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Analysis of Wave Load Combination Including Slamming

by

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1. Introduction

It is necessary to estimate extreme sea loads when probabilistic and reliability methods are used to analyse the ultimate strength of ship corresponding to the modes of failure due to yielding or inelastic buckling.

The ship response records taken in full-scale tests show that the vertical stress time history consists of a rapidly varying time history with random amplitude and frequency, oscillating about a mean value. The mean value itself is a weakly time-dependent function. The rapidly varying part includes a low frequency component due to the motion of ship as a rigid body, and a high frequency one due to the impact of the ship as a flexible body on the water, i.e. slamming ("springing" of the flexible ship excited by the energy present in the high frequency wave components is not considered in this paper).

Therefore, the total load on the flexible hull in a seaway can be decomposed into three components

$$M_{total} = M_0 + M_B + M_S \tag{1}$$

where M_o is the mean value of the total load, M_B is low frequency wave-induced load, and M_S is high frequency slamming load.

Unfortunately, the nature of present-day tools and procedures for calculating ship load is such that only separate individual components of the response can be calculated rather than the more relevant combined response /1/. A ship designer is, therefore, faced with the important problem of computing the total load from the individual components.

In addition, for design purposes it is not sufficient only to obtain the data of the combined load. One has to estimate the extreme value of the total load during the number of response cycles expected. This is obvious since the designer has to provide sufficient strength for the "worst condition".

It is the purpose of this paper to explore the combination of load components including slamming and estimate the extreme total load.

Slamming is generally a result of large pitch and heave motions in rough seas. It occurs under certain conditions only - depending on sea severity and on ship speed, heading and ship form. When it occurs the hull suffers large impact fluid load which excited a transient vibratory response in the hull, possibly causing serious local damage along with whipping of the hull. To reduce the danger of heavy damage or high vibratory response, the ship master will tend to change ship speed and heading under normal condition so as to keep the frequency and severity of such slamming load within reasonable bounds, according to his experience.

In view of the transient characteristics of slamming response and the practical operation of a ship in rough seas, it is, in the author's opinion, suitable to adopt short-term prediction in head seas and in one or several severe design sea states for computing ultimate strength of the hull including slamming response.

The period of time of short-term prediction is usually from 30 minutes to several hours, during which the mean value in equation (1) can be considered as a constant.

Owing to the nonlinear relation between slamming response and wave height, it is suitable to combine response components in time domain.

The time histories of the rigid body motion and low frequency wave-induced response for the ship in a seaway are calculated by the summation of responses obtained based on linear strip theory to individual regular sinusoidal wave components /2/. Although small variations may exist in the linear theoretical methods, much larger differences occur in the way the transient excitation is defined /3/. After the relative motion between the hull and the wave surface is determined, the transient excitation can be evaluated whenever a slam is encountered. The slamming response is thus superimposed on the time history of the wave-induced response to obtain the rapidly varying part in the total response of the flexible hull.

From a sufficiently long time history of rapidly varying response (usually 30 minutes), the extreme value of this response in a required time period can be estimated based on the principles of extreme value statistics.

Adding the extreme value to the mean value in equation (1), finally, the extreme total load is determined.

As an example, the method presented in this paper is used to predict the combined load and its extreme value for the S-175 container ship travelling in a specially designated long-crested irregular head seaway. The theoretically predicted results are compared with those of the model experiment made at China Ship Scientific Research Centre (CSSRC) by Sen Jinwei et al. /4/.

2. Model of a seaway

An irregular sea surface in deep water can be represented as the sum of a large number of regular waves, each component having a deterministic amplitude ζ_a , frequency ω , wave number k , statistically independent uniformly distributed phase angle ε ($0 \leq \varepsilon \leq 2\pi$) and being a solution of the linearised hydrodynamic equation for deep water waves. To a ship travelling with forward speed v in an irregular head seaway, the elevation of wave surface at any position x_b on the hull is (see Fig. 1)

$$\zeta_i(x_b, t) = \sum_{j=1}^{m} \zeta_{aj} \, e^{i(\omega_{ej} t + k_j x_b + \varepsilon_j)} \qquad (2)$$

where the real part of the expression is taken, ω_{ej} is the frequency of encounter of the j th wave given by

$$\omega_{ej} = \omega_j + \frac{\omega_j^2}{g} V \quad . \qquad (3)$$

The amplitude of the j th wave, ζ_{aj} , is determined from the wave spectrum $S_\zeta(\omega)$ as follows

$$\zeta_{aj} = \sqrt{2 \; S_\zeta(\omega_j) \; \Delta\omega_j} \qquad (4)$$

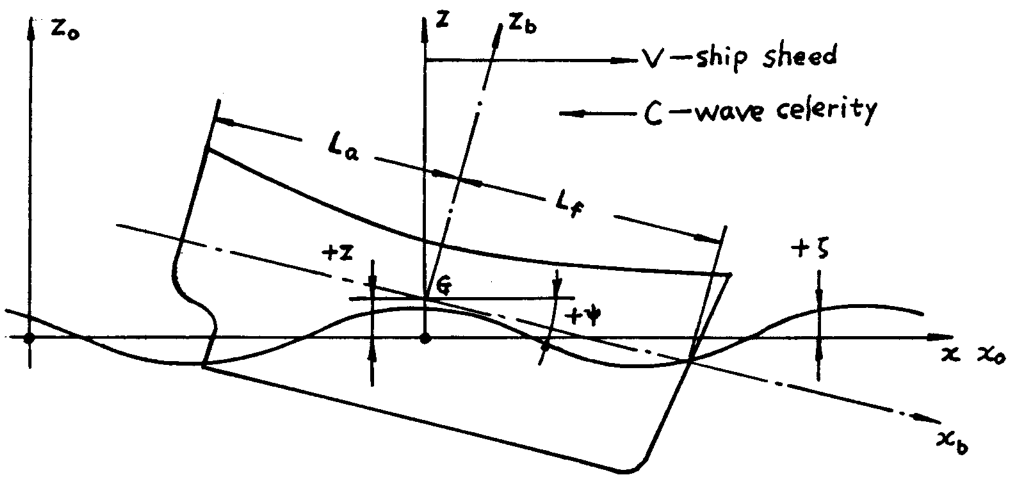

Fig. 1 Coordinate system and notations

Equation (2) represents a typical realisation of the surface wave profile of the long-crested unidirectional seaway defined by a given wave spectrum when particular values are assigned to the quantities ε_j . By changing the values of the component phase angles, ε_j , other typical realisations are constructed. It can be proved that $\zeta_i(x_b, t)$ is an ergodic random process in the mean, mean square and auto-correlation function statistics.

3. Hydrodynamic forces

The hydrodynamic forces on the hull consist of restoring, damping and inertia terms.

It is assumed that the vertical displacement of the ship's centre of gravity is z and the pitch angle with respect to horizontal plane is ψ when a ship travels in a regular head wave given by

$$\zeta = \zeta_a \, e^{i(\omega_e t + k x_b)} \tag{5}$$

By using the modified strip theory of Gerritsma and Beukelman /5/ and taking into account the non-vertical sides of the ship, the real hydrodynamic forces per unit length of the hull can be assumed in the form

$$\left.\begin{aligned}
F'_{1r} &= -\rho g \, (A' - A'_r) \\
&= F'_1 + \rho g \left[A'_r - A' + 2y_w \, (z - x_b\psi - \zeta^*)\right] \\
F'_{2r} &= -N'_r \frac{D}{Dt}\left(z - x_b\psi - \zeta^*\right) \\
&= F'_2 - (N'_r - N') \frac{D}{Dt}\left(z - x_b\psi - \zeta^*\right) \\
F'_{3r} &= -\frac{D}{Dt}\left[m'_r \frac{D}{Dt}\left(z - x_b\psi - \zeta^*\right)\right] \\
&= F'_3 - \frac{D}{Dt}\left[(m'_r - m') \frac{D}{Dt}\left(z - x_b\psi - \zeta^*\right)\right]
\end{aligned}\right\} \tag{6}$$

where F_1', F_2' and F_3' are the linear hydrodynamic forces obtained based on the assumption of vertical sides of the ship, i.e.

$$\left.\begin{aligned} F_1' &= -2\rho g y_w (z - x_b\psi - \zeta^*) \\ F_2' &= -N' \frac{D}{Dt}(z - x_b\psi - \zeta^*) \\ F_3' &= -\frac{D}{Dt}\left[m' \frac{D}{Dt}(z - x_b\psi - \zeta^*)\right] \end{aligned}\right\} \qquad (7)$$

A_r', N_r' and m_r' denote the instantaneous immersed area, damping coefficient and added mass of the section, respectively; A', N' and m' are the values of A_r', N_r' and m_r' corresponding to calm water condition, y_w denotes the half width of the section in calm water, and finally, ζ^* is the wave surface elevation including Smith correction.

