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A remeshing approach for the finite cell method applied to problems with large deformations

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The finite cell method (FCM) is based on an immersed boundary concept with high-order finite elements. When solving nonlinear problems using the FCM, it is often difficult to reach to the desired load step because of the large distortion of the mesh, particularly when badly broken cells are existing in the mesh. To overcome this problem, a global remeshing strategy is proposed to allow the nonlinear computation to proceed even for very large deformations where the distortion of the cells becomes significant. The core concept is to perform a computation up to a specific deformation state where the distortion of the cells becomes significant. Then, to continue the analysis, a new mesh is introduced. The performance of the proposed method is illustrated using two numerical examples of hyperelasticity.

1 A remeshing strategy for the finite cell method

The robustness of the FCM [1] for large deformation analysis can be significantly improved by applying a global remeshing approach, as proposed in [2]. The basic idea is to carry out the analysis using an initial mesh up to a certain deformation state where the mesh gets largely distorted. Afterwards, a new mesh is created that covers the deformed geometry taking advantage of the fictitious domain approach where the mesh generation is straightforward. Once the new mesh is created, a local radial basis function scheme is applied to interpolate the displacements and the displacement gradients from the old to the new mesh [3]. Then, the analysis is continued until a new mesh needs to be created. This process is repeated multiple times until the final desired load step is reached.

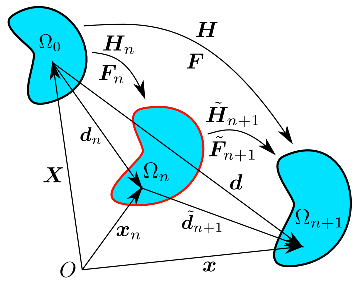

Fig. 1: Configurations involved in one remeshing step.

1.1 kinematics

In this section, the kinematics involved in the remeshing approach are briefly explained. Starting from an initial configuration Ω_0, the body is deformed until an intermediate configuration Ω_n is reached where a new mesh needs to be created. To this end, the displacement gradient and the deformation gradient can be computed as follow

$$\boldsymbol{H}_n = \frac{\partial \boldsymbol{d}_n}{\partial \boldsymbol{X}}, \qquad \boldsymbol{F}_n = \frac{\partial \boldsymbol{x}_n}{\partial \boldsymbol{X}} = \boldsymbol{H}_n + \boldsymbol{I}. \tag{1}$$

Here, $\boldsymbol{d}_n$ denotes the displacements leading to configuration Ω_n, while $\boldsymbol{x}_n$ and $\boldsymbol{X}$ refer to the current and initial position of the material points, respectively. Next, the configuration Ω_n can be further deformed until it reaches the configuration Ω_{n+1}, as can be shown in Fig. 1. For Ω_n, a new mesh can be created. In doing so, the total displacement gradient can be computed by applying the chain rule

$$\boldsymbol{H} = \frac{\partial \boldsymbol{d}}{\partial \boldsymbol{X}} = \frac{\partial (\boldsymbol{d}_n + \tilde{\boldsymbol{d}}_{n+1})}{\partial \boldsymbol{X}} = \frac{\partial \boldsymbol{d}_n}{\partial \boldsymbol{X}} + \frac{\partial \tilde{\boldsymbol{d}}_{n+1}}{\partial \boldsymbol{x}_n}\frac{\partial \boldsymbol{x}_n}{\partial \boldsymbol{X}} = \boldsymbol{H}_n + \tilde{\boldsymbol{H}}_{n+1}\left(\boldsymbol{H}_n + \boldsymbol{I}\right), \tag{2}$$

where $\tilde{\boldsymbol{d}}_{n+1}$ becomes the primary unknown. The total deformation gradient is computed as $\boldsymbol{F} = \tilde{\boldsymbol{F}}_{n+1}\,\boldsymbol{F}_n$. Here, the quantities $\tilde{\boldsymbol{H}}_{n+1}$ and $\tilde{\boldsymbol{F}}_{n+1}$ can be computed based on the new mesh. However, the quantities $\boldsymbol{H}_n$, $\boldsymbol{F}_n$ and $\boldsymbol{d}_n$ of the old mesh need to be interpolated since the new mesh does not carry any information about the old mesh.

1.2 Condition number

In order for the remeshing approach to perform well, the large condition number of the stiffness matrix caused by broken cells needs to be reduced. This is done using two approaches. Firstly, an artificial soft material is introduced in the fictitious domain by setting the indicator function to $\alpha = 10^{-5}$, see [2]. Secondly, a basis function removal scheme is applied to remove high-order basis functions that have a small support in the physical domain [4].

