Cable Dynamics – A Review

U. Starossek
Civil Eng., J. Muller Int., San Diego, CA, USA

Summary

The history of the theory of cable vibrations from the 18th century to the present is reviewed. Basic equations of linear dynamics of an extensible sagging cable are presented. A dynamic stiffness function that can be derived from these equations is given. Limiting transitions lead to other well known results: The static equivalent modulus of elasticity given by Ernst as well as the natural frequencies of motion given by Irvine and Caughey can be inferred from this function. A discussion of dynamic-excitation mechanisms and of dynamic interaction between cables and other structural elements conclude this contribution.

Introduction

The vibrating taut string was one of the first physical systems to which the new analytical tools of modern mechanics and mathematics were applied. Treatises on this problem were presented by Brook Taylor, d'Alembert, Euler, Johann and Daniel Bernoulli during the first half of the eighteenth century [41].

By 1788 Lagrange and others before had reached solutions of varying degrees of completeness for the vibrations of an inextensible, massless string, fixed at each end, from which numerous weights were hung. This corresponds to a discretization of the cable continuum. The general partial differential equations of the motion of a cable element under the action of a general force system were given by Poisson in 1820 [14]. However, apart from Lagrange's work on the equivalent discrete system, solutions for the sagging cable were unknown at that time.

In 1868, after preliminary work by Stokes and Röhrs, Routh gave exact solutions for an inextensible sagging cable which hung in a cycloid [29]. Cable elasticity was considered for the first time by Klöppel and Lie [18] in 1942. In 1949, Pugsley [27] proposed a semi-empirical theory for the natural frequencies of cables with great sag ($1{:}10 \leq$ ratio of sag to span $\leq 1{:}4$). In 1953, by assuming again that the cable was inextensible, Saxon and Cahn [31] gave theoretical solutions for cables with great sag.

However, until the 1970s there had neither theoretical nor experimental work been done which dealt with a remarkable discrepancy between the theories known by then: When reducing the sag to zero, the frequencies of the symmetric in-plane modes of an inextensible sagging cable do not equal the corresponding natural frequencies of a taut string. As shown by Irvine and Caughey [14] in 1974, description of that transition range requires consistent inclusion of cable elasticity. Their work [14] revealed an extensive comprehension of the linear theory of free vibrations of a rigidly supported horizontal cable with a ratio of sag to span from approximately 1:8 to zero (*horizontal* means that the cable ends are supported at the same level). Their basic and straightforward assumption was that the dynamic cable tension is a function of time alone (i.e., the elastic deformation is assumed to be quasi-static). They pointed out that the dynamic behavior of a cable essentially depends on only one geometric-elastic system parameter. For certain values of this parameter, the so-called 'cross-over' points, the natural frequencies of symmetric in-plane modes and the respective antisymmetric in-plane modes coincide.

Later on, Irvine extended the theory to inclined cables [16]. For this, the weight component parallel to the cable chord was neglected. The same author, jointly with Griffin, also made valuable contributions to the analysis of cable response to dynamic loading as it occurs in the case of support acceleration due to earthquake [15].

A more precise solution for the free vibrations of an inclined extensible and sagging cable was given by Triantafyllou in 1984 [43]. For this, spatial variability of dynamic tension and weight component parallel to chord were taken into consideration. For the frequency curves of *inclined* cables it follows that each 'cross-over' is replaced by an 'avoided crossing' (i.e., nearly a 'cross-over'). Nevertheless, validity of Irvine's theory was confirmed for a wide range of parameters.

The problem of dynamic interaction between cables and other system elements demands investigation of boundary induced cable vibrations. This point was treated by Davenport and Steels [7] in 1965. More refined theories were presented by Veletsos and Darbre [46], and Starossek [38, 39, 40]. In [38, 40], a complete linear theory of boundary induced vibration of a damped cable was given.

All works mentioned so far were limited to the investigation of small displacements (linear theory). A systematic exploration of nonlinear cable dynamics began only recently. As a representative of this new period, Tonis' work [42] can be considered.

