Development of the Differential Equations Concept Inventory to Assess Conceptual Understanding

TA Fuhrmann [1]
Merseburg University of Applied Sciences
Merseburg, Germany
0000-0002-3906-1631

C Kautz
Hamburg University of Technology
Hamburg, Germany
0000-0001-9665-8162

Conference Key Areas: Teaching foundational disciplines of Mathematics and Physics in engineering education
Keywords: *Concept Inventory, Differential Equations, Undergraduate Mathematics, Formative Assessment, Validity*

ABSTRACT

Differential equations (DEs) are an important mathematical concept for a wide variety of disciplines in engineering. Hence, students need to develop a good understanding of the basic concepts of DEs. To assess student conceptual understanding, Concept Inventories (CIs) are commonly used in engineering disciplines. They also serve to evaluate instruction especially when new teaching methods are tried out.
The Differential Equations Concept Inventory (DECI) was developed to be used to evaluate the average basic understanding. In a first stage it shall be used to compare the impact of different teaching methods. Existing frameworks were used to develop the DECI and statistical methods to assess its validity and reliability for the given purpose.

The DECI shows a good level of support to measure students' overall understanding of the concepts addressed by the DECI. It is not yet recommended to determine individual students' understanding of specific concepts, their difficulties or common errors. For this purpose, higher statistical standards would need to be met and individual influencing factors like gender or ethnicity would have to be investigated.

[1] *TA Fuhrmann*
tina.fuhrmann@hs-merseburg.de

Hence, the DECI can be used by instructors to examine the impact of their teaching and new teaching methods on the conceptual understanding of students in the field of differential equations.

1 INTRODUCTION

Differential equations (DEs) are an important mathematical concept for a wide variety of disciplines in engineering. Students must not only acquire (some) procedural skills but also a deep understanding of the basic concepts of DEs and their solutions. They need to be able to assign meaning to DEs, use them in the correct way and utilize their knowledge for complex problem solving. However, there is substantial research showing that many students have conceptual difficulties and do not achieve these goals (Zandieh and McDonald 1999; Habre 2000; Rasmussen 2001; Rowland and Jovanoski 2004; Trigueros 2004; Rasmussen and Kwon 2007; Raychaudhuri 2008; Mallet and McCue 2009; Arslan 2010; Camacho-Machín, Perdomo-Díaz, and Santos-Trigo 2012a; 2012b; Czocher, Tague, and Baker 2013; Camacho-Machín and Guerrero-Ortiz 2015; Hyland, Van Kampen, and Nolan 2018; 2021; Sijmkens et al. 2022; Habre 2023).

The term *difficulties* and not *misconceptions* is deliberately used, as students might not have conceptions about some of the addressed concepts at all. The difficulties are subdivided in those connected to (i) a severe lack of prior knowledge from algebra and calculus, (ii) the DE itself and (iii) the solution to the DE (Fuhrmann and Kautz 2022). For lecturers it can be challenging to successfully address students' difficulties in their teaching and provide for a good basic conceptual understanding.

To evaluate both, instruction and individual students' performance, Concept Inventories (CIs) are becoming increasingly popular in different fields of STEM education (e.g., Force Concept Inventory (Hestenes, Wells, and Swackhamer 1992)). CIs make one or more of the following claims about students' conceptual understanding and common student difficulties (Jorion et al. 2015): Students' CI scores can be used to indicate their overall understanding of all concepts identified in the CI, to indicate their understanding of specific concepts, and / or to indicate their propensity for difficulties or student errors. In this paper we discuss the design, reliability and validity of the Differential Equations Concept Inventory (DECI).

2 METHODOLOGY

2.1 Design of the DECI

The DECI was designed along the ten-step process laid out by Crocker and Algina (1986) which are in agreement in the essential aspects with Adams and Wieman (2011) and the AERA-Standards (2014).

Table 1. Ten-Step process for the design of the DECI (Crocker and Algina 1986)*. The steps are discussed in detail in the results section.*

Step 1	Define the purpose of the test
Step 2	Identify the concepts to be tested
Step 3	Define the test specifications

Step 4	Create an initial pool of items
Step 5	Have items reviewed by experts
Step 6	Hold preliminary item try-outs
Step 7	Field-test the items on a large representative student sample
Step 8	Conduct item analysis
Step 9	Design and conduct reliability and validity studies for the final test
Step 10	Develop guidelines for administration, scoring, and interpretation of the test scores

2.2 Statistical considerations

The most fundamental considerations for developing and evaluating a CI are its validity and reliability. The *Standards for Educational and Psychological Testing* (2014) define validity as the degree to which evidence and theory support the interpretations of test scores for the proposed uses of the inventory. They use the term reliability (precision) in the more general notion to denote consistency of the scores across instances of the testing procedure. Validity and reliability of the inventory can be tested through quantitative statistical and qualitative methods, e.g., external measures like interviews, measures from classical test theory (CTT), and item response theory (IRT).

