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SHIP WAVE RAY TRACING INCLUDING SURFACE TENSION.

von

Klaus Eggers
Huang De-bo

Februar 1990

Abstract

The aim of this work is to clarify the validity of ship wave ray theories at and near the ship's surface. As previous numerical investigations have led to ambiguities due to a breakdown of the ray analysis near the bow and stern stagnation points, we shall take care for the surface tension effect in order to milden such deficiencies; then the wave length never surpasses a positive minimum length which is attained at the boundary of a finite waveless zone around a stagnation point. It is found, however, that the ray equations degenerate at these boundaries, and that rays can be traced into the far field only if their starting point is selected outside a finite belt surrounding the waveless zone.

For a class of bi-circular prismatic struts of infinite downward extent, we investigated two alternative formulations of the free surface condition and their implications for the ray geometry. For low speeds we found in both cases an increase of the Kelvin wave cusp angle due to capillarity. We extended the ray tracing to capillary waves ahead of a blunt bow.

Introduction

The wave field at a point far away from a ship in stationary motion is well represented through Kelvin's pattern, found in a wedge-shaped region, with only a finite number of wave components, given through wave length, wave front angle and complex amplitude. The first two are constant along straight lines (characteristics) through a hypothetical origin, conceived as the locus of a point disturbance.– Observations suggest that under local modifications such a wave model may be adequate even *near* a ship; Ursell [1] hence generalised this approach for waves due to a point disturbance in a slightly non-uniform flow. From a set of physical assumptions, he replaced the intensity and direction of the uniform flow by the local components to obtain an analoguous spatially varying "dispersion relation" between wave angle and wave number; from a partial differential equation he obtained "rays" along the resultant of the local flow with a group velocity vector. To simplify the problem, Ursell considered only rays passing *through* the disturbance, though he admitted that his assumptions are questionable [1] there. Inui and Kajitani [2] used this approach for waves near a ship's bow, with the " double body flow " as the basic non-uniform flow.

Keller [3] derived Ursell's results from a more formal approach, tacitly assuming pertinence and uniform validity of his ray theory up to the ship's water line; he even concluded for certain ships that rays *must* originate from the double body flow stagnation points *only.* Yim[4] evaluated this approach numerically, but due to zero wave length at these points he had to start ray tracing using values at some distance. For certain rays carrying transverse waves he observed that they re-entered the hull; to avoid this, he introduced some mechanism of reflection.

Brandsma [5] investigated a class of bi-circular forms with varying entrance angle. Even with "back-shooting" from downstream, he found that no rays associated with *transverse* waves (as referred to the basic flow) emanating from the bow stagnation point can be found; (this is not necessarily in conflict with the calculations of Yim).

In the sequel, we shall demonstrate analytically that the rate of change of the wave front angle along a ray (and hence of the ray tangent via the dispersion relation) tends to infinity as the inverse distance from the stagnation point, unless the ray is starting tangentially to the water line. Thus at most *one* single ray can originate there, with the wave front normal pararallel to that ray. This explains part of Brandsma's dilemma, and one may question the validity of Keller's ray theory near the stagnation points.

Through our present investigation we want to clarify whether the inclusion of surface tension effects can improve the situation at least to the point that ray theory can give some qualitative information about the wave pattern geometry in accord with experimental observations for not so slender ships. We selected the class of bi-circular struts and thus have even the case of a blunt bow included.

[1] Otherwise rays could be extended to the domain far ahead through backward tracing, at least in case of a submerged disturbance.

The underlying analysis was presented by Eggers [6], where two alternative appoaches were considered: $(A)'$ based on the conventional surface condition (A) of slow ship theory, supplemented for surface tension following Maruo[7] and $(A^+)'$ based on a modified free surface condition, derived by Eggers[8][2] from certain invariance requirements for wave resistance, again supplemented for capillarity effects.

In both cases we obtain zones around the stagnation points where no steady waves can exist; at their boundaries, only waves of minimum wave speed, with wave front orthogonal to the double body flow, can occur. If we start rays from these boundaries rather than from the stagnation points, we apparently have a well defined initial value problem, even for blunt bow forms.-

In our computational investigations, we could confirm Maruo's experimental finding that capillarity effects can be significant even if the model speed exceeds the minimum wave speed considerably.

However, we found ourselves confronted with some instability phenomenon. Due to the strong rate of change of the wave angle along the ray near its origin, the wave length re-approached its minimum value after a short time and the analysis broke down. To find rays which can be continued into the far field, we had to select the starting point outside some "belt of short-livity" surrounding the waveless zone.

Derivation Of Dispersion Relation And Ray Equations From Free Surface Conditions.

For simplicity, we shall restrict ourselves to a 2-D flow around prismatic struts of infinite vertical extension. Let us consider a velocity potential of the form $U\phi_r + U\varphi$, where U stands for the far field uniform flow in the +x direction, $U\phi_r$ represents the "double body flow" (unbounded in the upward z-direction) and $U\varphi$ is the lowest order wavy potential. With $u \doteq U\phi_{rx}$, $v \doteq U\phi_{ry}$, with $\zeta_r \doteq (U^2 - u^2 - v^2)/2g$ and $D_r(x,y) \doteq (u\zeta_r)_x + (v\zeta_r)_y$, φ for $z = 0$ has to satisfy

$$u^2\varphi_{xx} + 2uv\varphi_{xy} + v^2\varphi_{yy} + g(\varphi_z + \zeta_r\varphi_{zz}) = 2g(\zeta_{rx}\varphi_x + \zeta_{ry}\varphi_y) + D_r(x,y) \tag{1}$$

(see Eggers [10]); for inclusion of surface tension, a term $\kappa\varphi_{zzz}$ has to be added on the l.h.s. (see Maruo [7]).

Seeking for wave-type solutions, we concentrate on the homogenous part of above d.e.; we further disregard the (amplitude modulating)[3] terms with φ_x and φ_y. Hence we consider the "modified approach" $(A^+)'$

$$u^2\varphi_{xx} + 2uv\varphi_{xy} + v^2\varphi_{yy} - g\zeta_r(\varphi_{xx} + \varphi_{yy}) = -\kappa\varphi_{zzz} - g\varphi_z \tag{2}$$

The neglect of $g\zeta_r\varphi_{zz}$ (the second term of a formally divergent Taylor expansion) leads to the approach $(A)'$ investigated by Maruo, which was developed from the "conventional" approach (A) underlying the ray analyses of Keller, Yim and Brandsma. Let us now consider u and v as slowly-varying (i.e.locally constant) quantities and let us disregard effects of phase and of amplitude, as they are of no concern for investigations on ray geometry. A potential of the form

$$\Phi \propto e^{(kz-iS)} \qquad \text{with} \qquad S = S(x,y) \tag{3}$$

represents a wave with wave number vector

$$\vec{k} \doteq \nabla S \doteq \{k_1, k_2\} \doteq \{k\cos\theta; k\sin\theta\} \tag{4}$$

if k_1 and k_2 are also slowly varying. Here θ is the angle of $\vec{k}$ against the x-direction. Let us define the speed ratio q and the flow angle β through $u \doteq Uq\cos\beta$ and $v \doteq Uq\sin\beta$. Then $\gamma \doteq \theta - \beta$ is the angle of $\vec{k}$ against the flow direction. In accord with Brandsma and Yim, we have selected the orientation of $\vec{k}$ such that $\cos\gamma$ is non-negative i.e. $\vec{k}$ is *opposite* to the propagation of a wave stationary to the ship.-

[2] Quite recently,this approach has been justified under new arguments by van Gemert[9].

[3] See Longuet-Higgins and Stewart [11], and [8].

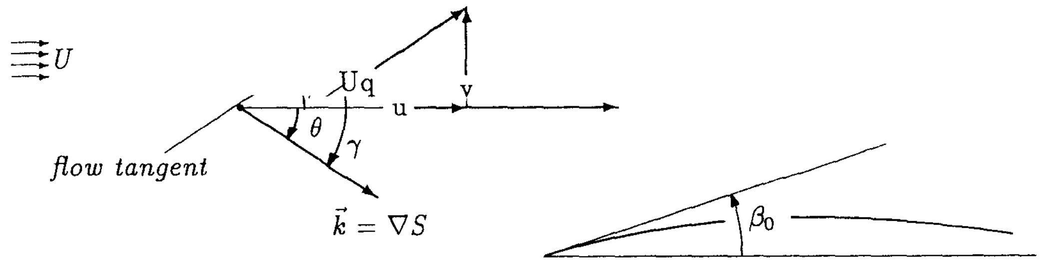

Fig.1 Sketch for flow angle β and wave angles θ and γ (both shown with negative values, typical for the starboard side of the bow).

