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The dynamic behaviour of a non-stationary elevator compensating rope system under harmonic and stochastic excitations

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Abstract. Moving slender elastic elements such as ropes, cables and belts are pivotal components of vertical transportation systems such as traction elevators. Their lengths vary within the host building structure during the elevator operation which results in the change of the mass and stiffness characteristics of the system. The structure of modern high-rise buildings is flexible and when subjected to loads due to strong winds and earthquakes it vibrates at low frequencies. The inertial load induced by the building motion excites the flexible components of the elevator system. The compensating ropes due to their lower tension are particularly affected and undergo large dynamic deformations. The paper focuses on the presentation of the non-stationary model of a building-compensating rope system and on the analysis to predict its dynamic response. The excitation mechanism is represented by a harmonic process and the results of computer simulations to predict transient resonance response are presented. The analysis of the simulation results leads to recommendations concerning the selection of the weight of the compensation assembly to minimize the effects of an adverse dynamic response of the system. The scenario when the excitation is represented as a narrow-band stochastic process with the state vector governed by stochastic equations is then discussed and the stochastic differential equations governing the second-order statistical moments of the state vector are developed.

1. Introduction

Slender structures such as ropes and cables employed in vertical transport applications are subjected to vibrations caused by various sources of excitation. They include excitations due to the irregularities of the guiding system as well as environmental phenomena such as strong winds and earthquakes [1]. The dynamic characteristics of the system are of a time-variant nature. The natural frequencies of the ropes change with the speed of the transport motion. More importantly, their length variation results in changes of their mass and stiffness characteristics so that *slow* variations of the resonance frequencies occur [2]. An adverse situation arises when the building is excited near its fundamental natural

frequency and vibrates periodically [3]. This in turn may result in a passage through resonance in the suspension and compensating rope systems when one of the slowly varying frequencies of the ropes approaches that of the inertial load resulting from the building sway. The study of vibration problems in elevator ropes in high-rise applications due to the low frequency building sway has attracted considerable attention. For example, an analysis of elevator ropes of fixed length in tall buildings subjected to excitation due to wind were investigated in [4] and an experimental study into rope sway control strategy in high-rise elevator installations was conducted in [5]. The main effort was put into the prediction of the overall rope system response and into the development of control strategies based in deterministic models.

In this paper, the results of a study to predict the sway of compensating ropes taking into account the time-variant (non-stationary) nature of the system are presented. The deterministic model is applied to investigate the passage through resonance during the elevator travel. Then, the methodology to account for the stochastic nature of the building motion is discussed and the differential equations governing the second-order statistical moments of the state vector are developed.

2. Dynamic model

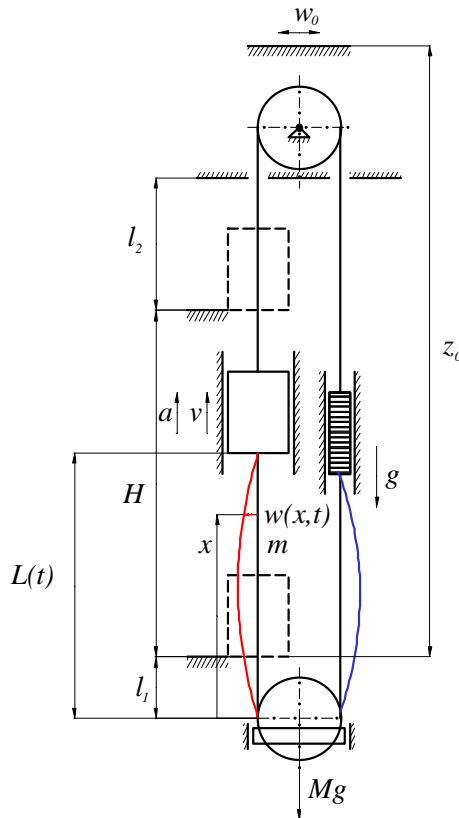


Figure 1

The dynamic response of compensating ropes is studied using the model depicted in Figure 1. This model represents an elevator car with compensating ropes of length $L(t)$ and mass per unit length m , moving at speed v with the acceleration rate a . The system is equipped with a compensator pulley assembly of mass M . The elevator travel height is denoted by H and represents the distance from the bottom (ground floor) landing to the top landing. The distance from the ground floor level to the centre of the compensator pulley is given as l_1 and the overhead height measured from the top landing to the top of the machine room floor is denoted by l_2 . The building sway is represented by motion $w_0(t)$ taking place at the height z_0 measured from the ground floor level.

