Received: 15 July 2020 | Accepted: 5 November 2020

DOI: 10.1002/pamm.202000307

Study on the Interaction of Nonlinear Water Waves considering Random Seas

Marten Hollm[1,*], **Leo Dostal**[1], **Hendrik Fischer**[1], and **Robert Seifried**[1]

[1] Institute of Mechanics and Ocean Engineering, Hamburg University of Technology, 21073 Hamburg, Germany

The nonlinear Schrödinger equation plays an important role in wave theory, nonlinear optics and Bose-Einstein condensation. Depending on the background, different analytical solutions have been obtained. One of these solutions is the soliton solution. In the real ocean sea, interactions of different water waves can be observed at the surface. Therefore the question arises, how such nonlinear waves interact. Of particular interest is the interaction, also called collision, of solitons and solitary waves.

Using a spectral scheme for the numerical computation of solutions of the nonlinear Schrödinger equation, the nonlinear wave interaction for the case of soliton collision is studied. Thereby, the influence of an initial random wave is studied, which is generated using a Pierson-Moskowitz spectrum.

1 Description of extreme water waves

A closer look at the ocean sea reveals that the nature of water waves is random. Irregularities like wind, swell and currents lead to a stochastic sea surface, which makes it hard to predict the amplitudes of the future incoming waves. Although most of the time the waves are not high enough to damage ships or offshore structures, extreme waves can occur. These pose a huge risk for these vessels and all persons present. Up to now, the origin of extreme waves is not fully understood and further studies are needed. In recent years the influence of stochastic wind on water waves has been studied [1]. Also it was investigated how waves change their behavior after a collision with other waves [2].

In a study of water waves, Dias et al. [3] have shown that in many cases it is enough to consider the Euler equation of fluid dynamics instead of the more complicated Navier-Stokes equations. But since solving the Euler equations is time-consuming as well, a further problem reduction has be done in our study. By using the method of multiple scales and taking into account terms up to order $\mathcal{O}(\varepsilon^3)$, the Nonlinear Schrödinger equation (NLS) can be derived in deep water as [1]

$$\mathrm{i}\psi_\tau = \alpha\psi_{\xi\xi} + \beta\,|\psi|^2\,\psi, \tag{1}$$

whereby, $\alpha = \frac{\omega}{8k^2}$, $\beta = \frac{1}{2}\omega k^2$ and the scaled spatial and temporal coordinates are given by $\xi = \varepsilon(x - c_g t)$ and $\tau = \varepsilon^2 t$. Thereby, $\psi(\xi, \tau) \in \mathbb{C}$ describes the wave envelope, $\varepsilon \ll 1$ is the wave steepness, x and t are the dimensional spatial and temporal coordinate, k is the wave number, ω is the frequency of the carrier wave and $c_g = \frac{\omega}{2k}$ is the deep water group velocity. Is has to be noted that in comparison to [1], any wind pressure or viscosity effects are here neglected. The corresponding wave period T, wavelength λ and free surface elevation η of a weakly nonlinear gravity wave are given by

$$T = \frac{2\pi}{\omega}, \qquad \lambda = \frac{g}{2\pi}T^2, \qquad \eta(x,t) = \varepsilon\mathrm{i}\frac{\omega}{g}\psi(x,t)\exp(\mathrm{i}(kx - \omega t)) + c.c. + \mathcal{O}(\varepsilon^2), \tag{2}$$

whereby g is the gravity constant and $c.c.$ denotes the complex conjugate.

One solution of Eq. (1) is the soliton solution. According to [4], a soliton solution which has an amplitude a_0 traveling in space with velocity v is given by

$$\psi(\xi,\tau) = a_0\mathrm{sech}\left[a_0\sqrt{\frac{\beta}{2\alpha}}(\xi - \xi_0 - v\tau)\right]\exp(\mathrm{i}(c\xi - w\tau)) \tag{3}$$

with a shift in space ξ_0 and the parameters $c = \frac{v}{2\alpha}$ and $w = \alpha c^2 - \frac{1}{2}\beta a_0^2$.

2 Collision of soliton solutions

In order to study the collision of different soliton solutions numerically, an initial condition must be used which contains information about the colliding solutions. For this, a superposition of soliton solutions of the form

$$\psi_0(\xi) = \sum_{i=1}^{N}\psi_i(\xi, 0) \tag{4}$$

* Corresponding author: e-mailmarten.hollm@tuhh.de, phone +00 49 40 42878 2308, fax +00 49 40 42878 2028

is used, whereby $\psi_i(\xi,0)$ is given by Eq. (3) and characterized by own velocity v^i, shift in space ξ_0^i and amplitude a_0^i.

In order to model the sea surface as realistic as possible, an irregular sea surface is added to the colliding solitons. A well-known model of random long-crested sea waves is given by the superposition of harmonic waves with frequencies ω, wave numbers $k(\omega)$ and amplitude A, which depends on the underlying sea state given by the corresponding one-sided spectral density $S(\omega)$. According to [2], the irregular sea surface is determined by

$$Z(\xi,\tau) = \int_0^\infty \cos\left(\omega\tau - \kappa(\omega)\xi + \varepsilon(\omega)\right)\sqrt{2S(\omega)\mathrm{d}\omega}, \tag{5}$$

whereby the integral is not a Riemann integral but a summation rule over different ω. The perturbation of the undisturbed solution can then be achieved by

$$\widetilde{\psi}(\xi,\tau) = (1+\Theta Z(\xi,\tau))\psi(\xi,\tau), \tag{6}$$

whereby Θ regulates the amount of perturbation.