Inserting (3) into (2) we obtain

$$k^2c^2 \doteq (ku\cos\theta + kv\sin\theta)^2 = gk + g\zeta_r k^2 + \kappa k^3 \tag{5}$$

where $c \doteq Uq\cos\gamma$ stands for the component of the basic flow parallel to $\vec{k}$, thus a wave must propagate opposite to $\vec{k}$ with the *local* phase velocity c in order to be stationary in a system moving against the flow. Note that in general c thus defined will *not* equal $U\cos\theta$, the phase velocity of this wave in an inertial system where the ship is advancing stationary with speed U.– Equation (5) leads to the "dispersion relation"[4]

$$c = \sqrt{g/k + g\zeta_r + \kappa k} \tag{6}$$

relating the *local* phase velocity with the wavelength for γ given. It may equivalently be expressed as

$$F(x,y,k_1,k_2) \doteq 1 - (k_1^2 + k_2^2)c^2/(uk_1 + vk_2)^2 = 0 \tag{7}$$

Noting that $k_1 = \frac{\partial S}{\partial x}, k_2 = \frac{\partial S}{\partial y}$, we may consider this as a partial d.e. for the function S(x,y) for which "characteristic stripes" (i.e. characteristic curves in the $\{x, y, k_1, k_2\}$-space) can be found from the equations

$$\frac{dx}{d\tau} = \frac{\partial F}{\partial k_1};\ \frac{dy}{d\tau} = \frac{\partial F}{\partial k_2};\ \frac{dk_1}{d\tau} = -F_x;\ \frac{dk_2}{d\tau} = -F_y \tag{8}$$

which define a curve parameter τ. Under multiple use of above relations, considering that

$$\frac{\partial(uk_1 + vk_2)}{\partial k_1} = u\ ;\ \frac{\partial(uk_1 + vk_2)}{\partial k_2} = v \tag{9}$$

$$\frac{\partial(ck)}{\partial k_1} = c_g\cos\theta\ ;\ \frac{\partial(ck)}{\partial k_2} = c_g\sin\theta \tag{10}$$

(where $c_g \doteq d(kc)/dk$ is in accord with the concept of group velocity related to $Uq\cos\gamma$ as phase velocity), we find

$$\begin{aligned} c_g &= \frac{d}{dk}(kc) = \frac{d}{dk}\sqrt{kg + k^2 g\zeta_r + k^3\kappa} \\ &= \frac{1}{2kc}(g + 2g\zeta_r + 3k^2\kappa) = \frac{c}{2}(1 + \frac{g\zeta_r + 2k\kappa}{c^2}) \end{aligned} \tag{11}$$

We obtain

$$\begin{aligned} \frac{dx}{d\tau} &= \frac{\partial F}{\partial k_1} = 2\cdot\frac{kc}{uk_1 + vk_2}\cdot\frac{\partial}{\partial k_1}\frac{kc}{uk_1 + vk_2} \\ &= \frac{2}{ck}\cdot\frac{\partial}{\partial k_1}(kc - uk_1) = \frac{2}{kc}(u - c_g\cos\theta) \end{aligned} \tag{12}$$

[4]Note that the term $g\zeta_r$ appears only under $(A^+)'$, not under $(A)'$; it may remind us of $c = \sqrt{gh}$ for waves on shallow water of depth h.

In a similar way

$$\frac{dy}{d\tau} = \frac{2}{kc}(v - c_g \sin\theta) \tag{13}$$

From (6) we find

$$\frac{\partial(kc)}{\partial x} = \frac{k}{2c}\frac{\partial}{\partial x}g\zeta_r \tag{14}$$

and thus

$$\begin{aligned} F_x &= -\frac{dk_1}{d\tau} = -2\frac{kc}{uk_1 + vk_2} \cdot \frac{\partial}{\partial x}\frac{kc}{uk_1 + vk_2} && (15)\\ &= -2\frac{\partial}{\partial x}(kc - uk_1 - vk_2) = -\frac{2}{kc} \cdot \frac{\partial}{\partial x}(\frac{k}{c}g\zeta_r - uk_1 - vk_2) \\ &= \frac{2}{c^2}((uu_x + vv_x)/2 + c(u_x \cos\theta + v_x \sin\theta)) && (16)\\ F_y &= \frac{2}{c^2}((uu_y + vv_y)/2 + c(u_y \cos\theta + v_y \sin\theta)) && (17) \end{aligned}$$

Then we find the rate of change of the wave angle θ from

$$\begin{aligned} \frac{kc^2}{2}\frac{d\theta}{d\tau} &= \frac{kc^2}{2}\frac{d}{d\tau}\arctan\frac{k_2}{k_1} \\ &= \frac{c^2}{2k}(k_1\frac{dk_2}{d\tau} - k_2\frac{dk_1}{d\tau}) = -c^2(F_y\cos\theta - F_x\sin\theta)/2 \\ &= ((g\zeta_r)_y\cos\theta - (g\zeta_r)_x\sin\theta)/2 + cu_x\sin 2\theta - cv_x\cos 2\theta && (18) \end{aligned}$$

where we have used $u_x = -v_y$, $u_y = v_x$ for our 2-D basic flow. In a similar way we obtain[5]

$$\begin{aligned} \frac{c^2}{2}\frac{dk}{d\tau} &= \frac{c^2}{2k}(k_1\frac{dk_1}{d\tau} + k_2\frac{dk_2}{d\tau}) \\ &= ((g\zeta_r)_x\cos\theta + (g\zeta_r)_y\sin\theta)/2 - cu_x\cos 2\theta - cv_x\sin 2\theta && (19) \end{aligned}$$

From equs.(12) and (13) we can easily confirm the general result

$$\tan(\beta + \alpha) \doteq \frac{dy}{dx} = \frac{v - c_g\sin\theta}{u - c_g\cos\theta} \tag{20}$$

(with α defined as the ray angle against the double body flow) which contains the choice of approach only through the explicit expression for c_g. We may thus recall Ursell's observation that the ray direction is along the resultant of the basic flow and the "group velocity" taken along the wave normal vector $-\vec{k}$, and that this does not require asymptotic analysis (see discussion to[8])[6]

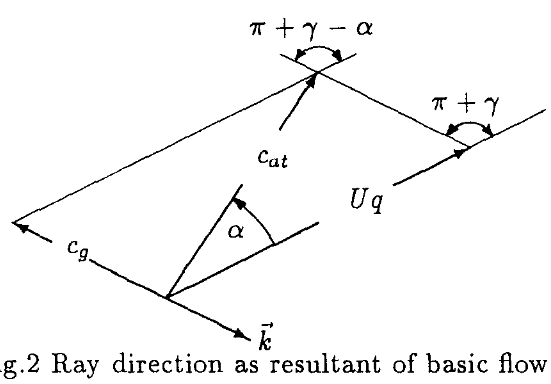

Fig.2 Ray direction as resultant of basic flow and c_g along direction of $-\vec{k}$ (with angle α against flow direction).

[5]The terms with $g\zeta_r$ (missing under approach $(A)'$) reflect the statement of Longuet-Higgins and Stewart [11] that short waves superposed a long wave shorten when climbing, increasing their length again when descending.

[6]For later use, we have introduced in an "action transport velocity" c_{at} along the tangent to the ray direction.–

Restrictions For The Wave Parameters.

From the dispersion relation (5) we have

$$c^2 = g/k + g\zeta_r + \kappa k \tag{21}$$

A minimum of c is found at $k = \sqrt{g/\kappa}$ giving

$$c = c_m\sqrt{1 + (1-q^2)/p^2} = U\sqrt{(1 - q^2 + 2p^2)/2} \tag{22}$$

where $c_m \doteq \sqrt[4]{4\kappa g}$ is the minimum velocity of capillary-gravity waves and $p \doteq c_m/U$ is a dimensionless parameter of surface tension. We introduce a dimensionless wave length $\Lambda \doteq g/(kU^2)$; then (5) is equivalent to

$$q^2\cos^2\gamma = \Lambda + (1-q^2)/2 + p^4/(4\Lambda) \tag{23}$$

In a plane of the variables q^2 and Λ , for p constant, (22) represents a family of hyperbolas between the asymptotes $\Lambda = 0$ and $\Lambda = \Lambda_\gamma \doteq (1 + 2\cos^2\gamma)q^2 - 1/2$.

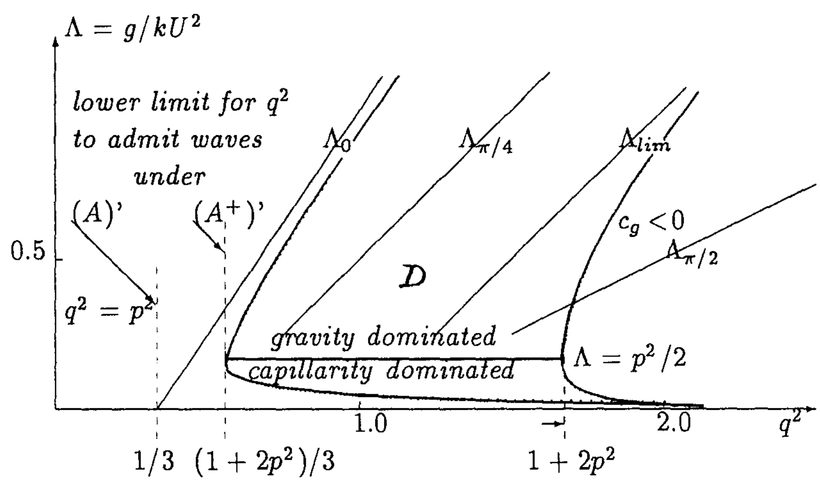

Fig.3 Domain D of admitted wave constellations in the plane of squared speed ratio q^2 and wave-length ratio Λ for $p^2 = 1/3$ corresponding to the case of $U = 0.23/p \approx 0.50$ m/s. (The boundary to the right is given through physical limits of q^2.)