The operator $\frac{D}{Dt}$ in equation (6) and (7) is the total derivative with respect to time t, that is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} - V \frac{\partial}{\partial x_b} \qquad (8)$$

Evidently, the actual hydrodynamic forces of the ship can be divided into a linear and a non-linear part. If we neglect the dependence of A_r', N_r' and m_r' on the relative motion between the hull and the wave surface, the force expressions correspond to the usual linear strip theory.

The non-linear part of the hydrodynamic forces, having impulsive character, constitutes the high frequency slamming load exciting transient vibratory responses in the hull.

Theoretical analyses and tests show that slamming has an insignificant influence on the displacement and velocity of conventional displacement ships. Therefore, the non-linear hydrodynamic forces may be neglected in practically calculating the rigid body motion of the ship. In this sense, slamming response is a correction for wave-induced responses obtained on the basis of the linear strip theory.

4. Rigid body motion and wave load

Heave, pitch and wave load of a ship are determined on the basis of considering the ship as a rigid body and taking into account only the linear part of the hydrodynamic forces.

The load per unit length on the hull to a single regular wave is

$$P' = -\frac{W'}{g}\left(\ddot{z} - x_b\ddot{\psi}\right) + F' + \rho g A' - W' \tag{9}$$

where F' denotes the sum of the hydrodynamic forces per unit length

$$F' = F_1' + F_2' + F_3' \tag{10}$$

and W' is the weight of the section per unit length.

The last two terms of P' correspond to the hull load in still water so that the dynamic part of the load is given by

$$P_d' = -\frac{W'}{g}\left(\ddot{z} - x_b\ddot{\psi}\right) + F' \tag{11}$$

The corresponding wave shear force Q_W and wave bending moment M_W at a specified cross section x' are (see Fig. 2)

$$\left.\begin{aligned} Q_W &= \int_{-L_a}^{x'} \left\{ \frac{W'}{g}\left(\ddot{z} - x_b\ddot{\psi}\right) - F' \right\} dx_b \\ M_W &= \int_{-L_a}^{x'} \left\{ \frac{W'}{g}\left(\ddot{z} - x_b\ddot{\psi}\right) - F' \right\} (x' - x_b)\, dx_b \end{aligned}\right\} \tag{12}$$

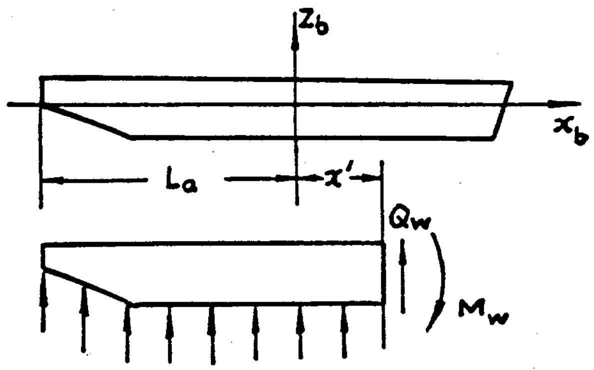

Fig. 2 Wave shear force and bending moment

Substituting (10) into equation (12) we may write

$$\left.\begin{aligned} Q_w &= (a_1+h_1)\ddot{z} + b_1\dot{z} + c_1 z - (d_1+n_1)\ddot{\psi} - e_1\dot{\psi} - g_1\psi \\ &\quad - F_{a1}\, e^{i(\omega_e t + \varepsilon_{F\xi 1})} \\ M_w &= x' Q_w + (A_1+H_1)\ddot{\psi} + B_1\dot{\psi} + C_1\psi - \\ &\quad (D_1+N_1)\ddot{z} - E_1\dot{z} - G_1 z - M_{a1}\, e^{i(\omega_e t + \varepsilon_{M\xi 1})} \end{aligned}\right\} \qquad (13)$$

where

$$\left.\begin{aligned} h_1 &= \int_{-L_a}^{x'} \frac{w'}{g}\, dx_b \\ n_1 &= \int_{-L_a}^{x'} \frac{w'}{g}\, x_b\, dx_b \\ H_1 &= \int_{-L_a}^{x'} \frac{w'}{g}\, x_b^2\, dx_b \\ N_1 &= \int_{-L_a}^{x'} \frac{w'}{g}\, x_b\, dx_b \end{aligned}\right\} \qquad (14)$$

The other coefficients are corresponding to those in /5/ one by one, and the only difference consists in that here the upper limit of the integral for the coefficients is the variable x' .

If the STF (Salvesen, Tuck, Faltinsen) strip theory is adopted, the above-mentioned coefficients will have some differences.

Noting $Q_W(L_f,t) = M_W(L_f,t) = 0$ for $x' = L_f$, then, (14) becomes the equation describing the ship's regid body motion in waves, which is frequently used in ship seakeeping studies.

To equation (5), ship motions in the regular wave may be written as

$$\left.\begin{aligned} z &= z_a \, e^{i(\omega_e t + \varepsilon_z)} \\ \psi &= \psi_a \, e^{i(\omega_e t + \varepsilon_\psi)} \end{aligned}\right\} \tag{15}$$

By solving the equation of ship motion, the frequency response functions for heave and pitch, i.a.

$$\left.\begin{aligned} \phi_z &= \frac{z_a}{\xi_a} \\ \phi_\psi &= \frac{\psi_a}{k \cdot \xi_a} \end{aligned}\right\} \tag{16}$$

and ε_z , ε_ψ can be found.

Similarly, Q_W and M_W are represented by

$$\left.\begin{aligned} Q_W &= Q_{Wa} \, e^{i(\omega_e t + \varepsilon_Q)} \\ M_W &= M_{Wa} \, e^{i(\omega_e t + \varepsilon_M)} \end{aligned}\right\} \tag{17}$$

From equation (13), we can obtain

$$\left.\begin{aligned}\phi_Q &= \frac{Q_{wa}}{\rho g L B \xi_a} \\ \phi_M &= \frac{M_{wa}}{\rho g L^2 B \xi_a}\end{aligned}\right\} \qquad (18)$$

and ε_Q , ε_M , where L is the ship's length, B is the ship's beam.

Owing to the fact that an irregular seaway may be described by the summation of a large number of deterministic regular waves, the time histories of ship motions corresponding to the typical realisation of the seaway given by equation (2) are

$$\left.\begin{aligned}z_i &= \sum_{j=1}^{m} \xi_{aj}\, \phi_{zj}\, e^{i(\omega_{ej}t + \varepsilon_{zj} + \varepsilon_j)} \\ \psi_i &= \sum_{j=1}^{m} \xi_{aj}\, k_j\, \phi_{\psi j}\, e^{i(\omega_{ej}t + \varepsilon_{\psi j} + \varepsilon_j)}\end{aligned}\right\} \qquad (19)$$

Further, the time history of the relative motion between the hull and the wave surface is given by

$$r = z_i - x_b \psi_i - \xi_i \quad , \qquad (20)$$

and the time history of the relative velocity between the hull and the wave surface (with respect to coordinate system xyz) is

$$\begin{aligned}\dot{r} &= \dot{z}_i - x_b \dot{\psi}_i - \dot{\xi}_i \\ &= \sum_{j=1}^{m} i \xi_{aj}\, \omega_{ej} \left[\phi_{zj}\, e^{i\varepsilon_{zj}} - x_b k_j \phi_{\psi j}\, e^{i\varepsilon_{\psi j}} - e^{ik_j x_b}\right] e^{i(\omega_{ej} + \varepsilon_j)}\end{aligned} \qquad (21)$$

In a similar manner, the time histories of the wave shear and bending moment in a seaway are found:

$$Q_W(x',t) = \rho g L B \sum_{j=1}^{m} \zeta_{aj}\, \Phi_{Qj}\, e^{i(\omega_{ej}t + \varepsilon_{Qj} + \varepsilon_j)}$$

$$M_W(x',t) = \rho g L^2 B \sum_{j=1}^{m} \zeta_{aj}\, \Phi_{Mj}\, e^{i(\omega_{ej}t + \varepsilon_{Mj} + \varepsilon_j)} \qquad (22)$$

5. Slamming response

Slamming is usually divided into bottom impact slamming and bow flare slamming according to the impact locations of a moving ship on the surface wave.