Validity can only be assessed for the specific proposed interpretations of test scores and therefore for the given purpose, target population, and setting. Qualitative measures were used to examine the four types of validity (face, content, criterion, and construct validity).

To asses reliability the following measures (table 2) were used (Direnga 2021):

The **average of score** $\bar{x}$ is calculated using equation (1) and the **standard error of the average** σ_{mean} is given for experimental data with unknown mean and standard deviation by (2). **Cronbach's-**α is a measure of internal consistency of the inventory and calculated using (3). Only given answers were included in the calculation of the variance, i.e., if an item was not answered by a student, it was not included rather than classified as being wrong. The **difficulty index** diff_j is a measure of "easiness" and ranges between 0 (always answered wrong; "difficult" item) and 1 (always answered correctly, "easy" item). The **discrimination index** disc_j compares the subgroup difficulty indices $\mathrm{diff}_{\mathrm{j,H}}$ of the 27 % high scorers (H) of that item to the one $\mathrm{diff}_{j,L}$ of the 27 % low scorers (L) and is defined by (5). Here, again, only students that answered the item of interest, are included in the calculation.

The **ability** of a student is defined as a (normalized) measure that predicts (or explains) the students' performance on the inventory (Hambleton, Swaminathan, and Rogers 1991). It also depends on parameters of the items, e.g., their difficulties and abilities to discriminate between students. Since the item parameters are not known and since not all items are in their final form yet, it was decided to calculate the **normalized score** ϑ as a simplified method to estimate a students' ability (6). ϑ does not take into account characteristics of the items. It therefore has a midpoint of 0 and a standard deviation of 1. The **probability** of answering an item correctly increases

Table 2. Equations used to calculate statistical measures.
n - number of test takers, x_i - score of each test taker, k - number of items on the test, $Var(\chi_j)$ - variance of item j, $Var(\chi)$ - variance of the entire test, $n_{j,correct}$ - fraction of test takers who responded to item j correctly, $n_{j,correct} + n_{j,incorrect}$ - pool of students that answered item j, x_n - score of individual n, σ_x - standard deviation of scores.

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$	(1)	$\text{diff}_\text{j} = \frac{n_{j,\text{correct}}}{n_{j,\text{correct}} + n_{j,\text{incorrect}}}$	(4)
$\sigma_\text{mean} = \sqrt{\frac{1}{n} \cdot \frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2}$	(2)	$\text{disc}_j = \text{diff}_{j,H} - \text{diff}_{j,L}$	(5)
$\alpha = \frac{k}{k-1} \cdot \left(1 - \frac{\sum_{j=1}^{k} \text{Var}(\chi_\text{j})}{\text{Var}(\chi)}\right)$	(3)	$\vartheta = \frac{x_n - \bar{x}}{\sigma_x}$	(6)

monotonically as the level of ability increases. This relationship can be shown in the item characteristic curve.

3 RESULTS

In the following, the relevant data for the DECI is presented along the steps of the design process as stipulated by Crocker and Algina (1986) and shown in table 1.

3.1 Definition of the purpose of the test

The Differential Equations Concept Inventory (DECI) was designed to measure students' basic conceptual understanding of DEs and their solutions. In a first stage it is intended to assess the success of instruction and instructional methods, meaning that the test scores indicate an overall understanding of all concepts identified in the DECI. Test score interpretation for individual students' learning is limited, since construct-irrelevant components (e.g., gender, ethnicities, or age) could influence the total score and were not investigated up to now. Also, higher levels of validity and reliability would have to be met. The DECI shall not be used as a placement tool, formative assessment tool, or to discover unknown difficulties. However, it can inform about the frequency of certain false ways of reasoning.