We are interested in the branch with $\Lambda > 0$; (otherwise, we would have an *increase* of the wave flow downwards, see(3)). Solving for Λ, we obtain

$$\Lambda = \frac{(1+2\cos^2\gamma)q^2 - 1}{2} \cdot \frac{1 \pm Sq}{2} \tag{24}$$

$$Sq \doteq \sqrt{1 - (2p^2/(2\cos^2\gamma + 1)q^2 - 1))^2} \tag{25}$$

Sq is real only for $q^2 \geq (1 + 2p^2)/(2\cos^2\gamma + 1) \geq (1 + 2p^2)/3$; this implies that in the zones around the stagnation points where $q^2 \leq (1 + 2p^2)/3$ *no* steady waves can exist. The upper sign of the root corresponds to gravity dominated waves ($\Lambda = \Lambda_g$); the capillarity dominated waves ($\Lambda = \Lambda_c$, lower sign) are better described by[7]

$$\Lambda = \frac{p^4}{4} \cdot \frac{2}{(1+2\cos^2\gamma)q^2 - 1} \cdot \frac{2}{1 \mp SQ} \tag{26}$$

[7]The relevant analysis has been established and profoundly discussed by Crapper [13] for the non-modified approach $(A)'$

For any γ, confluence of the two roots occurs for $\Lambda = p\ /2$ corresponding to the minimum of c found earlier (21). For the range $(1 + 2p^2)/3 \leq q^2 \leq 1 + 2p^2$ the angle γ is restricted through

$$|\gamma| \leq \frac{1}{2} \arccos\left(\frac{1+2p^2}{q^2} - 2\right) \tag{27}$$

(under approach $(A)'$, $|\gamma| \leq \arccos p$, independent from q^2). Beyond this range, for $q^2 > 1 + 2p^2$, the minimum of Λ_g is no longer $p^2/2$ but the value corresponding to $\gamma = \frac{\pi}{2}$. Then we have

$$\Lambda_g \geq \frac{q^2-1}{2} \cdot \frac{1 + \sqrt{1 - (2p^2/(q^2-1))^2}}{2} \approx \frac{q^2}{2} - \frac{1}{2} \tag{28}$$

If we accept the argument that stationary waves cannot propagate into areas where c_g/c is negative, the domain of admitted Λ values is further restricted (see(11)) through

$$\frac{2cc_g}{U^2} = \Lambda + 1 - q^2 + \frac{3p^4}{4\Lambda} \geq 0 \tag{29}$$

For gravity waves with $q^2 > 1 + \sqrt{3}p^2$ this implies

$$\Lambda > \Lambda_{lim} \doteq (q^2 - 1)\left(1 + \sqrt{1 - 3p^4/(1-q^2)^2}\right)/2 \tag{30}$$

One may observe that this limitation is automatically met if $|\gamma| \leq \pi/4$ with $d\Lambda/dq^2 \geq 1/2 + \cos^2\gamma \geq 1$ then (see Fig.3).– One may note that for any γ we find from (22), (25) and (5)

$$(c/U)^2 = \Lambda_g + \Lambda_c + (1 - q^2)/2 \tag{31}$$

All the above restrictions can be visualized through a display of the dependence between the wave front angle γ and the ray direction angle α with p and q held constant. From a geometrical interpretation of (19), invoking the sine theorem of elementary trigonometry (see Fig2.), we find

$$\frac{\sin\alpha}{c_g} = -\frac{\sin(\gamma - \alpha)}{Uq} = -\frac{\sin\gamma}{c_{at}} \tag{32}$$

and hence

$$\tan\alpha = \frac{\sin\gamma}{\cos\gamma - Uq/c_g} = \frac{\sin 2\gamma}{1 + \cos 2\gamma - 2c/c_g} \tag{33}$$

Setting $p = q = 0$, we have $c_g/c = 1/2$ in accord with Kelvin's results; we find that $|\alpha|$ will increase with $|\gamma|$ up to some maximum $\alpha_k = \arctan(1/\sqrt{8})$ and then fall off to zero with $\gamma = \pi/2$.– We may observe that for non-zero p, unless $q^2 \geq 1$, α approaches zero only together with γ, as $\cos\gamma$ will remain positive. Thus an outgoing ray ($\alpha > 0$) can turn inward again only if the wave front normal changes from inward ($\gamma < 0$) to outward at the "caustic" (in the terminology of Yim [17]) under a maximum of the wave length due to $\gamma = 0$.

In order to have α positive, we now consider the range of negative γ. If we exclude here those parts of curves where α turns negative due to $c_g \leq 0$ (only for $q^2 > 1$ under $(A^+)'$), we may observe that α as an odd function will in general have opposite sign to γ.

Let us refer to the range for which $|\alpha|$ is increasing from zero with $|\gamma|$ as to that of *transverse waves* and define the maximum value attained for α in this range as the *modified Kelvin angle* α_k. Then we find that for $q^2 \leq 1 + 2p^2$ each curve for gravity dominated waves turns under horizontal tangent (with $\alpha = \pi/2 + \gamma$) into one for capillarity dominated waves, so that after $\tan\alpha$ changed sign due to increase of $c_g/c > 1$ for such waves, γ falls off to zero again with the ray finally normal to the wave crest again, but opposite now to the flow direction.– For lower values of q^2, the maximum of α coincides with the transition to capillary waves, where the wave length attains it minimum. Here α_k can only be considered as a safe upper bound for the ray direction.

Note that our formulations and considerations throughout this paper are refered to the domain around the starboard side of the bow, where α is positive , hence γ negative in general. To deal with the other ship side, statements remain valid if β and θ, γ and α are counted *clockwise* against the x-axis and flow direction there.

The ratio c_g/c and hence $|\alpha|$ increases with decreasing speed U (i.e. increasing p) and with decreasing distance from the stagnation point (i.e. decreasing q) for γ held constant. This implies an increase of the modified Kelvin angle (which is measured against the flow direction!) especially *near the bow*, in particular under approach $(A^+)'$! This is well in accord with the experimental observations of Miyata[14] with wedge- shaped models with $U = 0.5$m/s, $(p \approx 0.462)$ and $U = 1$m/s, $(p \approx 0.231)$.

Under the approach $(A)'$, we can find from (11) the deviation of c/c_g from $1/2$ depends on the ratio p^* of c_m to $c(= Uq\cos\gamma)$,i.e. to the local phase velocity. With $g\zeta_r$ disregarded, we find from (11)

$$c/c_g = 1/2(1 + 2\kappa k/c^2) = 1/2(1 + p^*/(1 + SQ))$$

with $SQ \doteq \sqrt{1 - p^*}$.

Away from the bow, where $q^2 \approx 1$, thus $\zeta_r \approx 0$, there is little difference between $(A^+)'$ and $(A)'$. But again we observe an increase of α_k with decreasing U (i.e. with increasing influence of capillarity) in accord with Miyata's experiments with a rudder model (see Inui [15]) for the speeds $U = 1.15m/s, 1.72m/s$ and $U = 2.3m/s$ corresponding to p = 0.3, 0.14 and 0.1. We should acknowledge here that such widening of the Kelvin angle has already been predicted by Lord Kelvin in a footnote [16]

> ..In the case of even the highest speed attained by a duckling, this angle is perhaps perceptibly greater than $19^0 28'$ because of the dynamic effect of capillary surface tension on water...
> see letter to Wm. Froude, reprinted in "Nature" 1871.

In the domain where $q^2 > 1$ aside of the ship, a *reduction* of α_k is predicted under $(A^+)'$ including a termination of rays with short gravity waves with c_g approaching negative values. We may mention that for a vertical circular cylinder, q^2 increases up to 4.0, whereas for conventional forms q^2 will not exceed 1.2.–

It is only for not too small q and for not too large p that α is really stationary for $\alpha = \alpha_k$ and thus marks a transition from transverse to divergent waves, so that with $d\alpha/d\gamma = 0$ we may expect a wave cusp effect. If we search for the most forward point along the water line of a ship where stationarity occurs (i.e. immediately near the stagnation point for pointed bows) and trace a curve from there with $dy/dx = \tan(\beta + \alpha_k)$; this should define some outer boundary to the gravity wave pattern which coincides with Kelvin's cusp line in the far field. Such a line may be related to a shock front observed by Miyata (see discussion to Eggers [8]).

