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A COMPARISON OF THE CALCULATED AND MEASURED FREE-WAVE SPECTRUM OF AN INUID IN STEADY MOTION

by

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A COMPARISON OF THE CALCULATED AND MEASURED FREE-WAVE SPECTRUM OF AN INUID IN STEADY MOTION [1]

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Manuscript

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1) This work was performed at the Institute for Shipbuilding, Hamburg University, with the co-operation of the Hamburg Shipmodel Testing Tank (Hamburgische Schiffbau-Versuchsanstalt GmbH) and the Institute for Photogrammetry of the Technical University Berlin, under the auspices of the German Research Council (Deutsche Forschungsgemeinschaft).

2) Research fellow of the Alexander von Humboldt Trust, Hamburg University, Hamburg, Germany.

ABSTRACT

MICHELL's theory of ship wave resistance was founded on two simplifying assumptions: neglect of viscosity and the linearisation of boundary conditions. Even at the present state of the theory the only general way to appreciate the full import of these assumptions is the comparison with experiment. However, a recently developed theory and technique of wave analysis permit a detailed examination of wave spectrum in contrast with past experimental investigations which were restricted mostly to a measurement of resistance, and in a few cases of wave profiles. It is, therefore, now possible to check the validity of various proposals for semi-empirical corrections due to viscosity and nonlinearity, originally resulting from attempts to reconcile the measured and calculated resistance. We need refer only to INUI's corrections as they include essentially most former proposals due to WIGLEY, HAVELOCK and others.

The present paper begins with a discussion of the pertinence of the proposed method of wave analysis to check the validity of the linearised free-wave and ship-wave theories, and to solve the so-called separation (of viscous and wave drag) problem in ship-model correlation. The basic theory is then outlined, leading to formulas for the determination of wave spectrum and resistance from measured wave profiles either perpendicular or parallel to the direction of motion. Results of experiments with a mathematical model (called Inuid) follow. After establishing the validity of the linearised free-wave theory and the consistency of the results of wave analysis, ship-wave theory and experiment are compared in the light of INUI's corrections. Whereas the comparison of wave resistance appears to vindicate INUI's proposals, the detailed comparison of free-wave spectrum, however, reveals their inadequacy. Further research is necessary, before any final conclusions can be made.

TOPICAL OUTLINE

NOMENCLATURE

(Only the more important symbols are listed here, others are explained in the text where they are introduced.)

A	Control surface ahead of the ship
A_ν	Amplitude of an elementary wave
a_ν	Fourier coefficient of transverse wave section
B	Transverse control surface behind the ship
B	Breadth of the Michell ship corresponding to a given source distribution (it is used as a measure of source strength rather than as ship dimension)
b	Tankwwidth
C	Fourier cosine transform
$C_{F,Fo}$	Coefficients of frictional resistance
$C_{T,V,W}$	Coefficients of total, viscous and wave resistance
C, D	Lateral control surfaces
E, F	Undisturbed and deformed free surface
Fr	Froude number $(V/\sqrt{gL})$
$f(\vartheta), g(\vartheta)$	Spectrum of the free-wave system
g	Acceleration due to gravity
k_o	Velocity parameter (g/V^2)
k, k_ν	Circular wave number of elementary wave $(2\pi/\lambda_\nu)$
L	Length of source distribution = length of corresponding Michell ship ≈ length of corresponding Inuid
p	Fluid pressure
R	Resistance (or wave resistance in ideal fluid)
Re	Reynolds number (VL/ν)
$R_{T,V,W}$	Total, viscous and wave resistance
S	Fourier sine transform
S	Wetted surface area
T	Draft of source distribution = draft of corresponding Michell ship ≈ draft of corresponding Inuid at the ends
u, u_ν	Transverse circular wave number of elementary wave
V	Ship speed (or uniform stream velocity if ship at rest)
v	Perturbation velocity due to ship
$v_{x,y,z}$	Components of v
w, w_ν	Longitudinal circular wave number of elementary wave
X, Y, Z	Abbreviations for wx, uy, kz
x, y, z	Co-ordinates moving with the ship

NOMENCLATURE (contd.)

$\alpha, \beta, \gamma, \delta$	Semi-empirical viscosity and nonlinearity corrections proposed by INUI
α'	Attenuation factor for the bow wave due to self interference of the hull
$\alpha = \gamma\alpha'$	Total final attenuation factor for bow wave
β'	Attenuation factor for stern wave due to viscosity
$\beta = \gamma\beta'$	Total attenuation factor for stern wave
γ	Finite-wave-height correction factor
δ	Correction factor for additional phase shift between bow and stern wave due to viscosity
γ_0	Velocity parameter $(gL/2V^2)$
$\bar{\alpha}, \bar{\beta}, \bar{\delta}$, [illegible]	Semi-empirical viscosity and nonlinearity correction factors to free-wave spectrum interpreted in terms of virtual form parameters
$\bar{\alpha}$	Correction factor to B/L in forebody
$\bar{\beta}$	Correction factor to B/L in afterbody
$\bar{\delta}$	Correction factor to L: corrected $L = (1+\bar{\delta})L$
[illegible]	Correction factor for separation of stern wave
[illegible]	Wave height
[illegible]	Direction of propagation of elementary wave
[illegible]	Induced wave length in x-direction
[illegible]	Wave length in direction of propagation
ν	Wave number based on tank width
ξ_ν	Phase of elementary wave
ϱ	Fluid density
Σ	Usual symbol for a sum
Σ	Special symbol for a sum to denote that only a half of the zeroth term is included in the sum
[illegible]	Source intensity
φ	Perturbation potential
$\varphi_{x,y,z}$	Derivatives of φ = velocity components due to φ

I. INTRODUCTION

Background: This paper is an offshoot of a rather comprehensive investigation of the mutual interaction of viscous and wave resistance using submerged models, geosim tests, viscous wake survey and wave analysis as principal experimental tools. This work was assigned to the author by Professor Weinblum as a part of his extensive and persistent efforts to make the mathematical wave resistance theory applicable to practical problems of ship design. It was expected to make a modest contribution to our understanding of the nature and causes of the frequently observed discrepancy between experimentally measured and theoretically calculated resistance, specially wave resistance, which has probably been the main hurdle in way of an application of the theory. Emphasis was laid on the exploitation of known proposals and the development of new methods for direct determination of the viscous and wave resistance components by an analysis of the wake and wave pattern actually measurable in model experiment. For it was felt that possibilities of the simple resistance test, and its analysis on the basis of FROUDE's hypothesis, for the verification of wave resistance theory had been almost exhausted. After about three years of largely experimental and partly theoretical work this study has now reached a preliminary conclusion and a detailed general report will be published shortly. In this paper I shall be concerned only with wave analysis, in fact only with the type of wave analysis we have carried out in Hamburg in the framework of the aforesaid project. I shall begin with a brief discussion of the objectives of this analysis, then proceed to outline the basic theory in some detail and finally present a few experiment results closing with a comparison of the calculated and measured free-wave spectrum.

Definitions: It is not intended to begin the paper with a definition of the various concepts to be introduced later in the analysis. But I do believe that the formulation of the title of this paper demands some further elucidation at the very outset. The word "Inuid" is used to denote the unconventional model forms introduced by INUI [1] in an attempt to re-examine the validity of the existing wave resistance theory due to MICHELL and HAVELOCK on the basis of his new interpretation of the relation between hull form and the singularity distribution to be associated with it. INUI's concept is too well-known to be in need of a restatement here. Although from a strictly mathematical point of view it may still be controversial, yet I think it has been justified as a heurisitc step by INUI's successful experiments with a series of simple Inuids [2]. He contrived to reconcile the wave resistance predicted by a linearised theory in an ideal fluid with that derived from measurements in a real fluid by postulating a plausible set of semi-empirical corrections many of which were reinterpretations of former suggestions of WIGLEY [3], HAVELOCK [4], EMERSON [5] and others. As the present study was chiefly prompted by INUI's impressive but challenging work and aimed at a verification of his intuitive correction factors it was natural to execute new experiments with a form already tested by him. The one designated by INUI as S201 was chosen for it comes closest to a realistic hull form. Instead of using offsets published by INUI the form was recalculated from the exactly known linear source distribution. A 4 m long model of the Inuid S201 was used for all experiments. Only steady motion in smoothwater was investigated. All numerical results presented later in the paper refer, unless otherwise stated, to this model. (Needless to say that the theory of wave analysis as presented in Chapter II has nothing to do with INUI's hypothesis and its application is in no way restricted to Inuids.) The terms calculated and measured are to be understood in the following sense. The word calculated applied to the free-wave spectrum or resistance of an Inuid implies calculation for the generating source distribution by HAVELOCK's theory [6]. The calculated corrections are essentially those proposed by INUI [2]. The term "measured" free-wave spectrum actually means 'derived' from the measured wave profiles by spectral analysis based on the theory to be outlined here.

Purpose: The general purpose of wave analysis in clarification of the discrepancy between calculated and observed resistance should be evident from the foregoing. However, there are some aspects of the problem which deserve closer inspection. In view of the current interest in the direct determination of wave resistance by an analysis of the measured wave pattern it may appear superfluous to dilate upon the purpose of wave analysis. Yet I fear just because of this passionate interest it may be prudent to take a sober view of the prospects even at the risk of damping the enthusiasm of optimists who may expect too much of wave analysis. In attempting that I shall confine myself to the present method of analysis. Fortunately, the only assumption involved in it can be stated in rather simple terms. The actual, measurable deformation of the free surface at a sufficient distance from the ship is interpreted as a general linearised free-wave system of the type considered by HAVELOCK in [6]. The inherent mathematical simplicity is an attractive feature of this concept. However, it is not only tantamount to the linearisation of free-waves as such, but also postulates that at least in the region of interest all other disturbances, e.g. the local wave associated with the ship and the viscous decay and boundary-layer interaction are negligibly small. It should be noted that the basic assumption is not automatically invalidated by the certain occurrence of these distrubances in the vicinity of the ship. Another important feature is that the analysis is not bound to any special features of the wave system of a particular hull form, e.g. the KELVIN pattern due to a pressure point. Thus free-wave theory and ship-wave theory can be verified profitably in separate steps. In fact the results already obtained by the present method of analysis will be seen to serve a three-fold purpose.

Firstly, the concept of a linearised free-wave system provides a simple but sensitive criterion for self-consistence, namely the law of dispersion connecting the wave lengths of the component plane waves to the common speed of the steady system. Only a restricted class of functions can satisfy this criterion.

The spectrum of the system (i.e. amplitudes and phases of component waves) constitutes the degree of freedom which can be exhausted by comparatively little constraint. Hence by measuring redundant information on any actual free surface we can check to what degree of approximation the hypothetical model of a linearised free-wave system can be fitted to it. One of the basic experiment results to be presented later is evidence for the applicability of the linearised free-wave concept to wave system of the Inuid tested in a reasonable range.

The second step is the verification of the actual ship-wave theory by comparing the calculated free-wave spectrum and resistance with the measured values. Any conclusions drawn from such comparison will be subject to the assumptions in the theory, e.g. the use of an Inuid to represent a source distribution or vice versa. As the discrepancy in any case is considerable the real interest lies not in the verification of theory but of semi-empirical corrections proposed to reconcile it with experiment. INUI's corrections are particularly amenable to such study since they can be directly interpreted as modifications to the spectrum. Detailed results of such comparison will be given.

Finally there is another aspect of wave analysis capable of immediate practical use, viz its application for the separation of viscous and wave resistance from the point of view of geosim or ship model correlation. To avoid confusion let us define FROUDE resistance as that part of total resistance which seems to obey FROUDE's law when actual geosim results are analysed. The question is whether the wave resistance obtained from an analysis of the waves behind the ship should be identical with FROUDE resistance. The idea of attenuation of the bow wave implicit in many former proposals for the empirical correction of theoretical wave resistance would seem to deny this and the present investigation does indeed reveal a discrepancy of the expected order, thus apparently shattering a cherished hope. I shall revert to a detailed examination of this question while dealing with the analysis of experiment results.

II. BASIC THEORY

The basic theory involved in the present analysis consists of three simple steps:

a) Application of the momentum theorem to express ship resistance in terms of pressure and velocity distributions along suitable control surfaces in the fluid. Rearrangement of the expression by virtue of Bernoulli's equation and the condition of continuity.

b) Introduction of a linearised potential to describe the free-wave system to be expected at a control surface away from the ship by virtue of the radiation condition. Characterisation of the linearised free-wave system by means of a spectrum and exposition of the relation between ship resistance and free-wave spectrum by a).

c) Application of the inversion theorem for Fourier transforms to express free-wave spectrum and consequently wave resistance in terms of surface profiles.

In the actual application of the theory, of course, the sequence of these steps is reversed, i.e. we start by measuring surface profiles, derive the spectrum by Fourier analysis and finally calculate the wave resistance by using the known formula based on momentum considerations.