The following sections are dedicated to the linear dynamics of an extensible sagging cable. For sake of conciseness, only vibrations within the (vertical) cable plane are considered. Structural and/or fluid-dynamic damping is taken into account. The basic equations are given. A dynamic stiffness function that can be derived from these equations is presented. By means of such functions, the dynamic interaction between system elements and the dynamic behavior of composed systems can be analysed. By limiting transitions, the dynamic stiffness function given here leads to other well known results: The static equivalent modulus of elasticity given by Ernst as well as the natural frequencies of motion given by Irvine and Caughey can be inferred from it. Finally, dynamic-excitation mechanisms and dynamic interaction between cables and other structural elements are discussed.

Basic Equations

A horizontal cable with shallow sag is considered *(Fig. 1)*. Its static form is approximated by a quadratic parabola. The condition of dynamic equilibrium of the vertical forces *(Fig. 2)* leads to the equation of motion

$$H\frac{\partial^2 v}{\partial x^2}+h_\tau\frac{d^2 y}{dx^2}=m\frac{\partial^2 v}{\partial t^2}+c\frac{\partial v}{\partial t} \qquad (1)$$

where

H = the horizontal component of the static cable tension

m = the cable mass per unit length

c = the damping force per unit length and velocity.

The auxiliary quantity h_τ is defined as

$$h_\tau=\tau\frac{dx}{ds} \qquad (2)$$

where τ is the dynamic part of the total cable tension that is supposed to be invariable along the cable (quasi-static elastic deformation). The second basic equation

$$\frac{h_\tau}{EA}\left(\frac{ds}{dx}\right)^3=\frac{dy}{dx}\frac{\partial v}{\partial x}+\frac{\partial u}{\partial x} \qquad (3)$$

provides for the elastic and geometric compatibility of the cable element,

E = Young's modulus of elasticity

A = the effective cross-sectional area of the cable.

When the damping term is omitted and the quantity h_τ is substituted by h (dynamic part of horizontal component of total cable tension), equations (1) and (3) conform to the equations presented by Irvine and Caughey [14]. As shown in [38 and 40], these relations can be derived without limitation of the horizontal displacement u. They can therefore be taken as a basis for the analysis of a cable with displaceable boundaries presented herein.

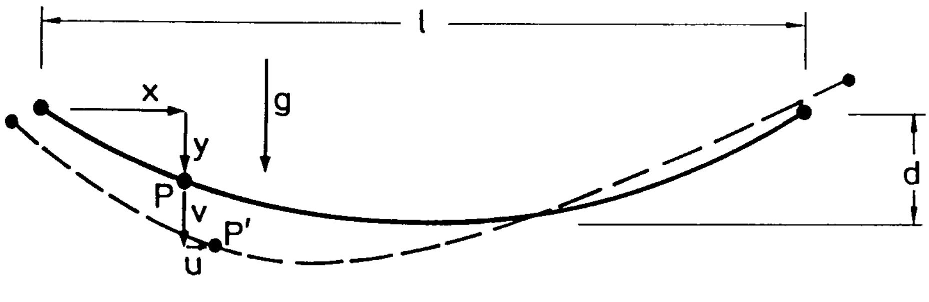


Fig. 1: Horizontal cable

It is assumed that the vibration is described by the products

$$v(x,t)=\tilde{v}(x)e^{i\omega t} \quad , \quad u(x,t)=\tilde{u}(x)e^{i\omega t} \qquad (4)$$

where

$$i^2=-1 \quad , \quad \tilde{u},\tilde{v},\omega\in\mathbb{C}$$

and in which ω is the circular frequency of motion. Consequently, the expression

$$h_\tau(x,t)=\tilde{h}_\tau(x)e^{i\omega t}; \quad \tilde{h}_\tau\in\mathbb{C}$$

will be valid, and analogous product descriptions are valid for the boundary forces and displacements. That is, only harmonic vibrations and vibrations with an exponentially variable amplitude (modified-harmonic vibrations) are considered. With this approach, both the steady-state system response to harmonic excitation as well as damped free vibrations can be investigated.