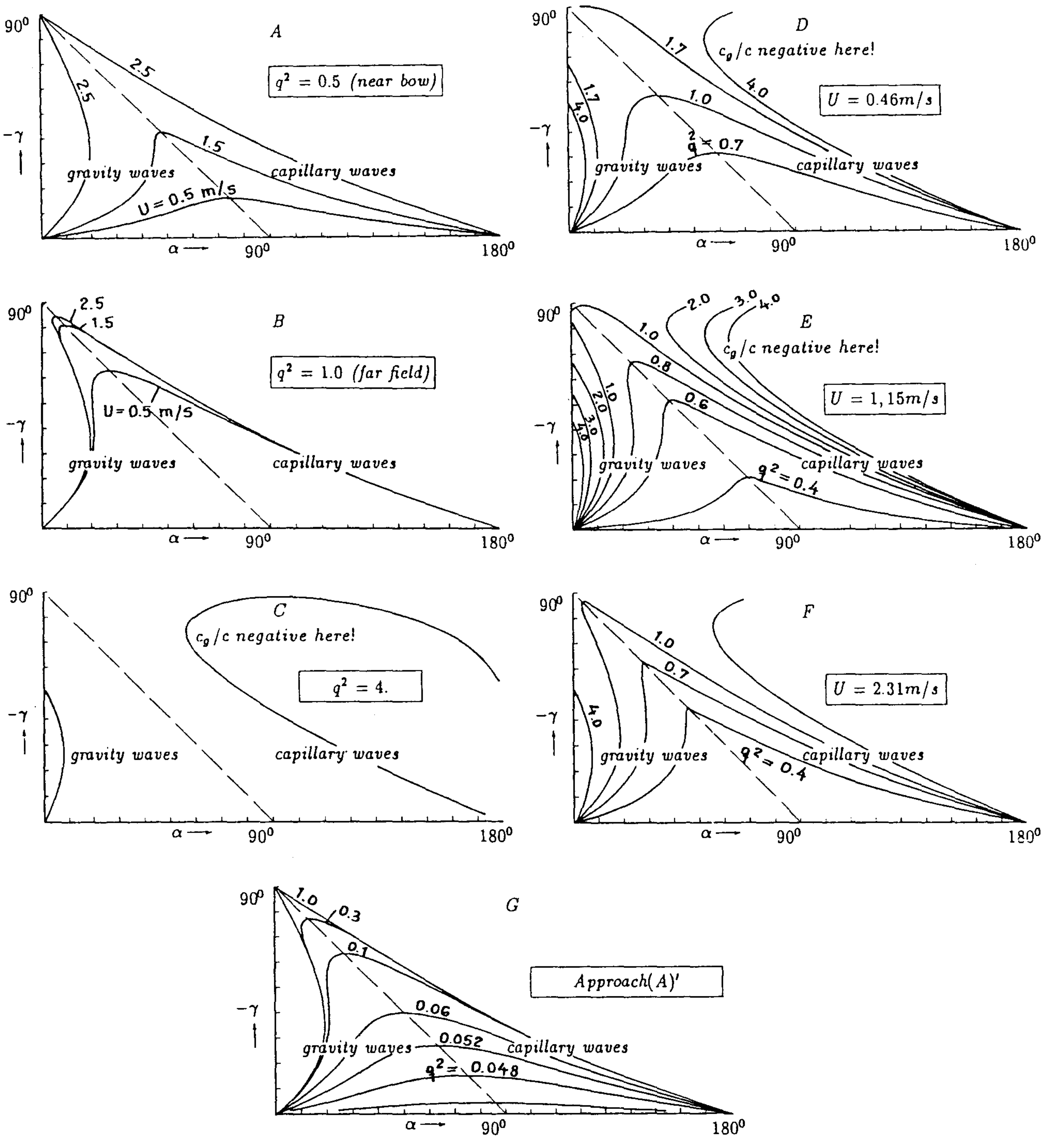

Fig.4Wave front angle γ versus ray angle α

A $q^2 = 0.5, U = 0.5, 1.5, 2.5m/s$ (near bow)

B $q^2 = 1.0, U = 0.5, 1.5, 2, 5m/s$ These curves hold as well for approach$(A)'$!

C $q^2 = 4., U = 0.5, 1.5, 2.5m/s$ (aside of blunt body).

For this extreme value, the range of γ for gravity rays is strongly restricted, if $c_g/c > 0$ is assumed.The influence of capillarity is not felt here.

D $U = 0,46$ (only sligthly larger than minimum wave speed c_m=0.23m/cs) for $q^2 = 0.7, 1.0, 1.7, 4.0$

E $U = 1.15m/s$ for $q^2 = 0.4, 0.6, 0.8, 1., 2., 3., 4.$

F $U = 2.3m/s$ for $q^2 = 0.4, 0.7, 1., 4.$

G Approach$(A)'$ for $U = 2.5m/s$ and $q^2 = 0.048, 0.052, 0.06, 0.10, 0.30, 1.$

(The relevant curve parameter here is Uq/c_m)

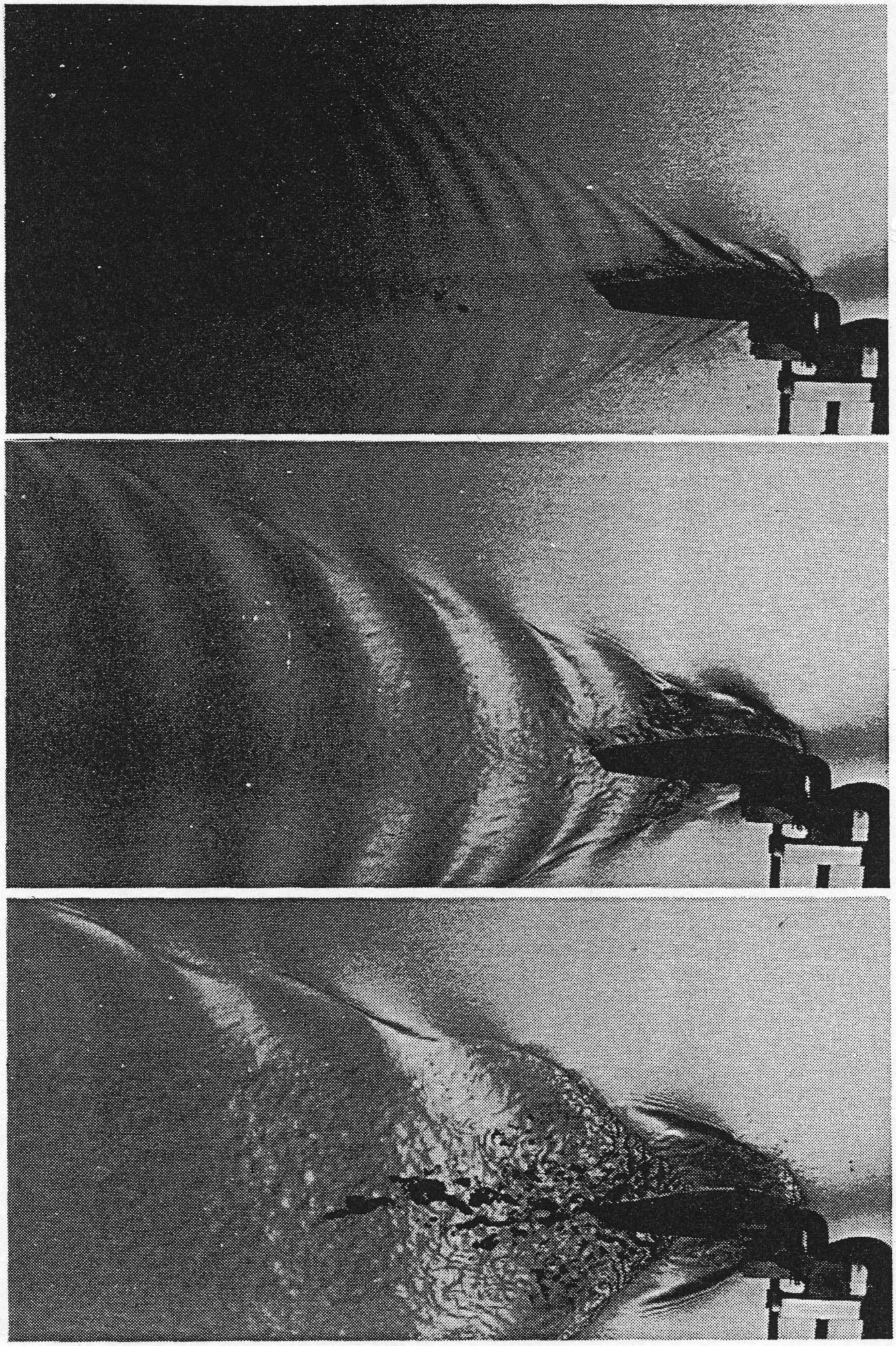

Fig.5. Wave pattern of a rudder model of length 0.3 m with speed U = 0.65, 0.5,, and 0.34 m/s. (By courtesy of Prof. T. Inui from [15]).

The Situation of Ray Tracing Near A Corner And The Short Life Of Rays Near The Waveless Zone.

Let us again consider 2-D potential flow as the basic flow, so that complex analysis can be used. We introduce

$$Z \doteq x + iy = r \cdot e^{i\delta} \quad V \doteq u - iv = Uq \cdot e^{-i\beta}, \tag{34}$$

$$K \doteq k_1 - ik_2 = k \cdot e^{-i\theta} \tag{35}$$

$$P \doteq (g\zeta_r)_x - i(g\zeta_r)_y = -(dV/dZ)V^\star \tag{36}$$

Then P stands for some gradient of the double body flow pressure.[8] The ray equations (12) and (13) may be written as

$$\frac{dZ^\star}{d\tau} = \frac{2}{kc}(V - \frac{c_g}{k}K) = \frac{2}{kc}c_{at} \cdot e^{-i(\alpha+\beta)} \tag{37}$$

With ds as differential of the arclength along the ray, this implies that $d\tau/ds = 1/\,|\,dZ/d\tau| = kc/2c_{at}$, so that we can write (17) and (18) as

$$\frac{1}{K}\frac{dK}{ds} = \left(\frac{1}{k}\frac{dk}{d\tau} - i\frac{d\theta}{d\tau}\right) \cdot \frac{d\tau}{ds} = -\frac{e^{i\theta}}{c_{at}} \cdot \frac{dV}{dZ}\left(\diamond\frac{V^\star}{2c} + e^{i\theta}\right) \tag{38}$$

Here $\diamond$ means 1 for (A^+) and $(A^+)'$, it means 0 for (A) and $(A)'$.– The flow in the vicinity of a stagnation point due to a corner is basically the flow near a corner between infinite planes as decribed by Milne-Thomson [12], we have

$$V \approx Q \cdot e^{(i\pi\beta_0/(\pi-\beta_0)} \,.\, Z^{\beta_0/(\pi-\beta_0)} \tag{39}$$

where Q is a real constant; this means that the range

$$\beta_0 \le \delta \le \pi$$

for the polar angle δ is mapped on the range

$$\beta_0 \ge \beta(\delta) = \beta_0(\pi - \delta)/(\pi - \beta_0) \ge 0$$

for the flow angle. In the special case of a bi-circular strut of opening angle $2\beta_0$ and length L, under parallel flow of strength U, we have

$$Q = U\frac{\pi}{\pi - \beta_0} \cdot L^{(\pi-\beta_0)/\beta_0} \tag{40}$$

so that $q = q(r) = \pi/(\pi - \beta_0) \cdot (r/L)^{(\beta_0/(\pi-\beta_0)}$ and hence

$$\frac{dV}{dZ} = \frac{\beta_0}{\pi - \beta_0} \cdot \frac{V}{Z} \tag{41}$$

$$P = -\frac{\beta_0}{\pi - \beta_0} \cdot \frac{1}{Z}V \cdot V^\star \tag{42}$$

Then the rate of change of θ, β, and k along the ray is found from

$$\frac{1}{K}\frac{dK}{ds} = -\frac{e^{i\theta}}{c_{at}} \cdot \frac{\beta_0}{\pi - \beta_o} \cdot \frac{V}{Z}\left(\diamond\frac{V^\star}{2c} + e^{i\theta}\right) \tag{43}$$

[8] Tulin [17] considered a quantity related to $|\,P\,|$ as a disturbance parameter and came to the vexing conclusion that ray theory does not apply for bow entrance angles $\beta_0 \le \pi/3$ as otherwise P is not bounded. On the other hand, Maruo [7] disclaimed the validity of ray theory for $\beta_0 \ge \pi/3$ due to divergence of an integral representing the phase.