3.2 Identification of the concepts to be tested

Student difficulties and key concepts with DEs were identified through literature, and complemented and partially confirmed by semi-structured interviews with 9 students (Fuhrmann and Kautz 2022). They were grouped, when necessary, and six concepts were chosen for the inventory based on the following criteria:

- **Completeness**: If all of the concepts covered by the inventory are understood by a student, the student may be considered as having gained a basic conceptual understanding of DEs. The inventory encompasses all essential concepts of DEs.

- **Parsimony**: If even one of the concepts covered in the inventory is not understood, one may not conclude, that this student gained a basic conceptual understanding of DEs. The inventory contains only essential concepts of DEs.

- **Testability**: It must be possible to test for the concepts with a multiple-choice questionnaire. E.g., it is hard to test for difficulties concerning the differences and similarities between functions, DEs and solutions of DEs with multiple choice questions since there has to be one definitely correct answer and four definitely incorrect ones with no room for interpretation.

The concepts included in the DECI are:

1) Recognition of DEs from a text-based problem

Students decide whether a text-based problem can be described by a DE and base their decision on correct arguments.

2) Formulation of a DE from a text-based problem

Students can formulate a DE from a text-based problem or associate the problem with an appropriate DE.

3) Understanding the meaning of variables and terms involved in the DE

Students are able to interpret the variables and terms involved in a DE, such as y, $\dot{y}$, x, and constants.

4) DEs and graphical representations (diagrams)

Students relate DEs and graphical representations of functions in diagrams. They interpret and draw conclusions from the graphical representation of a DE or its solution.

5) Application of solution methods and verification of solutions of DEs

Students decide for and use a correct solution method to solve basic-level DEs. They provide proper justification, using mathematically correct arguments, for determining whether a given solution can correspond to a given DE.

6) Prior knowledge

Students understand basic mathematical concepts that are considered to be important for the topic of DEs. Our choice was the understanding of functions, derivatives, infinitesimal quantities and graphical representations / diagrams.

Concepts regarding the topics of equilibrium solutions or homogeneous and inhomogeneous DEs were not considered by the test designers to be very basic concepts but themselves require a basic conceptual understanding of DEs. Therefore, they were not included in the DECI.

3.3 Definition of the test specifications

The DECI is designed to be administered as a post-test and should be paired with an appropriate pre-test as a reference for students' prior knowledge. It cannot be used as a pre-test, as the concepts are unlikely to be familiar to students from prior mathematical courses or everyday life experiences. When administering the DECI, students are expected to have covered the relevant materials on DEs in their study programs, but it can be implemented at any subsequent point in time.

Any CI that covers basic mathematical skills, including the understanding of functions, limits, derivatives, infinitesimal quantities, and integrals can be used as a pre-test (e.g., the Calculus Concept Inventory (Epstein 2013; Gleason et al. 2019) when ensured that the relevant concepts have been covered in the university courses beforehand).

The implementation is similar to other CIs (Madsen, McKagan, and Sayre 2017). The DECI has to be performed during class time (!) and lasts 30 min. An online form is possible, e.g., to ensure easy handling, comparability and data transfer between institutions. No tools are allowed except the test environment in the electronic device if administered online.

The DECI is a multiple-choice questionnaire for various reasons. In this way it is highly objective, allows for automated and fast scoring, enables assessing large student groups, and is feasible for the application of test-theoretical methods for validation and reliability checks. The concepts 1) to 5) relating to DEs are assessed with three items each, prior knowledge with four items, resulting in 19 items. The number of items per objective was guided by the principle of providing at least three items per concept (Engelhardt 2009) for a reliable and valid inventory while ensuring that the DECI remains within reasonable length constraints.

3.4 Creation of an initial pool of items

The questions were inspired by existing multiple-choice and open-ended questions from literature and online resources (e.g., http://mathquest.carroll.edu/de.html). For each of the concepts 1) to 5), three items were developed. The creation of distractors was based on known difficulties and students' typical thought patterns from literature and semi-structured interviews. The distractors may be more strongly linked with one of other concepts tested for in the DECI rather than the specific concept intended in the question. Prior knowledge was assessed with four items taken from the Calculus Concept Inventory.

3.5 Item review by experts and item revision

Four experts (i.e., professors and lecturers of mathematics) reviewed the items with a focus on ensuring mathematically correct wording and completeness of the items. They assured that the items encompassed all relevant concepts and were solvable with a basic conceptual understanding of DEs. Additionally, the authors strove to minimize complexity and facilitate understandability of the items for students.