Momentum theorem and wave resistance: Consider a ship at rest in a uniform stream of velocity V in the positive x-direction with the co-ordinate system as shown in Fig.1. Let the fluid region around the ship be enclosed by a control surface with the boundaries A - H as shown. If p be the fluid pressure, ρ the density and v_x , v_y , v_z the components of perturbation velocity v due to the presence of the ship, then the application of the momentum theorem under assumption of an ideal fluid obviously yields the following expression for the wave resistance:

(2.1) $R = \iint$ [illegible] $+ v_x)^2 \} dy\,dz - \iint \{ p + \rho (V + v_x)^2 \} dy\,dz$

$+$ [illegible] $dz\,dx$

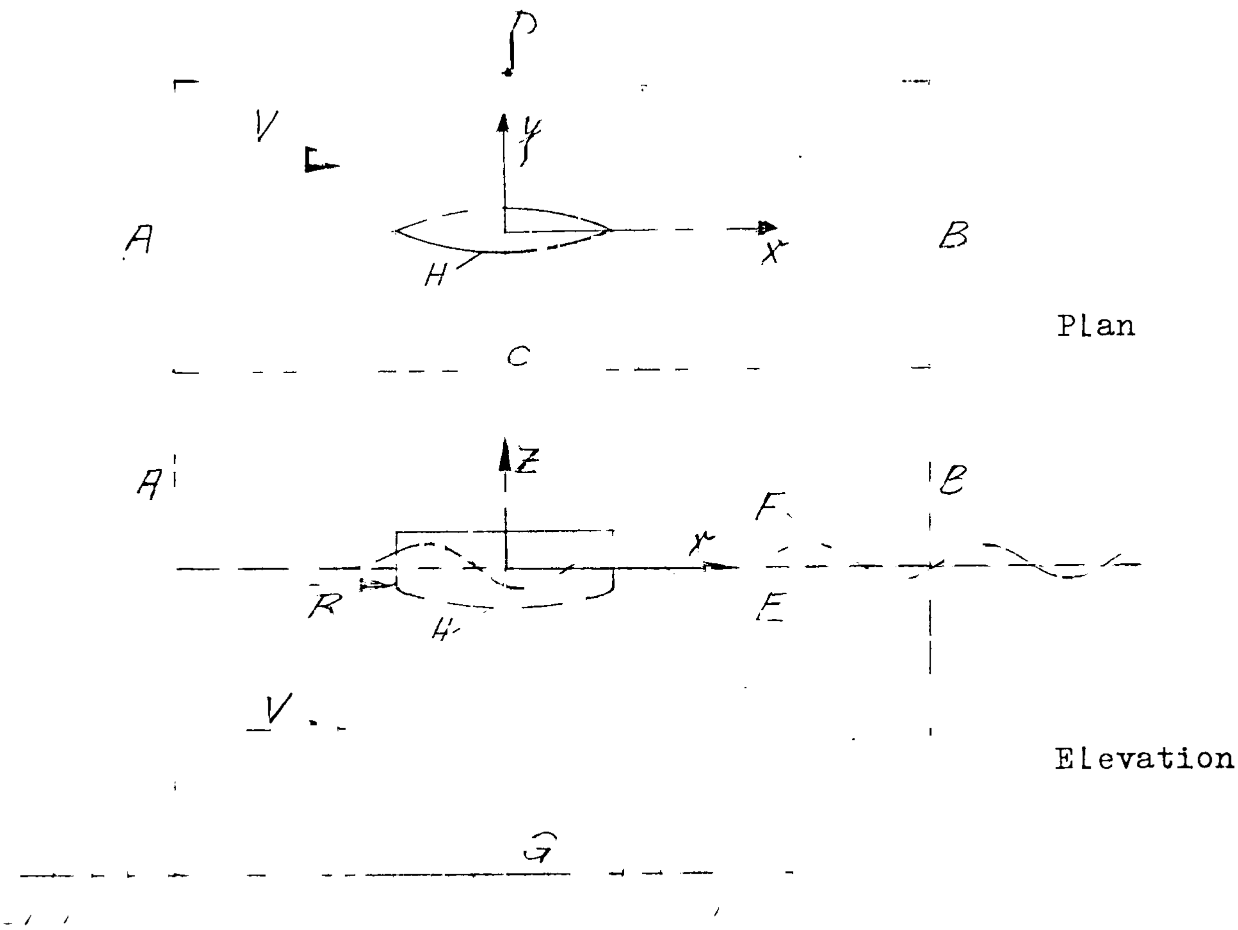


Fig.1 Co-ordinate system and control surface

This expression can be rearranged by virtue of Bernoulli's equation:

(2.2) $$\varrho g z + p + \frac{\varrho}{2}\{(V+v_x)^2 + v_y^2 + v_z^2\} = \frac{\varrho}{2}V^2$$

and the condition of continuity:

(2.3) $$\varrho\iint_A (V+v_x)\,dy\,dz + \varrho\iint_C v_y\,dz\,dx = \varrho\iint_B (V+v_x)\,dy\,dz + \varrho\iint_D v_y\,dz\,dx$$

A further simplification is possible by taking the boundary A so far ahead that the perturbance v can be neglected. Let us consider two special cases. First suppose that the transverse sections A and B extend in both directions to infinity (or eventually to the vertical side walls of a rectangular channel). Then the contribution of the lateral boundaries C and D will vanish and in the limit one obtains from the above equations the following formula for a pair of transverse sections:

(2.4) $$R = \varrho g\iint_{B-A} z\,dy\,dz + \frac{\varrho}{2}\iint_B (v_y^2 + v_z^2 - v_x^2)\,dy\,dz$$

or introducing a wave height ζ in the form:

(2.5) $$R = \frac{\varrho g}{2}\int_B \zeta^2\,dy + \frac{\varrho}{2}\iint_B (v_y^2 + v_z^2 - v_x^2)\,dy\,dz$$

Following HAVELOCK [6] this relation is usually derived by energy considerations. To understand HAVELOCK's interpretation rewrite (2.5) as:

$$(2.6) \quad RV = V\frac{\rho}{2}g\int_B \zeta^2 dy + V\frac{\rho}{2}\iint_B (v_x^2 + v_y^2 + v_z^2)\, dy\, dz - \rho V \iint_B v_x^2\, dy\, dz$$

If Fig.1 is now understood as showing the ship to be moving at a speed V through otherwise undisturbed fluid and B be a fixed geometrical surface, then the above equation represents the energy balance for the fluid to be left of B. The LHS is the rate of work done by the ship on the fluid. The first two terms on the RHS give the rate of increase of potential and kinetic energy due to the propagation of the wave system associated with the ship and the last term can be interpreted as the rate of work done by the pressure on the fluid to the left of B or as the transport of energy across the plane B due to wave motion.

Here I have deliberately chosen to derive the result by the momentum theorem for the following reasons: i) the analysis is easier to follow, ii) lateral sections can be handled with equal ease and iii) the importance of the condition of continuity is revealed (cf. Appendix).

Let us now consider the complementary case of the lateral sections C and D extending to infinity in both directions. Neglecting the contribution of A and B we now obtain from equations (2.1) to (2.3) the following formula for a pair of lateral sections:

$$(2.7) \quad R = \rho\iint_C v_y v_x\, dz\, dx - \rho\iint_D v_y v_x\, dz\, dx$$

Of course, in case of a symmetrical disturbance it can be further simplified to

$$(2.8) \quad R = 2\rho\iint_C v_y v_x\, dz\, dx$$

It is worth noting that this strikingly simple formula is applicable only in unrestricted water. In a channel of finite width it would not be permissible to neglect the contribution of B even in the limit.

The linearised free-wave potential: If the control surface is taken sufficiently far from the ship the flow there can be interpreted in terms of a free-wave potential which has the advantage of mathematical simplicity because it is only required to satisfy the boundary condition at the free surface but not atthe ship surface. The flow represented by a free wave potential, if extrapolated into the vicinity of the ship, would of necessity differ from the real flow there. But it is no handicap since we can caluulate resistance in terms of flow at the control surface. In fact that is the whole purpose of the introduction of the control surface. However, one concession must be made. For the present even the free waves can be studied only in terms of a linearised theory. In our co-ordinate system the most general linearised free-wave system in unrestricted deep water is represented by the perturbation potential:

(2.9) $$\varphi = V \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp(Z) \left[f \cos(X+Y) + g \sin(X+Y) \right] d\vartheta$$

with the notation

(2.10) $$Z = zk = z k_0 \sec^2\vartheta \qquad X = xw = x k_0 \sec^2\vartheta \cos\vartheta \qquad Y = yu = y k_0 \sec^2\vartheta \sin\vartheta, \qquad k_0 = g/V^2$$

Here ϑ and k can be interpreted as the direction of propagation and the circular wave number of a component plane wave respectively; u and w are the induced transverse and longitudinal wave numbers. Arbitrary functions $f(\vartheta)$ and $g(\vartheta)$ constitute the spectrum of the system.

The linearised potential is an exact solution of the Laplace equation:

(2.11) $$\nabla^2\varphi = \varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0 \qquad \text{for } z < 0$$

but satisfied the exact boundary conditions on the free surface:

(2.12) $$(V + \varphi_x)\zeta_x + \varphi_y \zeta_y - \varphi_z = 0 \qquad \text{on}$$

(2.14) $U \zeta_x - \varphi_z = 0$

(2.15) $g \zeta + U \varphi_x = 0$ on $z = 0$

or combined

(2.16) $\varphi_{xx} = - k_0 \varphi_z$

The surface elevation of the general linearised free wave system is given by

(2.17) $\zeta = - \frac{U}{g} \left[\varphi_x \right]_{z=0} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[f \sin(X+Y) - g \cos(X+Y) \right] d\vartheta$

The choice of the free-wave system for representation of flow at a control surface around the ship is governed by the radiation condition which states that free waves can trail only aft of the ship. The result is:

(2.18) $x \to - \infty$; $\zeta = 0$ (no waves)

(2.19) $x \to + \infty$; $\zeta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[f \sin(X+Y) - g \cos(X+Y) \right] d\vartheta$

(2.20) $y \to - \infty$; $\zeta = \int_{0}^{\pi/2} \left[f \sin(X+Y) - g \cos(X+Y) \right] d\vartheta$

(2.21) $y \to + \infty$; $\zeta = \int_{-\pi/2}^{0} \left[f \sin(X+Y) - g \cos(X+Y) \right] d\vartheta$

These are the appropriate models of free-wave systems for use at the control surfaces A, B, C and D respectively. Corresponding expressions for the potential can be derived from (2.9) by using the appropriate limits of integration. At first sight it may seem strange that different expressions are needed for B, C and D, although they all have the same spectrum. The different limits of integration are not only suggested by the radiation condition. As a typical example it has been shown in [11] for an isolated source that these limits are obtained by starting with the known full solution (including the potential due to the local wave) and taking asymptotic expansions for $x \to \pm \infty$ and $y \to \pm \infty$ using a method indicated by WEHAUSEN [9, p.477]. In fact the limits depend on the angle which the control surface makes with the x-axis in exactly the way prescribed by the radiation condition. Moreover, it will be seen that these expressions lead to consistent calculations of wave resistance. Again it must be emphasized that this differentiation holds only for horizontally unbounded fluid.

Free-wave spectrum and resistance: By using the relations already derived the wave resistance can now be expressed in terms of spectrum. Consider first the transverse section formula (2.5). Using (2.17) it can be written in the linearised form:

$$(2.22)\quad R = \frac{\rho V^2}{2g}\int_{B_E} \varphi_x^2\, dy + \frac{\rho}{2}\iint_{B_E}\left(\varphi_y^2+\varphi_z^2-\varphi_x^2\right) dy\, dz$$

where the suffix E denotes that the surface B_E extends only up to $z = 0$ and φ_x, φ_y, φ_z are to be derived from (2.9).

The well known result originally due to HAVELOCK [6] is:

$$(2.23)\quad R = \pi\rho V^2\int_0^{\pi/2}\left(f_e^2+g_e^2+f_o^2+g_o^2\right)\cos^3\theta\, d\theta = \frac{\pi}{2}\rho V^2\int_{-\pi/2}^{\pi/2}\left(f^2+g^2\right)\cos^3\theta\, d\theta$$

where f_e, g_e and f_o, g_o are the even and odd parts of f and g. This is the famous relation between spectrum and resistance. It will be seen that the same relation cam be obtained in a simpler way by using the lateral section formula (2.7) in the linearised form:

$$(2.24)\quad R = \rho\iint_{C_E}\varphi_y\varphi_x\, dz\, dx - \rho\iint_{D_E}\varphi_y\varphi_x\, dz\, dx$$

where different expressions are to be used for φ on C and D according to (2.20) and (2.21). Consider first the contribution from C.

$$(2.25)\quad \varphi = \int_0^{\pi/2}\exp(Z)\{[f\cos\gamma+g\sin\gamma]\cos wx+[g\cos\gamma-f\sin\gamma]\sin wx\}\, d\theta$$

$$(2.26)\quad \varphi_x = -k_0V\int_0^{\pi/2}\exp(Z)\{[f\cos\gamma+g\sin\gamma]\sin wx-[g\cos\gamma-f\sin\gamma]\cos wx\}\, d\theta$$

$$(2.27)\quad \varphi_y = k_0V\int_0^{\pi/2}\exp(Z)\{[g\cos\gamma-f\sin\gamma]\cos wx-[g\sin\gamma+f\cos\gamma]\sin wx\}\tan\theta\, d\theta$$

$$(2.28)\quad \int_{-\infty}^{\infty}\varphi_x\varphi_y\, dx = \pi k_0^2V^2\int_0^{\pi/2}\exp(2Z)[f^2+g^2]\tan\theta\left(\frac{d\theta}{dw}\right)d\theta$$

by use of the Fourier integral theorem. Further

$$(2.29)\quad R_C = \rho\int_{-\infty}^0 dz\int_{-\infty}^{\infty}\varphi_y\varphi_x\, dx = \frac{\pi}{2}\rho V^2\int_0^{\pi/2}\left(f^2+g^2\right)\cos^3\theta\, d\theta$$

Similarly from D

$$(2.30)\quad R_D = -\rho\int_{-\infty}^0 dz\int_{-\infty}^{\infty}\varphi_y\varphi_x\, dx = \frac{\pi}{2}\rho V^2\int_{-\pi/2}^0\left(f^2+g^2\right)\cos^3\theta\, d\theta$$

Finally

$$(2.31)\quad R = R_C + R_D = \frac{\pi}{2}\rho V^2\int_{-\pi/2}^{\pi/2}\left(f^2+g^2\right)\cos^3\theta\, d\theta$$

which is exactly the same as HAVELOCK's result (2.23).

Derivation of free-wave spectrum and resistance from measurements along transverse sections: The mehhodsof Fourier transforms can be used to express the spectrum in terms of surface profiles of wave height or its derivatives along transverse sections. For the sake of illustration let us consider two examples: i) a strip, i.e. ζ and ζ_x, ii) a pair of sections. Using (2.19) we have

$$(2.32)\quad \zeta = 2\int_0^\infty \left\{[f_e \sin X - g_e \cos X]\cos uy + [f_o \cos X + g_o \sin X]\sin uy\right\}\frac{d\vartheta}{du}\,du$$

$$(2.33)\quad \zeta_x = 2\int_0^\infty \left\{[f_e \cos X + g_e \sin X]\cos uy + [g_o \cos X - f_o \sin X]\sin uy\right\}\frac{d\vartheta}{du}\,du$$

Define Fourier transforms:

$$(2.34)\quad S = \int_{-\infty}^{\infty} \zeta \sin(uy)\,dy = 2\pi[f_o \cos X + g_o \sin X]\frac{d\vartheta}{du}$$

$$(2.35)\quad C = \int_{-\infty}^{\infty} \zeta \cos(uy)\,dy = 2\pi[f_e \sin X - g_e \cos X]\frac{d\vartheta}{du}$$

$$(2.36)\quad S_x = \int_{-\infty}^{\infty} \zeta_x \sin(uy)\,dy = 2\pi K_o[g_o \cos X - f_o \sin X]\sec\vartheta\,\frac{d\vartheta}{du}$$

$$(2.37)\quad C_x = \int_{-\infty}^{\infty} \zeta_x \cos(uy)\,dy = 2\pi K_o[f_e \cos X + g_e \sin X]\sec\vartheta\,\frac{d\vartheta}{du}$$

After some simplification the following formulas are obtained for the spectrum:

$$(2.38)\quad f_e = \frac{1}{2\pi}\frac{du}{d\vartheta}\left[C\sin X + \frac{\cos\vartheta}{K_o}C_x \cos X\right]$$

$$(2.39)\quad g_e = \frac{1}{2\pi}\frac{du}{d\vartheta}\left[-C\cos X + \frac{\cos\vartheta}{K_o}C_x \sin X\right]$$

$$(2.40)\quad f_o = \frac{1}{2\pi}\frac{du}{d\vartheta}\left[S\cos X - \frac{\cos\vartheta}{K_o}S_x \sin X\right]$$

$$(2.41)\quad g_o = \frac{1}{2\pi}\frac{du}{d\vartheta}\left[S\sin X + \frac{\cos\vartheta}{K_o}S_x \cos X\right]$$

For the calculation of wave resistance we need only

$$(2.41)\quad f_e^2 + g_e^2 + f_o^2 + g_o^2 = \frac{1}{4\pi^2}\left(\frac{du}{d\vartheta}\right)^2\left\{S^2 + C^2 + \frac{\cos^2\vartheta}{K_o}\left(S_x^2 + C_x^2\right)\right\}$$

and by (2.23) and (2.10)

$$(2.42)\quad R = \frac{\varrho g}{4\pi}\int_0^\infty \left\{(S^2 + C^2) + \frac{1}{kK_o}\left(S_x^2 + C_x^2\right)\right\}\left(2 - \frac{K_o}{k}\right)du$$

which is the desired result. It is of course understood that measurements are made so far behind the ship that the free-wave model (2.19) can be used. No further assumptions are necessary. The weighting factor $(2 - \frac{k_o}{k})$ allows a simple physical interpretation. The number 2 corresponds to the average distribution of energy in the free waves and k_o/k accounts for the rate of work done by pressure or the flow of energy due to wave motion.