Observing (34) and (5), separating real and imaginary part in (43), we find

$$\frac{d\theta}{ds} = -\frac{1}{r}\frac{c}{c_{at}} \cdot \frac{\beta_0}{\pi - \beta_0}\left(\diamond\frac{\sin(\theta-\delta)}{2\cos\gamma} + \sin(\gamma + \theta - \delta)\right) \tag{44}$$

$$\frac{d\beta}{ds} = -Im\frac{dV}{dZ}\frac{1}{V}\frac{dZ}{ds} = -Im\frac{\dot{\beta}_0}{\pi-\beta_0}\frac{1}{Z}\cdot e^{i(\alpha+\beta)} = -\frac{1}{r}\cdot\frac{\beta_0}{\pi-\beta_0}\cdot\sin(\alpha+\beta-\delta) \tag{45}$$

and thus

$$\frac{d\gamma}{ds} = \frac{d(\theta-\beta)}{ds}$$

$$= \frac{1}{r}\cdot\frac{\beta_0}{\pi-\beta_0}\left(\sin(\alpha+\beta-\delta) - \frac{c}{c_{at}}\left(\diamond\frac{\sin(\gamma+\beta-\delta)}{2\cos\gamma} + \sin(2\gamma+\beta-\delta)\right)\right) \tag{46}$$

In the viciniy of a stagnation point, for rays emanating from there, we have $\delta = \alpha+\beta$, hence $d\beta/ds = 0$, thus $d\alpha/ds = 1/r \cdot \beta_0/(\pi - \beta_0) \cdot 2\sin(2\gamma - \alpha) \cdot d\alpha/d\gamma$ with $\diamond = 0$ under (A). For $d\alpha/d\gamma$ non zero, this can tend to a finite limit without invalidation of (33) only if $\gamma = \alpha = 0$. Hence, unless showing infinite curvature, all rays must emanate tangentially to the hull from the stagnation point, with wave front normal in ray direction. The finite limit should dependend on δ i.e. on the direction of approach, with zero curvature if approached tangentially to the hull.– This explains the numerical difficulties as experienced by Brandsma.

A ray can not coincide with a streamline (or with the waterline in particuar) if there is curvature.—newline We would have to require $\alpha \equiv 0$, i.e.$\gamma \equiv 0$ hence $d\alpha/ds$ i.e. $d\gamma/ds \equiv 0$; with $c_{at} = c - c_g$ then, this implies

$$\frac{d\gamma}{ds} = Im\frac{dV}{dZ}\left(\frac{1}{c} - \frac{\diamond/2+1}{c-c_g}\right)e^{2i\beta} \tag{47}$$

valid even away from stagnation points; however, as $V = u - iv = Uqe^{-i\beta}$, this means that a change of the flow occurs in flow direction only, thus a ray can coincide with a streamline only if the rate of change of V is in the flow direction, i.e. that the streamline has no curvature!

For the rate of change of the wave number k we find

$$\frac{1}{k}\frac{dk}{ds} = -\frac{1}{r}\frac{c}{c_{at}}\frac{\beta_0}{\pi-\beta_0}\left(\diamond\frac{\cos(\gamma+\beta-\delta)}{2\cos\gamma} + \cos(2\gamma+\beta-\delta)\right) \tag{48}$$

If the value of Λ along a ray should equal the critical value $p^2/2$, this would correspond to the minimum for gravity waves; hence k then *must* decrease along the ray. However, due to the rapid increase of $|\gamma|$ near a stagnation point, the sum of cosine terms may change sign, so that k increases (in particular for $(A)'$ where the first cosine term is deleted) and Λ approaches $p^2/2$ again. Here the ray *must* terminate, as for $\Lambda = p^2/2$, even off the waveless zone boundary, the partial derivatives of Λ both regarding q^2 and γ vanish simultaneously in conflict with the ray equations, q^2 can not be varied independent from γ. This explains the previously mentioned occurence of short life rays. Hence the choice of initial points for rays is moot, quite apart from the ambiguity of assigning initial values there for amplitude and phase.

Further Considerations About The Ray Approach

A "ray" in the sense of our analysis is defined as a characteristic to a partial differential equation $F(x, y, S_x, S_y) = 0$ for a function $S(x, y)$, see(3). We know that for "hyperbolic" differential equations such lines display the influence of initial conditions in the sense of some flux of action from cause to result; and in certain cases we may even attribute some flux of energy to these lines. It is plausible to assume that such rays should emanate from a domain close to the ship as the creator of the wave pattern. But we should ascertain that the essential features of the complex 3-D flow near the ship, including sensitive variations, can really be modeled adequately through functions $S(x, y)$ with slowly varying gradient and associated complex amplitude functions $A(x, y)$ – Note that until Yim's [18] recent investigations, no effects of the Froude number on ray curvature could be modeled, and the variation of wave resistance only resulted from interference effects in the far field computed through integration along the rays.

We should keep in mind that a single ray can neither be measured nor even detected by whatever experimental set up, hence a validation of the ray approach by comparison of computational and experimental data is hardly feasable.–

Our investigation showed that certain *global* characteristics of the wave pattern, such as the variation of α_k, and hence of the tangential direction of the wave domain boundary (visible in the rich stock of Miyata's experimental results) can be predicted even near the bow with approach $(A^+)'$.

An evaluation of merits for the competitive approaches $(A)'$ and $(A^+)'$ may be attempted. But considering the fragile foundation of slow ship theory, it does not seem pertinent to discriminate between a "correct" and a "less consistent" approach, although it is obvious that with inclusion of surface tension effects a dogmatic rule of "automatic order change through differentiation" – essential for $(A)'$– can not be maintained.–

Actually, the omission of terms with $g\zeta_r$ (and hence with $1 - q^2$) under$(A)'$ has no fundamental consequences for our analysis in general. Certainly, the extent of zones without steady waves ahead of a blunt bow is considerably larger under (A^+), well in accord with data from experiments with a vertical circular cylinder, for which we have evaluated both approaches. However, the numerous recent investigations on the flow ahead of a blunt bow (see the survey by Mori [19]) make clear that between the bow and the stationary capillary wave zone we have to expect a finite domain with either a stationary plateau, a turbulent free surface or instationary waves propagating forward (Osawa [20]), and the flow visualisation experiments of Kayo et alii [21] display a system of instationary "necklace vorticies" in this domain. And the decay of capillary waves through viscosity, as investigated by Messick and Wu [22] should be taken account of.–

Numerical Calculations.

Our calculations have been performed both for approach $(A^+)'$ and approach $(A)'$. We considered the class of bi-circular cylinders which had already been investigated by Brandsma with conventional ray theory. The analytical expression for the velocity potential W is given in complex notation [12] in terms of bicircular coordinates ξ and η through

$$Z \doteq x + iy = L/2 \cdot \cot(\zeta/2) \tag{49}$$

$$W \doteq \varphi + i\psi = UL \cdot i/n \cot(\zeta/n) \tag{50}$$

with $\zeta = \xi + i\eta$, $\xi = \delta_1 - \delta_2$, $\eta = ln(r_1/r_2)$, $n = \frac{2(\pi-\beta_0)}{\pi}$, where β_0 stands for half the entrance angle and L for the length of the strut. The symbols $r_1, \delta_1, r_2, \delta_2$ stand for polar coordinates regarding the strut end points. For economical reasons, we deduced explicit expressions in real mode for u,v and their derivatives. We evaluated the ray equations by the Runge-Kutta method.

The aforementioned boundary of the short-life zone was determined numerically, assuming $q^2 = const$ there. It did not require much accuracy, we found that rays emanating outside such a border line were not sensitive to the choice of their origin. We selected $\gamma = 0$ as initial value, securing a maximum for the wave length and its rate of increase, hence minimum probability that it may become stationary and decrease again along the ray. Thus it is obviously adequate rather to operate with a continuous distribution of disturbances than with a concentration in the stagnation points.

Due to inclusion of surface tension, steady components of bow capillary waves could be investigated as well. We traced rays with capillary waves from that part of the outer boundary of the short-life belt where the basic flow is incoming, and rays with gravity waves from the part with outgoing flow; the two domains have no common boundary, save the point where the basic flow is tangent to the belt.–

For different speeds U and for different entrance angle β_0, rays for gravity- and for capillarity waves are shown in the following figures.

To show the difference between ours and the conventional ray theory and to test some part of our program, forward (downstream) and backward- (upstream from the far-field)tracing on the conventional ray theory were also performed. For the latter, considering the problems near the stagnation point, we changed the far fild initial conditions from those obtained by forward tracing until we could reach a pre-defined neighborhood of the stagnation point under obvious convergence of the wave front angle.

In acord with Yim[4], we have attached to each ray in our diagrams values of θ_i and θ_f, the initial and the final asymptotic values of the wave front angle θ, given in degrees rather than radians). The short segments on the rays show the local wave fronts.

The step width for the Runge-Kutta method was carefully chosen, comparing with results under half this width.