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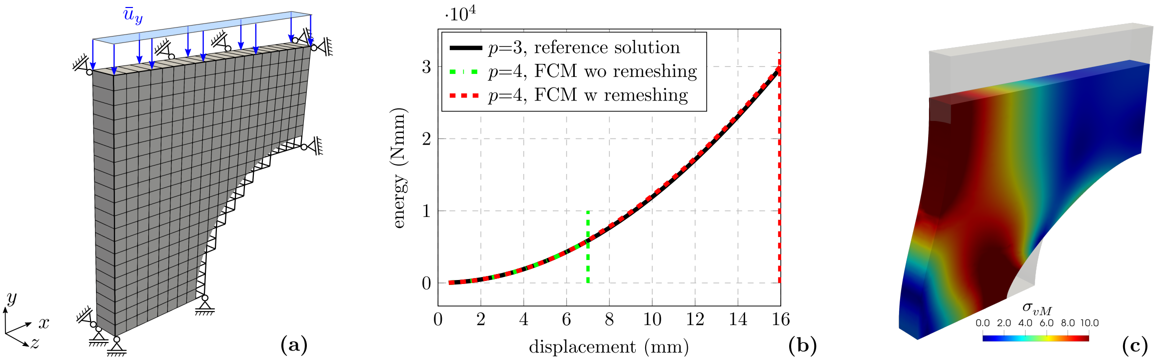

Fig. 2: Plate with a cylindrical hole. (a) FCM model and mesh. (b) Energy-displacement curves. (c) von Mises stress.

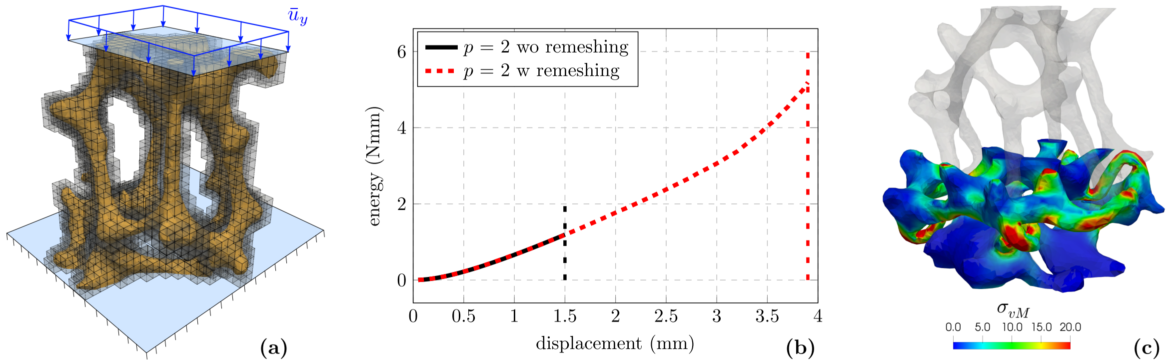

Fig. 3: Single pore of a foam. (a) FCM model and mesh. (b) Energy-displacement curves. (c) von Mises stress.

2 Numerical examples

In this section, we investigate two numerical examples using a hyperelastic material model [2] to demonstrate the performance of the presented method. In the first example, we consider a plate with a cylindrical hole. The geometry, mesh, and boundary conditions are shown in Fig. 2 (a). The geometry is modeled using 3420 elements to generate a reference solution with ansatz order of $p = 3$. In the FCM, 302 cells are used with ansatz order of $p = 4$. Next, we apply a number of load steps without remeshing and only a displacement of 7 mm can be reached. However, applying one remeshing step we can reach the last load step of 16 mm, as can be seen in Fig. 2 (b). Furthermore, a good agreement to the reference solution can be observed after remeshing which indicates that the interpolation of the data is accurate enough. In the second example, we consider a pore of a foam. The geometry, mesh and boundary conditions are shown in Fig. 3 (a). The geometry is discretized with 2721 cells using ansatz order of $p = 2$. The goal is to compress the foam and see how much it can be deformed. To this end, by plotting the energy-displacement curves in Fig. 3 (b), one can see that without remeshing only a displacement of 1.5 mm can be achieved, while utilizing the remeshing a deformation of 3.9 mm can be reached which is a factor of about 2.6 higher.

3 Conclusions

In this paper, we presented a remeshing approach for the FCM to improve its robustness for large deformation analysis. The main concept is to create a new mesh whenever the old mesh gets largely distorted. Afterwards, an interpolation scheme is applied based on a radial basis function to transfer the data between the old and the new mesh. By means of two numerical examples of hyperelasticity it is demonstrated that the proposed remeshing strategy helps to significantly improve the robustness of the FCM for large deformation analysis.

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