With the adoption of approach (4), the equation of motion (1) leads to the ordinary differential equation

where

$$i^2=-1 \quad , \quad \tilde{u},\tilde{v},\omega\in\mathbb{C},$$

$$h_\tau(x,t)=\tilde{h}_\tau(x)e^{i\omega t}; \quad \tilde{h}_\tau\in\mathbb{C}$$

$$H\frac{\partial^2\tilde{v}}{\partial x^2}+\omega_c^2 m\tilde{v}=\frac{8d}{l^2}\tilde{h}_\tau \qquad (5)$$

The introduction of the auxiliary parameter

$$\omega_c=\omega\sqrt{1-2\xi i} \qquad (6)$$

where $\xi=\dfrac{c}{2m\omega}$

provides a substantial simplification of further derivations. By utilizing trigonometrical solution functions with complex arguments, they can now be formally carried out as if damping were not present [38, 40].

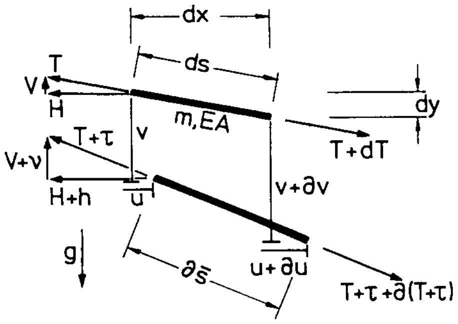


Fig. 2: Differential cable element

From the compatibility condition (3),

$$\frac{\tilde{h}_\tau}{EA}\left(\frac{ds}{dx}\right)^3=\frac{dy}{dx}\frac{\partial\tilde{v}}{\partial x}+\frac{\partial\tilde{u}}{\partial x} \qquad (7)$$

is obtained. In further derivations, displacement function $\tilde{u}(x)$ in equation (7) is usually eliminated by integration.

The equations given so far and the implied results are valid for a horizontal cable. By neglecting the weight component parallel to the chord, however, the entire theory can be extended to an inclined cable. The admissible ratio of sag to span depends on boundary conditions, cable inclination, and material. For a rigidly supported horizontal steel cable, the maximum ratio is approximately 1:8. An application to inclined cables and to cables with displaceable boundaries demands limitation of the sag-to-span ratio to 1:20 or less [38, 39, 40].

Dynamic Stiffness Functions

Static analysis of mechanical systems usually requires knowledge of the load-deformation behavior of the system elements. This behavior can be described in compact form by stiffness matrices. Limited to the steady-state response, it is possible to transfer this concept to the investigation of *dynamic* processes, which implies the development of dynamic stiffness matrices [4]. The coefficients of these matrices are time-independent relations between boundary forces and boundary displacements of the element as a part of a vibrating system. They are functions of the frequency of motion. From equations (5) and (7), a dynamic stiffness matrix for the sagging cable can be derived which is suitable for dy-

$$K_{11}^{*}=\frac{K_{11}}{K_{11}^{t,e}}=\frac{l}{L_e}\left(\frac{\left[1+\frac{1}{2}\varepsilon\tan\alpha\left(\kappa-1\right)\right]^2}{1+\frac{\lambda^2}{\Omega_c^2}\left(\kappa-1\right)}+\rho\Omega_c\cot\Omega_c\right) \tag{10}$$

namic direct-stiffness analysis of composed systems such as cable-stayed bridges or guyed masts [38, 39, 40].

Using an example, attention is focused on the coefficient

$$K_{11}=\frac{F_1}{\Delta_1} \tag{8}$$

of that matrix. If α *(see Fig. 3)* is chosen to be equal to the chord inclination, K_{11} represents the horizontal dynamic stiffness at the upper end of an inclined cable that is fixed at the lower end.