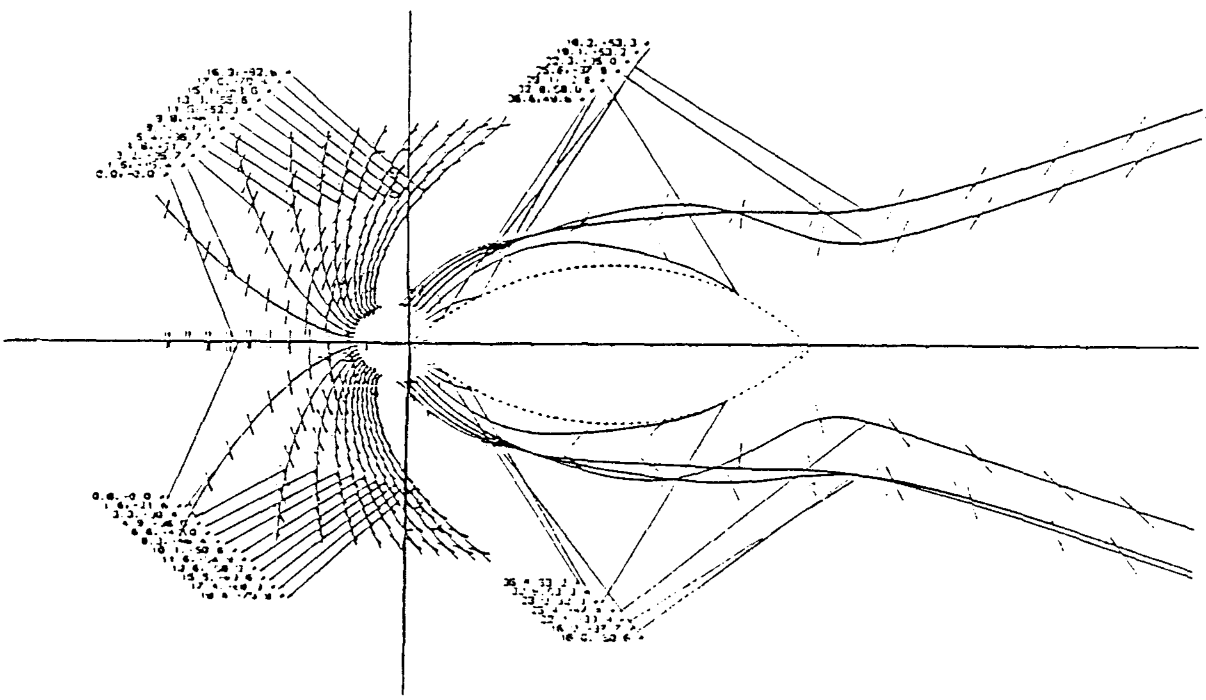

Fig.6 Rays of gravity and rays of capillarity for a strut of entrance angle $\beta_0 = 45^0$ under approach $(A^+)'$ upper half for $U = 0.57m/s$, lower half for $U = 2.31m/s$.

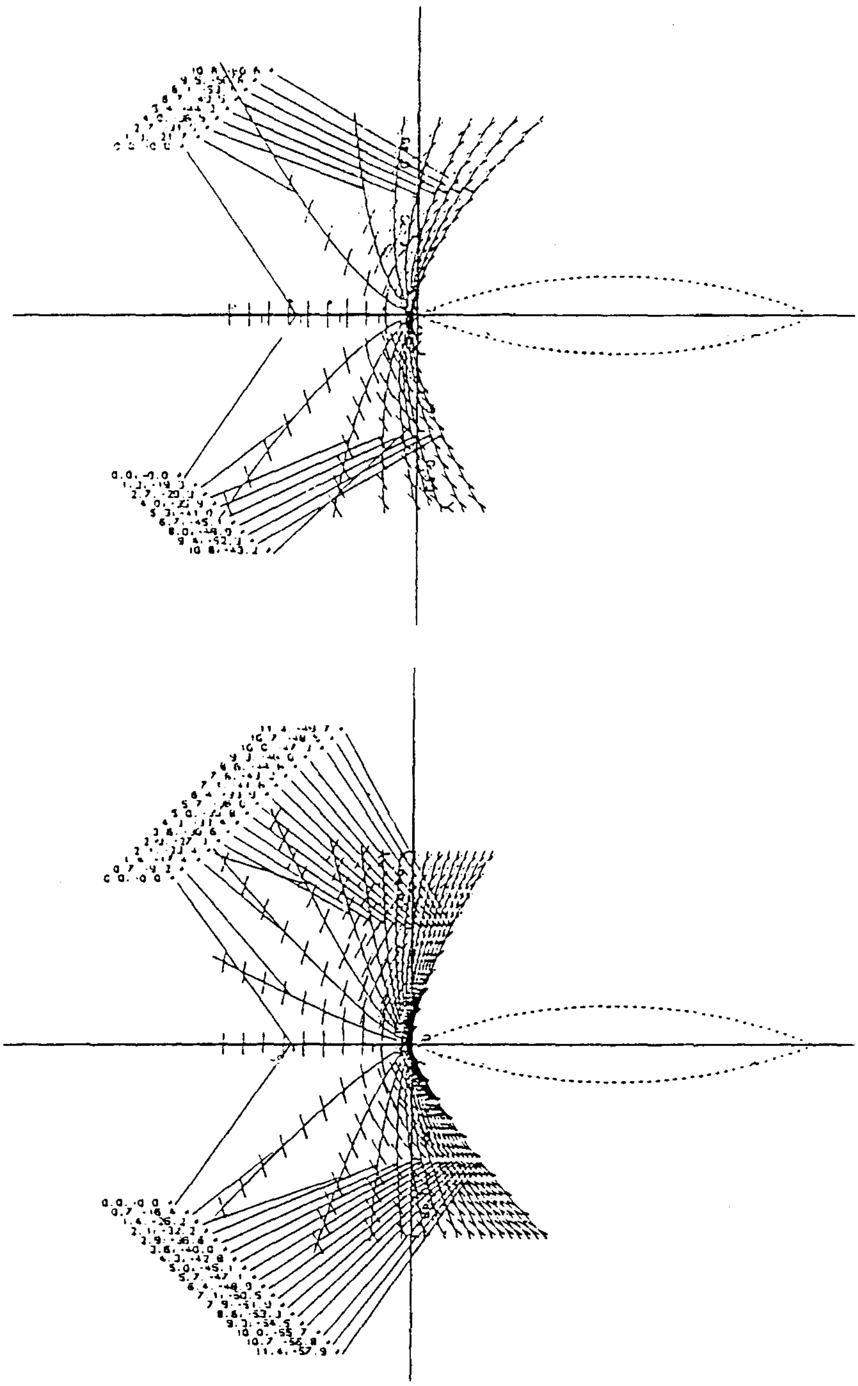

Fig.7 Rays of capillary waves for a slender strut ($\beta_0 = 22.5^0$) above for $(A^+)'$, below for $(A)'$. Upper halves are for speed $U = 0.62m/s (p = 0.37)$, lower halves are for $U = 0.92m/s (p = 0.25)$.

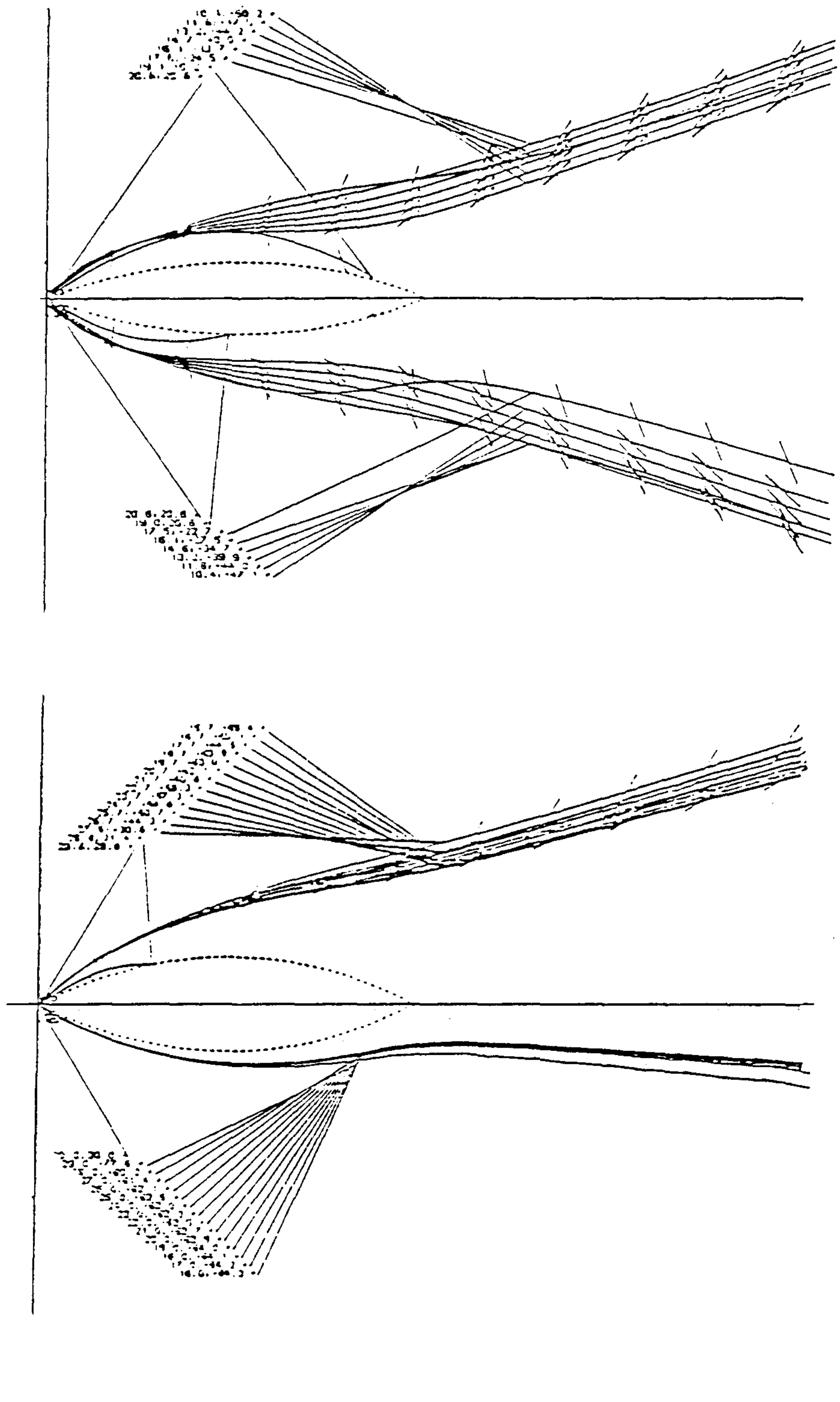

Fig.8 Rays of gravity waves for a slender strut ($\beta_0 = 22.5^0$) above for $(A^+)'$, below for $(A)'$. Upper halves are for speed $U = 0.57m/s(p = 0.4)$, lower halves are for $U = 2.31m/s(p = 0.1)$.

