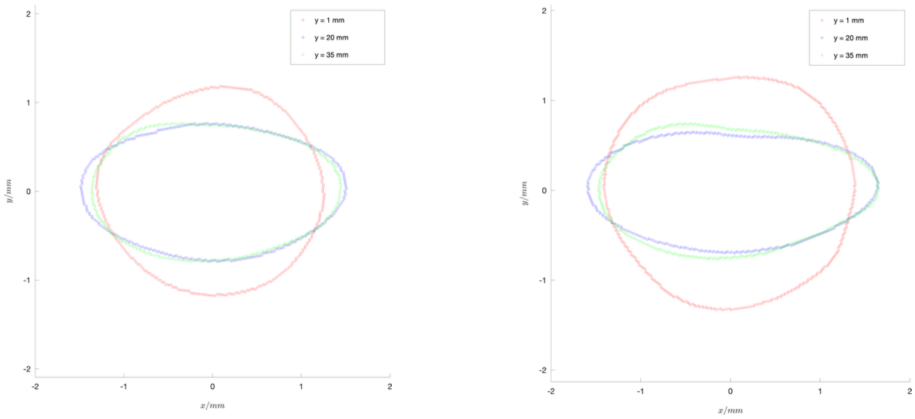
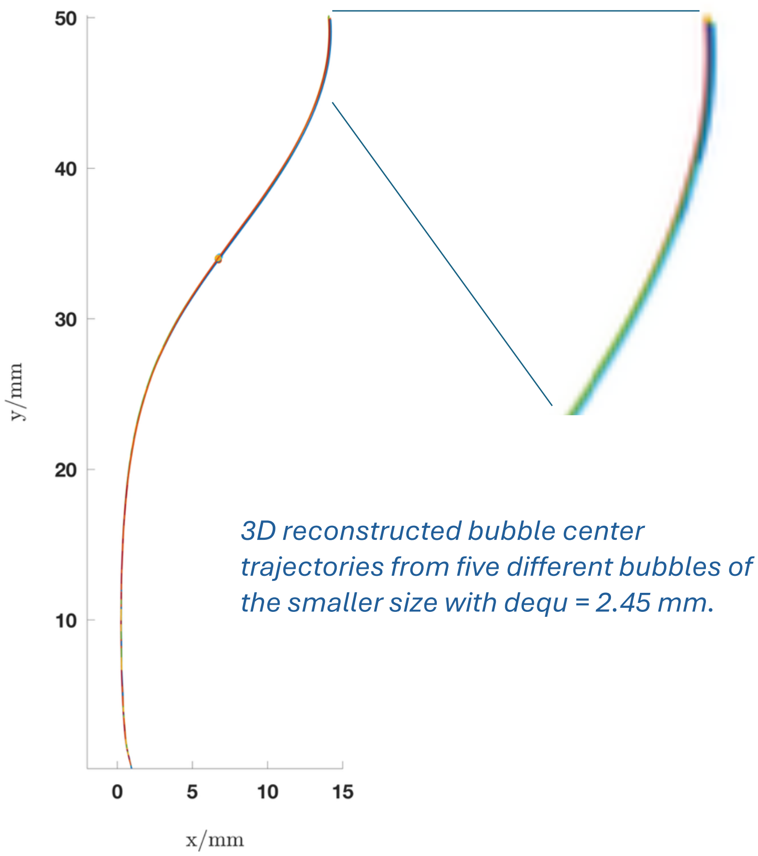
We differentiate in between the initial phase, when the bubble is still influenced by detachment and the developed phase after approx. 15-20 mm as Kong et al. (reference [9] in article) and this gives a more correct wobbling frequency. The developed rising phase was reached when the bubble contour clearly had an elliptic shape and did not accelerate anymore:



See the .gif movies of the bubbles rising on all four cameras for both bubble sizes and the lower measurement volume.

Subsequent different bubbles show identical rising behavior: See passage in the text in section 3.1., where the standard deviation of the inflection points from five different bubble-center trajectories is shown.



3D reconstructed bubble center trajectories from five different bubbles of the smaller size with dequ = 2.45 mm.

The data agrees well with the data of the Clift diagram as shown below. The bubbles rise as in other experiments with uncontaminated, clean water.