To determine the spectrum from a pair of wave height records $\zeta_1(x_1, y)$ and $\zeta_2(x_2, y)$ we again use (2.34) and (2.35) with suffixes and introducing the following abbreviations

$$(2.43)\quad \bar{X} = \tfrac{1}{2}(X_1 + X_2) = \tfrac{w}{2}(x_1 + x_2)\ ;\quad \delta X = \tfrac{1}{2}(X_1 - X_2) = \tfrac{w}{2}(x_1 - x_2)$$

obtain analogous formulas for the spectrum

$$(2.44)\quad f_e = \frac{1}{4\pi}\frac{du}{d\vartheta}\left\{\frac{\sin\bar{X}}{\cos\delta X}(C_1 + C_2) + \frac{\cos\bar{X}}{\sin\delta X}(C_1 - C_2)\right\}$$

$$(2.45)\quad g_e = \frac{1}{4\pi}\frac{du}{d\vartheta}\left\{-\frac{\cos\bar{X}}{\cos\delta X}(C_1 + C_2) + \frac{\sin\bar{X}}{\sin\delta X}(C_1 - C_2)\right\}$$

$$(2.46)\quad f_o = \frac{1}{4\pi}\frac{du}{d\vartheta}\left\{\frac{\cos\bar{X}}{\cos\delta X}(S_1 + S_2) - \frac{\sin\bar{X}}{\sin\delta X}(S_1 - S_2)\right\}$$

$$(2.47)\quad g_o = \frac{1}{4\pi}\frac{du}{d\vartheta}\left\{\frac{\sin\bar{X}}{\cos\delta X}(S_1 + S_2) + \frac{\cos\bar{X}}{\sin\delta X}(S_1 - S_2)\right\}$$

and for the wave resistance from

$$(2.48)\quad (f_e^2 + g_e^2 + f_o^2 + g_o^2) = \frac{1}{16\pi^2}\left(\frac{du}{d\vartheta}\right)^2\left\{\frac{(C_1+C_2)^2 + (S_1+S_2)^2}{\cos^2\delta X} + \frac{(C_1-C_2)^2 + (S_1-S_2)^2}{\sin^2\delta X}\right\},$$

the final formula using again (2.23) and (2.10)

$$(2.49)\quad R = \frac{\rho g}{16\pi}\int_0^\infty\left\{\frac{(C_1+C_2)^2 + (S_1+S_2)^2}{\cos^2\frac{w}{2}(x_1-x_2)} + \frac{(C_1-C_2)^2 + (S_1-S_2)^2}{\sin^2\frac{w}{2}(x_1-x_2)}\right\}\left(2 - \frac{k_0}{k}\right)du$$

All experiment results described in this paper were analysed on the basis of this formula. Hence the following remarks may be pertinent. As u varies from 0 to ∞, w varies from k_0 to ∞. Hence for any given value of $(x_1 - x_2)$ an infinite number of singularities of the weighting function will have to be dealt with. However, this difficulty is less serious than it seems to be. The actual spectrum of a normal ship form is such that the integral can be safely truncated at a finite value of u, and by choosing a sufficiently small value of $(x_1 - x_2)$ the first singularity can be pushed beyond the region of interest. In the limit, of course, this procedure leads to the strip described earlier which is however difficult to measure. In actual analysis, therefore, the difficulty was obviated by measuring more than two sections and calculating the spectrum by a least squares fit to exploit redundant information.

Derivation of free-wave spectrum and resistance from measurements along lateral sections: At least in the case of unrestricted water a Fourier transform approach similar to the preceding can be used to express the spectrum in terms of surface profiles of wave height or its derivatives along lateral sections sufficiently far from the ship. Let us consider a few examples involving ζ , ζ_x and ζ_y . Using (2.20) we have along a lateral section such as C in Fig.1:

(2.50) $\zeta = \int_0^{\pi/2} \{ f(\sin X \cos Y + \cos X \sin Y) - g(\cos X \cos Y - \sin X \sin Y) \} d\vartheta$

(2.51) $= \int_{k_0}^{\infty} [(f\cos Y + g \sin Y)\sin wx + (f \sin Y - g\cos Y)\cos wx] \frac{d\vartheta}{dw} dw$

Define Fourier transforms

(2.52) $S = \int_{-\infty}^{\infty} \zeta \sin(wx) dx = \pi (f\cos Y + g \sin Y) \frac{d\vartheta}{dw}$

(2.53) $C = \int_{-\infty}^{\infty} \zeta \cos(wx) dx = \pi (f\sin Y - g\cos Y) \frac{d\vartheta}{dw}$

Then the spectrum can be apparently derived from a single profile:

(2.54) $f = \frac{1}{\pi} \frac{dw}{d\vartheta} (S\cos Y + C \sin Y)$

(2.55) $g = \frac{1}{\pi} \frac{dw}{d\vartheta} (S\sin Y - C\cos Y)$

The same applies to wave resistance

(2.56) $f^2 + g^2 = \frac{1}{\pi^2} (S^2 + C^2) \left(\frac{dw}{d\vartheta}\right)^2$

and using (2.29)

(2.57) $R_c = \frac{\varrho g}{2\pi} \int_{k_0}^{\infty} (S^2 + C^2) \frac{k_0}{w} \sqrt{1 - \frac{k_0^2}{w^2}}\, dw$

In a similar way the following formulas can be obtained from Fourier transforms of ζ_x and ζ_y .

(2.58) $R_c = (\varrho g/2\pi) \int_{k_0}^{\infty} (S_x^2 + C_x^2) \frac{k_0}{w^3} \sqrt{1 - \frac{k_0^2}{w^2}}\, du$

(2.59) $R_c = (\varrho g/2\pi) \int_{k_0}^{\infty} (SC_y - CS_y) \frac{k_0^2}{w^3}\, dw$

(2.60) $R_c = (\varrho g/2\pi) \int_{k_0}^{\infty} (S_y^2 + C_y^2) \frac{k_0^2}{w^4} \frac{dw}{\sqrt{w^2/k_0^2 - 1}}$

An obvious difficulty arises if one tries to apply these formally derived formulas in practice. If the integrand is not to vanish at the point $w = k_0$ the formal Fourier transforms can not converge since the weighting factor becomes zero at that point in (2.57), (2.58) and (2.59). Only the formula (2.60) is immune.

On the other hand the weighting factor in formula (2.60) has a singularity at $w = k_o$. This is indeed quite welcome. For it is known from MICHELL's theory that for a normal hull form the integrand of the wave resistance integral in w has an integrable singularity at k_o, which disappears on transformation to u. Hence we rewrite (2.60) in u.

$$(2.61) \quad R_c = \frac{\varrho g}{2\pi} \int_0^\infty (S_y^2 + C_y^2) \frac{k_o^4}{w^6} \cdot \frac{1}{(2 - k_o^2/w^2)}\, du$$

Now we have a reasonable weighting function beginning with $1/k_o^2$ at u = o and dying out strongly as $u \to \infty$. Since the integrand is known to be finite everywhere we may expect the Fourier transforms S_y and C_y to converge, thus providing a method of wave analysis which may have a certain technical advantage over the transverse section method because lateral profiles are easier to measure. However, this result should be evaluated with caution as until now no practical experience has been gained with this formula. For the sake of completeness it should be noted that in case of unsymmetrical disturbance at least one lateral section on each side must be analysed to obtain the full spectrum and wave resistance.

Discussion: It may be of interest to compare the preceding theory with the various recent proposals of wave analysis such as those made by INUI [10], KORVIN-KROUKOVSKY [11], TAKAHEI [12], EGGERS [13], WARD [14] and GADD and HOGBEN [15], all based more or less on a similar free-wave concept. However, I am refraining from doing so because most of these authors will themselves present their viewpoint at the seminar and the discussions there should provide ample opportunity for comparison. But the analogy with the treatment of EGGERS will be obvious. Whereas the theory presented here applies to unrestricted fluid, EGGERS' analysis is based on the concept of a channel of rectangular cross-section. As far as the transverse sections are concerned the two analyses are equivalent since a gradual passage from one to the other is possible. But the treatment of the lateral sections is complementary rather than equivalent in so far as EGGERS' formula applies only to a tank of finite width and the present approach is valid exclusively in unrestricted water. No practical experience has been gained with either formula but I feel the following argument may serve to elucidate the common and opposing features of the two formulas. Both formulas have in common the fact that integrals must be taken strictly over an infinite length, and the hope that in practice a finite length would suffice to obtain fairly steady values. But the convergence criteria are quite different. EGGERS' formula takes advantage of the almost periodic character of the free-wave system in a tank and it is the quotient of the integral and the length of the profile which is supposed to converge in the limit. Hence it would probably work best in a long and narrow tank by making use of the repeated reflections at the tank walls. On the other hand the integral itself must converge in the fomula derived here and this is possible only in unrestricted water or in a sufficiently broad tank if steady values can be obtained before the first reflection. Finally it may be noted that EGGERS' formula postulates ideal reflection at the tank walls whereas the present formula requires ideal absorption.

III. EXPERIMENTS

It is considered beyond the scope of this paper to describe the details of experiment procedure or to reproduce the data obtained directly by experiment. But a few general remarks may be called for to facilitate understanding of the following analysis of results. Fig.2 shows the salient features of the wave measurement actually carried out. The same model was towed in two different tanks and surface elevation was recorded by different means. Fig.2 shows the range of speeds investigated and the region of measurement in each case. By the stereographic method as many as 31 transverse sections were evaluated from a single stereogram but in the large tank only about 5 transverse sections were recorded for each speed using a sonic wave transducer mounted on a cross-carriage on the towing carriage.

IV. ANALYSIS

We shall now deal with the analysis and evaluation of the results of experiments actually carried out with the Inuid S201 in the Hamburg Towing Tank. The basic theory needed for this purpose was outlined in a preceding chapter in terms of unbounded fluid because it enabled a symmetric treatment of transverse and lateral sections. Further it was mentioned that for transverse sections a passage to the case of a tank of rectangular cross-section is possible and would lead to results equivalent to those communicated earlier by EGGERS [13].

Introduction of tank formulas: Let us convert our formulas to the case of restricted water of infinite depth and obtain expressions essentially identical to those of EGGERS. The procedure is quite simple. The well-known effect of the introduction of tank walls is that the continuous spectrum degenerates into a discrete one uniformly spaced in transverse wave number u. The formal procedure consists of a variable transformation from ϑ to u and substitution of a sum instead of the integral with $\Delta u = \pi/b$. Thus we obtain from (2.9) the general linearised free-wave potential for a tank of width b

$$(4.1)\quad \varphi = \frac{g}{V}\sum_{\nu=0}^{\infty}\frac{A_\nu}{w_\nu}\cos w_\nu(x-\xi_\nu)\,e^{k_\nu z}\cos u_\nu y + \frac{g}{V}\sum_{2\nu=1}^{\infty}\frac{B_{2\nu-1}}{w_{2\nu-1}}\cos w_{2\nu-1}(x-\xi_{2\nu-1})\,e^{k_{2\nu-1}z}\sin u_{2\nu-1} y$$

from (2.17) the corresponding surface elevation

$$(4.2)\quad \zeta = \sum_{\nu=0}^{\infty} A_\nu \sin w_\nu(x-\xi_\nu)\cos u_\nu y + \sum_{2\nu=1}^{\infty} B_{2\nu-1}\sin w_{2\nu-1}(x-\xi_{2\nu-1})\sin u_{2\nu-1} y$$

and from (2.23) the wave resistance

$$(4.3)\quad R = \frac{\varrho g b}{8}\sum_{\nu=0}^{\infty} A_\nu^2\left(2-\frac{k_0}{k_\nu}\right) + \frac{\varrho g b}{8}\sum_{2\nu=1} B_{2\nu-1}^2\left(2-\frac{k_0}{k_{2\nu-1}}\right)$$

while the relations (2.10) yield the connections between the wave numbers

$$(4.4)\quad \begin{aligned} \frac{2\pi\nu}{b} &= u_\nu = k_0\sec^2\vartheta_\nu\sin\vartheta_\nu & \pi\frac{2\nu-1}{b} &= u_{2\nu-1} = k_0\sec^2\vartheta_{2\nu-1}\sin\vartheta_{2\nu-1}\\ w_\nu &= k_0\sec\vartheta_\nu & w_{2\nu-1} &= k_0\sec\vartheta_{2\nu-1}\\ k_\nu &= k_0\sec^2\vartheta_\nu & k_{2\nu-1} &= k_0\sec^2\vartheta_{2\nu-1}\end{aligned}$$

Here the stylised symbol $\sum$ is used to denote that only half the zeroth term enters the sum.