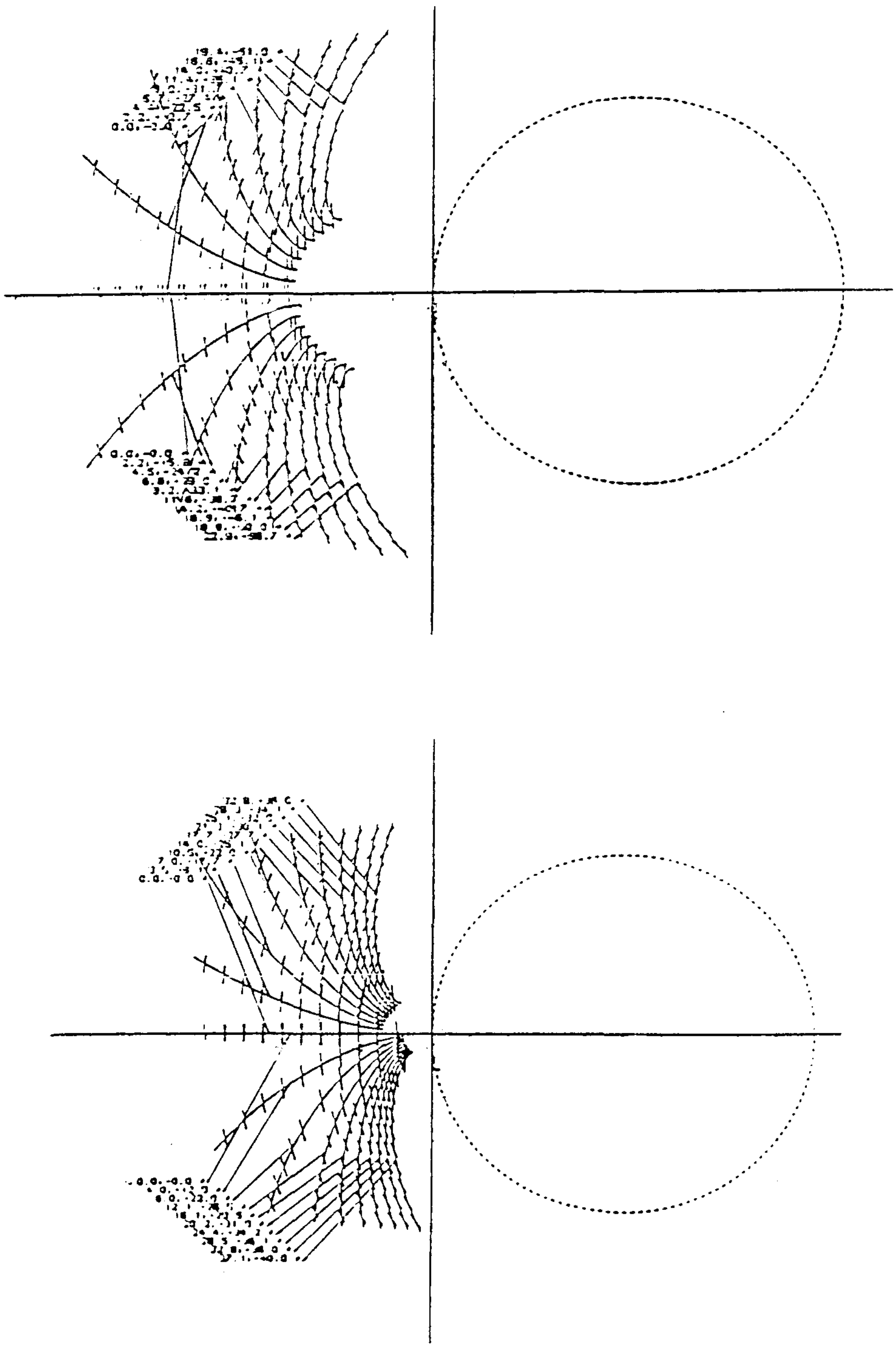

Fig.9 Rays of capillary waves for a circular cylinder ($\beta_0 = 90^0$) above for $(A^+)'$, below for $(A)'$. Upper halves are for speed $U = 0.62m/s(p = 0.37)$, lower halves are for $U = 0.92m/s(p = 0.25)$.

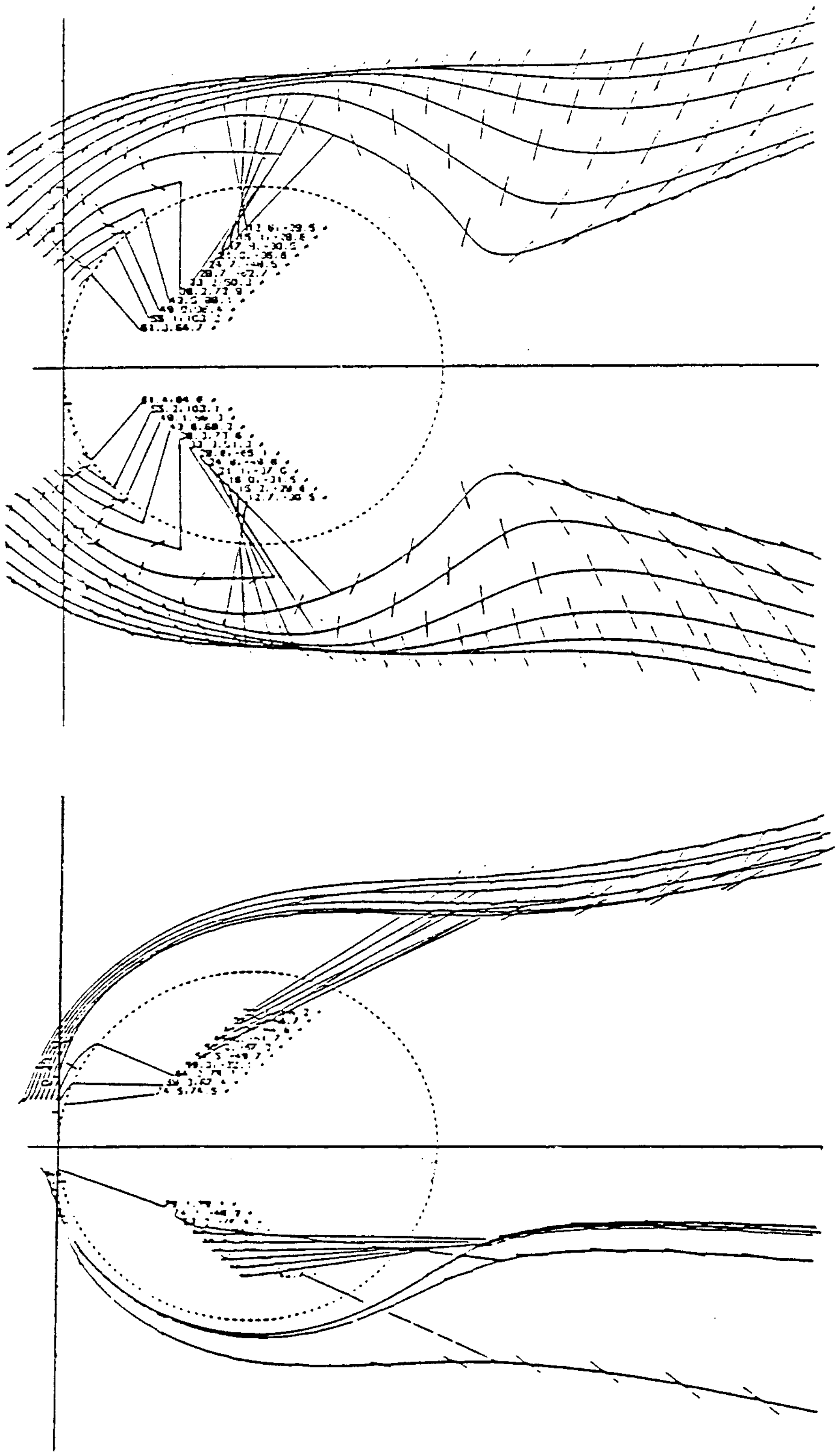

Fig.10 Rays of gravity waves for a circular cylinder ($\beta_0 = 90^0$) above for $(A^+)'$, below for $(A)'$. Upper halves are for speed $U = 0.57m/s(p = 0.4)$, lower halves are for $U = 2.31m/s(p = 0.1)$.

Conclusions

In the course of an assessement of the ray approach with regard to representing essential features of the wave pattern, we have incorporated capillarity effects in our analysis to overcome obvious shortcomings near the bow stagnation point, where otherwise the rate of change of the wave angle would tend to infinity for almost all ray directions with the inverse distance.

Our formulation $(A^+)'$ (see (2)) generalizes approach $(A)'$, where certain terms related to the double-body flow pressure are disregarded. Although within our work we could neither provide a rational model for ray generation nor even a justification for using the ray approach in the hull surface vicinity, the following facts have been discovered or confirmed.

