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On the origin of passive rotation in rotational joints, and how to calculate it

Shucen Du[1,*], **Josef Schlattmann**[1], **Stefan Schulz**[1], and **Arthur Seibel**[1]

[1] Workgroup on System Technologies and Engineering Design Methodology, Hamburg University of Technology, 21073 Hamburg, Germany

In this paper, a computation method for the output shaft's rotation of a passive rotational joint, especially of one that is deployed on a parallel mechanism, is investigated. As an alternative to typical geometric or screw theory methods, we propose a new approach that is based on the comparison between any passive rotational joint with a reference spherical joint. As examples, we apply our method to a universal joint and a novel double joint whose passive rotation is difficult to calculate with traditional methods.

1 Introduction

In a Stewart-Gough platform, the fixed base platform is usually connected to the moveable manipulator platform by six identical chains consisting of passive rotational joints, such as spherical or universal joints, and length-variable linear actuators, such as lead screws. Here, a passive rotation of the rotational joints is harmful because it causes additional length variations of the lead screws besides the demanded lengths (or additional stresses in the lead screws if the rotation is obstructed). In order to incorporate these changes in lengths into the kinematics, there is the necessity to analyze the origin of passive rotation and the equivalence relation of the input and output angles of rotational joints. In a previous paper [1], the authors made a comparison of three conventional algorithms to calculate the passive rotation of rotational joints and showed the equivalence conditions of these three algorithms by proposing equivalence constraints. In the present work, a generalized method for calculating the output rotation of passive rotational joints is provided.

2 Origin of passive rotation

In general, rotational joints are used to transmit rotations with different degrees of freedom. A passive rotation of these joints is caused by the rotation of the input shaft, or, if the input shaft is fixed, by the variation of the output shaft's orientation. In a drive-line setup, a more common name for this rotation is the driven rotation, which is, however, equal to the passive rotation if the input shaft's frame is taken as the reference coordinate system. The non-linearity of the input-output relation was discussed, e. g., by Hunt [2] and by Seherr-Thoß et al. [3].

3 Computation method for passive rotation

One important property of a spherical joint is that it does not transmit rotations along the axes of the input and output ends. Due to this irrelevance between the rotations of both sides of a spherical joint, the zero passive rotation of the output shaft is always ensured. This characteristic allows to use a spherical joint as the reference for calculating the output rotation of other passive rotational joints. Consider a rotation sequence as shown in Fig. 1. The rotation of the output shaft $\{O\}$ with respect to the input shaft $\{I\}$ can be obtained in a matrix form as follows:

$$\boldsymbol{R}_{\text{spherical}} = {}^{I}\boldsymbol{R}_{O} = \boldsymbol{R}(z,b)\boldsymbol{R}(x,a) = \begin{bmatrix}\boldsymbol{x}_{\text{spherical}} & \boldsymbol{y}_{\text{spherical}} & \boldsymbol{z}_{\text{spherical}}\end{bmatrix} , \tag{1}$$

where the rotation matrix $\boldsymbol{R}(*,**)$ means a rotation around $*$ with an angle $**$. It can be noticed that in this process, there is no rotation of $\{O\}$ around its axis; thus, the passive rotation remains zero under any orientation of $\{O\}$'s z-axis.

A universal joint contains two revolute pairs along two orthogonal axes, as shown in Fig. 2(b). Any orientation of $\boldsymbol{z}_{\text{Cardan}}$ referred to the input can be obtained through the following rotation sequence:

$$\boldsymbol{R}_{\text{Cardan}} = \boldsymbol{R}(y,\alpha_y)\boldsymbol{R}(x,\alpha_x) = \begin{bmatrix}\boldsymbol{x}_{\text{Cardan}} & \boldsymbol{y}_{\text{Cardan}} & \boldsymbol{z}_{\text{Cardan}}\end{bmatrix} . \tag{2}$$

The passive rotation, here indicated by the angle γ, is the angular difference between $\boldsymbol{x}_{\text{Cardan}}$ and $\boldsymbol{x}_{\text{spherical}}$. This value can be obtained by solving the dot product and cross product of these two vectors:

$$\begin{aligned}\cos(\gamma) &= \boldsymbol{x}_{\text{Cardan}} \cdot \boldsymbol{x}_{\text{spherical}} ,\\ \sin(\gamma) &= \boldsymbol{x}_{\text{spherical}} \times \boldsymbol{x}_{\text{Cardan}} \cdot \boldsymbol{z}_{\text{Cardan/spherical}} .\end{aligned} \tag{3}$$