The modern elevator control systems allow an accurately prescribed velocity and acceleration time profiles of the car / counterweight to be realized with their vertical transient oscillations being negligible. Bearing this in mind the tension T of the ropes is expressed as

$$T(x) = m(g + a)x + \frac{1}{2}Mg \quad (1)$$

The lateral displacement $w(x,t)$ of the ropes is governed by the following equation

$$m(w_{tt} + 2vw_{xt} + v^2w_{xx}) - T_0w_{xx} - m(g + a)xw_{xx} - mgw_x = 0, \quad 0 < x < L(t) \quad (2)$$

where $T_0 = \frac{1}{2}Mg$. Assuming that the lateral motions at the car end are not admitted the boundary conditions are given as

$$\begin{aligned} w(0,t) &= 0 \\ w(L,t) &= w_L(t) \end{aligned} \quad (3)$$

where $w_L(t)$ represents can be determined if the building sway $w_0(t)$ at z_0 above ground floor level is known (see Figure 1).

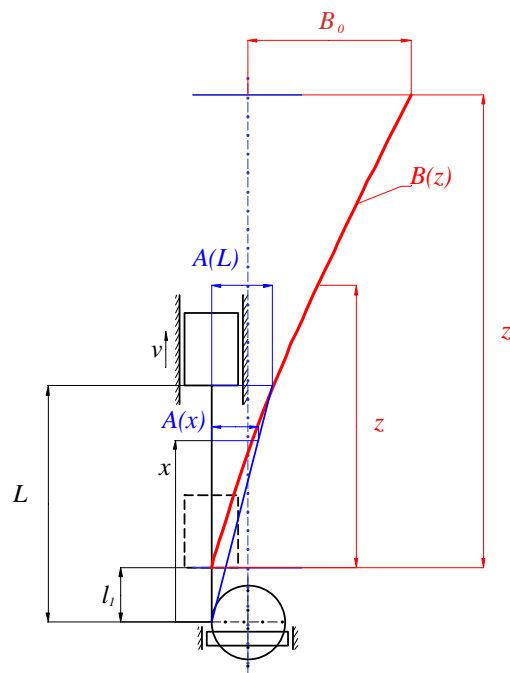


Figure 2.

3. Response to harmonic process

The motion $w_0(t)$ at the height z_0 above the ground floor level caused by the building sway can be treated as a harmonic process defined as

$$w_0(t) = B_0 \sin \Omega t \quad (4)$$

where B_0 is the maximum displacement at z_0 above the ground floor level and Ω is the fundamental natural frequency of the building, respectively.

3.1. Deformations

The building deformation shape $B(z)$ as shown in Figure 2, where z is the coordinate measured from ground floor level, is approximated by a cubic function as

$$B(z) = \frac{B_0}{z_0^3} z^3 \quad (5)$$

Thus, the displacement of the car (taking place together with the building) positioned in the well at $z = L - l_1$ above the ground floor is

$$A(L) = \frac{B_0}{z_0^3} (L - l_1)^3 = B_0 \left(\frac{L - l_1}{z_0} \right)^3 \quad (6)$$

It is evident that the corresponding shape (displacements) of the compensation rope in terms of the coordinate x (measured from the centre of the compensator pulley assembly, see Figure 2) is then given by a linear function of x as

$$A(x) = \frac{x}{L} A(L) = B_0 \left(\frac{L - l_1}{z_0} \right)^3 \frac{x}{L} \quad (7)$$