To obtain dimensionless graphs, K_{11} is related to the elastic part $K_{11}^{t,e}$ of the static stiffness K_{11}^{t} of a taut wire (i.e., a straight rod)

$$K_{11}^{t}=K_{11}^{t,e}\left(1+\frac{T_\Theta}{EA}\tan^2\alpha\right)$$

$$K_{11}^{t,e}=\frac{EA}{l}\cos^2\alpha \tag{9}$$

Eq. (10) results as an analytical function, which is mainly influenced by the fundamental cable parameter

$$\lambda^2=\left(\frac{mgl}{T_\Theta}\right)^2\frac{EAl}{T_\Theta L_e}\cos^2\Theta=$$

$$=\varepsilon^2\frac{EA}{T_\Theta}\frac{l}{L_e} \tag{11}$$

which was introduced by Irvine and Caughey [14]. Other characteristic cable parameters are

$$\varepsilon=\frac{mgl}{T_\Theta}\cos\Theta=\frac{8d}{l} \tag{12}$$

$$L_e\cong l\left[1+8\left(\frac{d}{l}\right)^2\right]=l\left(1+\frac{1}{8}\varepsilon^2\right) \tag{13}$$

$$\rho=\frac{\varepsilon^2}{\lambda^2}\tan^2\alpha\cong\frac{T_\Theta}{EA}\tan^2\alpha \tag{14}$$

The remaining quantities are defined as

$$\Omega_c=\omega_c l\sqrt{\frac{m}{T_\Theta}} \tag{15}$$

$$\kappa=\kappa\left(\Omega_c\right)=\frac{\tan\left(\Omega_c/2\right)}{\Omega_c/2} \tag{16}$$

$$T_\Theta=H/\cos\Theta \tag{17}$$

Furthermore, l denotes the chord length; and d is the sag perpendicular to the chord. The quantity T_Θ represents the static cable tension at the section where the cable is parallel to the chord, and corresponds approximately to the average cable tension. Angle Θ is the chord inclination (measured from horizontal line); and g is the gravitational acceleration.

The dynamic stiffness given by equation (10) is a complex function. Its real and imaginary parts correspond to the properties of a frequency-dependent spring-dashpot system. They are depicted in *Fig. 4* for a randomly selected set of parameters. Considerable variations occur within the range of natural frequencies (i.e., of a cable fixed at both ends). They are associated with increased transversal cable vibrations. This may lead to nonlinear effects [42], and could affect the validity of the here given linear theory. Recent measurements on a tall guyed mast under wind action [26], however, confirmed the relevance of linear analysis to practical applications.

Equivalent Static Cable Stiffness

Upon carrying out the limiting transition $\Omega_c\rightarrow 0$, equation (10) reduces to

$$K_{11}^{*}\Big|_{\Omega_c\rightarrow 0}=\frac{l}{L_e}\left(\frac{1}{1+\frac{1}{12}\lambda^2}+\rho\right) \tag{18}$$

The first term in brackets represents the elastic stiffness and the geometric stiffness due to cable sag. (The quantity $\frac{1}{12}\lambda^2$ is the ratio of both stiffness contributions.) It is a function of the fundamental cable parameter λ^2 defined by equation (11) and can be

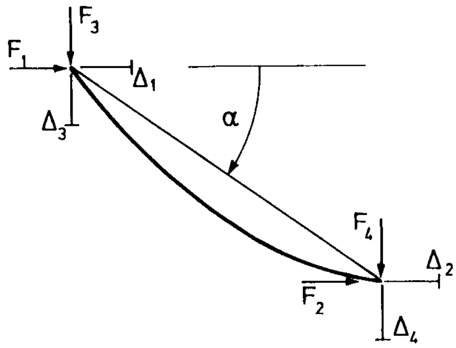
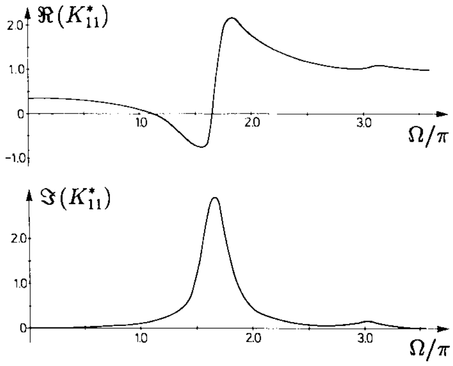