3.6 Preliminary item try-outs and item revision

This step was omitted because we found it to be of limited value in this development process due to the greatly diverse target group at Universities of Applied Sciences. Therefore, a first field test followed the item review with more field tests to come.

3.7 Field-test of the items on a large representative student sample

The DECI was used at different Universities of Applied Sciences in Germany. In contrast to traditional Universities or Technical Universities, the students' background is often more heterogeneous concerning their educational background and mathematical knowledge. We strongly believe, that the DECI is also valid in other types of universities, but did not yet test for it.

The DECI was carried out in six mathematics courses at five Universities of Applied Sciences with second semester students (Darmstadt, Landshut, Merseburg, Rosenheim, Regensburg). 121 students participated. No bonus points or rewards were given to students for participation.

3.8 Item analysis and item removal or revision

Based on the statistical analysis in section 3.9, test items will be rephrased and slightly rearranged before unrolling the 2nd version of DECI.

3.9 Design and conduct of reliability and validity studies

Reliability and validity of the DECI are assessed as far as possible in section 4.

3.10 Develop guidelines for administration, scoring, and interpretation of the test scores

Administration is equivalent to other CIs, as shown in section 3.3. Comparative scoring values need to be assessed for the final version.

4 RESULTS OF RELIABILITY AND VALIDITY STUDIES

General Statistical Considerations

The distributions of the total scores and of the number of answers are shown in Fig. 1 and Fig. 2. On average, 18.65 ± 0.12 items (out of 19) were answered and 7.30 ± 0.33 correct answers given (with standard errors of the mean as indicated). Fig. 3 and Fig. 4 show the percentage of test takers that did not answer each item and the percentage that answered an item correctly, respectively.

Reliability Considerations

Cronbach's-α is 0.728 and therefore larger than 0.7 which indicates that the DECI is sufficient for group measurements (Engelhardt 2009; Jorion et al. 2015). For each item of the DECI, α was also computed for the entire test without the given item. α should then be lower compared to the entire test. For items 1, 6, 10, 15 and 18 it was greater ($0.73 < \alpha < 0.74$), indicating the need for revision of these items.

In Fig. 5 the scatterplot of item difficulty and item discrimination is shown. Most of the items meet the recommended criteria for CIs intended for instructional (as opposed to individual) use, with the difficulty ranging between 0.2 and 0.8 and the item

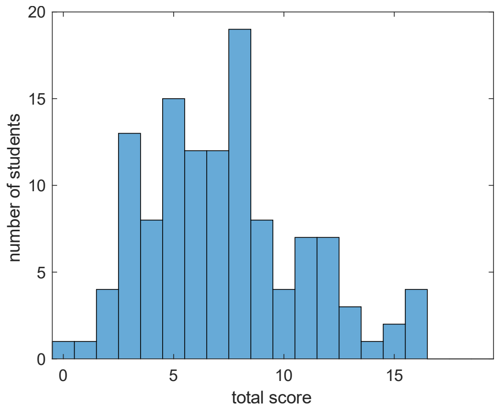

Fig. 1. Distribution of students based on the total scores, which is equal to the number of correct answers.

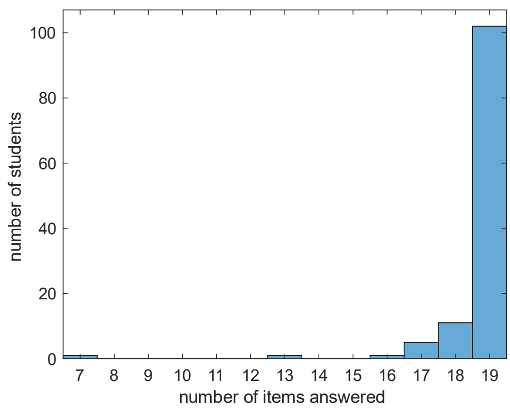

Fig. 2. Distribution of students based on the number of answers given in the DECI.

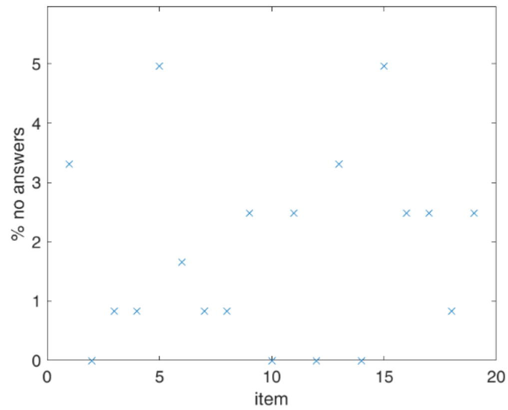

Fig. 3. Percentage of students that did not give an answer for each item.