When a ship travels in severe sea conditions, a certain length of hull (usually in the region of the forefoot) may emerge from the water. On re-entry the bottom suffers fluid loads in this region which excite a transient vibratory response in the hull. This phenomenon is called bottom impact slamming.

Bottom impact, however, is not a necessary condition for slamming to occur. Bow flare slamming occurs when the bow flare submerges into water, resulting in violent ship motions.

The main difference in these two slamming processes is that in bow flare slamming the process is of much longer duration than in bottom slamming.

The transient excitation associated with slamming is just the non-linear part of hydrodynamic forces, which is given by

$$\begin{aligned} P_S &= \rho g \left[A'_r - A' + 2 y_W (z - x_b \psi - \zeta^*) \right] \\ &\quad - (N'_r - N') \frac{D}{Dt} (z - x_b \psi - \zeta^*) \\ &\quad - \frac{D}{Dt} \left[(m'_r - m') \frac{D}{Dt} (z - x_b \psi - \zeta^*) \right] \end{aligned} \qquad (23)$$

It should be pointed out that equation (23) includes not only the bottom impact slamming load, but also the bow flare slamming load.

Of course, it is necessary to further analyse this expression for practical calculations.

5.1 Bow flare slamming load

In expression (23) , the immersed area A'_r , the damping coefficient N'_r and the added mass m'_r are functions of the location x_b , the shape of the immersed part of the section and the wave encounter frequency. Evidently, it will be extremely difficult to apply expression (23) to computations in irregular waves.

Note that:

(a) The contribution of the damping term is insignificant in comparison with restoring and inertia terms and can be neglected.

(b) In the frequency range expected to occur for greater motion, added mass is almost independent of frequency and close to the value for infinite frequency. Therefore m'_r and m' in equation (23) can be approximated by their high-frequency limits.

From the above assumptions, then, the expression for the bow flare slamming load at any position x_b on the hull in an irregular seaway is simplified as follows:

$$P_s(x_b,t) = \rho g \left(A'_{ri} - A' + 2y_w \gamma^*\right) - \frac{D}{Dt}\left[\left(m'_{\gamma i} - m'_o\right)\frac{D\gamma^*}{Dt}\right] \tag{24}$$

where

$$\left.\begin{aligned} \gamma^* &= z_i - x_b \psi_i - \xi_i^* \\ \xi_i^* &= \sum_{j=1}^{m} \xi_{aj}\, e^{-k_j T_j^*}\, e^{i(\omega_{ej} + k_j x_b + \varepsilon_j)} \\ T_j^* &= -\frac{1}{k_j} \ln \left(1 - \frac{k_j}{y_w} \int_{-T}^{0} y_w\, e^{k_j z_b}\, dz_b \right) \end{aligned}\right\} \tag{25}$$

z_i and ψ_i are ship motions in a seaway given by equation (19), A'_{ri} and m'_{ri} denote, respectively, the instantaneous immersed area and added mass at section x_b in a seaway determined with respect to the instantaneous section draft

$$T_i = T - r, \tag{26}$$

and, finally, m'_0 is the value of m'_{ri} as draft equals T.

The added mass of infinite frequency of a symmetrical hull sectional shape in vertical oscillatory motion may be calculated by means of a N-coefficients' conformal transformation.

5.2 Bottom impact slamming load

It is known that the time extent of bottom slamming is relatively short. Some effects, such as the influences of local water rise and spray due to the impact, the cushioning effect produced by the air trapped between water surface and impinging ship bottom, and the actual deadrise shape of the sections, etc., are not considered in the simple momentum slamming theory. Therefore, the magnitude of bottom impact load cannot be evaluated accurately by equations (23) or (24) (especially for ships with a wide flat bottom).

For this reason, in a previous paper /2/ written by author, the well established ship motion calculation was combined with information about experimental slamming pressure values given by

Ochi and Motter /6/ to determine the transient exciting load of bottom impact. The essential points are:

(a) Using ship motion time histories, the time instants satisfying the slamming condition ($r = T_j$; $\dot{r} \leq -\dot{\gamma}_*$), and the corresponding relative velocities are determined, where $\dot{r}_*$ denotes a threshold velocity.

(b) According to the magnitudes of relative velocities and section contours, the spatial and time distributions of slamming pressure are evaluated by means of available experimental results.

(c) Integrating along each section the vertical pressure components at corresponding time instants, the time-space distribution of the bottom impact load, $p_s(x_b, t)$, is found.

The threshold velocity can be neglected in practical calculations.

5.3 Calculation of Vibrations

Measurements have shown that the whipping of a hull excited by slamming is primarily in the form of a two node vibration mode. For the lower modes of vibrations, the deflection of the hull can be described appropriately by a free-free beam with non-uniform mass. By applying the modal superposition method, the resp0nse of the ship to the slamming load $p_s(x_b, t)$ is calculated.

For a particular mode i , the generalized coordinate is denoted by $q_i(t)$, the mode shape by $X_i(x_b)$, and the shear and moment spatial weighting functions by $V_i(x_b)$ and $M_i(x_b)$, respectively. Then the vertical elastic deflection normal to x_b , the shear force and the bending moment due to slamming are found by summing the contributions from all modes:

$$\left.\begin{aligned} z_s(x_b,t) &= \sum_{i=1}^{\infty} q_i(t)\, X_i(x_b) \\ Q_s(x_b,t) &= \sum_{i=1}^{\infty} q_i(t)\, V_i(x_b) \\ M_s(x_b,t) &= \sum_{i=1}^{\infty} q_i(t)\, M_i(x_b) \end{aligned}\right\} \qquad (27)$$

Substituting (27) into the system of partial differential equations describing the damped vertical response of a hull-beam to transient forces, neglecting the rotary inertia term and using the orthogonality principle, we get

$$\bar{\mu}_i \ddot{q}_i(t) + \bar{c}_i \dot{q}_i(t) + \bar{k}_i q_i(t) = Q_i(t) \tag{28}$$

where $\bar{\mu}_i$ is the generalized mass

$$\bar{\mu}_i = \int_{-L_a}^{L_f} \mu X_i \, dx_b \; , \tag{29}$$

$\bar{k}_i$ is the generalized spring constant

$$\bar{k}_i = \omega_i^2 \bar{\mu}_i \; , \tag{30}$$

and $Q_i(t)$ is the generalized excitation

$$Q_i(t) = \int_{-L_a}^{L_f} p_s(x_b, t) \, X_i \, dx_b \; . \tag{31}$$

$\bar{c}_i$, μ and ω_i denote the generalized damping, effective mass per unit length and natural frequency, respectively.

