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Stable Inversion for Flexible Multibody Systems Using the ANCF

Svenja Drücker[1,*] and **Robert Seifried**[1]

[1] Hamburg University of Technology, Institute of Mechanics and Ocean Engineering, Eißendorfer Str. 42, 21073 Hamburg, Germany

Flexible robots are often non-minimum phase systems. Thus, their inverse model must be computed by stable inversion. Here, a simplification to the stable inversion process is proposed, which enables its application to complex underactuated multibody systems. As an example, a flexible manipulator modeled by the absolute nodal coordinate formulation is considered.

1 Introduction

Modern light-weight robots are highly flexible systems. The flexible structure increases requirements on accurate control strategies and two-design degree of freedom control is a popular choice for such systems. A feedforward controller is responsible for large motion tracking, while a feedback controller makes the system robust against parameter uncertainties and disturbances. Ideally, the feedforward controller is an inverse model of the real system since it cancels out all known nonlinear dynamics. However, the derivation of an inverse model is not straight-forward for flexible multibody systems. The system dynamics is complex and analytical derivations are often burdensome. Moreover, the systems are often non-minimum phase, meaning their internal dynamics is unstable. Thus, the concept of stable inversion must be applied to obtain an inverse model. In this contribution, a simplification of the stable inversion problem is demonstrated, which makes it applicable to complex systems, such as manipulators modeled by the absolute nodal coordinate formulation (ANCF).

2 Stable Inversion for Flexible Multibody Systems

For underactuated multibody systems described by the generalized coordinates $\boldsymbol{y}$ and with system output $\boldsymbol{z}$, the inverse model for tracking the desired output trajectory $\boldsymbol{z}_\mathrm{d}(t)$ can be represented by the differential-algebraic equations (DAEs)

$$\boldsymbol{M}(\boldsymbol{y},t)\ddot{\boldsymbol{y}} + \boldsymbol{k}(\boldsymbol{y},\dot{\boldsymbol{y}},t) = \boldsymbol{q}(\boldsymbol{y},\dot{\boldsymbol{y}},t) + \boldsymbol{B}\boldsymbol{u} \tag{1}$$

$$\boldsymbol{s}(\boldsymbol{y},t) = \boldsymbol{z}(\boldsymbol{y}) - \boldsymbol{z}_\mathrm{d}(t) = \boldsymbol{0}\,. \tag{2}$$

Thereby, $\boldsymbol{M}$ is the generalized mass matrix, $\boldsymbol{k}$ is the vector of Coriolis, centrifugal and gyroscopic forces, $\boldsymbol{q}$ is the vector of applied forces and $\boldsymbol{B}$ is the distribution matrix of the input $\boldsymbol{u}$. The servo-constraints $\boldsymbol{s}$ force the system output to follow the predefined trajectory $\boldsymbol{z}_\mathrm{d}(t)$. The DAEs can be integrated forward in time to compute the inverse model of minimum phase systems [1]. However, an integration forward in time is not possible for non-minimum phase systems due to unstable internal dynamics. Stable inversion is proposed in [2] to compute the inverse model of non-minimum phase systems. It is proposed to define a boundary value problem to obtain a bounded solution to the unstable internal dynamics, which is described by the coordinates $\boldsymbol{\eta}$. The boundary conditions at initial simulation time T_0 and final time T_f are

$$\boldsymbol{B}_\mathrm{u}\left(\boldsymbol{\eta}(T_0) - \boldsymbol{\eta}_{\mathrm{eq},0}\right) = 0 \quad \text{and} \quad \boldsymbol{B}_\mathrm{s}\left(\boldsymbol{\eta}(T_f) - \boldsymbol{\eta}_{\mathrm{eq,f}}\right) = 0 \tag{3}$$

with the matrices $\boldsymbol{B}_\mathrm{s}$ and $\boldsymbol{B}_\mathrm{u}$ containing the eigenvectors of the stable and unstable eigenspaces of the internal dynamics linearized at the equilibrium denoted by the index *eq*. However, the tedious derivation of the eigenspaces makes the approach difficult to apply to complex systems. It is proposed to simplify the boundary conditions as

$$\boldsymbol{L}_0\,\boldsymbol{\eta}\,(T_0) = \boldsymbol{L}_0\,\boldsymbol{\eta}_{\mathrm{eq},0} \quad \text{and} \quad \boldsymbol{L}_f\,\boldsymbol{\eta}\,(T_f) = \boldsymbol{L}_f\,\boldsymbol{\eta}_{\mathrm{eq,f}}\,. \tag{4}$$