3.2. Discrete equations of motion

The boundary condition (3) are then written as

$$\begin{aligned} w(0, t) &= 0 \\ w(L, t) &= A(L) \sin \Omega t \end{aligned} \quad (8)$$

where $A(L)$ is given by equation (6). In order to investigate the response due to the building sway the overall lateral displacement of the ropes is expressed as

$$w(x, t) = \bar{w}(x, t) + A(x) \sin \Omega t, \quad 0 \leq x \leq L(t) \quad (9)$$

where $\bar{w}(x, t)$ represents the dynamic deflection relative to the building frame and $A(x)$ is given by equation (7). Thus, the overall displacement is represented as

$$w(x, t) = \bar{w}(x, t) + \left(\frac{L - l_1}{z_0} \right)^3 \frac{x}{L} w_0(t) \quad (10)$$

The transformation (10) is substituted into equation (2) and the dynamic deflections are expanded as

$$\bar{w} = \sum_{n=1}^N \Phi_n(x, L(\tau)) q_n(t) \quad (11)$$

where $q_n(t)$ represent the modal coordinates, $\Phi_n(x) = \sin \frac{n\pi}{L(\tau)} x$, $n = 1, 2, \dots, N$, are the slowly-varying

natural vibration modes of the corresponding taut string. In this formulation the length of the ropes is considered to vary slowly which is observed on the *slow time* scale defined as $\tau = \varepsilon t$, where ε represents a small parameter [2]. After orthogonalising with respect to the natural modes, the following system of ordinary differential equations describing the dynamic behaviour of the compensating ropes is obtained

$$\ddot{q}_r(t) + \sum_{m=1}^N C_m(\tau) \dot{q}_m(t) + \sum_{m=1}^N K_m(\tau) q_m(t) = Q_r(t; \tau), \quad r = 1, 2, \dots, N \quad (12)$$

where the coefficients C_m and K_m are slowly varying. These coefficients and the excitation function Q_r are given as

$$C_m(\tau) = \begin{cases} 2\zeta_r \omega_r(\tau), & n = r \\ \frac{4nr\nu}{(n^2 - r^2)L(\tau)} \left[(-1)^{r+n} - 1 \right], & n \neq r \end{cases}, K_m(\tau) = \begin{cases} \omega_r^2(\tau) - \frac{r^2\pi^2}{L} \left[\frac{v^2}{L} - \frac{g+a}{2} \right], & n = r \\ \frac{2}{(n^2 - r^2)L(\tau)} \left[\frac{2rn^3(g+a)}{n^2 - r^2} - gm \right] \left[(-1)^{r+n} - 1 \right], & n \neq r \end{cases}$$

$$Q_r(t; \tau) = \frac{2}{r\pi L(\tau)} \left(\frac{L(\tau) - l_1}{z_0} \right)^3 \left\{ (-1)^r L \ddot{w}_0(t) - [g w_0(t) - 2v \dot{w}_0(t)] \left[(-1)^r - 1 \right] \right\}$$

(13)

where $r = 1, 2, \dots, N$, $\omega_r(\tau) = \frac{r\pi}{L(\tau)} \sqrt{\frac{T_0}{m}}$ and ζ_r represents the modal damping ratios of the ropes.

3.3. Results

The dynamic response of the compensating system is determined for an installation roped 1:1 of rated speed 8 m/s, travel height $H = 390$ m equipped with 8 compensating ropes of mass per unit length $m = 2.11$ kg/m each. The building is subjected to a sway of frequency 0.1 Hz and amplitude $B_0 = 0.75$ m measured at $z_0 = 387.5$ m above the ground floor. The coupled differential equations (12) are solved numerically using $N = 5$ modes in the expansion (11) for masses $M = 500, 15,000$ kg with damping ratios assumed as $\zeta_r = 3\%$ across all modes. The numerical tests are conducted for the lift car ascending with the acceleration/ deceleration rate of $a = 1.2$ m/s² from the lowest landing upwards to the highest level. The MATLAB implementation of an explicit Runge-Kutta (4,5) formula is used to integrate the equations of motion on the fast time scale with the slowly varying coefficients C_m and K_m pre-calculated and evaluated at each integration step. The dynamic deformations relative to the building frame are then determined from the solution using the expansion (11). The fundamental resonance frequencies are determined from the eigenvalues of matrix of the coefficients K_m and accommodate the influence of speed and acceleration of the ropes. They are plotted vs. the length of the ropes in Figures 3(a) and Figure 4(a), respectively, where the frequency of the excitation corresponding to the building sway frequency is also shown. The dynamic deformations of the ropes are then presented in Figures 3(b) and Figure 4(b), respectively. These curves illustrate the deformations and maximum amplitudes of the ropes at various stages of the elevator travel.