In most cases, it is difficult to obtain dependable data on hull structural damping. If there are no experimental results for the ship in question, the following expression can be used /2/:

$$c_i = \frac{\bar{c}_i}{\bar{\mu}_i} = 4.7 \cdot 10^{-4} \, \omega_i^{2.45} - \frac{V}{\bar{\mu}_i} \int_{-L_a}^{L_f} \frac{dm_o'}{dx_o} X_i^2 \, dx_b \tag{32}$$

Assuming that, at time t = 0, $\dot{q}_i(0) = q_i(0) = 0$, the solution of equation (28) is

$$q_i(t) = \int_0^t \frac{Q_i(\tau)}{\lambda_i \bar{\mu}_i} e^{-\frac{c_i}{2}(t-\tau)} \sin \lambda_i (t-\tau)\, d\tau \tag{33}$$

where

$$\lambda_i = \sqrt{\omega_i^2 - \frac{1}{4} c_i^2} \tag{34}$$

Therefore, from $q_i(t)$ and the spatial weighting functions defined by

$$\left.\begin{aligned} V_i(x') &= -\int_{-l_a}^{x'} \mu \omega_i^2 X_i \, dx_b \\ M_i(x') &= -\int_{-l_a}^{x'} \mu \omega_i^2 X_i (x'-x_b)\, dx_b \end{aligned}\right\} \tag{35}$$

the slamming shear force $Q_S(x',t)$ and slamming bending moment $M_S(x',t)$ at any position x' on the hull can be calculated.

Belik and Price /3/ indicated that the Ochi-Motter impact theory produces very small contributions to the steady state responses in comparison with the momentum theory. This conclusion is not, in the author's opinion, generally applicable. It depends mainly on the types of ship whether the contribution is large or small. The author calculated slamming responses for two ships of different forms in /2/. Ship A is a fast ship with bulbous bow and a large bow flare, while ship B is a full ship with flat bottom and light draft. The calculation is performed for the time history of a seaway given by equation (2). Ochi-Motter impact theory is used when the hull bottom impinges on the water surface, and momentum slamming theory is used when the hull immerses into the water after the impact. The results calculated show that the transient response is dominated by bow flare slamming while the contribution of bottom impact slamming is relatively small for ship A, whereas exactly the reverse is true for ship B. Full-scale experimental results for these two ships agree quite well with this calculated conclusion.

6. Extreme value of rapidly varying response

Superimposing the wave and slamming responses calculated in the time domain corresponding to a typical realisation of the irregular seaway, the rapidly varying part in the total response (1) is obtained, as shown in Fig. 3.

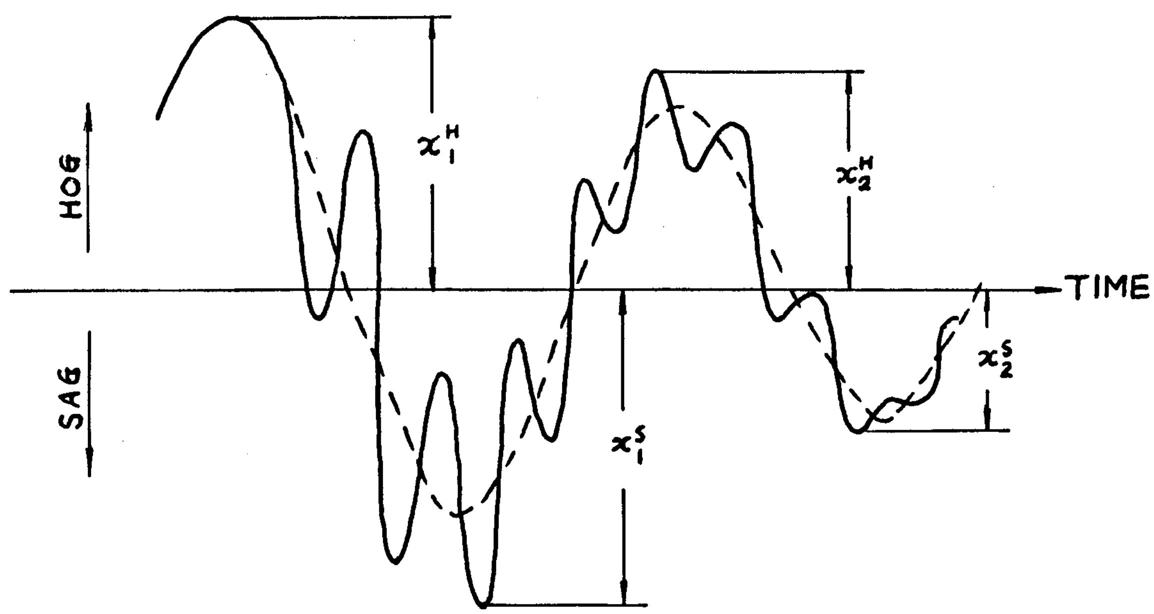

Fig. 3 Time history of bending moment (wave-induced + slamming)

Since the mean value in equation (1) is constant for short-term predictions, in order to obtain the extreme total response, the extreme value of the rapidly varying part has to be estimated first.

The peak amplitude of the rapidly varying response in a cycle of encounter with the wave is denoted by a random variable X (see Fig. 3); its cumulative distribution and probability density functions are $F_X(x)$ and $f_X(x)$, respectively.

Let Y_n represent the maximum amplitude of the rapidly varying response in n wave encounters. By using order statistics, the cumulative distribution function $F_{Y_n}(y_n)$ and the probability density function $f_{Y_n}(y_n)$ are given by

$$F_{Y_n}(y_n) = \left[\left\{ F_X(x) \right\}^n \right]_{x=y_n} \tag{36}$$

$$f_{Y_n}(y_n) = n \left[f_X(x) \left\{ F_X(x) \right\}^{n-1} \right]_{x=y_n} \tag{37}$$

The model value of the probability density function $f_{Y_n}(y_n)$, $\bar{y}_n$, is called the probable extreme value in n encounters. Its value can be obtained by letting the derivative of $f_{Y_n}(y_n)$ with respect to y_n be zero:

$$f'_X(\bar{y}_n) F_X(\bar{y}_n) + (n-1) \left\{ f_X(\bar{y}_n) \right\}^2 = 0 \tag{38}$$

The value $\bar{y}_n$ is the most likely maximum value expected to occur in n encounters. It should be noted, however, that the probability is quite large that the actual maximum value exceeds $\bar{y}_n$.

It is highly desirable from a design point of view, therefore, to predict the extreme value whose probability of being exceeded is very small. In other words, let us choose a small number α

and obtain the extreme value $\hat{y}_n(\alpha)$ from the following relationship:

$$\int_0^{\hat{y}_n(\alpha)} f_{Y_n}(y_n)\,dy_n = \left\{F_X(\hat{y}_n)\right\}^n = 1-\alpha \tag{39}$$

The response $\hat{y}_n(\alpha)$ is called design extreme value in n encounters. The parameter α is at the designer's discretion.

Unfortunately, the probability distribution for the combined wave and slamming response is not known so far. One way to overcome this difficulty is to evaluate the extreme response by an approximate method /7/. That is, the extreme response can be estimated approximately by using the accumulation of the observed or computed data over a sufficiently long period of time.

Assuming that the initial cumulative distribution function can be written in following form

$$F_X(x) = 1 - e^{-q(x)} \tag{40}$$

where $q(x)$ is a positive real-valued function satisfying the conditions for $F_X(x)$ being a cumulative distribution function, then, equation (38) becomes

$$\frac{q''(\bar{y}_n)}{\left\{q'(\bar{y}_n)\right\}^2}\left\{1-e^{-q(\bar{y}_n)}\right\} + n e^{-q(\bar{y}_n)} - 1 = 0 \tag{41}$$

Since the first term is small in comparison with other terms for large n, (41) yields

$$e^{-q(\bar{y}_n)} = \frac{1}{n} \tag{42}$$

From equation (40) and (42), we obtain

$$1 - F_X(\bar{y}_n) = \frac{1}{n} \tag{43}$$

A similar analysis shows that the design extreme value is given by

$$1 - F_X(\hat{y}_n) = \frac{\alpha}{n} \tag{44}$$

Equations (43) and (44) imply that it is not necessary to know the function $q(x)$ for evaluating the extreme responses in practice. The values $\bar{y}_n$ and $\hat{y}_n(\alpha)$ can be determined by extrapolating the initial cumulative distribution function.

For precisely estimating extreme responses, it is highly desirable to reliably extrapolate the initial cumulative distribution function beyond the given data points.

For this purpose, the function $q(x)$ in (40) may be expressed as a combination of an exponential and a power-of- x - function:

$$q(x) = a x^m e^{-p x^k} \tag{45}$$

The expression is of sufficient generality to include various possible cases. It was used by Ochi and Whalen /7/ in estimating the probability of extreme sea states (significant wave height).