Our numerical investigations have displayed several global effects on the wave pattern geometry resulting from the inclusion of capillartity to our analytical model; they gain practical relevance for small speeds, say for U less than 2 m/s (if we consider a minimum capillary wave speed of 0.23 m/s.) :

(1) Both the far-field Kelvin angle and the "modified Kelvin angle" near the bow (i.e. the angle between tangents to the wave region boundary and to the hull water line) are found to increase with decreasing U.

(2) With increasing bow entrance angle, both the zones of no steady waves and the surrounding short-life belt, from which no rays proceeding into the far field can be found, grow in size.

(3) The outward extent of this belt is decreasing with increasing U, i.e. the stronger the capillarity, the larger the short life belt.

The above findings are in qualitative accord with some tendencies one may observe from experimental visualizations of flow and wave pattern as presented by Inui[15] (his Fig.2-2 is reproduced in our Fig.5), of Miyata[14], of Maruo[7] and of Osawa[19] (see Fig.11). It is true that we can not expect our analytical model to cover all features of the complex phenomena observed, effects of viscosity and finite wave elevation in particular, though the latter may be assumed to be less significant considering the low speeds of the models. Thus it seems that in this regard ray theory displays a certain value for predicting ship wave phenomena, although the re-entrance of rays or their reflection at the water line must be considered an open problem, among others. In any case, the authors would like to emphasize the need to take account of surface tension at low speeds, well in accord with Maruo[7]. We hope that our work reported here can add some further weight on this aspect.–

Acknowledgements

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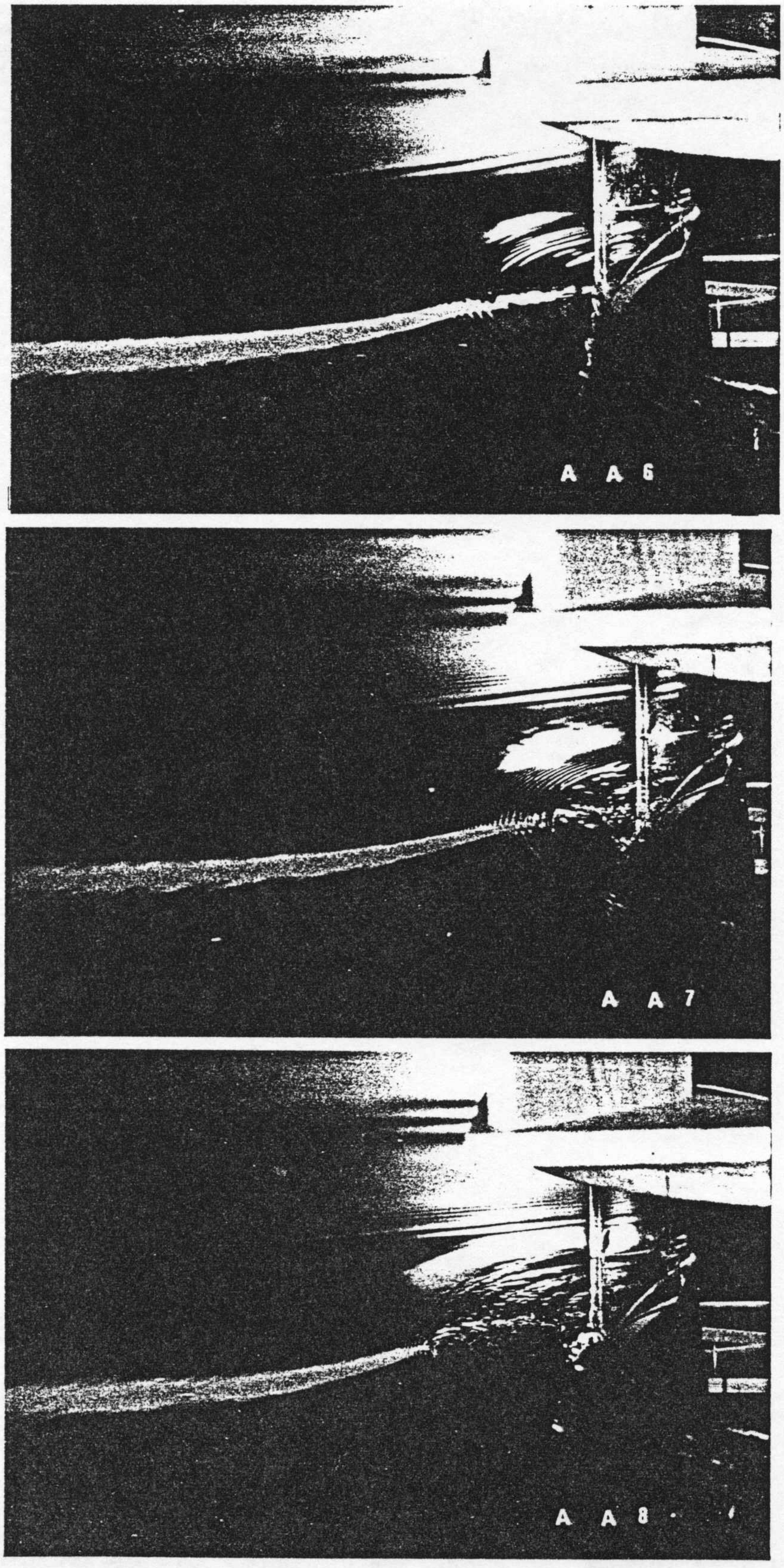
The essential content of this paper has been presented at the Fifth International Symposium on Numerical Ship Hydrodynamics in Hiroshima, Japan in September 1989.

The paper has been dedicated to Prof. Theodore Yao-tsu Wu
to the occasion of his sixty fifth birthday.–



0.6 m/s

0.7 m/s

0.8 m/s

Fig.11. Bow waves in front of a circular cylinder (advancing to the left) with $U=$ 0.6, 0.7, and 0.8 m/s.(From Osawa[20])

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Discussions at the Hiroshima Conference.

Prof. H.S.Choi, National University of Seoul, Korea:

First of all, I would like to congratulate the authors that the surface tension effect has been successfully included in the ray theory to clarify the wave pattern aroaund the bow more clearly. It may be specially useful for work with small models. It is in this case that the local phase velocity reaches the minimum celerity of $0.23m/s$, and the capillary wave breaks. It implies that a new source of singularity has been invited by your method. I would be happy if you comment on it.

Prof. H.Kajitani, University of Tokyo, Japan:

1. I suppose that ray tracing is a kind of low speed theory. I am not sure that a pretty high Fn applied in Fig (5) is available or not.
2. Could you comment on what difference can be observed on the traced characteristic lines between with and without surface tension effects?
3. The wave length of capillary waves in front of a ship's bow changes with its distance. Have you computed the capillary phase?

Prof.A.J.Hermans, Technical University of Delft, Netherlands:

I congratulate the authors with the interesting extension of my theory. I agree with them that in the region of very short waves (near the stagnation points) surface tension is dominant and that its influence on the ray problem is seen in the whole field. It makes the model for the ray problem more accurate than the one described by Brandsma and myself.

It is a pity that the authors do not say any word on the influence of the wave exutation coefficients and the corresponding wave amplitude. It is my philosophy that one must try to balance all compounds of the building. To my oppinion, our approach has such a balance in our level. Do the authors expect that our approach to the amplitude problem is applicable in their case? If so, do they expect that the influence of surface tension is noticable there just so well?

Reply to Discussions.

Reply to Prof. Kajitani:

Prof. Kajitani's worrying about applying the ray theory to high Fn is certainly natural. We use Fig.5 (from Inui and Miyaya) , where the highest Fn is 0.4, to show the qualitative confirmation with the test results,[9] we do not think that ray theory (at present) can predict strong npn-linear effects. Keller [3] . claimed that ray theory may be used up to Fn = 0.7; we are more conservative in this regard. As to the differences with and without surface tension, they could be listed as follows:

In the conventional theory, without surface tension, (i) we have to deal with a *point disturbance*, i.e. all waves start from the stagnation points, stationary waves exist even near these points and (ii) Rays and wave patters are essentially independent from the ship's speed. In our present theory (including surface tension) there is a region of disturbance such that rays can reach the far field only when starting from outside this region. No stationary waves can exist inside this region.[10] For(ii), in our approach wave patterns depend on U, they change their tendencies with the change of the short-life belt, local and far-field Kelvin angles come to closer accord with experiments.
The capillary wave phase could be calculated, once reliable initial values were given.–

Reply to Prof. Choi:

Thank you for your congratulation and comment. If the length of a gravity wave decreases during propagation, it may break before the local phase velocity reaches its minimum. In our method, we start rays from the short life belt boundary. Then waves in general become *longer*, with $1/k\frac{dk}{ds} < 0$, see eq.(48). In case that c decreased and reapproached c_m, we stopped ray tracing and took the starting point more outside. This does not imply the occurence of a new source singularity.

Reply to Prof. Hermans:

We are happy to acknowledge the fundamental work you refer to, which has inspired our analytical proceedure. The purpose of this paper is to find out if, once surface tension is accounted for, the ambiguity and difficulty of the conventional ray theory could be overcome. We appreciate all endeavours up to now to develop ray theory and make it applicable, but obviously some improvement is still needed for a clear concept of where a ray can and should originate (provided that ray theory *is* valid up to the ship's surface); only then can we assign initial values to amplitude and phase and consider the exitation problem.–

[9]In Fig.5, the rudder being small, Fn=0.4 still means a *slow* speed in relation to the phase velocity c_m in uniform flow

[10]Observing carefully the region in front of a slowly advancing body with blunt bow, say a circular cylinder as shown in Fig 11, one could in some speed range find *instationary* waves between the stationary capillary ind the instable turbulent region close to the bow. We deal with stationary waves only.