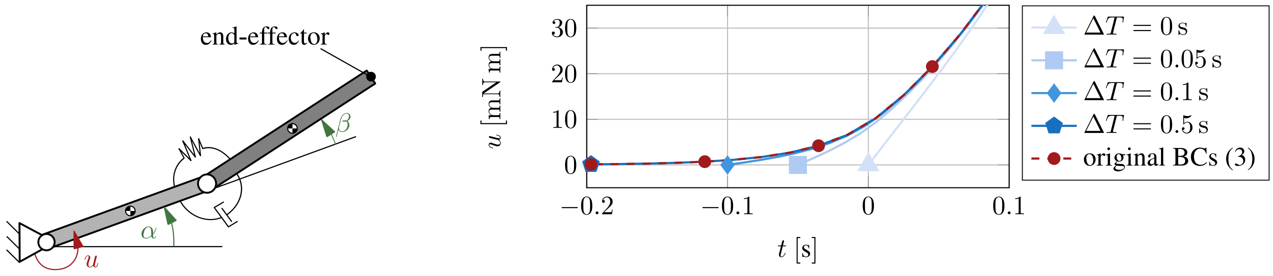
The binary matrices $\boldsymbol{L}_0$ and $\boldsymbol{L}_f$ select a number of states to be equal to the initial equilibrium and a number of states to be equal to the final equilibrium, such that the total number of boundary conditions matches the number of states $\boldsymbol{\eta}$. Moreover, they can be reformulated in the original coordinates $\boldsymbol{y}$. Thus, there is no need to derive the internal dynamics explicitly.

3 Application Examples

First, a manipulator with one passive joint is considered, see Fig. 1(a). Its system output z is defined as the angle between the end-effector and the horizontal. Due to simplicity of the system, its internal dynamics can be derived analytically [1]. Thus,

* Corresponding author: svenja.druecker@tuhh.de, +49 (0)40 42878-4895

a comparison between the stable inversion problem using the original boundary conditions (3) and the simplified boundary conditions (4) is possible. For the inversion, the desired trajectory $z_{\mathrm{d}}(t)$ is chosen as a smooth transition from $z(t_0 = 0\,\mathrm{s}) = 0°$ to $z(t_f = 1\,\mathrm{s}) = 30°$. The inverse model is computed using the stable inversion approach for different simulation time intervals $[t_0 - \Delta T\,;\, t_f + \Delta T]$ with $t_0 - \Delta T = T_0$ and $t_f + \Delta T = T_f$. Increasing ΔT shows convergence of the solution using simplified boundary conditions to the solution using correct boundary conditions, see Fig. 1(b). Thereby, only the beginning of the resulting input trajectory is shown since it shows the differences between the two solutions. It is noted that the convergence speed at time t_0 and t_f is given by the positive and negative eigenvalues of the internal dynamics, respectively.

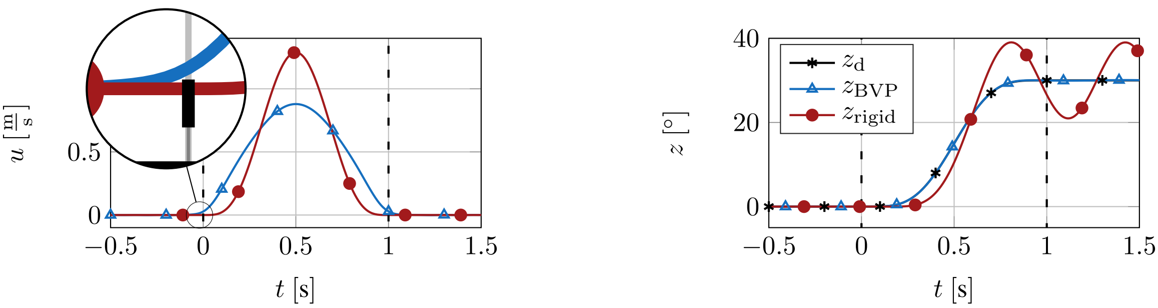


(a) Robot model.

(b) System input u for different pre- and postactuation phases ΔT.

Fig. 1: Simulation results for a manipulator with one passive joint.

The manipulator with one passive joint may not be able to accurately model a highly flexible manipulator. Thus, the same manipulator is now modeled using four ANCF elements with the beam model of [3]. Due to the complex equations of motion, an analytical derivation of the internal dynamics and therefore the derivation of the correct boundary conditions (3) is not possible. Thus, the simplified boundary conditions (4) are applied to solve the stable inversion problem. The simulation results show the system input in Fig. 2(a) and the simulated system output in Fig. 2(b). Thereby, the index *BVP* denotes the solution from the stable inversion problem. The index *rigid* denotes the solution obtained from inverting an equivalent rigid manipulator and applying the input u_{rigid} to the flexible manipulator in a forward time simulation. The results show that an inversion of an equivalent rigid system is not sufficient for accurate tracking, since an oscillation of the end-effector remains. When taking the flexible model into account in the inversion process, very good tracking performance can be accomplished.



(a) Computed system input u_{d} from stable inversion.

(b) Output z from a forward simulation with input u_{d}.

Fig. 2: Simulation results for a flexible manipulator modeled by four ANCF elements.

4 Conclusion

In this contribution, simplified boundary conditions for the stable inversion problem are proposed. Convergence results for a simple manipulator with one passive joint support the use of such simplified conditions. They enable the application of the stable inversion method to more complex systems, for which the internal dynamics cannot be derived analytically.

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