The parameters in $q(x)$ are determined numerically by a non-linear least squared fitting procedure (for example, Gauss-Newton or Marquardt method). The form used in this minimization procedure is given by

$$G = \ln\left\{-\ln\left(1 - F_X\right)\right\} = \ln a + m \ln x - p x^k \tag{46}$$

The parameters are optimized such that the objective function, i.e. the sum of the squared differences between G in (46) and the corresponding data values, becomes minimal.

Once the parameter values are determined, the extreme responses are evaluated from (43) and (44).

7. Mean value of total load

The dominant factors which affect the mean value of the total load on the hull in a seaway are 1. the stillwater load, 2. thermal effects, and 3. the wave-making effect created by a ship's own wave system when moving at high speed.

The stillwater load depends only on the weight distribution and the shape of the underwater hull and can be calculated statically. If a statistical description of the stillwater bending moment is adopted, data have shown that the general trend assumes a normal distribution for conventional types of ships /8/.

Records of midship stress obtained on bulk carriers show that thermal effects are surprisingly high. The temperature gradients that procedure such thermal stresses are considered as loads here because they have similar effects. Once the temperature difference along a ship hull is determined, the thermal stresses can be calculated using a general purpose finite element computer program.

The stillwater and thermal responses are weakly time-dependent variables. For short-term prediction, they can be considered constants during a design storm. Therefore, these two responses may easily be combined for one or several postulated design conditions.

When a ship is advancing in calm water, there is a deviation in the pressure distribution compared with the value at zero speed. Therefore a bending moment is induced which is called, in this paper, the wave-making bending moment. Experimental results for series 60 models /9/ are presented in Fig. 4. The data published by Moor /10/ show a similar tendency.

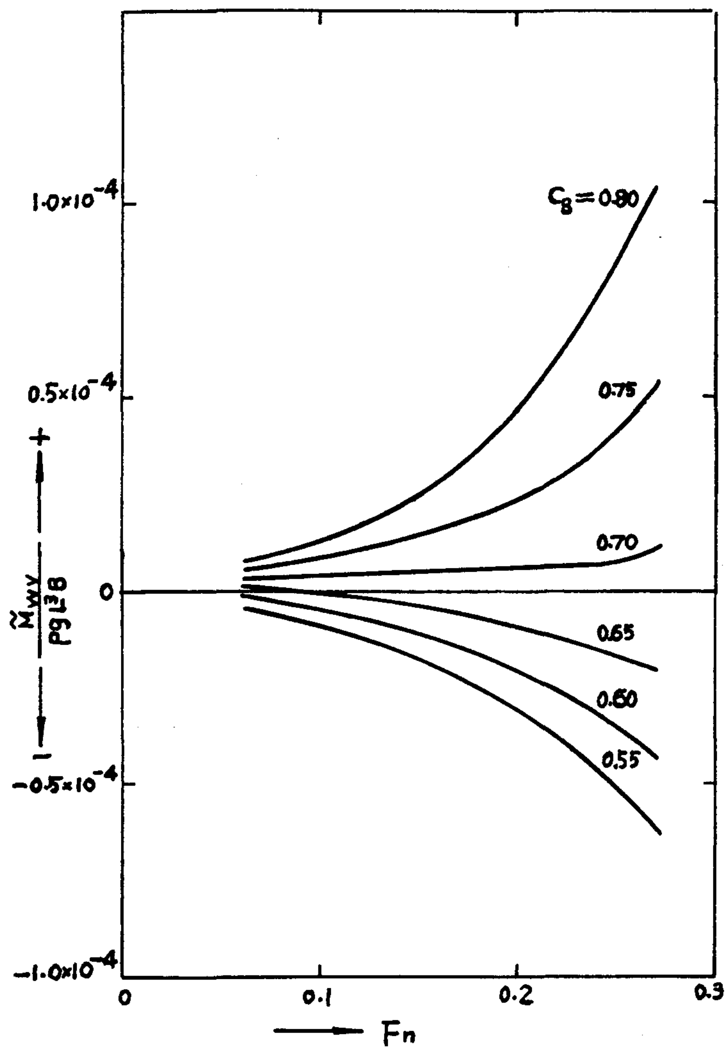

Fig. 4 Wave-making bending moment
(L/H = 17.50, L/B = 7.00) /9/

In order to clarify the contribution of the wave-making effect to the total response, it is necessary to carefully analyse these experimental results.

A record of the longitudinal bending moment measured on a model in a regular head wave is shown in Fig. 5 /11/. In this figure, the different hogging and sagging bending moment response curves are replaced by three equivalent components (note that the still-water response is taken as zero in these experiments). One component is that caused by the model's own wave system in calm water. The bending moment excited by the wave is the sum of two parts, an oscillating part which corresponds to the wave-induced bending moment obtained on the basis of the linear strip theory, and a shift of the mean value in the oscillation from the level expected in calm water with the model running at the same speed. It is obvious that the magnitude of this shift depends on the wave height, wave length, ship speed, section flare, etc.

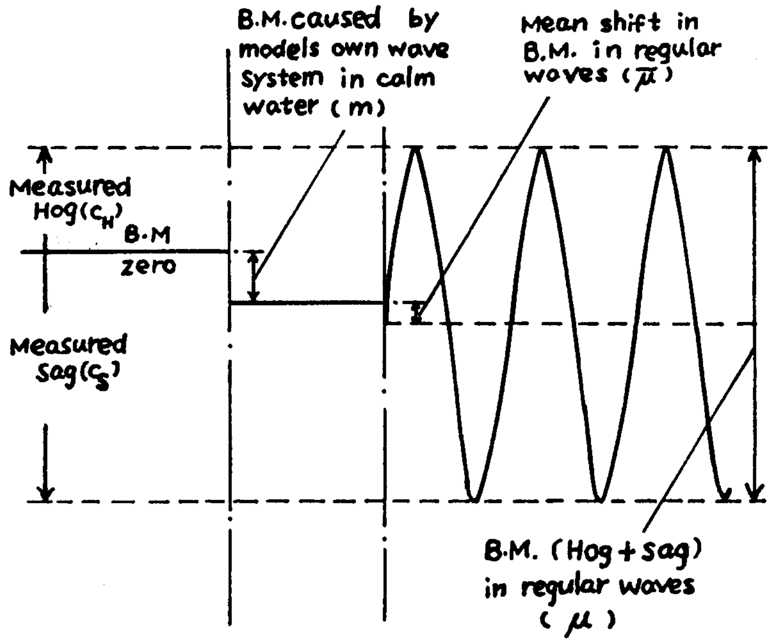

Fig. 5 Components of bending moment in regular waves /11/

That is to say, the hogging and sagging bending moments in regular waves are not simply equal to the sum of one half of the double amplitude and the wave-making bending moment, because the effect of the bending moment shift has to be considered.

Comparisons of double amplitudes of the low frequency wave bending moment with corresponding experimental data have been carried out extensively; they showed satisfactory agreement. The experimental results for the S-175 container ship model given by Sen Jinwei et al. /4/ confirm again this conclusion. So, before a theoretical method including the effect of the ship's own wave system is established, the mean value between hogging and sagging bending moments measured on a model in waves may be determined and taken as the third mean component resulting from the wave-making effect. This way of doing matches the above computations for wave and slamming responses.

If there are no experimental data of the ship to be designed, the third mean component is estimated according to the following equations which are derived from results of published model experiments.

If dimensionless coefficients c_H, c_S, m and $\bar{\mu}$ are defined by

$$c_H \text{ or } c_S = \frac{\text{hog or sag bending moment, resp.}}{2 \rho g L^2 B \zeta_a}$$

$$m = \frac{\text{wave-making bending moment}}{\rho g L^3 B}$$

and

$$\bar{\mu} = \frac{\text{shift in bending moment}}{2 \rho g L^2 B \zeta_a}$$

(m and $\bar{\mu}$ are taken to be positive in hogging and negative in sagging; c_H and c_S are taken to be positive), then the mean value of the bending moment amidships measured on models in regular head waves can be derived as follows:

$$\begin{aligned} M_{om} &= \rho g L^3 B \left(m + \bar{\mu} \frac{2 \zeta_a}{L} \right) \\ &= \rho g L^2 B \zeta_a \left(c_H - c_S \right) \end{aligned} \tag{47}$$

The dimensionless coefficients in (47) may be estimated according to reference /10/.