* Corresponding author: e-mail shucen.du@tuhh.de

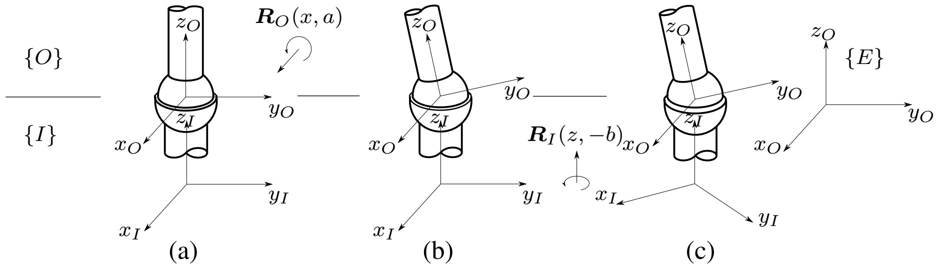

Fig. 1: Reference rotation of a spherical joint: any z-orientation of the output shaft referred to the input shaft can be obtained through the process from (a) to (c).

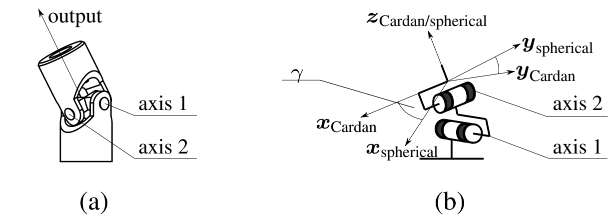

Fig. 2: 3D model (a) and kinematic diagram (b) of a universal joint.

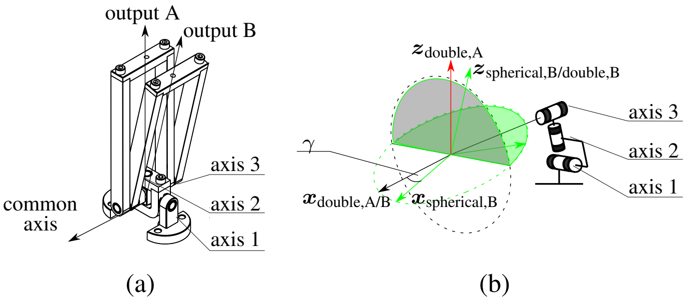

Fig. 3: 3D model (a) and abstracted kinematic diagram (b) of a double joint.

In fact, considering the arbitrary condition that both $\boldsymbol{z}_{\text{spherical}}$ and $\boldsymbol{z}_{\text{Cardan}}$ are identical and the inner products of the x-axes and y-axes are equal, the classical result of a universal joint's passive rotation can derived, see, e. g., [1]. It is worth mentioning that it is possible to measure $\boldsymbol{z}_{\text{Cardan/spherical}}$ in real-time by using inertial measurement units as proposed, e. g., in [4].

In order to reduce the complexity of the direct kinematics problem of parallel mechanisms, the linear actuators are often installed in the way that they always intersect in pairs at the base and the manipulator platform joints, see, e. g., [5]. Hence, it is obligate to utilize rotational joints with multiple outputs, e. g., by using a double joint as illustrated in Fig. 3(a). Similarly to Eq. (3), the passive rotation of output A and output B, shown in the Fig. 3(b), are γ_{A} and γ_{B}, respectively, and the passive rotation γ_{B}, e. g., can be calculated as follows:

$$\left.\begin{aligned}\cos(\gamma_{\text{B}}) &= \boldsymbol{x}_{\text{spherical,B}} \cdot \boldsymbol{x}_{\text{double,A/B}}\ ,\\ \sin(\gamma_{\text{B}}) &= \boldsymbol{x}_{\text{spherical,B}} \times \boldsymbol{x}_{\text{double,A/B}} \cdot \boldsymbol{z}_{\text{double,B}}\end{aligned}\right\} \Rightarrow \tan(\gamma_{\text{B}}) = \frac{\cos(\gamma_{\text{B}})}{\sin(\gamma_{\text{B}})} = -\frac{\boldsymbol{x}_{\text{spherical,B}} \cdot \boldsymbol{z}_{\text{double,A}}}{\boldsymbol{y}_{\text{spherical,B}} \cdot \boldsymbol{z}_{\text{double,A}}}\ . \tag{4}$$

We can conclude from Eq. (4) that the passive rotations in both output shafts highly depend on each others z-axes. The relation between two outputs is the only factor that determines the passive rotations, but not the input rotation. A more comprehensive mathematical derivation of the computation method for calculating the passive rotation of rotational joints is given in [6].

4 Conclusion

In this paper, we distinguished the definitions of passive rotation in parallel mechanisms from the driven rotation in a drive-line setup. The two rotations have the same origin, which is the relative orientation variation of the output shaft with respect to the input shaft, but the kinematic properties are different and studied for different applications. We therefore proposed a new method for computing the passive rotation based on vector comparison. This method shows advantages in handling the passive rotation problem especially for multi-end joints like double joints.

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