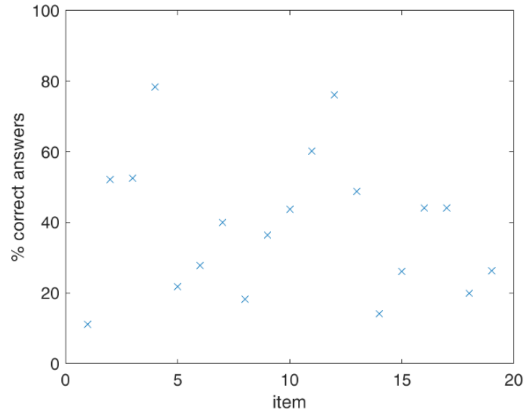

Fig. 4. Percentage of students that gave correct answers in relation to the number of students answering each item.

discrimination being larger than 0.8 (Jorion et al. 2015). The diagram indicates the need for revision for items 1, 8, 14 and 18. A rarely chosen option in item 6 will be removed leaving 5 answering options and leading to a better alignment with the other items with mostly five options.

For each item, the item characteristic curve (ICC, e.g., Fig. 6) and the distribution of chosen answer options was assessed. ICCs were constructed by grouping students in 10th percentiles (i.e., aggregating 12 to 13 students per data point on the ICC). This methodology facilitated the collection of 10 data points for each ICC and allowed for a comprehensive examination of item quality. The observed slopes for the correct answers across all items were positive, showing that students with a higher total test score were more likely to answer each item correctly. Despite attempts at curve fitting and model validation, such as utilizing a 3-parameter logistic model, these efforts were impeded by the limited amount of data available as well as

the necessity for slight adaptions in certain items.

The process of assessing the ICC and the given answers is illustrated using item 3. The question and answer options are shown in Fig. 6. Even though the correct option a) is the first response option and the only one with a derivative of y, only 52 % of students chose this answer. Option b) was chosen by 26 % of students and may result from two possible ways of thinking. First, for some students, DEs are inextricably linked to exponential functions. Second, some students do not or not

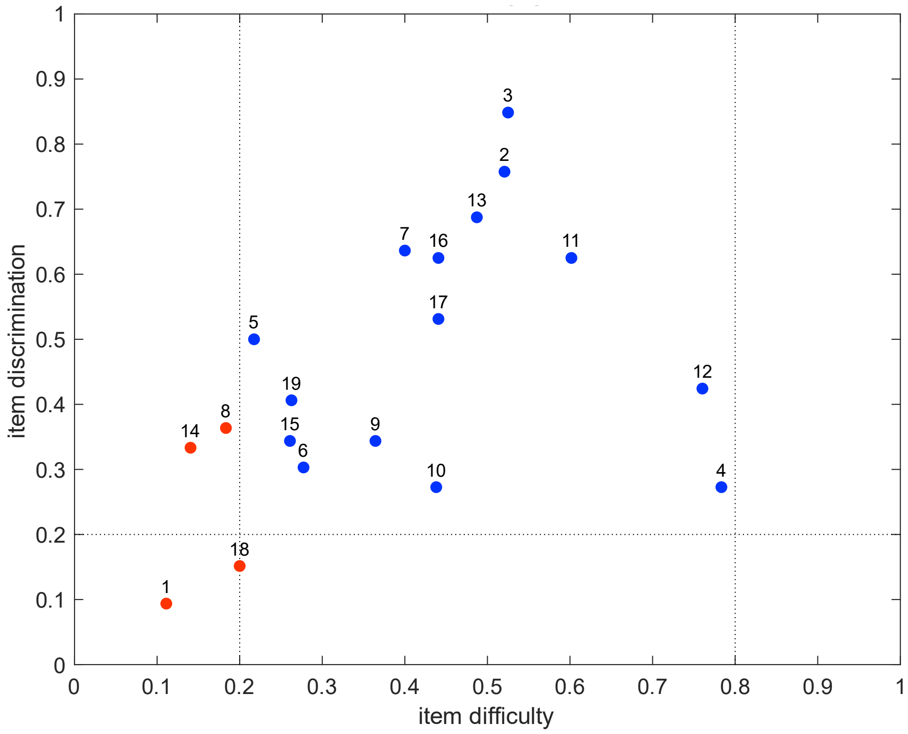

Fig. 5 Scatterplot of item difficulty and item discrimination. Recommended minimum and maximum values are denoted by the dotted lines. Three items (1,8,14) did not meet the recommended minimum item difficulty of which two had a good discrimination index.