It is evident from the plots shown in Figure 3(a) that for the case $M = 15,000$ kg the frequency of the load coincides with the first (fundamental) frequency of the ropes within the length range of about 382 - 383 m, when the car approached the top landing. This leads to large motions of the ropes with the amplitudes exceeding 1.5 m as shown in Figure 3(b). The frequency plots shown in Figure 4(a) for the case $M = 500$ kg demonstrate that when a smaller mass of the compensator assembly is used the natural frequencies are lower and in addition to the fundamental resonance (taking place when the rope length is about 105 m) the possibility of higher (second) resonance exists. The second resonance is predicted when the rope length is about 365 m. This leads to large displacements of the ropes demonstrating a strong influence of the second mode shape. As shown in Figure 4(b), in this case the amplitudes reached 1.4 m when the car approached the top landing.

4. Stochastic motion of the building

The motion $w_0(t)$ resulting from the building sway is seldom exactly harmonic. The excitation due to environmental phenomena such as wind action is usually a wide band stochastic process. The response in the fundamental mode of the building is then a narrow-band process with the center frequency equal

to the fundamental natural frequency Ω . The stochastic motion of the building can be obtained from the analysis of the building response.

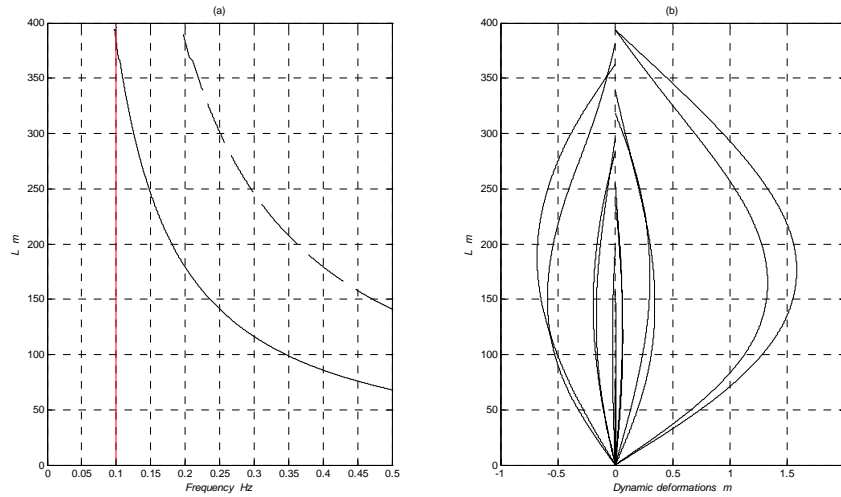


Figure 3. $M = 15,000 \text{ kg}$ (a) The variation of the first (solid curve) and second (dashed curve) natural frequencies together with the frequency of the sway (vertical solid line); (b) Deformation shapes of the ropes.

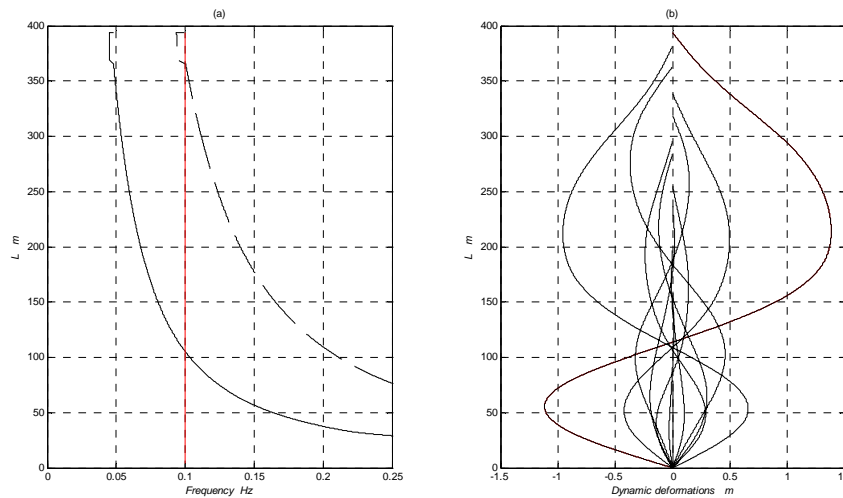


Figure 4. $M = 500 \text{ kg}$ (a) The variation of the first (solid curve) and second (dashed curve) natural frequencies together with the frequency of the sway (vertical solid line); (b) Deformation shapes of the ropes.