The above expressions can best be interpreted in terms of elementary waves in a tank (Fig.3). Each elementary wave is composed of a pair of conjugate plane waves moving with velocities $V\cos\vartheta$ in the directions $\pm\vartheta$. The resulting pattern moves with a speed V along the x-direction and satisfies the tank wall condition along the longitudinal anti-nodal planes. Different configurations are shown in Fig.3 which also illustrates the different wave numbers. It is noted that each term in (4.2) represents an elementary wave, the integral values of tank wave number ν leading to symmetrical cross-sections and semi-integral values to anti-metric cross-sections. A general free-wave system in a tank can be considered as a linear combination of an infinite number of discrete elementary waves, each of which is characterised by a wave number ν, amplitude A_ν or $B_{2\nu-1}$, and phase ξ_ν or $\xi_{2\nu-1}$. The last two quantities together constitute the spectrum. The concept of an elementary wave is fundamental to the subsequent analysis.

The equations (4.1) through (4.4) reveal the immense advantage of the tank formulas over the unbounded fluid as far as numerical treatment is considered. Integrals are reduced to sums and Fourier transforms of ζ, ζ_x can be calculated by Fourier analysis as pointed out by EGGERS. As far as practical application is concerned there may be two interpretations to the use of tank formulas. If the wave sections to be analysed are perceptibly affected by tank walls as in the case of the small tank in Fig.2 the use of tank formulas with the correct tank width is imperative and exact. However, if the region in question is apparantly unaffected by real tank walls as in the case of the large tank in Fig.2 a tank formula with a sufficiently large hypothetical width can still be employed by way of numerical approximation. For example, a given set of measured sections was analysed using three different hypothetical tank widths including the asymmetrical case shown in Fig.2 and completely consistent results were obtained. This result was discovered rather accidentally, but now with the advantage of hindsight it appears to be perfectly natural.

Verification of linearised free-wave theory: A criterion for checking the applicability of the linearised free-wave theory to actually measured wave sections mentioned in the Introduction will now be derived. Rewrite (4.2) using (4.4) as

$$(4.5) \quad \zeta = \sum_{\nu=0}^{\infty} a_\nu \cos\frac{2\pi\nu}{b}y + \sum_{2\nu=1}^{\infty} b_{2\nu-1} \sin\frac{\pi(2\nu-1)}{b}y$$

It is evident that the Fourier cosine and sine coefficients a_ν , $b_{2\nu-1}$ must be prescribed functions of x

$$(4.6) \quad a_\nu = A_\nu \sin w_\nu(x-\xi_\nu) \quad ; \quad b_{2\nu-1} = B_{2\nu-1} \sin(x-\xi_{2\nu-1})$$

that is sine waves of predetermined periods $2\pi/w_\nu$, $2\pi/w_{2\nu-1}$. Two measured sections will in general suffice to determine the unknowns A_ν , ξ_ν etc. and fix the whole spectrum. If more than two sections are measured the consequent redundant information can be exploited to verify the applicability of the assumption (4.2). For one particular case (γ_0 = 10) as many as 31 stereophotographically measured sections were analysed in this way and the dispersion criterion applied to the first 30 Fourier cosine and sine coefficients. In all cases where any significant amplitude was measurable the correct wave length was observed although for higher coefficients the test was of necessity inconclusive. Fig.4 shows a typical result. The importance of this result for the present analysis can hardly be overestimated. Firstly it can be interpreted as a justification of the basic assumption although it must be pointed out that nonfulfilment of the above criterion would not have discredited the linearised free-wave theory as such, in view of other possible disturbing factors like i) local wave of the ship, ii) viscous effects and iii) experimental errors. Secondly it points a way to eliminating just such errors by analysing more than two sections and calculating the spectrum by a least squares fit. An examination along these lines brought out the important result that with our data at least five sections were necessary to get steady values for the spectrum.

Fig.5 shows a comparison of the spectrum obtained by testing the same model at about the same speed but in two different tanks and measuring the wave sections by different techniques.

The small tank results shown by open dots were averaged from the 31 stereophotogrammetrical sections just mentioned. The large tank results were derived from six sections recorded by a sonic wave transducer by using the real and a hypothetical tank width. In order to correlate the different tank wave numbers the spectrum is plotted on the transverse wave number u. However, the actual ν-scales as well as scales for w and θ are added for the sake of comparison. Correlation of amplitudes was achieved by plotting $k_o b A_\nu$ instead of A_ν. It will be noticed from (4.6) that the phase ξ_ν cannot be determined uniquely. By convention the value closest to the point of reference (in this case the stern) is plotted in Fig.5 thus explaining the steps. Following conclusion is made. Provided a sufficient number of sections (five or more) is available to eliminate statistical errors the present analysis yields a fairly consistent spectrum which is reproducible, insensitive to the measuring technique and invariant with respect to change of actual or hypothetical tank width.

The next logical step would be the verification of ship-wave theory by comparing the calculated and measured spectrum. But let us first pause to compare the integral effect of the spectrum, namely the wave resistance.

<u>Comparison of wave resistance</u>: The comparison of wave resistance is a ticklish affair because apparently there is no rigid definition of wave resistance in a real fluid (cf. BIRKHOFF et al. [7]). Nevertheless an attempt has been made to evaluate the significance of the resistance calculated from the measured free-wave spectrum by comparing it with what are conventionally called calculated and measured wave resistance. Curve 1 in Fig.6 represents the wave resistance of the generating source distribution of the Inuid S201 calculated by HAVELOCK's theory. Curve 2 has been derived from the total measured resistance by use of the HUGHES [8] (1954) friction formulation for estimating the viscous component. The assumed form factor of 1.18 is essentially confirmed by resistance measurements at low Froude numbers, tests with submerged models, geosim analysis

and wake analysis. Semi-empirical corrections have been devised by INUI [2] to reconcile calculated and measured curve by taking account of viscosity and nonlinearity in terms of modifications to the free-wave system. They can also be used to predict the resistance of the free-waves behind the model. The curve 3 has been obtained in this way by using the values published by INUI [2] for the model S201. The reason for the discrepancy between 2 and 3 is the attenuation of the bow wave during the propagation along the ship. According to INUI if ζ_B be the theoretical bow wave then the actual bow wave is $\gamma\zeta_B$ at the bow and $\alpha\zeta_B$ at the stern. Consequently if R_B be the resistance of the bow wave alone, the discrepance between 2 and 3 will be $(\gamma^2 - \alpha^2)R_B$. Although this idea is implicit in the proposals of INUI and his predecessors, who have almost all used a bow-wave attenuation factor, it has rarely, if ever, been stated. This is rather surprising in view of its implications to the prospects of a successful waved analysis behind the ship.

Finally Fig.6 shows the points 4 being the result of the wave analysis actually carried out for 15 speeds over the entire interesting range of Froude number. It is seen that the resistance obtained from wave analysis is generally lower than the conventional measured wave resistance, the percentage discrepance being greater for lower Froude numbers. On the other hand the agreement between the predicted and actual result of wave analysis is remarkable thus supporting the above argument and apparently vindicating INUI's proposals. However, it should be pointed out that the above argument is only one way of explaining the difference between 2 and 4 . The influence of other assumptions involved in 2 and 4 should not be disregarded. The agreement between 3 and 4 should be checked not only in resistance but over the whole spectrum before drawing final conclusions. This will be done presently but it is first necessary to study the theoretical free-wave spectrum of the Inuid and how it is affected by the various correction factors proposed by INUI.

Theoretical free-wave spectrum: The source distribution associated with the S - series Inuids is given by (cf. [2], p.193)

$$\sigma = (8BV/L^2)x \tag{4.7}$$

over the surface $-L/2 \le x \le L/2$; $-T \le z \le 0$; $y = 0$ and zero elsewhere. Here B is the breadth of the corresponding Michell ship and not the Inuid. We shall however retain this symbol as a convenient measure of source strength, especially in the form B/L. The free-wave system of this distribution in a symmetric tank of width b is easily calculated (cf. EGGERS [13, p. 105])

$$\zeta = -\sum_{\nu=0}^{\infty} \frac{64B}{k_0 b L^2}\left(\frac{\sin\left(\frac{\omega_\nu L}{2}\right) - \left(\frac{\omega_\nu L}{2}\right)\cdot\cos\left(\omega_\nu \frac{L}{2}\right)}{k_\nu^2 + u_\nu^2}\right)\left(1-e^{-k_\nu T}\right)\sin(\omega_\nu x)\cos\frac{2\pi\nu y}{b} \tag{4.8}$$

In analogy with (4.2) the amplitude spectrum is given by

$$A_\nu = \left|\frac{64B}{k_0 b L^2}\left(\frac{\sin\left(\omega_\nu \frac{L}{2}\right) - \left(\omega_\nu \frac{L}{2}\right)\cos\left(\omega_\nu \frac{L}{2}\right)}{k_\nu^2 + u_\nu^2}\right)\left(1-e^{-k_\nu T}\right)\right| \tag{4.9}$$

and the phase by

$$\xi_\nu = \begin{cases} 0 \\ \pm\pi/\omega_\nu \end{cases} \quad -\sin\left(\frac{\omega_\nu L}{2}\right) + \left(\omega_\nu \frac{L}{2}\right)\cos\left(\omega_\nu \frac{L}{2}\right) \begin{matrix} > 0 \\ < 0 \end{matrix} \tag{4.10}$$

It is this spectrum which we are going to compare with the measured spectrum of the Inuid S201 at all speeds tested. In order to show the general features the amplitude spectrum for five selected values of the velocity parameter γ_0 has been plotted in Fig.7. For this purpose a nondimensional form has been chosen but dimensional scales applying to our model are added. This diagram shows the advantage of multiplying the amplitude by k_0. The function plotted has the same order of magnitude in the entire range of interest. Nondimensional longitudinal wave number has been chosen as the base because the zeros are then independent of γ_0. But separate scales of transverse wave number have also been shown for each γ_0.

Corrections to free-wave spectrum: It was stated at the outset of the paper that the ultimate aim would be to scrutinize the validity of INUI's elaborate system of corrections by comparing the theoretical and measured spectra. Before beginning with such comparison let us first enumerate them and then illustrate their effect by taking one at a time. The corrections are [2, p. 205-- 215]:

Finite wave height correction to bow and stern wave	γ
Hull's self interference correction to bow wave	α'
Total correction to bow wave on reaching the stern	$\alpha = \gamma\,\alpha'$
Viscosity correction to stern wave	β'
Total correction to stern wave	$\beta = \gamma\beta'$
Phase shift correction due to viscosity etc.	δ

It is obvious that as far as our analysis of the waves behind the ship is concerned we need consider only three factors, namely α, β und δ. To illustrate their effect formally let the theoretical free-wave system be composed of bow and stern waves as follows

(4.11) $$\zeta(x,y) = \zeta_B(x,y) + \zeta_S(x,y)$$

then the corrected free-wave system would be

(4.12) $$\tilde{\zeta}(x,y) = \alpha\,\zeta_B(x,y) + \beta\,\zeta_S(x-\delta L, y)$$

or rewriting (4.8) as

(4.12) $$\zeta = \zeta_B + \zeta_S = \sum_{\nu=0}^{\infty} \frac{32B}{k_0 b L^2}\left(\frac{1-e^{-k_\nu T}}{k_\nu^2+u_\nu^2}\right)\left\{\left(w_\nu \frac{L}{2}\right)\sin\left(x+\frac{L}{2}\right)w_\nu + \cos\left(x+\frac{L}{2}\right)w_\nu\right\}\cos\frac{2\pi\nu y}{b}$$
$$+\sum_{\nu=0}^{\infty} \frac{32B}{k_0 b L^2}\left(\frac{1-e^{-k_\nu T}}{k_\nu^2+u_\nu^2}\right)\left\{\left(w_\nu \frac{L}{2}\right)\sin\left(x-\frac{L}{2}+\delta L\right)w_\nu - \cos\left(x-\frac{L}{2}+\delta L\right)w_\nu\right\}\cdot\cos\frac{2\pi\nu}{b}\cdot y$$

the corrected system would be

(4.13) $$\tilde{\zeta} = \alpha\sum_{\nu=0}^{\infty} \frac{32B}{k_0 b L^2}\left(\frac{1-e^{-k_\nu T}}{k_\nu^2+u_\nu^2}\right)\left\{\left(w_\nu \frac{L}{2}\right)\sin\left(x+\frac{L}{2}\right)w_\nu + \cos\left(x+\frac{L}{2}\right)w_\nu\right\}\cos\frac{2\pi\nu}{b}y$$
$$+\beta\sum_{\nu=0}^{\infty} \frac{32B}{k_0 b L^2}\left(\frac{1-e^{-k_\nu T}}{k_\nu^2+u_\nu^2}\right)\left\{\left(w_\nu \frac{L}{2}\right)\sin\left(x-\frac{L}{2}+\delta L\right)w_\nu - \cos\left(x-\frac{L}{2}+\delta L\right)w_\nu\right\}\cdot\cos\frac{2\pi\nu}{b}y$$

It will be seen from the above that the corrections α and β applied alone can also be interpreted as corrections to virtual breadth B or better to source strength parameter B/L. But δ cannot be regarded strictly as a virtual length correction because it is not applied consistently to L in (4.13). The following alternative corrections are proposed:

Correction factor to B/L of forebody	$\bar{\alpha}$
Correction factor to B/L of afterbody	$\bar{\beta}$
Percentage correction to L	$\bar{\delta}$

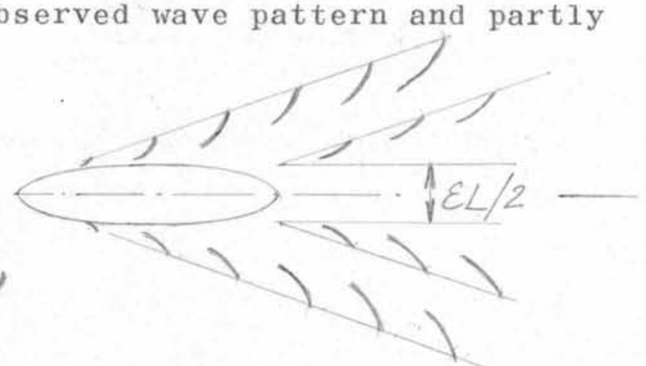
To illustrate the subtle difference consider the corrected system

(4.14) $$\hat{\zeta} = \sum_{\nu=0}^{\infty} \frac{32\bar{\alpha}(B/L)}{k_0 b L'} \left(\frac{1-e^{-k_\nu T}}{k_\nu^2+u_\nu^2}\right)\left\{\left(w_\nu \frac{L'}{2}\right)\sin\left(x+\frac{L'}{2}\right)w_\nu + \cos\left(x+\frac{L'}{2}\right)w_\nu\right\}\cos\frac{2\pi\nu}{b}y$$

$$+\sum_{\nu=0}^{\infty} \frac{32\bar{\beta}(B/L)}{k_0 b L'} \left(\frac{1-e^{-k_\nu T}}{k_\nu^2+u_\nu^2}\right)\left\{\left(w_\nu \frac{L'}{2}\right)\sin\left(x-\frac{L'}{2}\right)w_\nu - \cos\left(x-\frac{L'}{2}\right)w_\nu\right\}\cos\frac{2\pi\nu}{b}y$$

with $L' = (1+\delta)L$. It is clear that for $\delta = 0$ the new factors $\bar{\alpha}$ and $\bar{\beta}$ are equivalent to α and β. But even for small values of δ the difference in $\tilde{\zeta}$ and $\hat{\zeta}$ would be small except when $(w_\nu L/2)$ is small i.e. at high speeds. Although the alternative proposal may be practically no better than the original the following argument is suggested. When applying empirical corrections to a chain of causes and effects the more general and perspicuous would be a correction the earlier it is applied. If wave resistance is the effect of the form then HAVELOCK's [6] corrections are more general than WIGLEY's [3] because they are applied to the form rather than directly to resistance. INUI's corrections are intermediate in character for they are applied to the waves and the above approach may push them back one step along the causal chain by interpreting them as consistent corrections to virtual form parameters.