sufficiently distinguish between a DE and its solution. Option b) is a very well-known solution for students to the stated problem. Option c) (7.4 %) might be selected by students who do not have a deep understanding of the meaning of the variables and terms involved in a DE. They may see that the rate of change, which they think of as the dependent variable, is proportional to the number of rabbits, which they consider to be the independent variable. Therefore, a linear function results. Option d) (6.6 %) was included as an option for students who reason similarly but might additionally think that the answer must be more “complicated” than a linear function. Option e) was designed to attract students who build an isolated, completely decoupled knowledge about DEs and think of them as something “totally different” to what they have learned before. For those students it can be nonsensical to write down a DE (part of the new knowledge) for an everyday, well-known phenomenon (part of the previous knowledge).

If the following problem can be described by a differential equation, which differential equation can correspond to the problem ($a, b \in \mathbb{R} \setminus \{0\}$)?

A small number of rabbits are located in a very large open area without predators. The rate of change in the number of rabbits is proportional to the number of rabbits present.

a) $\dot{y}(t) = a \cdot y(t)$

b) $y(t) = a \cdot e^{bt}$

c) $y(t) = a \cdot t$

d) $y(t) = a \cdot t^2$

e) The problem is not described by a differential equation.

Fig. 6: Item 3 with the correct answer being a).

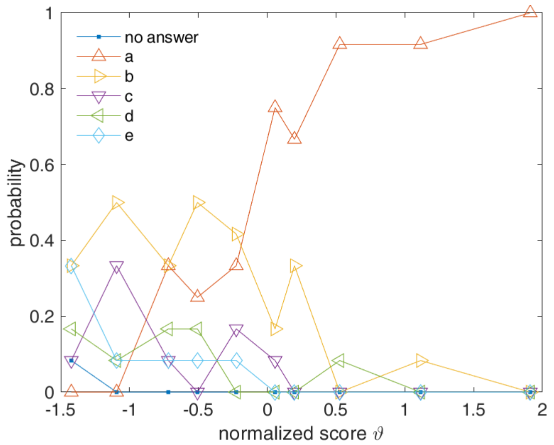

Fig. 7. Item Characteristic Curve of item 3. The probability of giving a specific answer is shown for all answer options. The correct answer is a.

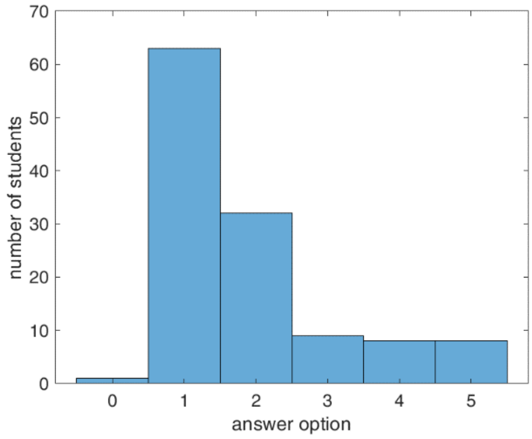

Fig. 8. Distribution of student responses: index 0 indicates the count of students who did not respond to the item, indices 1 to 5 correspond to options a to d.

In Fig. 7 it can be seen, that the probability for answering item 3 correctly increases for students with a higher normalized score. This is the preferred characteristic for an item in a Concept Inventory. Fig. 8 shows, that the correct option is the preferred one, but also, that all options were chosen by students. Therefore, all options appeal to students and no option has to be exchanged.

Validity Considerations

Face Validity*:* Mathematicians (experts) verified that the DECI appears to measure the intended constructs.

Content Validity*:* During steps 2 and 5 of the design process, attention was devoted to ensuring that the concepts and items of the DECI fully represent the overall construct, namely, the basic conceptual understanding of DEs. However, for a final version of the DECI, interviews with experts should be conducted.