Fig. 3: Global force and displacement quantities



$\lambda^2=23.48$ $\alpha=55.86°$

$\epsilon=0.1218$ $\xi=\frac{0.14}{\Omega/\pi}$

Fig. 4: Dynamic stiffness function

transformed into the formula for the so-called equivalent modulus of elasticity given by Ernst [8]. The second term – cable parameters ρ – corresponds to the ratio of rotational to elastic stiffness of a taut wire (see equations (9) and (14)). In the static case, this parameter is usually of minor influence on the total stiffness.

In order to obtain the equivalent modulus of elasticity in a more representable form, some substitutions are made, and l/L_e is approximated by unity. The fundamental cable parameter λ^2 can now be written

$$\lambda^2=\varepsilon^2\frac{E}{\sigma}=\left(\frac{\gamma\bar{l}}{\sigma}\right)^2\frac{E}{\sigma} \tag{19}$$

where

γ = mg/A = the cable weight per unit length and effective cross-sectional area
σ = T_Θ/A = roughly the average cable stress
$\bar{l}$ = $l\cos\Theta$ = the horizontal cable span
ϵ = $\gamma\bar{l}/\sigma$.

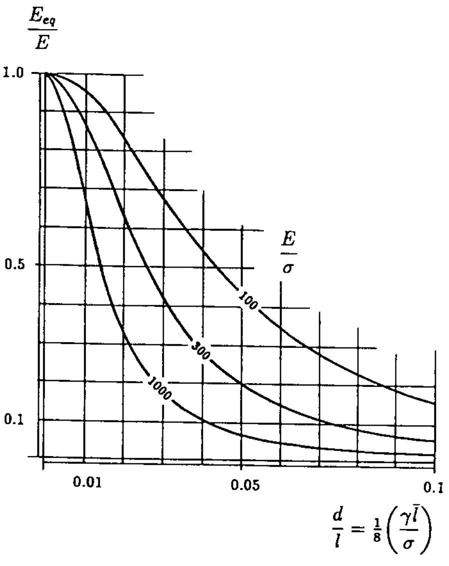


Fig. 5: Dimensionless equivalent modulus of elasticity

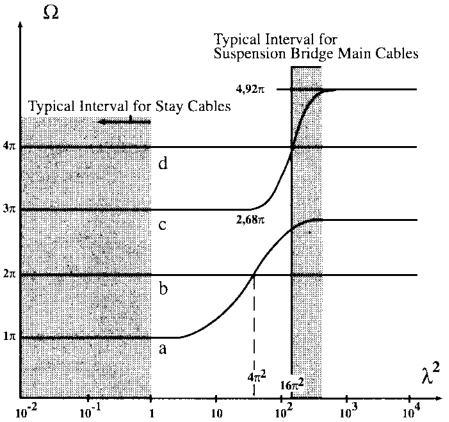


Fig. 6: The first two natural frequencies of symmetric (a, c), and of antisymmetric (b, d), in-plane vibrations [10]

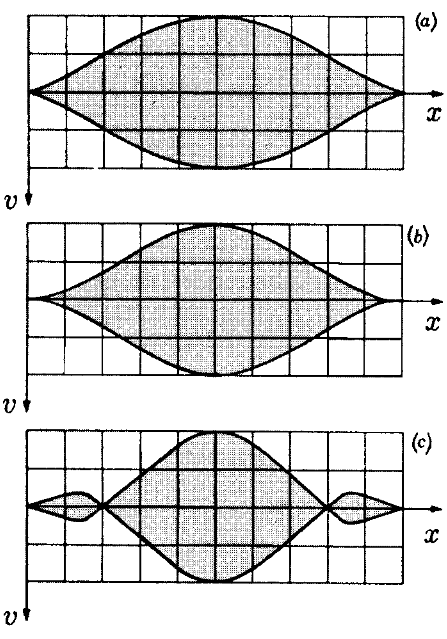


Fig. 7: Possible forms of the first symmetric in-plane mode [14]
a) $\lambda^2 < 4\pi^2$, b) $\lambda^2 = 4\pi^2$, c) $\lambda^2 > 4\pi^2$