Further a new separation correction ε to the stern wave was considered prompted partly by the observed wave pattern and partly by the measured spectrum.



As can be readily seen it tends to cause a phase shift of the stern wave by

(4.15) $$(\varepsilon L/2)\,tg\,\vartheta = (\varepsilon L/2)\,u_\nu/w_\nu$$

It thus tends to compete with the correction δ for higher values of ν.

The effect of all these corrections on amplitude and phase spectrum has been illustrated by concrete examples in self-explanatory Figs. 8 to 10. As is usual in nature the effect of α and β is to round off the corners and the steps of the theoretical amplitude and phase spectra. The quite different effects of δ and ε in Fig.9 as well as Fig.10 should be noted.

Comparison of calculated and measured free-wave spectrum:
The calculated and measured spectrum of the Inuid S201 has been compared for 18 speeds in the light of the foregoing corrections. Four typical results are reproduced in Figs. 11 to 14. (These cases are marked by crosses in Fig.6 to show that INUI's corrections appear completely justified if only resistance is compared.) Each Fig. shows besides the measured amplitude spectrum (two sets in Figs. 11 and 12) three calculated curves. Firstly the uncorrected theoretical curve lying as expected wide of the mark specially at lower speeds. Secondly a corrected curve calculated strictly according to the values of the correction factors given by INUI for his model S201 [2, p. 342-4].disregarding any scale effect between his model (L = 1,75 m) and ours (L = 4 m). And lastly a second corrected curve found by adjusting the values of the factors just defined to fit the measured spectrum. This is not a systematic least squares fit but found rather by inspection and trial and error. However, it is unlikely that a substantially better fit can be obtained by using these factors alone.

On the whole INUI's correction factors lead to a fair qualitative agreement with the result of our wave analysis, which is quite remarkable in view of the fact that his analysis was based almost only on a study of the resistance curve. The weak points of his set of corrections seem to be the controversial factors δ and α', and it is doubtful whether his interpretations in terms of the transverse wave ($\theta = 0$) is compatible with the results of our analysis. However, it should be admitted that his transverse wave may mean something different from the elementary wave considered here. His locally determined transverse wave would be more subject to the influence of viscous wake than ours which has been averaged from the entire wave system.

Good quantitative agreement can be obtained by adjusting the values of the correction factors to the measured data. However it becomes increasingly difficult for higher values of Froude number and for higher θ at any given Froude number.

V. CONCLUSIONS

In view of the fluid state of the question of wave analysis it would be premature to draw final conclusions without hearing the other evidence to be presented at the seminar. Nevertheless it may be expedient to underline the main results of this paper.

Theoretically the possibility of determining the wave resistance of a ship in horizontally unbounded fluid from surface profile measurements along sections transverse or lateral to the direction of motion has been shown using a uniform approach based on the concept of the linearised free-wave potential.

Experimental evidence has been brought from numerous stereo-photogrammetric and sonic wave measurements that the actual surface elevation behind a model in a towing tank can be represented by a general linearised free-wave system with good approximation.

Comparison of the calculated and measured free-wave spectrum of an Inuid in steady motion in the light of INUI's elaborate semi-empirical corrections for viscosity and nonlinearity have revealed features which could hardly have been detected by a comparison of resistance alone.

ACKNOWLEDGEMENTS

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APPENDIX

The variable resistance paradox: While deriving the relation (2.23) between the spectrum of the linearised free-wave system (2.9) and the associated wave resistance with the help of the rearranged momentum equations (2.6) and (2.7) it was tacitly assumed that the condition of continuity (2.3) is exactly fulfilled. Paradoxical results may be obtained if the following complete momentum relations are used to calculate the wave resistance of the linearised potential.

From (2.1) and (2.2) we get for transverse sections

(A.1) $$R = \frac{\varrho g}{2}\int_B \zeta^2 dy + \frac{\varrho}{2}\iint_B (\varphi_y^2 + \varphi_z^2 - \varphi_x^2)\, dy\, dz - \varrho V^2 \int_B \zeta\, dy - \varrho V \iint_B \varphi_x\, dy\, dz$$

and for lateral sections

(A.2) $$R = \varrho \iint_C \varphi_y \varphi_x\, dz\, dx - \varrho \iint_D \varphi_y \varphi_x\, dz\, dx + \varrho V \iint_C \varphi_y\, dz\, dx - \varrho V \iint_D \varphi_y\, dz\, dx$$

The reason is that the last two terms in each equation, usually neglected on grounds of continuity, may actually yield contributions of the same order as the remaining terms if the linearised potential is used. I fear, this paradox can be genuinely resolved only in terms of a higher order theory (cf. WHITHAM, Journ.Fl.Mech., Jan.1962) but the following modus vivendi with the linearised potential is suggested. Consistent results can be obtained if the linearised potential is regarded as the exact solution of the linearised problem, in which the upper boundary is not the physical free surface F, but the geometrical surface E (z = o) with the hypothetical property (2.16); there is no wave height and no variation of potential energy but there is a diffusion of fluid and momentum through the boundary E, and ζ can be retained as a symbol for $-(V/g)\varphi_x$ on E to show the analogy with the real physical problem.

To illustrate the above argument let us consider the simple case of a transverse plane wave in a tank of unit width. The linearised potential is

(A.3) $$\varphi = A_0 V \exp(k_0 z) \cos(k_0 x)$$

and the linearised surface elevation

(A.4) $$\zeta = A_0 \sin(k_0 x)$$

The usual procedure would be to start with the "linearised" momentum or energy relation

(A.5) $$R = \frac{\rho g}{2}\zeta^2 + \frac{\rho}{2}\int_{-\infty}^{0}(\varphi_z^2 - \varphi_x^2)\,dz$$

introduce ζ, φ_x, φ_z according to (A.4) and (A.3) and obtain the standard result

(A.6) $$R = \frac{\rho g}{4}A_0^2$$

However, if we start from the exact momentum equation

(A.7) $$R = \rho g\int_{B-A} z\,dz + \frac{\rho}{2}\int_{B}(\varphi_z^2 - \varphi_x^2)\,dz - \rho V\int_{B-A} V\,dz - \rho V\int_{B}\varphi_x\,dz$$

use (A.3) for φ and calculate consistently to second order in amplitude A_0 we get the variable resistance paradox

(A.8) $$R = \frac{\rho g}{4}A^2 + \rho g A_0^2 \sin^2(k_0 x)$$

The error resulting from the violation of the condition of continuity does not vanish even when averaged over x. A higher order theory like that of STOKES would resolve the paradox by demonstrating a small difference in mean level and/or velocity between the control surfaces A and B. But the alternative problem approach suggested above will also yield consistent results. It can be verified using (A.3) that the following equation of continuity is exactly satisfied.

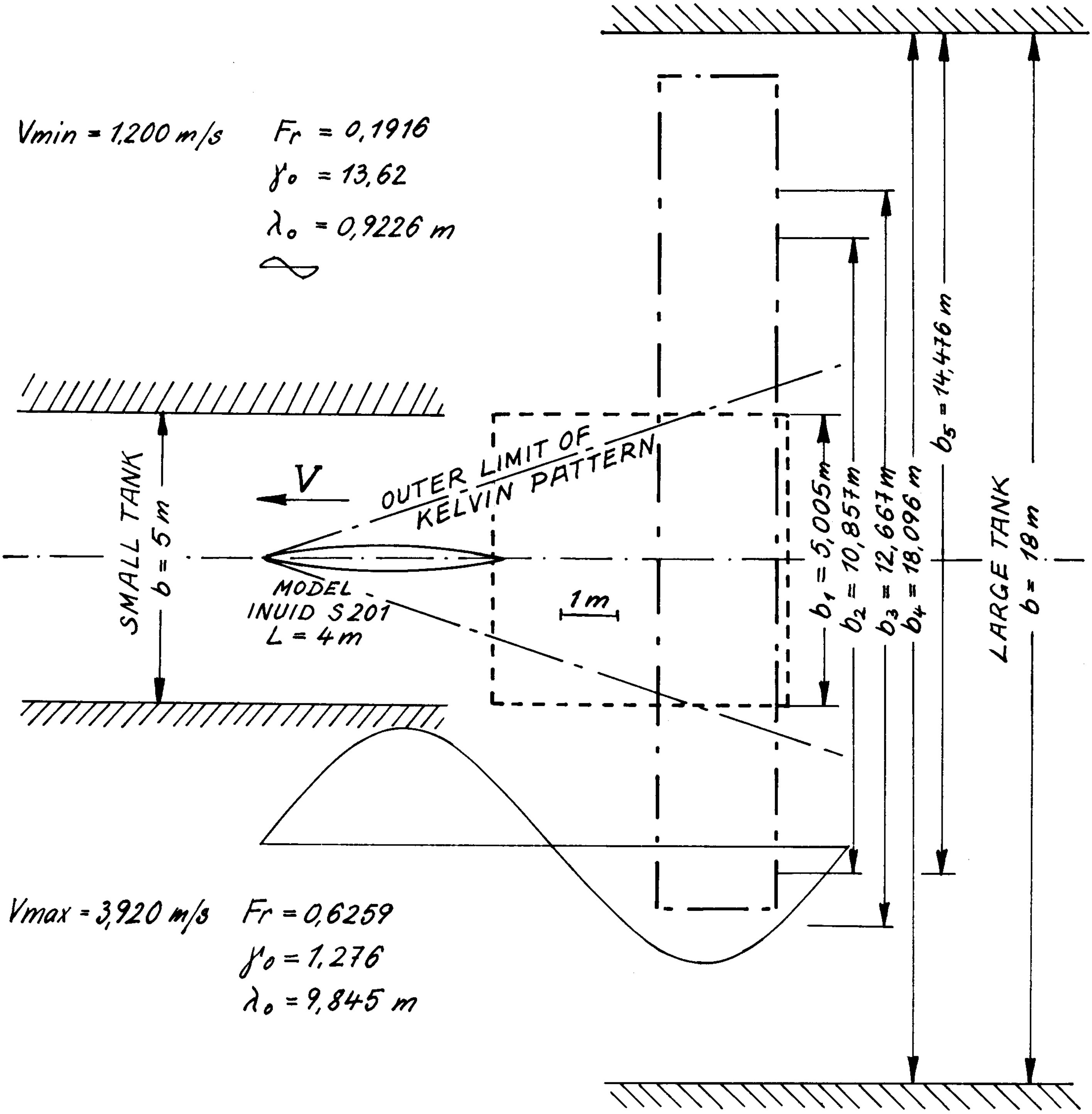
(A.9) $$\rho\int_{AE} V\,dz - \rho\int_{E}\varphi_z\,dx - \rho\int_{BE}(V + \varphi_x)\,dz = 0$$

Further the appropriate momentum equation

(A.10) $$R = \int_{BE}(\varphi_z^2 - \varphi_x^2)\,dz - \rho\int_{E}\varphi_z\varphi_x\,dx$$

using (A.3) and the boundary condition $\varphi_{xx} = -k_0\varphi_z$ on E again leads to the standard result (A.6). It is interesting that the contribution to R from diffusion through E in the artificial problem corresponds to the potential energy term in the natural problem.

It may be noted that for all oblique elementary waves in a tank the paradox does not arise. The unbounded fluid problem is more complicated.