Criterion Validity*:* The results of the DECI cannot be compared to other quantities since there is no "gold standard" inventory available (concurrent validity). Due to strong data protection regulations in Germany, the DECI was performed anonymously, asking students only for self-generated identity codes (SGICs, (Timmermann et al. 2016)). Therefore, it cannot be correlated to e.g., exam results (predictive validity) or other measures.

Construct validity*:* Construct validity assesses whether the DECI and its individual items measure the intended overall construct. Some questions were given to students in semi-structured interviews in a modified form. The students' reasoning was then observed and analyzed, suggesting that the questions were correctly interpreted by the students. However, the exact questions of the DECI were not given to students in interview settings.

5 CONCLUSIONS

The Differential Equations Concept Inventory (DECI) is not in its final form yet, but our data indicate that it is a valid and reliable instrument to assess students' basic conceptual understanding of DEs and their solutions. Accordingly, it can be utilized to compare the effectiveness of different teaching methods.

REFERENCES

Adams, Wendy K., and Carl E. Wieman. 2011. "Development and Validation of Instruments to Measure Learning of Expert-Like Thinking." *International Journal of Science Education* 33 (9): 1289–1312. https://doi.org/10.1080/09500693.2010.512369.

American Educational Research Association, American Psychological Association, and National Council on Measurement in Education. 2014. *Standards for Educational and Psychological Testing*. Washington, DC: American Educational Research Association.

Arslan, S. 2010. "Do Students Really Understand What an Ordinary Differential Equation Is?" *International Journal of Mathematical Education in Science and Technology* 41 (7): 873–88. https://doi.org/10.1080/0020739X.2010.486448.

Camacho-Machín, M., and C. Guerrero-Ortiz. 2015. "Identifying and Exploring Relationships between Contextual Situations and Ordinary Differential Equations." *International Journal of Mathematical Education in Science and Technology* 46 (8): 1077–95. https://doi.org/10.1080/0020739X.2015.1025877.

Camacho-Machín, M., J. Perdomo-Díaz, and M. Santos-Trigo. 2012a. "An Exploration of Students Conceptual Knowledge Built in a First Ordinary Differential Equations Course (Part I)." *The Teaching of Mathematics* 15 (1): 1–20.

Camacho-Machín, M., J. Perdomo-Díaz, and M. Santos-Trigo. 2012b. "An Exploration of Students Conceptual Knowledge Built in a First Ordinary Differential Equations Course (Part II)." *The Teaching of Mathematics* 15 (2): 63–84.

Crocker, Linda M., and James Algina. 1986. *Introduction to Classical and Modern Test Theory*. New York: Holt, Rinehart, and Winston.

Czocher, Jennifer A., Jenna Tague, and Greg Baker. 2013. "Where Does the Calculus Go? An Investigation of How Calculus Ideas Are Used in Later Coursework." *International Journal of Mathematical Education in Science and Technology* 44 (5): 673–84. https://doi.org/10.1080/0020739X.2013.780215.

Direnga, Julie. 2021. "Assessing the Effectiveness of Research-Based Active Learning Materials for Introductory Engineering Mechanics." Hamburg University of Technology. https://doi.org/10.15480/882.3229.

Engelhardt, Paula V. 2009. "An Introduction to Classical Test Theory as Applied to Conceptual Multiple-Choice Tests." *Getting Started in PER* 2 (1): 1–40.

Epstein, Jerome. 2013. “The Calculus Concept Inventory–Measurement of the Effect of Teaching Methodology in Mathematics.” *Notices of the American Mathematical Society* 60 (08): 1018. https://doi.org/10.1090/noti1033.

Fuhrmann, Tina A., and Christian Kautz. 2022. “Understanding of Differential Equations in a Highly Heterogeneous Student Group.” In *Towards a New Future in Engineering Education, New Scenarios That European Alliances of Tech Universities Open Up*, 288–97. Universitat Politècnica de Catalunya. https://doi.org/10.5821/conference-9788412322262.1280.

Gleason, Jim, Spencer Bagley, Matthew Thomas, Lisa Rice, and Diana White. 2019. “The Calculus Concept Inventory: A Psychometric Analysis and Implications for Use.” *International Journal of Mathematical Education in Science and Technology* 50 (6): 825–38. https://doi.org/10.1080/0020739X.2018.1538466.

Habre, S. 2000. “Exploring Students’ Strategies to Solve Ordinary Differential Equations in a Reformed Setting.” *The Journal of Mathematical Behavior* 18 (4): 455–72. https://doi.org/10.1016/S0732-3123(00)00024-9.