4.1. Narrow-band process

It may be assumed that $w_0(t)$ is a narrow-band process mean-square equivalent to the harmonic process with the amplitude A_0 and the frequency Ω as given by equation (4). The function $w_0(t)$ together with its first and second derivatives $\dot{w}_0(t)$ and $\ddot{w}_0(t)$ must be continuous [6]. This scenario is adequately idealized by assuming that the motion $w_0(t)$ is the response of the second order auxiliary filter to the process $X(t)$, which in turn is the response of the first-order filter to the Gaussian white noise excitation $\xi(t)$ [6] as illustrated in Figure 5. The governing equations are

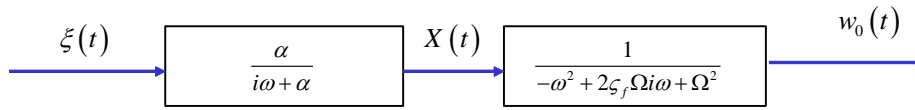


Figure 5. Narrow-band process scenario.

$$\begin{aligned} \ddot{w}_0(t) + 2\zeta_f\Omega\dot{w}_0(t) + \Omega^2w_0(t) &= X(t) \\ \dot{X}(t) + \alpha X(t) &= \alpha\sqrt{2\pi S_0}\xi(t) \end{aligned} \tag{14}$$

where ζ_f denotes the damping ratio of the filter which defines its band width, α is the filter variable, S_0 is the constant level of the power spectrum of white noise $\xi(t)$. Precisely speaking, the response $X(t)$ to the Gaussian white noise excitation $\xi(t)$ is a process which is not differentiable in the usual sense, hence the notation $\dot{X}(t)$ is not mathematically meaningful. The second equation in (14) has a clear mathematical sense when it is written in terms of the differential. Accordingly, the governing equations for both filters are written down in a state space form as

$$d \begin{bmatrix} X \\ w_0 \\ \dot{w}_0 \end{bmatrix} = \begin{bmatrix} -\alpha & 0 & 0 \\ 0 & 0 & 1 \\ 1 & -\Omega^2 & -2\zeta_f\Omega \end{bmatrix} \begin{bmatrix} X \\ w_0 \\ \dot{w}_0 \end{bmatrix} dt + \begin{bmatrix} \alpha\sqrt{2\pi S_0} \\ 0 \\ 0 \end{bmatrix} dW(t) \tag{15}$$

where $dW(t)$ is the increment of the Wiener process $W(t)$ and the Gaussian white noise excitation $\xi(t)$ is the generalized (not in usual sense) derivative of the Wiener process. It is important to note that the choice of parameters in equation (15) should guarantee that the filter variable α is real. The stochastic process $w_0(t)$ is mean-square equivalent to the harmonic process (4) if its standard deviation and variance are given as

$$\sigma_{w_0} = \frac{\sqrt{2}}{2} A_0, \quad \text{var}(w_0) = \sigma_{w_0}^2 = \frac{A_0^2}{2} \tag{16}$$

respectively. It can be shown that this equivalence is achieved if the filter variable α is given as [6]

$$\alpha = \Omega \left(-\zeta_f + \sqrt{\zeta_f^2 + \frac{\zeta_f\Omega^3 A_0^2}{\pi S_0 - \zeta_f\Omega^3 A_0^2}} \right) \tag{17}$$