The parameter ε corresponds approximately to the ratio of the weight component perpendicular to the chord to the average cable tension. E/σ is roughly the reciprocal of the average cable strain. By utilizing equations (18) and (19), the dimensionless equivalent modulus of elasticity (tangent modulus) can be written as follows:

$$\frac{E_{eq}}{E} = K_{11}^{*}\Big|_{\Omega_c \to 0,\ \alpha=0} = \frac{1}{1+\frac{1}{12}\lambda^2} = \frac{1}{1+\frac{1}{12}\left(\frac{\gamma \bar{l}}{\sigma}\right)^2 \frac{E}{\sigma}} \qquad (20)$$

This expression depends on only two dimensionless parameters. Hence a general graphic presentation is possible as it is shown in *Fig. 5*.

Free Vibration Response

Solutions for the free vibrations of a cable that is fixed at both ends were given by Irvine and Caughey [14] and Irvine [16]. Their results can easily be derived from equation (10). The stiffness function that is described by this equation becomes infinite for certain values of Ω_c. In the case of real Ω_c, these values coincide with the dimensionless natural frequencies of an undamped cable suspended from rigid end supports. This condition leads to the frequency equations

$$\tan\frac{\Omega}{2} \doteq 0 \qquad (21)$$

$$\tan\frac{\Omega}{2} - \frac{\Omega}{2} + \frac{4}{\lambda^2}\left(\frac{\Omega}{2}\right)^3 \doteq 0 \qquad (22)$$

where the dimensionless natural frequency Ω is defined as

$$\Omega = \omega l \sqrt{\frac{m}{T_\Theta}} \qquad (23)$$

and in which ω is the natural circular frequency. The solution of equation (21) is

$$\Omega_n = 2n\pi \ ; \ n = 1,\ 2,\ 3,\ \ldots \qquad (24)$$

and corresponds to the natural frequencies of the antisymmetric in-plane modes. The roots of the transcendental equation (22) correspond to the natural frequencies associated with the symmetric in-plane modes. Alternatively to a numeric solution of (22), the following interpolation formulae may be used.

For large λ^2 (inextensible sagging cable), the expression

$$\Omega_n \cong (2n+1)\pi - \frac{4}{(2n+1)\pi} \qquad n = 1,\ 2,\ 3,\ \ldots \qquad (25)$$

can be utilized [17]; maximum deviation from exact values is 0.15 % (for $\lambda^2 = \infty$). For sufficiently small λ^2 (taut string), it is valid that

$$\Omega_n \cong (2n-1)\pi \ ; \ n = 1,\ 2,\ 3,\ \ldots \qquad (26)$$

this formula becomes exact for $\lambda^2 = 0$. As an approximation for the frequency of the first symmetric in-plane mode and for any given λ^2, Irvine [17] derived the expression Eq. (27), as by means of a Fourier series approach; frequencies evaluated by this formula deviate at most 0.5 % from exact values. An approximation by Bauer [2] for the first symmetric in-plane frequency, adapted to the notation introduced here reads

$$\Omega_1 \cong \pi\sqrt{1+\frac{8\lambda^2}{\pi^4}} \qquad (28)$$

this expression, however, is valid for sufficiently small λ^2 only; for $\lambda^2 = 4\pi^2$, deviation from exact value is +3 %. (In [2], the influences of flexural stiffness and elastic supports on natural frequencies are additionally investigated.)