Habre, S. 2023. “Qualitative Aspects of Differential Equations in an Inquiry-Oriented Course.” *International Journal of Mathematical Education in Science and Technology* 54 (3): 351–64. https://doi.org/10.1080/0020739X.2021.1954250.

Hambleton, Ronald K., Hariharan Swaminathan, and H. Jane Rogers. 1991. *Fundamentals of Item Response Theory*. Measurement Methods for the Social Sciences Series 2. Newbury Park, Calif: Sage Publications.

Hestenes, David, Malcolm Wells, and Gregg Swackhamer. 1992. “Force Concept Inventory.” *The Physics Teacher* 30 (3): 141–58. https://doi.org/10.1119/1.2343497.

Hyland, D., P. Van Kampen, and B. Nolan. 2021. “Introducing Direction Fields to Students Learning Ordinary Differential Equations (ODEs) through Guided Inquiry.” *International Journal of Mathematical Education in Science and Technology* 52 (3): 331–48. https://doi.org/10.1080/0020739X.2019.1670367.

Hyland, D., P. Van Kampen, and Brien C. Nolan. 2018. “Outcomes of a Service Teaching Module on ODEs for Physics Students.” *International Journal of Mathematical Education in Science and Technology* 49 (5): 743–58. https://doi.org/10.1080/0020739X.2017.1410736.

Jorion, Natalie, Brian D. Gane, Katie James, Lianne Schroeder, Louis V. DiBello, and James W. Pellegrino. 2015. “An Analytic Framework for Evaluating the Validity of Concept Inventory Claims.” *Journal of Engineering Education* 104 (4): 454–96. https://doi.org/10.1002/jee.20104.

Madsen, Adrian, Sarah B. McKagan, and Eleanor C. Sayre. 2017. “Best Practices for Administering Concept Inventories.” *The Physics Teacher* 55 (9): 530–36. https://doi.org/10.1119/1.5011826.

Mallet, Dg, and Sw McCue. 2009. “Constructive Development of the Solutions of Linear Equations in Introductory Ordinary Differential Equations.” *International Journal of Mathematical Education in Science and Technology* 40 (5): 587–95. https://doi.org/10.1080/00207390902759626.

Rasmussen, C. 2001. “New Directions in Differential Equations: A Framework for Interpreting Students’ Understandings and Difficulties.” *The Journal of Mathematical Behavior* 20 (1): 55–87. https://doi.org/10.1016/S0732-3123(01)00062-1.

Rasmussen, C., and O. N. Kwon. 2007. “An Inquiry-Oriented Approach to Undergraduate Mathematics.” *The Journal of Mathematical Behavior* 26 (3): 189–94. https://doi.org/10.1016/j.jmathb.2007.10.001.

Raychaudhuri, Debasree. 2008. “Dynamics of a Definition: A Framework to Analyse Student Construction of the Concept of Solution to a Differential Equation.” *International Journal of Mathematical Education in Science and Technology* 39 (2): 161–77. https://doi.org/10.1080/00207390701576874.

Rowland, D. R., and Z. Jovanoski. 2004. “Student Interpretations of the Terms in First-Order Ordinary Differential Equations in Modelling Contexts.” *International Journal of Mathematical Education in Science and Technology* 35 (4): 503–16. https://doi.org/10.1080/00207390410001686607.

Sijmkens, Elien, Nico Scheerlinck, Mieke De Cock, and Johan Deprez. 2022. “Benefits of Using Context While Teaching Differential Equations.” *International Journal of Mathematical Education in Science and Technology*, March, 1–21. https://doi.org/10.1080/0020739X.2022.2039412.

Timmermann, D, J Direnga, J Lund, and C Kautz. 2016. “Design and Application of Self-Generated Identification Codes (SGICs) for Matching Longitudinal Data.” In *Proceedings of the 44th SEFI Annual Conference*. Tampere, Finnland.

Trigueros, M. G. 2004. “Understanding the Meaning and Representation of Straight Line Solutions of Systems of Differential Equations.” In *Proceedings of the Twenty-Sixth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 127–34.

Zandieh, M., and M. McDonald. 1999. “Student Understanding of Equilibrium Solution in Differential Equations.” In *Proceedings of the 21st Annual Meeting of the Northern American Chapter of the International Group for the Psychology of Mathematics Education*, 253–58. Columbus, OH: ERIC.