For irregular waves, Murdey et al. /11/ used in the analysis, instead of $\bar{\mu}$, a coefficient $\bar{\mu}_S$ defined by

$$\bar{\mu}_S = \frac{\text{mean shift in bending moment}}{\rho g L^2 B H_S}$$

where H_s denotes significant wave height.

The mean value of bending moment amidships measured on models in irregular head waves, then, is given by

$$M_{oms} = \rho g L^3 B \left(m + \bar{\mu}_s \frac{H_s}{L}\right) \tag{48}$$

Employing the analysis technique of multiple regression, Murdey et al. produced the following expressions for m and $\bar{\mu}_s$ based on the results of model experiments:

$$m = 10^{-4} F_n^2 \left(116.3\, c_B^2 - 3.46\, c_B \frac{L}{T} + 3.04 \frac{L}{T} - 20.6\, c_W - 2.01 \frac{L}{B} + 0.6\, LCB - 39.5 \right) \tag{49}$$

$$\bar{\mu}_s = 10^{-2} F_n^2 \left(A_0 + A_1 c_B + A_2 \frac{L}{T} + A_3 \frac{L}{B}\right) \tag{50}$$

where F_n is Froude number, c_B is block coefficient, c_W is waterplane area coefficient, LCB is longitudinal position of the centre of buoyancy from midships (percent of L), and values of the coefficients $A_0 \ldots A_3$ are given in Table 1. In this table, $\bar{T}_z$ is the non-dimensional average zero crossing period defined as $T_z \sqrt{g/L}$, where T_z denotes the dimensional period value.

Once the stillwater response, the thermal effect and the third mean component are determined, they are added to the extreme value of the rapidly varying part obtained in a certain number of wave encounters. Finally, the extreme total load on the hull in the same number of wave encounters is evaluated.

$\bar{T}_z$	A_0	A_1	A_2	A_3	Standard Error
0·6	(0·07)	(0·00)	(0·001)	−0·014	0·00002
0·8	(0·21)	(0·04)	(0·005)	−0·050	0·00006
1·0	(0·35)	(0·37)	0·018	−0·135	0·00012
1·2	(−0·40)	1·90	0·057	−0·284	0·00019
1·4	−2·36	4·23	0·112	−0·403	0·00024
1·6	−4·39	6·18	0·155	−0·454	0·00027
1·8	−5·63	7·21	0·178	−0·464	0·00029
2·0	−5·94	7·37	0·183	−0·464	0·00030
2·2	−5·75	7·08	0·176	−0·448	0·00029
2·4	−5·29	6·58	0·163	−0·428	0·00027
2·6	−4·71	5·98	0·148	−0·408	0·00025
2·8	−4·09	5·36	0·133	−0·390	0·00024
3·0	−3·49	4·79	0·119	−0·373	0·00023
3·2	−2·90	4·21	0·105	−0·357	0·00023
3·4	−2·41	3·74	0·094	−0·345	0·00022
3·6	−2·01	3·36	0·085	−0·333	0·00021
3·8	−1·67	3·01	0·076	−0·320	0·00021
4·0	−1·40	2·73	0·069	−0·307	0·00020

Values in brackets are statistically non-significant

Table 1 Coefficients in equation (50) /11/

8. Numerical example

In this section the main results of a numerical application to the S-175 container ship are given.

S-175 model experiments were made on bending moments, shear forces and motions in regular and irregular head waves at the seakeeping basin of CSSRC by Sen Jinwei et al. /4/. An aluminium model of 1:55 scale was designed by employing the principle of similarity in hydrodynamics and structural dynamics. The hull was divided into six separate parts joined by a longitudinal circular steel beam through the whole length of the hull. The model principal characteristics are listed in Table 2. Investigations were made for Froude numbers of 0.20 and 0.275; the height of regular waves and the significant height of irregular waves were about L/42 and L/30, respectively. The duration of each model experiment in irregular waves corresponded to about 15 minutes of the full scale time.

Length between perpendiculars (L)	3.1818 m
Beam (B)	0.4618 m
Depth (D)	0.2800 m
Draft (T)	0.1727 m
Displacement (Δ)	145.085 kg
Block coefficient (C_B)	0,5716
Midship section coefficient ($C_{\boxtimes}$)	0.970
Longitudinal position of the centre of gravity from midships (X_g)	0,01417 L
Longitudinal radius of gyration (K_{yy})	0,24 L
First mode natural frequency	9.2 Hz

Table 2 General characteristics of S-175 model /4/

In the computations of this paper, for comparison with the model experiments it is assumed that S-175 container ship travels at Froude number 0,275 in a long-crested irregular head seaway described by the experimental wave spectrum with a significant wave height $H_S = \frac{L}{30} = 5.83$ m.

The following results are full-scale values.
The vibratory characteristics are calculated first using a flexible body vibration program. The lowest natural frequency is 63.4 cpm, which is lower than the experimental value of 74 cpm.

From the experimental wave spectrum, a typical wave elevation realisation of about 30 minutes duration is constructed. Superimposing calculated midship wave and slamming bending moments corresponding to this realisation, the time history of the rapidly varying part of the total bending moment is obtained. The histogram of the double amplitudes of the rapidly varying part is shown in Fig. 6. This figure includes also the experimental result at the location of the ship's centre of gravity.

Corresponding statistical values are presented in Table 3.

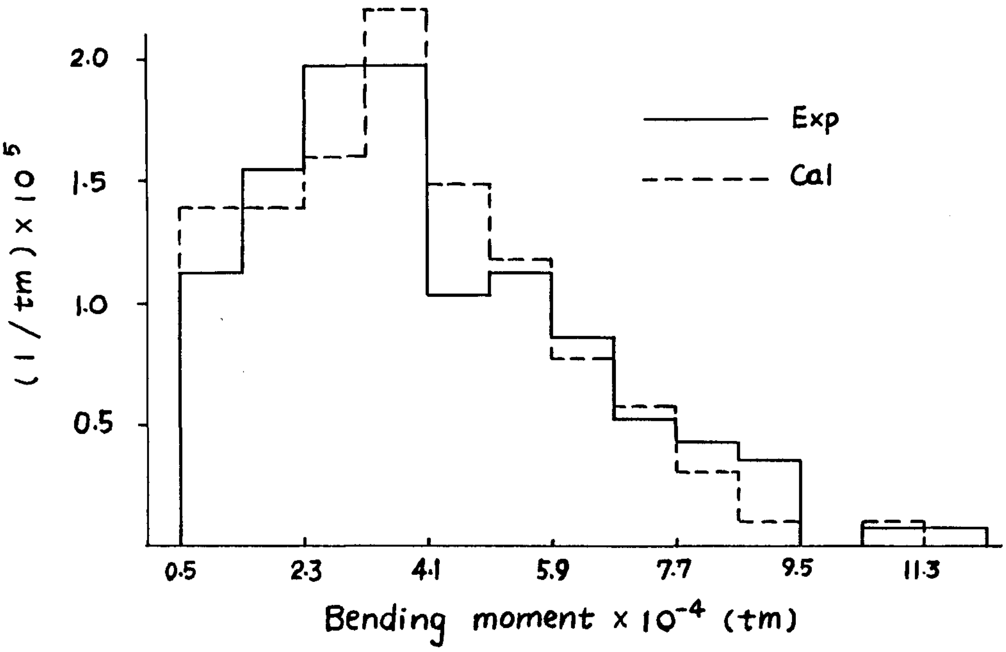

Fig. 6 Histogram of double amplitudes of bending moment (wave-induced + slamming)

		average period (s)	mean X 10^{-4} (tm)	rms X 10^{-4} (tm)	max X 10^{-4} (tm)
hog + sag	exp	7.134	4.034	4.656	12.244
	cal	6.944	3.859	4.395	10.976
hog	exp	7.134	0.328	0.899	2.855
	cal	6.944	0.243	1.099	3.481
sag	exp	7.134	3.706	4.063	9.389
	cal	6.944	3.602	3.797	7.495

Table 3 Statistical values of bending moment

To get separate predictions for hogging and sagging responses, the mean value of the total bending moment has to be estimated.