The first four in-plane natural frequencies are depicted in *Fig. 6* as functions of λ^2. The first symmetric in-plane mode is shown in *Fig. 7* for three different values of λ^2. These pictures illustrate the importance of the fundamental cable parameter and the significance of the cross-over points

$$\lambda_j^2 = (2j\pi)^2 \ ; \ j = 1,\ 2,\ 3,\ \ldots \qquad (29)$$

that mark the transition from the dynamics of a taut string ($\lambda^2 = 0$) to the dynamics of an inextensible sagging cable ($\lambda^2 = \infty$).

$$\Omega_1 \cong \left\{\frac{1}{2}\left\{\left(10\pi^2 + \frac{80\lambda^2}{9\pi^2}\right) - \left[\left(10\pi^2 + \frac{80\lambda^2}{9\pi^2}\right)^2 - 4\left(9\pi^4 + \frac{656\lambda^2}{9}\right)\right]^{\frac{1}{2}}\right\}\right\}^{\frac{1}{2}} \qquad (27)$$

It follows from *Fig. 6* that the natural frequencies of tightly stretched cables as they are employed in cable-stayed bridges correspond to the natural frequencies of taut strings. Suspension bridge main cables, however, are in the transition range. That is, their free-vibration characteristics depend on both cable sag and cable elasticity.

Mechanisms of Dynamic Excitation

On the one hand, cable vibrations may be excited by distributed fluid forces caused by turbulence, vortex shedding, self-induction (galloping), fluid-elastic interaction between neighboring cables, or even by interaction between rain, wind, and cable. Special attention should be focused on all kinds of self-excited vibrations (galloping, and interaction phenomena [13, 30, 34]). On the other hand, cable vibrations induced by support motion have been observed [48]. Specific nonlinear mechanisms (parameter effect) might be of importance [20], although, boundary induced vibrations occur within the borders of linear theory as well.

Galloping

Across-wind galloping is a dynamic-aeroelastic instability similar to flutter of airfoils or bridges. It is excited by fluid forces that are induced by the system motion itself. These forces act in phase with the system velocity and correspond to a negative aerodynamic damping; the equation of motion is homogeneous. Circular cables cannot gallop because of their cross-sectional symmetry. However, only small deviations from a perfectly circular shape (dodecagon) or icing may imply galloping instability.

Approaches to predict critical wind speeds and vibration amplitudes for galloping are described in [24, 30, 34, 36]. All known methods require cross-sectional shape coefficients to be established experimentally by means of wind tunnel tests.

Dynamic Interaction between Cable and Beam

The dynamic analysis of systems such as cable-stayed bridges or guyed masts should allow for the dynamic interaction between cables and beams. That is, the vibration characteristics of the cables should be taken into consideration [1, 22, 40].

To include cable dynamics into analysis, a linear theory of boundary induced cable vibration was developed in [38, 39, 40]. This theory can be utilized for a dynamic analysis of composed systems in which the behavior of each cable is described by a dynamic stiffness matrix. (One coefficient of that matrix is given by equation (10)). Based upon this approach, the eigenvalue problem as well as the response problem can be solved. Furthermore, frequency-response functions (admittance functions) for a composed system can be obtained. These functions are suitable for linear stochastic analysis of random vibrations induced by, for example, wind forces. Such an analysis was performed in [21, 22] for a multiple-guyed mast. The impact that dynamic interaction has on both the overall system behavior and the vibration of the individual cables became evident in these studies.

Conclusions

The beginnings of cable dynamics date back to the birth of modern mechanics and mathematics. After some significant progress made recently, the linear theory of vibrating cables can now be considered a simple but powerful tool to examine small vibrations of single cables, and composed systems containing cables, for a large range of system parameters. In the last couple of years, nonlinear cable dynamics also has made considerable progress. While nonlinear theory is inherently much more involved, and less applicable to everyday engineering analysis, the theories developed to date seem capable of reproducing the very unique phenomena that appear when large cable vibrations are considered. Further effort is needed not only to fully understand phenomena such as rain-wind-induced cable vibrations or dynamic interaction between cables and other structural elements, but also to make the gained findings more easily accessible to the design engineer.

This article is dedicated to Prof. Jörg Schlaich on the occasion of his 60th birthday.

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