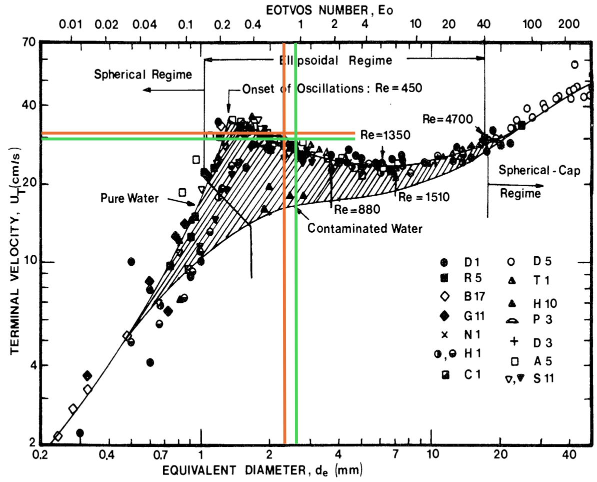
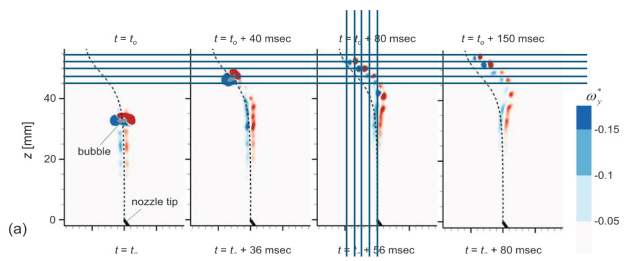


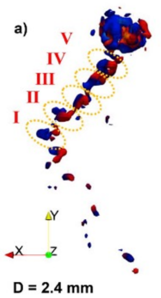
FIG. 7.3 Terminal velocity of air bubbles in water at 20°C.

Direct comparison of the distances in between the vortex cores with existing literature is in the manuscript for a comparison with the numerical work of Cano-Lozano in section 3.3.
The inter-vortex distance Λ (in this article called vortex structure wavelength) from the cited experimental study of Chang et al. and is more difficult to compare to, because the article does not provide a good scale along the corresponding figures but roughly it seems to be of the same order. It is possible to approximately derive the inter-vortex distance from figure 6 in the work of Lee and Parker, however. We find it to be slightly lower than in our case yielding $\Lambda = 2.5$ mm for their single bubble of d_b = 2.41 mm (see discussion in the article in section 3.3.)

The vortex sizes are not readily comparable for the mentioned studies as they depend on the quality of the experimental data and the threshold of the visualization of the second-generation vortex measures. Here, references to more modern, third generation vortex detection methods for future studies have been added when the Q-criterium is introduced in section 3.3.



Adapted from J. Lee, H. Park, Flow induced by the single-bubble chain depending on the bubble release frequency, Physics of Fluids 34 (3) (2022) 033312.



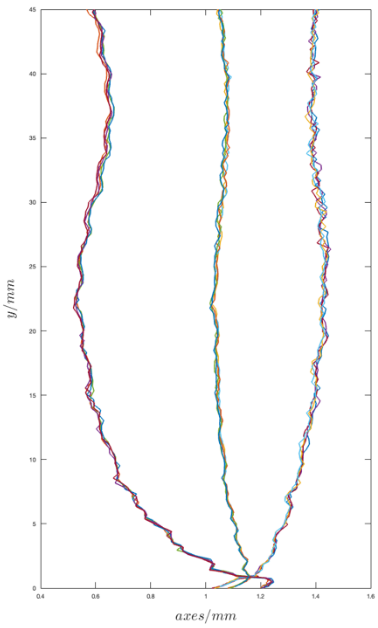
Y. Chang, C. Müller, P. Kováts, L. Guo, K. Zähringer, Hydrodynamics and shape reconstruction of single rising air bubbles in water using high-speed tomographic particle tracking velocimetry and 3d geometric reconstruction, Exp Fluids 65 (1) (2023) 6. doi:10.1007/s00348-023-03746-0.

The equivalent radius from the 2D-camera images slightly surpasses the estimated value of $r_b = 0.5\, d_b = 1.225$ mm over the whole bubble rise in the case of the smaller bubble (figure 3(d)). On the contrary, the estimated value from the 2D-camera projections slightly falls short of $r_b = 0.5\, d_b = 1.31$ mm over the whole bubble rise in the case of the larger bubble (figure 4(d)). For both figures, the axes and the equiv. diameter are determined from taking the average over all four camera views along the bubble rise.

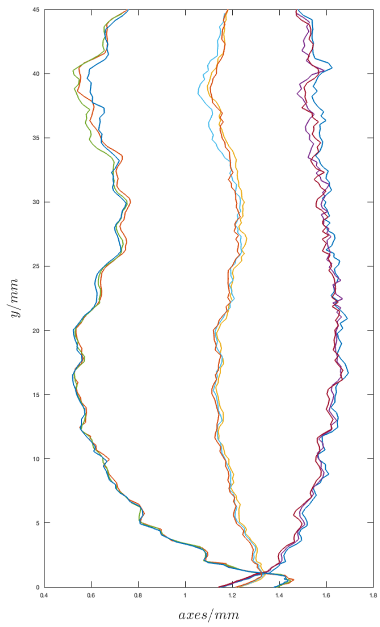
For the 3D bubble reconstruction, the reconstructed bubble in object space was projected onto the x-y plane to fit an ellipse to this projection. The bubble was then approximated to be axisymmetric and of oblate shape. This assumption, the bubble being axisymmetric, made for the 2D and the 3D bubble size estimates, however, is not valid along the full bubble rise, since the bubble has three wobbling axis which also rotate regarding the x-y plane. This assumption is only reasonable, when the bubble is still nearly spherical, right after the bubble detachment. Therefore, the bubble size was approximated at this detachment point for both methods.