An initial condition of two soliton waves with the same amplitude $a_0 = 1\,$m, speed $v = \pm 1$ m/s and frequency $\omega = 1$ rad/s is shown in Fig. 1 with and without perturbation of an irregular sea surface, respectively. The corresponding numerical solutions are shown in Fig. 2. These results have been computed by using finite differences in time and a spectral scheme in space as well as the Pierson-Moskowitz spectrum $S(\omega)$. Figure 2a illustrates that a significant wave elevation can be observed around the time of collision. After the collision the solitons regain their structure. An initial disturbance by irregular waves as shown in Fig. 2b does not change this behavior much and only leads to perturbed solitons.

3 Conclusion

The obtained results indicate that soliton solutions can exist in irregular seas. An interaction leads to higher waves and the soliton collision can appear also under realistic sea conditions.

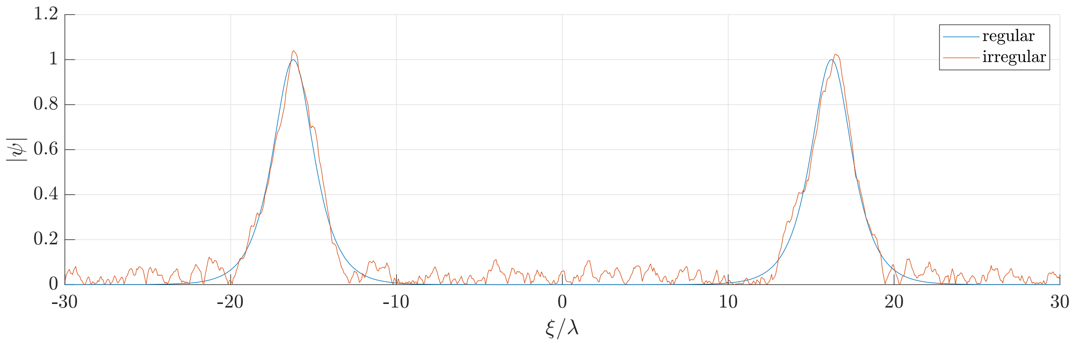

Fig. 1: Initial condition for the soliton wave interaction with and without disturbance. Thereby, the Pierson-Moskowitz spectrum has been used with significant wave height $H_s = 1.2\,$m and the modal frequency $\omega_m = 0.5\,$rad/s. The scaling factor Θ was chosen as $\Theta = 0.2$.

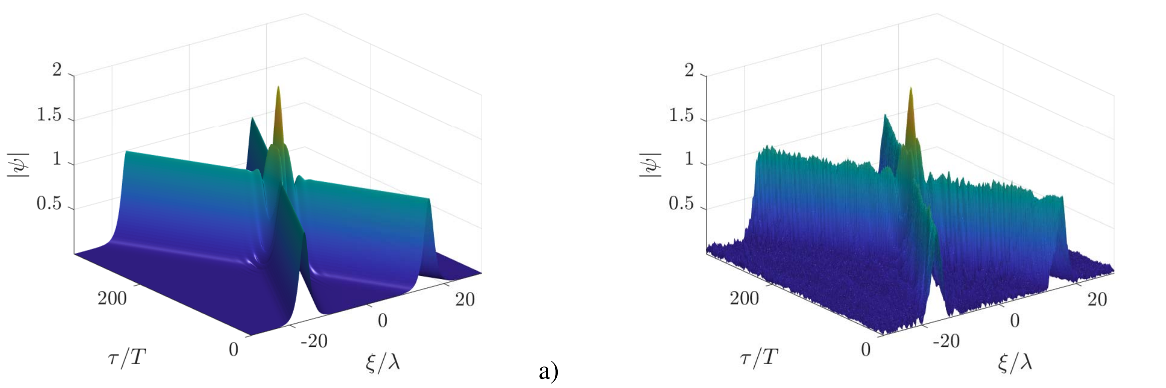

Fig. 2: Soliton collision **a** without and **b** with perturbations by a random sea. In both cases, the initial condition from Fig. 1 has been used.

Acknowledgements Open access funding enabled and organized by Projekt DEAL.

References

[1] L. Dostal, M. Hollm and E. Kreuzer, Nonlinear Dyn. **99(3)**, 2319-2338 (2020).

[2] H. Fischer, M. Hollm and L. Dostal, Soliton Collision in Random Seas, submitted, preprint available at arXiv.
[3] F. Dias, A. I. Dyachenko and V. E. Zakharov, Phys. Lett. A **372(8)**, 1297–1302 (2008).
[4] W. Bao, Q. Tang and Z. Xu, J. Comput. Phys. **235**, 423–445 (2013).

