal opinion that the new I.T.T.C. friction line may improve the model/ship correlation but being physically inconsistent it cannot lead to a final solution, which should be sought on lines suggested by Foettinger, Horn, Hughes, Granville, Inui, etc. Tanks must accept temporary difficulties if progress in knowledge can thereby be expected.

Theoretical work on bulbous bow design has been done by W. C. S. Wigley and the Author. The results are stimulating but not final; because of the limitations of theory experimental methods are indispensable.

Prof. Sretenski's paper on wave resistance in a viscous fluid, presented to the Wageningen Symposium 1957, is presumably the first bold task in this new and difficult field. No numerical evaluations are so far available; the physical limitations mentioned by the distinguished Russian author are serious. Therefore the Author is unable at the moment to make any useful comment.

The Author thanks Mr. Wigley for his statement on Michell's attitude. Knowing the Author's deep admiration for Michell, Mr. Wigley will accept the explanation that nothing deprecating was meant by the pertinent remarks.

Mr. Wigley correctly points out that the viscous drag problem has been only slightly touched upon in the paper. This was because of a recent paper* by Lap presented to this Institution on a similar subject although the nomenclature used by him is different.

Prof. Conn raises many important topics. The Author is grateful for his first remark on the actual form of the sectional area curve when the vessel is in motion, which raises a new aspect of the problem. However, he does emphasize once more that the distribution curves so far obtained are intermediate products only if one wishes to apply theory to forms which cannot be considered as thin ships.

* Trans. Inst. Eng. & Shipbldrs. in Scot., 1957-58, vol. 101, p. 369.

* *loc cit.*

For the purpose of wave resistance calculations and within the limits of accuracy attainable the analytical representation of ship lines does not present serious difficulties. Conditions are different when one wishes to avoid fairing in the mould loft by basing the design on mathematical curves. Reference may be made, for example, to recent work by Dr. Pien of the Taylor Model Basin. Some serious difficulties with regard to bilge curvature have, however, still to be overcome.

The use of the viscous drag concept is indispensable when the aim is a physically consistent explanation of the ship resistance phenomena. The total viscous drag includes the tangential resistance which may be split up into frictional resistance of the equivalent plate and the frictional form resistance and the viscous pressure drag—a concept which to some extent coincides with Froude's eddy resistance. The double model technique, although open to some criticism especially because of experimental difficulties, was very helpful in elucidating basic physical facts. In the long run correct power prediction as well as investigations on improving ship forms require the use of the viscous drag concept. Reference is made to recent work by Inui, Hughes and Granville—just to mention a few names.

In principle Tulin's work* is a definite step forward. It is to be regretted that to the Author's knowledge no Taylor Model Basin reports have been issued on the subject, although work was initiated a long time ago. The scientific merits of the method justify considerable expense to reach conclusive results.

The Author concurs with Mr. Emerson's remarks on Prof. Inui's achievements. More definite statements will be possible after a greater number of Inui's bodies are available. Unfortunately the work involved is more cumbersome than originally assumed. To supplement the important research, cylindrical bodies and bodies of revolution are being evaluated systematically. In addition one has to realize that further steps will be required in the range of large wavemaking as clearly pointed out by H. Pond.

The Author highly appreciates Mr. Ferguson's quotation from R. E. Froude's important paper, which elucidates in a most concise manner the dependency of (wave) resistance upon form, especially upon the sectional area curve. Once more the Author wishes to acknowledge that the use of the concept " sectional area curve in motion " may lead to interesting tentative working hypotheses for investigating properties of the ship forms which are not thin. Mr. Ferguson's provocative thought in his last paragraph should be tested experimentally as within the present linear theory the formal optimization leads directly to distributions which possess the desired minimum properties.

It is to be regretted that the results of the harmonic analysis mentioned are no longer available; by introducing Fourier series instead of polynomials the problems should be susceptible to a solution within the validity of present methods.

The Author fears that the application of the Yourkevitch rule to high speed/length ratios overstrains the underlying empirical formula.

The Author wishes to express his gratitude to all contributors for the kind interest shown in the paper.