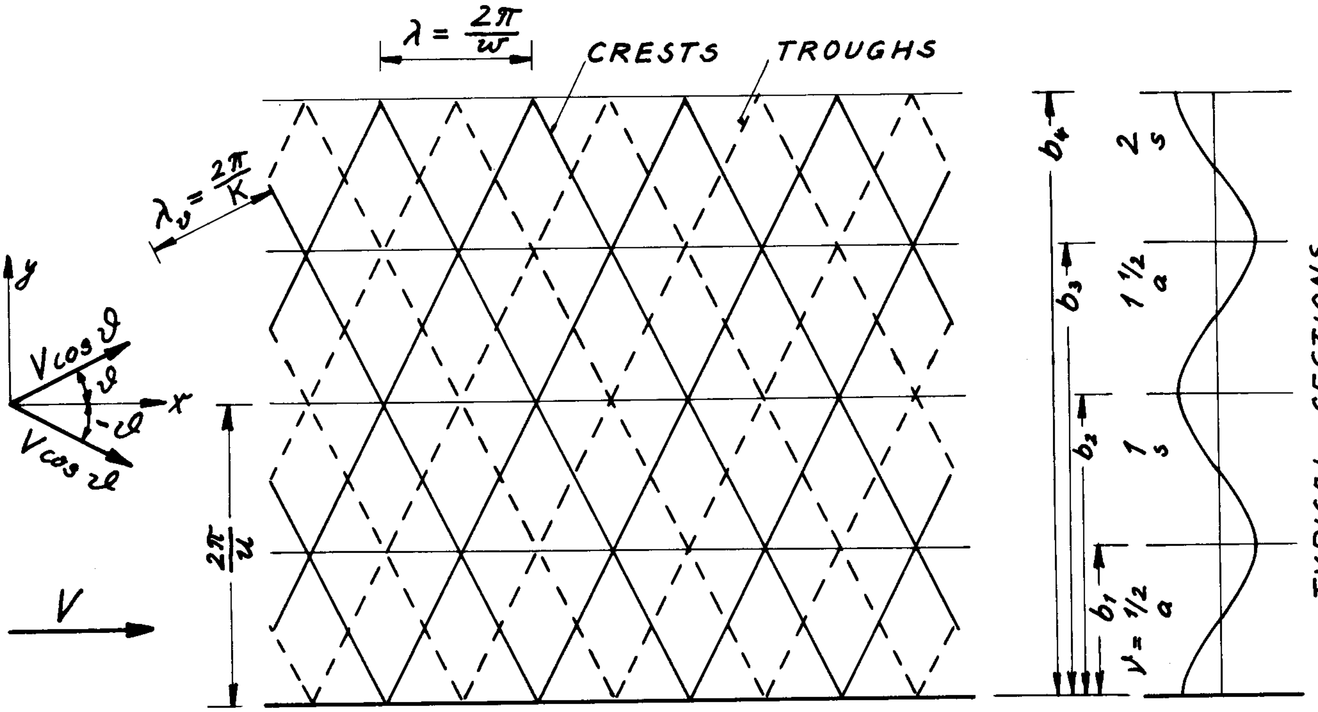
REGION OF WAVE MEASUREMENTS IN

SMALL TANK BY STEREOPHOTOGRAMMETRY - - - - - - -

LARGE TANK BY SONIC TRANSDUCER ——— - ——— -

$b_2 \div b_5$ HYPOTHETICAL TANK WIDTH FOR SPECTRAL ANALYSIS

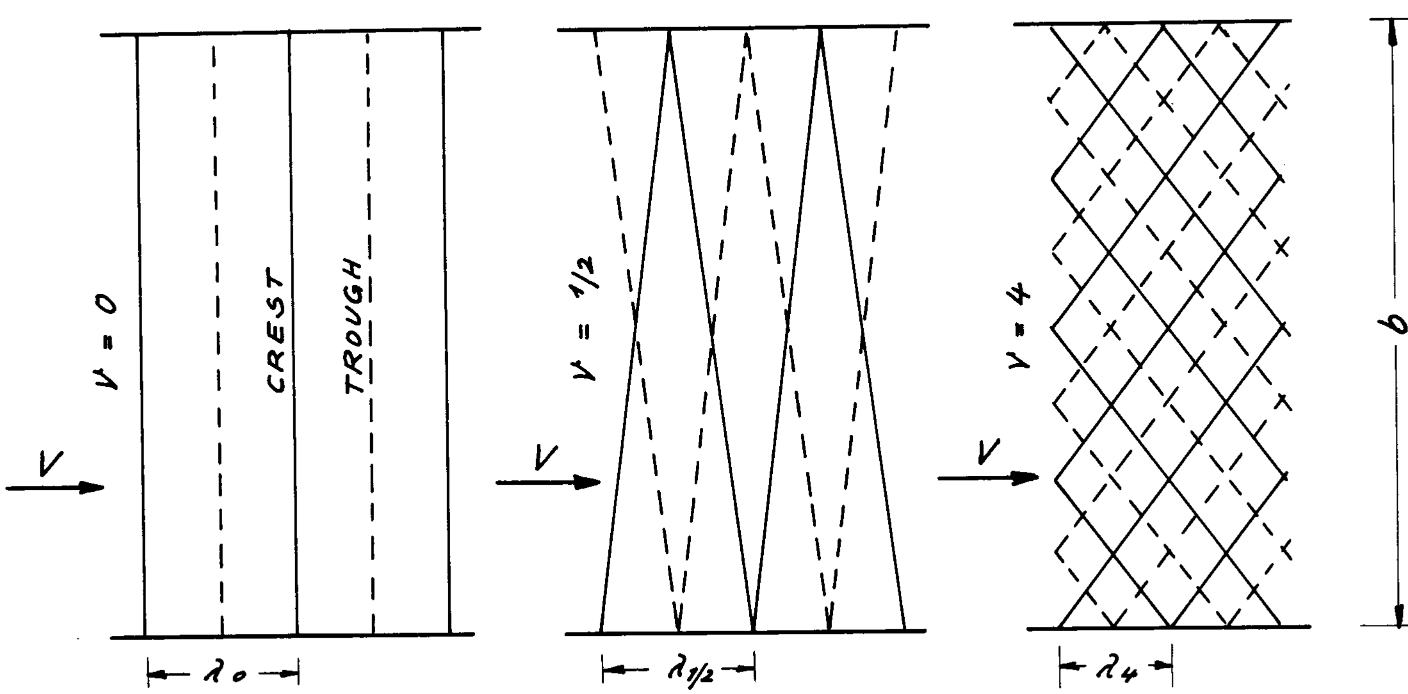
FIG. 2 RANGE OF WAVE ANALYSIS



$K_o = g/V^2$; $K = K_o \sec^2 \vartheta$; $w = K_o \sec \vartheta$; $u = K_o \sec^2 \vartheta \sin \vartheta = 2\pi\nu/b$

a) POSSIBLE TANK WIDTHS FOR A GIVEN WAVE

(FIXED V AND ϑ ; VARIABE b)



b) POSSIBLE WAVES IN A GIVEN TANK

(FIXED V AND b ; VARIABLE ϑ)

FIG. 3 ELEMENTARY WAVE IN A TANK

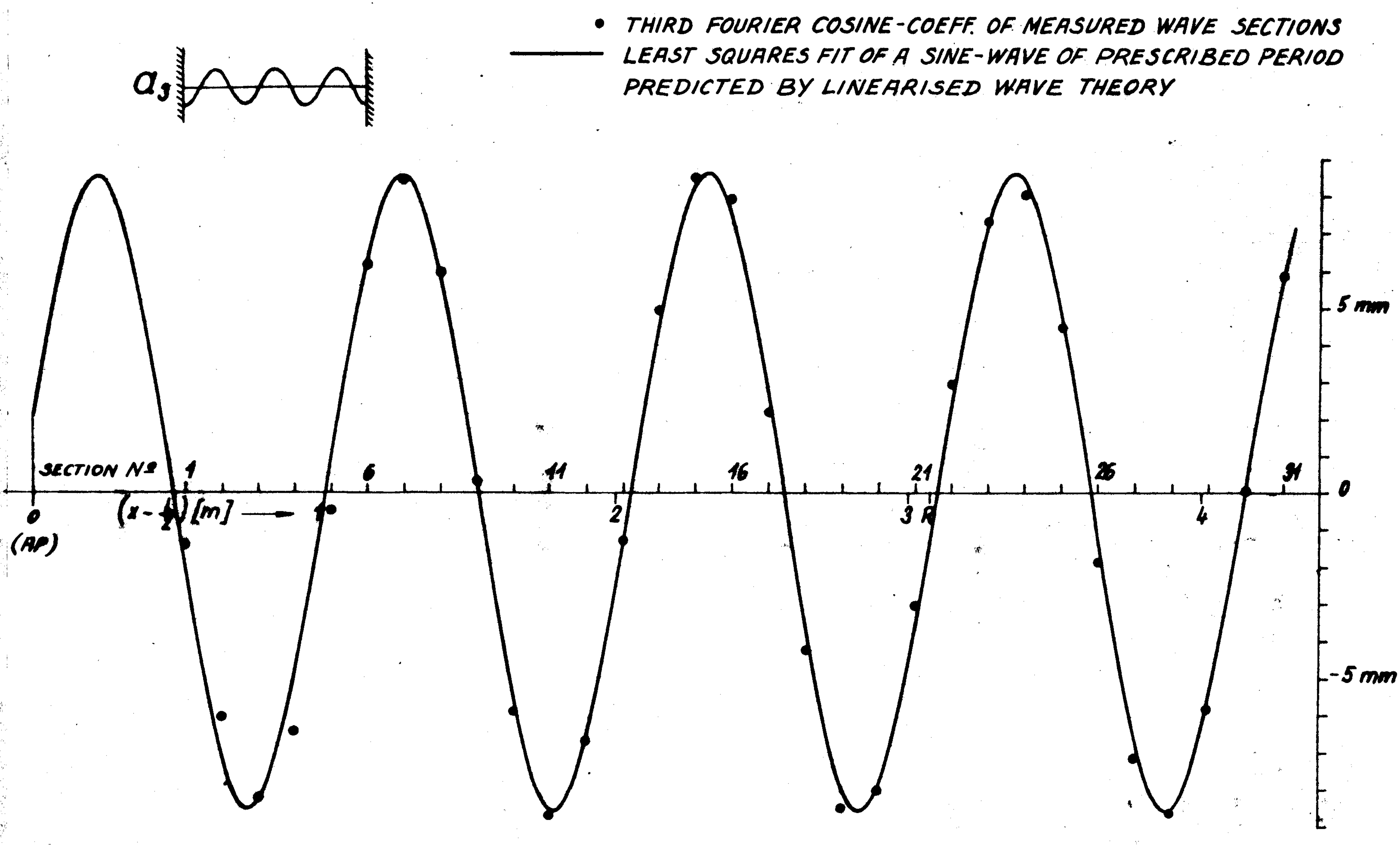


FIG. 4 TYPICAL PLOT OF FOURIER COEFFICIENT FOR CHECKING APPLICABILITY OF LINEARISED FREE-WAVE THEORY

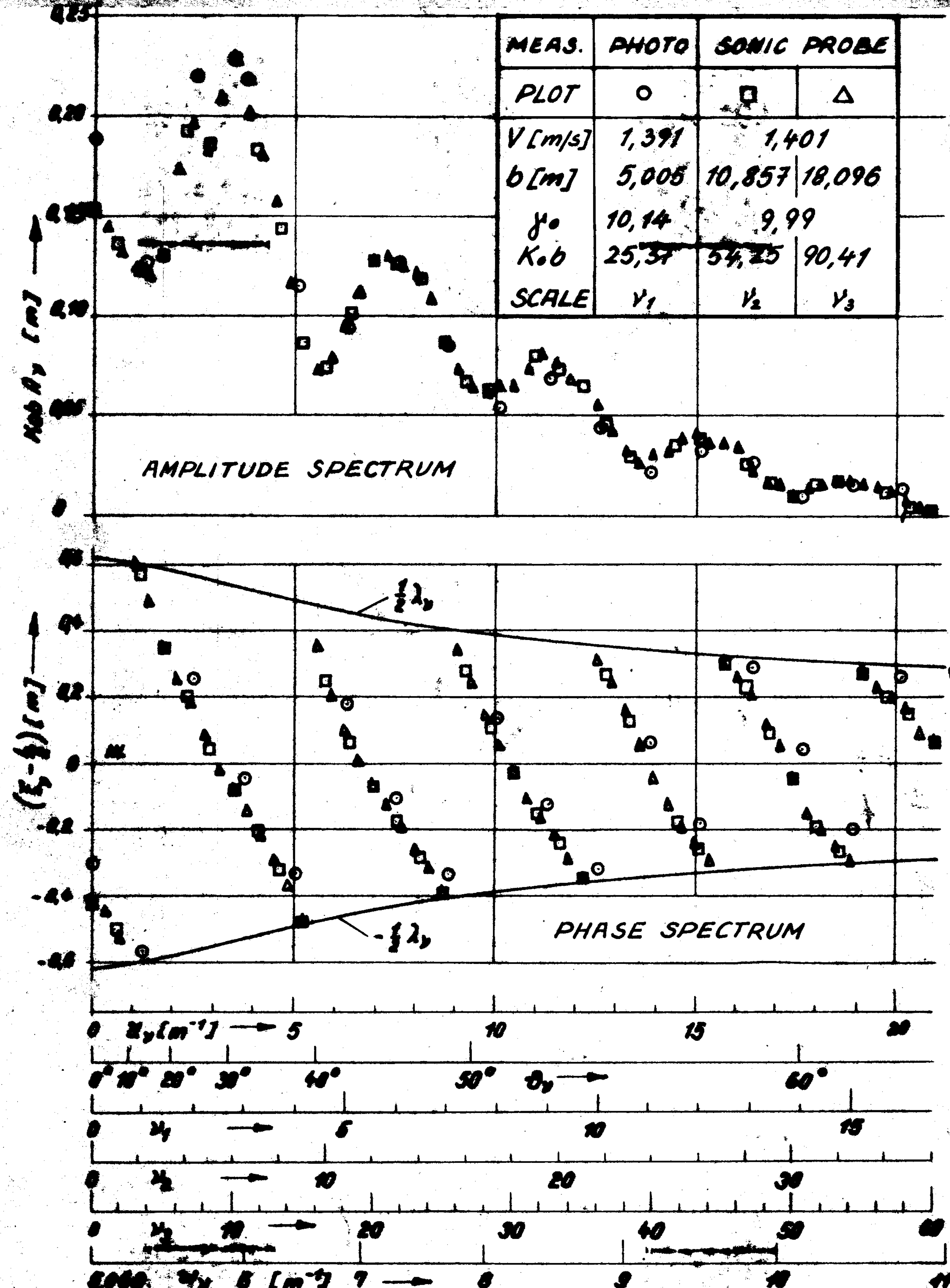


FIG. 5 PLOT OF MEASURED FREE-WAVE SPECTRUM [redacted] REPRODUCIBILITY AND INVARIANCE

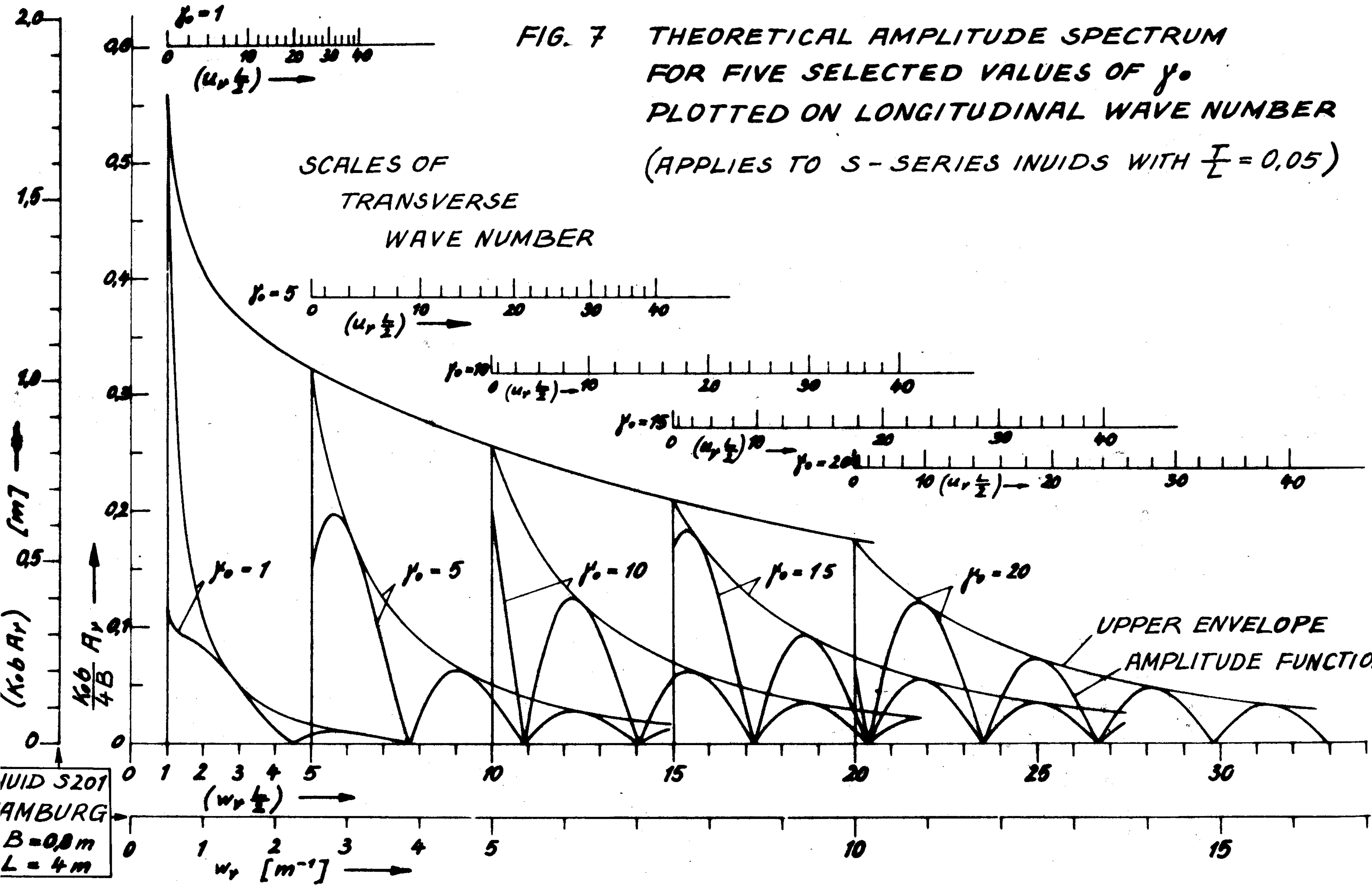


FIG. 7 THEORETICAL AMPLITUDE SPECTRUM FOR FIVE SELECTED VALUES OF γ_0 PLOTTED ON LONGITUDINAL WAVE NUMBER (APPLIES TO S-SERIES INUIDS WITH $\frac{T}{L} = 0,05$)