The stillwater and thermal responses can be calculated by routine methods without difficulty.

The third mean component caused by the ship's own wave system is, at midship section, given by equation (50) as $1.57 \cdot 10^4$ tm (sag). On the other side, by using the experimental means of the low frequency hogging and sagging bending moments at the location of the centre of ship gravity, this mean component is given as $1.53 \cdot 10^4$ tm (sag).

By adding the third component $1.57 \cdot 10^4$ tm (sag) to the calculated rapidly varying part, separate comparisons for hogging and sagging moments are obtained as presented in Figs. 7 and 8, and in Table 2.

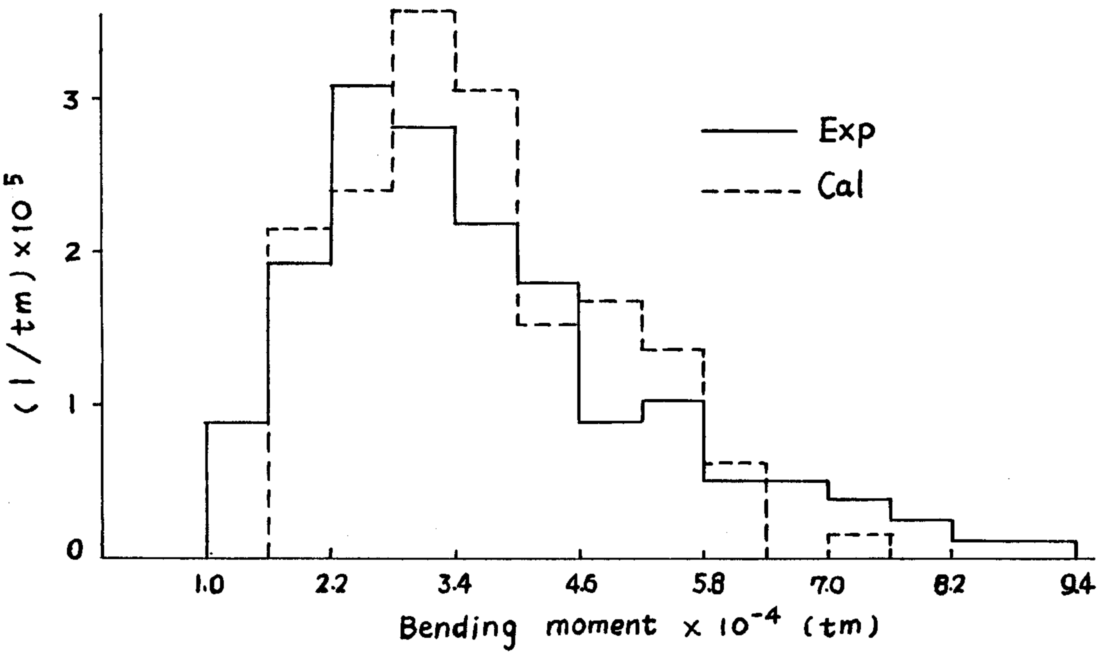

Fig. 7 Histogram of hogging bending moment

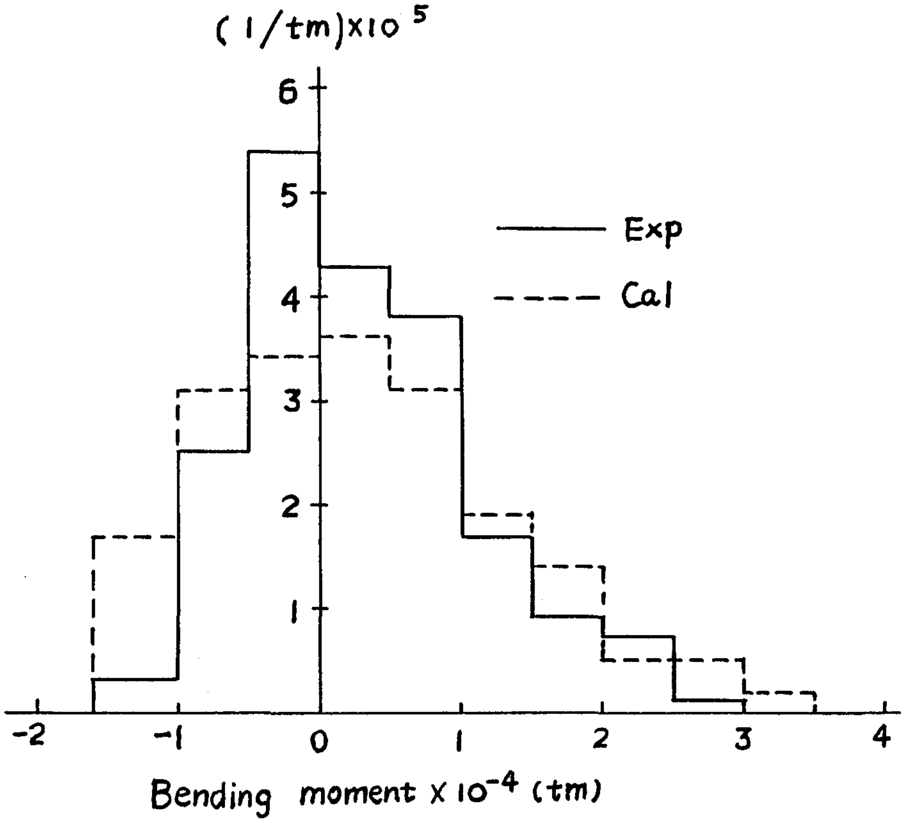

Fig. 8 Histogram of sagging bending moment

In order to obtain the extreme value of the calculated rapidly varying moment (wave-induced + slamming), the parameters in equation (45) are determined numerically by a nonlinear least squared fitting procedure. The cumulative distribution functions obtained by using these parameters are plotted on Weibull probability paper as shown in Fig. 9 and 10, together with the data points. As a comparison, the cumulative distribution functions given by a Weibull distribution fitted to the data points are also plotted in these figures. Evidently, the cumulative distribution functions obtained according to equations (40) and (45) represent very well the data over the entire range of the values, and therefore it can be used to estimate the extreme responses (see Fig. 11).

In a similar manner, by using the data given by the model experiment, Fig. 12 is obtained.

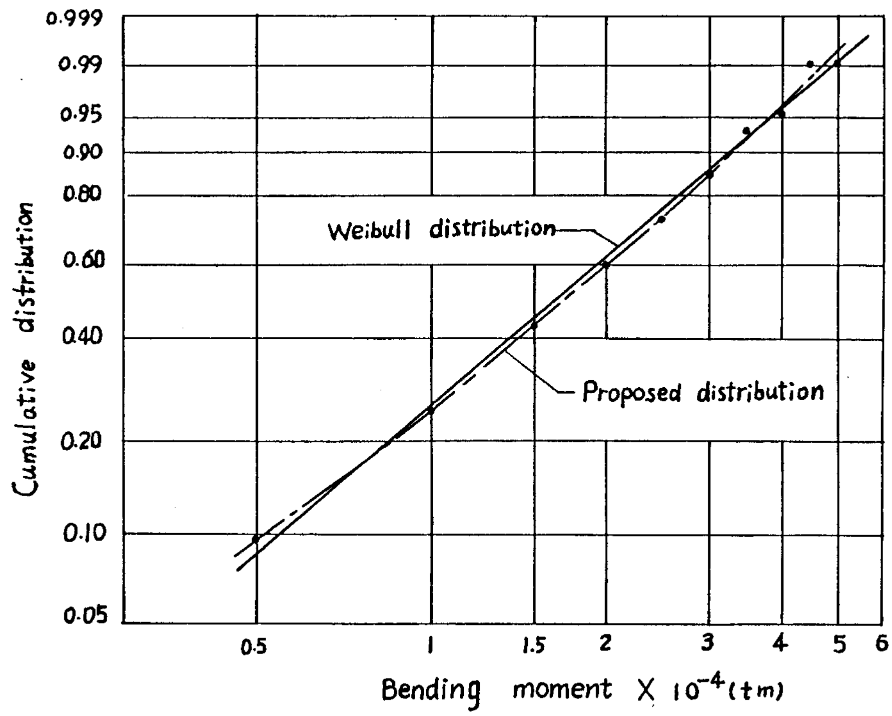

Fig. 9 Calculated cumulative distribution functions plotted on weibull probability paper (hogging)