The equivalent diameter was read out right after bubble detachment from both bubble reconstructions (2D-> $d_{\mathrm{b}} = 2.58$ mm and 3D-> $d_{\mathrm{b}} = 2.32$ mm) and the average value is stated in the article-> $d_{\mathrm{b}} = 2.45$ mm (table 2).
The same was done for the larger bubble. Here, the equivalent diameter from 2D-> $d_{\mathrm{b}} = 2.60$ mm and 3D-> $d_{\mathrm{b}} = 2.64$ mm) and the mean was the value stated in the article-> $d_{\mathrm{b}} = 2.62$ mm. The mean standard deviation of the equivalent diameter of different bubbles at along the height is below $\sigma = 0.022$ mm for both bubble sizes and both techniques (now stated in the text on page 11).

3D reconstruction for the lower field of view:

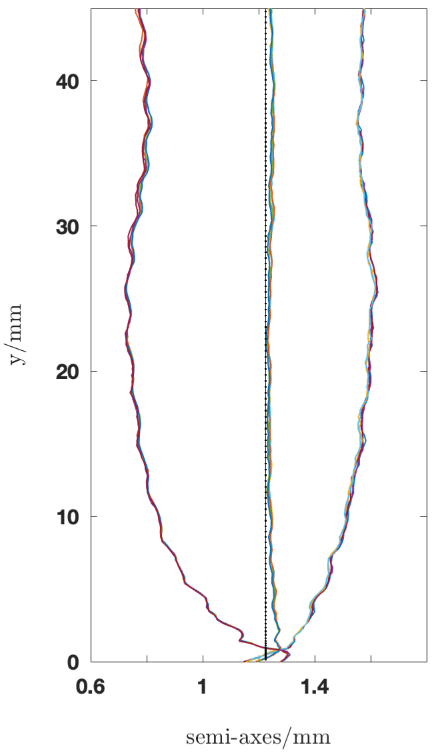


Small bubble semi-axes and equiv. diameter for 3d reconstr. for five bubbles

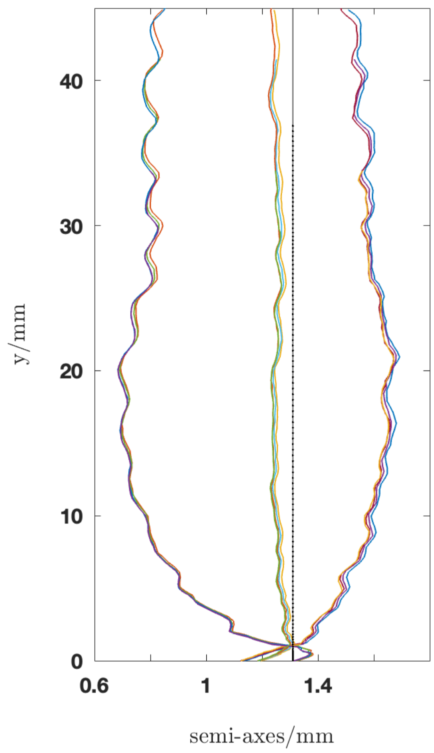


Large bubble semi-axes and equiv. diameter for 3d reconstr. for three bubbles

2d reconstruction for the lower field of view:



Small bubble semi-axes and equiv. diameter for 2d reconstr. for five bubbles



Large bubble semi-axes and equiv. diameter for 3d reconstr. for four bubbles

The bubble volume should be constant over the whole bubble trajectory, since there was no mass transfer because the air bubbles rose in air saturated bidistilled water. The variation of the bubble volume with height is negligible since pressure change is only of approx. 0.008 bar. This corresponds to a bubble volume change of $\Delta V = 0.061\ \mathrm{mm}^3$ resulting in a change in equivalent diameter of $\Delta R = 0.0033\, mm$ over the full measurement height (80 mm) using the ideal gas equation as an approximation. Any change in equivalent diameter over height should therefore be an effect of the bubble motion, which causes the angle in between the projection plane of the bubble's major axis and the camera's x-y plane to vary, or a measurement error.
The volume flow rate of the syringe pump could not be used for the determination of the exact bubble volume calculation. The bubble generation was very periodic but every now and then no bubble was produced. Also, the pressure drop across the needle is unknown and due to the compressibility of the air it would have to be considered. We therefore opted to estimate the bubble size from the measurement of the bubble. Also, similar experiments have chosen the same route (Lunde and Perkins, 1998, [8] in the manuscript).

Flow chart – scripts for track treatment