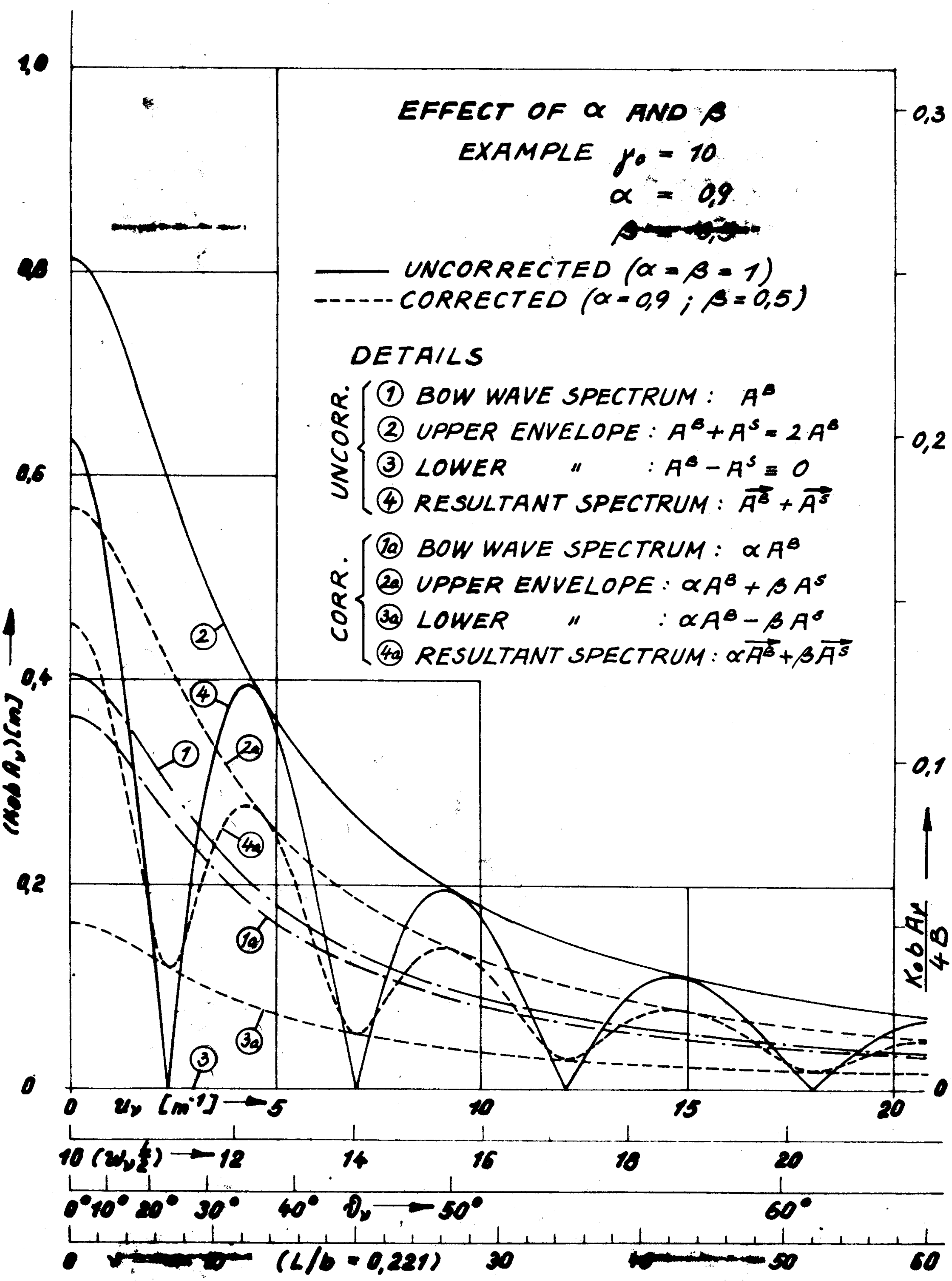


FIG. 8 CORRECTIONS TO AMPLITUDE SPECTRUM

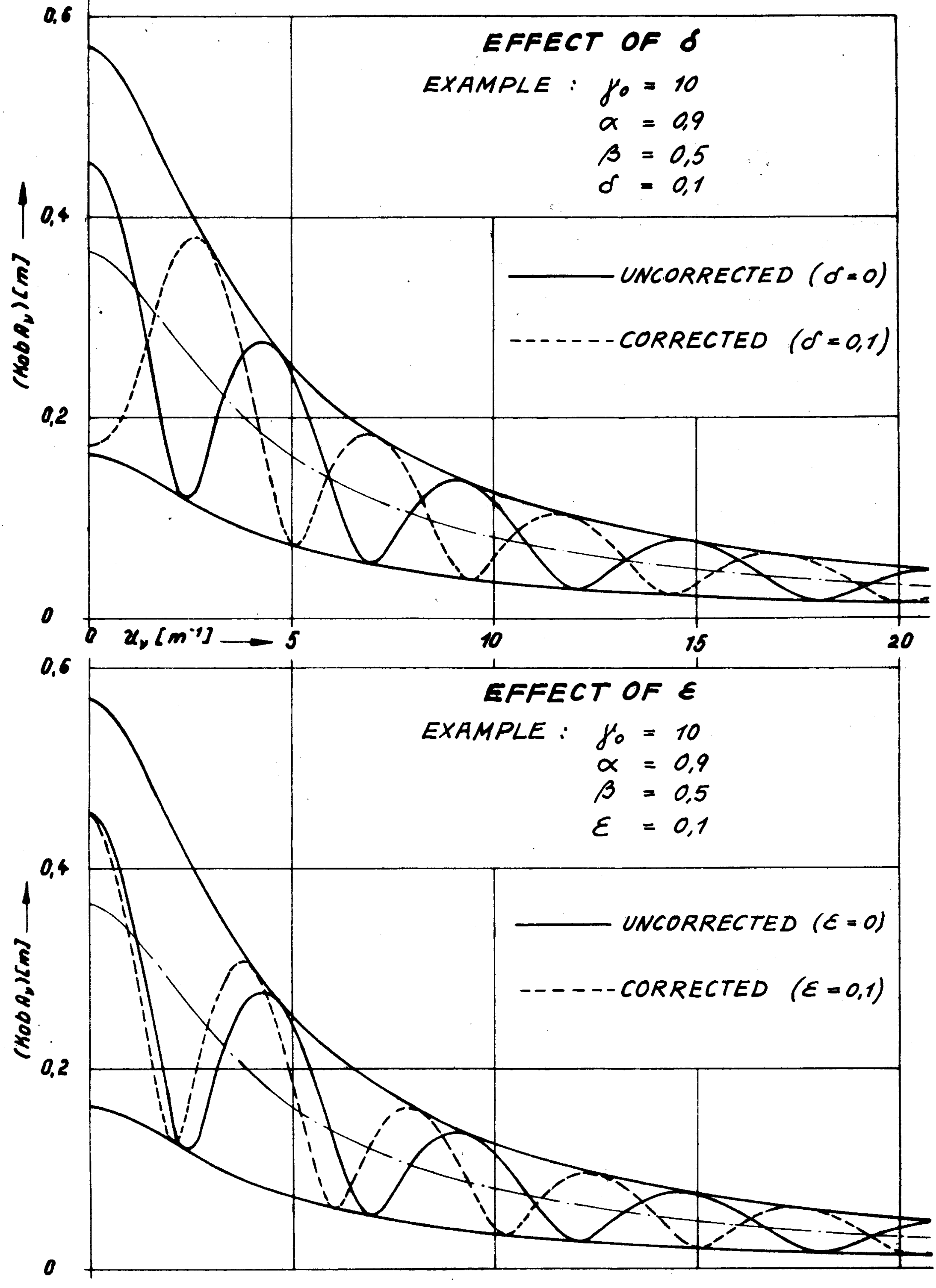


FIG. 9 CORRECTIONS TO AMPLITUDE SPECTRUM

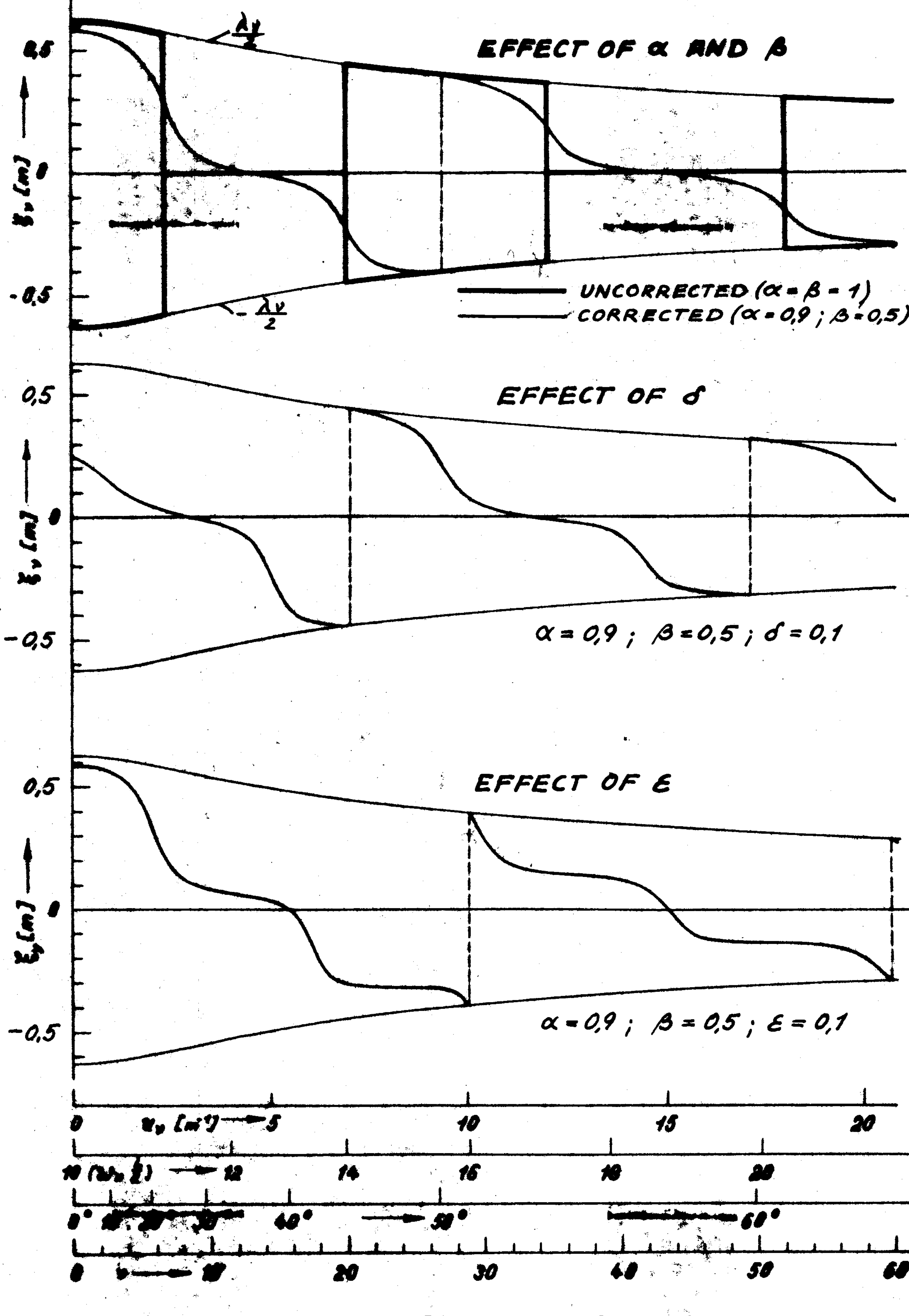


FIG. 10 CORRECTIONS TO PHASE SPECTRUM EXAMPLE $\gamma_0 = 10$

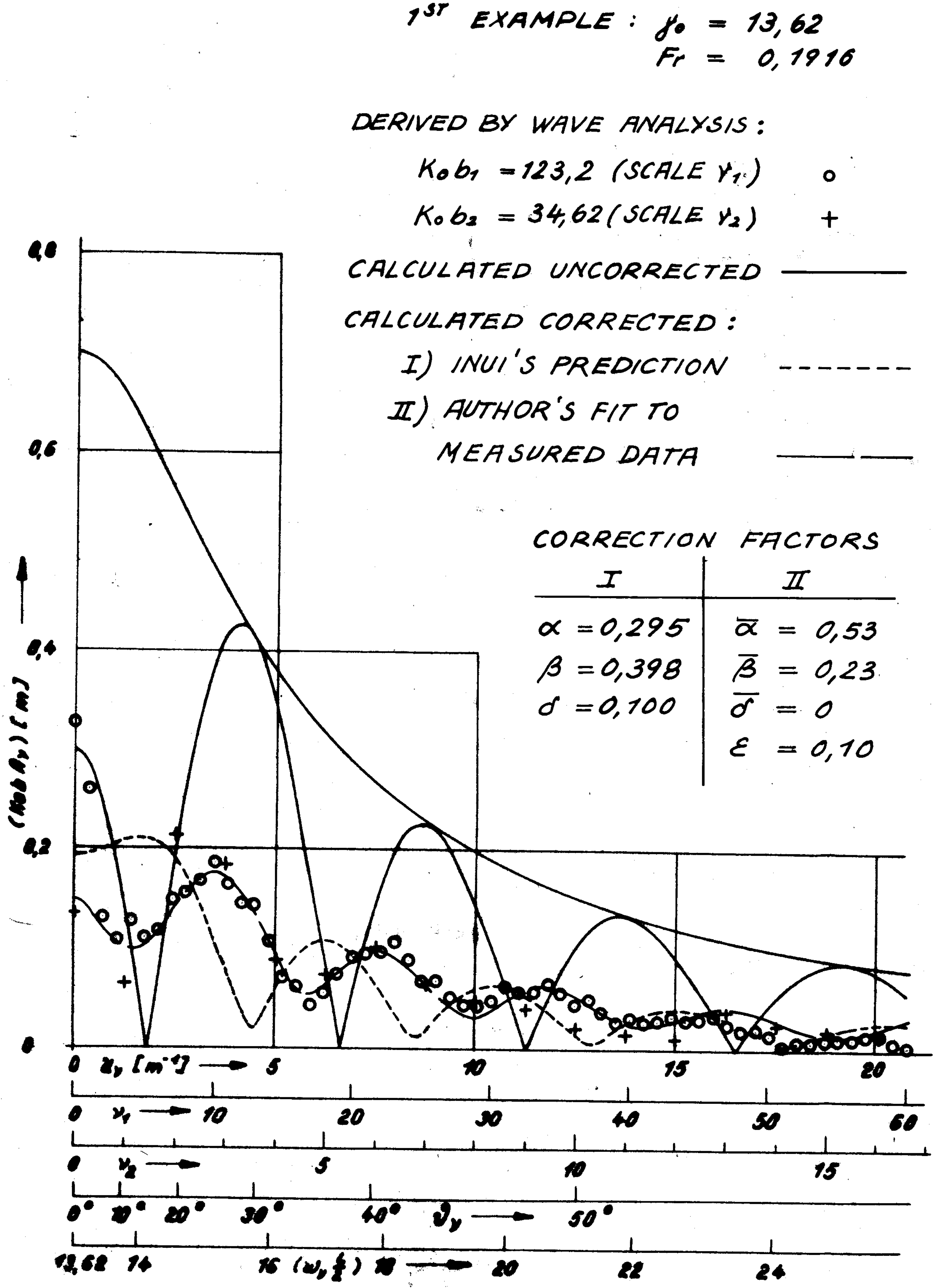