4.2. Stochastic equations

It is evident from the expressions (13) that the excitation function $Q_r(t; \tau)$ can be expressed in terms of the motion $w_0(t)$ and its time derivatives as $Q_r(t; \tau) = \beta_r^{(1)}(\tau)w_0(t) + \beta_r^{(2)}(\tau)\dot{w}_0(t) + \beta_r^{(3)}(\tau)\ddot{w}_0(t)$. Using equation (14) the second derivative of $w_0(t)$ can be expressed as $\ddot{w}_0(t) = X(t) - \Omega^2w_0(t) - 2\zeta_f\Omega\dot{w}_0(t)$ so that the excitation function $Q_r(t)$ can be rewritten as $Q_r(t; \tau) = \gamma_r^{(1)}(\tau)w_0(t) + \gamma_r^{(2)}(\tau)\dot{w}_0(t) + \beta_r^{(3)}(\tau)X(t)$, where $\gamma_r^{(1)} = \beta_r^{(1)} - \Omega^2\beta_r^{(3)}$ and $\gamma_r^{(2)} = \beta_r^{(2)} - 2\zeta_f\Omega\beta_r^{(3)}$.

The state vector defined as $\mathbf{Y}(t) = [\mathbf{q}(t) \quad \dot{\mathbf{q}}(t) \quad X(t) \quad w_0(t) \quad \dot{w}_0(t)]^T$ is then governed by the following set of stochastic equations

$$d\mathbf{Y}(t) = \mathbf{A}\mathbf{Y}(t)dt + \mathbf{b}dW(t) \quad (18)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -\mathbf{K} & -\mathbf{C} & \mathbf{B} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_f \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{b}_f \end{bmatrix} \quad (19)$$

with \mathbf{I} denoting the identity matrix and \mathbf{K} and \mathbf{C} representing the matrices of coefficients $K_m(\tau)$ and $C_m(\tau)$ appearing in equations (12), respectively. Furthermore, \mathbf{B} is a matrix with rows comprising coefficients $\beta_r^{(3)}, \gamma_r^{(1)}, \gamma_r^{(2)}$ and the matrix \mathbf{A}_f and vector \mathbf{b}_f in (19) are defined as follows

$$\mathbf{B} = \begin{bmatrix} \beta_1^{(3)} & \gamma_1^{(1)} & \gamma_1^{(2)} \\ \beta_2^{(3)} & \gamma_2^{(1)} & \gamma_2^{(2)} \\ \dots & \dots & \dots \\ \beta_N^{(3)} & \gamma_N^{(1)} & \gamma_N^{(2)} \end{bmatrix}, \quad \mathbf{A}_f = \begin{bmatrix} -\alpha & 0 & 0 \\ 0 & 0 & 1 \\ 1 & -\Omega^2 & -2\zeta_f\Omega \end{bmatrix}, \quad \mathbf{b}_f = \begin{bmatrix} \alpha\sqrt{2\pi S_0} \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

The differential equations governing the second-order statistical moments of the state vector, i.e. the covariance matrix $\mathbf{R}_{\mathbf{Y}\mathbf{Y}} = E[\mathbf{Y}\mathbf{Y}^T]$, are then obtained from equation (18) as

$$\frac{d}{dt}\mathbf{R}_{\mathbf{Y}\mathbf{Y}} = \mathbf{A}\mathbf{R}_{\mathbf{Y}\mathbf{Y}} + \mathbf{R}_{\mathbf{Y}\mathbf{Y}}\mathbf{A}^T + \mathbf{b}\mathbf{b}^T \quad (21)$$

Furthermore, higher-order joint statistical moments of the state vector $\mathbf{Y}(t)$ can be determined with the aid of the Itô's differential rule.

5. Conclusions

The analysis of the response to excitation represented as harmonic process showed that when the fundamental mode of the building is tuned to the fundamental mode of the ropes large dynamic deformations of the ropes occur. Large weights of the compensating pulley assembly would be needed to shift the fundamental resonance frequency of the ropes above the frequency of the building excitation far enough to steer away from the fundamental resonance. If a small mass of the compensator assembly is used, a passage through the fundamental resonance will occur at an earlier stage of the elevator up travel and the resulting dynamic displacements are smaller. But, in this case the second mode of the ropes is activated when the car approaches the top landing. The designer of high-rise elevator systems needs both the mean value and the variance to establish bounds for the compensating rope response. Thus, the excitation mechanism can be represented as a narrow-band process mean-square equivalent to the harmonic process and used in the dynamic model to predict the probabilistic characteristics of the rope response.

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