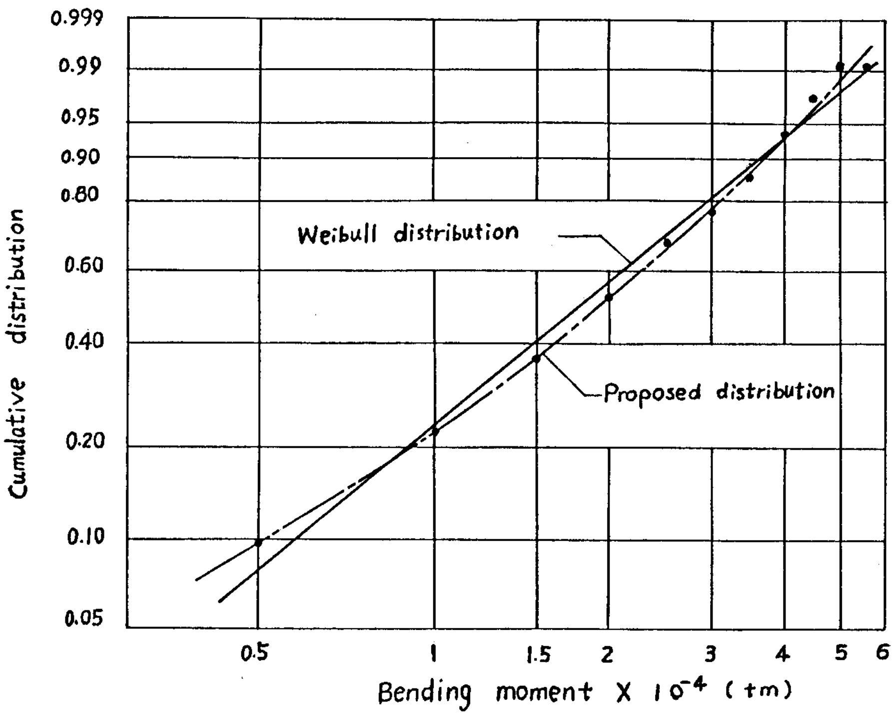

Fig. 10 Calculated cumulative distribution functions plotted on Weibull probability paper (sagging)

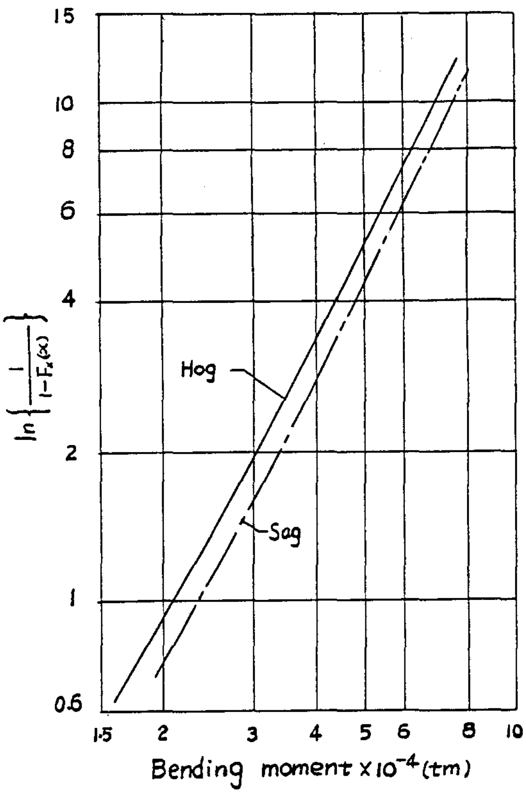

Fig. 11 Prediction of extreme bending moment (wave-induced + slamming) (computation)

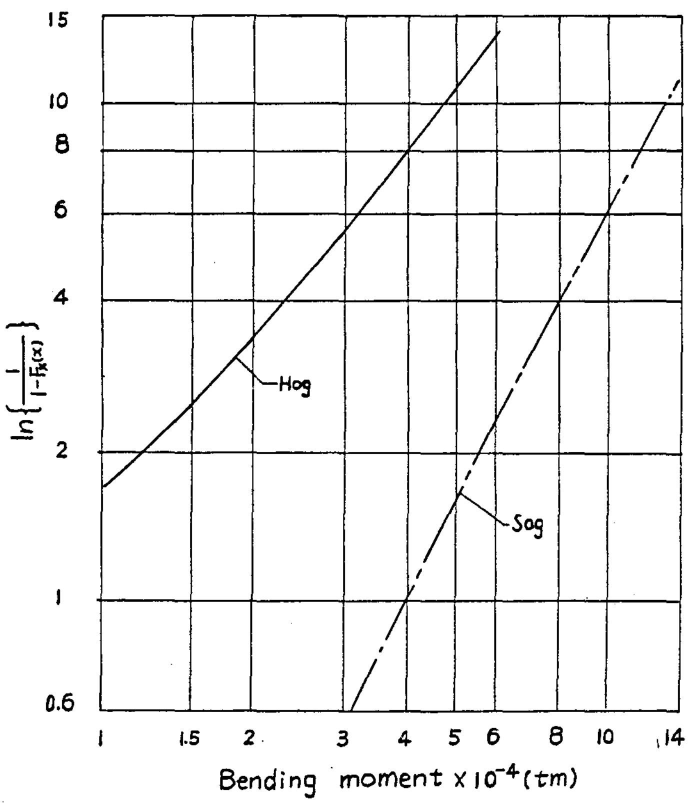

Fig. 12 Prediction of extreme bending moment (experiment)

From Figs. 11 and 12, the extreme responses can be evaluated.

For example, since the average period of wave encounter is 6.944 s (see Table 3), the number of the amplitude of the rapidly varying bending moment expected in 5 hours will be 2592. By taking the logarithm of this number, it is obtained from Fig 11 that the probable extreme value of the sagging rapidly varying bending moment expected in 5 hours will be $6.70 \cdot 10^4$ tm. After adding the third mean part of $1.57 \cdot 10^4$ tm (sag) to the above value, the probable extreme total sagging bending moment expected in 5 hours, excluding stillwater and thermal responses, will be $8.27 \cdot 10^4$ tm. On the other side, from the expected number of cycles (2592) and Fig. 12, a corresponding value of $10.50 \cdot 10^4$ tm is obtained.

As can be seen in comparison, the agreement between computation and experiment is satisfactory in general. Some differences are caused not only by some approximations in the method presented in this paper, but also by the following factors: The vibratory characteristics used in the computation are different from the measured values; equation (50) and computed values refer to the midship section, whereas experimental results apply at the ship's centre of gravity; and the duration of each model experiment in irregular waves is only 15 minutes which may be too short to accurately analyse the response including slamming /2/.

9. Conclusions

A procedure has been presented for combining the wave load components including slamming and estimating the extreme total load. The procedure is then applied to the S-175 container ship travelling at a Froude number of 0,275 in a specially chosen irregular head seaway. The computation is compared with model experiments, and it is shown that the agreement is satisfactory in general.

It should be pointed out that, for practical predictions, some important factors, such as the frequency of occurence of seaways of different severities, the shape of wave spectra, the persistence of each sea state and the ship's speed in a seaway etc., have to be considered.

The following problems should be further investigated:

(a) The definition of the slamming excitation.

(b) The determination of realistic structural damping data.

(c) The bending moment caused by ship's own wave system.

10. Acknowledgement

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