FIG. 11 COMPARISON OF AMPLITUDE SPECTRUM

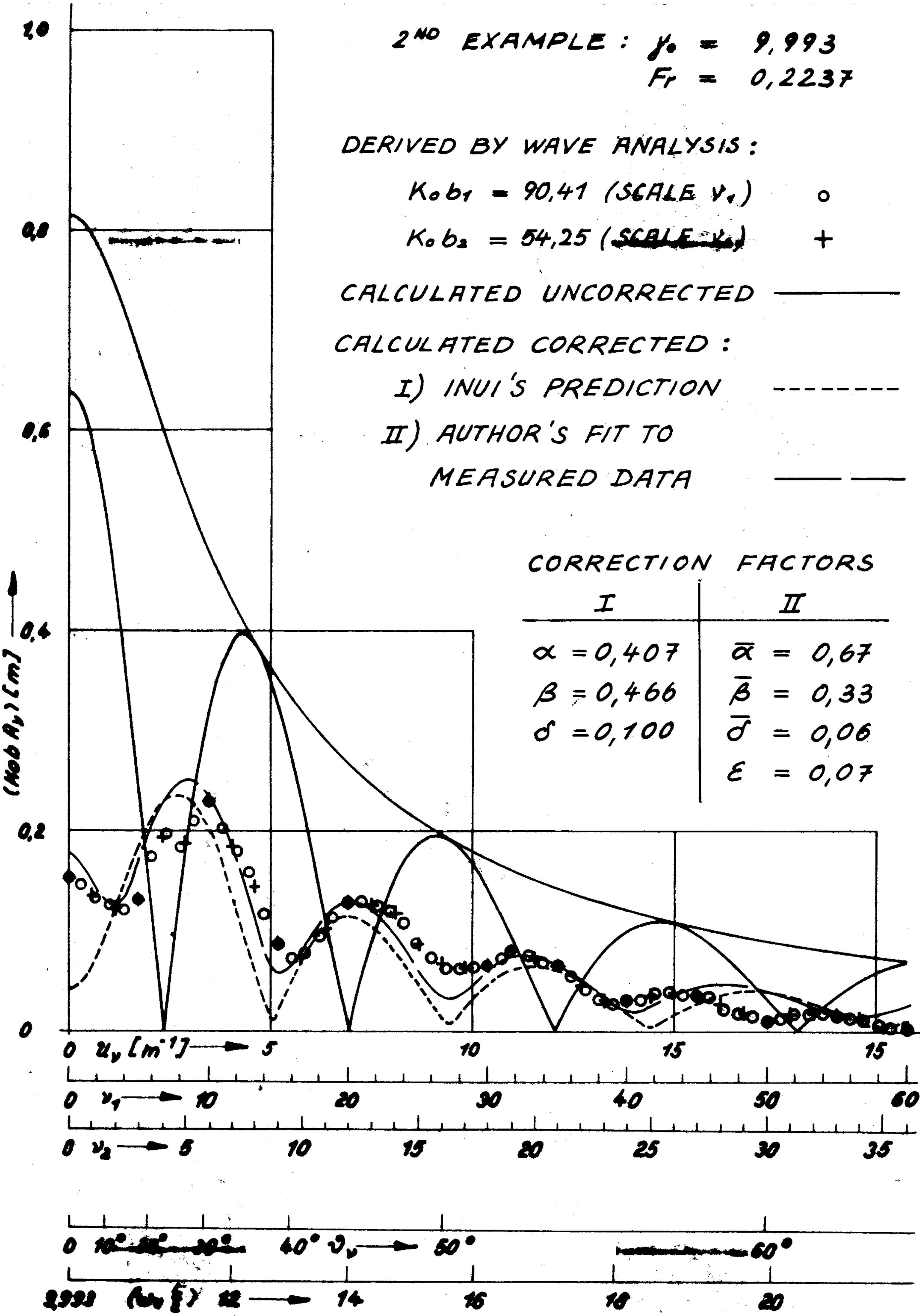


FIG. 12 COMPARISON OF AMPLITUDE SPECTRUM

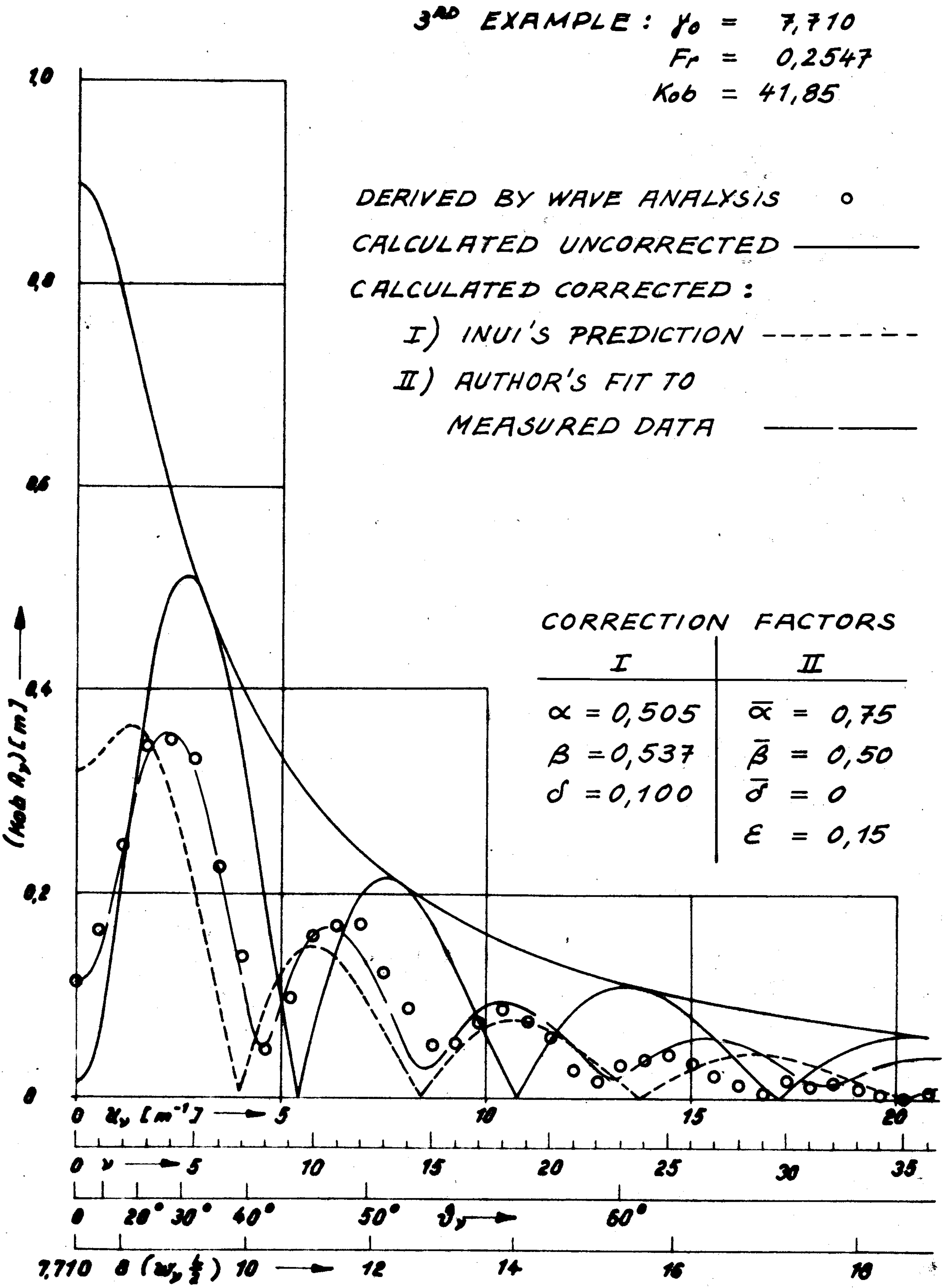


FIG. 13 COMPARISON OF AMPLITUDE SPECTRUM

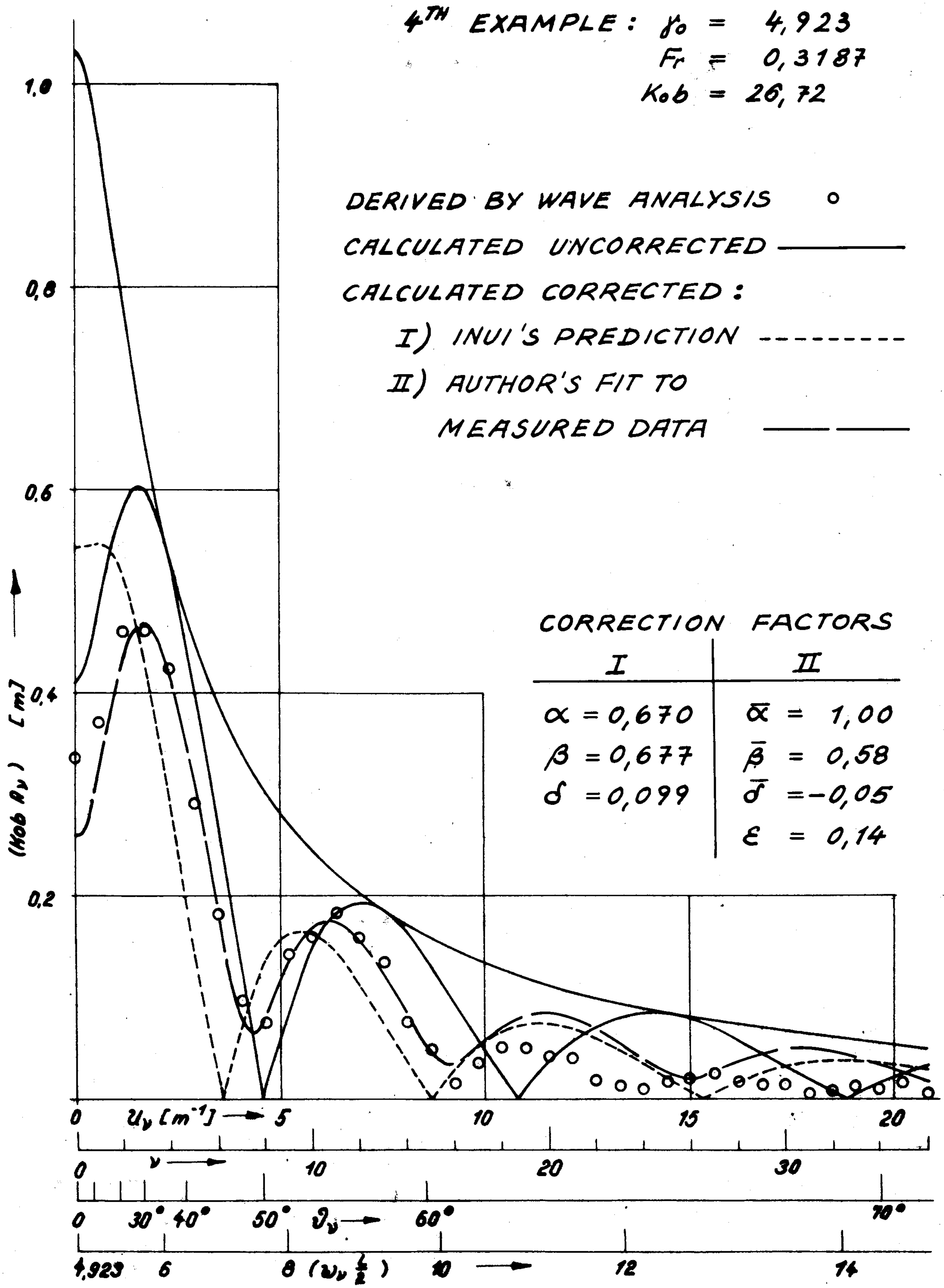


FIG. 14 COMPARISON OF AMPLITUDE SPECTRUM